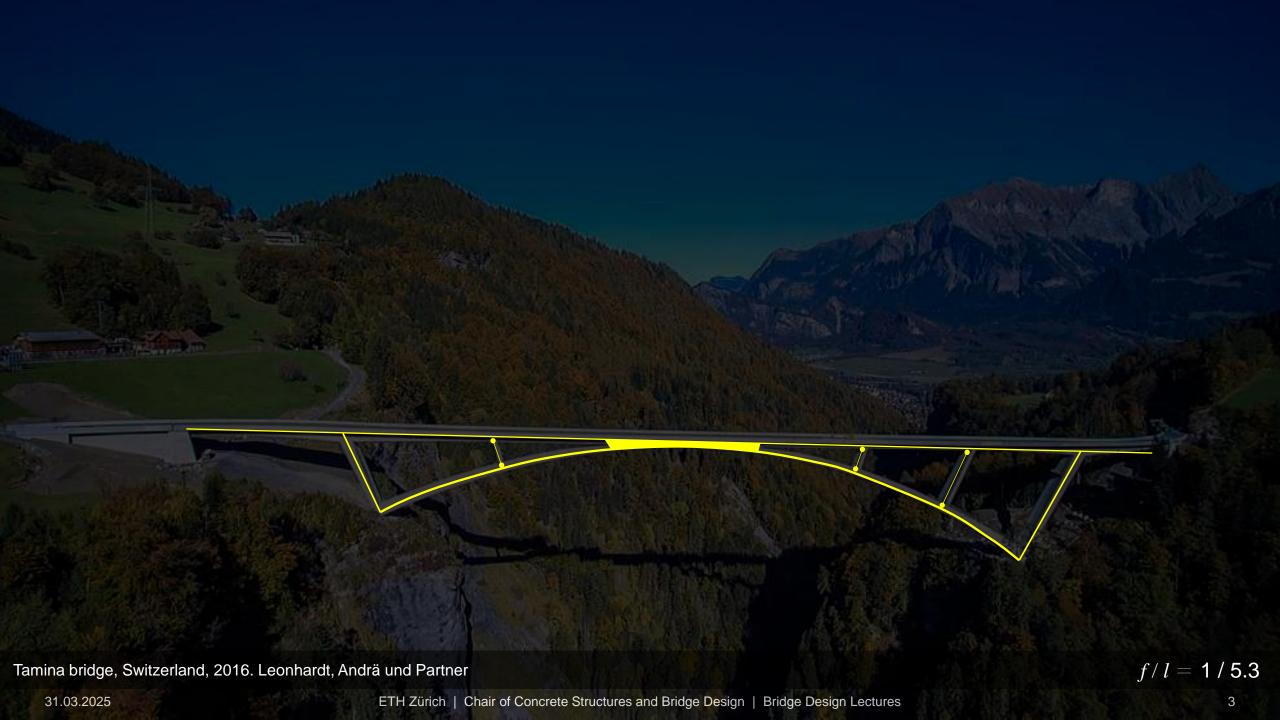
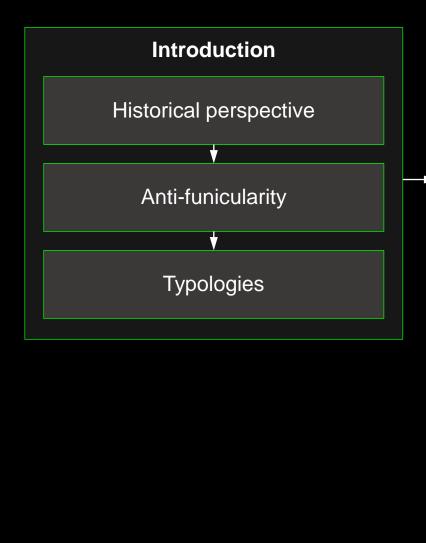
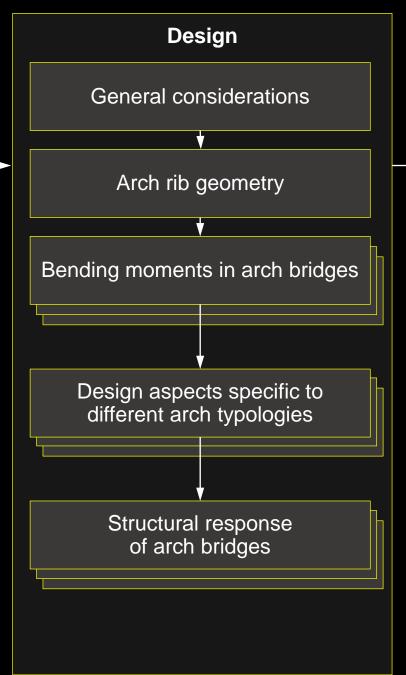
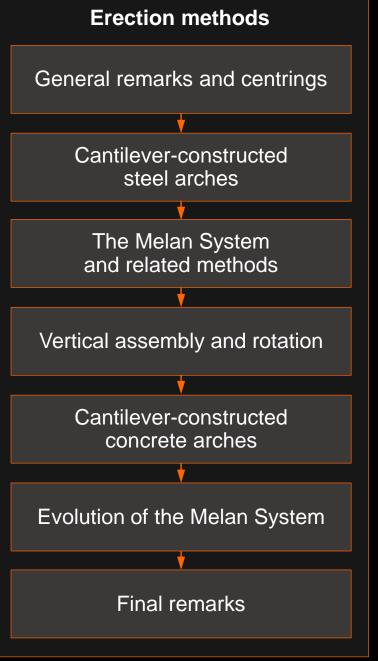
(Bogenbrücken)







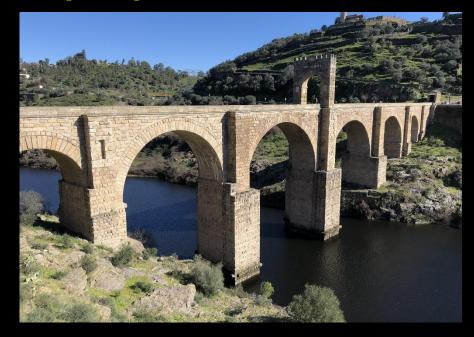


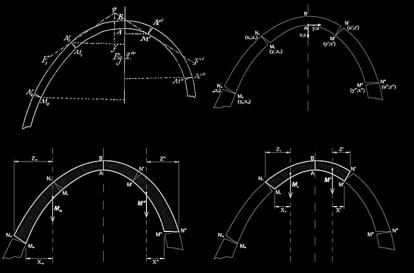


Introduction

Introduction – Historical perspective and terminology

- Masonry arches, and masonry arch brides, have been built for centuries, or rather, millennia (photo)
 - → analysis of arches was one of the first topics studied in the history of the theory of structures
 - → da Vinci already studied and measured the horizontal thrust of arches
 - → Coulomb was one of the pioneers, followed by many other (Monasterio, Culmann, Poleni, Heyman, ...) (figure)
- Since there is no tensile strength in the joints, masonry structures act primarily in compression → anti-funicular arch geometry (axis geometrically similar to funicular polygon of forces, i.e. corresponding to thrust line = Druck-/Stützlinie) is ideal.





