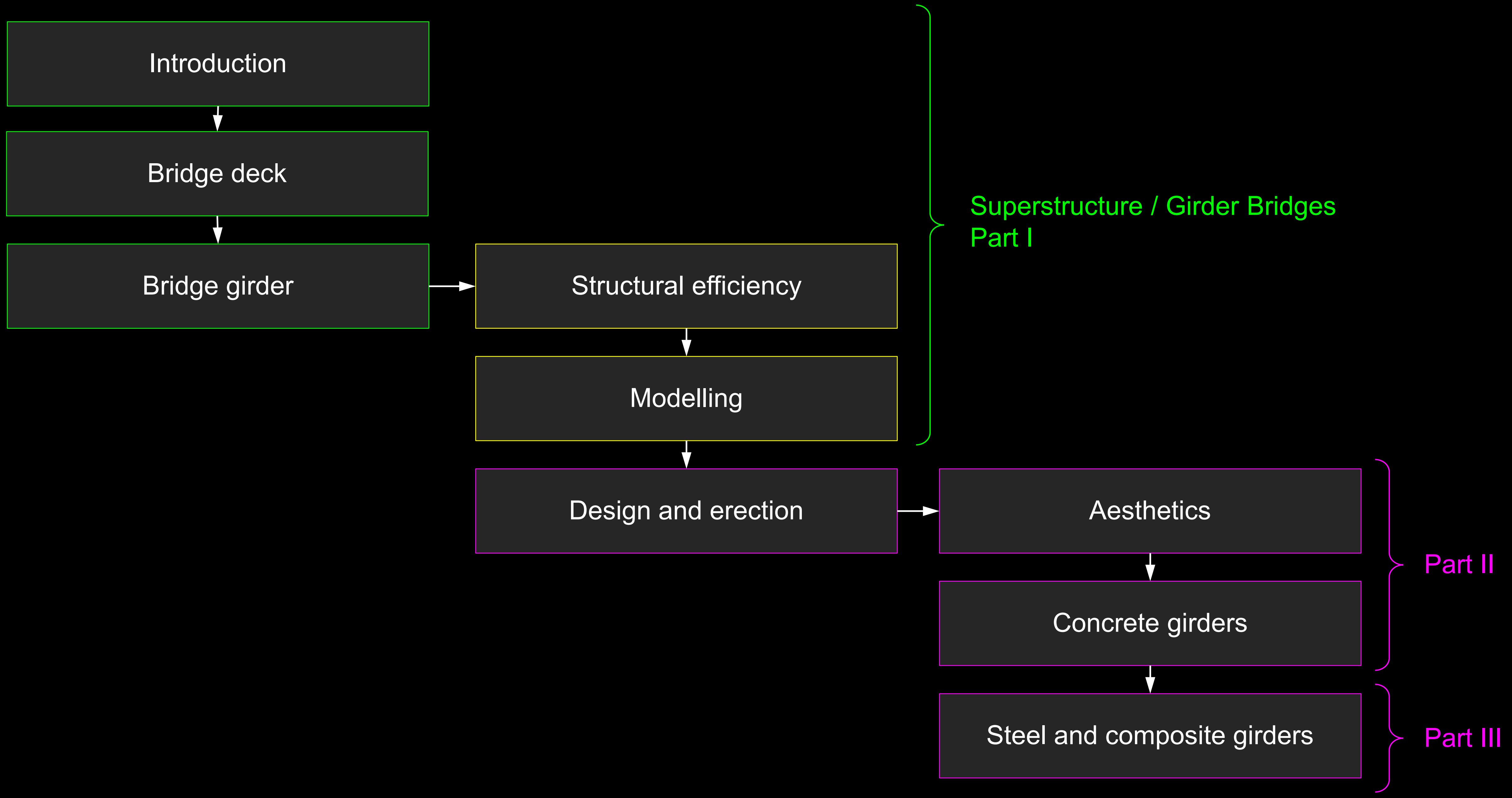
Superstructure / Girder Bridges
Überbau / Balkenbrücken


Introduction
Bridge deck
Bridge girder
Structural efficiency
Modelling
Superstructure / Girder Bridges
Part I
Design and erection
Aesthetics
Concrete girders
Part II
Steel and composite girders
Part III

Superstructure / Girder bridges

Introduction

Introduction: Terminology and content

A girder bridge consists of one or several girders, that carry loads primarily by vertical shear and longitudinal bending.

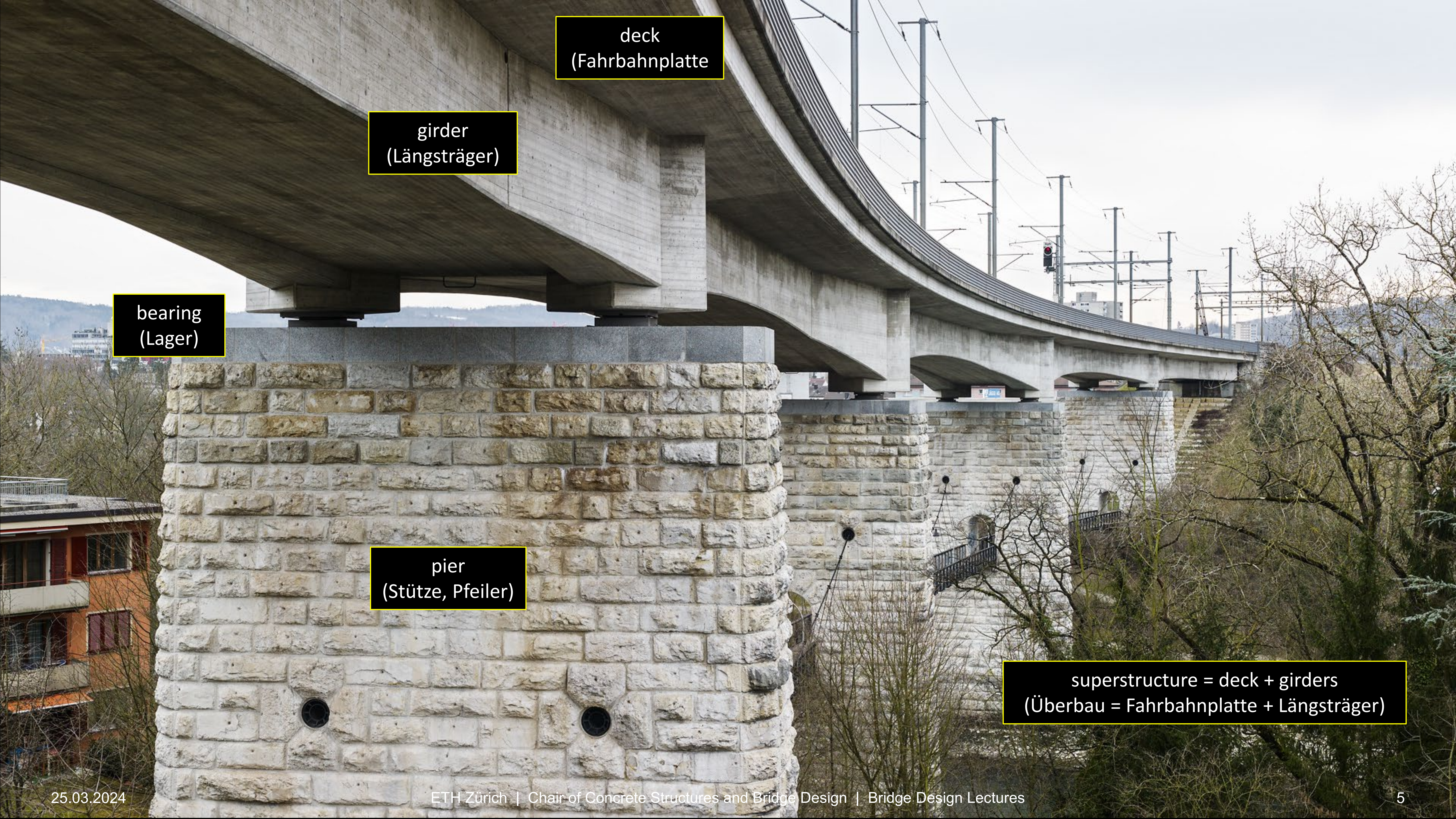
The girders are supported at the bridge ends (abutments) and often also on intermediate supports (piers).

In a girder bridge, the bridge girder including the bridge deck is equivalent to the superstructure.

In other bridge types (arches, cable-stayed bridges, …), additional elements constitute the superstructure together with the girder, that carries the loads to these elements similar as the girder in a girder bridge.

After a brief introduction to girder bridges, this chapter therefore treats bridge girders.

Kochertalviadukt Geislingen, 1979. Fritz Leonhardt


deck
(Fahrbahnplatte
girder
(Längsträger)
bearing
(Lager)
pier
(Stütze, Pfeiler)
superstructure = deck + girders
(Überbau = Fahrbahnplatte + Längsträger)

Introduction: Aesthetic quality of girder bridges

Girder bridges are often seen as inelegant. Indeed, there are many dull girder bridges.

However, if carefully proportioned and detailed, they often provide good solutions in situations where a calm and unpretentious, unobtrusive bridge is appropriate.


Buñol viaduct, Spain


Steinbachviadukt Sihlsee, Switzerland 2014. dsp Ingenieure + Planer

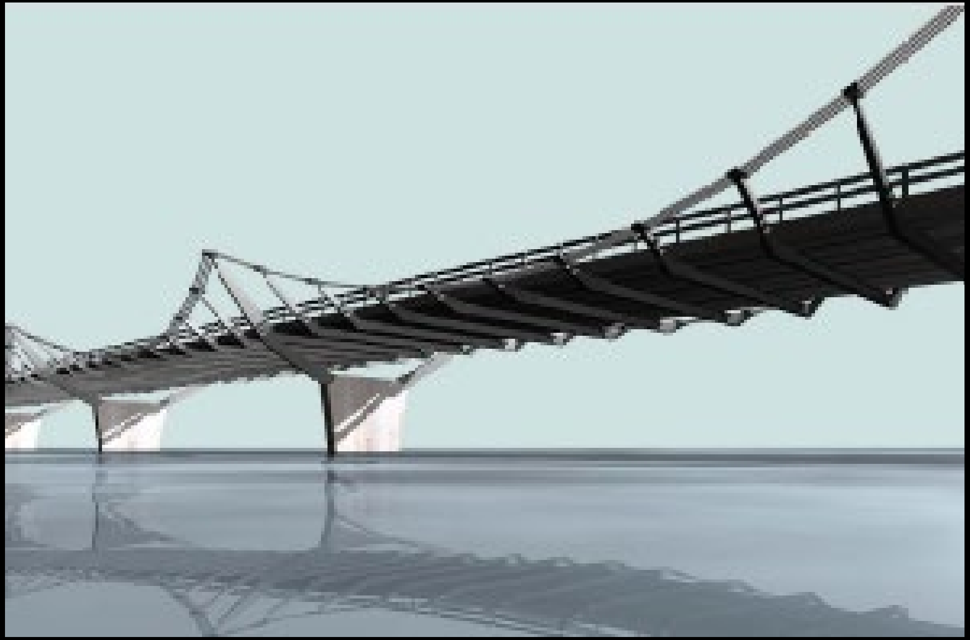

Isthmus Viaduct, Spain, 2009. Carlos Fernandez Casado, S.L.

Introduction: Aesthetic quality of girder bridges

Girder bridges are often seen as inelegant. Indeed, there are many dull girder bridges.

However, if carefully proportioned and detailed, they often provide good solutions in situations where a calm and unpretentious, unobtrusive bridge is appropriate.

Steinbachviadukt Sihlsee, Switzerland 2014. dsp Ingenieure + Planer



Introduction: Advantages and drawbacks of girder bridges

Advantages and drawbacks of girder bridges

- ✓ Economically competitive for short and medium spans
  (deck significantly contributes to longitudinal load transfer)
- ✓ Repetitive, simple and efficient construction process
  (multiple use of formwork etc.)
- ✓ Standard construction equipment and know-how sufficient
- ✓ Well suited for prefabrication and fast erection
  (using special equipment)
- ✓ Low level of complexity in the design phase
- ✓ Calm and unobtrusive appearance

- ➢ Inefficient longitudinal structural system (bending)
  … limited span range, particularly for constant depth
  … high use of materials
- ➢ Massive and dull appearance
- ➢ Bridge not perceived by users crossing it
  (if girders are positioned underneath the deck as usual)

Introduction: Design parameters

Geometry
- Bridge length
- Deck width
- Alignment in plan (straight, curved, skew, polygonal)
- Transverse gradients
- Vertical alignment

Spans
- Number and position of piers (intermediate supports)

Supports
- Vertical supports and continuity (simply supported / continuous)
- Horizontal supports (bearing layout & dilatation concept)
  … articulated
  … integral or semi-integral
  … position movement centre
- Torsional support system

Materials
- Concrete
- Steel
- Steel-concrete composite
- Timber
- …

Construction method
- Conventional scaffold
- Balanced cantilevering
- Advanced shoring
- Incremental launching
- Precast span-by-span
- Lifting
- …

Cross-section
- Single or multi-girder
- Closed cross-section (single-cell or multicellular box girder)
- Open cross-section (T, double-T, multi-girder, trough)
- Slab or voided slab (with or without cantilevers)
- …
- Depth / slenderness
- Constant or variable depth
- Constant or variable width

Design criteria

Iterate until satisfactory result is found

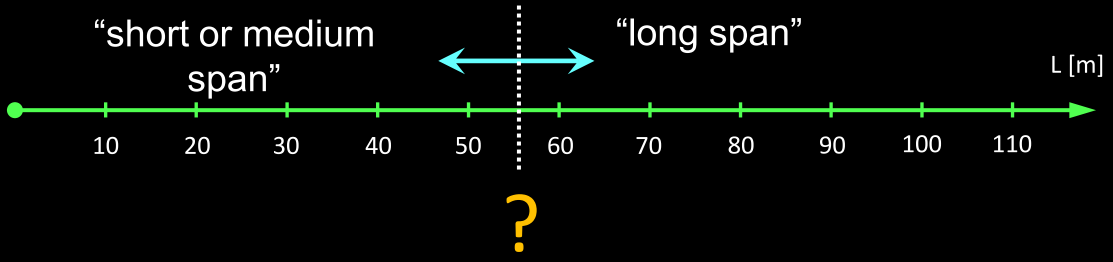
Introduction: Span ranges

The span (axis distance between supports) is an important parameter in the design of girder bridges and bridge girders, as it is decisive for the choice of

- suitable / economical construction processes
- the girder layout (materials, cross-section, …)

Typical spans of girder bridges are in the range of 25…100 m, depending on the structural system and the materialisation (more information see *structural efficiency / optimum span*). Bridge girders in other typologies often have shorter spans.

In literature, reference is frequently made to “short and medium span” or “long span” bridges. However, there is no clear limit between short, medium or long spans. Often, bridges with a span up to 50…60 m are referred to as «medium span bridges».



Ulla viaduct, Spain, 2015. IDEAM

Introduction: Bridge use / traffic loads

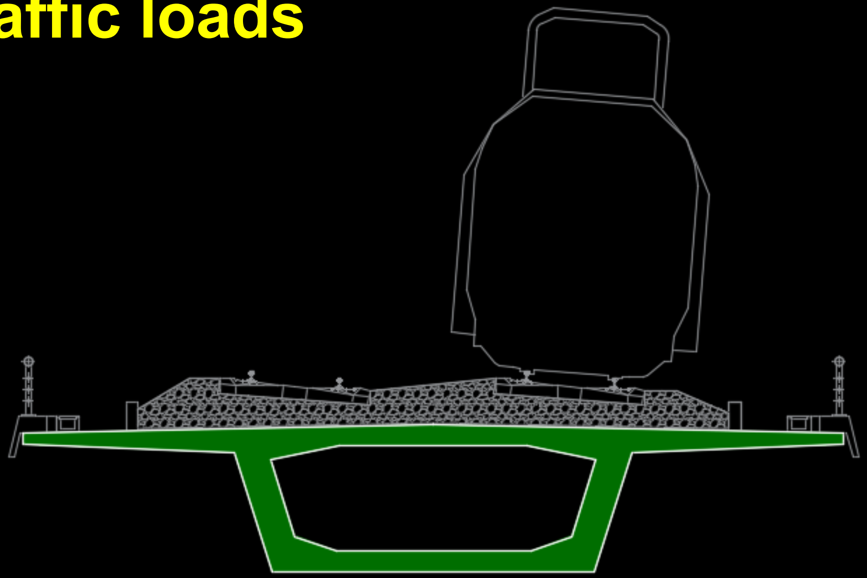
As discussed in the chapter on conceptual design, there are substantial differences between

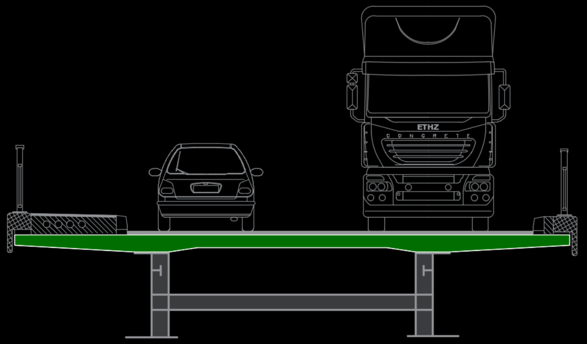
- Road bridges
- Railway bridges
- Footbridges

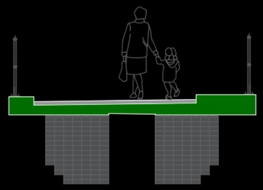
in terms of

- Traffic loads (see functions of bridge deck).
- Exposure (e.g. chlorides)
- Functionality and serviceability criteria

These differences, summarised on the next slide, are decisive for the conception of a bridge and the bridge girder and explain why there is much more variety in the design of footbridges.





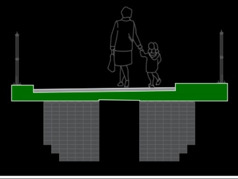


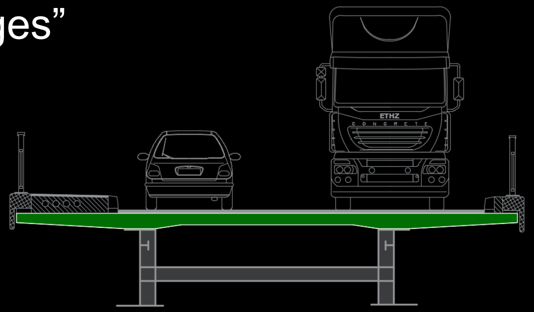
Introduction: Bridge use / traffic loads

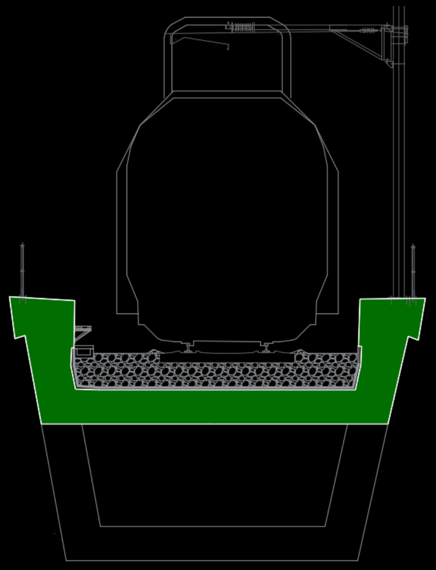
The loads depend heavily on the use of the bridge

→ design of "footbridges" differs significantly from "bridges"

→ focus of lecture: road and railway bridges







Bridge use	**Pedestrian / Bicycle**	**Road** ($\alpha_Q=\alpha_q=0.9$)	**Railway** ($\alpha=1.33$, $\Phi_{dyn}=1.67$ for typ. deck)
Concentrated loads "*Q*"	low (service vehicles only) [CH: 10 kN]	high / var. position of vehicle axis [CH LM1: $4 \cdot \alpha_Q \cdot (150+100)$ kN = 900 kN]	very high / distributed by ballast [CH LM1: $4 \cdot \alpha \cdot \Phi_{dyn} \cdot 250$ kN = 2220 kN, per track]
Distributed loads "*q*"	moderate [CH: 4 kPa, full width]	moderate-high (on limited width) [CH LM1: $\alpha_q \cdot 9$ kPa = 8.1 kPa, 3 m width]	high [CH LM1: $\alpha \cdot \Phi_{dyn} \cdot 80$ = 178 kN/m, per 3.80 m]
Longitudinal horizontal loads	low	moderate (braking / traction)	high (braking / traction)
Transverse horizontal loads	low	low-moderate (centrifugal)	moderate-high (centrifugal / nosing)
Fatigue	usually irrelevant	moderate (local elements)	highly relevant
Dynamic effects	slender bridges often sensitive to vibrations	included in traffic loads (most codes)	dynamic factor depending on structural element / dynamic analysis for high speed rail
Deflections (vertical)	moderate $w \leq l / 600$ (LM1)	moderate $w \leq l / 500$ (LM1)	highly relevant $w \leq l / 2000$, v = 160 km/h (LM1-2)
Durability issues	moderate (de-icing)	high (de-icing, heavy load on joints)	low (no de-icing, joints not directly loaded)

Introduction: Materialisation

The materialisation of the bridge girder is an important choice in the design, depending primarily on the use and the span of the girder.

Usual materialisations for road / railway bridges:

- prestressed concrete girders
  $\rightarrow$ frequently used for economic reasons
- steel-concrete composite girders
  $\rightarrow$ fast erection, but usually more expensive
- steel girders (orthotropic deck on steel girders)
  $\rightarrow$ higher cost, only used if weight is critical

Timber is rarely used due to limited durability (or environmental issues if CCA-impregnated, see timber decks)

Usual materialisations for footbridges:

- steel and timber used more frequently
- new materials are gaining importance (fibre-reinforced polymers, ultra-high performance fibre-reinforced concrete)


Sir Leo Hielscher bridges, Australia, 2010. Maunsell Group and SMEC


HS Riudellots de la Selva Viaduct, Spain, 2009. Fhecor Ingenieros


Archidona viaduct, Spain, 2012. IDEAM


Neckartenzlingen, Germany, 2017. Ing. Miebach

Introduction: Static system

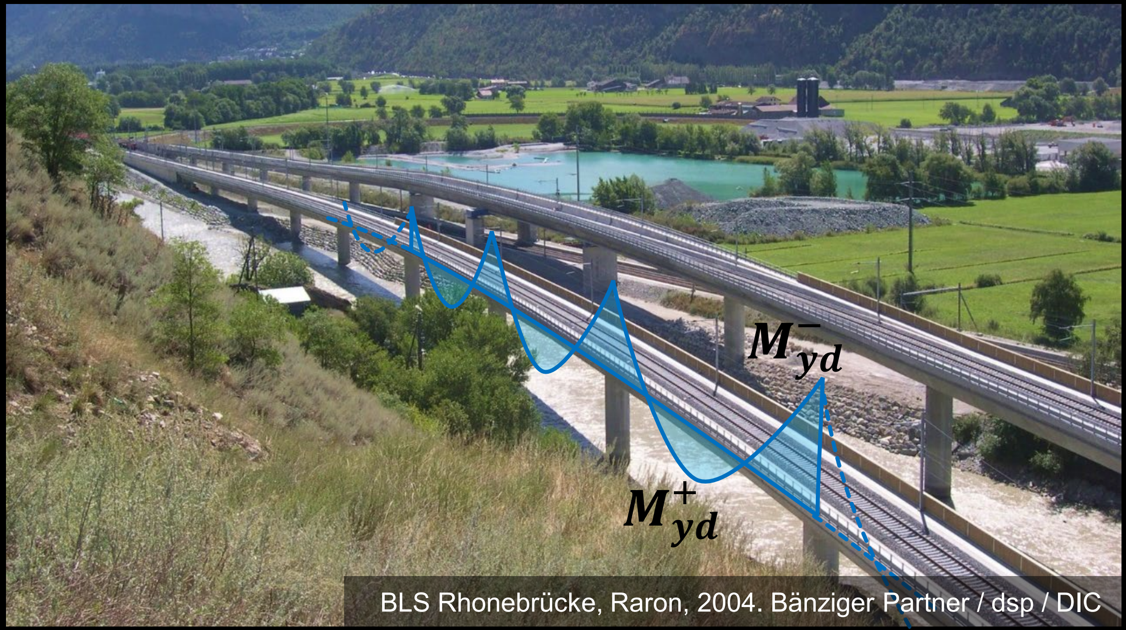
Another important choice is the longitudinal static system of the bridge girder.

Bridge girders can be simply supported or continuous over two or more spans.

In multispan bridges, continuous girders are much more efficient and durable, but their erection (if prefabricated) is more complicated.

More details see *strategies for efficient bridge girders* and *bearing layout and dilatation concept*.

Melchaabrücke, Sarnen, 2008. dsp



BLS Rhonebrücke, Raron, 2004. Bänziger Partner / dsp / DIC

Introduction: Cross-section

The typology of the cross-section is another relevant decision in conceptual design.

Common solutions are

(a) Box-girders (single-cell closed cross-sections, concrete, steel or composite)

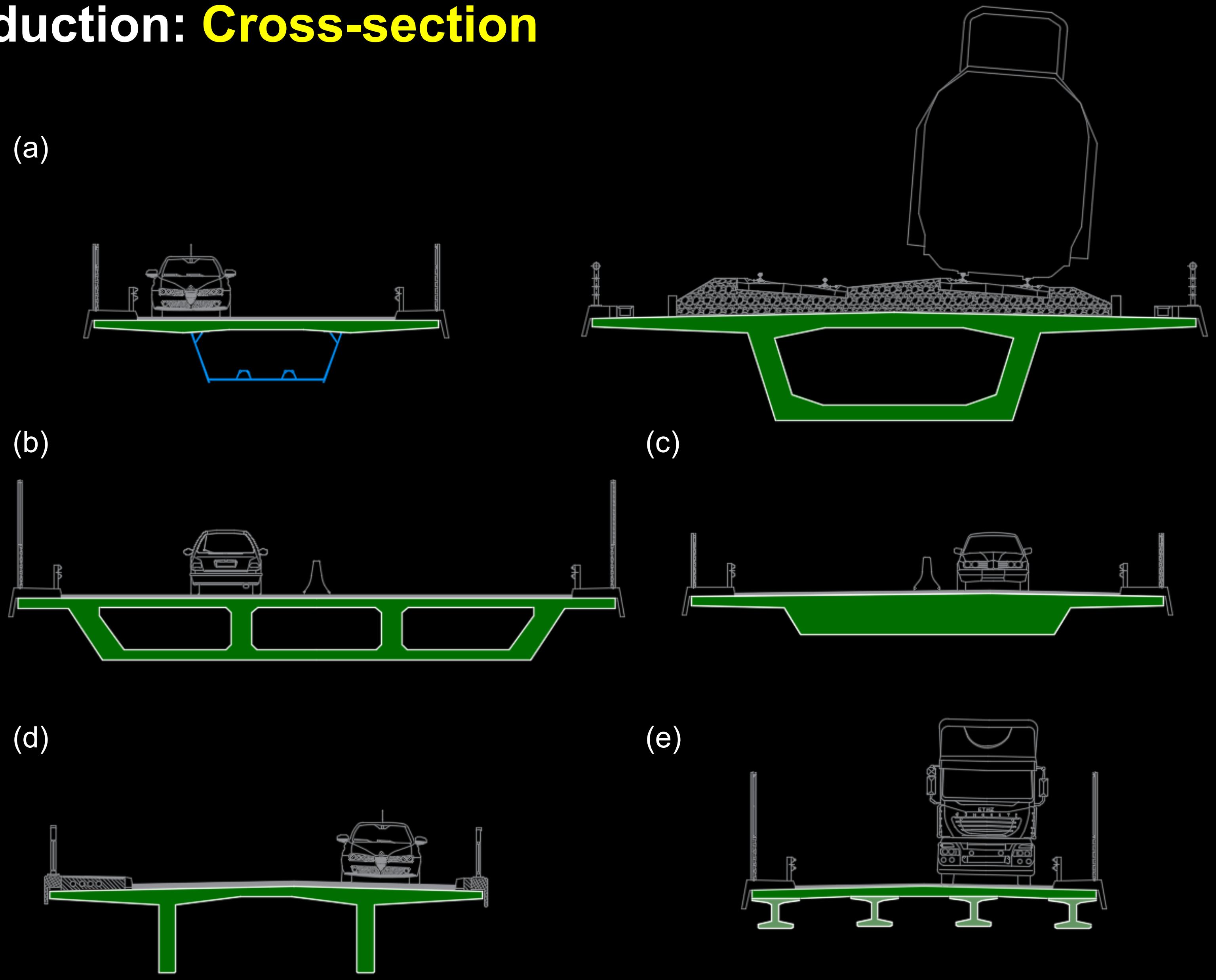
(b) Multicell box girders (multicellular closed cross-sections)

(c) Slabs (solid cross-sections, often tapered or provided with overhangs to save weight)

(d) Double-T girders (open cross-sections with two girders)

(e) Multi-girder deck (open cross sections with several girders, typically steel or prefabricated I-beams)

See More details see *strategies for efficient bridge girders* for selection criteria.



Introduction: Erection method

Many different erection methods are used depending on span, accessibility and height above ground, number of spans (repetitiveness), materialisation etc. In major girder bridges, the erection method is a decisive aspect of the conceptual design.

Concrete girders are often cast in place using:

- conventional scaffold / falsework
- (balanced) cantilevering
- movable scaffold system (also referred to as advanced shoring)

Girders can also be precast in segments erected span by span or by (balanced) cantilevering. This is more frequent in concrete girders, but also possible in steel or composite bridges, see photo.

Alternatively, entire bridge girders can be launched or lifted in. The latter is usual for steel or timber girders; concrete girders are often too heavy to be transported as a whole, but can be cast behind an abutment and incrementally launched.

In composite bridges, the steel girders are often lifted in, and the concrete deck is cast on the steel girder(s), without additional scaffold.

Details on erection methods see material-specific sections.

Balanced cantilevering

Ulla viaduct, Spain, 2015. IDEAM

Movable scaffold system (MSS)

Isthmus viaduct, Spain, 2009. CFCSL

Superstructure / Girder bridges

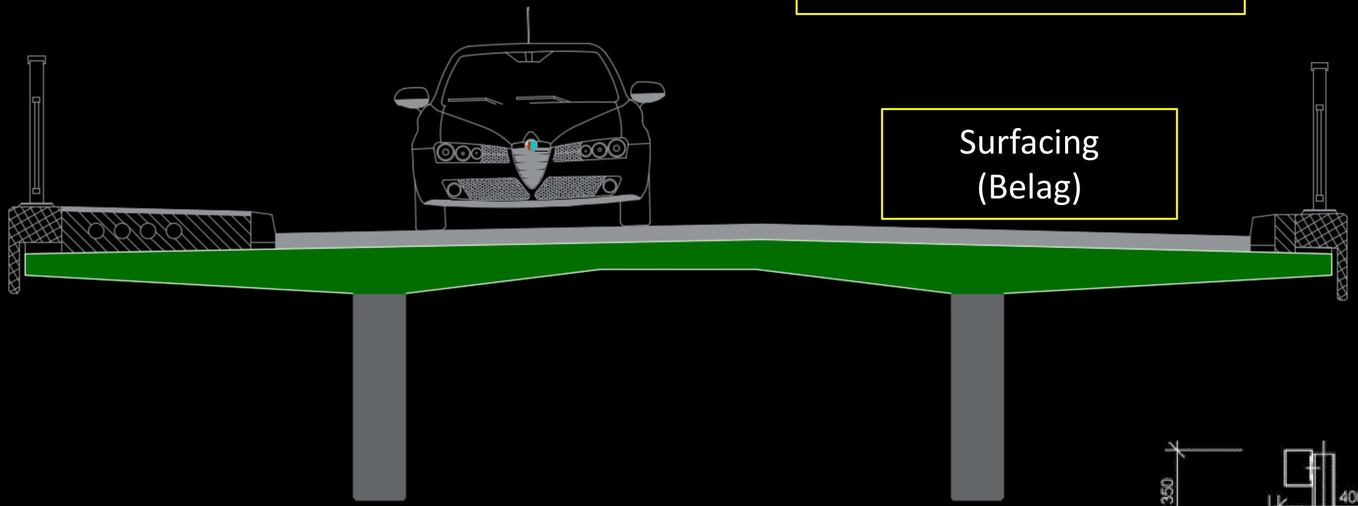
Bridge deck

Bridge deck: Functions

- Carry the traffic loads (and deck self-weight)
- Transfer these loads to the longitudinal girder(s)
- Contribute to the longitudinal stiffness of the girder (acting as flange)
  $\rightarrow$ consider effective widths (if transverse span is long compared to girder span)
- Integrate all elements required to comply with the functionality of the road, railway or pedestrian way it carries:

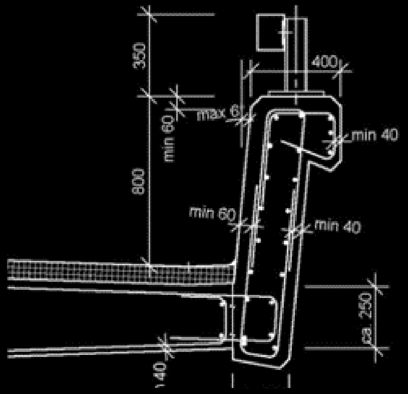
  … surfacing (or ballast on railway bridge)
  … drainage
  … noise protection
  … guardrails and handrails
  … etc.

guardrail/ handrails
(Leitschranke / Geländer)

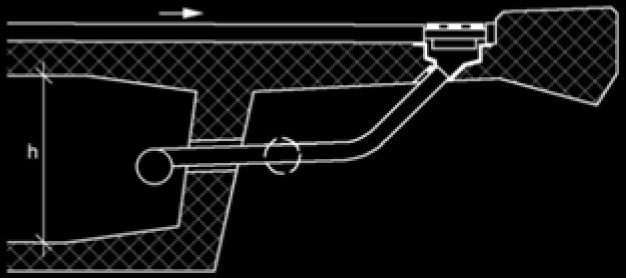


Surfacing
(Belag)

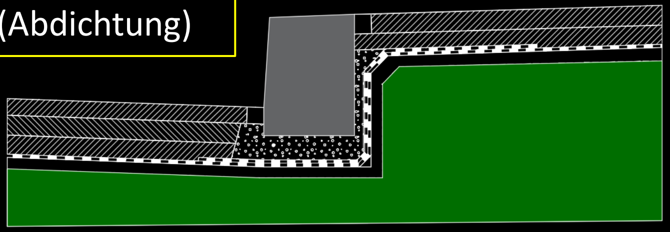
guardrail
(Leitschranke)



drainage
(Entwässerung)



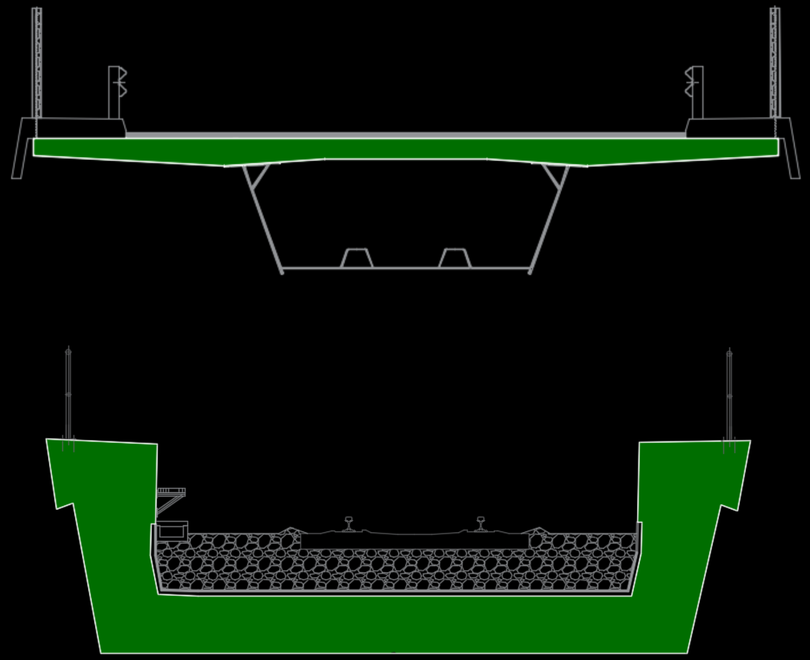
waterproofing
(Abdichtung)



Bridge deck: Concrete deck

Concrete deck (standard solution)

- Slenderness ca. $L/15\ldots L/20$ (L = transverse span between webs or girders, often tapered to save weight
- Minimum thickness $t_{min} \approx 200$ mm (4 reinforcement layers, concrete cover)
- Usually thicker ($t_m \approx 300$ mm), governed by shear strength (no shear reinforcement) and fatigue checks



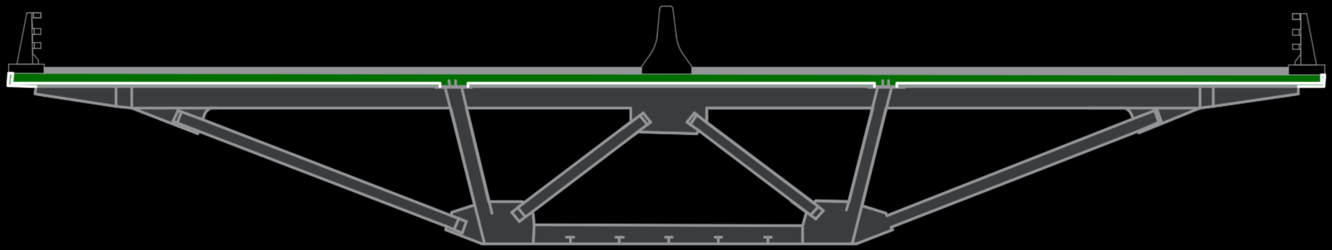
- ✓ economical solution
- ✓ robust and durable (with proper waterproofing)
- ✓ fatigue usually not problematic
- ➢ relatively thick and heavy (7.5 kN/m^2 for t_m = 300 mm, for deck **without** girders)

Bridge deck: Concrete deck (reduced weight)

Concrete deck (options to save weight)

- Slenderness ca. $L/15 \ldots L/20$ (L = transverse span between webs or girders, often tapered to save weight
- Minimum thickness $t_{min} \approx 200$ mm (4 reinforcement layers, concrete cover)
- Usually thicker ($t_m \approx 300$ mm), governed by shear strength (no shear reinforcement) and fatigue checks
- Possible options to save weight in decks with wide cantilevers and/or large internal spans:
  … transverse prestressing of deck
  … provision of transverse ribs
  … provision of additional supports (longitudinal ribs) supported by struts, e.g. on cantilever edge

- ✓ economical solution
- ✓ robust and durable (with proper waterproofing)
- ✓ fatigue usually not problematic
- ✓ relatively lightweight (photo on right side: ca. 9 kN/m^2 i.e. t_m = 360 mm **including** long.+transv. girders)



Bridge deck: Steel deck

Steel deck

- Orthotropic steel deck, usual in road bridges:
  … deck plate $t = 12…16$ mm
  … trapezoidal stiffeners @ 600 mm, approx.
  $H = 300 \times b = 300/150$ mm, $t = 6…8$ mm
  … stiffener span (crossbeams spacing) ca. 4 m

- Steel plate with or without flat plate stiffeners, for pedestrian and bicycle bridges (not shown)

✓ relatively lightweight (ca. 2.5 kN/m^2)

✓ thin, saves depth in case of low clearance

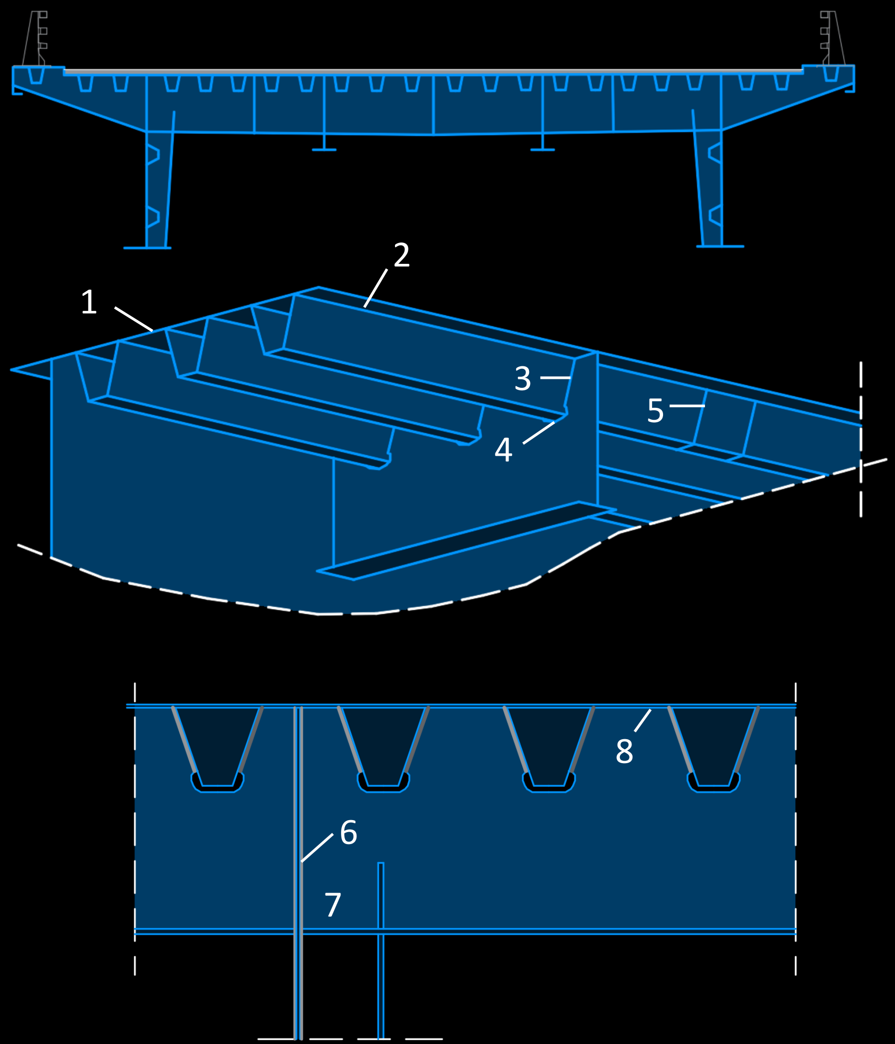
✓ large transverse spans possible

➢ expensive (high fabrication effort)

➢ susceptible to fatigue problems (many welds, proper detailing essential)

➢ noise emissions (particularly in railway bridges)

Orthotropic steel deck (OSD):



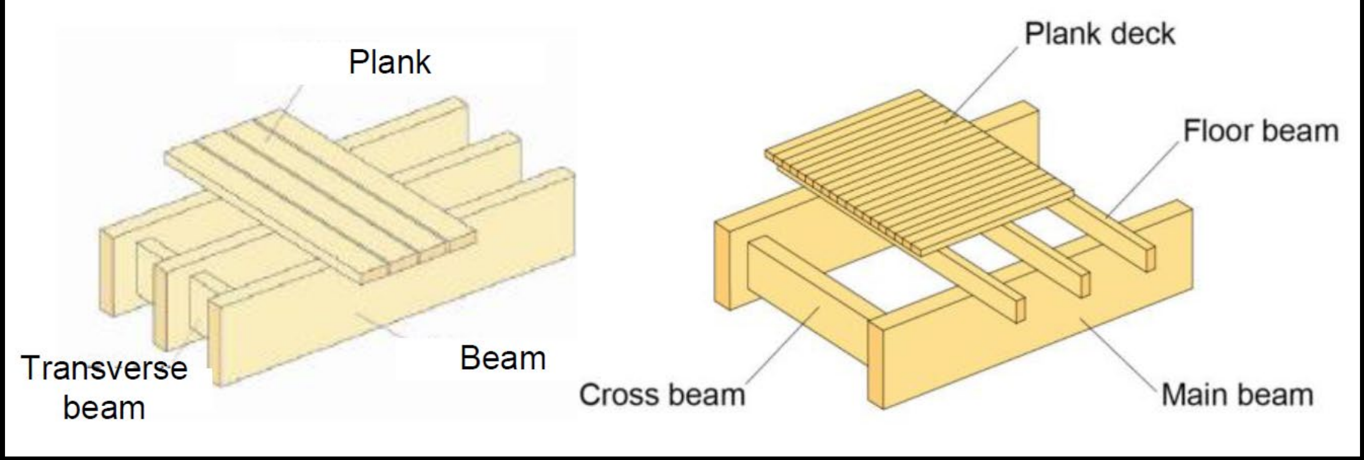
Legend
1) deck plate
2) welded connection of stiffener to deck plate
3) welded connection of stiffener to web of crossbeam
4) cut out in web of crossbeam
5) splice of stiffener
6) splice of crossbeam
7) welded connection of crossbeam to main girder or transverse frame
8) welded connection of the web of crossbeam to the deck plate

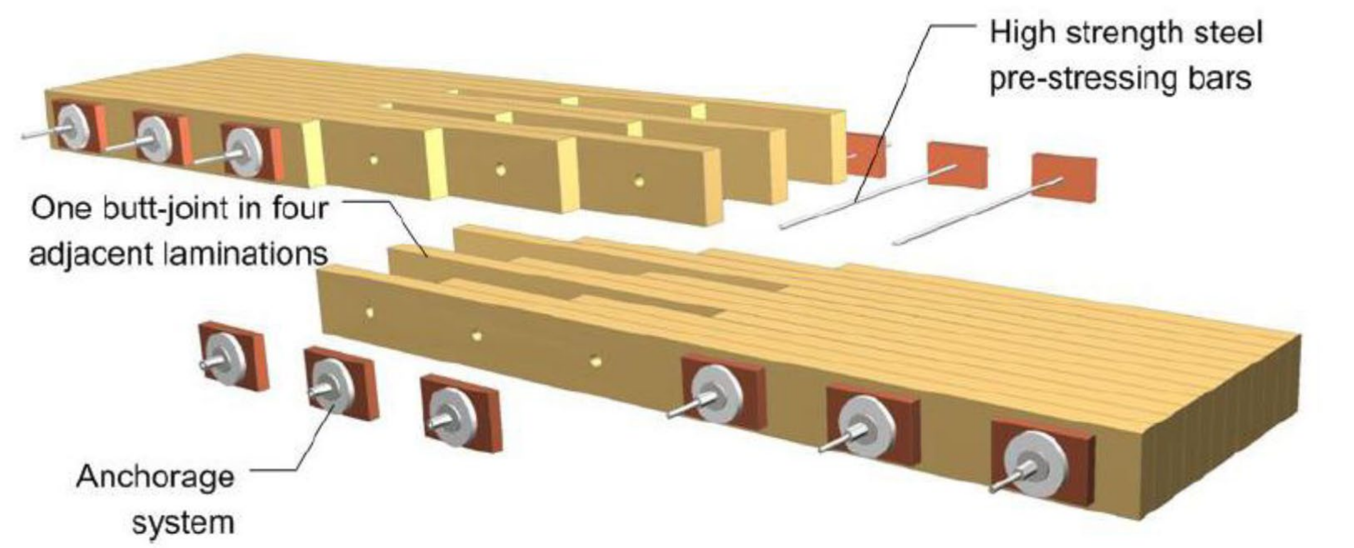
Bridge deck: Timber deck

Timber deck

- Detailing depends on use (loads, exposure) and local preferences
- Possible solutions:
  … transverse planks (US: glulam) on
  longitudinal girders
  … longitudinal boards on transverse floor beams
- Additional wear planks (→ protection, roughness) or membrane and surfacing (road bridges)
- transverse prestressing for biaxial load transfer (account for prestress losses due to temperature and humidity variations)

✓ lightweight
✓ appealing to pedestrian use
✓ sustainability …unless impregnated
➢ limited load capacity
➢ predominantly uniaxial load transfer
➢ limited durability (unless protected or impregnated
→ severe environmental issues, see notes)

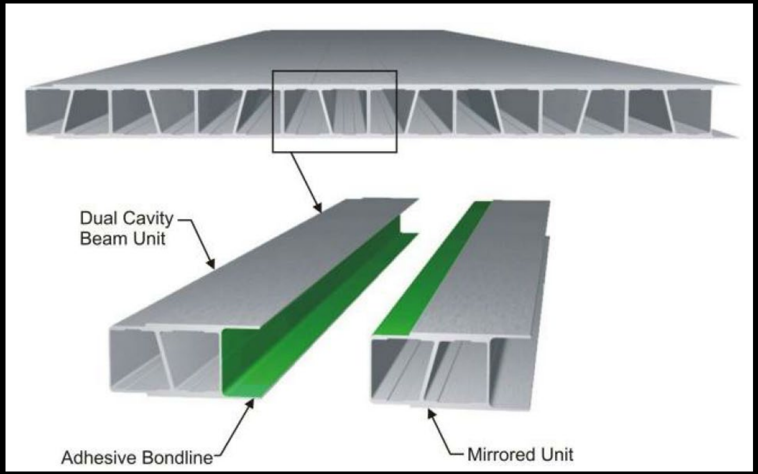




Bridge deck: GFRP deck

GFRP deck

- Pultruded GFRP profiles, assembled with adhesives and/or clamps
- Beam units for larger spans (usually transverse direction) or planks



- ✓ ultra-lightweight
- ✓ durable (no corrosion)
- ➢ lack of standardisation
- ➢ lacking long-term experience (fatigue, UV exposure)
- ➢ primarily uniaxial load transfer (usually)
- ➢ brittle material behaviour
- ➢ expensive

Bridge deck: **Design**

The deck slab is usually modelled as a slab supported by

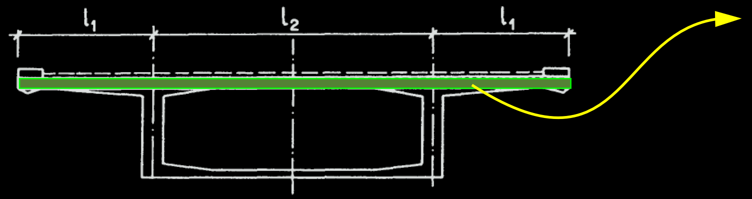
- longitudinal girders or webs
- cross-beams if they support the deck

Linear elastic FE slab analyses are standard today for the design of bridge decks. Often, rigid supports are assumed, but a refined analysis may be appropriate in special cases (e.g. thick slabs on slender cross-beams).

The rotational restraint of the supports depends on the type of girder. For concrete girders, the boundary conditions shown in the figure (adapted from Menn, 1990) may be assumed. Steel girders and cross-beams usually do not provide significant fixity (deck much stiffer than webs) as also shown in the figure.

For the investigation of transverse bending of the longitudinal girders, the support moments obtained from the deck slab analysis are applied to the box girder and the webs of open cross sections, respectively, and superimposed to transverse bending of the cross-section due to other causes (torque introduction), see *bridge girder*.

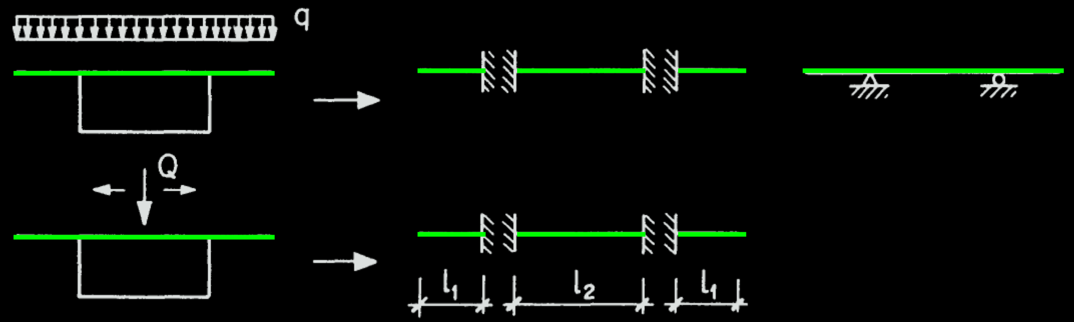
Deck model (constant depth for analysis)



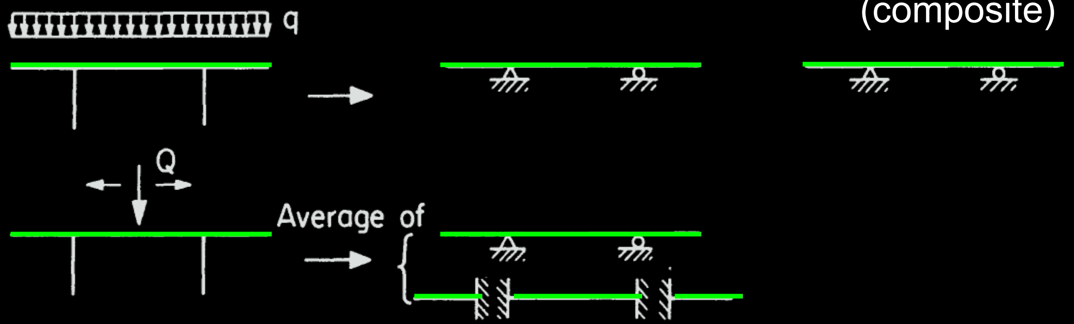
$$\frac{\partial^2 m_x}{\partial x^2} + 2\frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} + q = 0$$

design of slabs see e.g. courses «Stahlbeton II», «Flächentragwerke», …

Deck on box girder … concrete box … steel box (composite)



Deck on double-T beam … concrete beams … steel beams (composite)



Bridge deck: Design

In the analysis of the deck slab, concentrated loads are often spread as shown in the upper figure. Strictly speaking, this spreading would require reinforcement, and according to SIA 262, only a spreading in the surfacing should be considered (see AGB Report 636).

In preliminary design, bending moments in the deck may be estimated:

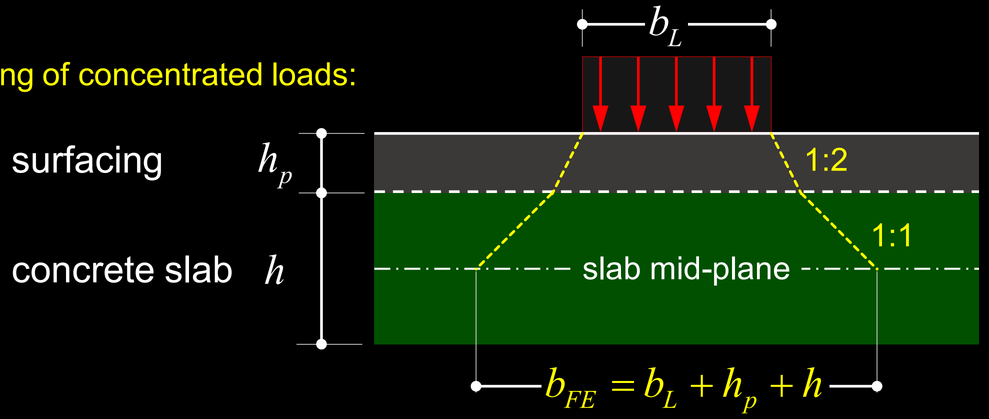
- assuming a spreading under 45° in-plane for concentrated loads (lower figure)
- distributed loads are transferred in the transverse direction

Note that this simplified treatment of concentrated loads

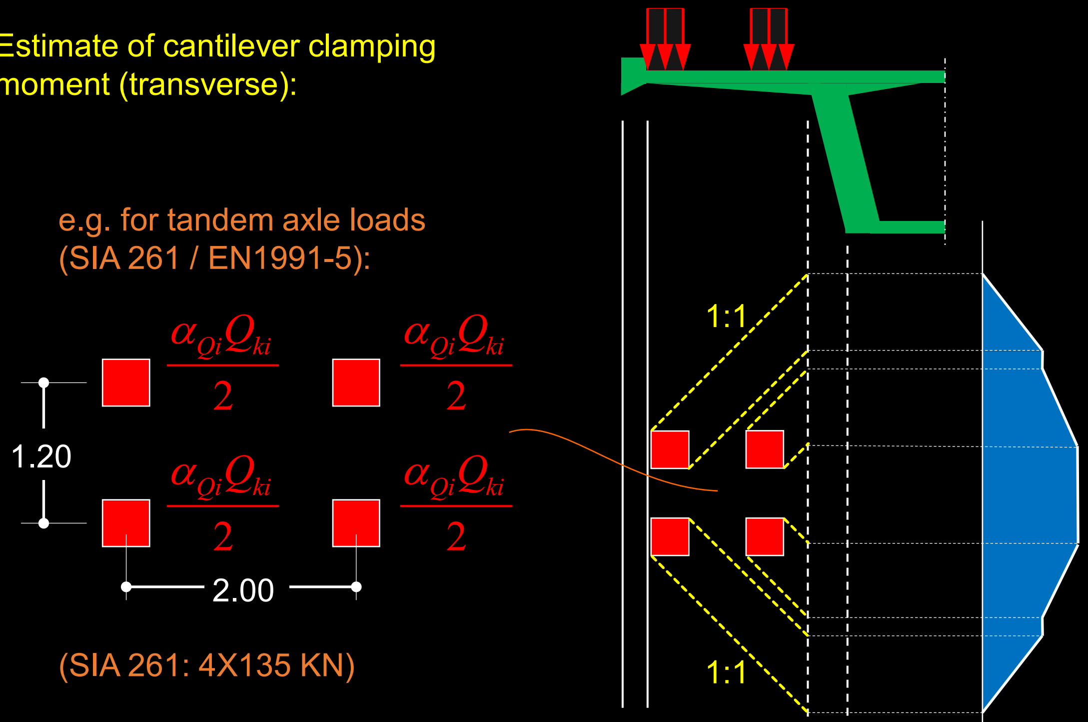
- presumes sufficient longitudinal resistance (usually ok)
- is not suitable for fatigue verifications
- is not suitable (potentially unconservative) for shear strength verification

According to SIA 262, the shear capacity depends on the utilisation of the bending resistance m_d/m_{Rd} → see AGB Report 636 (notes) for verification in final design (notes).

Spreading of concentrated loads:



Estimate of cantilever clamping moment (transverse):



Bridge deck: Design

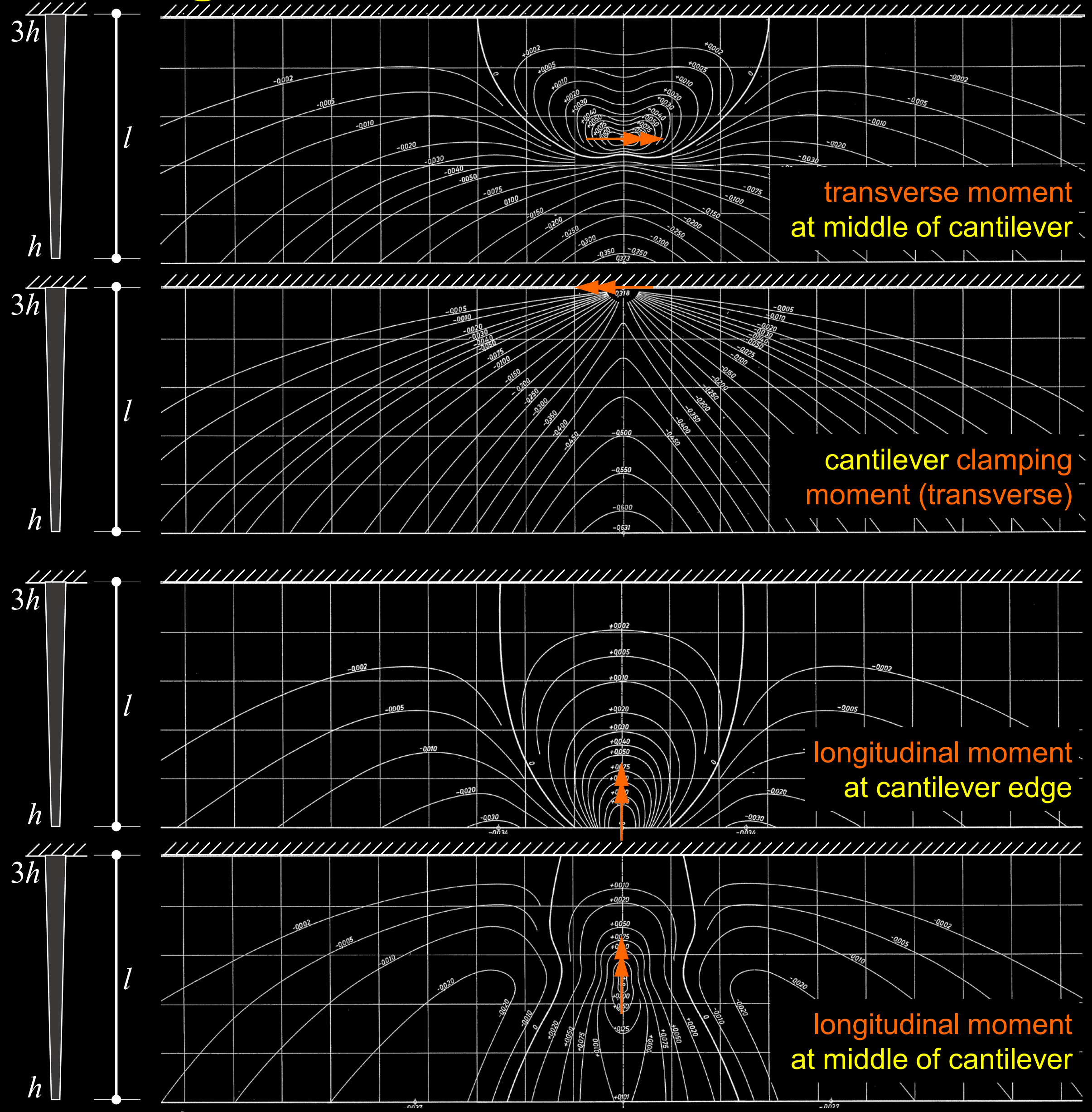
Before the advent of affordable, user-friendly FE-analyses of slabs, determining the internal actions caused by concentrated loads was challenging.

Influence surfaces (published by Homberg, Pucher and others, see notes) were used to this end until few decades ago. These show

- the bending moment (or shear force)
- at a specific point of a slab
- in a specific direction of a slab
- for a unit load (sometimes to be divided by 8π)
- assuming linear elasticity

The design actions are obtained from the influence surfaces by integration (using approximations, often by eye). Homberg's publications include evaluations for the load models used at the time of publication.

The figures on the right show influence surfaces for bending moments in an infinitely long cantilever with variable thickness (adapted from Homberg, 1965).



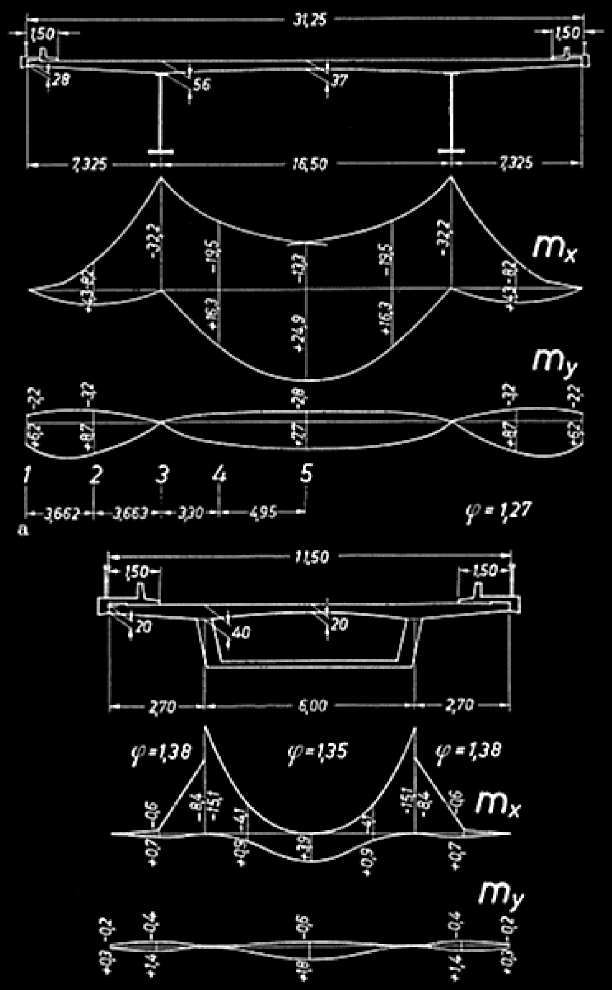
Bridge deck: Design

When designing using influence surfaces, the distribution of bending moments between the points covered in the charts need to be accounted for.

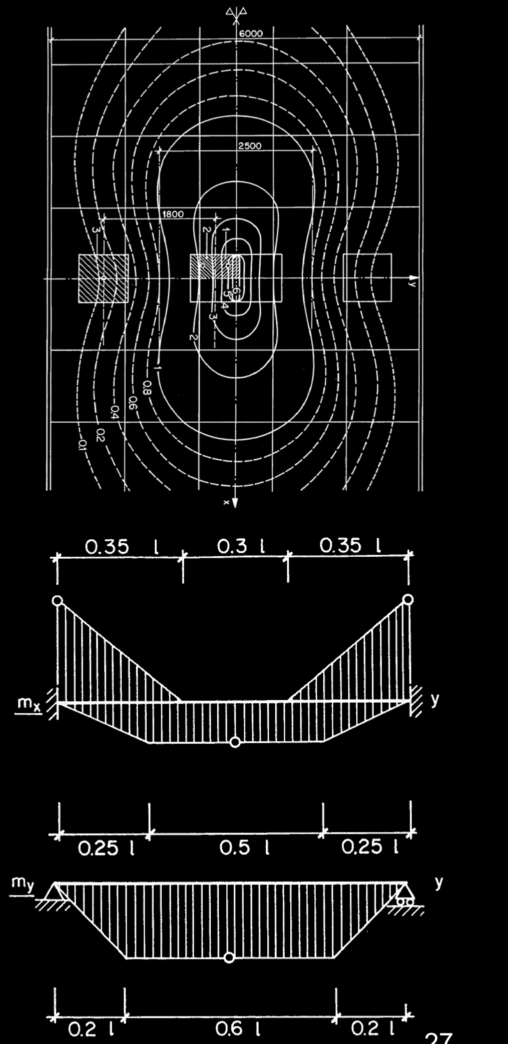
The figures on the right show possible assumptions to this end.

From today's perspective, they are obsolete for design, as FE-analyses of slabs yield this information much more efficiently. They are still useful to get an intuitive understanding, e.g. regarding the possible curtailment of reinforcement.

Transverse variation of bending moments (from Homberg+Ropers):



Influence surface for interior slab and transverse variation of bending moments (from Menn)



Superstructure / Girder bridges

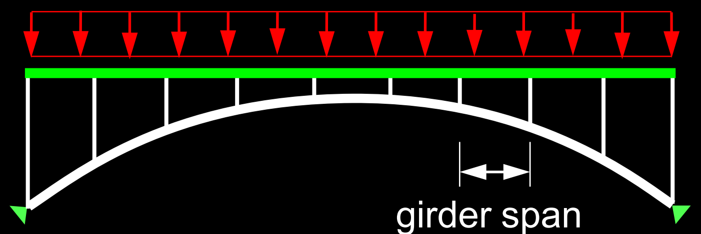
Bridge girder – Structural efficiency

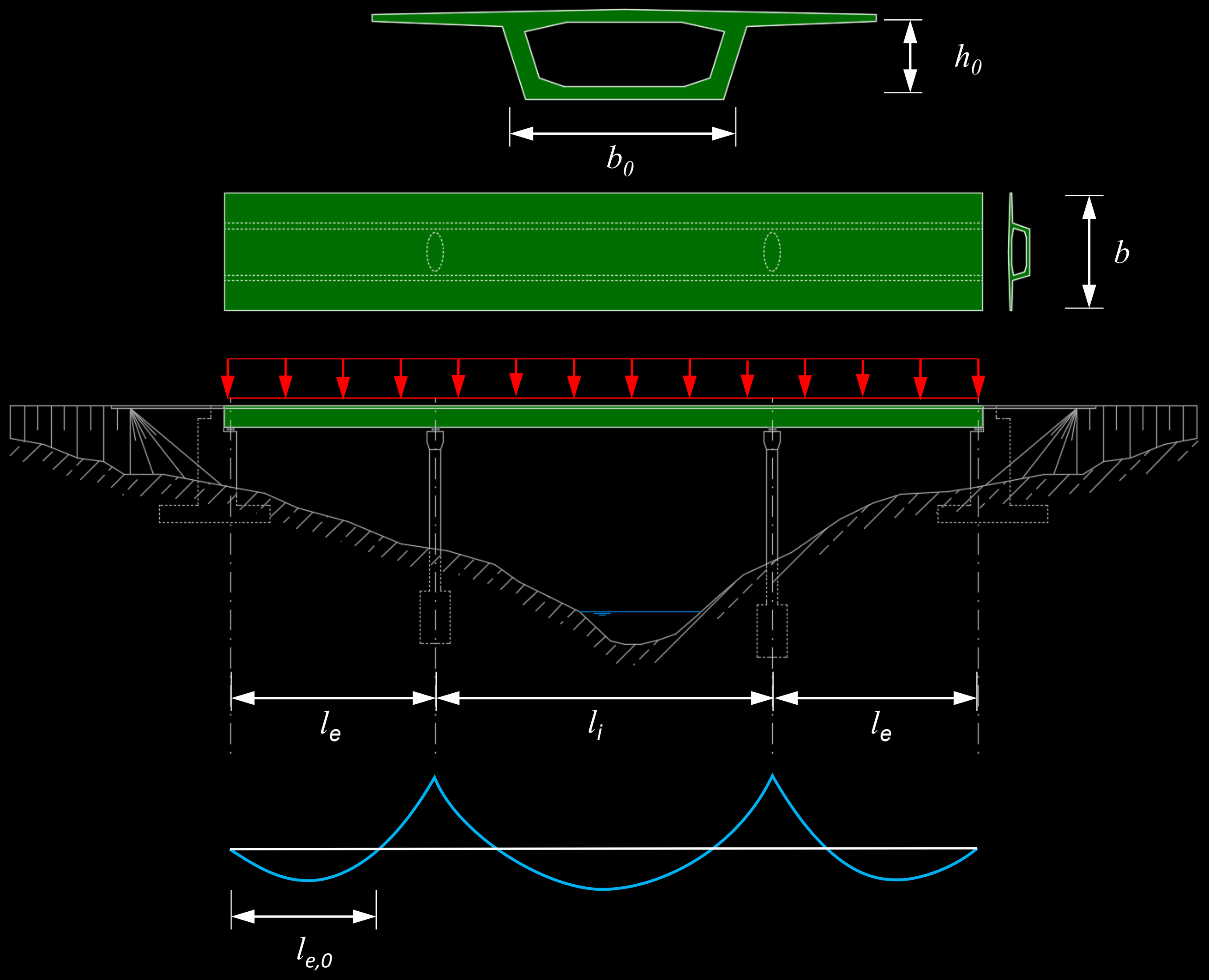
Bridge girder – Structural efficiency: Dominant internal action

The bridge girder transfers loads longitudinally to its supports (piers, abutments or elements of the superstructure supporting the girder).

In girder bridges, the spans l are significantly longer than the depth h_0 and the width b_0 of the girder. Hence, longitudinal bending is governing the design.

Note: Effective girder spans are typically much shorter in bridges types where the superstructure consists of more elements than the girder, e.g. arch bridges:





Bridge girder – Structural efficiency: Dominant load

Self-weight of the girder = large portion of the total load, bending moments due to self-weight increase with the span

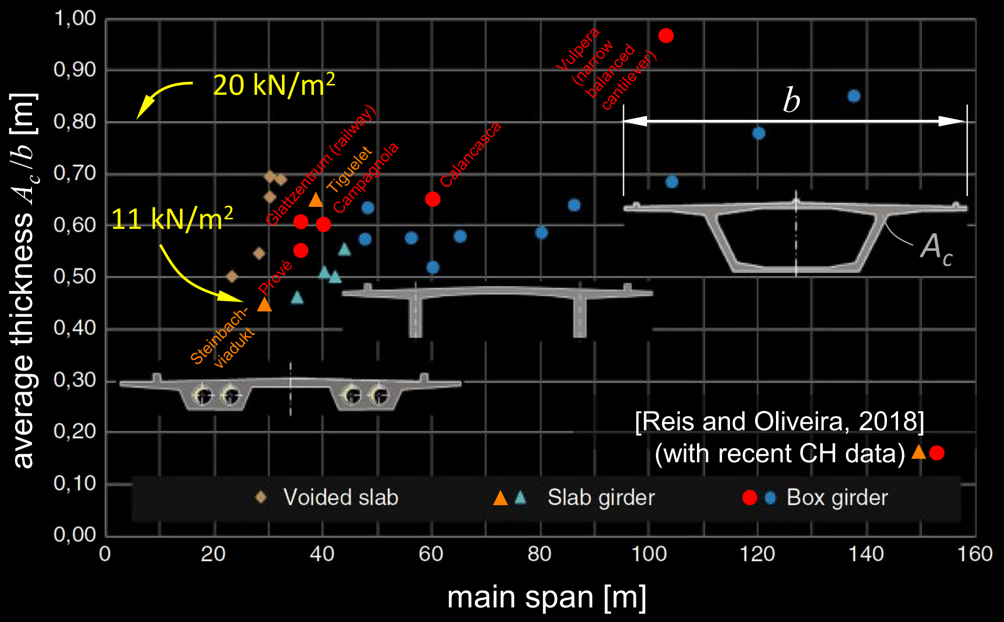
→ deeper girders (= more weight) required with increasing spans
→ self-weight is highly relevant

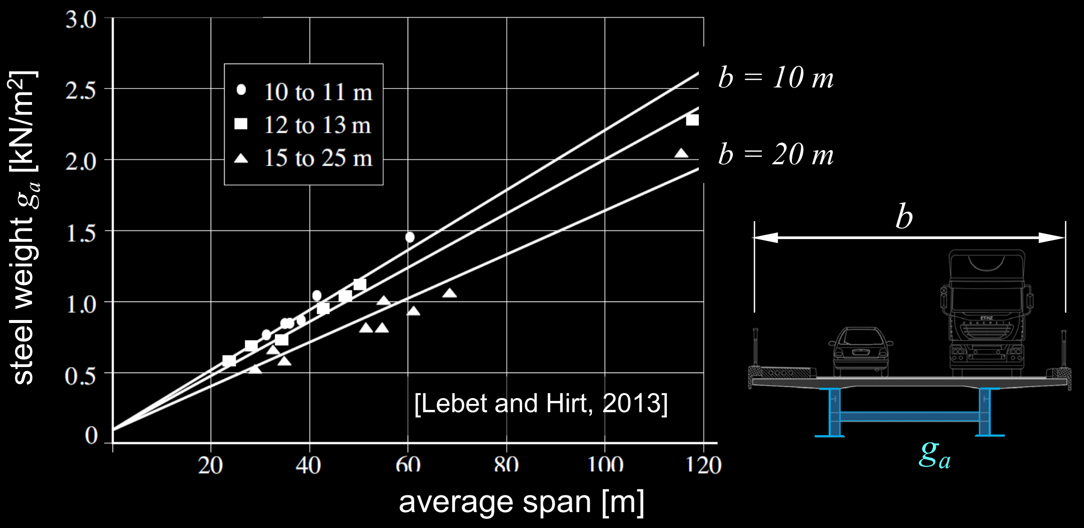
Equivalent girder thickness $t_{eq} = A_c/b$ (cross-section divided by deck width) for recent concrete girder bridges (upper figure):

- $t_{eq,min} \approx 0.45$ m at short spans → 0.45 · 25 = 11 kN/m²
- $t_{eq} > 0.80$ m for long spans → 0.80 · 25 = 20 kN/m²
- moderate increase since the deck (ca. 0.3 · 25 = 7.5 kN/m²) is always required; weight increase without deck more pronounced

Steel weight of composite girders (with concrete deck, lower figure):

- minimum ca. 0.75 kN/m² at short spans
- more than 2.2 kN/m² for long spans
- pronounced increase but steel weight = only 10…30% of the weight of the concrete deck





Bridge girder – Structural efficiency: Static system

The efficiency of a girder bridge primarily depends on

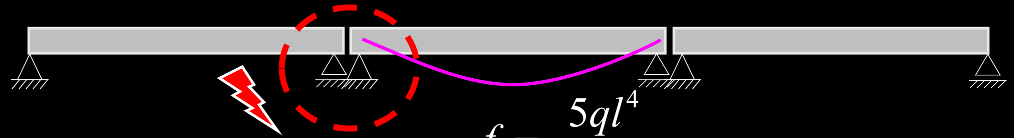
- the static system
- the cross-section and its materialisation
- the erection process

Simply supported girders can be erected very fast, particularly if prefabricated girders are used, and are often the cheapest solution (neglecting service life costs).

Therefore, despite many drawbacks (see figure), simply supported girders have been used in countless bridges, and are still popular in many countries worldwide.

Continuous girders are statically much more efficient than simply supported girders, and have further advantages (see figure).

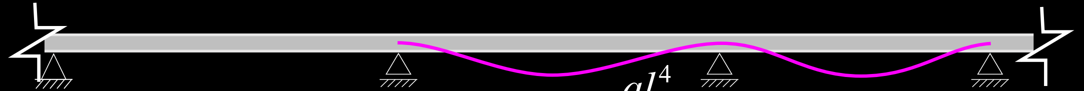
Simply supported girders:



$$f = \frac{5ql^4}{384EI}$$

✓ fast and simple erection (by lifting in)
➢ high maintenance demand
➢ lack of durability (mainly in road bridges)
➢ unsatisfactory user comfort (road bridges)
➢ lack of robustness

Continuous girder:



$$f = \frac{ql^4}{384EI}$$

✓ high stiffness → higher slenderness possible
→ less material consumption
✓ activation of negative bending resistance
✓ lower maintenance demand
✓ higher durability
➢ more complicated construction

Bridge girder – Structural efficiency: Variable depth

The depth of the girder is both

- beneficial (higher stiffness and bending resistance) as well as
- harmful (higher self-weight and thus bending moments)

→ maximise depth while minimising bending moments

→ adjust depth to required bending resistance

Simply supported girders

- high bending moments only in span

→ reduce depth near the supports

→ limited increase in efficiency (reduced self-weight near supports has little effect on the bending moments)

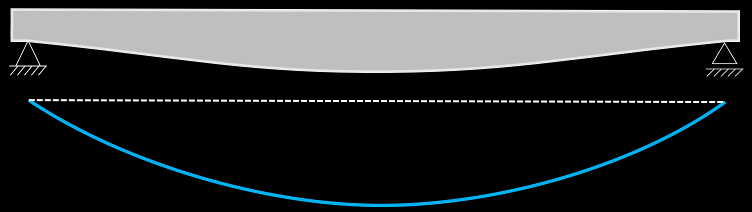
Continuous girders

- highest bending moments over intermediate supports

→ reduce depth at midspan

→ pronounced increase in efficiency (self-weight is reduced where it causes high bending moments)

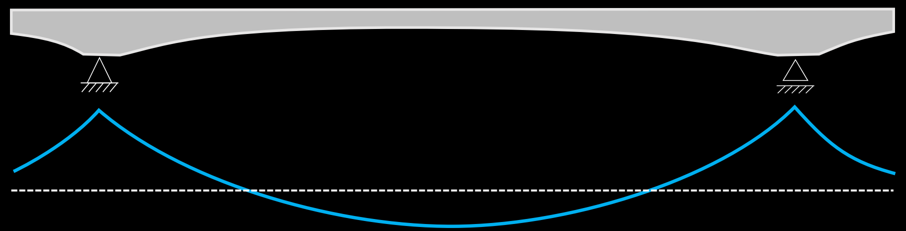
Simply supported girder:



✓ maximum depth where bending moments are highest

➢ full weight where it causes high bending moments

Continuous girder:



✓ maximum depth where bending moments are highest

✓ reduced weight where it causes high bending moments

➢ positive (sagging) bending moments may become governing, particularly in end-spans (traffic loads), if depth is reduced too much

➢ more expensive to build, but economical for larger spans or in case of specific requirements (clearance, …)

Bridge girder – Structural efficiency: Efficient cross-section

Since longitudinal bending is the dominant action and self-weight is the dominant load at large spans, efficient solutions require cross sections that combine

high bending stiffness & strength	+	low self-weight

while ensuring sufficient stiffness and capacity for other loads, particularly non-symmetric traffic loads.

→ use suitable material with high ratios of stiffness and strength to specific weight (E/γ, f_y/γ)

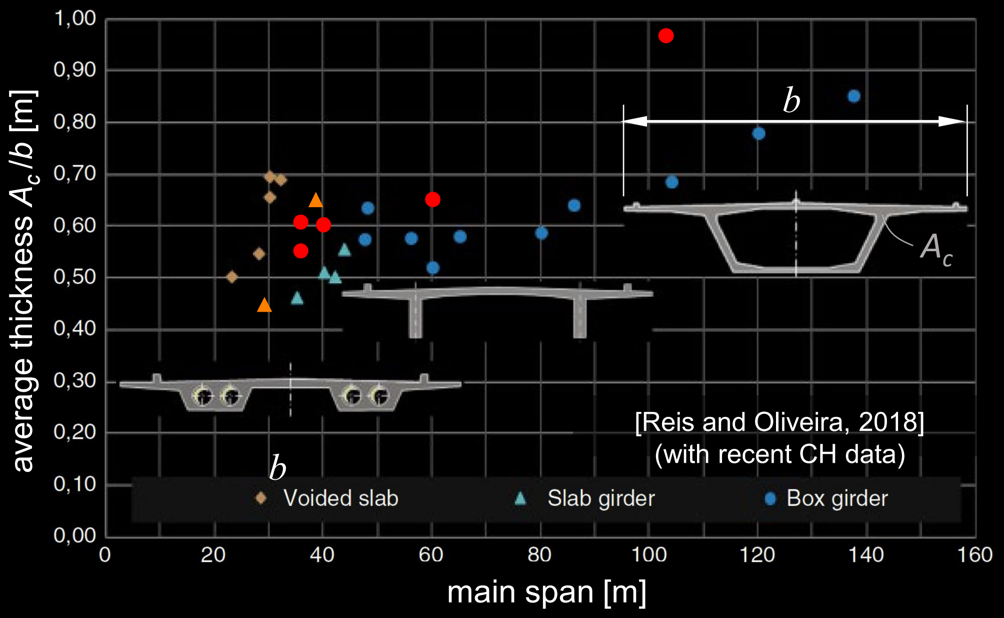
→ optimise cross-section, i.e. maximise ratios of bending stiffness and strength to cross-section (EI_y/A_{tot}, M_{Rd}/A_{tot})

Theoretically, a pure stringer cross-section would be ideal:

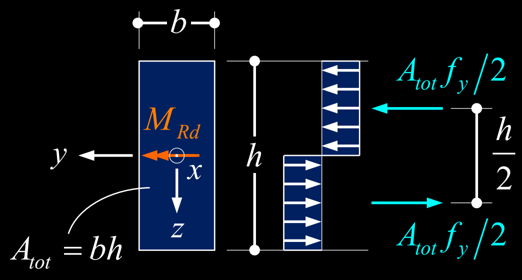
→ 3 x stiffer

→ 2 x stronger

than a rectangular cross-section (for linear elastic - ideally plastic materials)



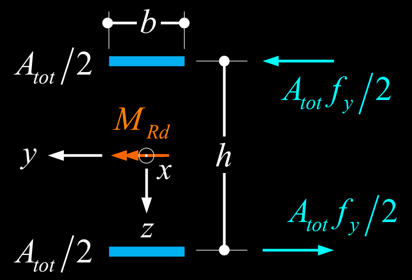
Rectangular cross-section



$$EI_y = E\frac{bh^3}{12} = E\frac{A_{tot}h^2}{12}$$

$$M_{Rd} = f_y\frac{bh^2}{4} = f_y\frac{A_{tot}h}{4}$$

Stringer cross-section



$$EI_y = 2\frac{EA_{tot}}{2}\left(\frac{h}{2}\right)^2 = E\frac{A_{tot}h^2}{4}$$

$$M_{Rd} = \frac{f_y A_{tot}}{2}h = f_y\frac{A_{tot}h}{2}$$

Bridge girder – Structural efficiency: Efficient cross-section

Pure stringer cross-sections are not feasible, but

- Concentrating the material far from the neutral axis is beneficial for the ratios EI_y / A_{tot}, M_{Rd} / A_{tot}
- In prestressed concrete girders, reducing the weight by doing so even increases the decompression moment (figure)

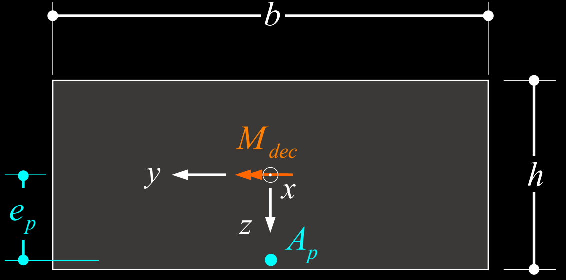
Efficient cross-sections should therefore have wide flanges but only narrow webs, and the deck should be activated as flange:

→ locate deck at top or bottom of cross-section

→ minimise web thickness, with limitations given by:
… required shear strength
… space requirement for casting of webs
(particularly for internal prestressing cables)
… maximum slenderness of steel plates

→ use trusses instead of solid webs
… only economical in long-span bridges
… may be aesthetically beneficial (transparency)

Rectangular cross-section:



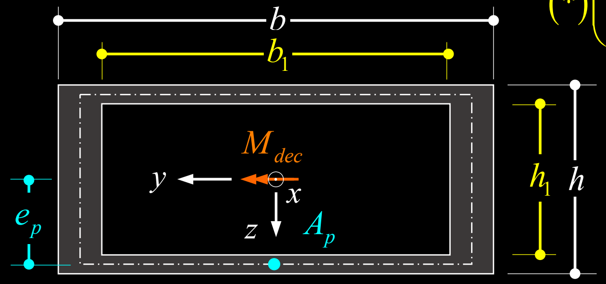
$A = bh; \quad P = A_p \sigma_p$

$I_y = \frac{bh^3}{12}; \; W_y = \frac{I_y}{h/2} = \frac{bh^2}{6}; \; k = \frac{W_y}{A} = \frac{h}{6}$

$M_{dec} = P(e+k) = P\left(e + \frac{W_y}{A}\right) = P\left(e + \frac{h}{6}\right)$

Box girder:

$(*)\left(\frac{b_1}{b} = \frac{h_1}{h}\right)$



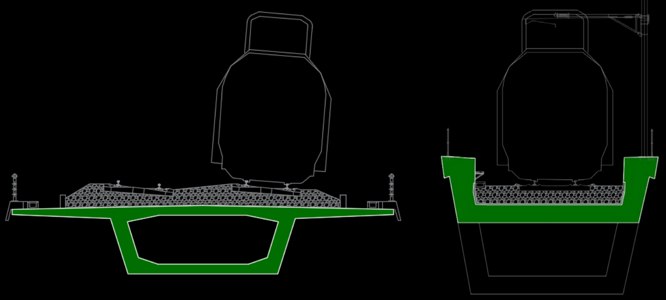
$A_n = bh - b_1 h_1 = A - A_1; \qquad P = A_p \sigma_p$

$I_{y,n} = \frac{bh^3 - b_1 h_1^3}{12}; \; W_{y,n} = \frac{I_{y,n}}{h/2} = \frac{bh^2 - b_1 h_1^3 / h}{6}; \quad k = \frac{W_{y,n}}{A_n}$

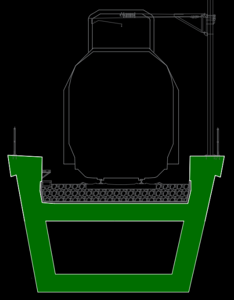
$M_{dec} = P\left(e + \frac{W_{y,n}}{A_n}\right) = P\left(e + \frac{h}{6} \cdot \frac{1 - \frac{A_1}{A}\frac{h_1^2}{h^2}}{1 - \frac{A_1}{A}}\right) \overset{(*)}{=}$

$= P\left(e + \frac{h}{6} \cdot \left[1 + \frac{A_1}{A}\right]\right)$

Efficient cross-sections:



Inefficient c.s.



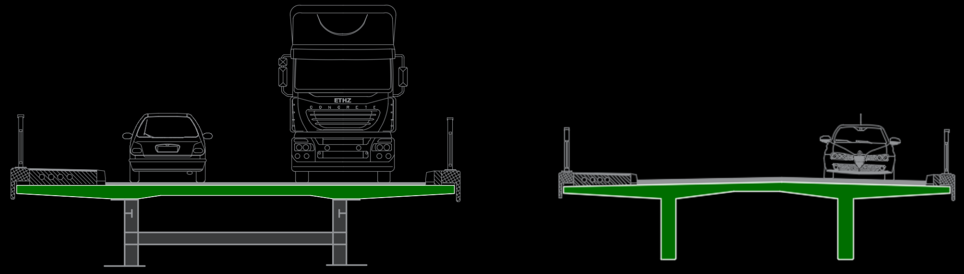
Bridge girder – Structural efficiency: Efficient cross-section

Whether an open cross-section or a box girder is appropriate depends on the static system and spans (particularly magnitude of hogging moments and torsional moments).

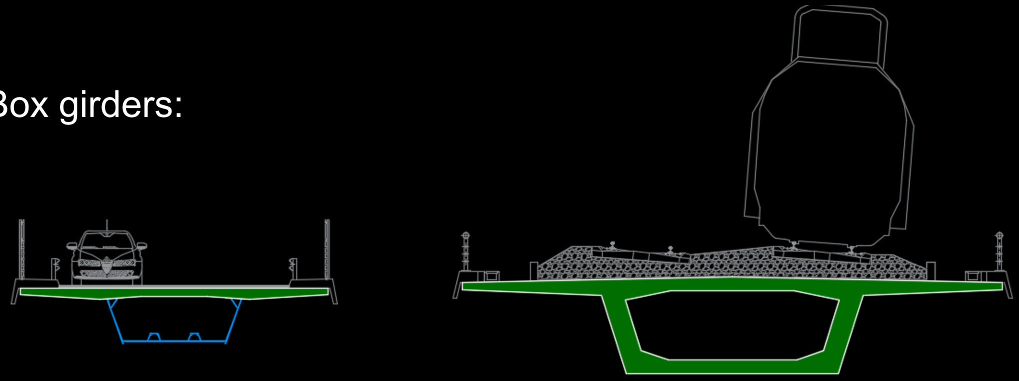
Regarding bending, the following should be considered:

- Concrete decks are particularly effective where subjected to longitudinal compression (usually sagging moments).
- Open cross-sections without a bottom slab are efficient in regions of sagging moments (compression in concrete deck, tension concentrated in bottom chord = narrow steel flange or prestressing cables at bottom of web).
- A bottom slab may be required over the supports, in order to resist the compressive forces caused by the hogging moments (particularly in concrete girders, respecting ductility criteria for the depth of the compression zone (e.g. $x/d<0.35$).

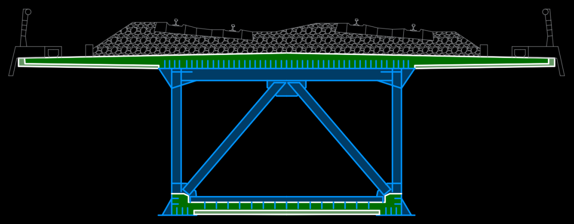
Open cross-sections:



Box girders:



Double composite action:



Bridge girder – Structural efficiency: Efficient cross-section

Bending is dominant, but sufficient stiffness and capacity for other loads, particularly torsional moments, is also required. Therefore, box girders (closed cross-sections) are frequently used in bridges with

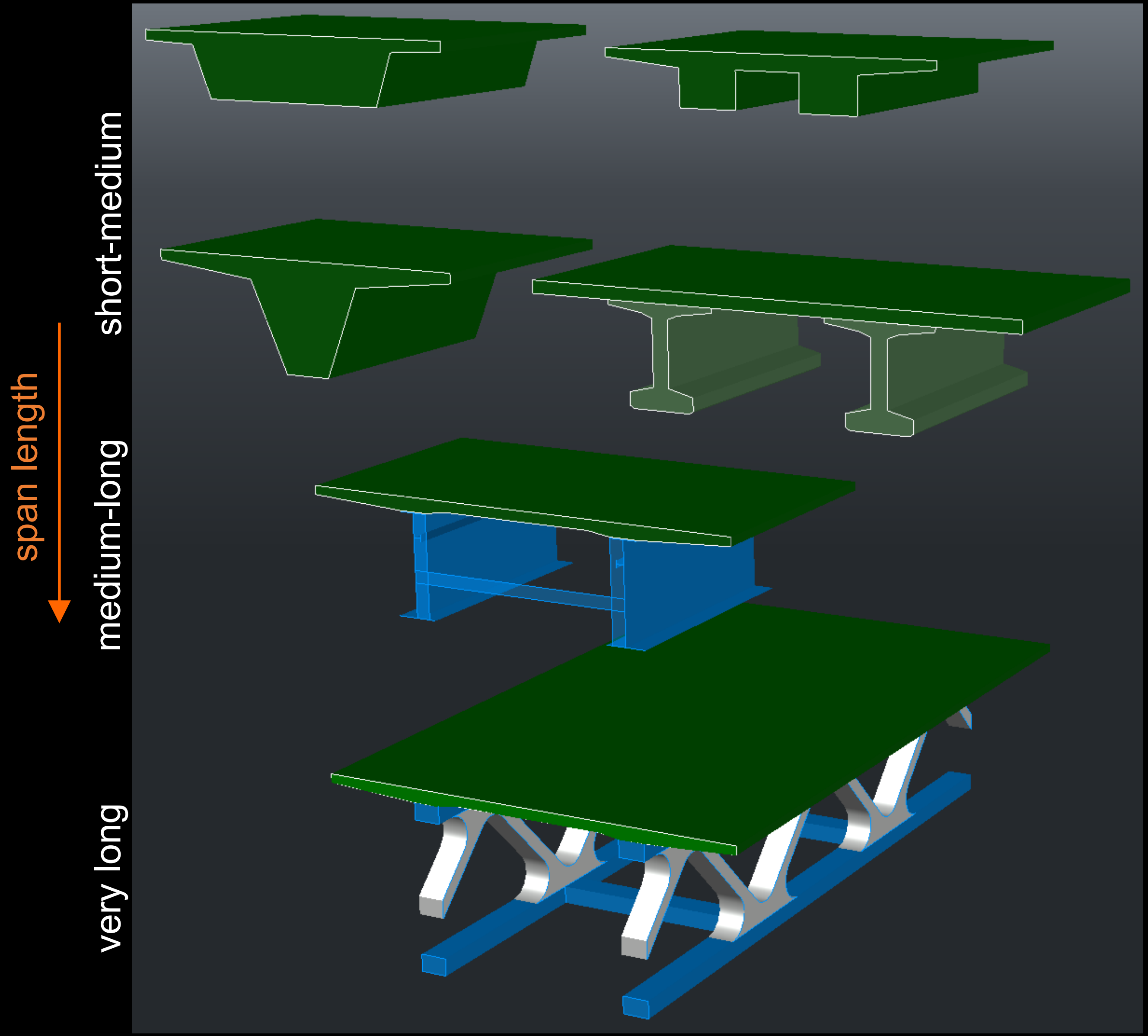
- high eccentric traffic loads
- strong curvature or skew supports

Statically efficient cross-sections often require significantly more labour or more expensive materials than simpler, less efficient solutions.

With increasing spans, structural efficiency becomes more relevant and aligned with economy.

narrow / mod. wide deck
- straight / mod. curved
- strong curvature

wide deck
- straight / mod. curved
- strong curvature



Bridge girder – Structural efficiency: Efficient cross-section

Bending is dominant, but sufficient stiffness and capacity for other loads, particularly torsional moments, is also required. Therefore, box girders (closed cross-sections) are frequently used in bridges with

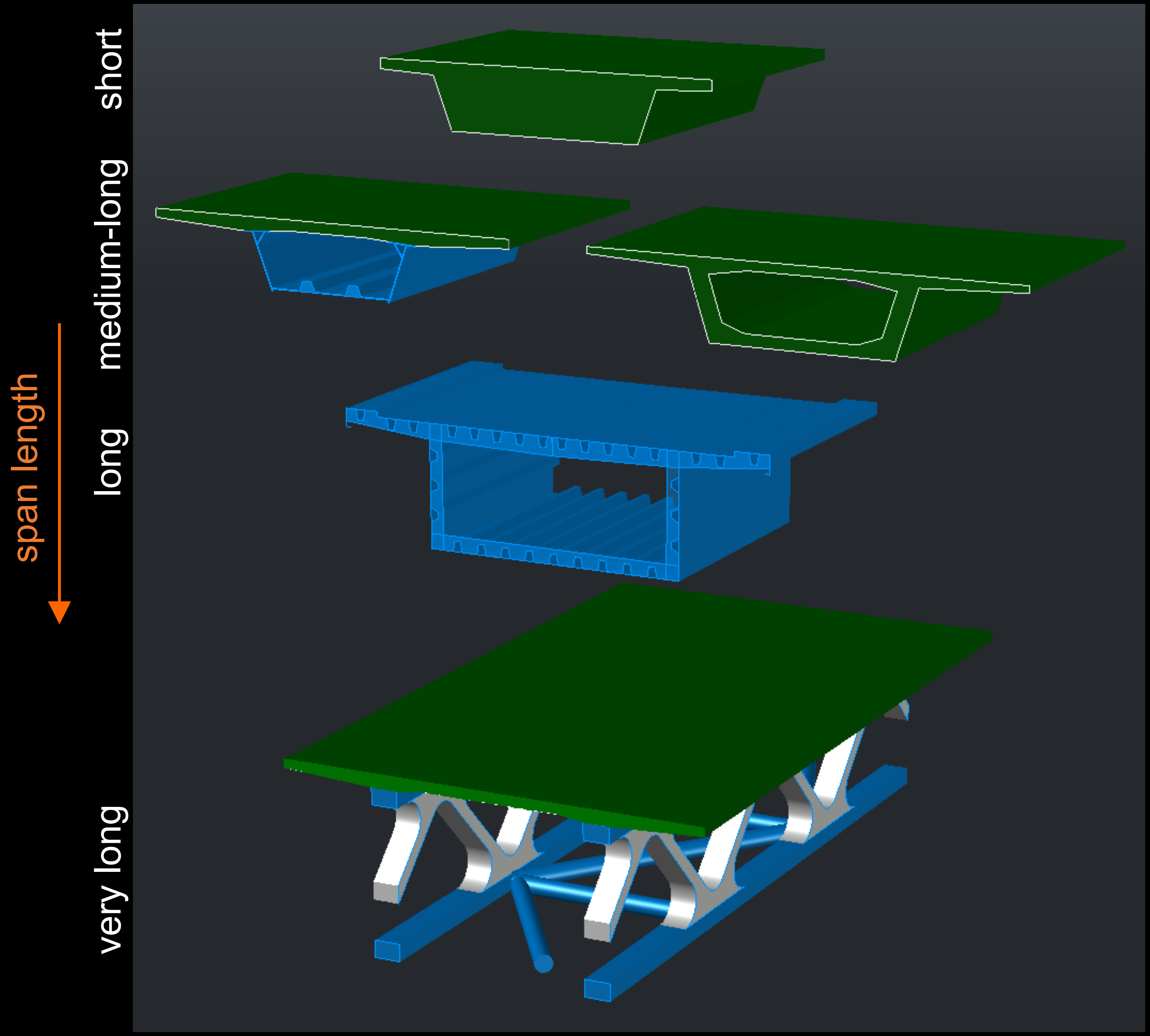
- high eccentric traffic loads
- strong curvature or skew supports

Statically efficient cross-sections often require significantly more labour or more expensive materials than simpler, less efficient solutions.