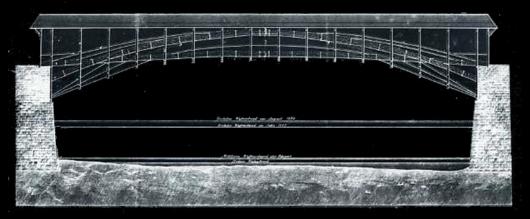
- Masonry arch bridges are part of the cultural heritage of our society and, more specifically, the Swiss railway network.
- For example, the Albula and Bernina lines of RhB are UNESCO World Cultural Heritage, the consistent use of standardised stone masonry arch bridges being one of their main characteristics.





Timber arches have also been built for many centuries.
 Johannes Grubenmann was one of the pioneers (photo).

- About two centuries ago, iron (photo), steel and concrete arches became economical, significantly increasing the feasible spans.
- With its high compressive, but negligible tensile strength, concrete is perfectly suited for arch bridges.



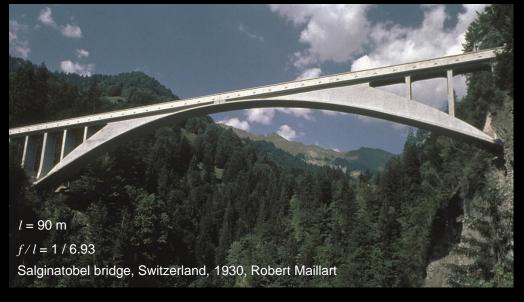


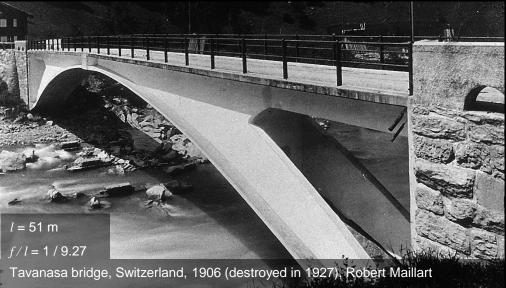
- The first concrete arch bridges were mimicking masonry arches (unreinforced concrete used as inexpensive stone surrogate). More slender, efficient and elegant concrete arches emerged about a century ago (photo).
- Switzerland was at the forefront in these developments, mainly due to:
 - → its topography with many steep valleys being wellsuited for arch bridges
 - → the early development of cement production (with very limited domestic steel production)
 - → competent and innovative structural engineers
- The following Swiss bridge designers are internationally recognised as pioneers in concrete arch bridge design:
 - → Robert Maillart
 - → Alexandre Sarrasin
 - → Christian Menn

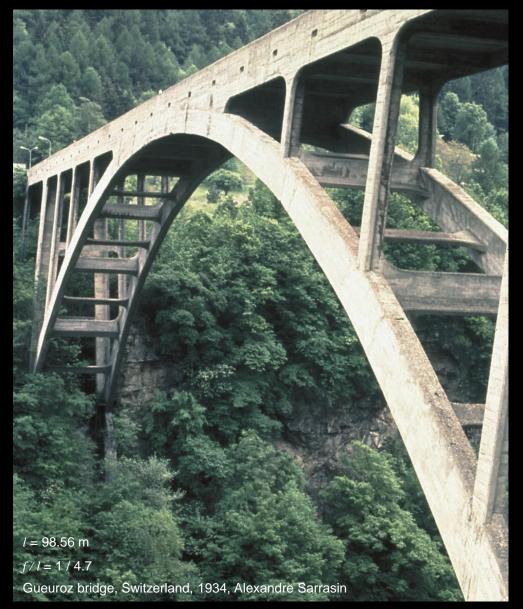
The next slides show some of their most prominent bridges. For more examples, see respective presentation "Eminent bridge designer of the week".





















- Of course, spectacular concrete arch bridges were also designed by designers in many other countries.
- As an example, the Tara
 Bridge (aka Đurđevića-Tara
 Bridge) designed by Mijat S.

 Trojanović, opened in 1940



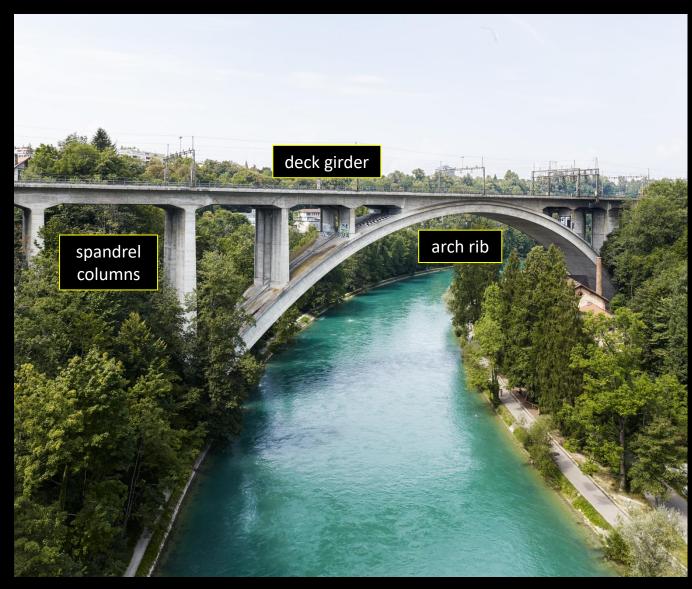
- Due to their high erection costs and the progress of more economical typologies (cantilever-constructed bridges for shorter, cable-stayed bridges for longer spans), only few large arch bridges were built in the 2nd half of the 20th century.
- The last three decades have, however, seen a revival of long-span arch bridges, driven by the development of CFST-arches in China (CFST = concrete-filled steel tube).
- Since the first CFST bridge with a moderate span of 115 m built in 1990 (Wanchang Bridge), more than 400 such arches were built.
- Currently, the Third Pingnan Bridge is the longest CFST arch @ 575 m span (2020, see photo, succeeding to the Bosideng Bridge, 2013 @ 530 m span, animated photo).



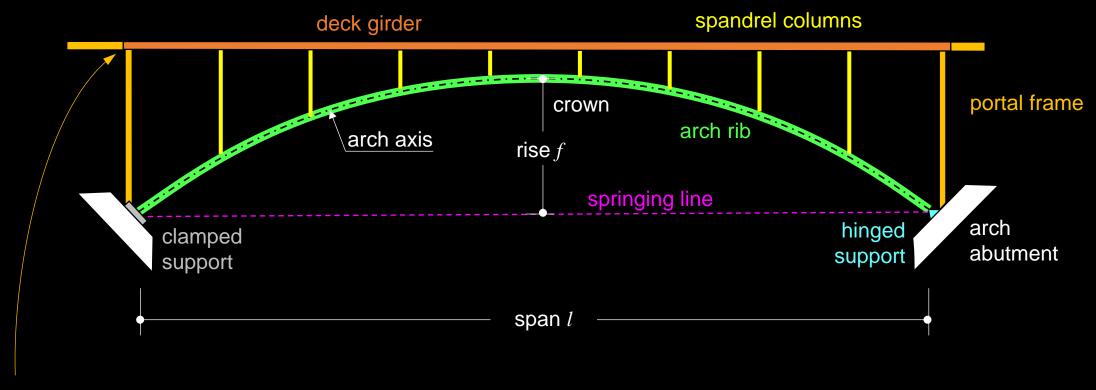
Arch bridges – Introduction: Terminology