With increasing spans, structural efficiency becomes more relevant and aligned with economy.

narrow / mod. wide deck
- straight / mod. curved
- strong curvature

wide deck
- straight / mod. curved
- strong curvature



Bridge girder – Structural efficiency: Efficient cross-section

Bending is dominant, but sufficient stiffness and capacity for other loads, particularly torsional moments, is also required. Therefore, box girders (closed cross-sections) are frequently used in bridges with

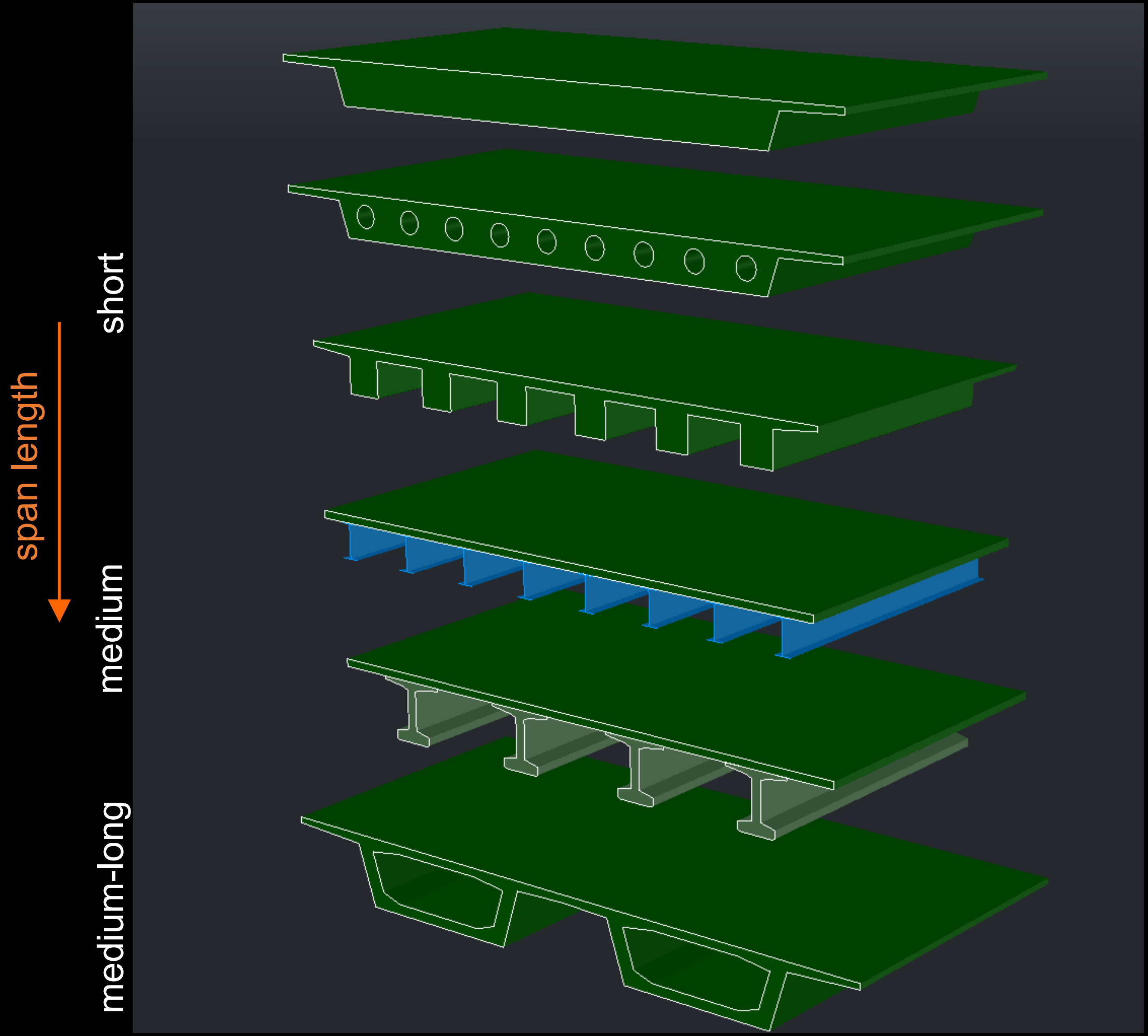
- high eccentric traffic loads
- strong curvature or skew supports

Statically efficient cross-sections often require significantly more labour or more expensive materials than simpler, less efficient solutions.

With increasing spans, structural efficiency becomes more relevant and aligned with economy.

narrow / mod. wide deck
- straight / mod. curved
- strong curvature

wide deck
- straight / mod. curved
- strong curvature



Bridge girder – Structural efficiency: Efficient cross-section

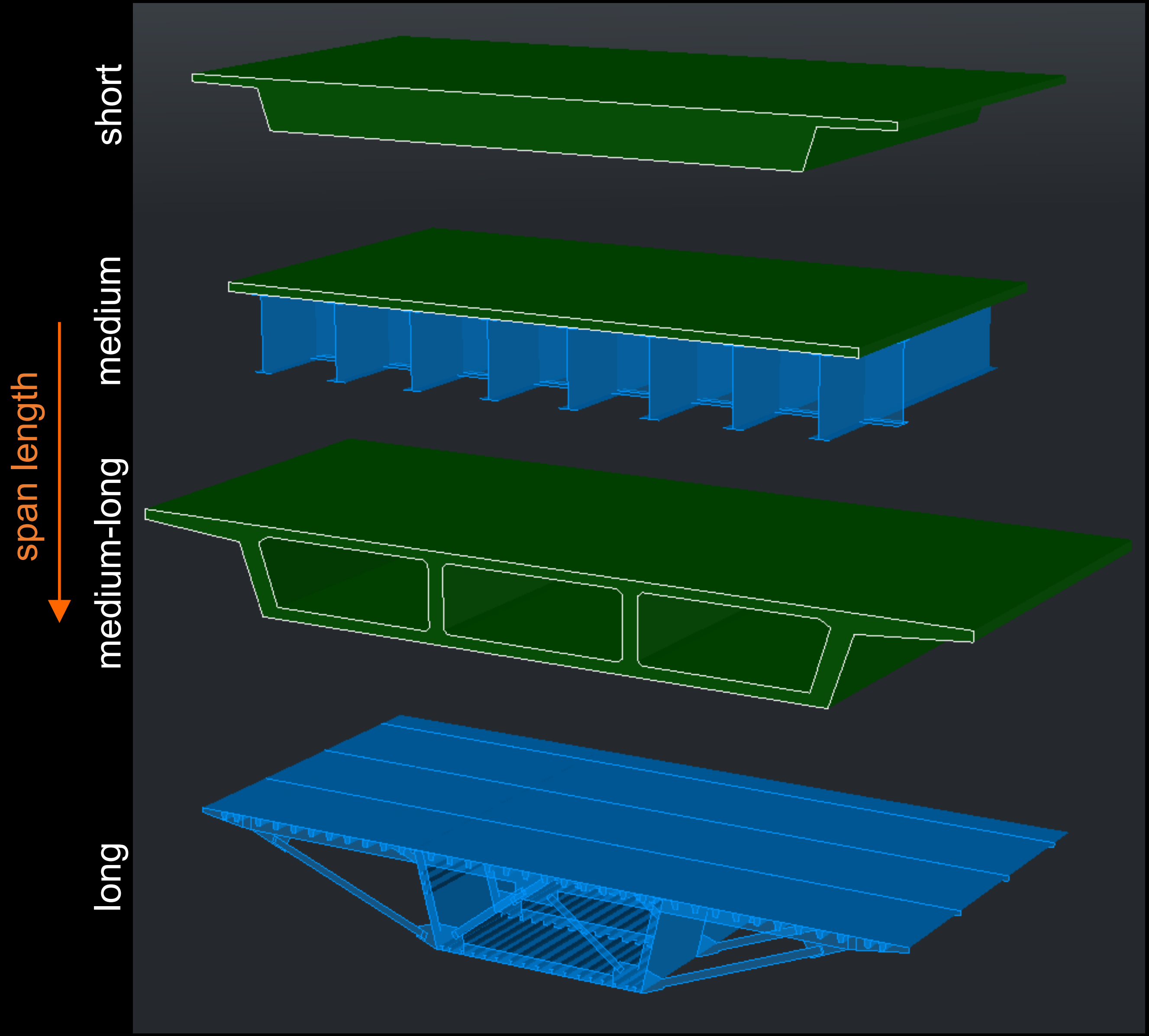
Bending is dominant, but sufficient stiffness and capacity for other loads, particularly torsional moments, is also required. Therefore, box girders (closed cross-sections) are frequently used in bridges with

- high eccentric traffic loads
- strong curvature or skew supports

Statically efficient cross-sections often require significantly more labour or more expensive materials than simpler, less efficient solutions.

With increasing spans, structural efficiency becomes more relevant and aligned with economy.

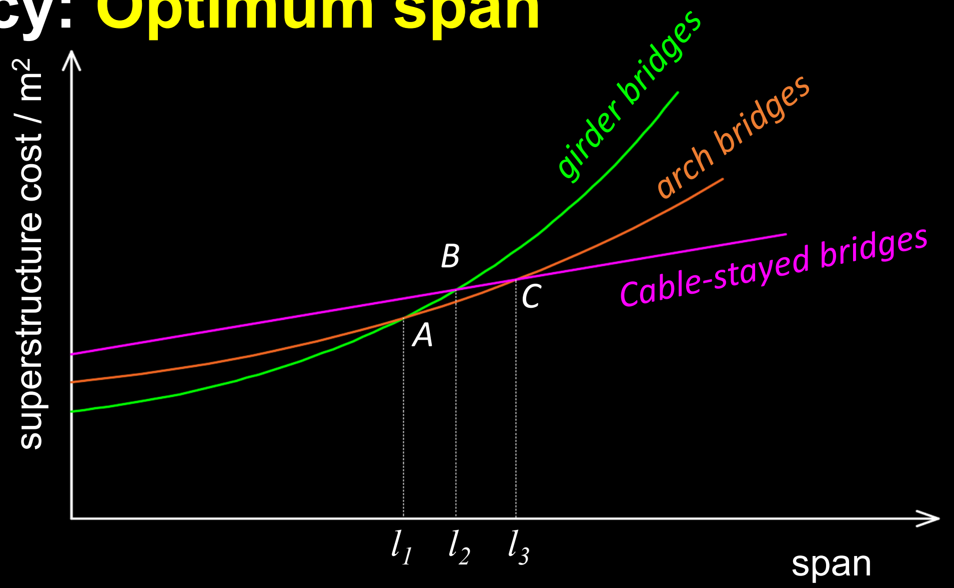
narrow / mod. wide deck	straight / mod. curved
	strong curvature
wide deck	straight / mod. curved
	strong curvature



Bridge girder – Structural efficiency: Optimum span

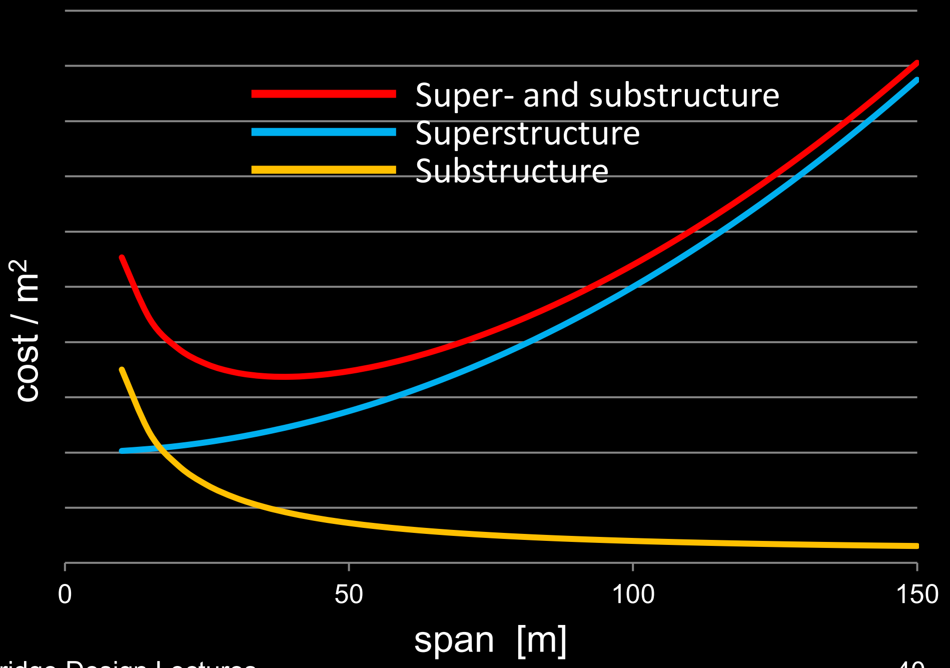
Upper figure:

- Since more depth is required at larger spans, the costs of the bridge girder increase with its span
- Girder bridges are economical at smaller spans than other, inherently more efficient typologies (since these also require a girder and are thus less efficient at small spans).



Lower figure:

- Contrary to the costs of the girder (superstructure), the substructure costs decrease with span (short spans = many piers and foundations)
- The cost of super- and substructure of a girder bridge therefore exhibit a minimum at the optimum economic span
- This optimum span is usually around 30 m
- The minimum is rather flat, leaving considerable freedom for economic solutions considering other aspects, such as aesthetics.



Bridge girder – Structural efficiency: Optimum span

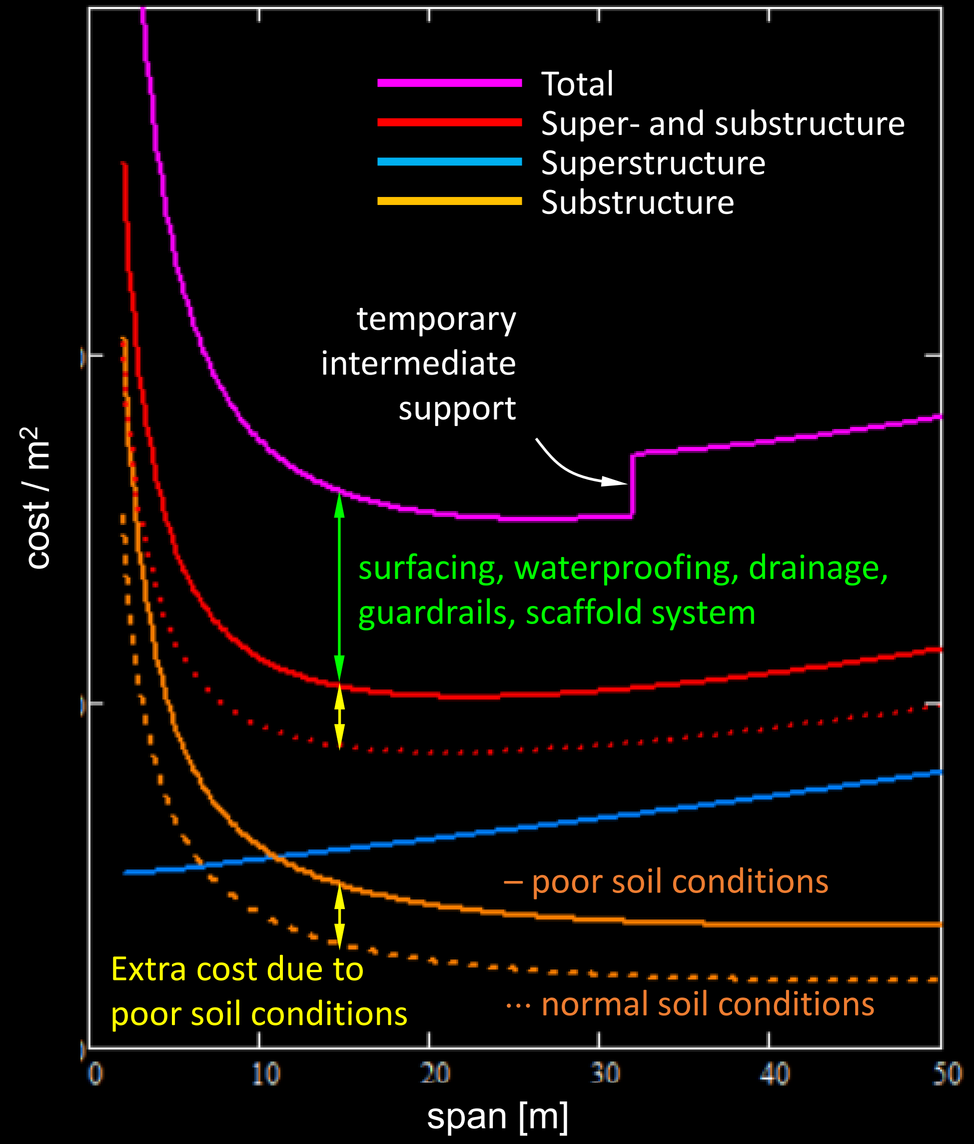
The optimum economic span of a girder bridge is rather insensitive to the soil conditions, see figure:

- Substructure costs are compared for normal (dotted) and poor soil conditions (solid), with 3x higher foundation cost
- The optimum span is only slightly increased by very poor soil conditions

Apart from superstructure and substructure, other components contribute significantly to the total cost, such as

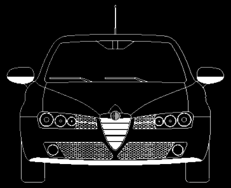
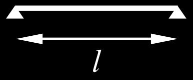
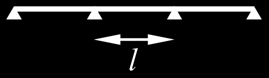
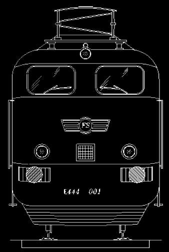
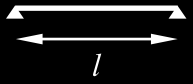
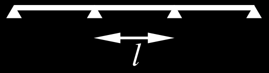
- surfacing, waterproofing and drainage
- guardrails
- scaffold

These are largely independent of the span except for the scaffold costs. The latter decrease slightly with the span, since more scaffolding operations are required at smaller spans if the scaffold is re-used (more spans for same bridge length), up to the point where the span requires a more expensive scaffold system.



Bridge girder – Structural efficiency: Optimum span

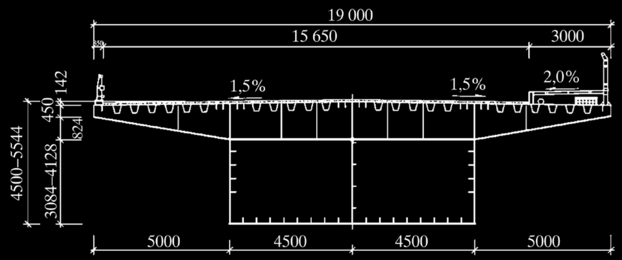
The following spans are generally considered economical for girder bridges:

		Concrete	Steel / Composite
		$l \approx 30 \ldots 35$ m	$l \approx 50 \ldots 60$ m
		$l \approx \ldots 100$ m	$l \approx \ldots 120$ m
		$l \approx 25 \ldots 30$ m	$l \approx 40 \ldots 45$ m
		$l \approx \ldots 70$ m	$l \approx \ldots 100$ m

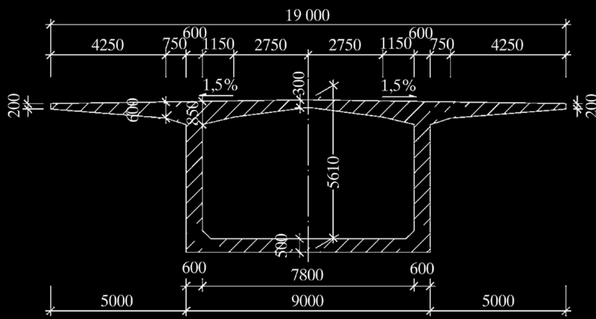
Note that these are no strict or exact limits. Rather, they depend on many site-specific aspects and are indicated here for guidance only. The bridge shown on the right, with much longer spans (max. 330 m), illustrates this.

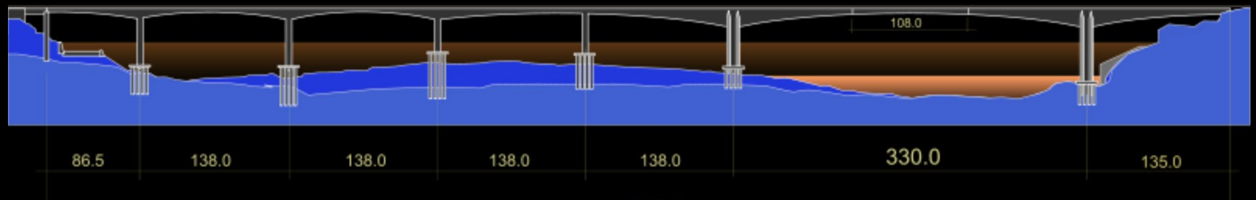
The New Shibanpo Bridge, Chongqing, China, 2006. T. Y. Lin International

Midspan 103 m of main span:



Typical cross-section:





Bridge girder – Structural efficiency: Span ratios

Criteria for the length of end spans:

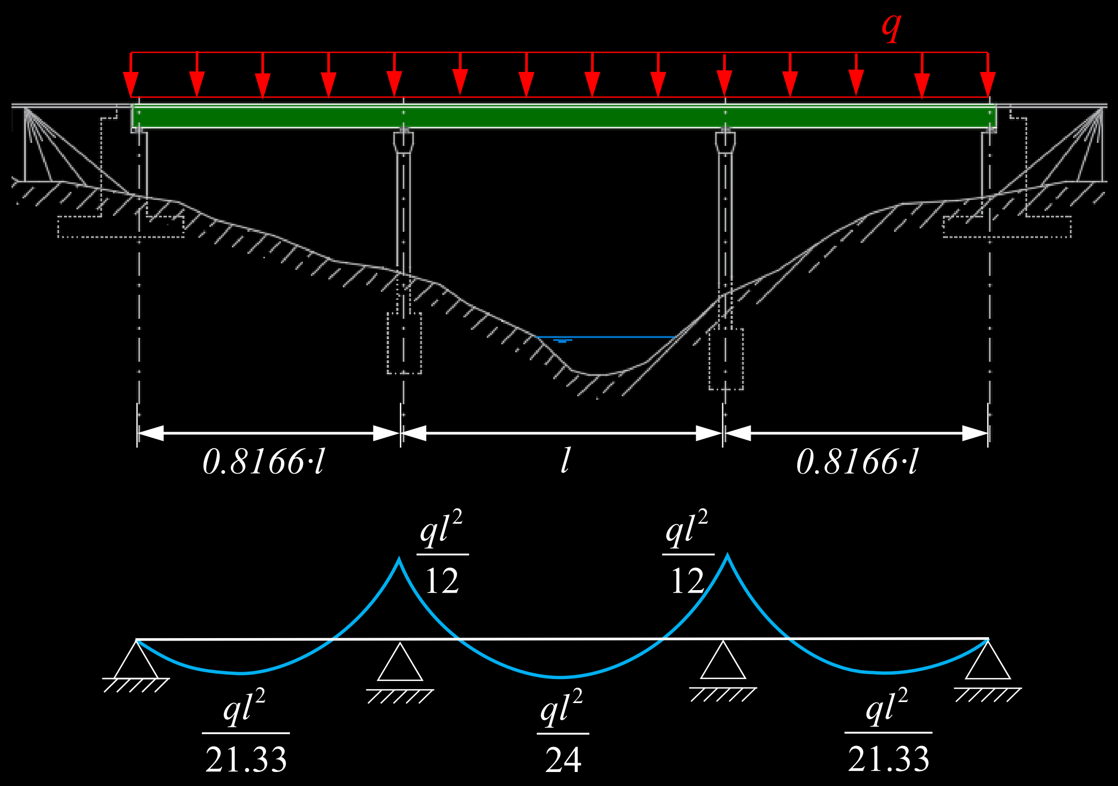
- Ensure similar magnitude of bending moments as in interior spans → $l_{end} \approx (0.70 \ldots 0.85) \cdot l_{int}$ (*)
- Prevent uplift of bearings (no negative support reactions in service conditions)
- If possible, ensure vertical support reactions at the abutments large enough to transfer horizontal forces with standard bearings (avoid separate horizontal bearings)

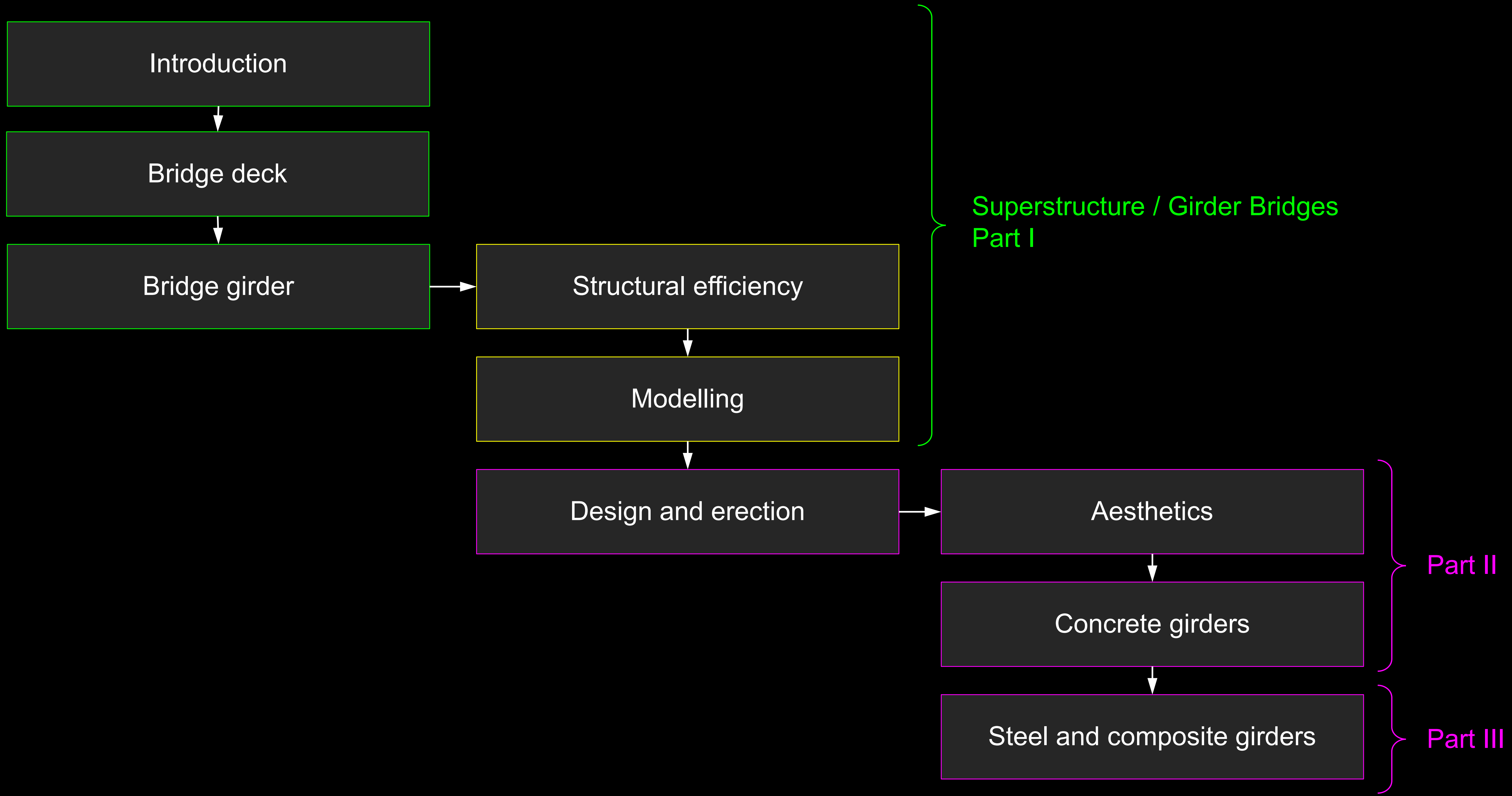
The governing load combination for the minimum support reaction includes a significant contribution from torsion:

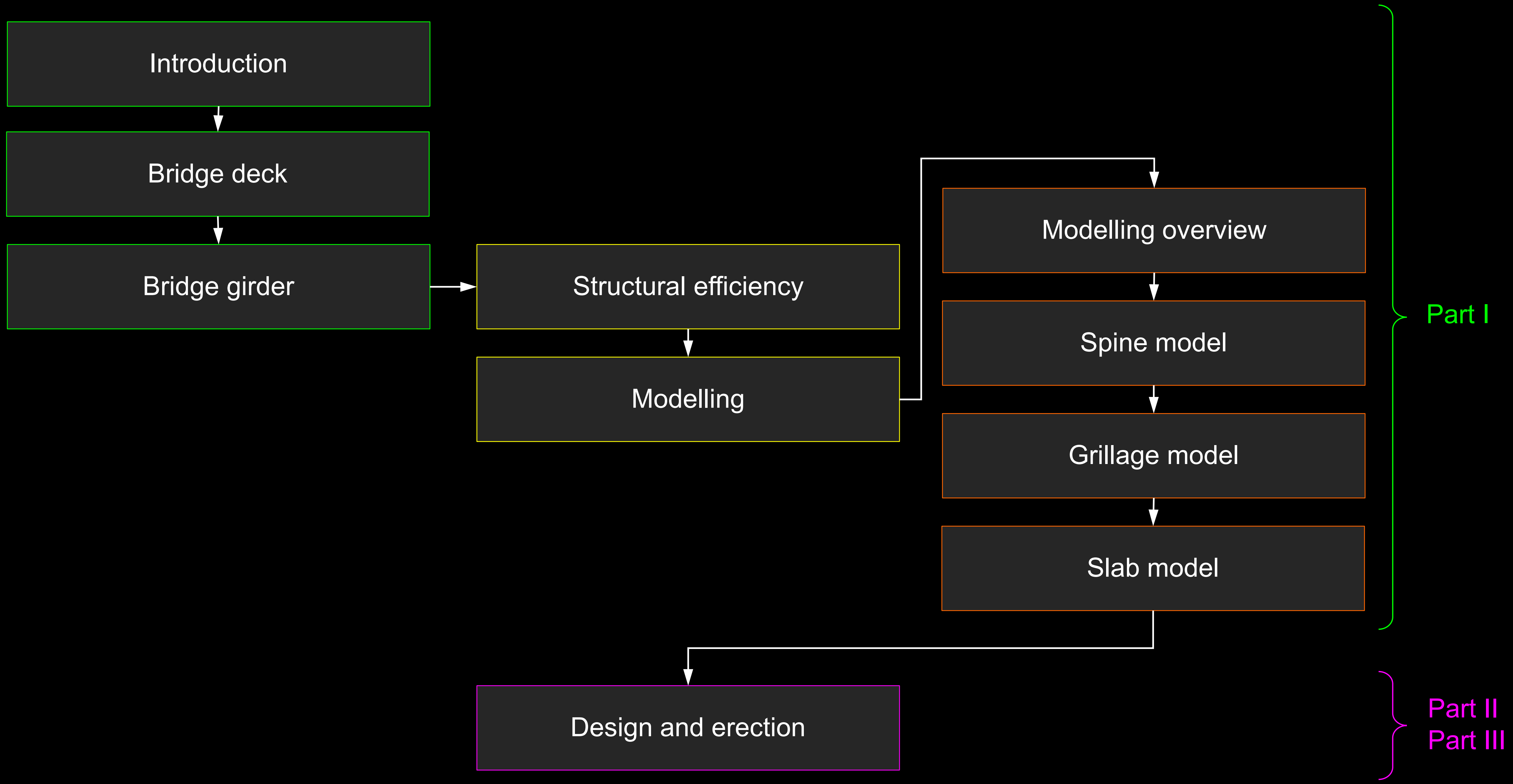
→ The minimum end span to prevent uplift depends on torsional behaviour (no specific value can be given; textbook recommendations often neglect torsion)

→ The transverse spacing of bearings at the abutment should be as large as possible

(*) In a girder with constant EI_y subjected to uniform load, the bending moment over the intermediate supports equals that of an infinite continuous girder if $l_{end} = 0.8166 \cdot l_{int}$.




Introduction
Bridge deck
Bridge girder
Structural efficiency
Modelling
Design and erection
Aesthetics
Concrete girders
Steel and composite girders
Superstructure / Girder Bridges
Part I
Part II
Part III


Introduction
Bridge deck
Bridge girder
Structural efficiency
Modelling
Modelling overview
Spine model
Grillage model
Slab model
Design and erection
Part I
Part II
Part III

Superstructure / Girder bridges

Bridge Girder – Modelling overview

Bridge Girder – Modelling overview: General remarks

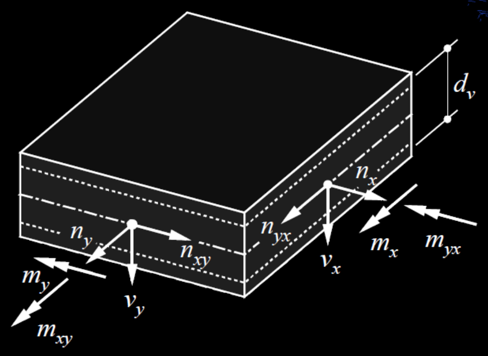
A good model is simple, yet captures the relevant phenomena and enables a safe and efficient design. Hence, a model should be

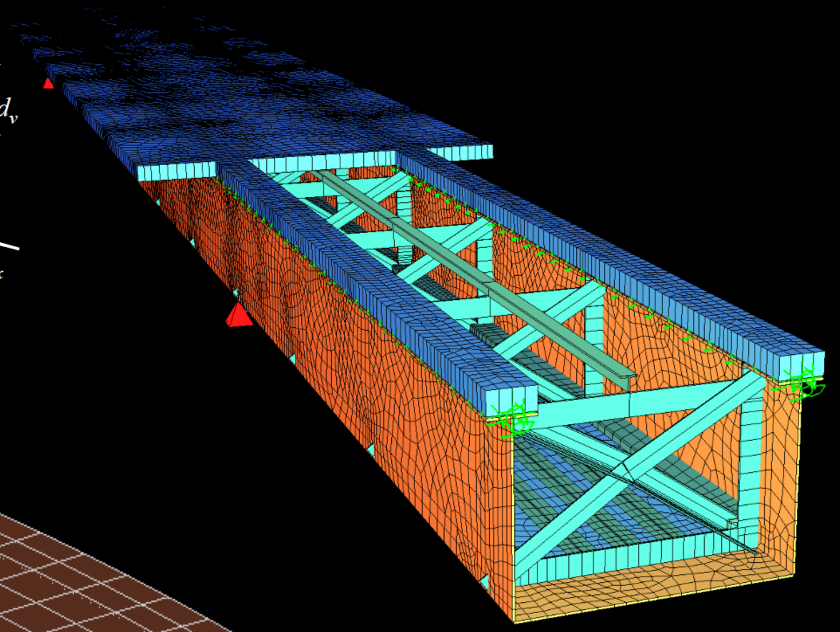
- as simple as possible, but not simpler

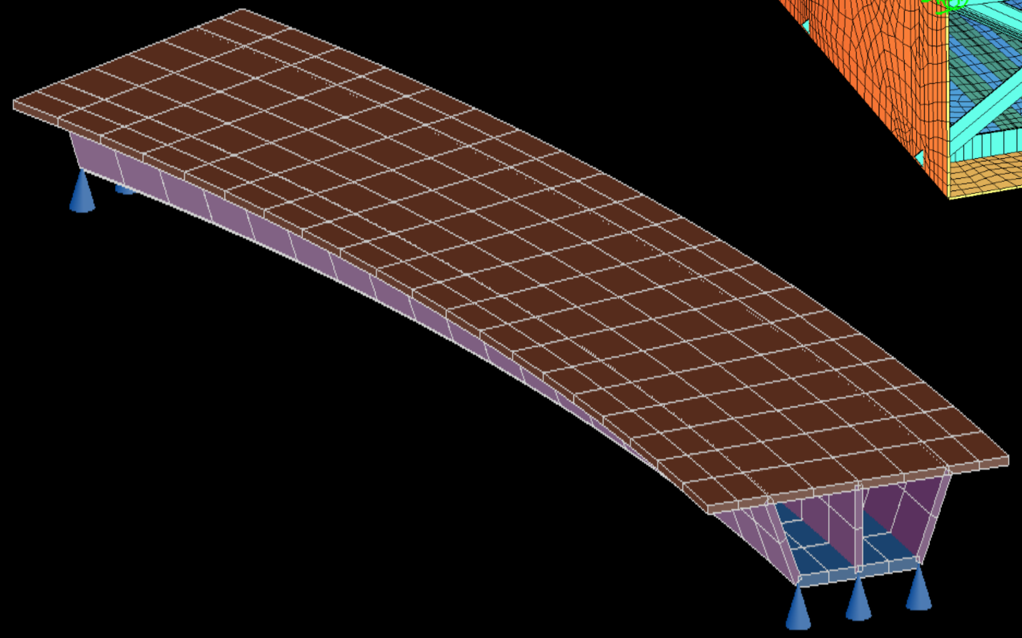
With today's computing power at the hands of engineers, it is tempting to use a more complex model than required.

However, it must be kept in mind that highly complex models may limit the designer's insight into the behaviour ("black box models"). If modelling errors remain undetected, overly complex models lead to worse (or even dangerous) results than simple models, which are inherently approximate but transparent. Hence, keep in mind that

- it is better to be roughly right than exactly wrong





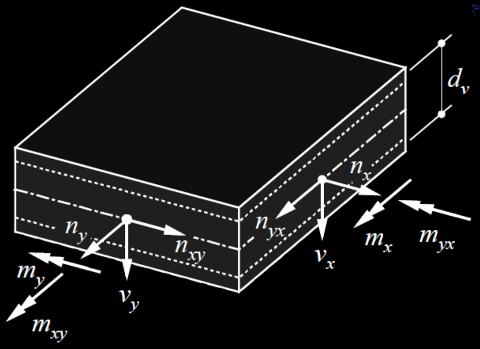


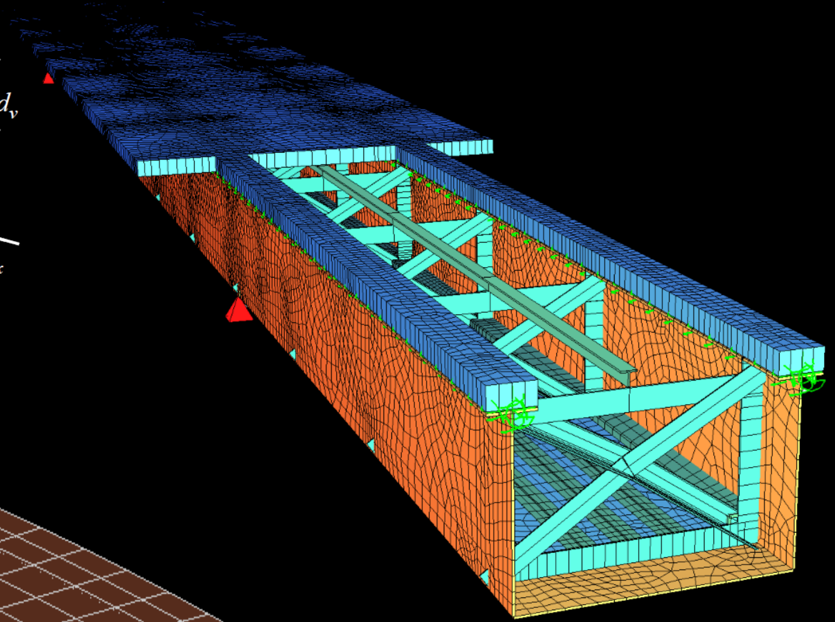
Bridge Girder – Modelling overview: Folded plate models (FE analyses)

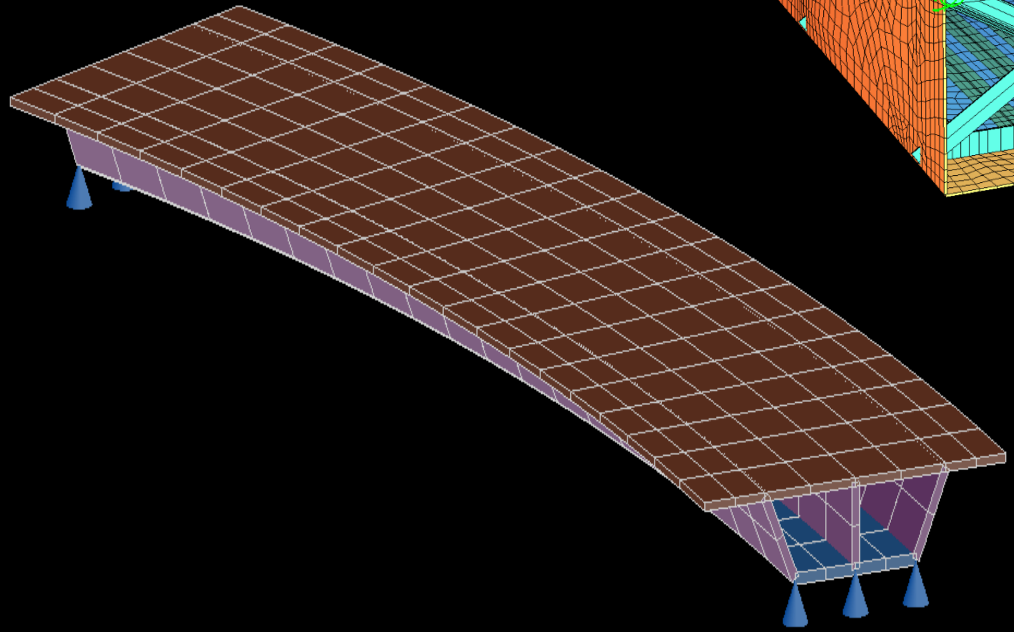
Most bridge girders consist of thin, planar elements. Hence, folded plate models (shells in the case of curved bridges) would be most “realistic”.

In spite of the progress in computational tools, such models are rarely used for design today, for the following reasons:

- highly complex models (8 stress resultants in shells)
  - very time consuming (inefficient design process)
  - lacking transparency, prone to errors
- limited use for design as despite high computational effort
  - linear elastic analysis does not capture the real behaviour (cracking, other nonlinearities)
  - detailing based on output is not straightforward (particularly for concrete elements)







Simpler models are therefore still preferred for design purposes and presented in the lecture:

- spine models (single / line beam model = Stabmodell)
- grillage models (Trägerrostmodell)
- slab models (Plattenmodell)

Bridge Girder – Modelling overview: Simplified models

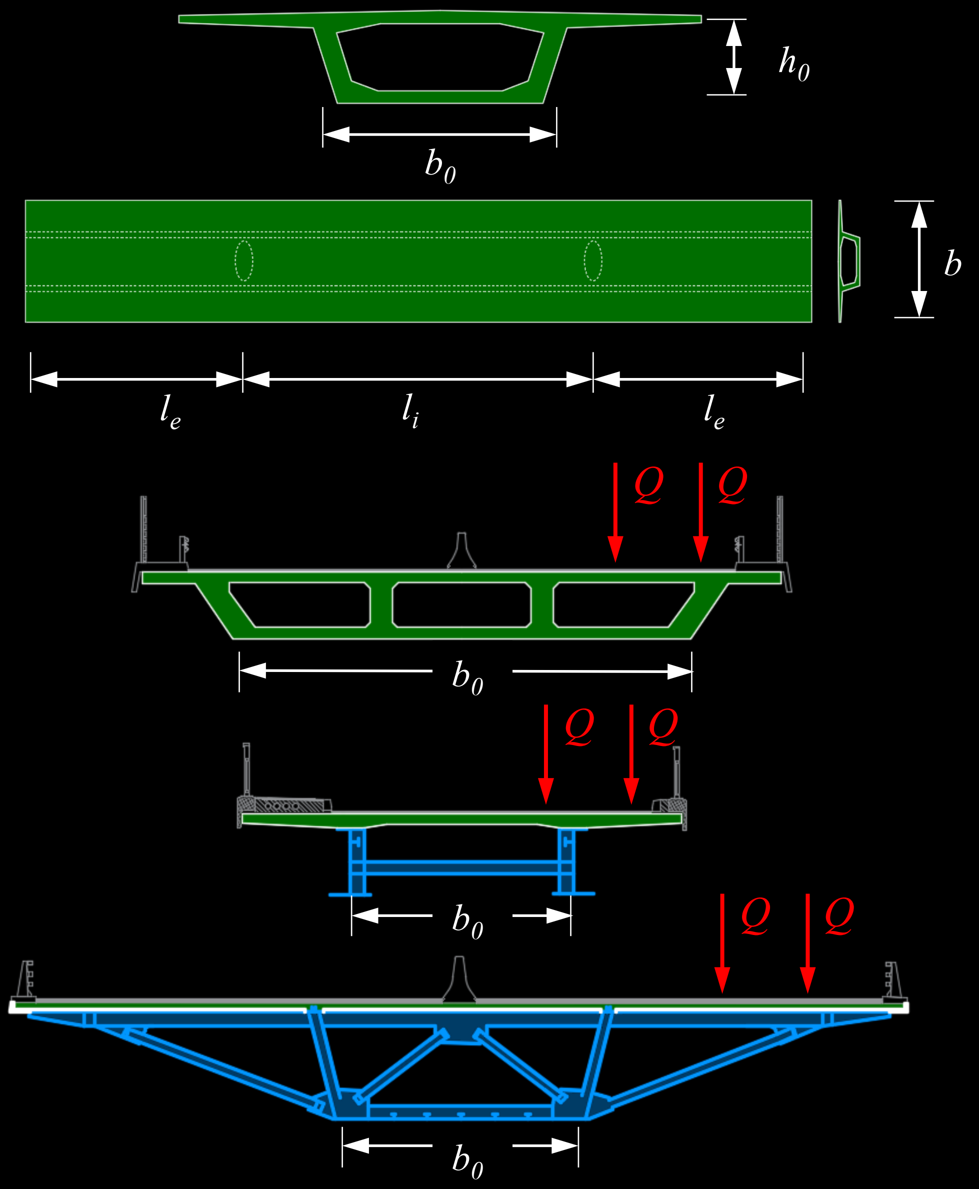
Among the simplified models (spine, grillage, slab), the simplest one that is adequate should be used. If possible, a spine model is therefore chosen.

Whether a spine model can be used depends primarily on the following criteria:

- The ratio between the width b_0 of the girder ($b_0 < b$) and the effective girder span (l_0); a spine model (single beam or line beam) is usually appropriate if

$$l_0 \geq 2\left(b_0 + h_0\right)$$

- The type of cross-section, which defines the behaviour of the girder under eccentric load; a spine model is usually appropriate for box girders



Bridge Girder – Modelling overview: Simplified models

Girders with open or closed cross-section behave fundamentally different in torsion (see *spine model for open cross-sections* for more details, including Factor κ).

Accordingly, different models are adequate:

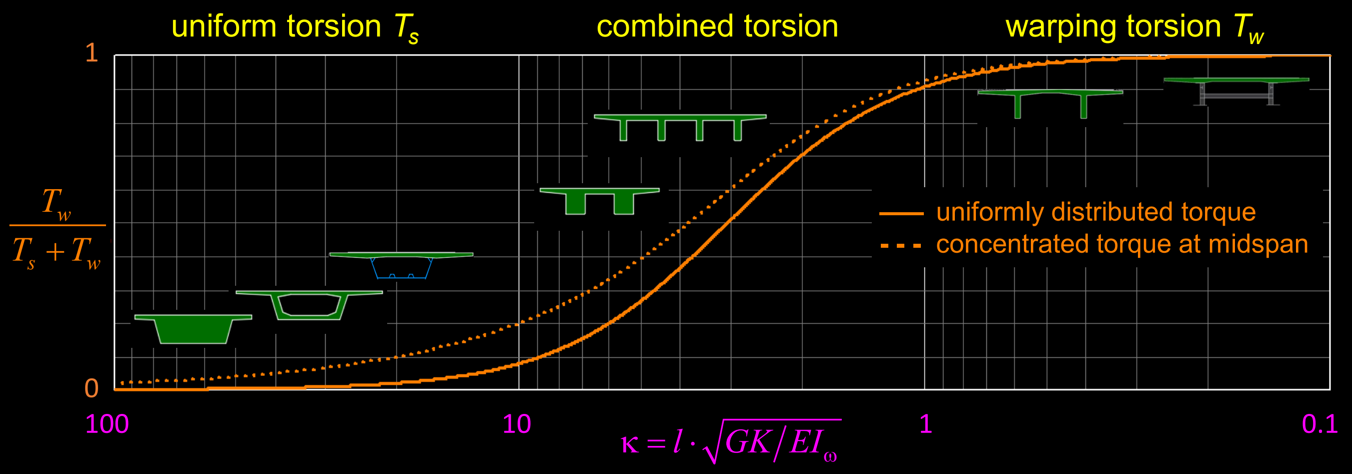
- Uniform torsion T_s prevails in girders with solid, convex cross-section and in box girders since $GK >> EI_\omega / l^2$

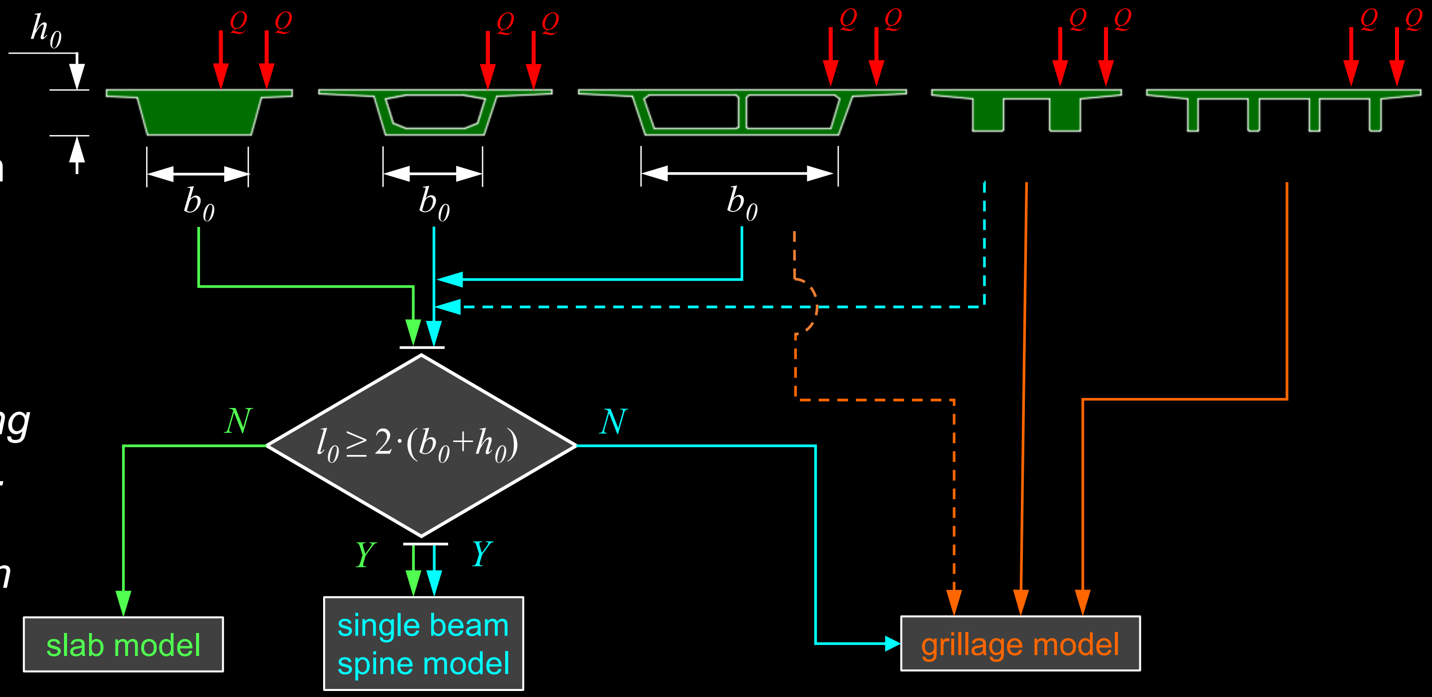
$\rightarrow$ spine model applicable

- Warping torsion T_w ("antisymmetric bending" with corresponding distortions) prevails in girders with an open cross-section since $GK << EI_\omega / l^2$

$\rightarrow$ grillage model appropriate

Note: Warping torsion can be analysed analytically using a spine model as well (see Marti, Theory of Structures). However, this is tedious for general cross-sections and considering many load-cases, and yields no information on the transverse behaviour.