An arch bridge essentially consists of three fundamental structural elements:

- Arch rib (or simply arch)
 - → main structural element
 - ... supporting the deck
 - ... transferring the loads to the arch abutments
 - → anti-funicular geometry for permanent loads (pure compression under these actions)
- Deck girder (or just deck / girder, all are commonly used for arches)
 - → usually continuous girder, transferring its selfweight and the traffic loads to the spandrel columns or hangers
- Spandrel columns or hangers
 - → structural elements connecting deck and arch, acting primarily in
 - ... compression (spandrel columns)
 - ... tension (hangers)



Arch bridges – Introduction: Terminology

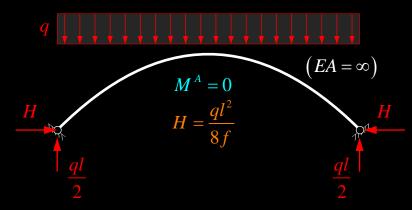


separation of deck girder above arch abutments (portal frames) common in historical bridges, not adequate for modern bridge design

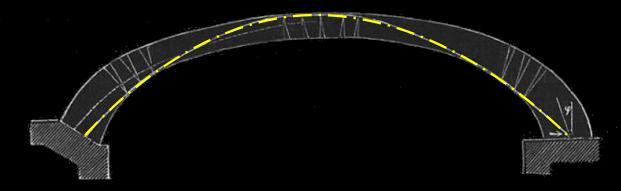
Introduction — Anti-funicularity

- Arches are highly efficient structures, since they are able to carry loads by "compression only" – provided that the thrust line lies inside the arch cross-section.
 - → the ability of arches to carry high loads is primarily due to their shape
- Structures whose axis coincides with the thrust line (i.e., is geometrically similar to the funicular polygon) under a certain load are anti-funicular for that specific load, i.e., they act in pure compression.
- Anti-funicular arches are thus analogous to funicular structures (latin funiculus = rope), but with opposite sign (compression instead of tension).
- In the analysis of masonry arches, and masonry structures in general, graphic approaches are very useful (see notes, figure and next slide).
- The thrust line shows the resultant of compression (in the example on the next slide, for traffic load on the right half of the span).

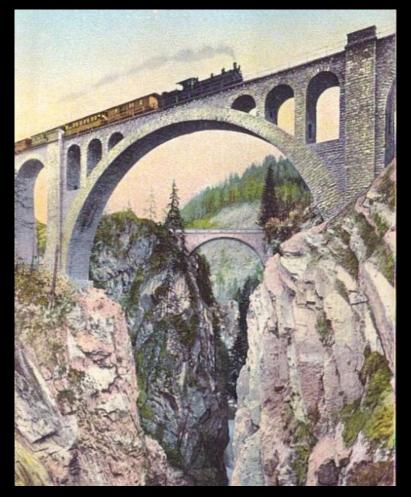
Parabolic arch under uniform load: Arch axis geometrically similar to funicular polygon, pure compression in arch

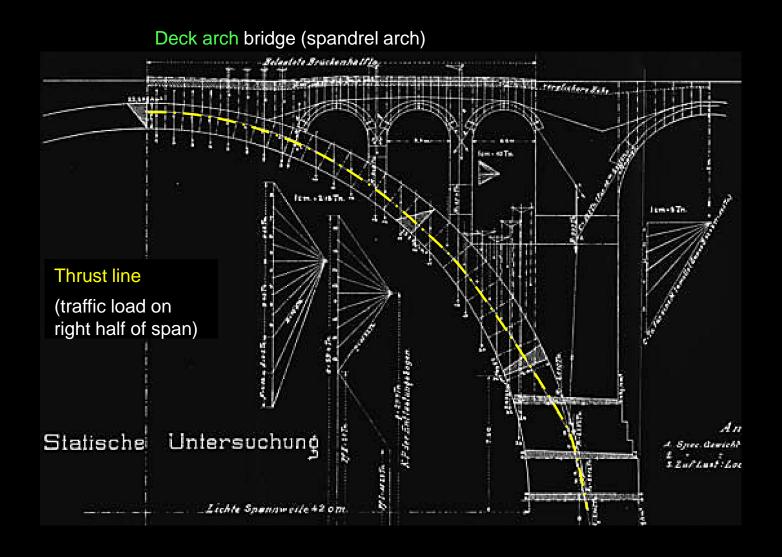


Culmann (1866): Explanation of arch thrust and support conditions



 Most existing masonry viaducts, such as the Soliser Viadukt (clear span 42 m), were designed using graphical statics.

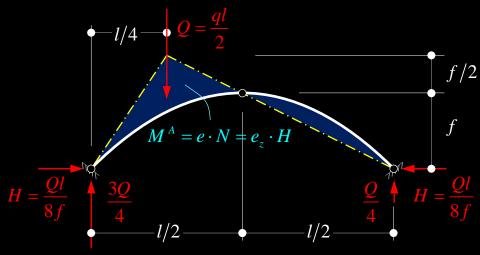


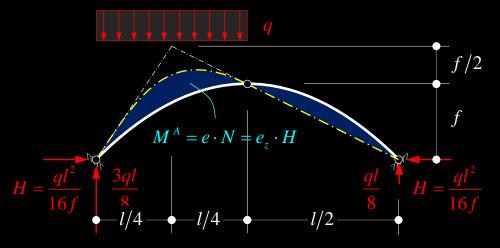


- However, other than ropes and funicular structures in general, arch ribs (as anti-funicular structures for a specific load)
 - → do not adjust their shape to varying configurations of applied loads
 - \rightarrow need to resist arch bending moments $M = e \cdot N = e_z \cdot H$ caused by loads causing deviations e (with vertical component e_z) of thrust line and arch axis

(*M* can be resisted jointly by arch and deck, see behind)

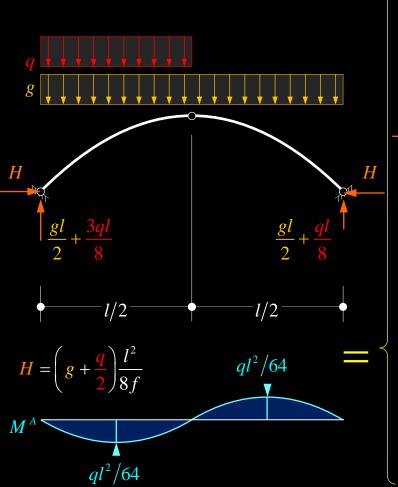
→ in any case require a bending stiffness to prevent buckling (even if globally stabilised by other elements, local buckling must be prevented) Three-hinged arch and thrust line for half-sided load (illustration adapted from Marti, 2014)