Superstructure / Girder bridges

Bridge Girder – Spine model – Global analysis

(Einstabmodell, Längsrichtung)

Spine model – Global analysis: General remarks

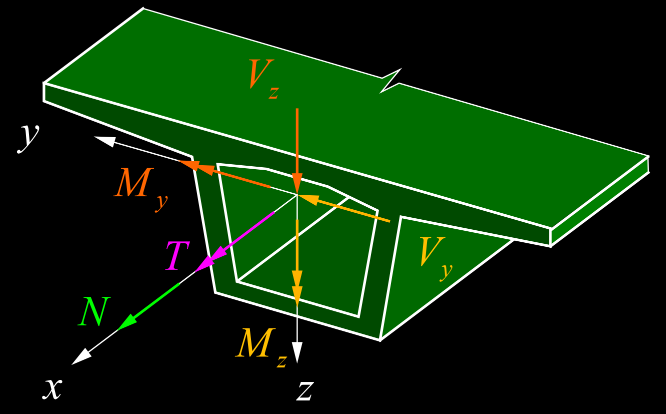
In a *spine model* (also referred to as *single beam* or *line beam model*), the girder = spine has to resist:

- Bending moments M_y and shear forces V_z caused by gravity loads (self-weight, traffic loads, …)
- Bending moments M_z and shear forces V_y caused by transverse horizontal loads (wind, centrifugal forces, earthquake loads)
- Torsional moments T caused by the eccentricities of the applied loads (with respect to the girder axis or the shear centre), as well as by curvatures in plan.
- Axial forces N are usually small in girder bridges, even if integral abutments are used.

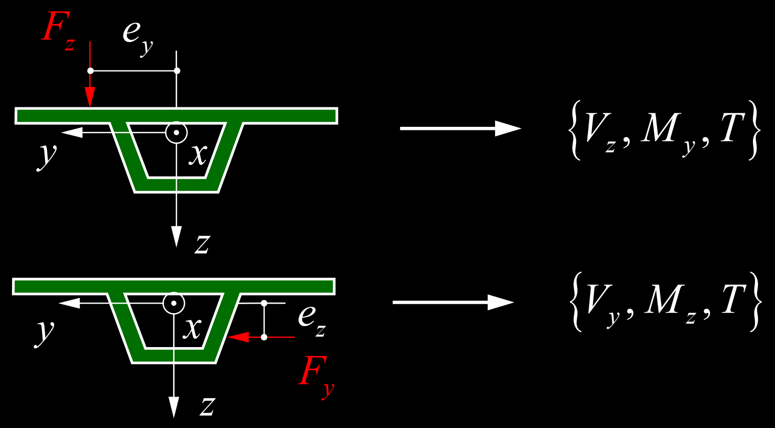
In many cases, gravity loads and the corresponding internal actions V_z, M_y and T, govern the design.

Torsion is treated much less in other courses than shear and bending, and using a spine model requires special considerations regarding the introduction of torques.

Therefore, torsion and load introduction are treated in this lecture in more detail, whereas it is assumed that students are proficient in the structural analysis and the design for shear and bending.



Internal actions (stress resultants) in a single beam model



Spine model – Global analysis: General remarks

In a general cross-section with arbitrary material behaviour, internal actions (stress resultants) and deformations are related by integration or iteration (see e.g. *Stahlbeton I*).

The analysis is greatly simplified by the usual assumption of linear elastic behaviour using

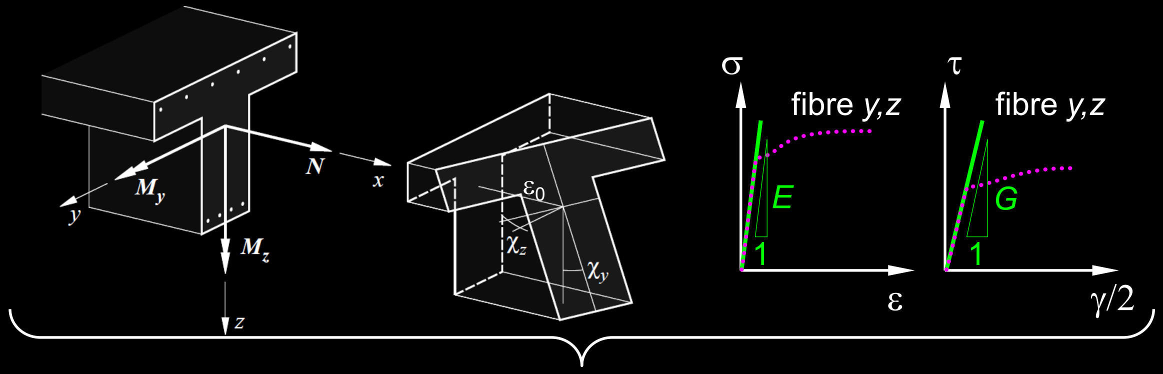
- axial stiffness EA
- bending stiffnesses EI_y and EI_z
- torsional stiffness GK (= GI_p for circular cross-sections)

Shear deformations are usually neglected ($GA^* \rightarrow \infty$). However, torsional deformations are taken into account (see notes).

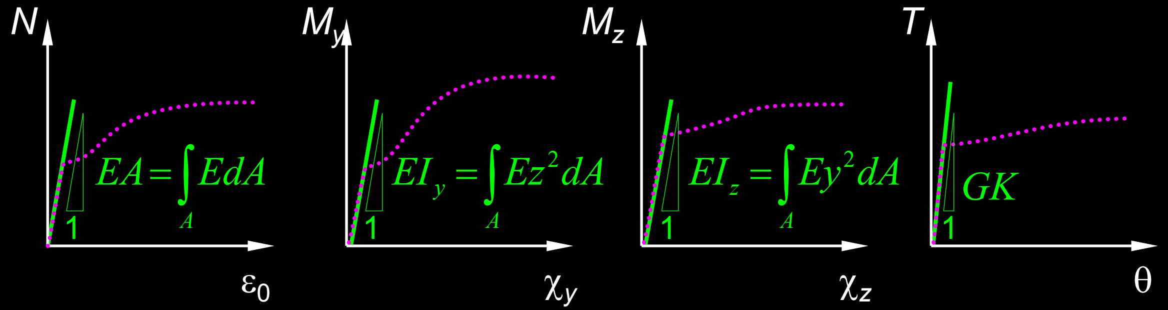
While effective flange widths are often accounted for, further simplifications are usually adopted in the structural analysis (but not in the design of the members!):

- use of uncracked stiffnesses EI^{I} for concrete members (cracking could be considered by the cracked stiffness EI^{II})
- consideration of full section of slender steel plates (webs)

The determination of axial and bending stiffnesses is straightforward (see formulas in figure). The torsional stiffness GK is treated later in this lecture in more detail.



Cross-section: «real» behaviour / linear elastic idealisation



$$\left\{ \begin{aligned} N &= \int_A \sigma_x dA \\ M_y &= \int_A \sigma_x z dA \\ M_z &= \int_A \sigma_x y dA \\ T &= \int_A \left(\tau_{zx} y - \tau_{yx} z \right) dA \end{aligned} \right\} \underset{\text{integrate}}{\overset{\text{iterate}}{\rightleftarrows}} \left\{ \begin{matrix} \varepsilon_0 \\ \chi_y \\ \chi_z \\ \theta \end{matrix} \right\} \qquad \left\{ \begin{aligned} N &= EA \cdot \varepsilon_0 \\ M_y &= EI_y \cdot \chi_y \\ M_z &= EI_y \cdot \chi_z \\ T &= GK \cdot \theta \end{aligned} \right\}$$

Spine model – Global analysis: Decomposition of eccentric loads

For the analysis in the spine model, eccentric loads can be substituted by a statically equivalent combination of

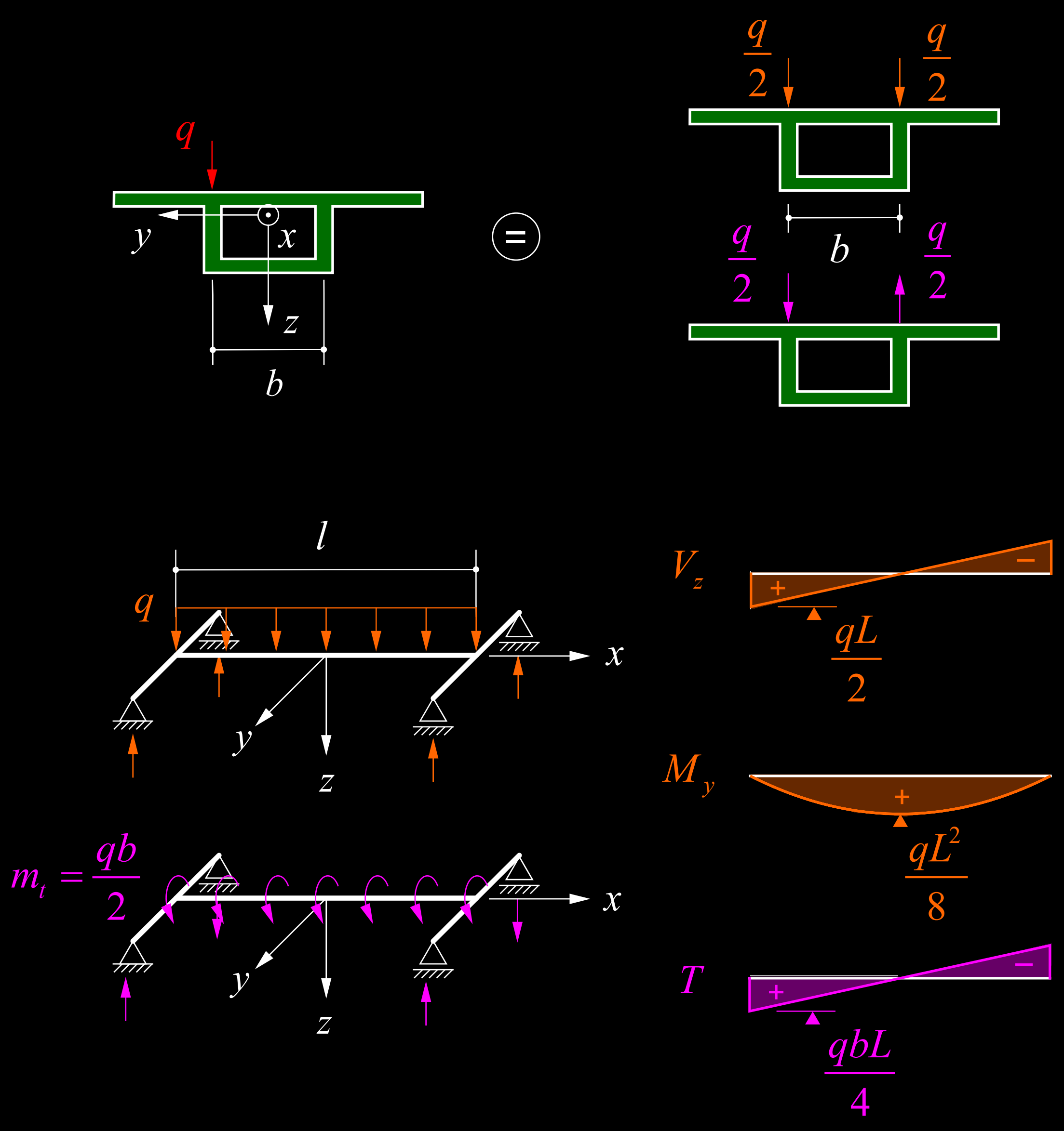
- symmetrical load causing (acting in the girder axis) } shear forces, bending moments

and

- torque or force couple causing (“anti-symmetrical load”) } torsional moments

Bending and torsion can then be analysed separately, and the resulting forces (e.g. shear forces per element) superimposed for dimensioning. This is illustrated here for vertical loads with horizontal eccentricity, but equally applies to vertically eccentric transverse horizontal loads.

Generally, eccentric loads do not act in the axis of a web. However, the decomposition in a symmetrical load and a torque is also possible. This is illustrated in the following slides for a box girder, but also applies to solid and open cross-sections (although local load introduction is different, see behind).



Spine model – Global analysis: Decomposition of eccentric loads

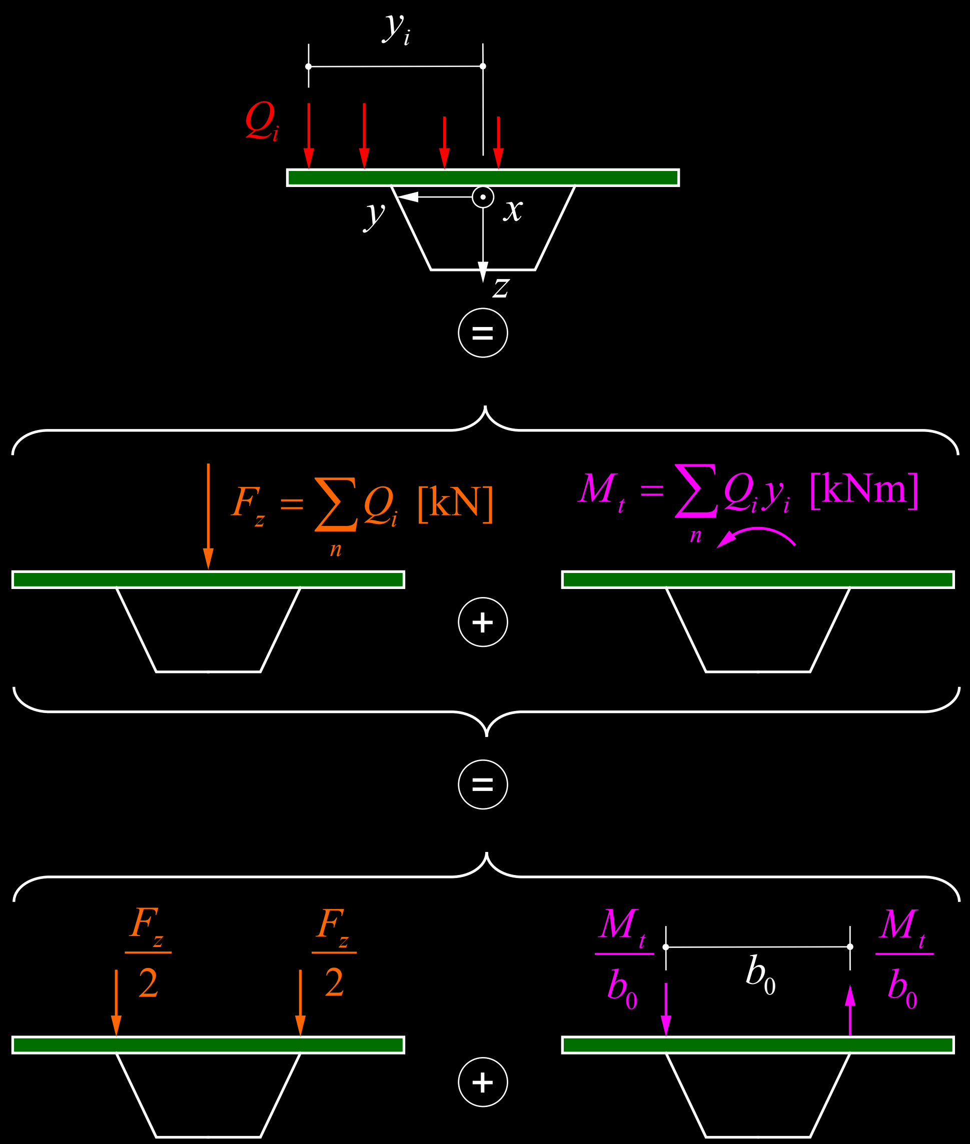
Eccentric concentrated loads [kN] are usually due to traffic loads (concentrated loads representing vehicle axle loads).

They are substituted by a statically equivalent combination of

centric concentrated load [kN] and concentrated torque [kNm] (used for global analysis)

or

two equal concentrated vertical forces and a concentrated force couple, where the forces [kN] act in the axes of the webs (used for load introduction analysis)



Spine model – Global analysis: Decomposition of eccentric loads

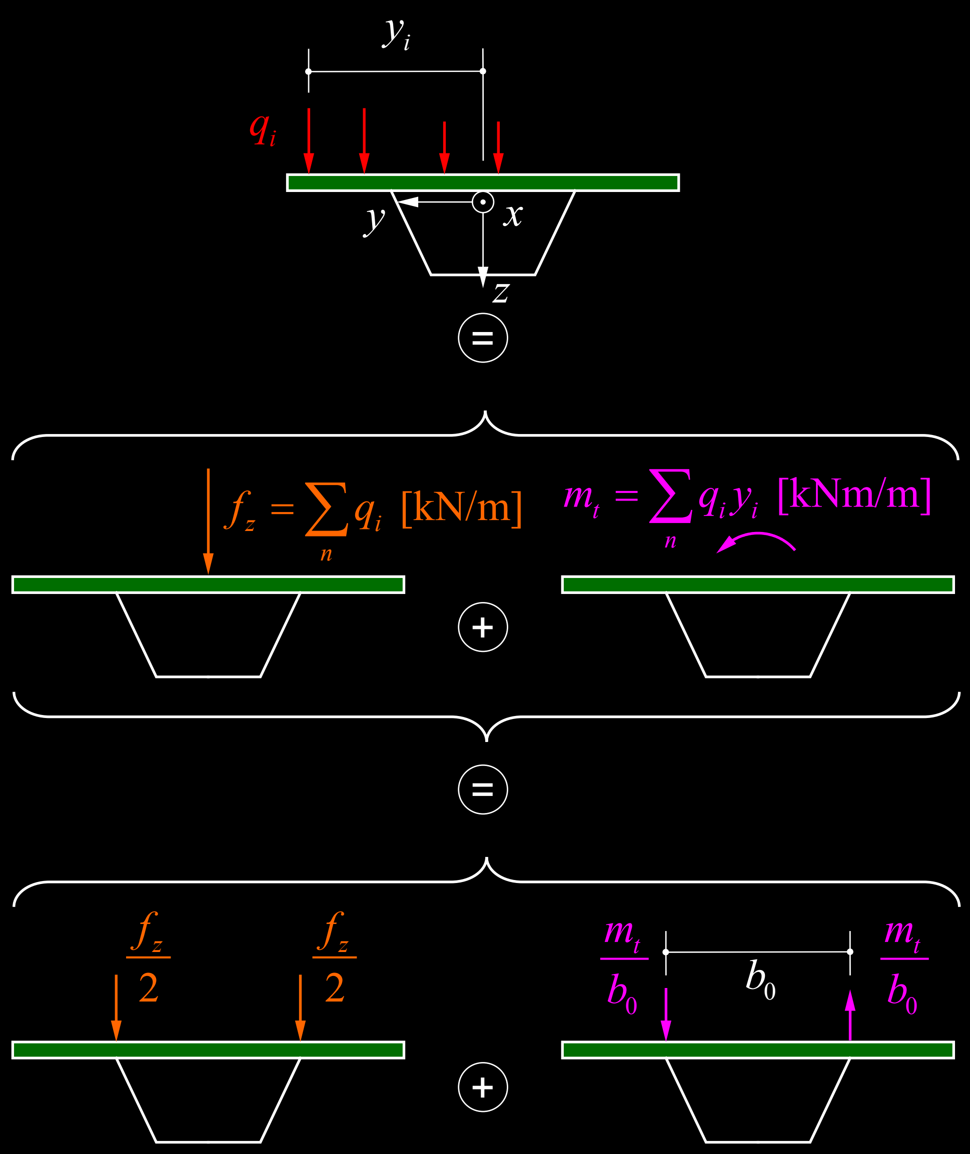
Eccentric line loads [kNm^{-1}] may be due to traffic loads (e.g. line load of ballastless track rail) or superimposed dead loads (e.g. crash barriers).

They are substituted by a statically equivalent combination (obtained by summation) of

centric line load [kNm^{-1}] and
distributed torque [kN]
(used for global analysis)

or

two equal line loads and a
line load couple, where the forces [kNm^{-1}] act in the axes of the webs
(used for load introduction analysis)



Spine model – Global analysis: Decomposition of eccentric loads

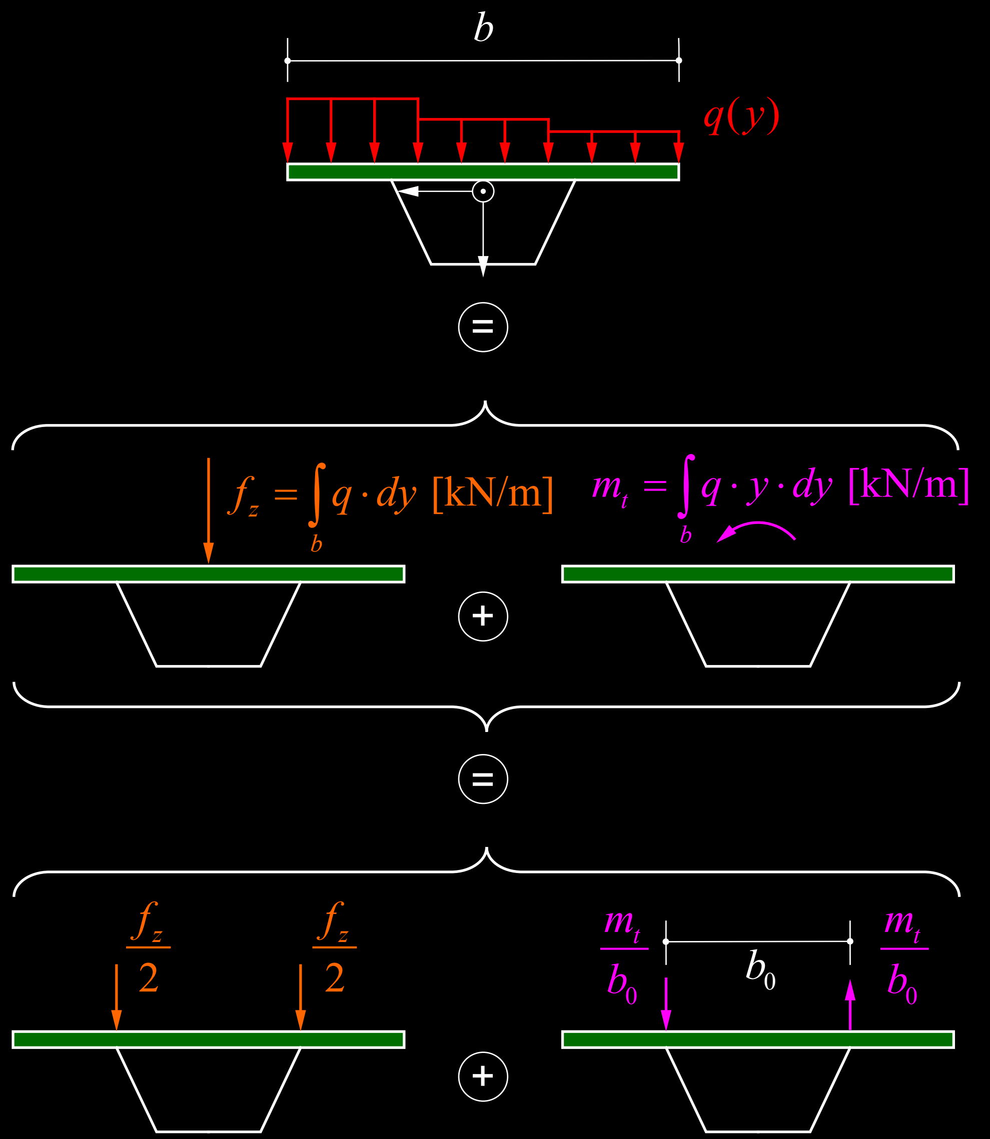
Distributed (surface) loads [kNm^{-2}] are due to self-weight, superimposed dead loads (e.g. surfacing), or distributed traffic loads.

They are substituted by a statically equivalent combination (obtained by integration) of

centric line load [kNm^{-1}] and
distributed torque [kN]
(used for global analysis)

or

two equal line loads and a
line load couple, where the forces [kNm^{-1}] act in the axes of the webs
(used for load introduction analysis)



Spine model – Global analysis: Torsion span

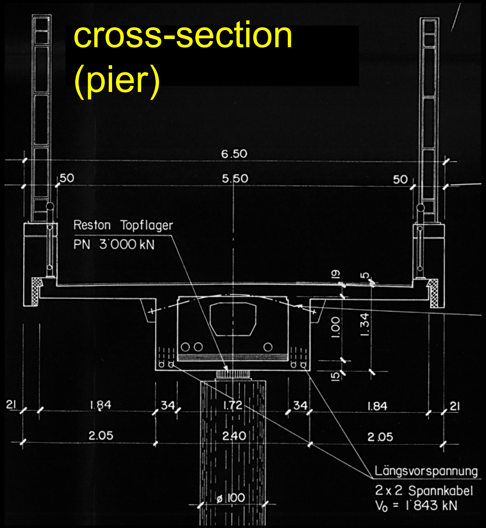
The torsional support system usually differs from the static system for vertical loads:

- Torsional fixity must be provided at the abutments (avoid torsional rotations of the girder ends and associated vertical offsets), with hardly any exception possible.
- Intermediate supports (piers) need not always provide torsional fixity. In particular, box girders have a high torsional stiffness, enabling large torsional spans without excessive twist.

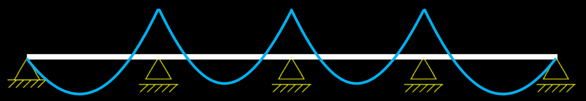
Accordingly, the torsion span = distance between supports impeding torsional rotation does not necessarily correspond to the shear span, e.g.

- Piers with torsional fixity → torsion span = shear span
- Piers as point supports → torsion span = bridge length (e.g. single articulated bearing in girder axis)

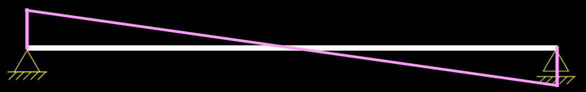
Single supports without torsional fixity enable slender piers, which may be advantageous, see example (less obstruction of river, elegance); main span 31.5 m, torsion span 115 m.



vertical support system and bending moments (uniform load)



Torsional support system and torsional moments (uniform torque)



Aarebrücke Zuchwil-Solothurn, Ingenieurbüro Th. Müller, 1986

Spine model – Global analysis: Torsion caused by curvature in plan

Torsion is not only caused by eccentric loads, but also by curvature of the girder in plan. M_y and T in curved girders are coupled → 2nd order inhomogeneous differential equation.

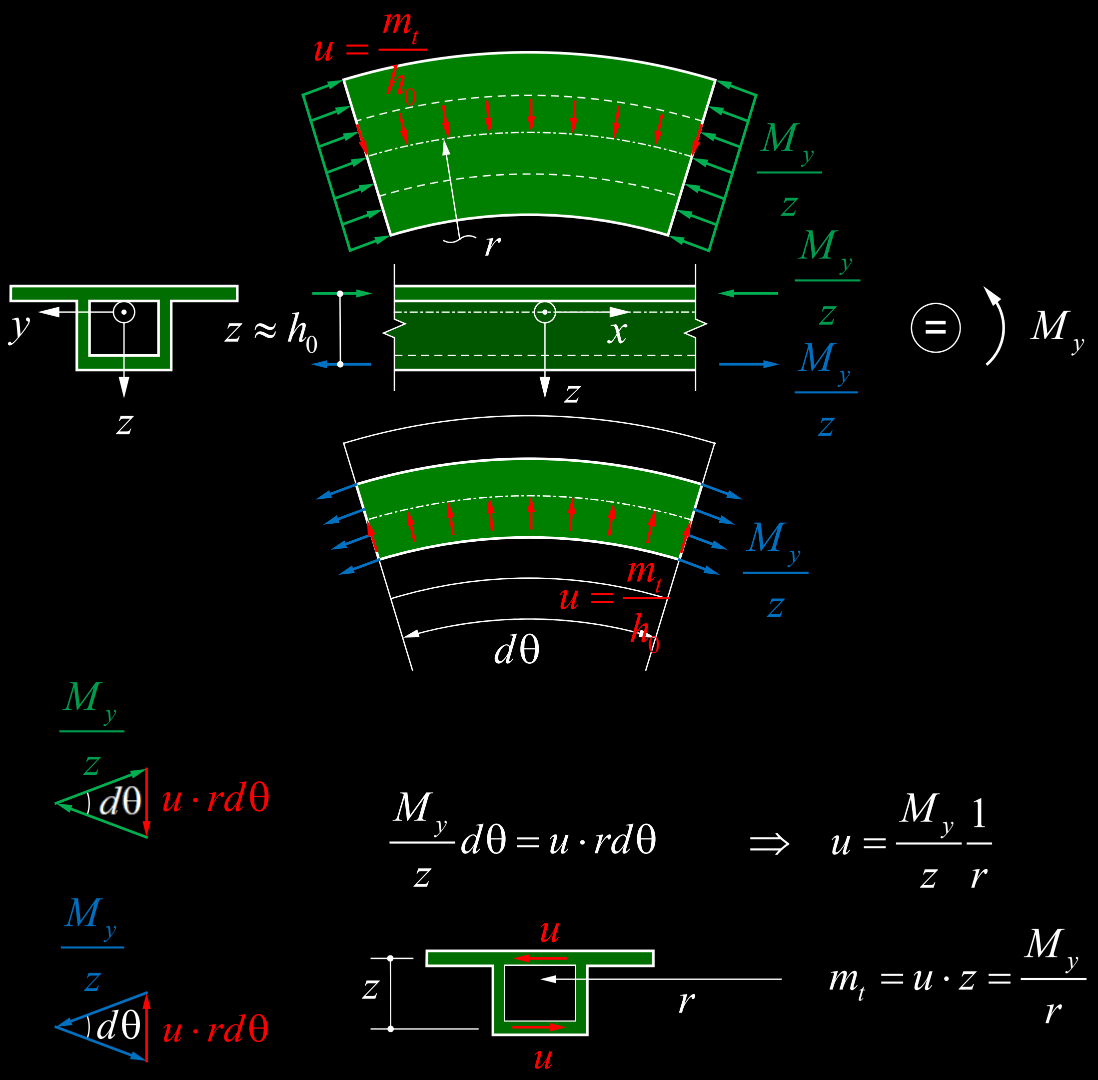
For a more direct understanding of the behaviour one may determine M_y for the straight girder (developed length) and consider the torques due to the chord forces deviation:

- M_y is resisted by chord forces $\pm M_y/z$, with lever arm z
- chords are curved → deviation forces $u = \pm M_y/(r \cdot z)$

→ distributed torque $$m_t = \frac{M_y}{r}$$

applied to the girder by a horizontal line load couple with lever arm $z \approx h_0$ $$\pm\frac{M_y}{z \cdot r} \approx \pm\frac{M_y}{h_0 \cdot r}$$

The girder has to transfer the distributed torque (→ torsion). The cross-section (or intermediate diaphragms) must introduce the horizontal line load couple, i.e., convert it to uniform torsion (see behind and *curved bridges*).



Spine model – Global analysis: Torsion caused by skew supports

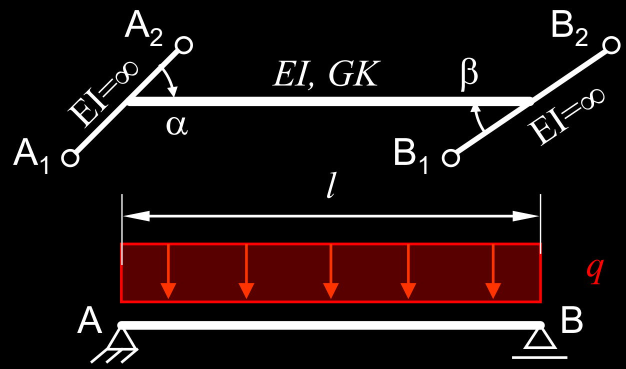
Torsion is also caused by skew supports, since eccentric vertical support reactions are applied.

If stiff diaphragms and articulated bearings are provided, the behaviour can be analysed using models as shown on the right for a simply supported girder:

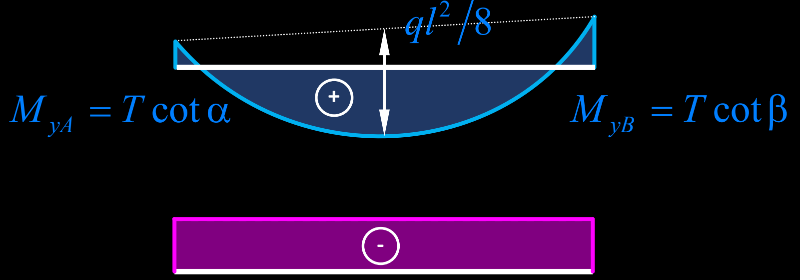
- diaphragms rigid ($EI=\infty$), simply supported (no torsion in diaphragms, can rotate around their axis!)
- determine internal actions analytically or using force method (see Stahlbeton I) or frame analysis software
- skew supports provide a partial fixity, where M_y and T are coupled geometrically
- supports on side of acute angles (A2, B1) receive higher reactions than those on side of obtuse angles (A1, B2)

The girder has to transfer the concentrated torque (→ torsion). Support diaphragms introduce the concentrated vertical force couple applied by the support reactions, i.e., convert it to uniform torsion (see behind and *skew bridges*).

Static system and loading (plan):



Internal actions (elevation):



$$T = -\frac{ql^2}{8} \cdot \frac{\cot\alpha + \cot\beta}{\cot^2\alpha + \cot\alpha\cot\beta + \cot^2\beta + \frac{3EI}{GK}}$$

Spine model – Global analysis: Torsion in box girders (stiffness)

The torsional stiffness for thin-walled, homogeneous hollow cross-sections (steel “a” or uncracked concrete “c”) is

$$GK = \frac{4A_0^{\,2}G}{\oint \frac{ds}{t}} = \frac{4A_0^{\,2}G}{\sum \frac{l_i}{t_i}} \qquad \left(G = \frac{E}{2\cdot(1+\nu)} \right)$$

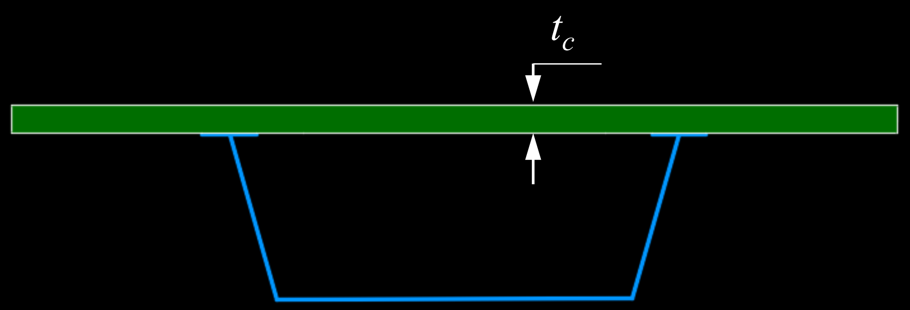
In composite cross-sections, using the steel as reference material (E_a), accordingly

$$GK = \frac{4A_0^{\,2}G_a}{\sum \frac{n_i \cdot l_i}{t_i}} \qquad \left(G_a = \frac{E_a}{2\cdot(1+\nu)}, n_i = \frac{E_a}{E_i} \right)$$

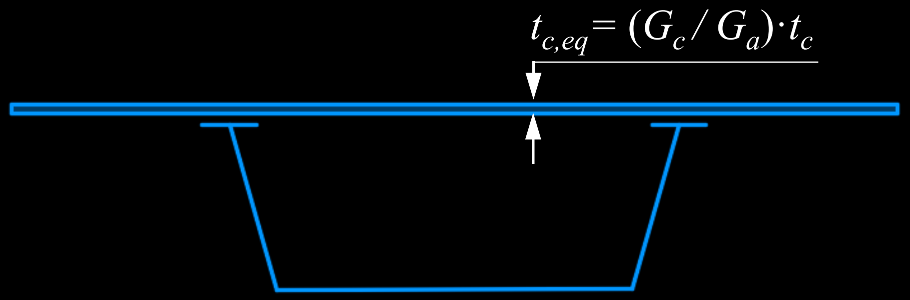
For cracked concrete, the determination of GK is more complicated. For a concrete box girder with constant wall thickness, having a uniformly distributed stirrup reinforcement ρ_w and longitudinal reinforcement ρ_l:

$$GK^{II} = \frac{4A_0^{\,2}E_s}{\frac{\cot^2\alpha}{\rho_l} + \frac{\tan^2\alpha}{\rho_w} + n\cdot(\tan\alpha + \cot\alpha)^2} \frac{t}{\sum l_i} \qquad \left(\tan\alpha = \sqrt[4]{\frac{\rho_l^{-1} + n}{\rho_w^{-1} + n}} \right)$$

see lecture notes Stahlbeton I (E_s = stiffness of reinforcement).



real section



equivalent section

Spine model – Global analysis: Torsion in box girders (stiffness)

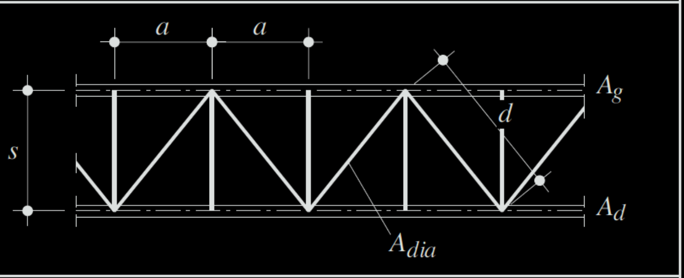
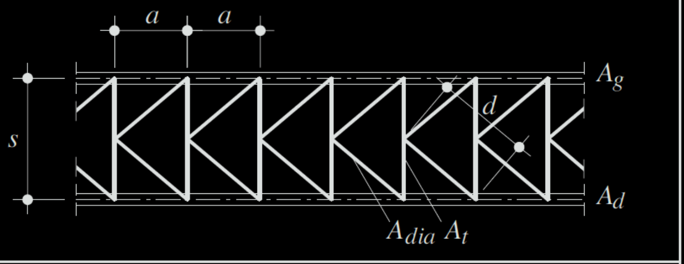
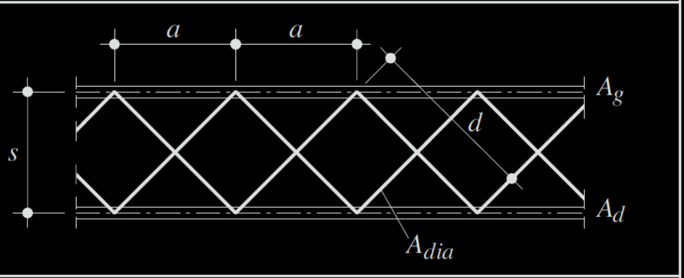
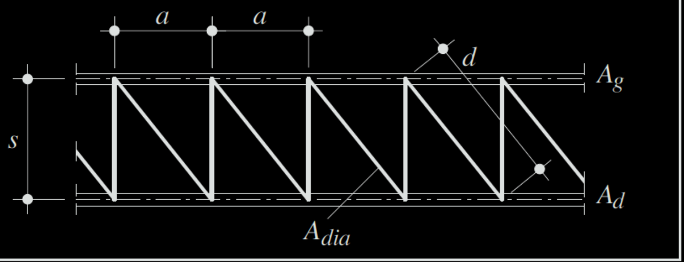
If the bottom slab is replaced by trusses, being part of a closed cross-section, the torsional stiffness may be calculated using an effective thickness.

The corresponding values of the equivalent thicknesses may be obtained e.g. using the work method.

The table on the right gives values for usual truss typologies (from Lebet and Hirt, 2013).

Trussed webs may be treated similarly.

Equivalent thicknesses of other truss layouts are obtained by applying the virtual work equation (for a unit shear deformation) and equating the deformation of the solid plate to that of the truss.

t_{eq}	Plan Bracing Geometry
$\frac{E}{G}\,\dfrac{as}{\dfrac{d^3}{A_{dia}}+\dfrac{a^3}{3}\left(\dfrac{1}{A_g}+\dfrac{1}{A_d}\right)}$	
$\frac{E}{G}\,\dfrac{as}{\dfrac{2d^3}{A_{dia}}+\dfrac{s^3}{4A_t}+\dfrac{a^3}{12}\left(\dfrac{1}{A_g}+\dfrac{1}{A_d}\right)}$	
$\frac{E}{G}\,\dfrac{as}{\dfrac{d^3}{2A_{dia}}+\dfrac{a^3}{12}\left(\dfrac{1}{A_g}+\dfrac{1}{A_d}\right)}$	
$\frac{E}{G}\,\dfrac{as}{\dfrac{d^3}{A_{dia}}+\dfrac{s^3}{A_t}+\dfrac{a^3}{12}\left(\dfrac{1}{A_g}+\dfrac{1}{A_d}\right)}$	

Spine model – Global analysis: Torsion in box girders (shear flow)

Box girders can be treated as thin-walled hollow cross sections. Torsional moments T are primarily resisted by uniform torsion ("St.-Venant torsion"), i.e., a circumferential shear flow of constant magnitude $\tau \cdot t$ (Bredt):

$$\tau \cdot t = \frac{T}{2A_0} \quad \text{with } A_0 = b_0 \cdot h_0 \quad \left(\tau \cdot t = \tau_i \cdot t_i \forall i\right)$$

→ shear force per element of the cross-section, with thickness t_i and length l_i: $V_i = \tau \cdot t \cdot l_i$

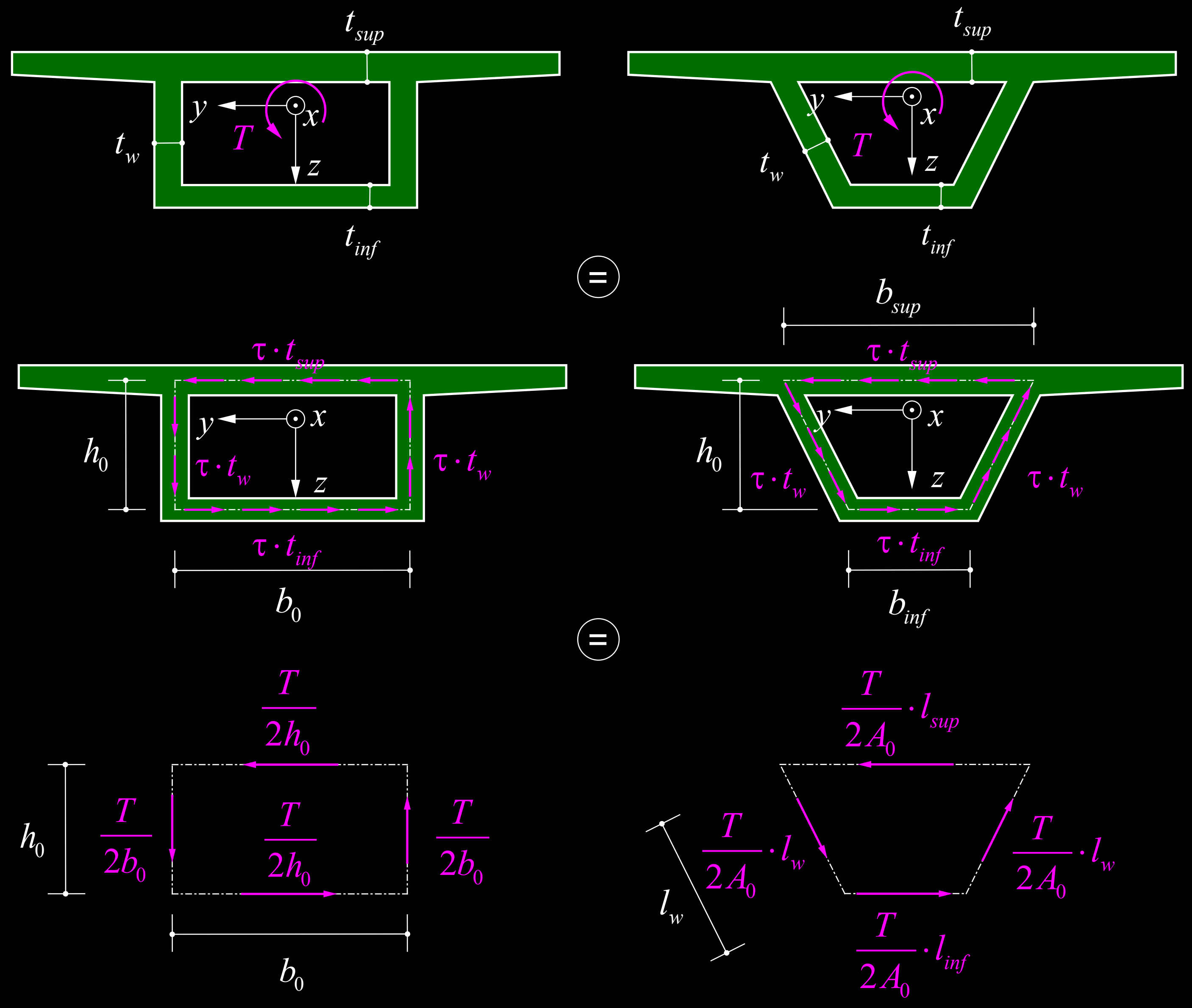
→ shear forces in webs and top / bottom slab of an orthogonal box girder:

$$V_w = \pm \tau \cdot t \cdot h_0 = \pm \frac{T}{2b_0} \quad V_{\text{sup,inf}} = \pm \tau \cdot t \cdot b_0 = \pm \frac{T}{2h_0}$$

→ ditto, for box girder with inclined webs:

$$\tau \cdot t = \frac{T}{2A_0} \quad \text{with } A_0 = \frac{b_{\text{sup}} + b_{\text{inf}}}{2} h_0$$

$$V_i = \tau \cdot t \cdot l_i \quad \text{with } l_w = \sqrt{h_0^2 + \left(\frac{b_{\text{sup}} - b_{\text{inf}}}{2}\right)^2}$$



Superstructure / Girder bridges

Bridge Girder – Spine model – Transverse analysis

(Einstabmodell, Querrichtung)

Spine model – Transverse analysis: Limitations of spine model

In the spine model, the girder is idealised as a beam:
→ results of the global analysis are the internal actions
= stress-resultants acting on the entire cross-section.

In reality, the girder is not a beam that merely transfers loads applied to its axis longitudinally. Rather

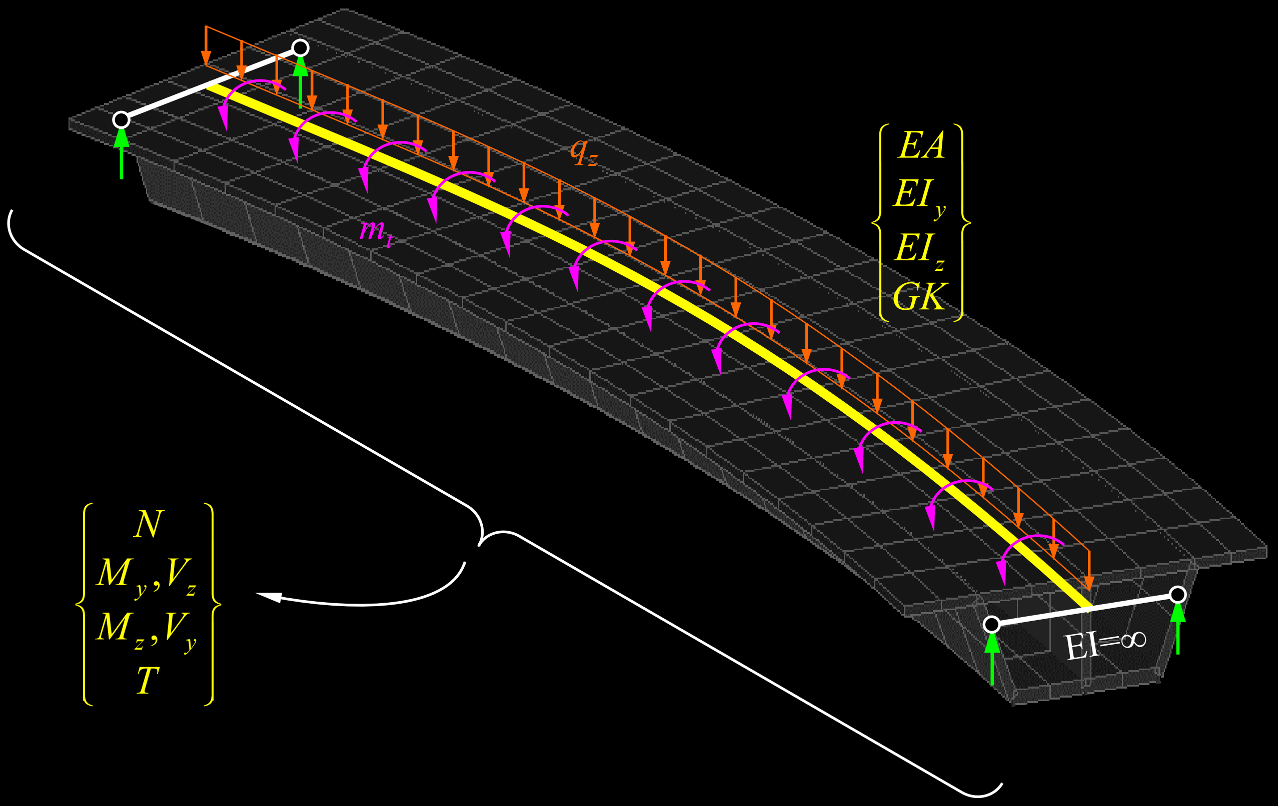
- loads also need to be carried in transverse direction
- The cross-section is not rigid but may be distorted

The spine model does not yield direct information on this transverse behaviour, particularly regarding:

- local bending of the deck
- introduction of torques
- warping torsion

Hence, these effects need to be investigated separately. This is feasible with reasonable effort and accuracy for box girders and solid cross-sections, see following slides.