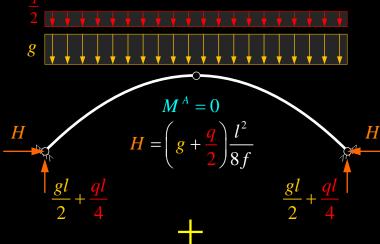


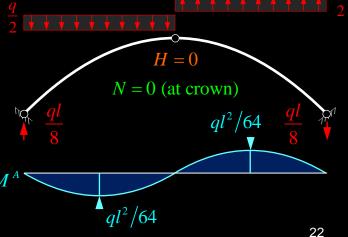


- Any arch geometry is obviously antifunicular for one specific load configuration only.
- All other loads need to be carried by bending
 - → of the arch itself ("stiff arch"), figures
 - → of the deck girder ("deck-stiffened arch")
 - → of arch and deck girder combined (usual)
- In analysis, applied loads can be divided into loads causing pure compression (those for which the geometry was chosen) and loads causing pure bending, see figure.
- Self-weight is the dominant load in bridges
 - → the arch geometry should closely match the thrust line under permanent loads
 - → arches are then still very efficient as they carry a large portion of the total loads in compression (figures)

Arch, anti-funicular for uniform load, under non-symmetrical load (illustration adapted from Marti, 2014)



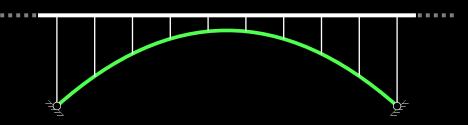


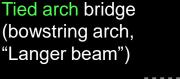


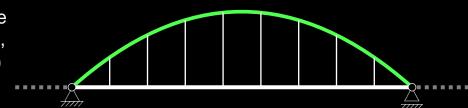
Introduction – Typologies

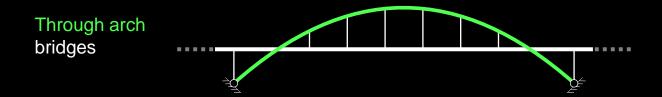
- The typology of arch bridges is commonly related to the position of the deck with respect to the arch.
- Accordingly, the following types of arches can be distinguished:
 - → Deck arch bridge: deck above arch
 - → Tied arch bridge: deck below arch (bowstring arch, "Langerscher Balken")
 - → Through arch bridge: deck and arch intersect (with or without connection)
- Each typology has its structural particularities, but with a common element: The arch.



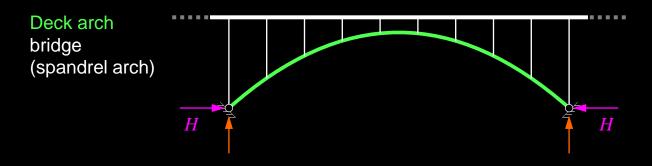








- Structurally, it makes more sense to distinguish arch typologies based on the way the arch thrust *H* (horizontal component of arch normal force) is resisted.
- Arches are most efficient if the arch thrust is carried by the ground ("true arches"), which requires stiff soil
 - → principle of masonry arch bridges (note: high self-weight is beneficial for foundations as it reduces the inclination of the support reaction)
 - → principle of deck arch bridges



Deck arch bridge

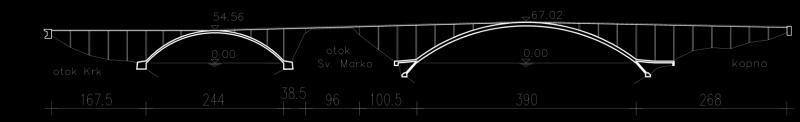
- → Deck girder positioned at top of arch
- → Arch supports deck via spandrel columns
- → Solid-spandrel arches or trussed arches are also used (figures)
- → Full arch thrust transferred to arch abutments

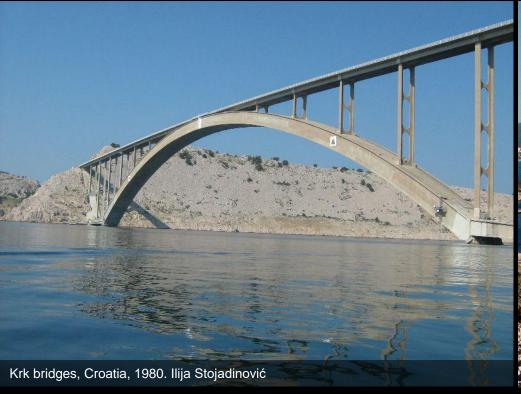




Deck arch bridge example

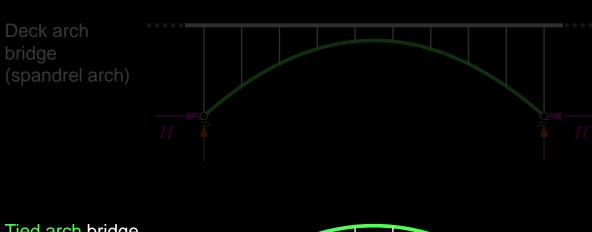
- Reinforced concrete
- Clamped arches
- l = 390 and 244 m
- f/l = 1/5.82 and 1/4.47

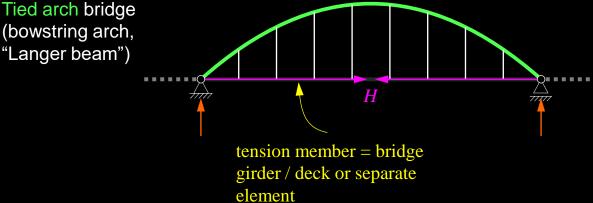






- Structurally, it makes more sense to distinguish arch typologies based on the way the arch thrust *H* (horizontal component of arch normal force) is resisted.
- Arches are most efficient if the arch thrust is carried by the ground ("true arches"), which requires stiff soil
 - → principle of masonry arch bridges (note: high self-weight is beneficial for foundations as it reduces the inclination of the support reaction)
 - → principle of deck arch bridges
- Alternatively, the arch thrust can be resisted by a tension member connecting the supports (along springing line)
 - → structurally less efficient, since arch thrust must be resisted in tension
 - → principle of tied arch bridges:
 - ... deck = tension member (more efficient) or
 - ... separate tension member parallel to deck (less efficient)
 - → externally, a tied arch is a simply supported beam





Tied arch bridge:

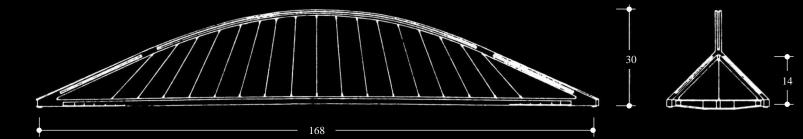
- → Arch positioned above deck
- → Deck suspended by arch via hangers
- → Arch thrust fully resisted by deck
 (→"externally", it is a simply supported beam)
- → Known in German speaking countries as Langer beam (Langerscher Balken) or "versteifter Stabbogen"





Tied arch bridge example

- Steel arch
- Simply supported (arch + deck = "girder")
- l = 168 m
- f/l = 1/5.60



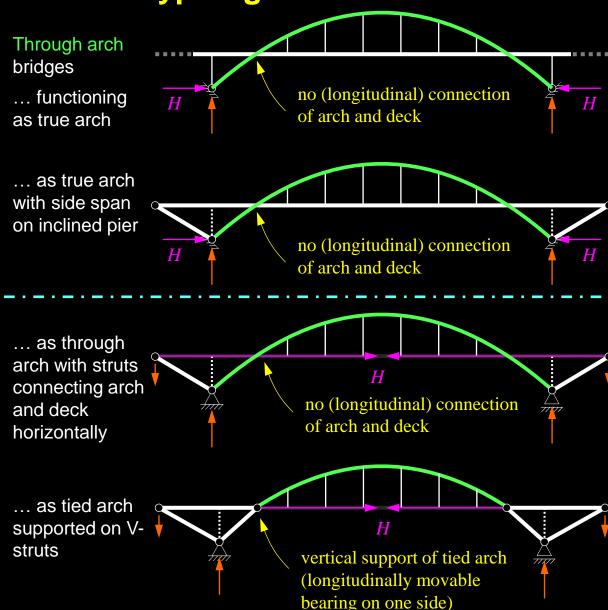




- In through arch bridges, the thrust may be resisted
 - → by the foundations as in a deck arch (true arch)
 - → by a tension member connecting the supports as in a tied arch
- If the thrust is resisted by the foundations (true arch, upper figures), the structural system corresponds to a deck arch, with the following aspects to be considered:
 - → arch must pass deck without transferring longitudinal forces
 - → mix of hangers+spandrel columns (different stiffness)
- If the thrust is resisted by the deck, different layouts are possible (bottom figures):
 - → through arch with struts transferring thrust to deck
 - → tied arch supported on cantilevered structure

In either case, such through arches are significantly more complex in design and construction than deck or tied arches.

 The structural concept of through arches is often hard to identify: They lack the logic of form other arches



Through arch bridge:

- → Deck and arch overlap in elevation
- → Midspan part suspended from arch via hangers, side spans supported by spandrel columns (if required)
- → Arch thrust resisted by
 - ... foundation (= true arch) or
 - ... deck (= tied arch) or
 - ... both depending on stiffnesses (deliacte to quantify)



Tardis bridge, Mastrils-Landquart, Switzerland, 2003. dsp Ingenieure + Planer



Through arch bridge example

- Steel arch
- Clamped true through arch
- l = 329 m
- f/l = 1/4.7

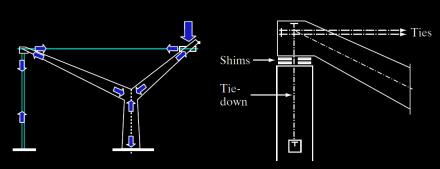


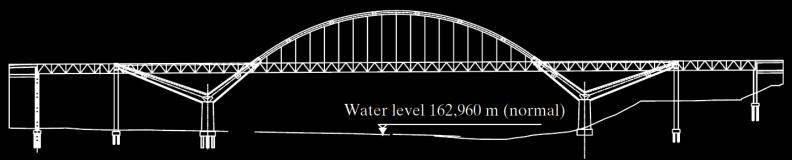


Through arch bridge example

- Steel arch on concrete V-struts
- Tied through arch
- l = 420 m
- f/l = 1/4.4





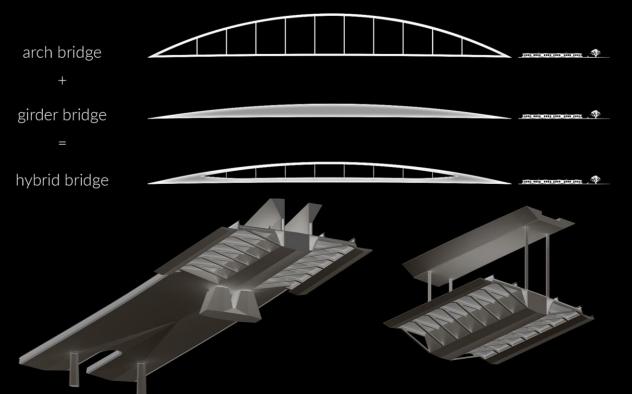




Slender tied arches are sometimes termed "hybrid arch bridges".

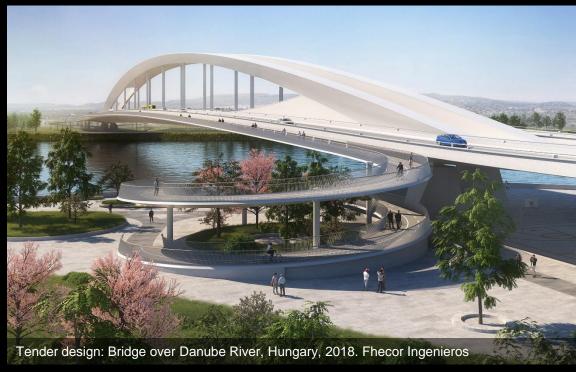
However, while the solutions are attractive, this term is technically ill-founded, see structural response).

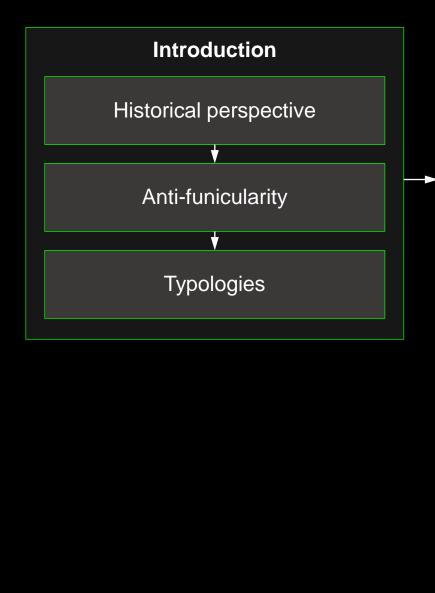
(in any arch bridge, arch and deck share the applied loads (arch in bending and arch action, girder in bending). In flat arches, the deck simply carries a larger portion of the applied loads).