For girders with open cross-sections, this does not apply, and a spine model is therefore usually inappropriate (see *spine model for open cross-sections*).



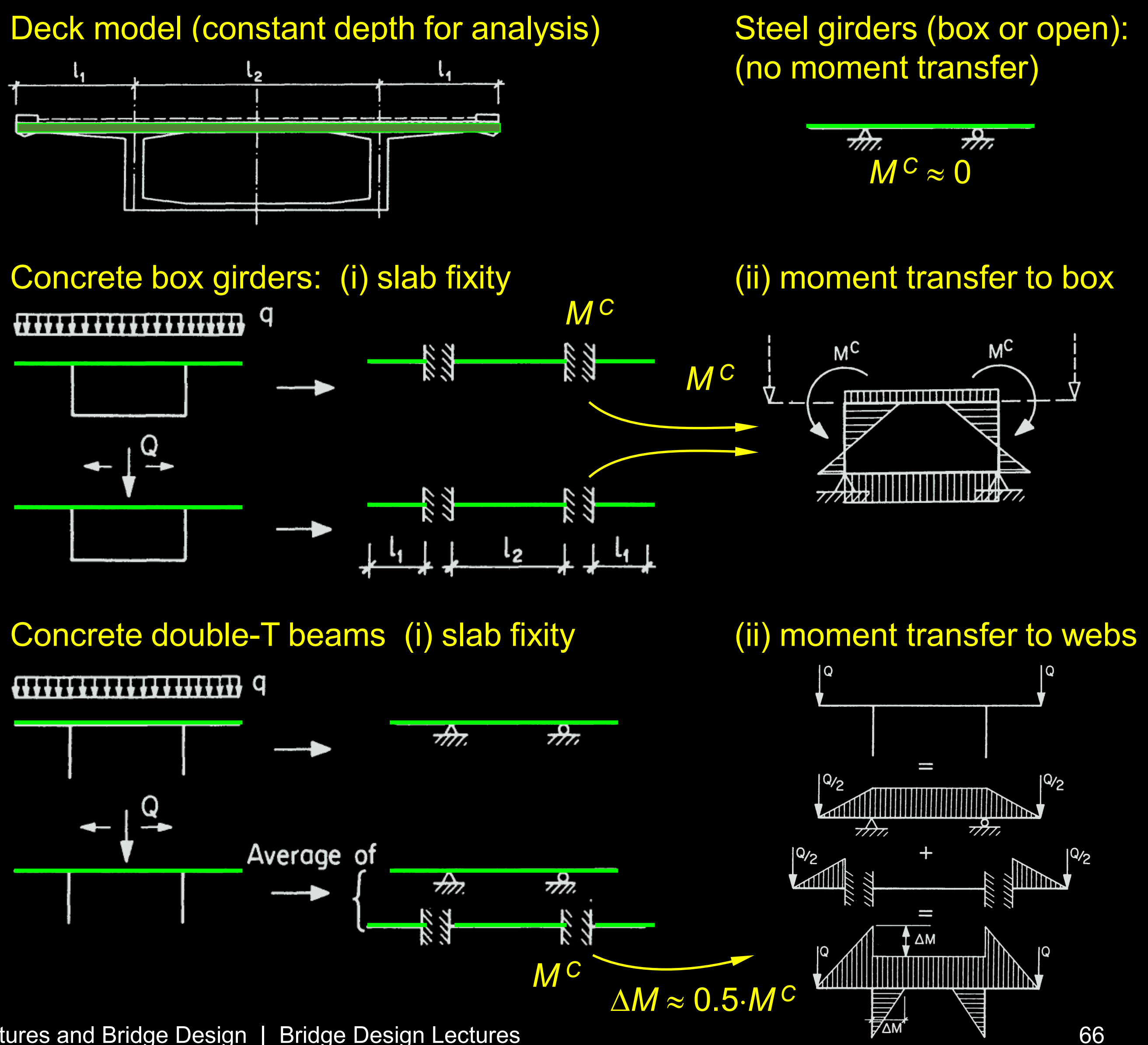
Spine model – Transverse analysis: Transverse bending

Local bending of the deck has been dealt with in *bridge deck*. The bottom slab of box girders can be modelled accordingly (primarily carries self-weight).

The support moments obtained from the deck slab analysis (usually only in concrete girders) need to be applied to the girder to ensure equilibrium. Usually, primarily the cantilever moment M^C is relevant.

These moments cause transverse bending of the longitudinal girders as illustrated in the figure for symmetrical load on the cantilevers.

In box girders, more general load combinations can be analysed using the frame model shown in the figure. For open cross-sections, this is more complicated, see e.g. [Menn 1990, 5.3.1].



Spine model – Transverse analysis: Transverse bending

The web of concrete box girders is typically much thicker, and therefore stiffer than the deck:

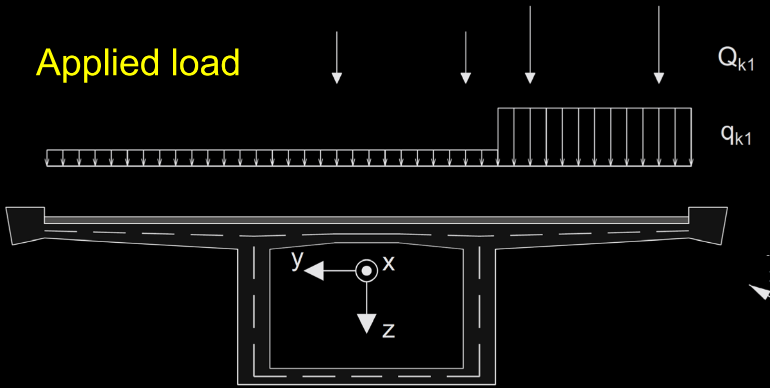
→ most of the cantilever moments are transferred to the web

→ further transverse bending moments are caused by torque introduction, see behind

→ webs of concrete box girders need to be designed for the combination of longitudinal shear and transverse bending

NB. Neglecting moment transfer from the deck to the webs may be unsafe even if the deck is designed to resist the full bending moments (see notes for details).

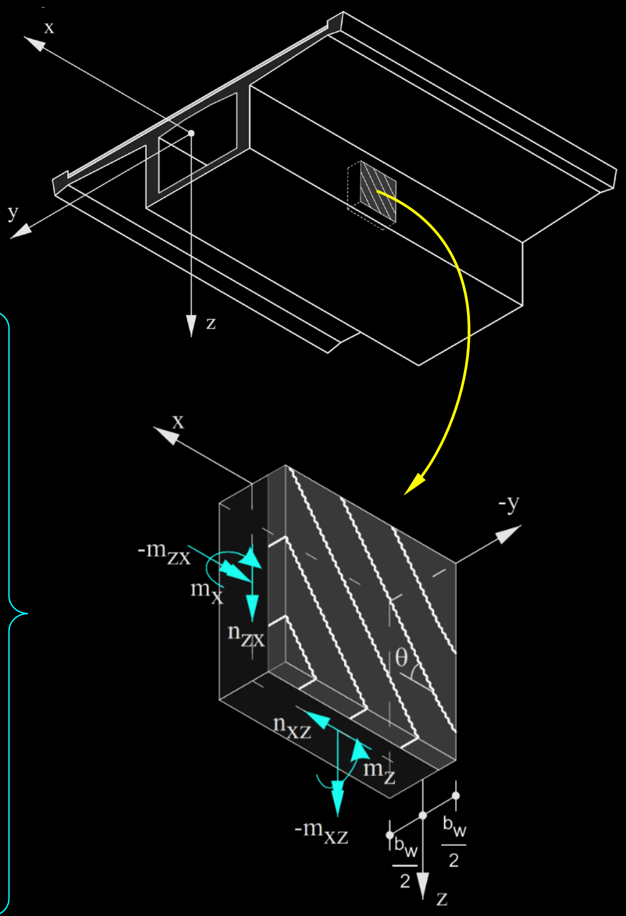
Applied load



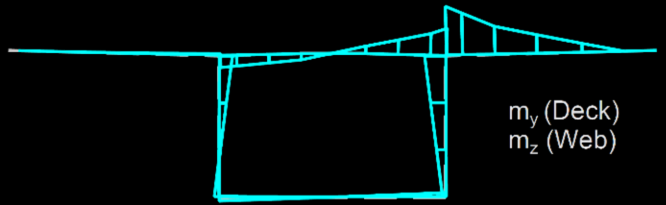
Combined loading of web:

… longitudinal shear (V+T)

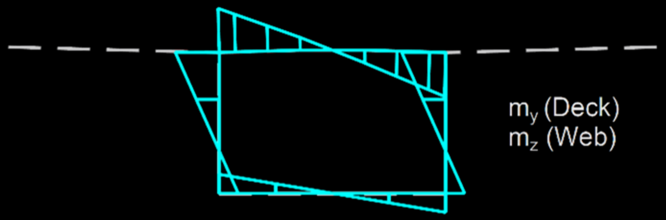
… transverse bending



Moment transfer from deck



Distortion (see behind)



Spine model – Transverse analysis: Transverse bending

The combined application of transverse bending and in-plane shear leads to a simultaneous:

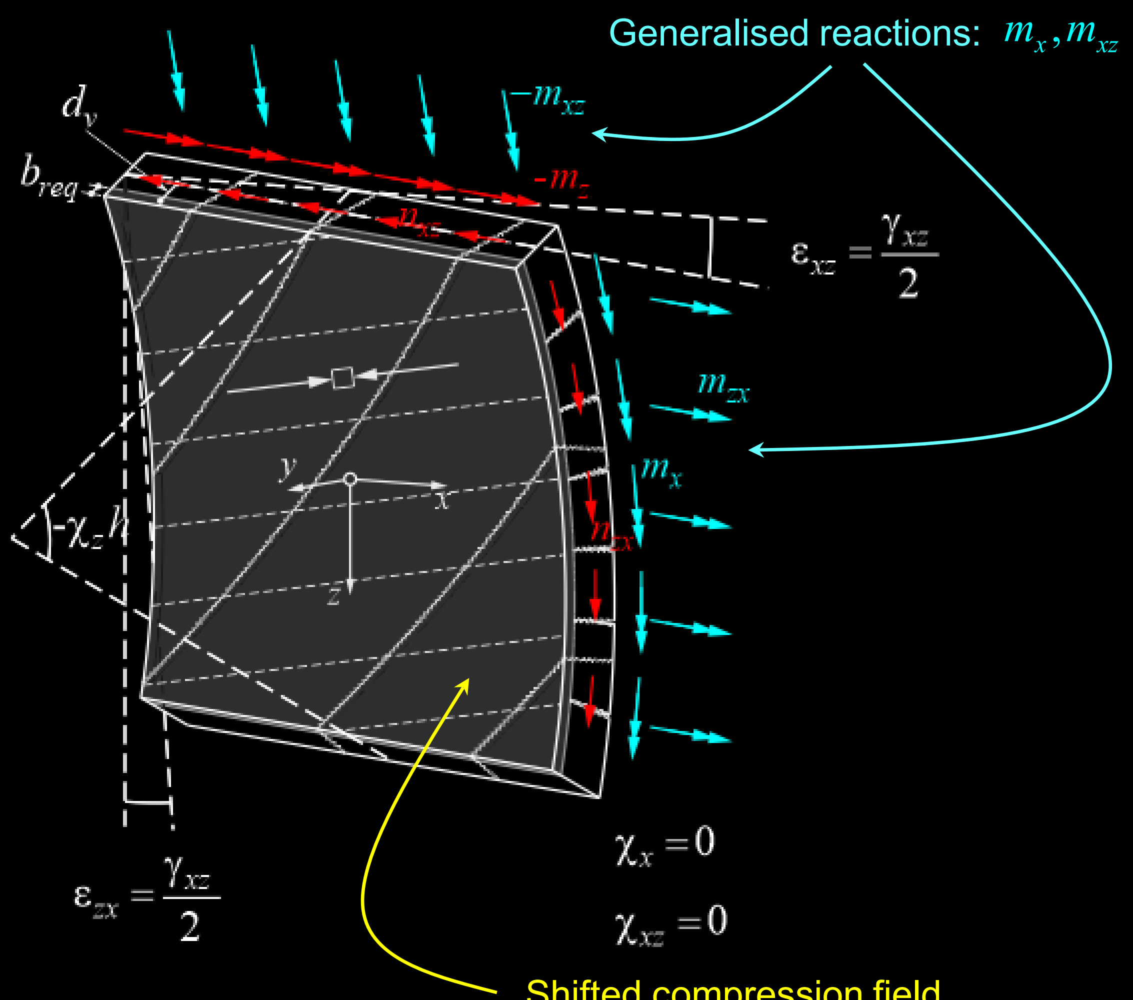
→ shift of the compression field towards the flexural compressive side of the web, which in turn is facilitated by / requires…

→ generalised reactions (the shift of the compression field corresponds to twisting moments m_{zx} and bending moments m_x)

These generalised reactions are able to develop due to the web being restrained against twisting and longitudinal bending by the deck and bottom flange.

Generally, the principal compressive direction varies across the thickness of the web (see reference in notes). In the following, two simpler equilibrium models proposed by Menn (1990) based on the works of Thürlimann and Marti, assuming a compression field of constant inclination shifted to the flexural compression side of the web, is considered (see notes for additional remarks).

Web element loaded in in-plane shear and transverse bending



Spine model – Transverse analysis: Transverse bending

The resistance under combined longitudinal shear and transverse bending can be checked using interaction diagrams, which are commonly normalised with respect to the two reference cases of pure longitudinal shear and pure transverse bending.

1) Pure longitudinal shear (see figures): The diagonal compression field extends over the entire web width, with the corresponding upper limit to the shear resistance (web crushing depending on the axial strains ε_x and θ_c or stirrup yielding):

$$\sigma_{c3} = \frac{n_{xzd}}{\cos\theta_c \sin\theta_c} = n_{zxd}\left(\tan\theta_c + \cot\theta_c\right) \le k_c f_{cd} \qquad \left(n_{xzd} = \frac{V_{zd}}{d_v}\right)$$

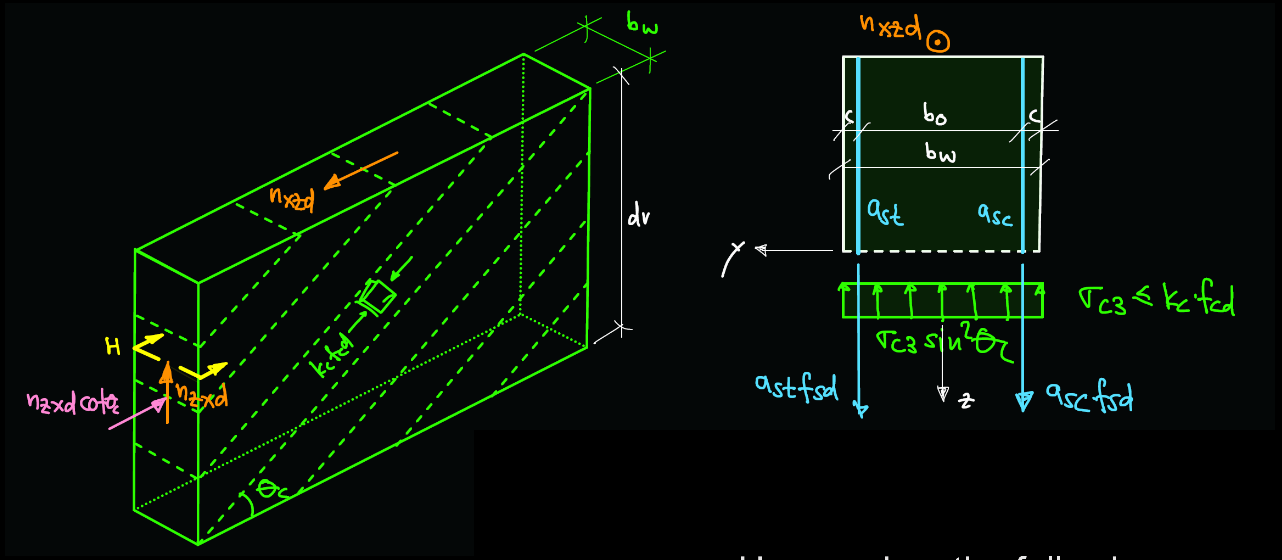
$$n_{zx,Rd} = \min\begin{Bmatrix} n_{zx,Rdc} \\ n_{zx,Rds} \end{Bmatrix} = \min\begin{Bmatrix} \dfrac{k_c f_{cd} b_w}{\tan\theta_c + \cot\theta_c} \\ \left(a_{st} + a_{sc}\right) f_{sd} \cot\theta_c \end{Bmatrix}$$

2) Pure transverse bending :

$$m_{z,Rdy} \approx a_{st} f_{sd}\left(b_w - c - \frac{a_{st} f_{sd}}{2 f_{cd}}\right)$$

Web element

Horizontal section (longitudinal) section



Here and on the following slides, the indices *t* and *c* (with the stirrup forces) denote the flexural tensile and compressive side of web, respectively, due to transverse bending.

Spine model – Transverse analysis: Transverse bending

In the case of predominant shear force, the diagonal compression field is shifted as much as possible, with the minimum required width to transfer the shear force:

$$b_{req} = \frac{n_{zxd}}{k_c \cdot f_{cd} \cos\theta_c \sin\theta_c} = \frac{n_{zxd}\left(\tan\theta_c + \cot\theta_c\right)}{k_c \cdot f_{cd}}$$

Equilibrium (compression field shifted as much as possible to the flexural compression side) requires:

$$\frac{n_{zxd}}{\cot\theta_c} - a_{sc} f_{sd} - a_{st} f_{sd} = 0$$

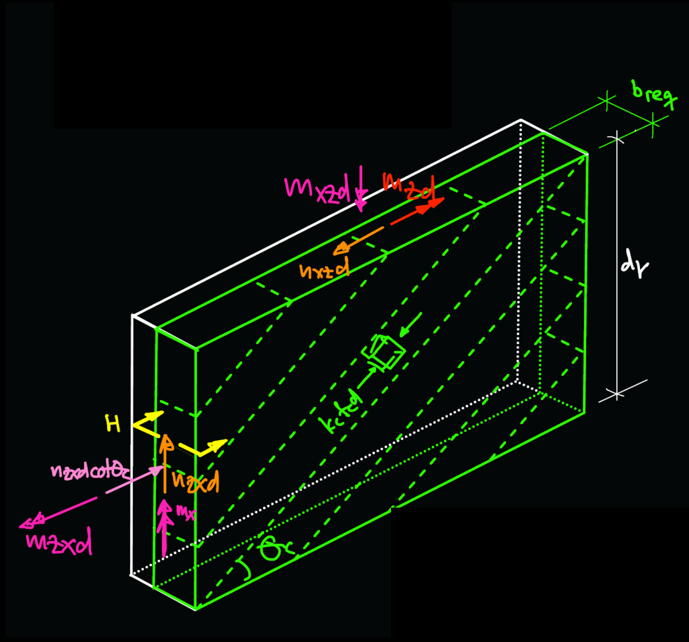
$$a_{st} f_{sd} \cdot b_0 - \frac{n_{zxd}}{\cot\theta_c}\left(\frac{b_{req}}{2} - c\right) - m_{zd} = 0$$

which can be solved for the stirrup forces:

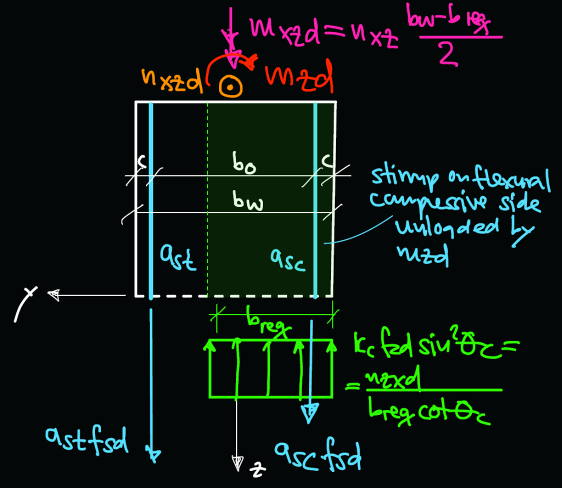
$$a_{sc} f_{sd} = \frac{n_{zxd}}{b_0 \cot\theta_c}\left(b_0 - \frac{b_{req}}{2} + c\right) - \frac{m_{zd}}{b_0},$$

$$a_{st} f_{sd} = \frac{n_{zxd}}{b_0 \cot\theta_c}\left(\frac{b_{req}}{2} - c\right) + \frac{m_{zd}}{b_0}$$

Web element



Horizontal section (longitudinal) section



Spine model – Transverse analysis: Transverse bending

In the case of predominant transverse moment, the force in the stirrups on the compressive side is assumed to be zero, and a vertical concrete compression zone of width $b_m \leq b_w - b_{req}$ (typically much narrower) is added; in this zone, $k_c = 1$ is assumed as for pure bending. The concrete compression transferring longitudinal shear is thus shifted towards the centreline of the of the web compared to the model on the previous slide.

The two equilibrium equations are then:

$$\frac{n_{zxd}}{\cot\theta_c} - a_{st} f_{sd} + b_m f_{cd} = 0, \quad \rightarrow \quad b_m = \frac{a_{st} f_{sd} - \dfrac{n_{xzd}}{\cot\theta_c}}{f_{cd}}$$

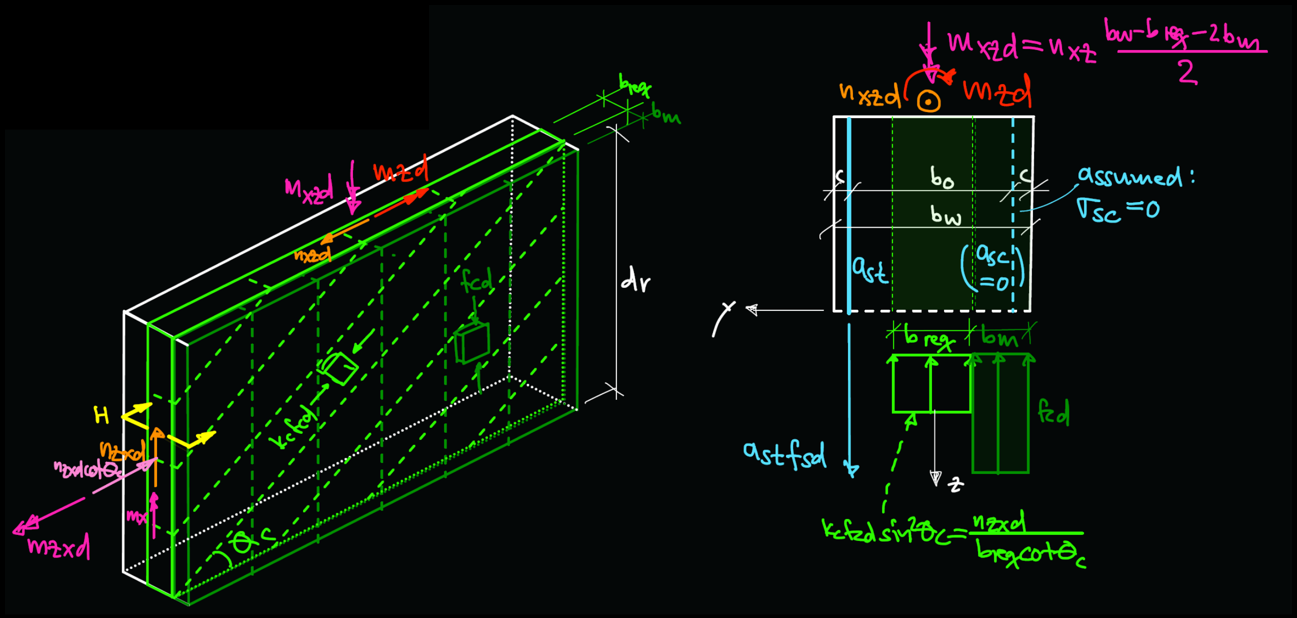
$$m_{zd} - a_{st} f_{sd}\left(b_w - c - \frac{b_m}{2}\right) + \frac{n_{zxd}}{\cot\theta_c}\left(\frac{b_{req}}{2} + \frac{b_m}{2}\right) = 0$$

and the stirrup force on the tensile side is given by:

$$a_{st} f_{sd} = \frac{m_{zd} + \dfrac{n_{zxd}}{\cot\theta_c}\left(\dfrac{b_{req}}{2} + \dfrac{b_m}{2}\right)}{b_w - c - \dfrac{b_m}{2}} \quad (check: b_m \leq b_w - b_{req})$$

Web element

Horizontal section (longitudinal) section



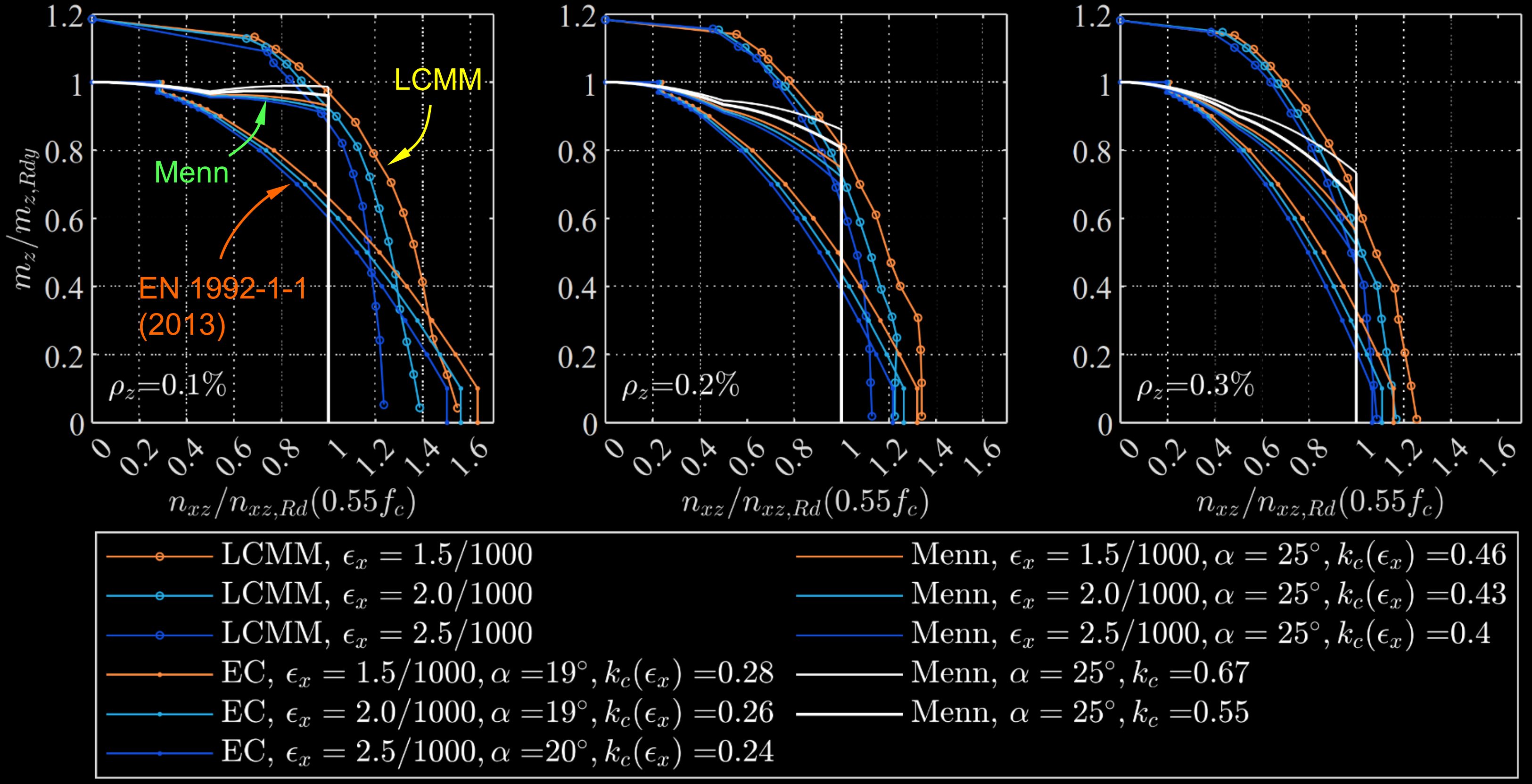
Spine model – Transverse analysis: Transverse bending

More accurate interaction diagrams can be obtained using a layered shell element model (LCMM, see notes).

The figure compares these interaction diagrams with those obtained by Menn's simple model (previous slides).

- Menn yields good results if $k_c\ f_{cd}$ and α (= θ_c) are chosen suitably
- Menn is unsafe if k_c = 0.67 [Menn 1990, 5.3.2] is used with a flat α

The diagrams include the simple quadratic interaction proposed by EN1992-1-1, which is seen to be overly conservative (for elements subject to restraints which lead to the development of generalised reactions).



Spine model – Transverse analysis: Torsion in box girders (general)

Box girders resist torsion primarily by uniform torsion but torques are typically applied by eccentric vertical or horizontal forces (rather than circumferential loads). Hence

→ introduction of torques tends to distort the cross-section (see upper figures and next slides), causing

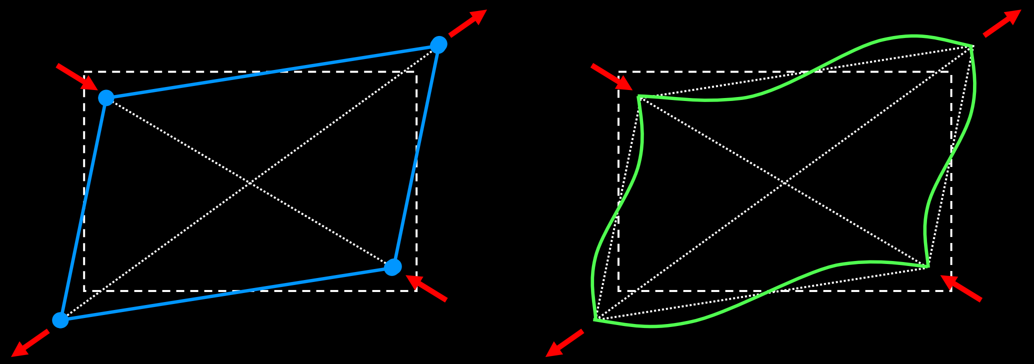
→ significant warping torsion and corresponding longitudinal stresses unless distortion of the cross-section is impeded

Longitudinal stresses due to distortion of box girders are difficult to quantify and distortion of the section is undesirable

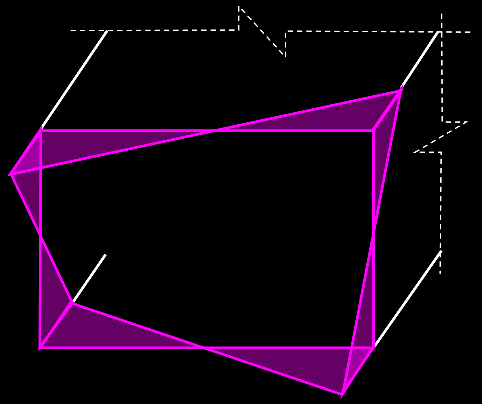
→ box girders are usually designed to avoid significant distortion, which can be achieved
… by a transversely stiff cross-section acting as frame (upper right figure)
… by an adequate number of sufficiently stiff diaphragms if the girder lacks transverse stiffness (upper left figure)

Note: Even without distortional loading, the cross-section of box girders generally warps, see bottom figure. However, this does not cause significant stresses even if warping is restrained (see notes).

Distortion of a rectangular cross-section with hinged connections (left) and stiff corners (right): displacements in the transverse direction



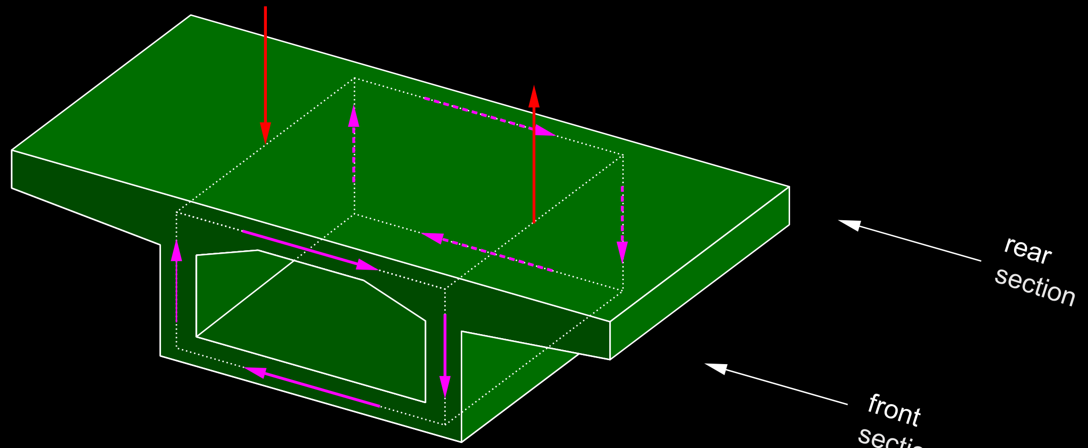
Warping of a rectangular cross-section: longitudinal stress-free displacements (unless warping is restrained)



Spine model – Transverse analysis: Introduction of eccentric loads

In the following slides, the introduction of torques in box-girders due to different types of load (concentrated, distributed, horizontal, vertical) is outlined. In all cases,

- applied torques and circumferential shear flow are statically equivalent (= in equilibrium)
- the load introduction (the transformation of torques to a circumferential shear flow) causes a self-equilibrated set of distortional forces



Depending on static system and load position along girder

- the percentage of the applied torque transferred in positive and negative *x*-direction varies, but
- the change of the torsional moments (resultant of the circumferential shear flows) in two sections in the span always corresponds to the torque applied between these sections.

Spine model – Transverse analysis: Introduction of eccentric loads

Concentrated torques due to vertical force couples are usually caused by traffic loads (concentrated loads representing vehicle axle loads).

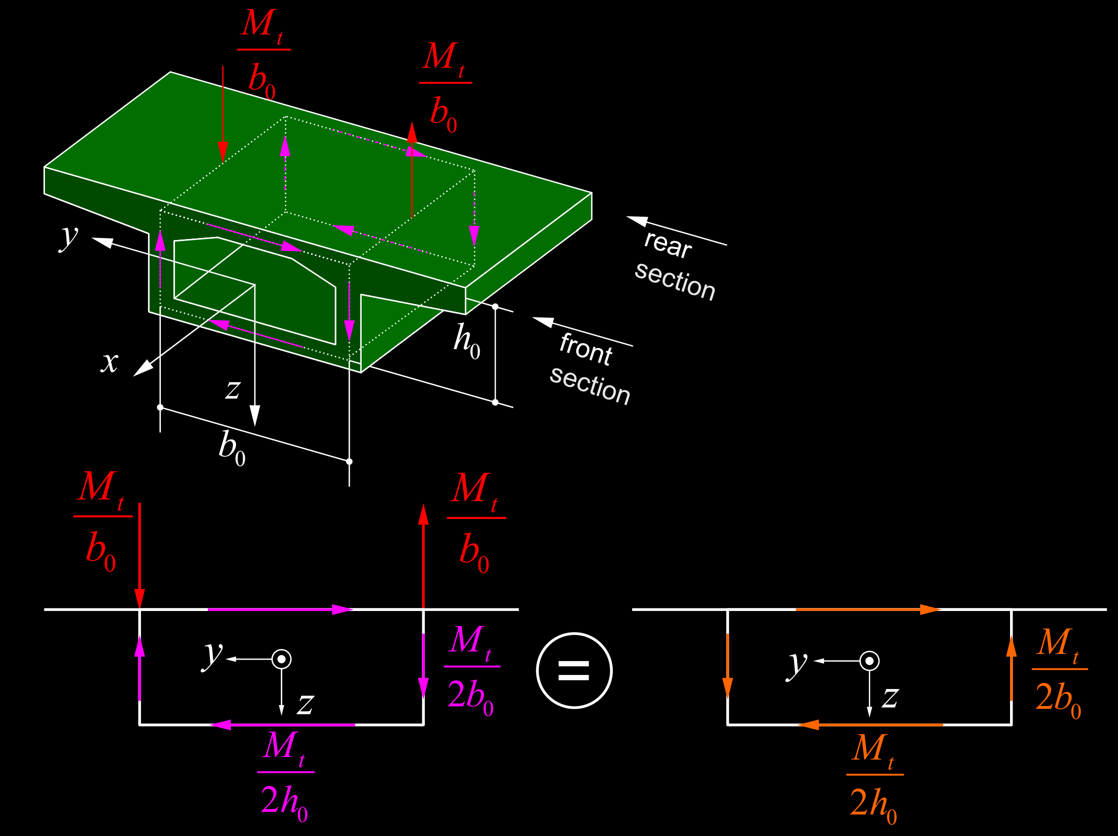
The figure illustrates the forces acting on the free body (girder between front and rear sections):

- applied loads
- circumferential shear flow

The sum of these forces (per side of the cross-section) are the distortional forces, which can alternatively be represented by two equal diagonal distortional forces of opposite sign (passing through the corners since loads are applied in the web axes).

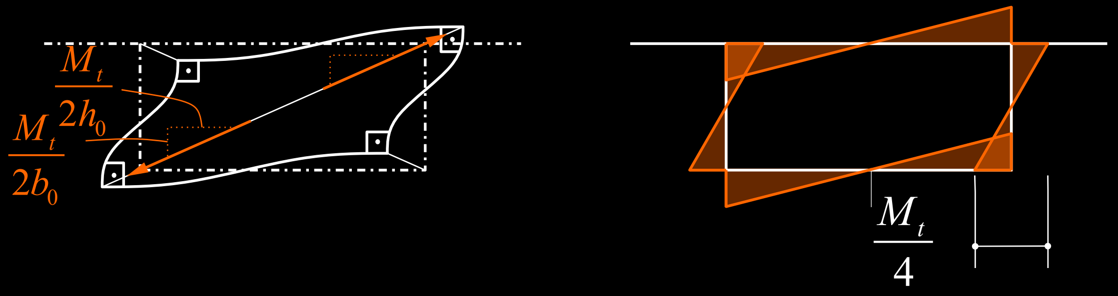
The cross-section tends to distort rhombically due to the distortional forces. If it has a transverse bending resistance, distortion is restrained by transverse bending.

Otherwise, distortion of the cross-section is hindered only by longitudinal bending of its elements, i.e., warping torsion, over the distance to the next intermediate diaphragm impeding distortion.



→ distortion of cross-section

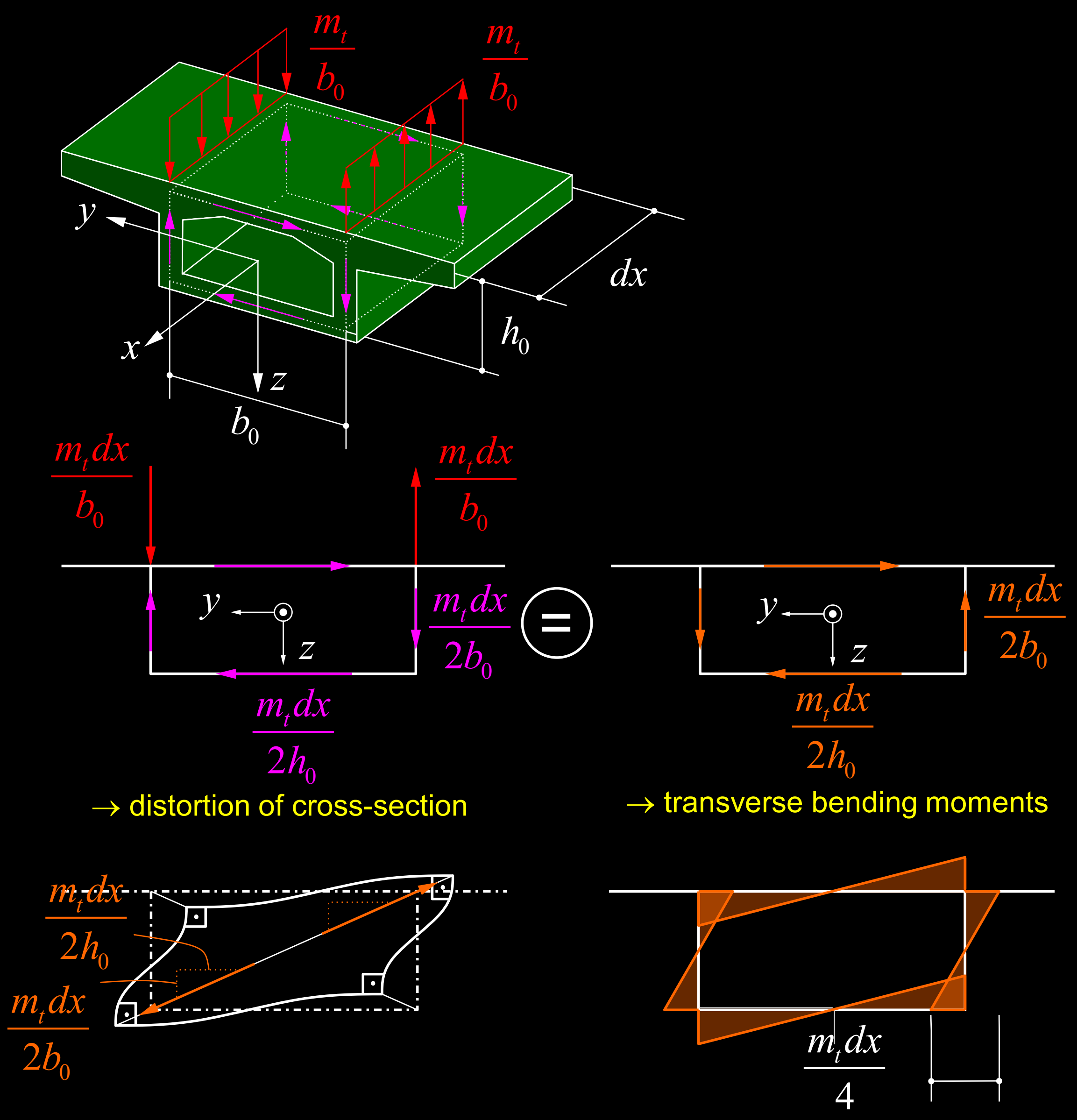
→ transverse bending moments



Spine model – Transverse analysis: Introduction of eccentric loads

Distributed torques due to vertical line load couples
may be due to traffic loads (e.g. line load of ballastless track rail) or superimposed dead loads (e.g. crash barriers).

(further comments see previous slide)



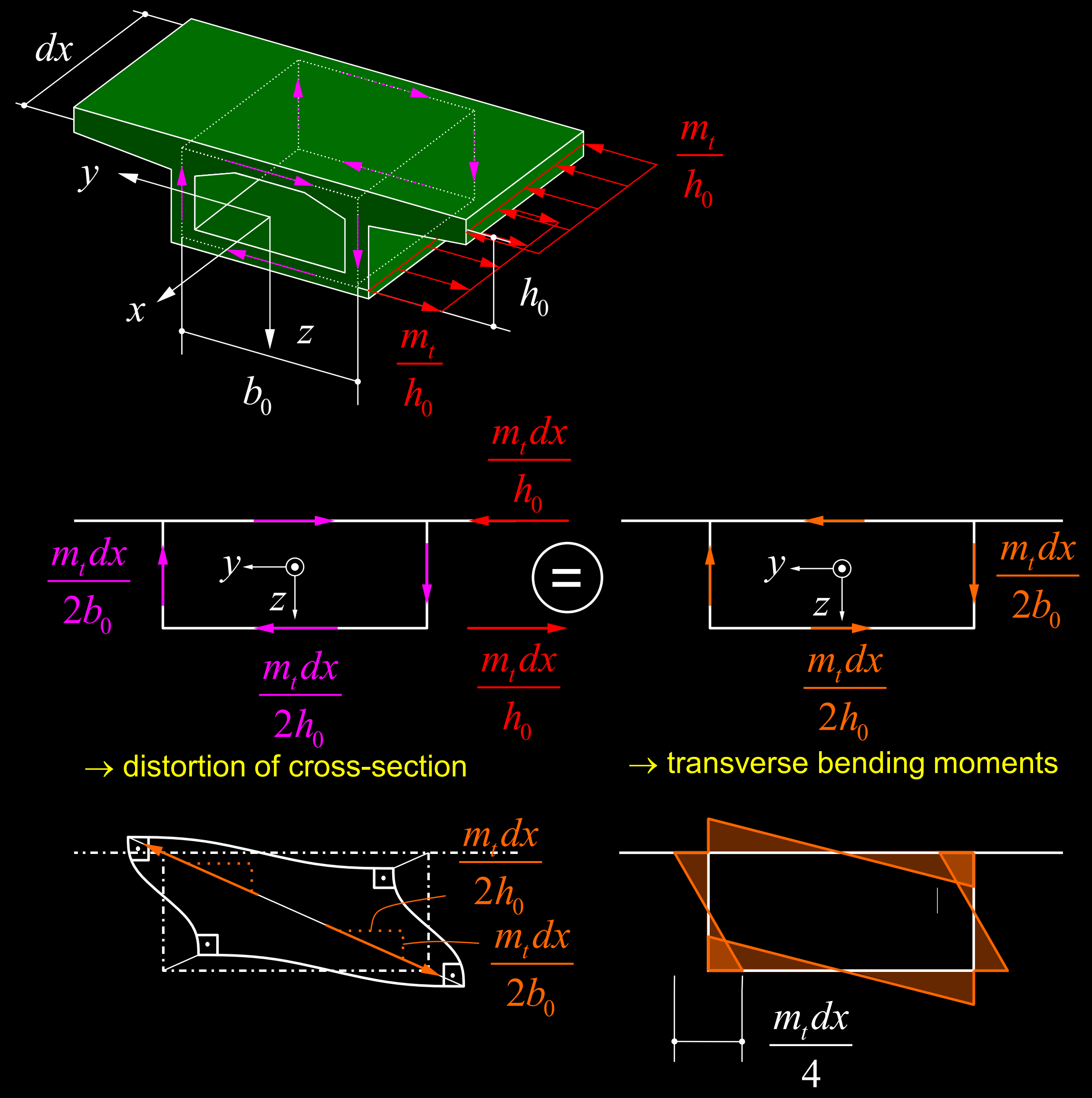
Spine model – Transverse analysis: Introduction of eccentric loads

Distributed torques due to horizontal line load couples may be due to wind or girder curvature in plan.

Torques applied by horizontal forces couples are particularly relevant in curved bridges, as commented on slide on torsion in curved bridges (general).

Distortional forces caused by a torque applied through a horizontal force couple have opposite signs compared to those caused by a torque of equal sign applied through a vertical force couple.

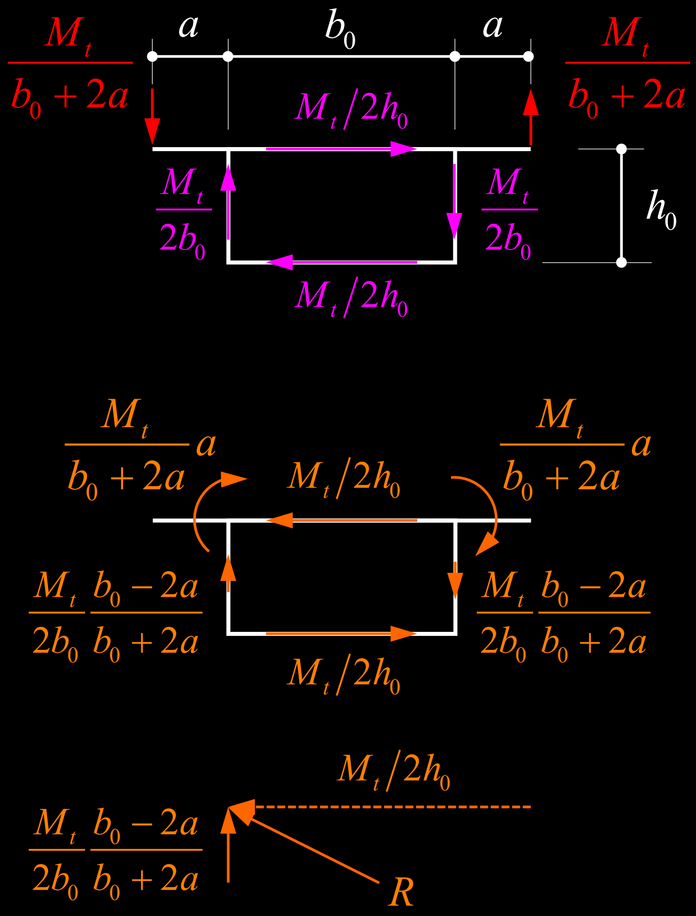
(further comments see previous slide)



Spine model – Transverse analysis: Introduction of eccentric loads

The distortional forces obtained by applying vertical force couples in the web axes (as in the previous slides) are usually on the safe side.

If the loads are applied on the cantilever, a smaller distortional force results (see figure on the right, noting that R is aligned to the diagonal of the section with its vertical component being smaller).



$$R = \underbrace{\frac{M_t}{2}\frac{b_0-2a}{b_0+2a}\frac{\sqrt{h_0^2+b_0^2}}{b_0 h_0}}_{\text{forces applied at cantilever ends}} \leq \underbrace{\frac{M_t}{2}\frac{\sqrt{h_0^2+b_0^2}}{b_0 h_0}}_{\text{forces applied in web axes}}$$

Spine model – Transverse analysis: Torsion design of box girders

Concrete box girders are significantly stiffer in the transverse direction than steel and composite box girders.

Straight or slightly curved concrete box girders usually have

- sufficient strength to introduce torques applied in the span
- sufficient stiffness to prevent significant distortion of the cross-section without intermediate diaphragms

→ intermediate diaphragms are only required in strongly curved concrete box girders.

Contrary to concrete box girders, steel or composite box girders are usually unable to resist significant torques applied in the span, nor to provide adequate restraint to distortion of the cross-section, without intermediate diaphragms.