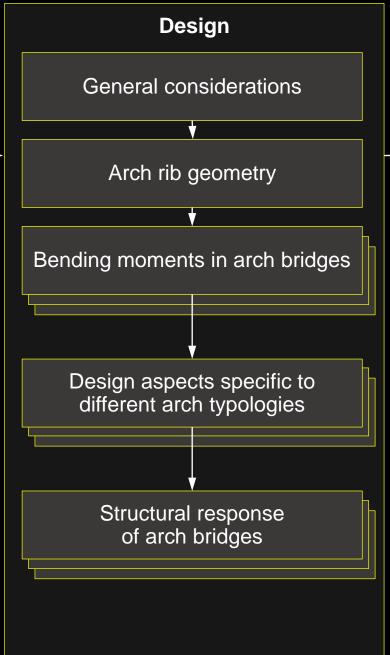


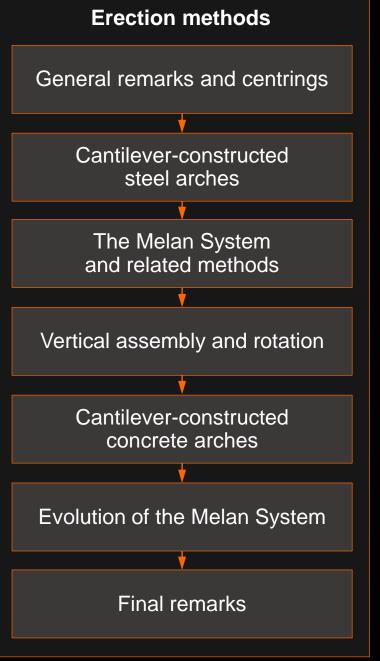
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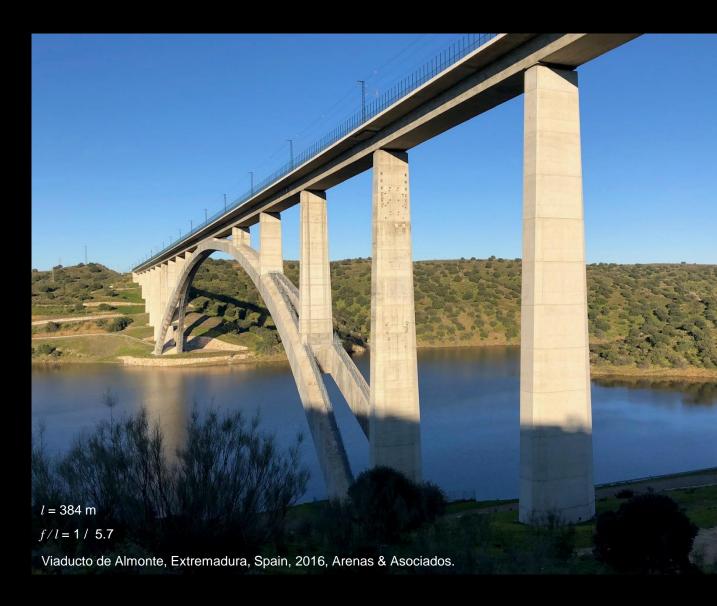




Design

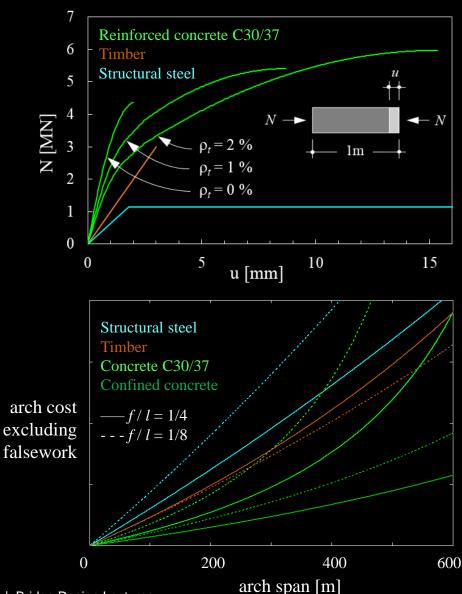
Design – General considerations

- Arches are very efficient structures in their final configuration, but
 - → arch action is only activated at closure
 - → arch centrings are expensive (tailor-made falsework and formwork)
 - → efficient erection methods important in any structure – are particularly important in arches
- Arch bridges built by cantilevering are considered economical for spans 100 m ≤ l ≤ 300 m (200 m for concrete arches)
 - → for shorter spans, girders are more economical (cost of arch is not compensated by savings in the deck girder)
 - → for longer spans, cable-stayed bridges are more economical due to the efficient erection method
 - → longer spans may be economical if an optimised erection method is used (e.g. CFST arches, see erection methods)
- Other reasons, particularly aesthetical considerations, may still justify arch bridges



Material cost vs erection method

- The figures compare different materials for conventional short span structures (no complicated falsework for concrete, nor large cranes for steel and timber)
 - → the load-deformation characteristics of compression members costing 100 CHF/m
 - → the total cost of an arch for two rise-to-span ratios
- The concrete compression members are significantly stiffer and stronger at the same cost.
- Even for very large spans, and using normal strength concrete, concrete is by far the most economical material in the final configuration due to its low cost and high compressive strength (despite the better weight/strength ratio of steel and timber, which could be improved for concrete using high-strength concrete).
- However, falsework for long-span arches is very expensive
 - → concrete arches built on conventional scaffold are only economical for short spans
 - → unless efficient arch construction methods are used, steel arches are thus more economical for medium-large spans

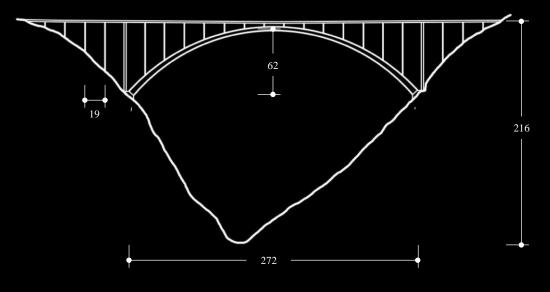


• An example of a bridge where higher cost of an arch bridges was justified by the superior aesthetics quality and where a steel truss arch was more economical than a concrete arch (lighter weight = erection by stayed cantilevering of the arch possible, see erection methods – is the New River Gorge Bridge (1977, record span arch bridge until 2012).



Deck arch bridges (and true arch through arch bridges) transfer important horizontal forces – the arch thrust – to the foundations, which is the most efficient solution. However

- → the viability of deck arch bridges depends on the soil conditions
- \rightarrow the arch thrust increases with decreasing rise-to-span ratios f/l
- → Long span and slender arches require solid rock at the arch abutments



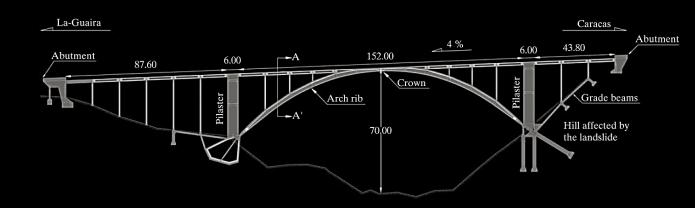


"Perfect" site for deck arch bridge:

- strong soil (solid rock)
- steep valley
- relatively large span

- If true arch bridges are built in inadequate sites (soft soil, unstable slopes), consequences may be drastic.
- This is particularly due to their sensitivity to (horizontal) movements of the arch abutments
 - → horizontal movements of the arch abutments cause changes in the horizontal reaction = deviations of the thrust line and corresponding bending moments along the arch
 - \rightarrow the importance of these effects depends on the magnitude of the movements and the rise-to-span ratio f/l (see structural response).

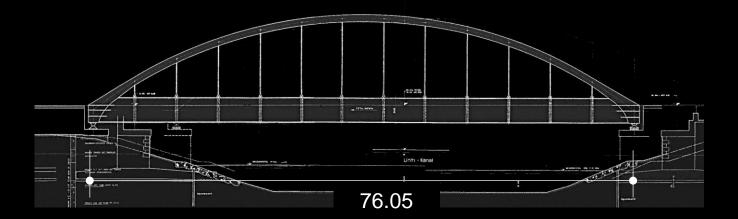








- Tied arch bridges, on the contrary, are simply supported girders "externally" (the deck girder acts as tension member = tie, only vertical reactions under gravity load)
 - → suitable for locations with soft soil
 - → generally worth considering in single-span bridges (more transparent than simply supported standard girder bridges)
 - → generally appropriate for single-span bridges with low clearance above traffic lines
 - → particularly suitable for bridges spanning rivers where often the following conditions apply:
 - ... low clearance above flood level
 - ... no piers in river possible
 - ... soft soil layers to considerable depth
- The elements connecting deck and arch are often pin-jointed, acting in pure tension
 - → referred to as "hangers" (even if they carry bending moments, see design)





Materials

- Concrete
- Steel
- Composite
- Timber
- •

Shape

- Single arch
- Double arch (in cross-section)
- Straight in plan
- Curved in plan
- Polygonal in plan
- Spatial arch
- ...

Cross section of the arch rib(s)

(constant or depth and/or width increasing towards abutments)

- Box
- Solid rectangular
- Tubular
- Truss
- ...

Cross section of the deck (usually constant)

- Box
- Slab
- T or double T
- ...

Hinges in the arch rib

- Clamped ("zero-hinge") arch
- Two-hinged arch
- Three-hinged arch

Rise-span ratio f/l

- High arch $f/l \approx 1/2$
- Standard arch $f/l \approx 1/6$
- Low arch f/l < 1/10

Distributions of rigidities

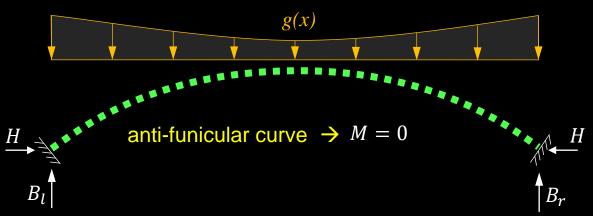
- Stiff arch flexible deck
- Flexible arch stiff deck
- Intermediate solutions

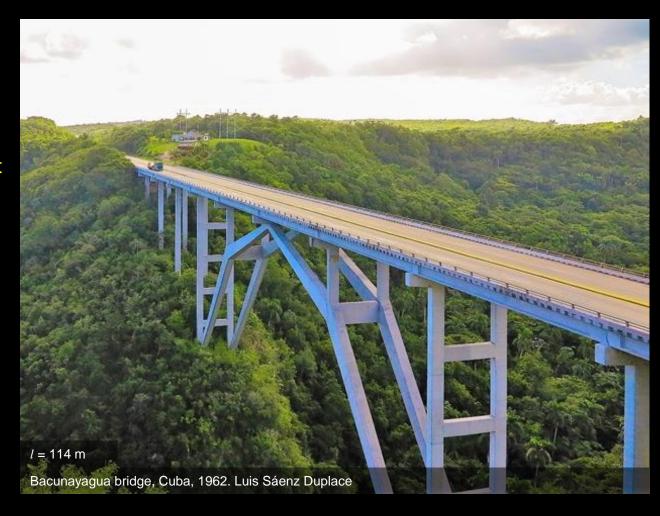
Geometry of hangers / spandrel columns

- Number
- Inclination
- Hinges at top and/or bottom

Design – Arch rib geometry

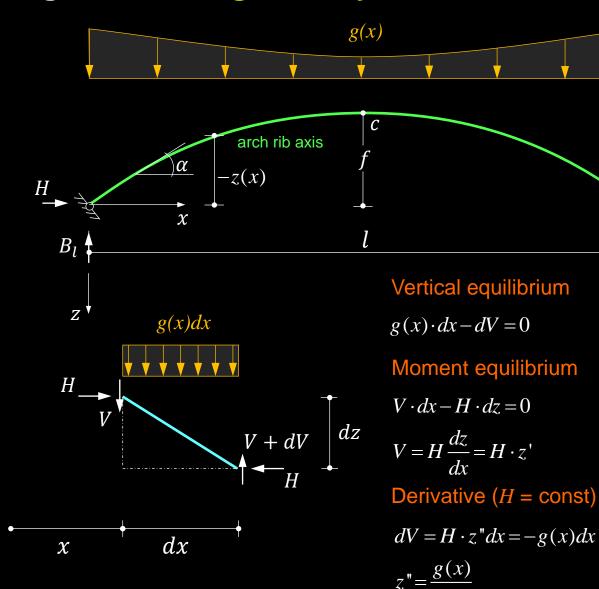
- Most of the following slides show deck arches, but they equally apply to tied arches unless indicated otherwise.
- The arch axis should closely correspond to the thrust line due to permanent load, such that no bending moments are caused by this (usually most important) action
 - → arch geometry geometrically similar to funicular polygon of permanent loads
- The arch is not uniformly loaded, but rather, receives most loads via the spandrel columns
 - → "classic" curved arch reasonably anti-funicular only for closely spaced columns (8...10 over span)
 - → if fewer spandrel columns or hangers are provided, a polygonal arch geometry should be chosen





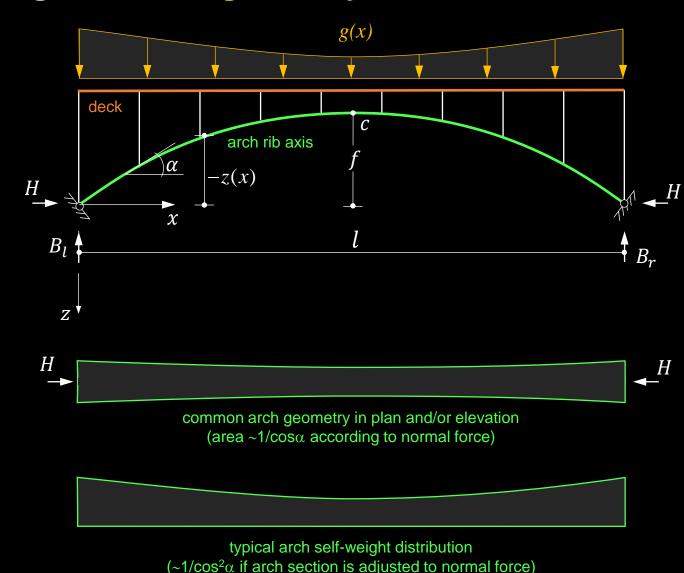
Determination of the geometry of the arch

- The analytical equation to determine the antifunicular geometry for a given load g(x) is a 2nd order ordinary differential equation (see figure).
- The arch thrust *H* (horizontal component of arch normal force) is constant if only vertical loads act.
- For any value of the arch thrust H > 0, (positive H = compression in arch rib), an anti-funicular geometry is obtained (all are geometrically similar):
 - \rightarrow small $H \leftrightarrow$ large rise f (high arch)
 - \rightarrow large $H \leftrightarrow$ small rise f (low arch)
- If the arch axis (centre of gravity of the arch rib) coincides with the resulting curve, the load g(x) causes pure compression in the arch rib.



Determination of the geometry of the arch

- Generally, the differential equation has to be integrated numerically since g(x) is not constant:
 - \rightarrow the self-weight of the arch is proportional to $1/\cos\alpha$ (relevant weight: per horizontal length)
 - → the arch normal force for constant thrust H is also proportional to $1/\cos\alpha$; the arch section is often increased towards the springing lines accordingly (→ arch self weight increasing ≈ with $1/\cos^2\alpha$)
 - → point loads applied by spandrel columns differ even if "smeared" over column spacing due to varying column height
- The "exact" anti-funicular geometry can be determined numerically in many different ways, even accounting for arch compression / second order effects (geometrical non-linearity).
- On the following slides, a method for determining the funicular curve by simple hand calculations, useful for pre-dimensioning, is presented.