→ several intermediate diaphragms (usually about 5) per span are therefore provided even in straight steel and composite box girders

Hence, there are considerable differences in the torsion design of concrete and steel or composite box girders, see next slide.

Arrollo de las Piedras viaduct, Spain, 2006. IDEAM

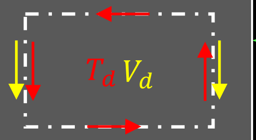
Spine model – Transverse analysis: Torsion design of box girders

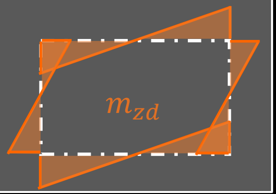
Torsion design of box girders

transversely stiff cross-section?

YES → **Design (concrete) girder**

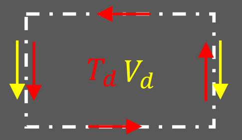
- shear force per element of cross-section due to torsional moment T_d and longitudinal shear V_{zd} and V_{yd}
- higher shear force over length required to convert concentrated torques to uniform torsion
- interaction with transverse bending moments m_z caused by moment transfer from deck and torque introduction





NO → **Design (steel and composite) girder**

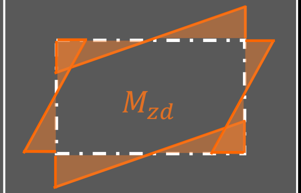
- shear force per element of cross-section due to torsional moment T_d and longitudinal shear V_{zd} and V_{yd}
- higher shear force over distance to adjacent diaphragm



Design intermediate diaphragms

Design each intermediate diaphragm to:

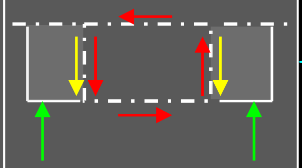
- resist the full distortional forces over its share of the span (neglect contributions from cross-section)
- provide adequate stiffness to prevent significant distortion of the cross section



Design support diaphragms

Design the support diaphragms to introduce the support reactions (green) considering:

- high concentrated torque applied by vertical reactions
- indirect support if bearings are not located in web axes



Spine model – Transverse analysis: Torsion design of box girders

Irrespective whether intermediate diaphragms are provided, the box girder is designed to resist the full applied torsional moment in uniform torsion, combined with vertical and horizontal shear forces.

The figure shows schematically how the governing shear forces per element of the cross-section are determined.

If no intermediate diaphragms are provided, the design needs to account for transverse bending moments particularly due to torque introduction.

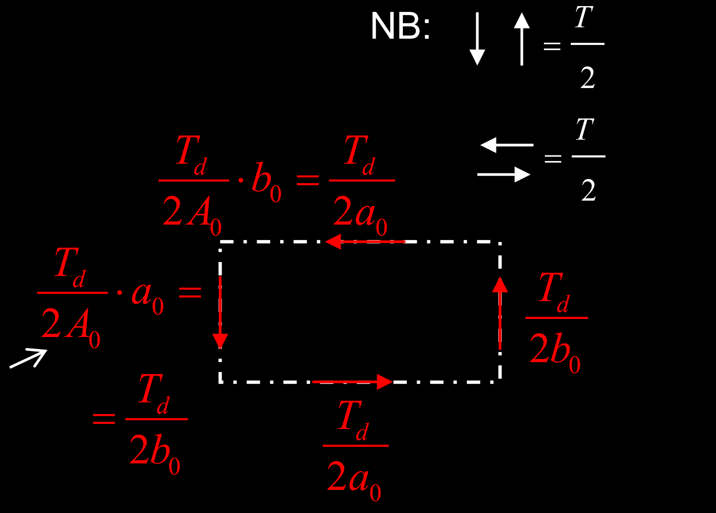
The design needs to account for the higher shear forces caused by eccentric loads in the longitudinal shear design i.e. design for higher shear forces over distance to next diaphragm (or length required to convert torques to uniform shear), see next slide.

Shear flow (Bredt):

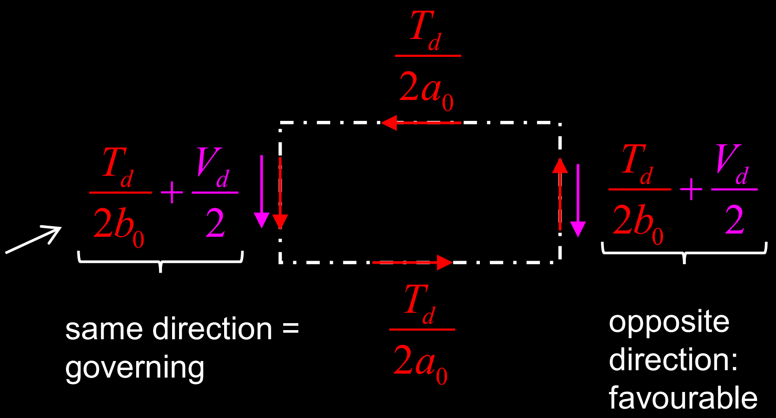
$$'\tau \cdot t' = \frac{T_d}{2A_0}\left[\frac{\text{kN}}{\text{m}}\right]$$

forces per wall:

$$'\tau \cdot t' \cdot z_i = \frac{T_d}{2A_0} \cdot z_i \quad [\text{kN}]$$



superposition of forces due to T_d and V_d



Spine model – Transverse analysis: Torsion design of box girders

Since the applied torques are only converted to a circumferential shear flow

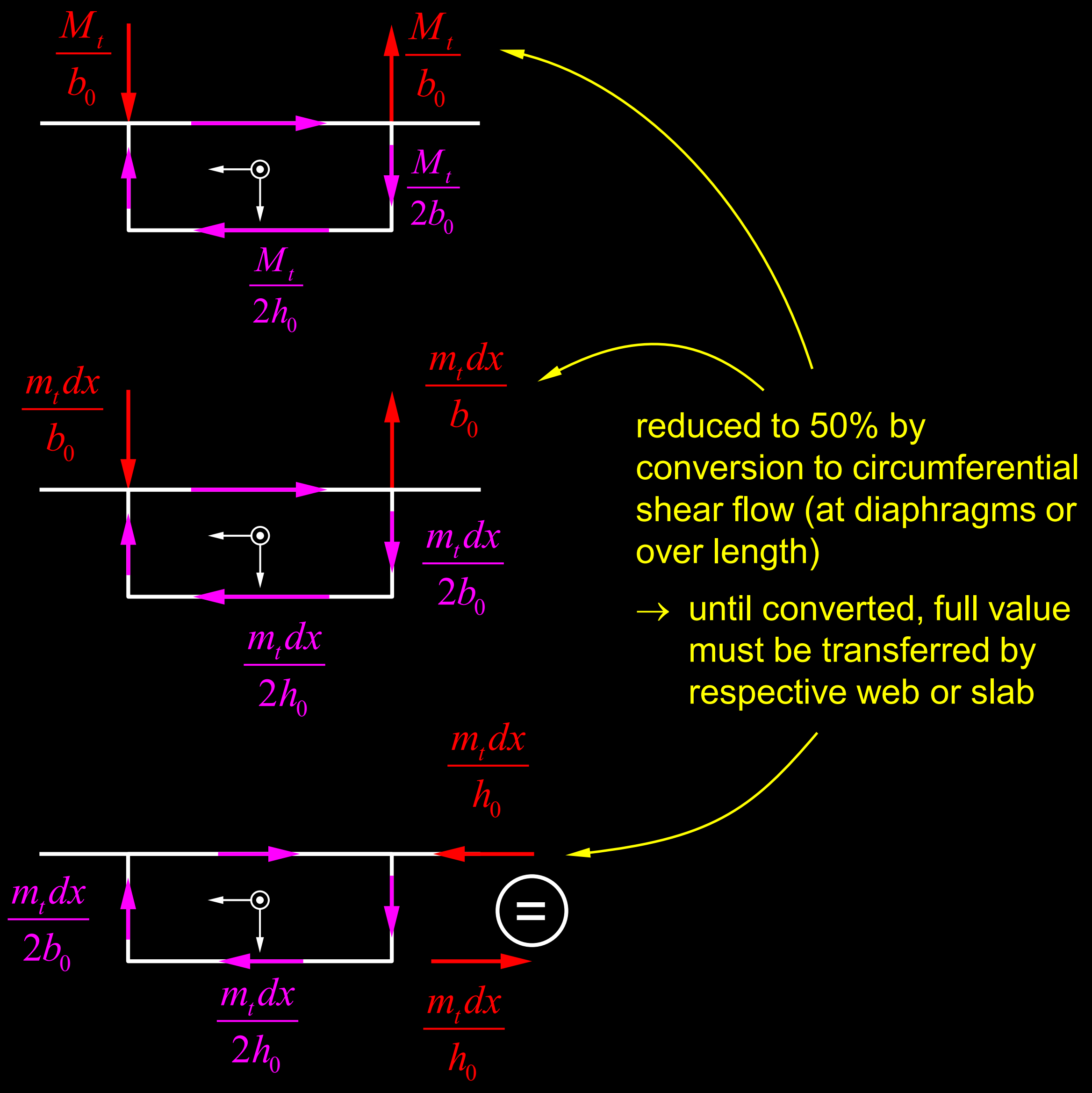
- by intermediate diaphragms, or
- by transverse bending of the cross-section, which requires a certain length for concentrated torques

→ higher shear forces than obtained assuming a circumferential shear flow need to be accounted for in longitudinal shear design:

in girders with intermediate diaphragms:
… for concentrated and distributed torques
… over the distance to the next intermediate diaphragm

in concrete box girders without intermediate diaphragms
… for concentrated torques (*)
… over the distance required to introduce concentrated torques by transverse bending (strength-dependent)

(*) If transverse bending moments due to distributed torque introduction exceed the shear+transverse bending capacity of a concrete girder, intermediate diaphragms are required.



Spine model – Transverse analysis: Design of intermediate diaphragms

Intermediate diaphragms are designed to

- introduce torques applied in the span

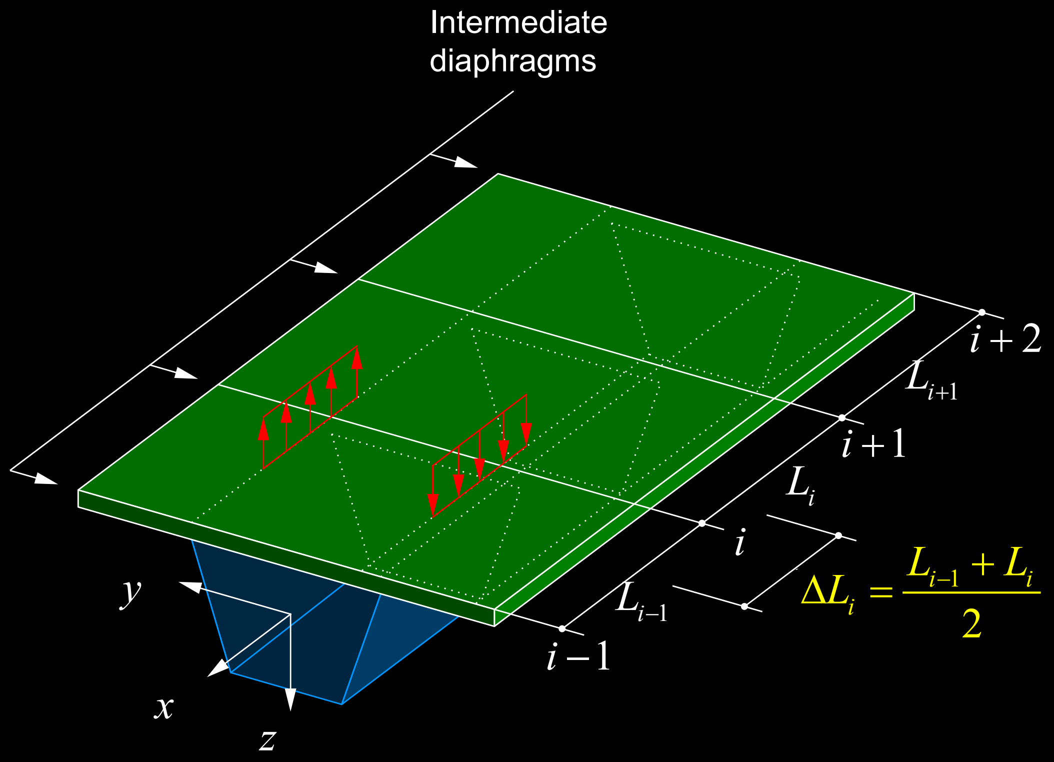
→ each diaphragm needs to resist the distortional forces over its respective share of the span ΔL_i (see figure)

→ neglecting contributions from the cross-section between the diaphragms (even in concrete girders)

- provide adequate stiffness to prevent significant distortion of the cross section of steel and composite box girders; commonly accepted criteria (based on numerical studies) to achieve this are:

→ minimum stiffness shall limit normal stresses due to warping torsion (caused by distortion) to ≤ 5% of the normal stresses due to global bending, which is in turn

→ deemed to be satisfied if the following is provided
… 5 solid steel plate diaphragms per span or
… 5 cross-bracings per span, each with a distortional stiffness of ≥ 20% of a 20 mm steel plate diaphragm (see e.g. Lebet and Hirt, 2013 for more details)



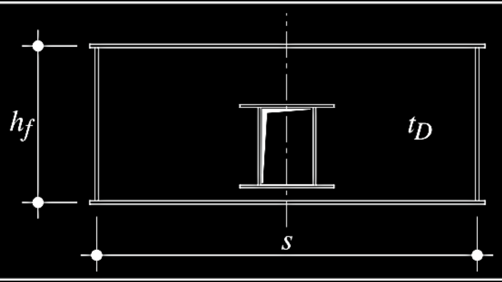
Spine model – Transverse analysis: Design of intermediate diaphragms

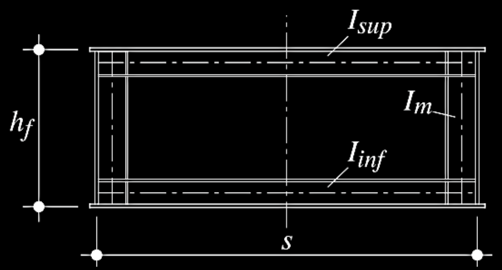
In summary, the design of the intermediate diaphragms is determined by:

- Minimum stiffness to control longitudinal stresses due to distortion

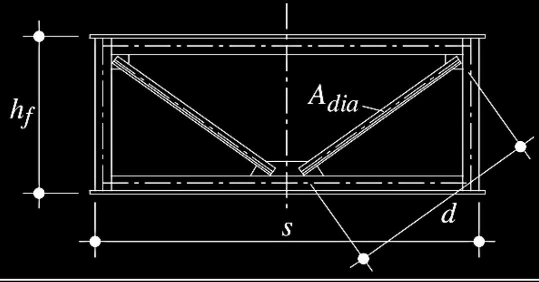
→ the table shows the distortional stiffnesses of the most used cross bracings in a steel or steel-concrete composite box section

- Resistance required for torque introduction (and bending if used as support for deck)

<table>
<tr>
<td>
</td>
<td>Diaphragm cross bracing

$$K_D = G t_D s h_f$$</td>
</tr>
<tr>
<td>
</td>
<td>Frame cross bracing

$$K_D = \frac{24 E I_m}{\alpha_0 h_f}$$

$$\alpha_0 = 1 + \frac{(2s)/h_f + 3(I_{inf} + I_{sup})/I_m}{(I_{inf} + I_{sup})/I_m + (6h_f/s) \cdot (I_{inf} I_{sup}/I_m^2)}$$</td>
</tr>
<tr>
<td>
</td>
<td>V Truss cross bracing

$$K_D = \frac{E A_{dia} s^2 h_f^2}{2 d^3}$$</td>
</tr>
</table>

Spine model – Transverse analysis: Design of intermediate diaphragms

The minimum stiffness requirement (≥ 20% of a 20 mm steel plate diaphragm) given on the previous slide is simple, but strict and arbitrary.

Alternatively, the minimum stiffness of intermediate diaphragms to comply with the "≤ 5% normal stress" criterion can be determined by modelling the box girder as illustrated schematically in the figure on the right:

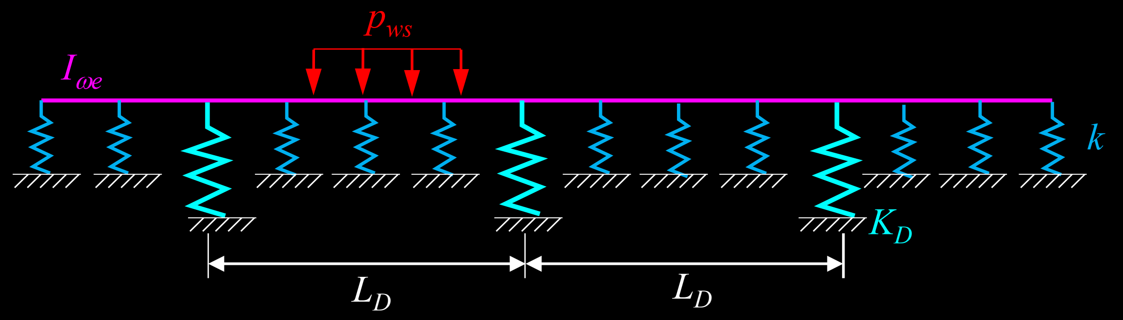
→ the distortion of a box girder, elastically restrained by the distortional stiffness of the cross-section (transverse frame) and cross-bracings

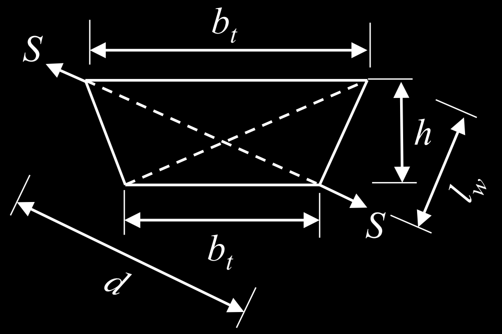
$$EI_{\omega e}\frac{d^4w}{dx^4}+kw=p_{ws}$$

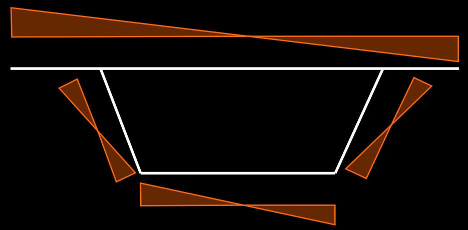
$I_{\omega e}$ = warping moment of inertia
w = web movement contained in its plane
k = distortional stiffness

→ is analogous to a beam on elastic foundation

$$EI\frac{d^4w}{dx^4}+kw=q$$







K_D = cross-bracing distortional stiffness
k = box distortional stiffness
L_D = diaphragm spacing
M_Q = concentrated torsion moment
m_q = distributed torsion moment
M_f = bending moment
R = radius in plan

$$S=p_{ws}\frac{l_w}{d}$$

$$p_{ws}=\frac{M_t}{(b_t+b_b)}\frac{b_b}{b_t}\frac{l_w}{h}$$

$$p_{ws}=\frac{M_t}{2b}\ \text{(for rectangle)}$$

$$M_t=M_Q+m_qL_D+\frac{M_f}{R}L_D$$

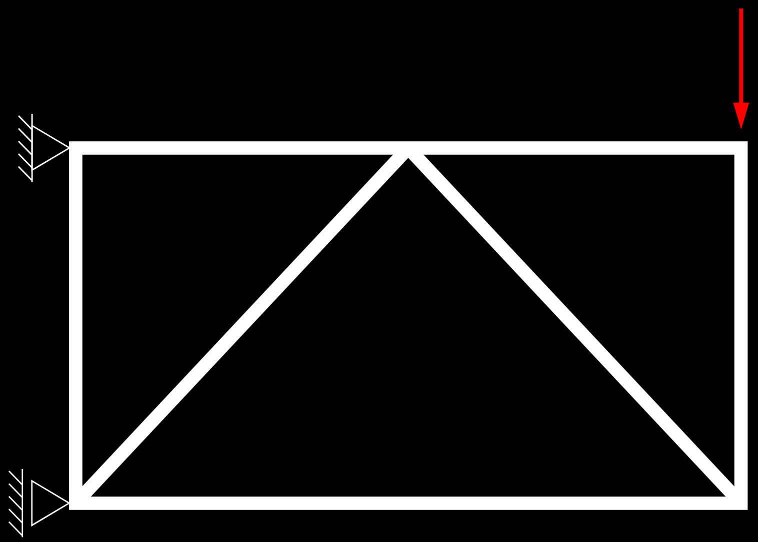
Spine model – Transverse analysis: Design of intermediate diaphragms

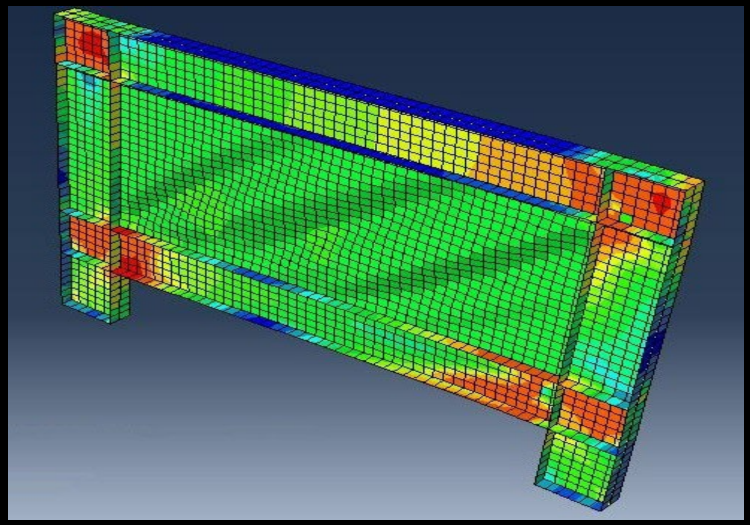
To design an intermediate diaphragm by resistance, the structural element is isolated and all actions acting on it are applied (ensuring that all forces are globally in equilibrium):

- (positive): force couples in webs due to torques applied by vertical loads, and force couples in slabs due to torques applied by horizontal loads and curvature, respectively
- (negative): forces in webs and slabs corresponding to circumferential shear flow
- loads acting directly on the diaphragm (positive) with corresponding forces in webs or slabs (negative)
- forces due to its function as transverse stiffener (steel and steel-concrete composite cross-section)

→ Truss, frame or cross-bracing diaphragms:
Truss analysis (usually using frame analysis software)

→ Solid diaphragm: Strut-and-tie models / stress fields, or FE analysis (membrane element, linear elastic for steel diaphragms, nonlinear analysis e.g. Idea Statica Detail for concrete diaphragms, see Advanced Structural Concrete)





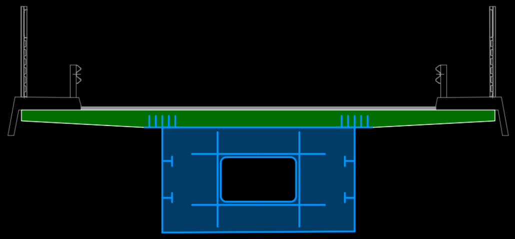
Spine model – Transverse analysis: Intermediate diaphragm types (steel)

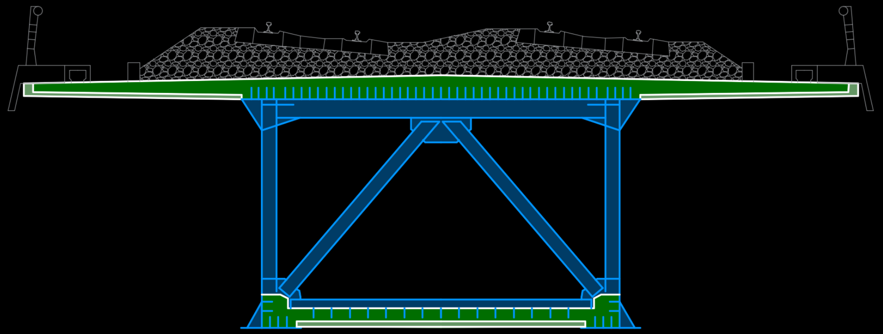
Intermediate diaphragms should

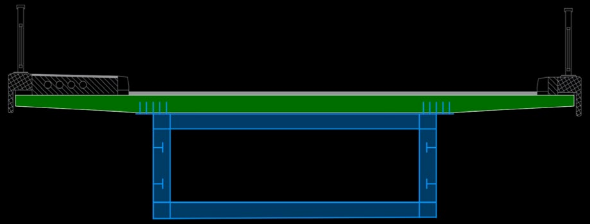
- be lightweight (minimise self-weight)
- allow access (passage) for inspection

The following are used in steel and composite bridges:

- Solid diaphragm (steel plate)
  + high stiffness
  – high weight → cost
  – usually inefficient (minimum thicknesses)
  – limited access (manholes reduce stiffness)
- V-truss cross-bracing
  ± moderate stiffness
  ± moderate weight
  + efficient
  + good access
  – many connections
- Frame cross-bracing
  – low stiffness
  ± moderate weight
  + good access





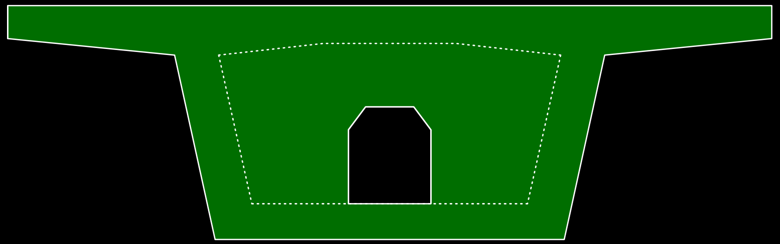


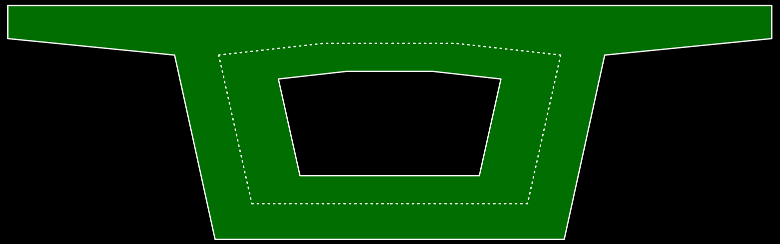
Spine model – Transverse analysis: Intermediate diaphragm types (concrete)

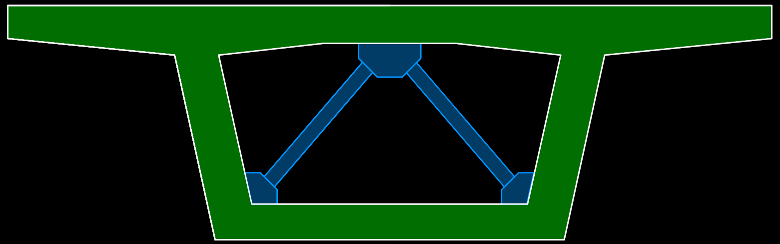
Intermediate diaphragms in concrete box girders should be avoided. If required, complication of the construction process should be minimised (moving internal formwork).

The following solutions are used in concrete bridges:

- Solid with manhole
  + high stiffness
  – high weight
  – completely obstructs moving of internal formwork
  – complicated removal of diaphragm formwork
- Concrete frame
  ± moderate stiffness
  ± moderate weight
  ± easier moving of internal formwork
  – complicated diaphragm formwork
- Steel bracing (post-installed)
  – low stiffness
  + low weight
  + perfect solution for moving internal formwork
  – complicated connections







Spine model – Transverse analysis: Support diaphragms

Piers and abutments provide:

- vertical support (virtually always) …
- torsional restraint (abutments always, piers often) …
- transverse horizontal fixity (usually) …
- longitudinal horizontal fixity (in some cases) …

to the girder, see *bearing layout and dilatation concept*.

The support reactions (applied by bearings or monolithic connections) must be transferred to the girder (converted to forces acting in the planes of the webs and slabs of the cross-section)

→ Support diaphragms

Note: Since the vertical reactions are smaller at the abutments (end support of continuous girder) than at intermediate supports, the transverse distance between the bearings b_R should be as large as possible to avoid uplift (despite the transverse bending caused by the eccentricity of vertical supports to the web axes).

Spine model – Transverse analysis: Design of support diaphragms

Torsional restraint is usually provided by vertical support reactions, hence support diaphragms need to resist

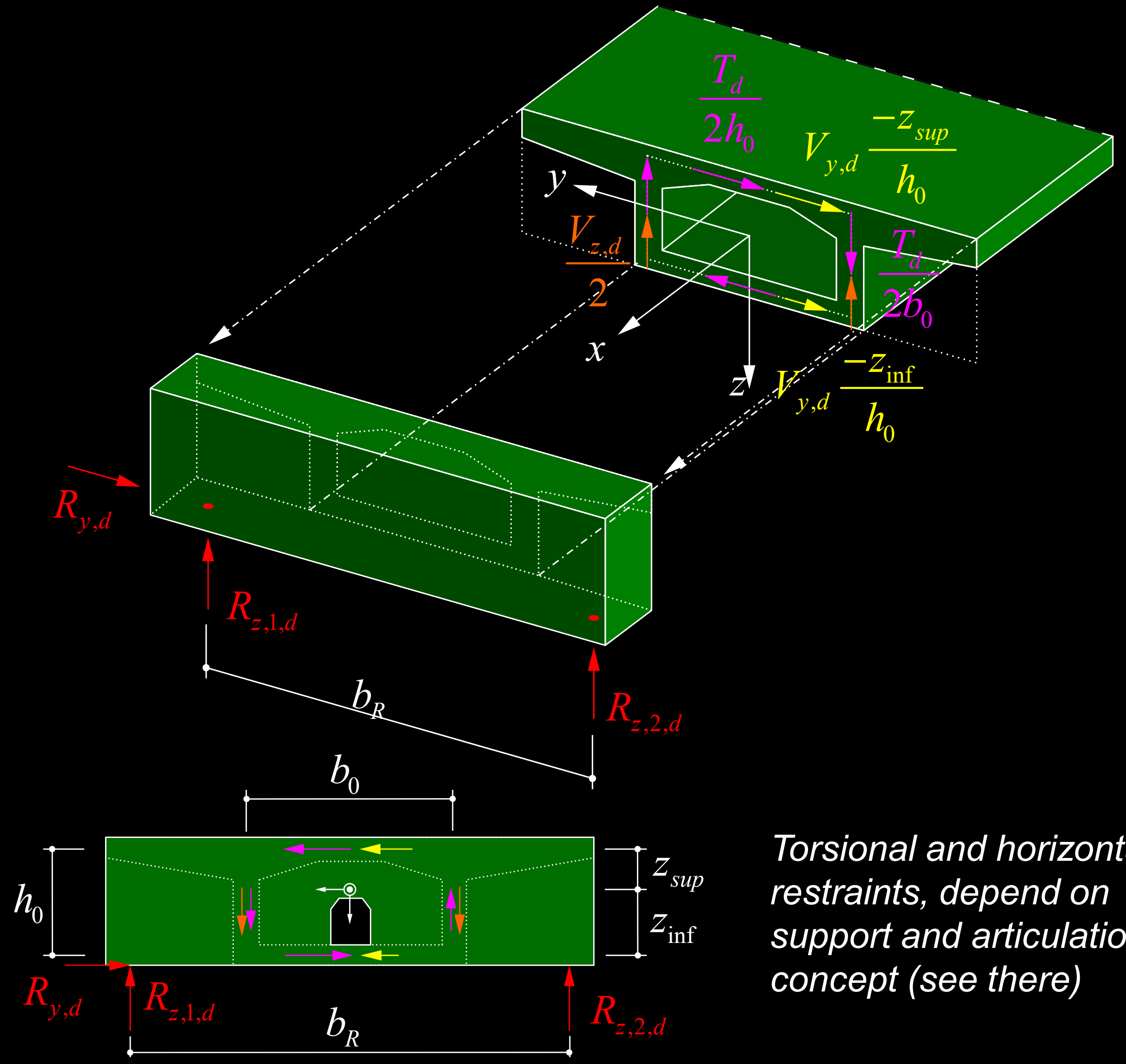
→ distortion due to torque introduction (analogous to intermediate diaphragms) and

→ significant transverse bending (resisted by cross-section in the span) unless bearings are located in the web axes

The support diaphragms have to resist much higher forces than intermediate diaphragms, since

- support torques correspond to the integral of torques applied over half the torsion span
- support reactions correspond to the integral of loads applied over the distance to the point of zero shear.

→ support diaphragms required also in straight concrete girders



Torsional and horizontal restraints, depend on support and articulation concept (see there)

Spine model – Transverse analysis: Design of support diaphragms

Torsional restraint is usually provided by vertical support reactions, hence support diaphragms need to resist

→ distortion due to torque introduction (analogous to intermediate diaphragms) and

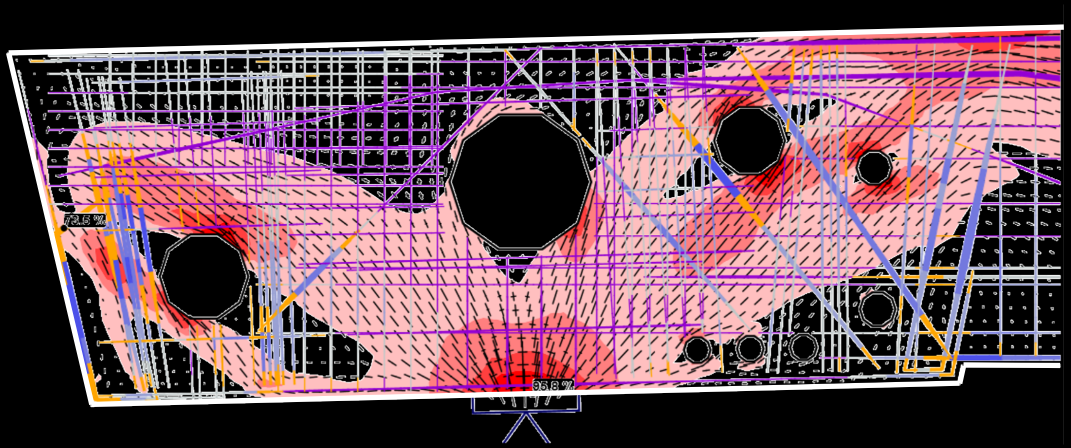
→ significant transverse bending (resisted by cross-section in the span) unless bearings are located in the web axes

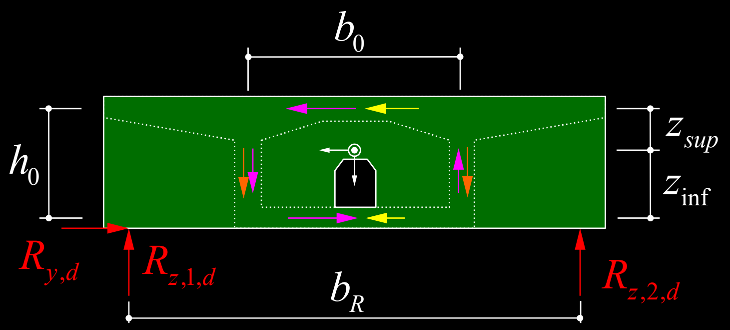
The support diaphragms have to resist much higher forces than intermediate diaphragms, since

- support torques correspond to the integral of torques applied over half the torsion span
- support reactions correspond to the integral of loads applied over the distance to the point of zero shear.

→ support diaphragms required also in straight concrete girders

Solid end diaphragms are therefore often required. These are usually designed based on a plane stress analysis (concrete diaphragms → stress fields by hand or CSFM, see advanced structural concrete, steel diaphragms → FEM).





Torsional and horizontal restraints, depend on support and articulation concept (see there)

Superstructure / Girder bridges

Bridge Girder – Spine model for open cross-sections

Spine model for open cross-sections: General remarks

Using a spine model for girders with open cross-section is inefficient, because (as outlined on the following slides):

- the contributions of uniform torsion and warping torsion to the total torsional moment vary along the span and depend
  … on the static system and
  … the position of applied torques

→ design for several load-cases tedious

→ analysis cannot be carried out efficiently (using e.g. structural analysis software for 2D or 3D frames)

Furthermore, investigating the transverse behaviour of girders with open cross-section based on the results of a spine model is even more demanding than for box girders (which is already demanding, twice as many slides as for global analysis …):

- transfer of a significant part of torsional moments by warping torsion results in

→ substantial distortion of the cross-section (by torsion, not only by torque introduction as in box girders)

→ significant longitudinal stresses due to torsion

→ high transverse bending moments due to torsion

Spine model for open cross-sections: General remarks

In spite of these inconveniences, spine models were frequently used in the past for the analysis of girders with open cross-section, since more complex 2D or 3D-models required a much higher computational effort (which was critical before the advent of modern, user-friendly structural analysis software and affordable personal computers).

Today, running a grillage analysis (see *grillage model*), or even using a folded plate model, is

- more efficient and
- yields more detailed insight into the structural behaviour, particularly regarding transverse load transfer

→ Use of grillage models is recommended for girders with open cross-section

The application of spine models to girders with open cross-section is treated here only to the extent required for understanding the basic concepts of older design recommendations and codes, and because it is still useful for preliminary design of double-T girders, as illustrated on the following slides.

Spine model for open cross-sections: General remarks

Girders with open cross-section transfer eccentric loads primarily by warping torsion (antisymmetric bending), rather than uniform torsion

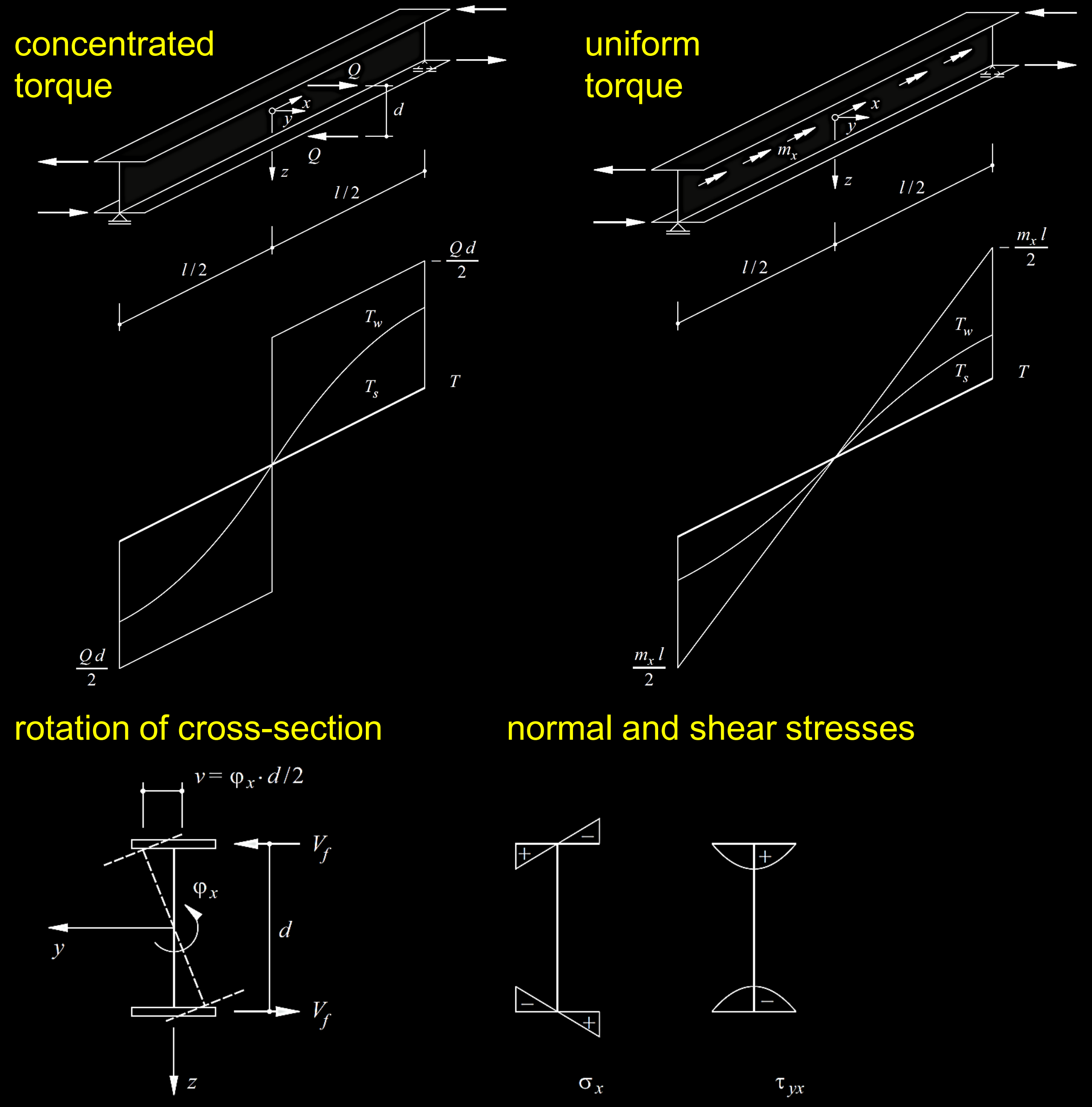
→ cross-section is significantly distorted by torsional moments

→ share of torque transferred by warping torsion T_w and uniform torsion T_s, respectively, varies …
… depending on position of applied torque
… along the span

→ complicated analysis, particularly in the case of wide bridges with more than two webs (idealisation as spine not reasonable!)

In simple cases the longitudinal behaviour of girders with open cross-section can though be analysed with a spine model.

As an example, see figure on the right (from P. Marti, Theory of Structures, Section 13.4.3). The behaviour of girders with two webs will be treated in the following as the I-beam in this example, but rotated by 90°.



Spine model for open cross-sections: Equilibrium model

Generally, eccentric loads acting on girders with open cross-section can be decomposed analogously as in box girders. For example (figure), distributed loads are decomposed in a symmetrical force f_z and a torque m_t.

In symmetric girders (with respect to the *z*-axis), carrying torsion by a combination of uniform and warping torsion

$$T = T_s + T_w \qquad m_t = m_{t,s} + m_{t,w}$$

→ equivalent design loads applied to half-girders:

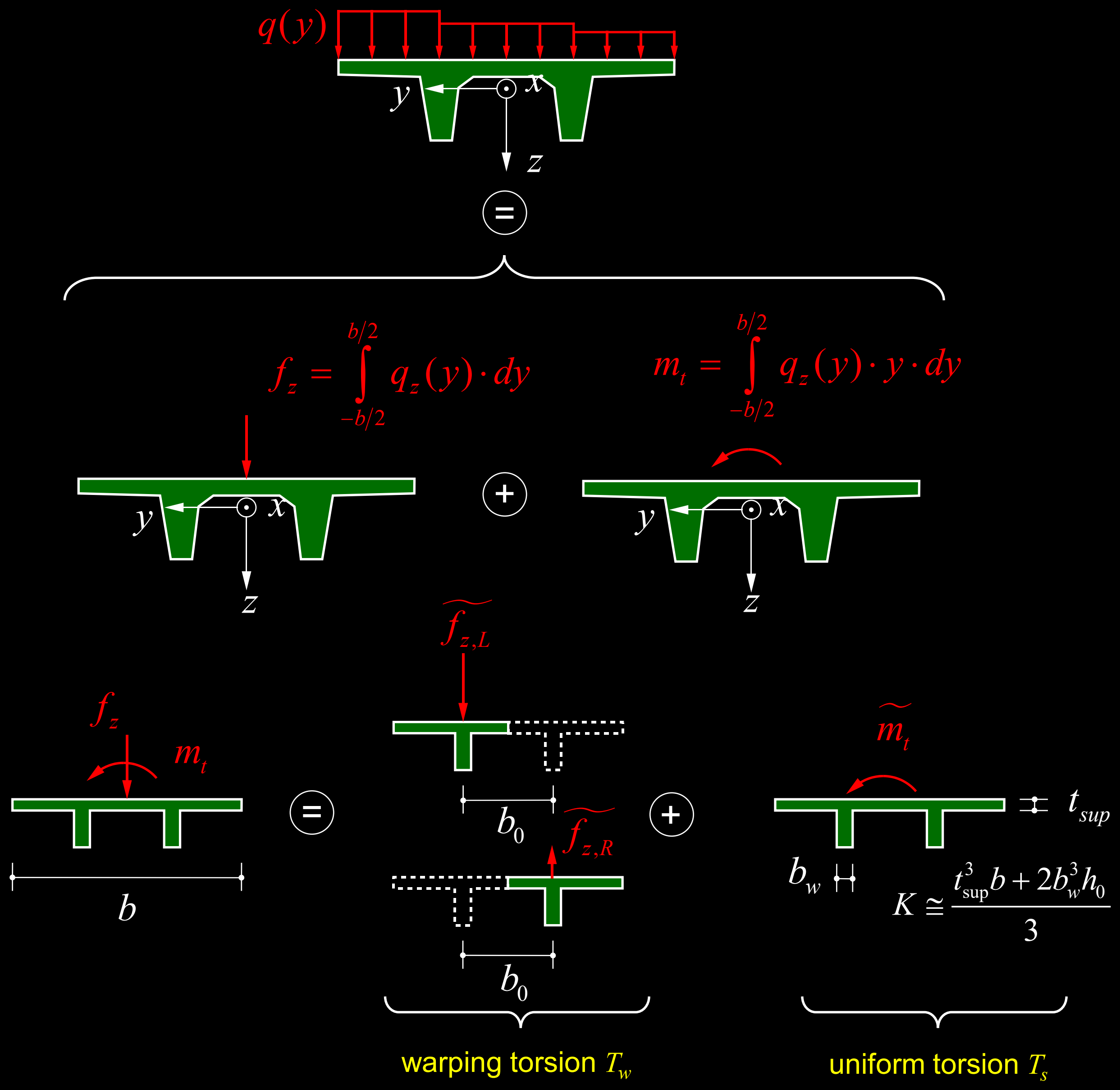
- half the applied vertical load f_z and an additional vertical load corresponding to the torques transferred by warping torsion T_w

$$\tilde{f}_{z,L,R} = \frac{f_z}{2} \pm \frac{m_t}{b_0} \cdot \frac{m_{t,w}}{m_{t,w} + m_{t,s}} = \frac{f_z}{2} \pm \frac{m_{t,w}}{b_0}$$

- half of the torques transferred by uniform torsion T_s

$$\tilde{m}_t = m_t \cdot \frac{m_{t,s}}{m_{t,w} + m_{t,s}} = m_{t,s}$$

  the latter being carried by the web and the part of the deck belonging to each half girder (by uniform torsion of the components constituting the cross-section).



Spine model for open cross-sections: Equilibrium model

As mentioned above, the ratio $m_{t,s}/m_{t,w}$ varies along the span and depends on the position of applied loads.

The distribution $m_{t,s}/m_{t,w}$ can theoretically be determined by the condition that the rotations of the cross-section caused by $m_{t,s}$ and $m_{t,w}$ be equal along the entire span:

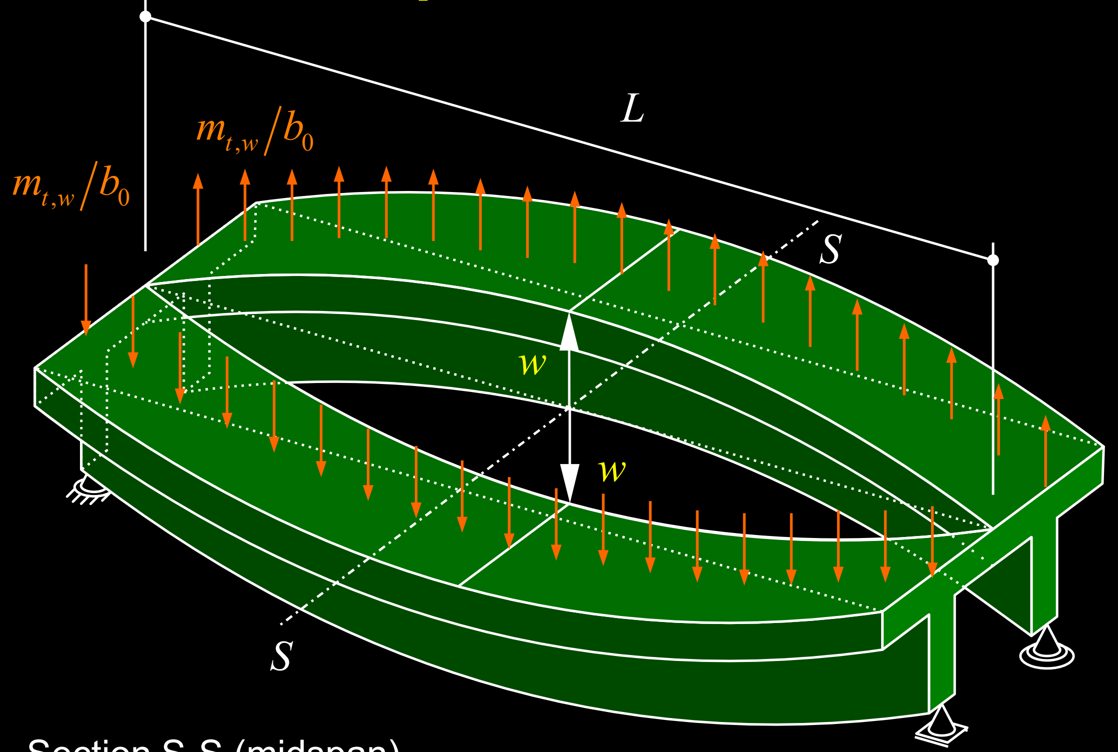
$$\forall x: \quad \varphi_s(x) = \varphi_w(x) = \frac{w_L - w_R}{b_0}$$

Nevertheless, these calculations are complicated and time-consuming, and “accurate” results are hardly ever required (nor obtained, linear elasticity ≠ reality).

Therefore, in concrete girders

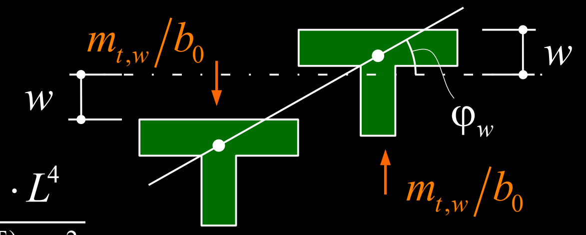
- a constant ratio $m_{t,s}/m_{t,w}$ over the entire girder length is usually assumed
- which may be determined by compatibility at midspan (see figure) or using the chart on the next slide
- or simply estimated using typical values
  … $m_{t,s}/m_{t,w} \approx 0.5$ for long spans
  … $m_{t,s}/m_{t,w} \approx 0.25$ for short spans

In steel and composite girders, refined calculations may be required (limited ductility due to stability issues).



Section S-S (midspan)
→ simply supported girder and uniformly distributed torque

$$w = \frac{5 \cdot m_{t,w}/b_0 \cdot L^4}{384 EI^{(T)}}$$



$$\varphi_w = \frac{2w}{b_0} = \frac{5 \cdot m_{t,w} \cdot L^4}{192 EI^{(T)} \cdot b_0^2} = \frac{5 \cdot m_{t,w} \cdot L^4}{96 EI^{(TT)} \cdot b_0^2}$$

Spine model for open cross-sections: Equilibrium model

As mentioned above, the ratio $m_{t,s}/m_{t,w}$ varies along the span and depends on the position of applied loads.

The distribution $m_{t,s}/m_{t,w}$ can theoretically be determined by the condition that the rotations of the cross-section caused by $m_{t,s}$ and $m_{t,w}$ be equal along the entire span:

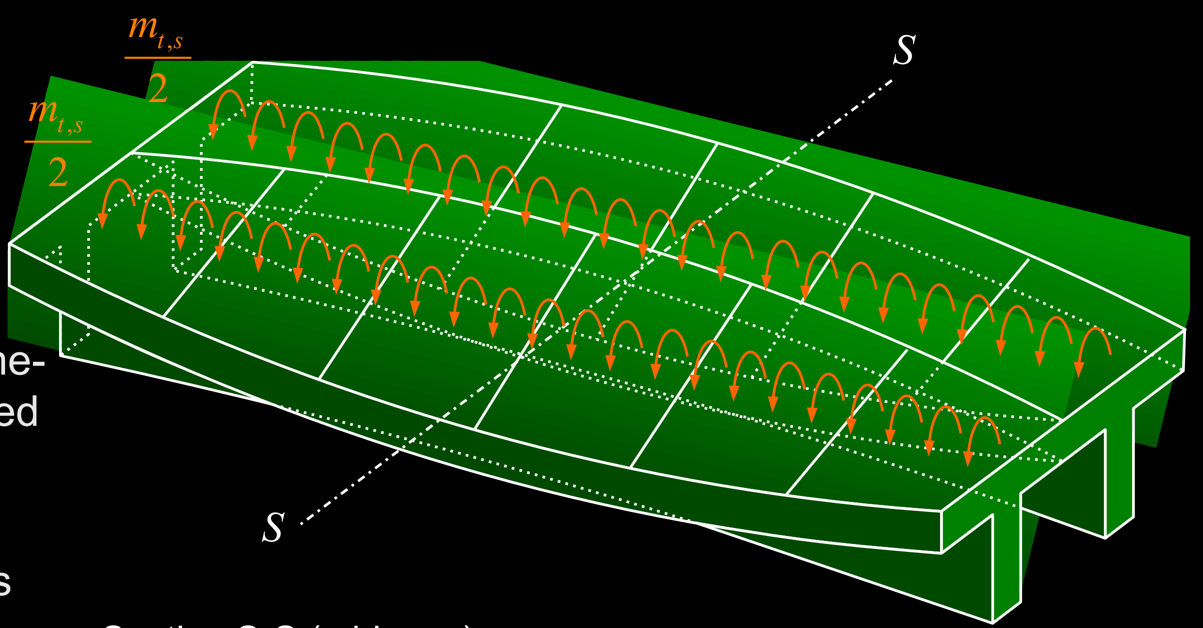
$$\forall x: \quad \varphi_s(x) = \varphi_w(x) = \frac{w_L - w_R}{b_0}$$

Nevertheless, these calculations are complicated and time-consuming, and "accurate" results are hardly ever required (nor obtained, linear elasticity $\neq$ reality).

Therefore, in concrete girders

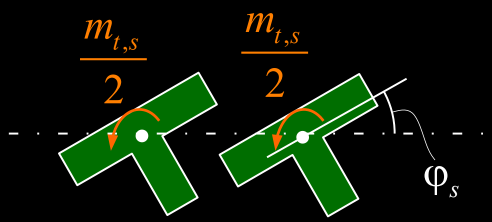
- a constant ratio $m_{t,s}/m_{t,w}$ over the entire girder length is usually assumed
- which may be determined by compatibility at midspan (see figure) or using the chart on the next slide
- or simply estimated using typical values
  … $m_{t,s}/m_{t,w} \approx 0.5$ for long spans
  … $m_{t,s}/m_{t,w} \approx 0.25$ for short spans

In steel and composite girders, refined calculations may be required (limited ductility due to stability issues).



Section S-S (midspan)
→ simply supported girder and uniformly distributed torque

$$GK^{(TT)} \cong G\frac{t_{sup}^3 b + 2b_w^3 h_0}{3}$$



$$\varphi_s = \int_0^{L/2} \frac{T_s \cdot \overline{T}}{GK^{(T)}} dx = \frac{2}{GK^{(T)}} \cdot \frac{L}{2} \cdot \frac{(m_{t,s}/2)L}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{(m_{t,s}/2)L^2}{8GK^{(T)}} = \frac{m_{t,s}L^2}{8GK^{(TT)}}$$

Spine model for open cross-sections: Equilibrium model

As mentioned above, the ratio $m_{t,s}/m_{t,w}$ varies along the span and depends on the position of applied loads.

The distribution $m_{t,s}/m_{t,w}$ can theoretically be determined by the condition that the rotations of the cross-section caused by $m_{t,s}$ and $m_{t,w}$ be equal along the entire span:

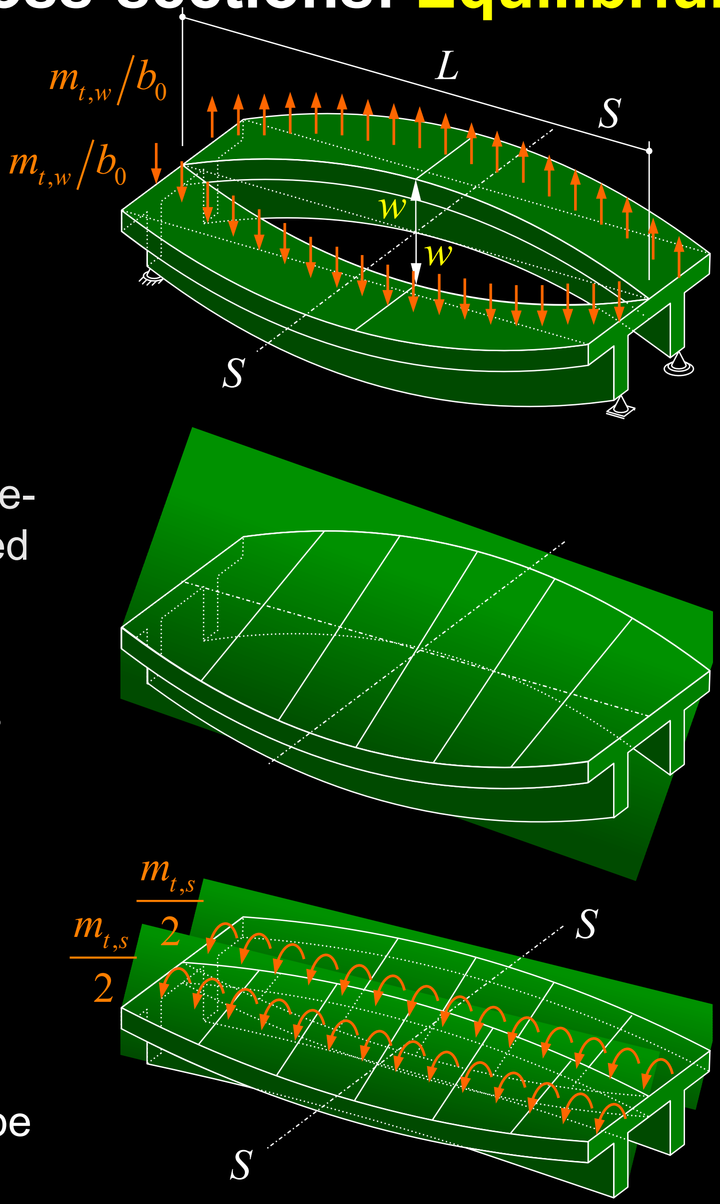
$$\forall x: \quad \varphi_s(x) = \varphi_w(x) = \frac{w_L - w_R}{b_0}$$

Nevertheless, these calculations are complicated and time-consuming, and “accurate” results are hardly ever required (nor obtained, linear elasticity $\neq$ reality).

Therefore, in concrete girders

- a constant ratio $m_{t,s}/m_{t,w}$ over the entire girder length is usually assumed
- which may be determined by compatibility at midspan (see figure) or using the chart on the next slide
- or simply estimated using typical values
  … $m_{t,s}/m_{t,w} \approx 0.5$ for long spans
  … $m_{t,s}/m_{t,w} \approx 0.25$ for short spans

In steel and composite girders, refined calculations may be required (limited ductility due to stability issues).



Section S-S (midspan)
→ simply supported girder and uniformly distributed torque

$$\varphi_w = \frac{2w}{b_0} = \frac{5 \cdot m_{t,w} \cdot L^4}{96 EI^{(TT)} \cdot b_0^2}$$

$$\varphi_w \overset{!}{=} \varphi_s \rightarrow \frac{m_{t,s} L^2}{8 GK^{(TT)}} = \frac{5 \cdot m_{t,w} \cdot L^4}{96 EI^{(TT)} b_0^2}$$

$$\rightarrow \frac{m_{t,s}}{m_{t,w}} = \frac{5}{12} L^2 \frac{GK^{(TT)}}{EI^{(TT)} b_0^2}$$

$$\varphi_s = \int_0^{L/2} \frac{T_s \cdot \overline{T}}{GK^{(T)}} dx = \frac{m_{t,s} L^2}{8 GK^{(TT)}}$$

Spine model for open cross-sections: Equilibrium model

On the previous slide, the $m_{t,s}/m_{t,w}$ was estimated as

$$\frac{m_{t,s}}{m_{t,w}} = \frac{5}{12} L^2 \frac{GK^{(\mathrm{TT})}}{EI^{(\mathrm{TT})} b_0^2} \quad \left(= \frac{T_s}{T_w} \text{ if } \frac{m_{t,s}}{m_{t,w}} = \text{const } \forall x \right)$$

where $EI^{(TT)}$ = bending stiffness of full section and

$$GK^{(\mathrm{TT})} \cong G \frac{t_{\mathrm{sup}}^3 b + 2 b_w^3 h_0}{3}$$

is the uniform torsional stiffness of the entire cross-section. The warping constant of the cross-section [m^6] is approximately

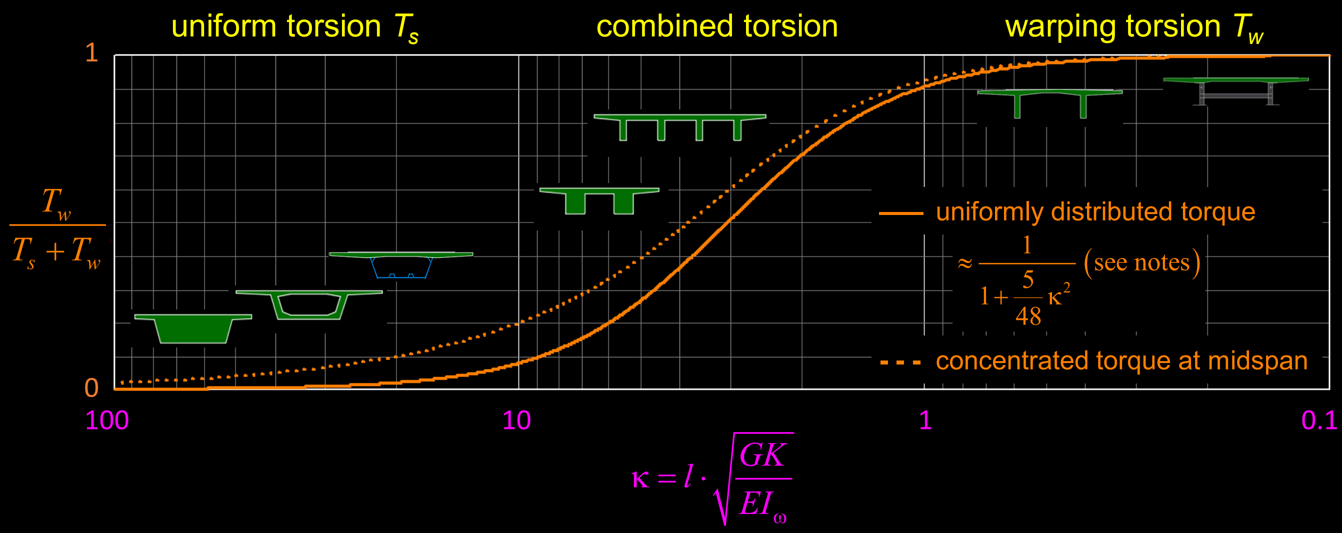
$$I_\omega \approx 2 \cdot I^{(\mathrm{T})} \cdot \frac{b_0^2}{4} \approx \frac{I^{(\mathrm{TT})} b_0^2}{4} \quad \left(I^{(\mathrm{T})} \approx \frac{I^{(\mathrm{TT})}}{2} \right)$$

and hence, the ratio m_s/m_w is equal to:

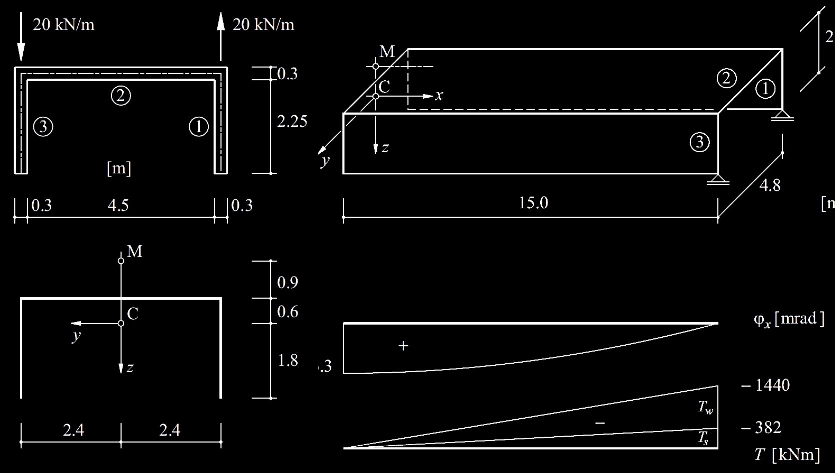
$$\frac{m_{t,s}}{m_{t,w}} = \frac{5}{48} L^2 \frac{GK}{EI_\omega} = \frac{5}{48} \kappa^2; \quad \frac{m_{t,w}}{m_{t,w} + m_{t,s}} = \frac{1}{1 + \frac{5}{48}\kappa^2} \qquad \kappa = L\sqrt{\frac{GK}{EI_\omega}}$$

The parameter κ (used before) is thus indeed a measure for the ratio of uniform to warping torsion.

Note: The equations and the diagram apply to a simply supported girder under uniform torque. For other configurations, similar results are obtained.



Example (figures and exact result see Marti, Theory of structures)



E = 30 GPa

G = 12.5 GPa

$I^{(T)} = 0.87$ m^4

[m] $I_\omega \approx I^{(T)} \cdot (b_0)^2/2 = 10.06$ m^6

$K^{(TT)} = 0.0864$ m^4

$\rightarrow \kappa \approx 1.79$

$\rightarrow T_w/(T_s+T_w) \approx 0.75$ (diagram)

(«exact»:(1440-382)/1440 = 0.73)

Spine model for open cross-sections: Equilibrium model

The assumption of a constant ratio of uniform torsion to warping torsion $m_{t,s}/m_{t,w}$, without strictly satisfying compatibility, can be justified in ULS design by the lower-bound theorem of the theory of plasticity (see notes) if

- ductile behaviour is ensured and
- the dimensioning for T_s and T_w is carried out consistently

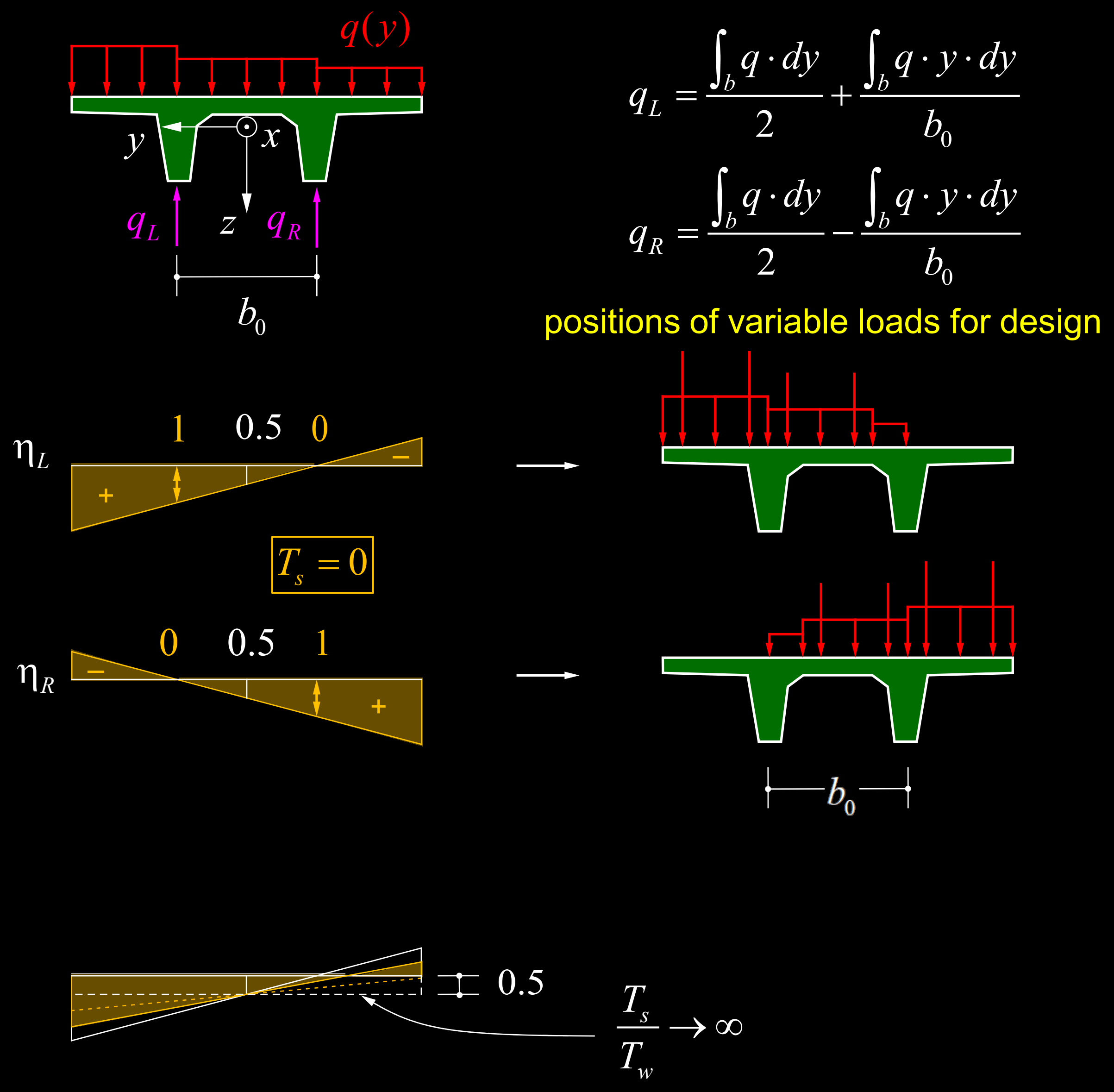
For example, in preliminary design one may (see figure)

- assume $T_s = 0$ (i.e. pure warping torsion) (analogous to assuming $T_w = 0$ in box girders)
- design each half of a double-T girder for the loads corresponding to the support reactions of a deck simply supported on the two webs (q_L and q_R)

→ governing load combinations (positioning of variable loads) for each half girder obtained using the influence line for the support reactions of a simply supported beam, which can be interpreted as "transverse influence line"

Assuming $T_s \neq 0$ the influence lines remain straight but become flatter, with lower extreme values.

Regarding transverse loads and bending stiffness, see notes.

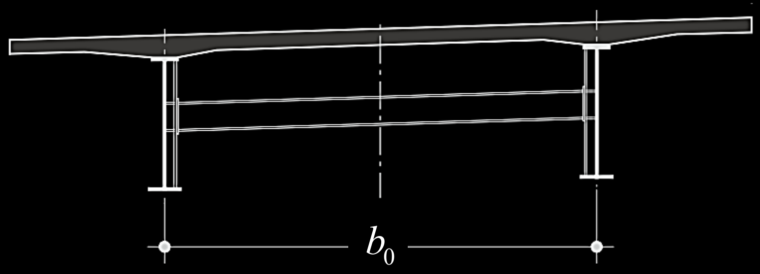


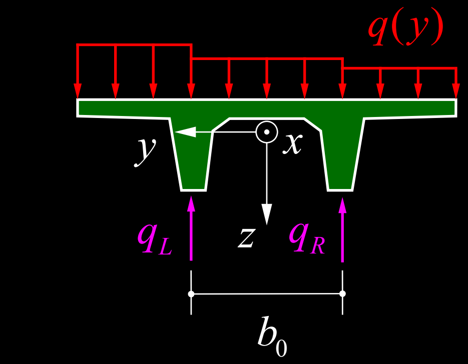
$$q_L = \frac{\int_b q \cdot dy}{2} + \frac{\int_b q \cdot y \cdot dy}{b_0}$$

$$q_R = \frac{\int_b q \cdot dy}{2} - \frac{\int_b q \cdot y \cdot dy}{b_0}$$

Spine model for open cross-sections: Equilibrium model

The assumption $T_s = 0$ (pure warping torsion) is particularly appropriate for the design of steel and steel-concrete composite bridges with two plate girders, since the torsional stiffness of the latter is indeed negligible.

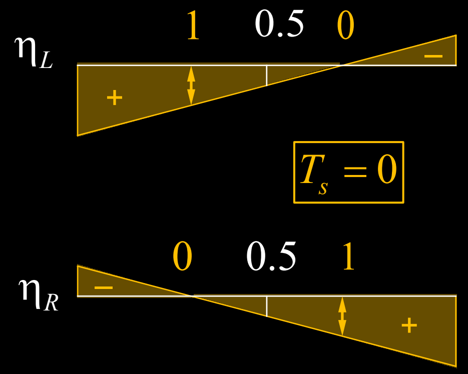


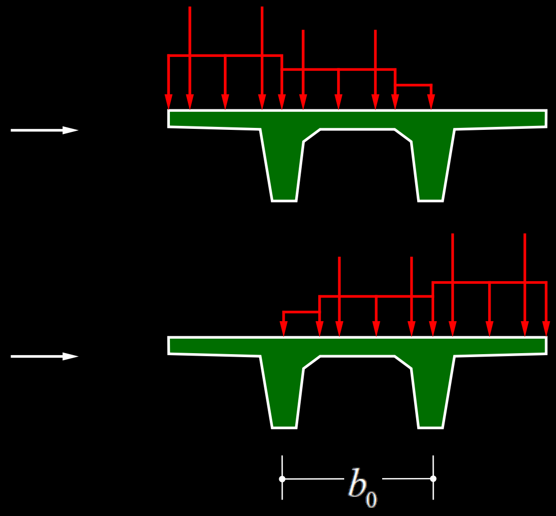


$$q_L = \frac{\int_b q \cdot dy}{2} + \frac{\int_b q \cdot y \cdot dy}{b_0}$$

$$q_R = \frac{\int_b q \cdot dy}{2} - \frac{\int_b q \cdot y \cdot dy}{b_0}$$

positions of variable loads for design





The simplified model assuming $T_s = 0$ is on the safe side for the design of the longitudinal girders, and thus often sufficient for their ULS and SLS design in straight bridges with such cross-sections.

However, in skew or curved steel and composite bridges, determining camber requires more refined models to avoid fit-up issues, see respective chapters (final slides in skew / curved bridge presentations).

Spine model for open cross-sections: Multi-girder bridges

In multi-girder bridges (open cross-section with more than two webs/beams):

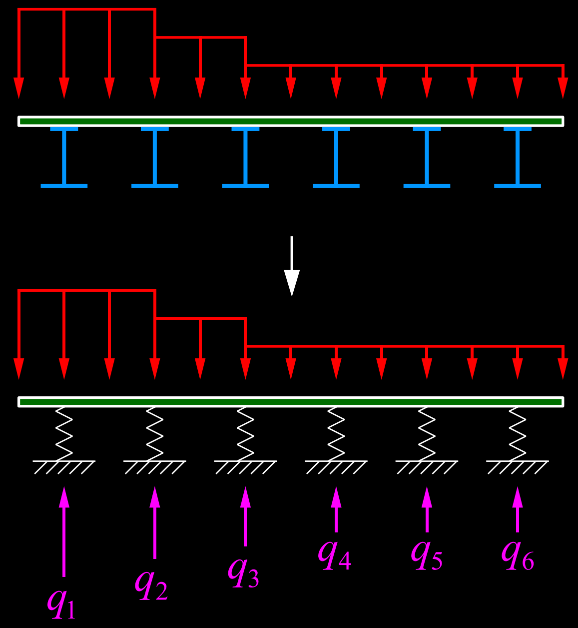
- determination of $m_{t,s}/m_{t,w}$ is further complicated since the deck is statically indeterminate in the transverse direction (even if $GK = 0$ is assumed for individual webs/beams, see top figure)

→ loads carried by each web cannot be determined by equilibrium even for $T_s = 0$

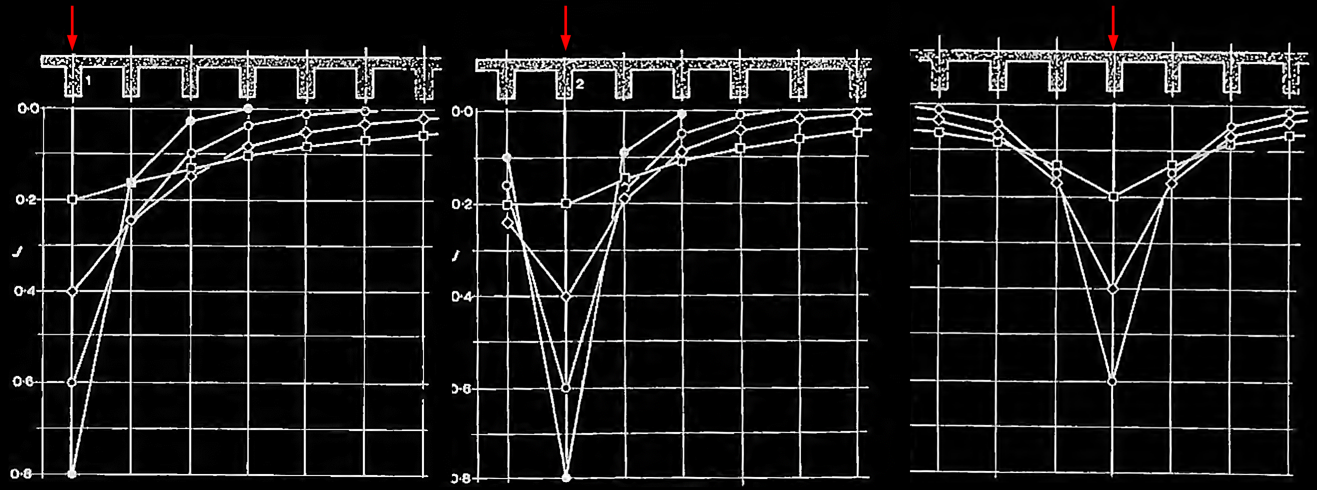
→ determination of the loads q_i carried by each web requires several assumptions, but remains complicated

→ still no direct information on transverse behaviour needs to be analysed

→ grillage models should be used for multi-girder bridges



Edge beam loaded | Beam next to edge loaded | Interior beam loaded



Older textbooks and design recommendations, and several existing bridge design codes, contain detailed information on the analysis of multi-girder bridges. These are outlined on the following slide without entering into details.

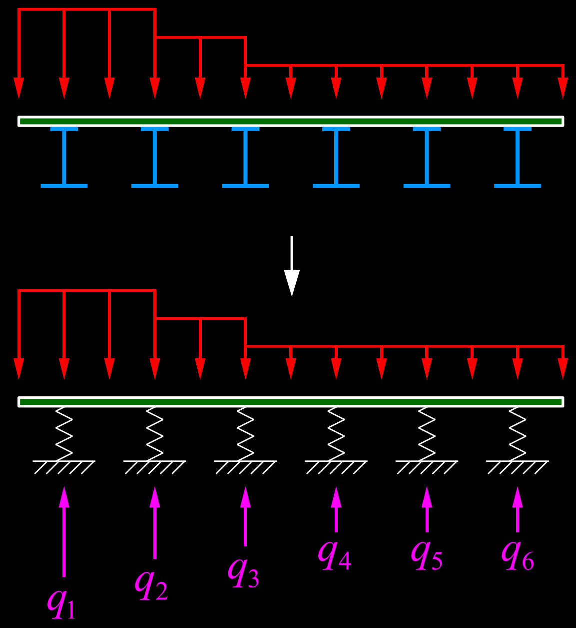
Spine model for open cross-sections: Multi-girder bridges

Design charts (bottom figure) show load distribution factors that may be used to determine the loads acting on each single web/beam of a multi-girder bridge.

These factors may be used in design for determining e.g.

→ longitudinal shear and bending moments

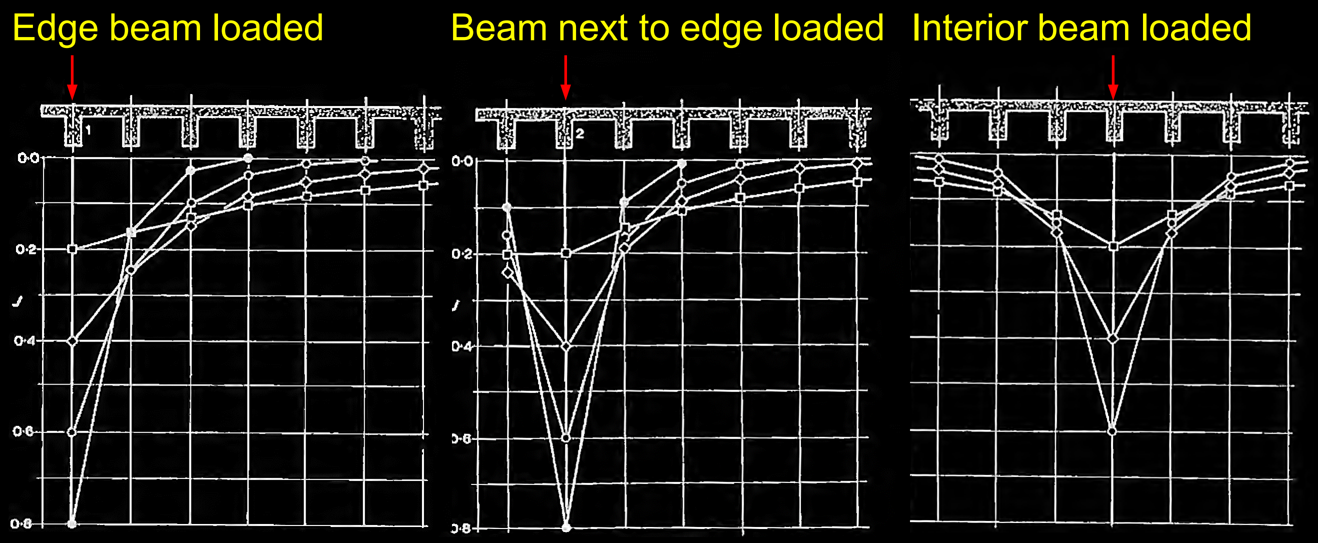
→ damage factor λ_4 for fatigue verifications (bending moments due to fatigue load in different positions)



The values given by the design charts

- essentially correspond to transverse influence lines
- show that, depending on the deck configuration (cantilevers, beam spacings) the edge beams and adjoining interior beams receive significantly higher load than the standard interior beams.

Note that the peak values of the design charts (influence lines) depend on the flexural and torsional stiffness ratios in the longitudinal and transverse directions. Separate charts exist for determining these peak values.



Superstructure / Girder bridges

Bridge Girder – Grillage model (Trägerrostmodell)

Grillage model – General aspects

Girders with open cross-section, as well as multi-cell box girders, can be analysed with grillage models.

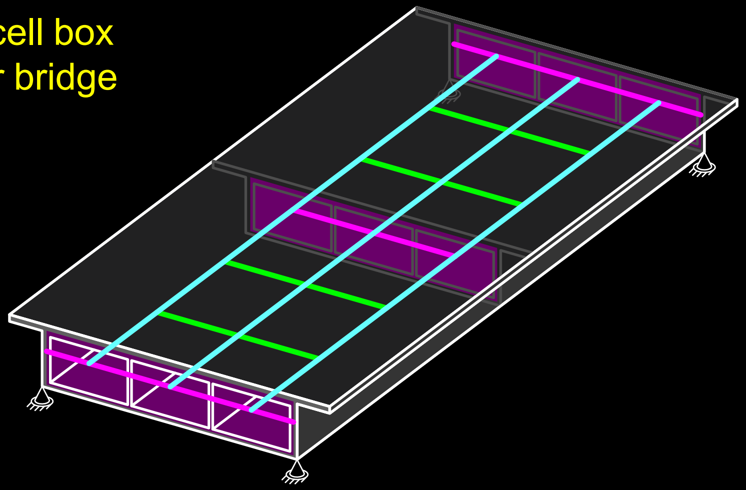
In a grillage model, the girder is idealised as a grid of longitudinal and transverse beams, where

- longitudinal beams “*LB*”
  → represent webs (concrete), beams (steel) or cells of box girders
- transverse beams (usually no more than 3 to 5 per span)
  → represent diaphragms or transverse ribs “*D*”
  → simulate the stiffness of the deck and (if applicable) the bottom slab (“virtual diaphragms”) “TB”

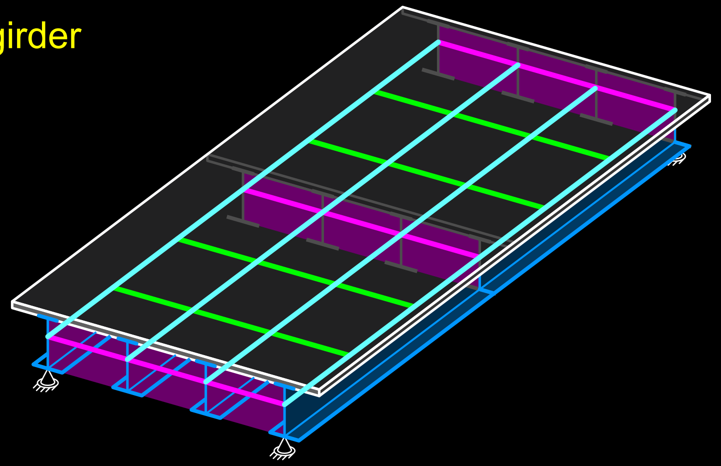
Usually, an orthogonal grid is chosen, and consideration of a plane (two-dimensional) grillage is sufficient.

In specific cases, three-dimensional analysis may be useful, particularly to account for membrane action of the deck slab in girders with open cross-section.

Multicell box girder bridge



Multi-girder bridge



Grillage model – General aspects

The stiffnesses of the longitudinal and transverse members should reasonably represent the real bridge girder.

To this end, member stiffnesses are essentially determined as for the girder of a spine model, accounting for

- cracking (in non-prestressed members)
- long-term effects
- composite action in composite members

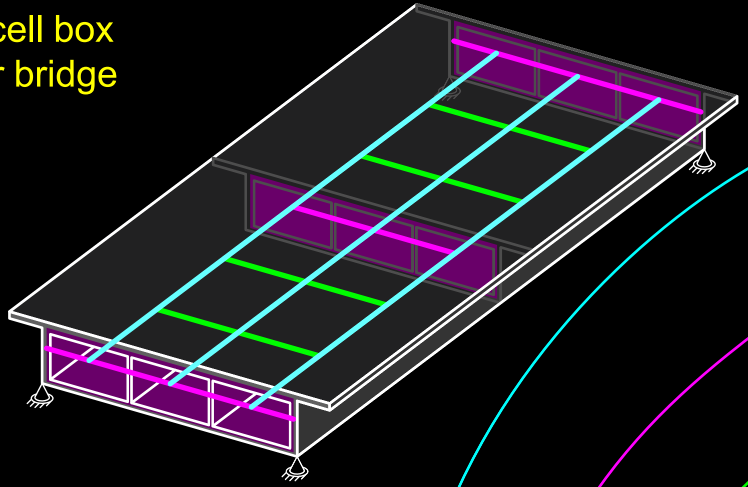
Even the most complex model will not be able to represent the "true" behaviour, particularly due to

- nonlinearities due to cracking
- time dependent effects

→ grillage models should be as simple as possible to capture the dominant phenomena

→ in preliminary design and ULS design of concrete girders, a torsionless grillage ($GK = 0$ for all members) is often sufficient (this can be justified by the lower bound theorem of plasticity theory if ductile behaviour is guaranteed, see *spine model for open cross-section – equilibrium model*)

Multicell box girder bridge



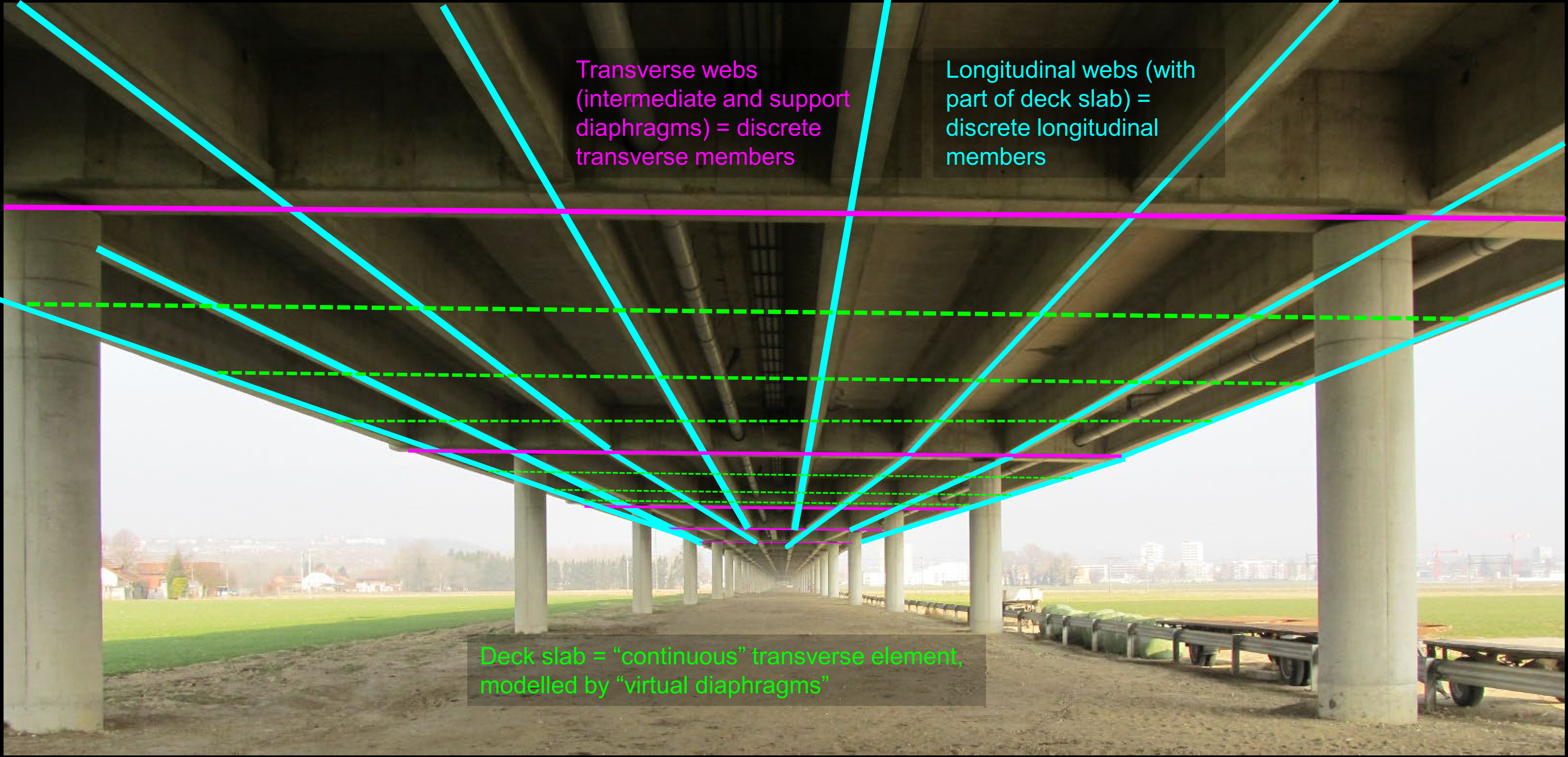
webs = longitudinal members

webs = transverse members

deck slab = transverse member

Multi-girder bridge

Grillage model – General aspects



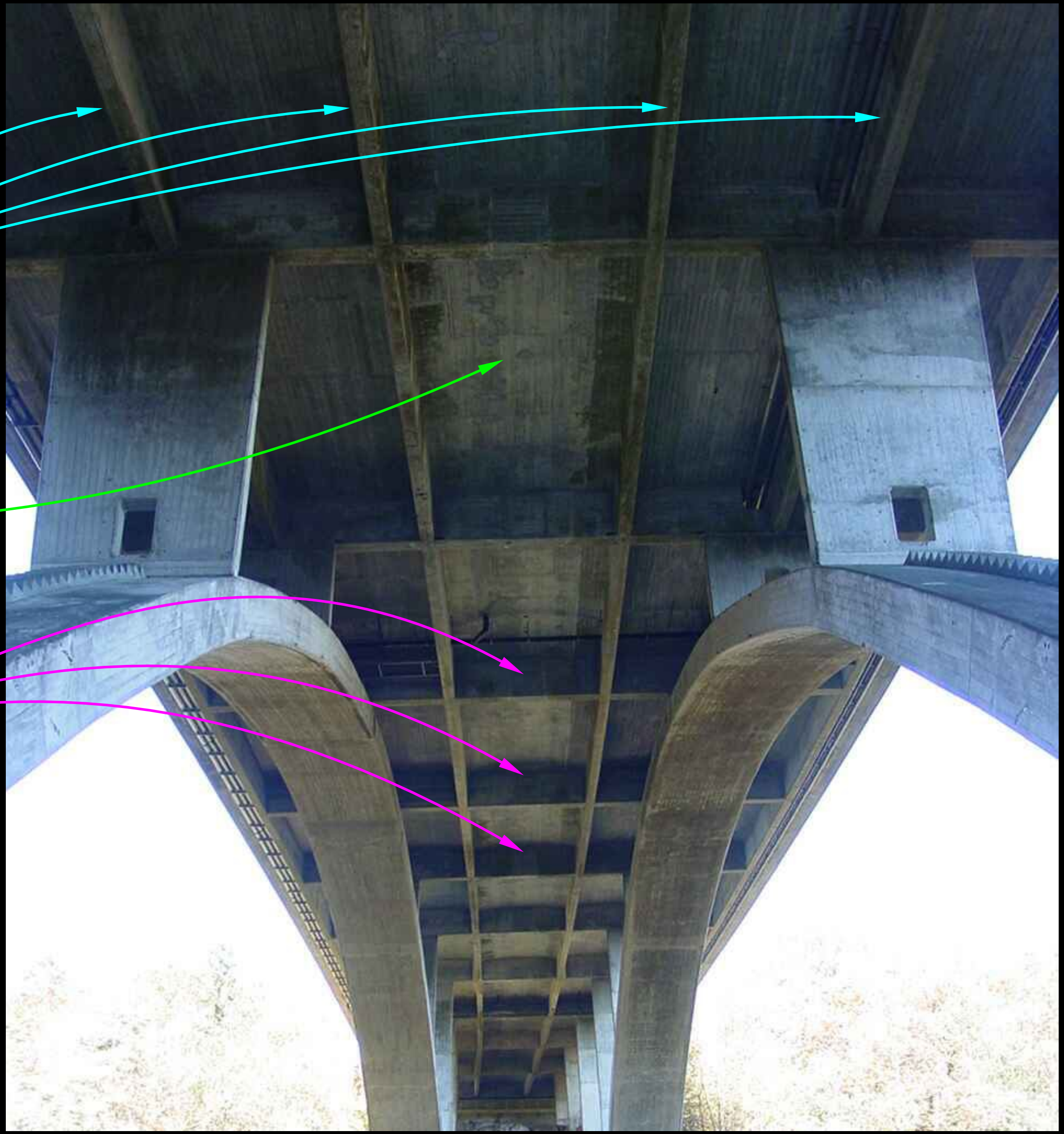
Grillage model – General aspects

Grillage models can also be used for analysing bridge girders of other bridge types

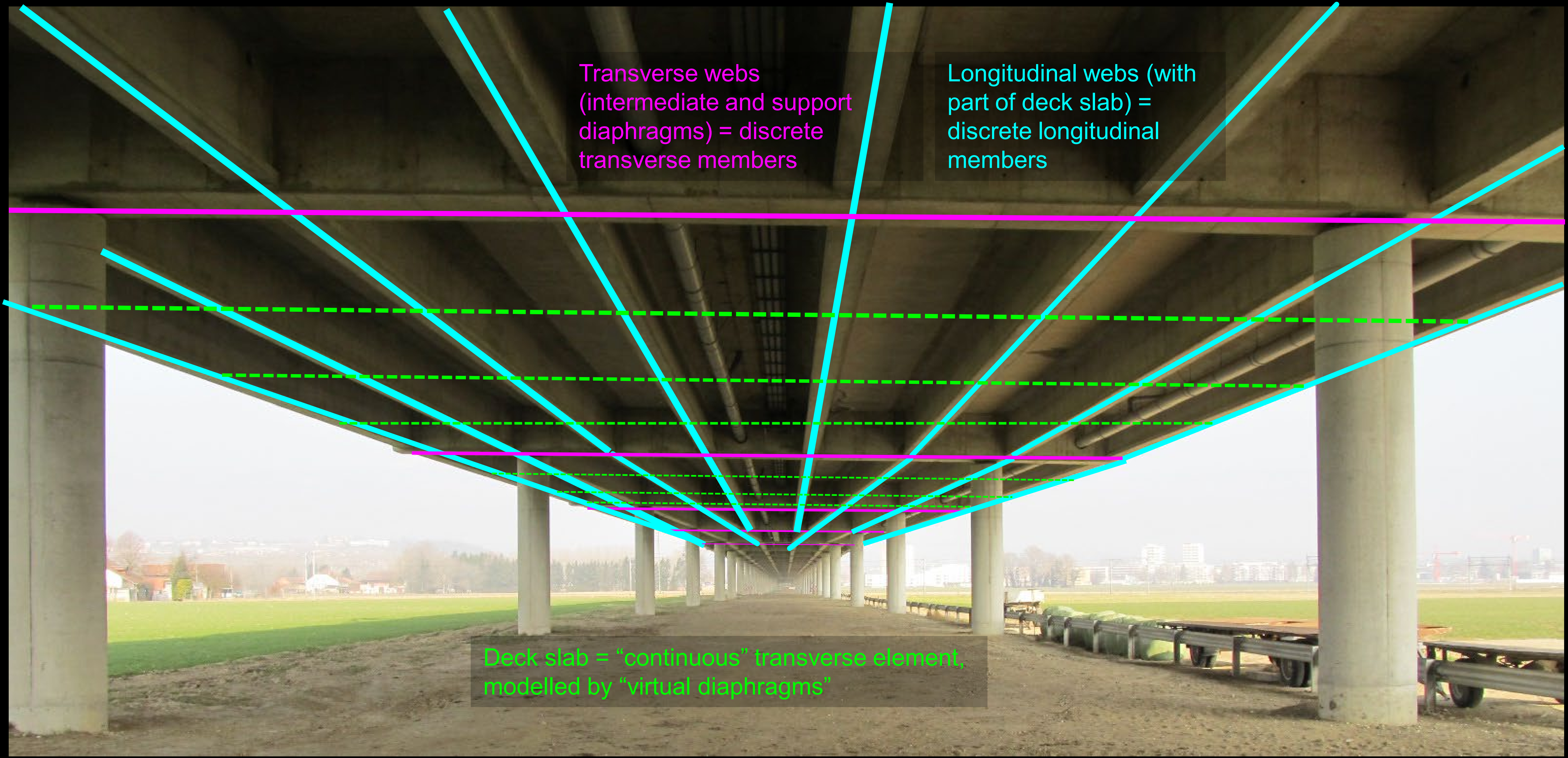
Longitudinal webs (with part of deck slab) = discrete longitudinal members

Deck slab = “continuous” transverse element, modelled by “virtual diaphragms”

Transverse webs
(intermediate and support diaphragms) = discrete transverse members



Grillage model – General aspects



Grillage model – General aspects

The definition of loads (particularly traffic loads) in grillage models may be quite time-consuming since loads have to be defined with respect to the grillage members

→ introduce additional, virtual beams along traffic lanes (connected to grillage) and apply loads to these

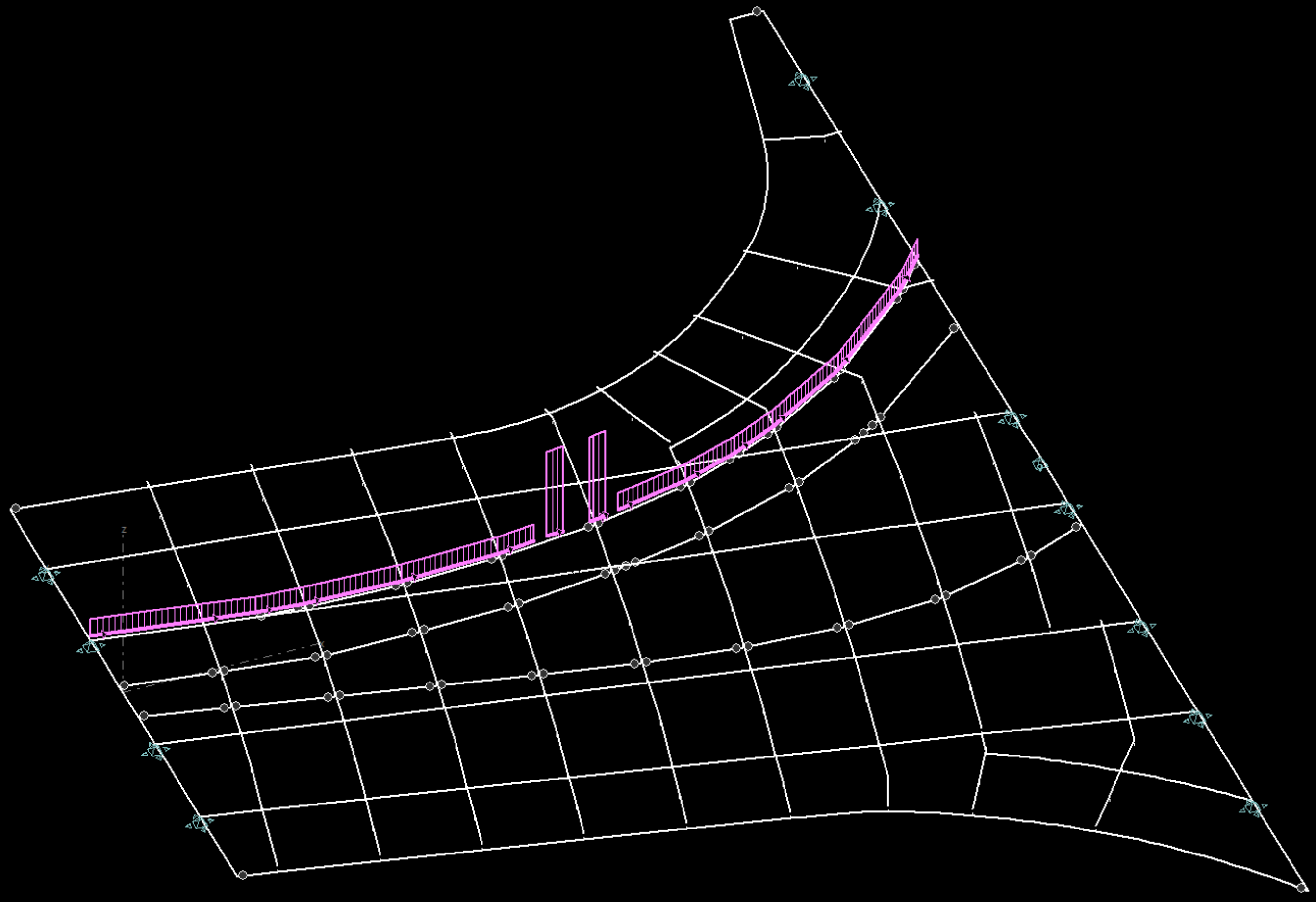
→ some software programs offer the possibility to define a virtual surface simulating the deck, to which the loads can be applied in their actual position (internally, a slab calculation is run)

In all cases, it must be ensured that the self-weight of the girder is correctly modelled: Avoid that the deck weight is accounted for twice

→ assign weight to longitudinal beams and diaphragms

→ model transverse beams representing deck and bottom slab ("virtual diaphragms") as weightless

If cross-sections are defined in a frame analysis software, stiffnesses and weights are assigned automatically. They need to be partially overwritten (stiffnesses) or deleted (weight assigned to the transverse beams).



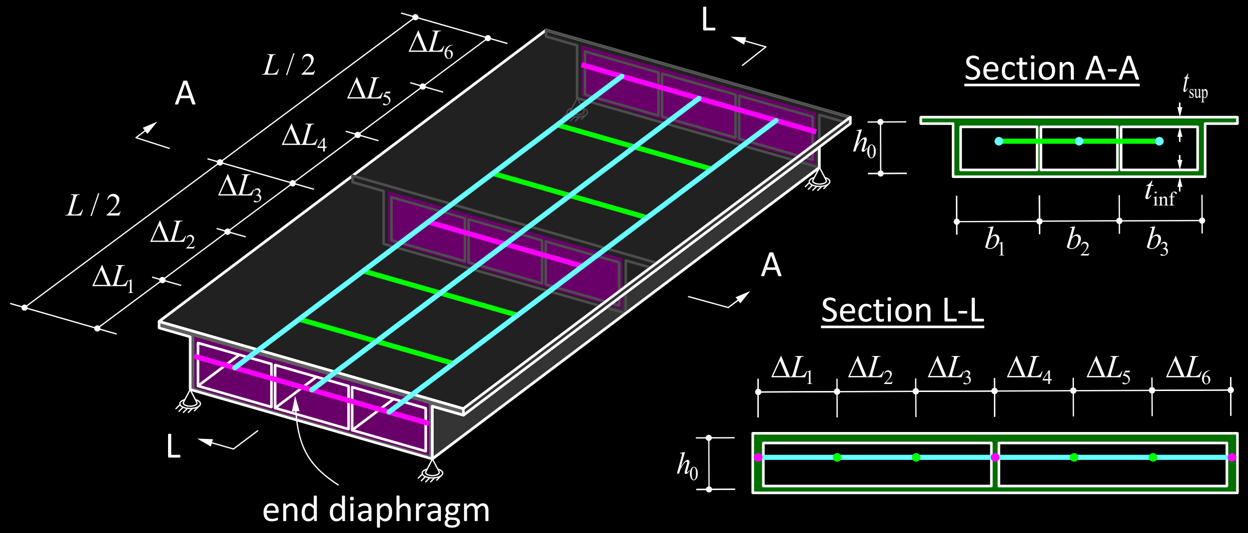
Grillage model – Multi-cell box girders and voided slabs

In multi-cell box girders and voided slabs, there are two options for defining the longitudinal beams LB” of the grillage.

- Option A (prioritise longitudinal beams):

→ one beam per cell → $n_{LB} = n_{cells}$

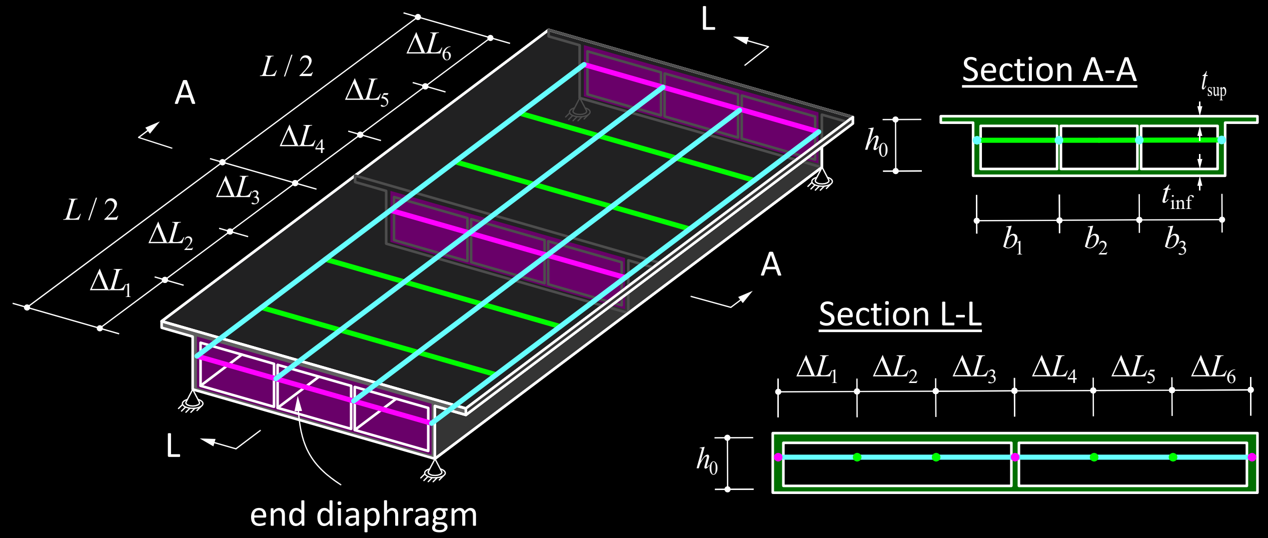
→ full torsional stiffness of cross-section GK_{tot} assigned to (distributed among) longitudinal beams



- Option B (treat torsion as in a slab):

→ one beam per web → $n_{LB} = n_{cells} + 1$

→ torsional stiffness of the cross-section GK_{tot} shared
$GK_{tot}/2$ → distributed among longitudinal beams
$GK_{tot}/2$ → assigned to transverse beams



Similar results are obtained using both options. Option A appears more appropriate for box girders with few cells, and option B for voided slabs.

Grillage model – Multi-cell box girders and voided slabs

Bending and shear stiffnesses of longitudinal beams

- In grillage option A and B, each longitudinal beam is assigned its share of the stiffness $EI_{y,tot}$ and EA_{tot} of the entire girder:

$$EI_{y,LBi} \cong \frac{b_i}{\Sigma b_i} EI_{y,tot}, \qquad EA_{LBi} \cong \frac{b_i}{\Sigma b_i} EA_{tot}$$

  alternatively, each longitudinal beam can be assigned the stiffness of its cross-section (see notes)

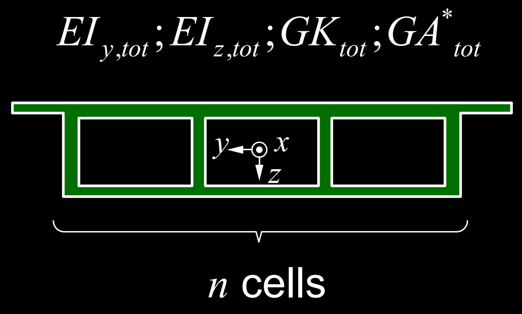
$$EI_{y,LBi} \cong \int_{A_{LBi}} Ez^2 dA_{LBi}, \qquad EA_{LBi} \cong \int_{A_{LBi}} EdA_{LBi}$$

- In grillage option A and B, each longitudinal beam is assigned the stiffness $EI_{z,LBi}$ corresponding to its cross-section (much smaller than $EI_{z,tot} \cdot b_i / \Sigma b_i$, see notes) :

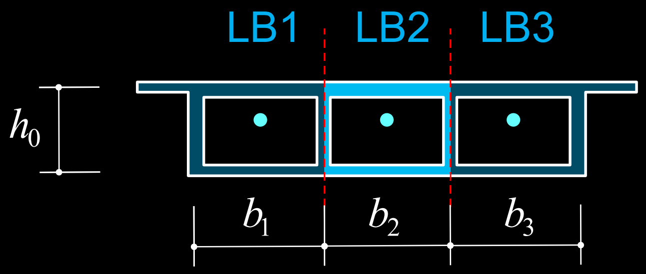
$$EI_{z,LBi} \cong \int_{A_{LBi}} Ey^2 dA_{LBi}$$

- In grillage option A and B, each longitudinal beam is assigned its share of the total shear stiffness GA^*_{tot} of the entire girder, usually neglecting shear deformations:

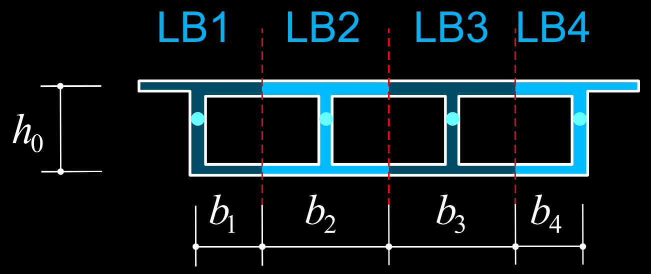
$$GA^*_{LBi} = \frac{b_i}{\Sigma b_i} GA^*_{tot} \rightarrow \infty$$



Grillage option A



Grillage option B



Grillage model – Multi-cell box girders and voided slabs

Torsional stiffness of longitudinal beams

- In grillage option A, each longitudinal beam is assigned its share of the full total torsional stiffness GK_{tot} of the entire girder

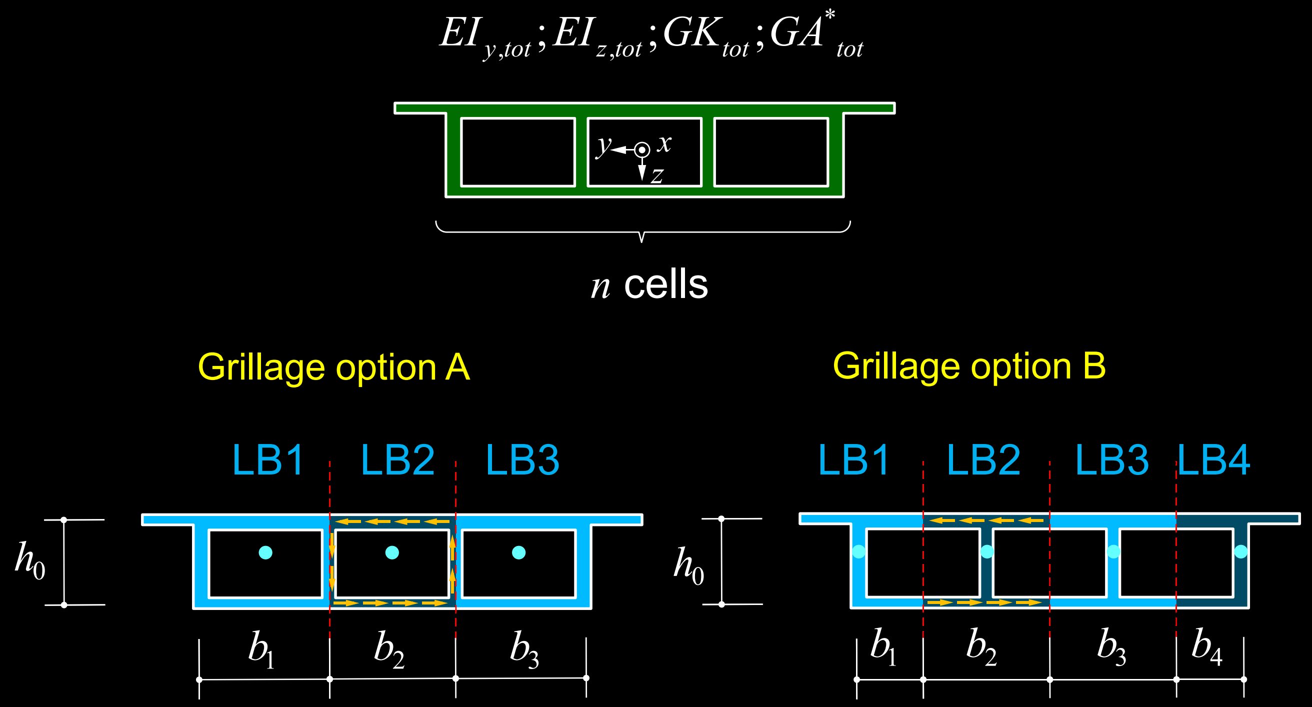
$$GK_{LBi}^{A} = \frac{b_i}{\sum b_i} GK_{tot} \qquad (\text{but } GK_{TBi}^{A} = 0, \text{ see behind})$$

and the resulting torsional moments are assigned to the box section of each longitudinal beam as in a single cell box girder (see notes)

- In grillage option B, each longitudinal beam is assigned only the total torsional stiffness corresponding to the deck and bottom slab, which roughly corresponds to half the total torsional stiffness, i.e.

$$GK_{LBi}^{B} = \frac{b_i}{\sum b_i} \frac{GK_{tot}}{2} \qquad (\text{but } GK_{TBj}^{B} \approx GK_{LBi}^{B} \frac{\Delta L_j}{b_i}, \text{ see behind})$$

and consequently, the resulting torsional moments are assigned to the deck and bottom slab of each longitudinal beam (see notes)

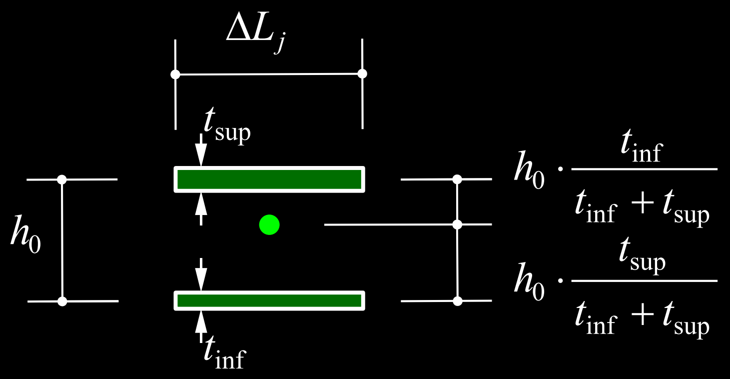


Grillage model – Multi-cell box girders and voided slabs

Bending stiffnesses of transverse beams

- In grillage option A and B, each transverse beam is assigned the bending stiffness EI_y corresponding to the stringer cross-section of deck and bottom slab over the length ΔL = transverse beam spacing):

$$EI_{y,TBi} \cong E\Delta L_j h_0^2 \frac{t_{\inf} t_{\sup}}{t_{\inf} + t_{\sup}}$$



$$EI_{y,TBi} \cong E\Delta L_j \left(\frac{t_{\inf}^3 + t_{\sup}^3}{12} \approx 0 + t_{\sup} \frac{t_{\inf}^2 h_0^2}{\left(t_{\inf} + t_{\sup}\right)^2} + t_{\inf} \frac{t_{\sup}^2 h_0^2}{\left(t_{\inf} + t_{\sup}\right)^2} \right)$$

- In grillage option A and B, each transverse beam is assigned its share of the bending stiffness $EI_{z,tot}$ of the entire girder (deck and bottom slab over full span length):

$$EI_{z,TBi} \cong E \frac{\Delta L_j}{L} \left(t_{\sup} + t_{\inf}\right) \frac{L^3}{12} \qquad (\approx \infty)$$

which is much larger than the sums of the stiffnesses EI_z of the individual beams. This high transverse stiffness ensures that the axial stiffness of the longitudinal beams, and the corresponding higher effective transverse bending stiffness of the entire deck, can be activated (see notes on EI_z of longitudinal beams).

Grillage model – Multi-cell box girders and voided slabs

Shear stiffness of transverse beams

- In grillage option A and B, the transverse beams consist only of the deck and bottom slab, without web

→ assumption $GA^*_{tot} \rightarrow \infty$ is inappropriate for vertical shear

→ act vertically as Vierendeel girders with stiff posts; neglecting deformations of webs GA^* is:

$$GA^*_{z,TBi} = E\frac{\Delta L_j}{b_i^2}\left(t_{sup}^3 + t_{inf}^3\right) \qquad \left(\text{but } GA^*_{y,TB} \rightarrow \infty\right)$$

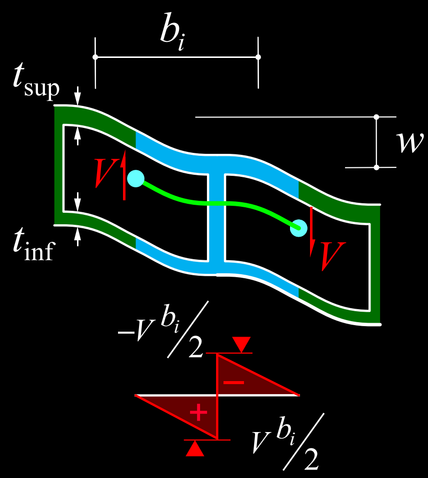
- Despite neglecting deformations of the web, the shear stiffness GA^* of transverse beams is underestimated if the webs are wide or the slabs tapered towards the webs

→ better approximation: replace b_i by clear span of slabs between webs

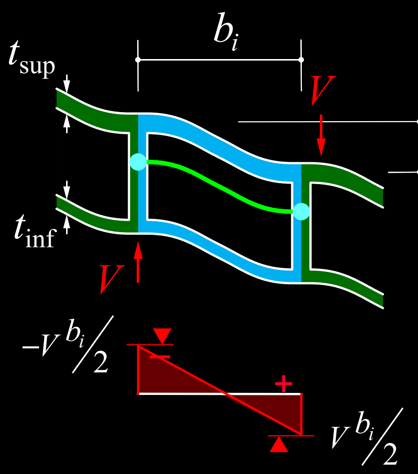
→ use tapered section in virtual work equation

- In voided slabs, the shear stiffness GA_z^* of transverse beams can be estimated by replacing the circular voids by square ones of equal area.

Grillage option A



Grillage option B

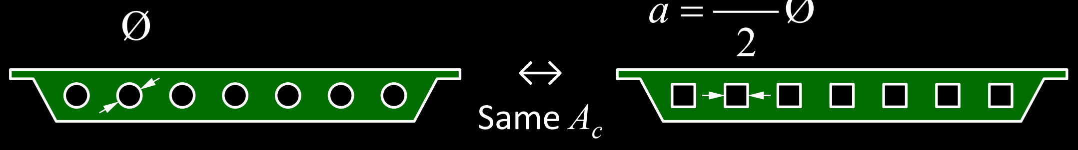


$$V_{sup} = V\frac{EI_{sup}}{EI_{sup} + EI_{inf}} \quad \text{,with } I_{sup} = \frac{\Delta L_j \cdot t_{sup}^{\;3}}{12}\text{, } I_{inf} = \frac{\Delta L_j \cdot t_{inf}^{\;3}}{12}$$

$$w = \int \frac{M\bar{M}}{EI}dx = V_{sup}\frac{b_i}{2}\cdot\frac{b_i}{2}\cdot\frac{b_i}{3EI_{sup}} = \frac{V \cdot b_i^3}{12\cdot(EI_{sup} + EI_{inf})}$$

$$GA^* = \frac{V}{\gamma} = \frac{V \cdot b_i}{w} = E\frac{\Delta L_j}{b_i^2}(t_{sup}^{\;3} + t_{inf}^{\;3})$$

$$a = \frac{\sqrt{\pi}}{2}Ø$$



Grillage model – Multi-cell box girders and voided slabs

Torsional stiffness of transverse beams

- In grillage option A, the entire torsional stiffness GK_{tot} of the girder is assigned to the longitudinal beams, i.e.

$$GK_{TB}^{A} = 0$$

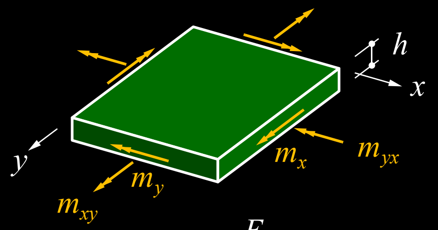
- In grillage option B, about half of the torsional stiffness GK_{tot} is assigned to longitudinal and transverse beams each, similar as in a slab (whose torsional stiffness per direction is half that of a uniaxial beam, see top figure).

→ Transverse beams are assigned the same torsional stiffness per unit length as longitudinal beams, i.e.

$$GK_{TBj}^{B} = GK_{LBi}^{B} \frac{\Delta L_j}{b_i} = \frac{b_i}{\sum b_i} \frac{GK_{tot}}{2} \frac{\Delta L_j}{b_i} = \frac{\Delta L_j}{\sum b_i} \frac{GK_{tot}}{2}$$

- A more refined approach (applicable e.g. if e.g. slab thicknesses vary strongly over the width) consists in using the torsional stiffness of the deck and bottom slab, i.e. (see lower figure)

$$GK_{TBj}^{B} \cong G \cdot \Delta L_j \cdot h_0^{\,2} \frac{2 \cdot t_{inf} \cdot t_{sup}}{t_{inf} + t_{sup}} \qquad \left(\approx EI_{y,TBj} \text{ since } G \approx \frac{E}{2} \right)$$



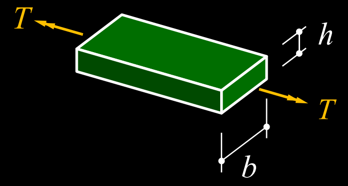
$$G = \frac{E}{2(1+\nu)}$$

$$m_x = \frac{Eh^3}{12(1-\nu^2)}(\chi_x + \nu\chi_y)$$

$$m_y = \frac{Eh^3}{12(1-\nu^2)}(\chi_y + \nu\chi_x)$$

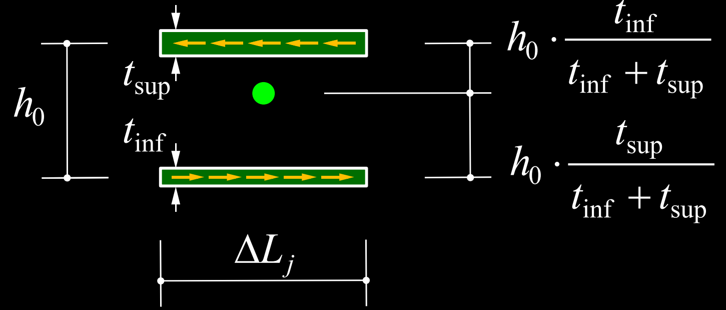
$$m_{xy} = \frac{Eh^3}{12(1+\nu)}\chi_{xy} = G\frac{h^3}{6}\chi_{xy}$$

$$\rightarrow \text{"}K\text{"} = \frac{h^3}{6} \text{ per unit width}$$



$$GK \triangleq \frac{bh^3}{3} \text{ for } h \ll b$$

$$\rightarrow K = \frac{h^3}{3} \text{ per unit width}$$



Grillage model – Multi-cell box girders and voided slabs

Stiffnesses of diaphragms

- Diaphragms are modelled as beams, with an effective width of the deck and bottom slab

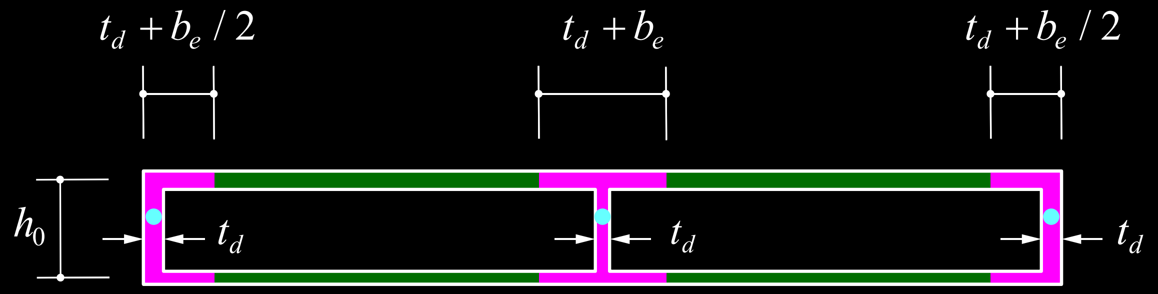
→ Stiffnesses determined accordingly, as for the girder in a spine model, usually neglecting shear deformations:

$$EI_{y,D} = \int_A Ez^2 dA$$

$$EI_{z,D} = \int_A Ey^2 dA$$

$$GA_D^* \to \infty$$

$$GK_D \approx G\left(\frac{h_0 t_D^3}{3} + \frac{\left(t_D + b_{e,\sup}\right)t_{\sup}^3}{3} + \frac{\left(t_D + b_{e,\inf}\right)t_{\inf}^3}{3}\right)$$



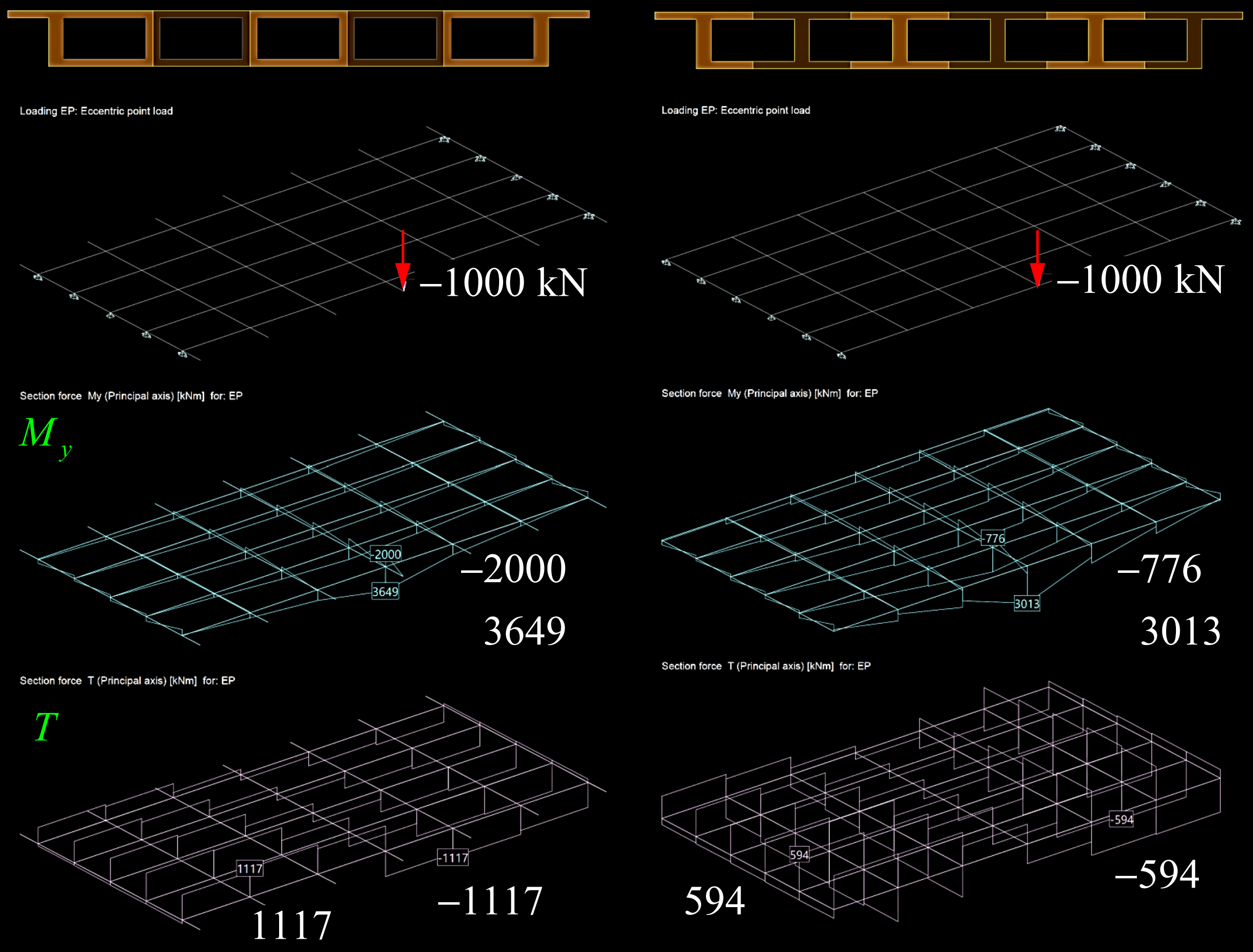
Grillage model – Multi-cell box girders and voided slabs

The figure compares the results of grillage analyses using the options A (left) and B (right) for a single-span girder with a multi-cell box cross-section, loaded by an eccentric concentrated load at midspan.

The results are as expected:

- Deformations are approximately equal in both models (difference < 10%)
- Bending moments are approximately equal in both models (sum over 5 and 6 longitudinal beams)
- Torsional moments result only in longitudinal beams in Model A, but also in transverse beams in Model B
- Torsional moments in the longitudinal beams of Model B are roughly 50% of those in Model A
- Torsional moments in longitudinal and transverse beams of Model B are approximately equal at intersections

→ Both models yield the same results



Grillage model – Open cross-sections (plane grid)

In girders with open cross-sections, the determination of the stiffnesses of longitudinal and transverse beams is much simpler than for multi-cell box girders:

- Longitudinal beams = webs (concrete) / steel beams

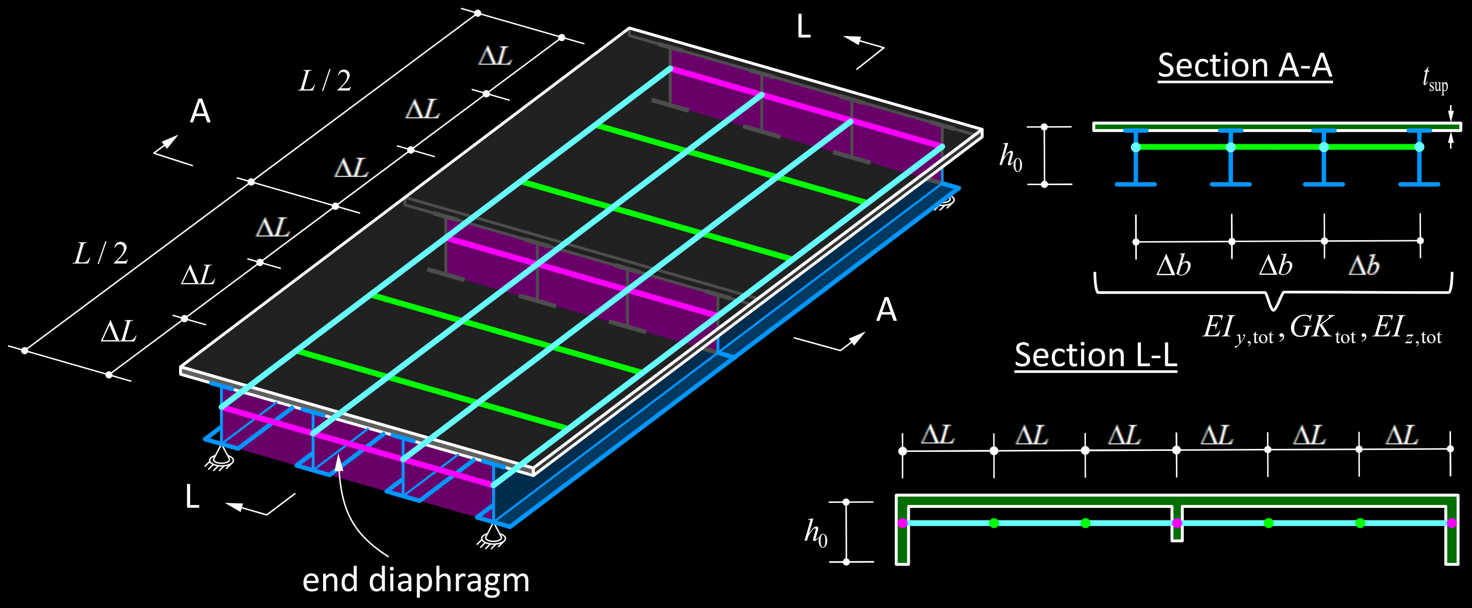
→ one beam per web → $n_{LB} = n_{web}$

- Transverse beams (virtual diaphragms)

→ Simulate the deck stiffness

- Diaphragms = "physical" transverse beams

→ Similar as multi-cell box girder



Longitudinal beams
Each beam is assigned its corresponding EI_y and GK of the girder, i.e. approximately:

$$EI_{y,LB} \approx \frac{EI_{y,tot}}{n} \cong \int_{A_{LBi}} Ez^2 dA_{LBi}$$

$$EI_{z,LBi} \cong \int_{A_{LBi}} Ey^2 dA_{LBi}, \quad EA_{LBi} \cong \int_{A_{LBi}} EdA_{LBi}$$

$$GK_{LB} \approx \frac{GK_{tot}}{n}, \qquad G\bar{A}_{LB} \approx \infty$$

Transverse beams

$$EI_{y,TB} \approx E \cdot \Delta L \cdot \frac{t_{sup}^3}{12}$$

$$EI_{z,TB} \approx E \cdot \frac{\Delta L}{L} \cdot t_{sup} \cdot \frac{L^3}{12}$$

$$G\bar{A}_{TB} \approx \infty$$

$$GK_{TB} \approx \frac{\Delta L \cdot t_{sup}^3}{3} \approx 0$$

Grillage model – Open cross-sections (plane grid)

In the case of wide webs or beams (e.g. separated box sections)

→ transverse stiffness of the deck is significantly underestimated by the formulas given on the previous slide

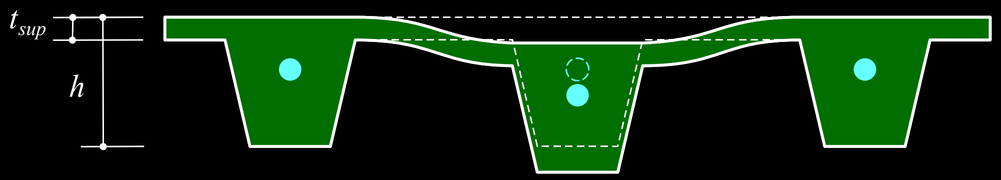
Example: three-web girder

- middle longitudinal beam is displaced downwards
- edge beams remain in their original, unrotated position

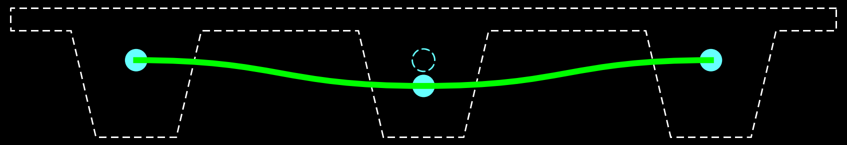
→ to match real behaviour, transverse beam stiffness needs to be corrected over the length corresponding to the width of the webs

→ Use higher average value, or tapered section with stiff part over longitudinal beam (usual in computer programs)

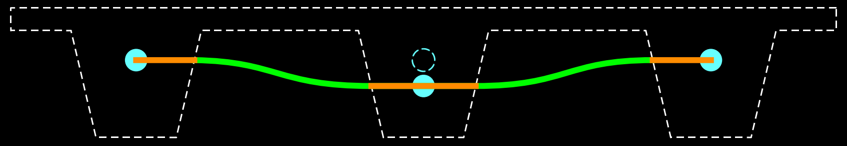
assumed girder distortion



grillage deformations with transverse beams having a constant stiffness → underestimates deck stiffness



grillage deformations with stiff transverse beams over the width of the webs



$$EI_{y,TB} \cong E\Delta L \frac{h^3}{12}$$

$$\gg$$

$$EI_{y,TB} = E\Delta L \frac{t_{sup}^{\;3}}{12}$$

Grillage model – Open cross-sections (membrane action of deck / 3D grid)

Membrane action of deck slab

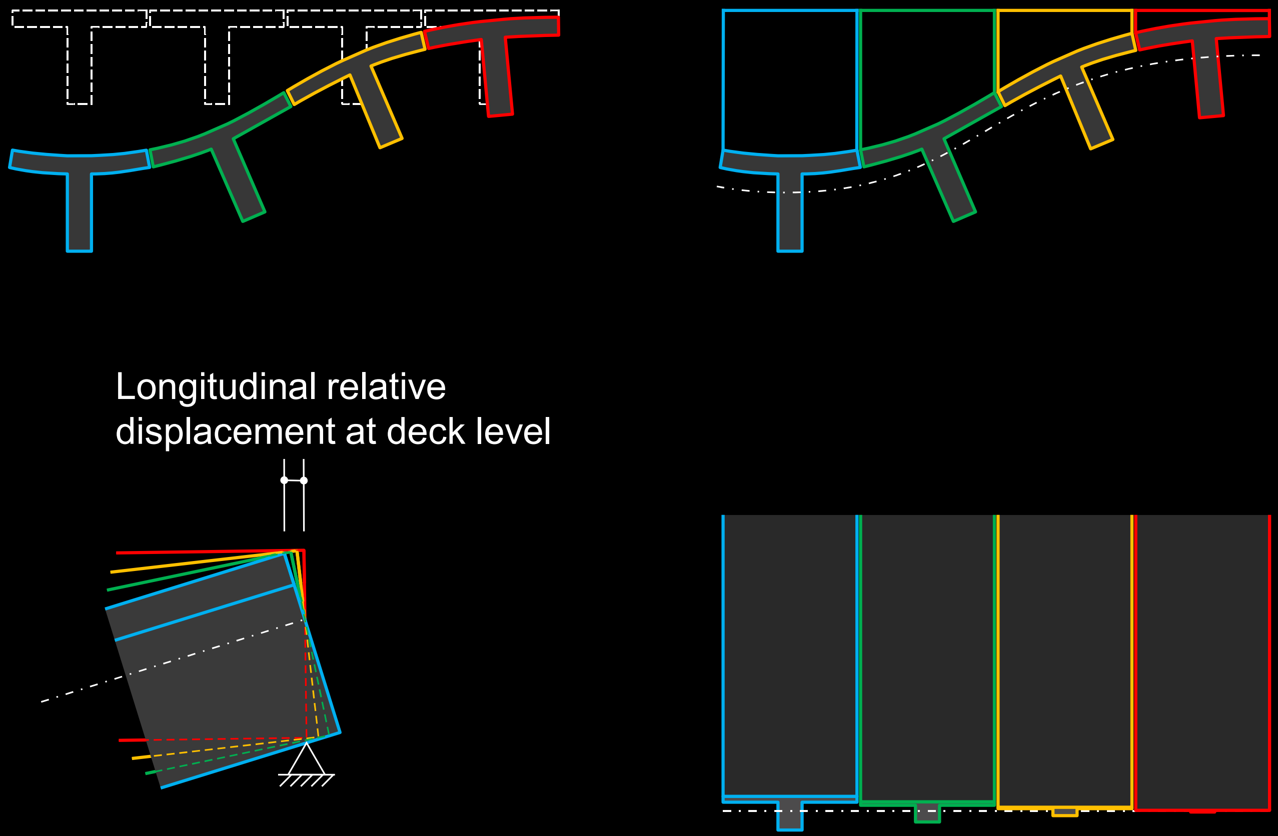
Plane grillages cannot reproduce in plane shear transfer between the parts of the deck assigned to each longitudinal beam. However

- such membrane forces are however required to avoid longitudinal relative displacements in the "longitudinal joints" between the beams
- which occur in plane grillages despite that the distortions of the girder are well reproduced

This is illustrated by the figure:

- distortion of the cross-section (a) is correctly represented by the plane grillage model and its individual longitudinal beams (b), since the transverse beams ensure compatibility
- However, longitudinal relative displacements at the level of the deck result, as shown in elevation (c) and plan (d).

→ 2D grillage underestimates stiffness of the girder.



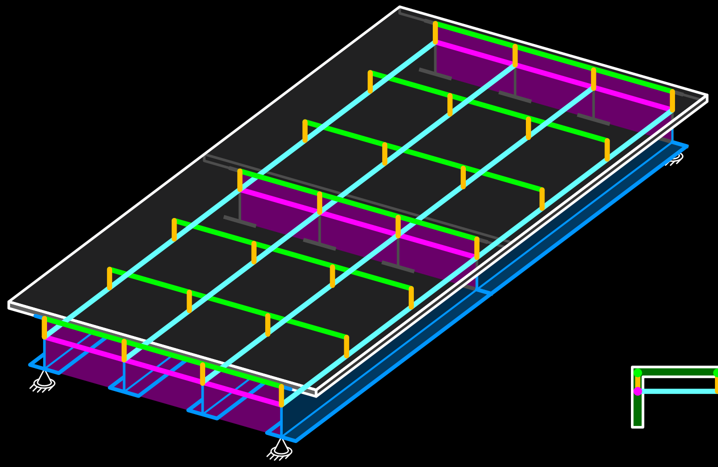
Grillage model – Open cross-sections (membrane action of deck / 3D grid)

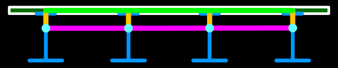
Membrane action of deck slab

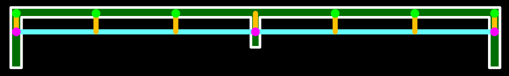
The underestimation of girder stiffness due to neglecting the compatibility between adjacent longitudinal beams is often accepted, as it gives results on the safe side.

If required, the membrane action of the deck slab can be accounted for by using a 3D grillage model, where

- longitudinal and transverse beams are positioned at the levels of their centres of gravity ($\rightarrow$ transverse beams are positioned above the longitudinal beams, which causes membrane action) and
- connected by means of vertical rigid link elements
- stiffnesses of the longitudinal and transverse beams are essentially the same as in the plane grid but
- if transverse beams are introduced at locations of diaphragms, the stiffness of the diaphragms is defined by their cross-section without deck slab (effective width = 0, avoid accounting for deck slab stiffness twice)







| rigid connections:

$$EI_y \rightarrow \infty$$

$$EI_z \rightarrow \infty$$

$$GK \rightarrow \infty$$

$$GA \rightarrow \infty$$

Superstructure / Girder bridges

Bridge Girders – Slab model (slab bridges)

Bridge Girders – Slab model (slab bridges): Modelling

Modelling of slab bridges

In slab bridges, deck and bridge girder are combined, i.e., loads are carried in two directions (slab):

$$\frac{\partial^2 m_x}{\partial x^2} + 2\frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} + q = 0$$

For the design of slabs, see e.g. courses «Stahlbeton II», «Flächentragwerke».

Linear elastic FE analyses are standard today for slab bridges:

- Spreading of concentrated loads see section on bridge deck analysis
- Support conditions corresponding to bearing layout

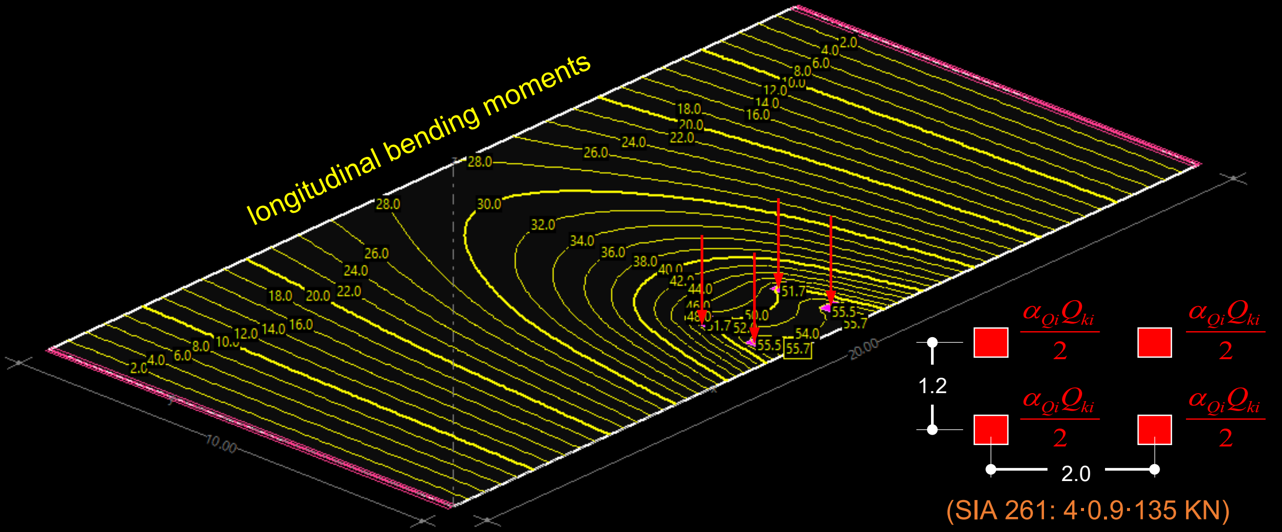
Before the advent of user-friendly, affordable FE slab analysis programs, grillage models were used to analyse slab bridges (using similar stiffnesses as in grillage option B for multi-cell box girders). Today, this is obsolete and therefore not further outlined here.

Slab dimensioning

$$m_{x,Rd} \geq m_{x,d} + k \cdot \left| m_{xy,d} \right|$$
$$m_{y,Rd} \geq m_{y,d} + \frac{1}{k} \cdot \left| m_{xy,d} \right|$$

$$m'_{x,Rd} \geq -m_{x,d} + k' \cdot \left| m_{xy,d} \right|$$
$$m'_{y,Rd} \geq -m_{y,d} + \frac{1}{k'} \cdot \left| m_{xy,d} \right|$$

$$v_{0d} \leq v_{Rd} = k_d \cdot \tau_{cd} \cdot d_v$$

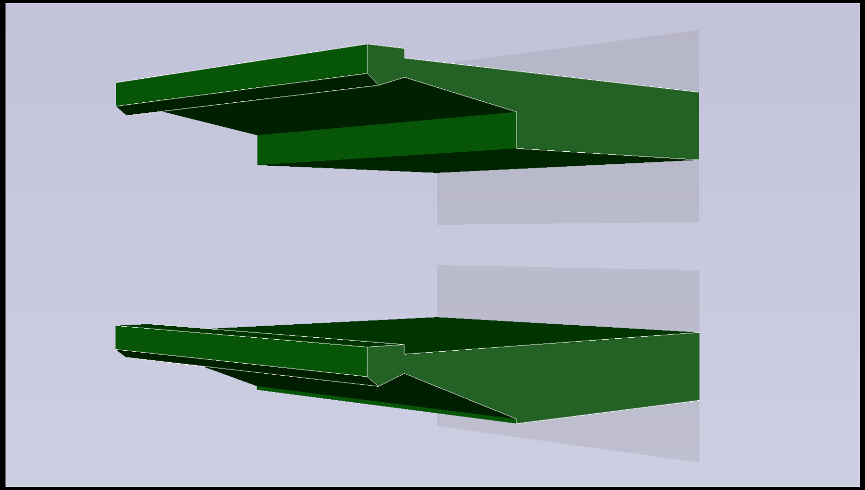


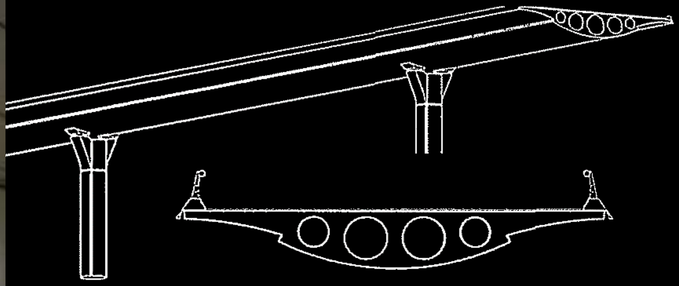
(SIA 261: 4·0.9·135 KN)

Bridge Girders – Slab model (slab bridges): Selected aspects

Specific aspects of slab bridges / slab models

- It is recommended to treat prestressing in slabs as anchor, deviation and friction forces, acting on the subsystem "reinforced concrete structure without prestressing", see lectures “Stahlbeton II”, “Advanced structural concrete” and notes.
- Slab bridges are often supported on several bearings per abutment (“line support”)
  … make sure the intended distribution of support reactions is reasonably achieved
  … particularly if using precast elements (tolerances!)
- Uplift may be a problem at supports near acute corners of skew slabs
- To enhance visual slenderness, it is recommended to reduce the thickness along the free edges.





Reyes de Aragón overpass, Spain, 2005. CFCSL