Determination of the geometry of the arch

- 1. Determine the bending moments $M_0(x)$ in a simply supported girder (span = arch span), loaded by all permanent loads of the arch (arch rib, spandrel columns, deck girder, superimposed dead load)
- 2. The bending moments in the arch rib $M^{A}(x)$, differ from $M_{0}(x)$ by the moment due to the horizontal thrust H:

$$M^A(x) = M_0(x) + H \cdot z(x)$$

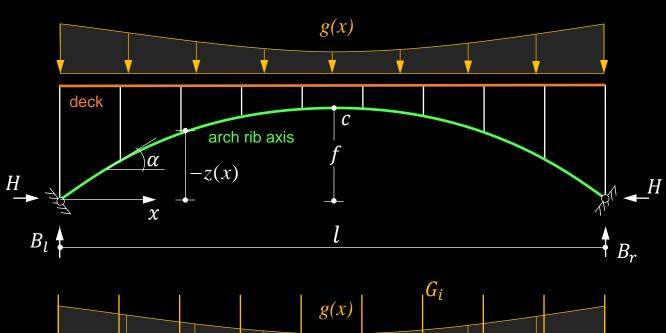
3. Imposing the condition $M^A = 0$ (anti-funicularity), with the bending moment at the crown $M_0^c = M_0(l/2) = H \cdot f$, the arch thrust = reaction H and the anti-funicular geometry z(x) follow for a chosen value of the rise f:

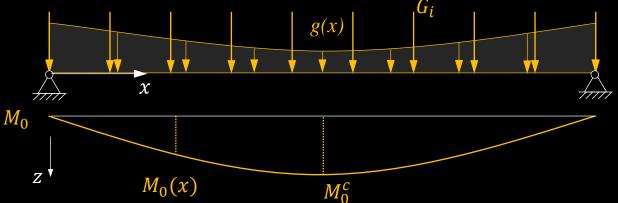
$$z(x) = -\frac{M_0(x)}{H}$$

$$H = \frac{M_0^c}{f}$$

$$z(x) = -\frac{M_0(x)}{M_0^c} f$$

(as postulated, the anti-funicular geometry is geometrically similar to the funicular polygon)





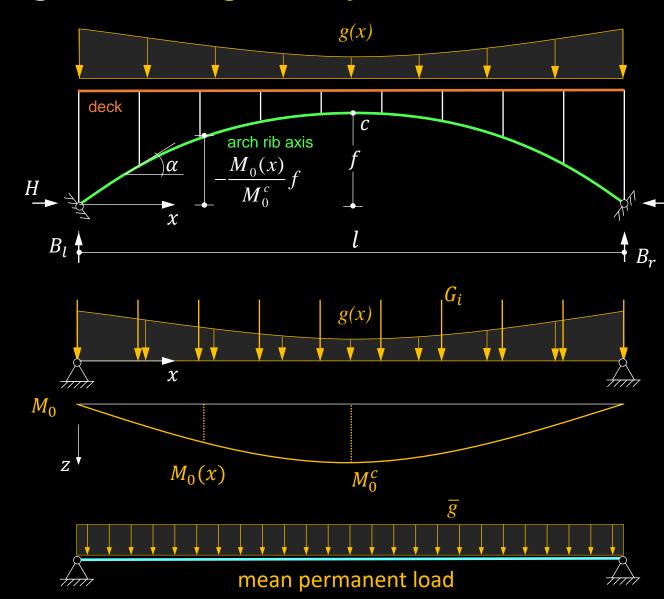
Determination of the geometry of the arch

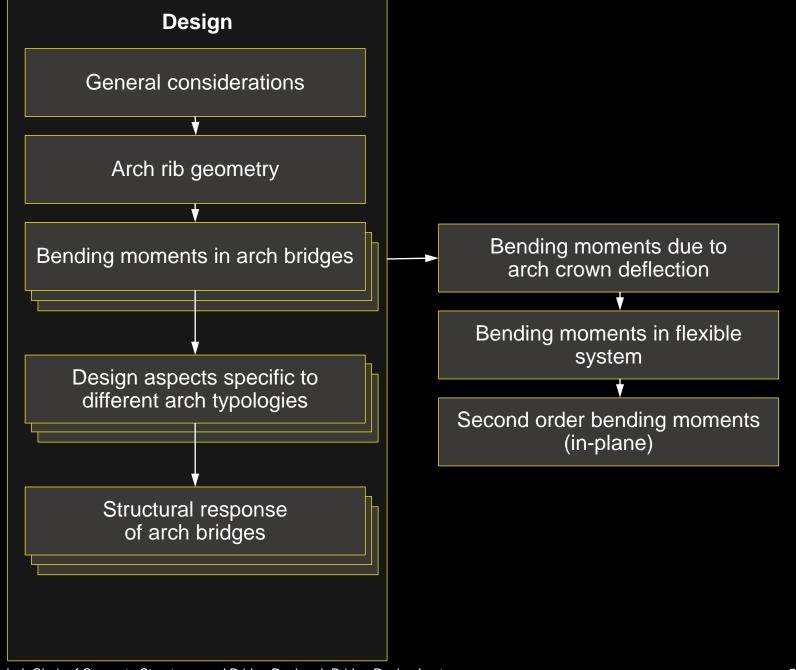
- An iterative procedure is required since the weights of the arch rib and spandrel columns depend on the geometry of the arch rib.
- As a first approximation in preliminary design, the mean permanent loads \overline{g} over the entire length of the arch can be used for further simplification.
- Hence, the arch is subjected a uniformly distributed load (corresponding to the total permanent load of the structure supported by the arch divided by its span), resulting in a quadratic parabola for the arch axis:

$$H(\overline{g}) \cong \frac{\overline{g} \cdot l^2}{8f}$$
 $z(x) = -\frac{8f}{\overline{g} \cdot l^2} M_0(x)$

And the axial force in the arch is:

$$N(\overline{g}) = -\frac{H(\overline{g})}{\cos \alpha}$$





Design – Bending moments due to arch crown deflection

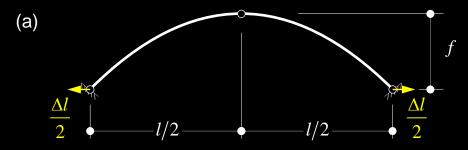
("Biegemomente infolge Scheiteleinsenkung")

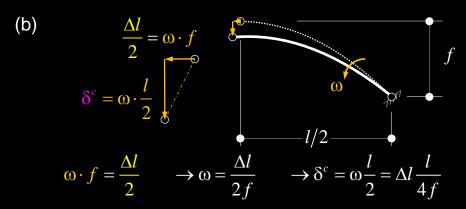
Deflections due to horizontal support displacements ΔI

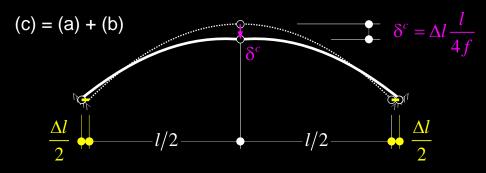
- Arches can accommodate horizontal support displacements Δ/ with little axial restraint by adjusting their shape
 - \rightarrow deflection at arch crown δ^c
 - \rightarrow stress-free arch crown deflection in three-hinged arches (see figures): $\delta^c = \Delta l \frac{l}{4}$
 - → causing bending moments in two-hinged and fixed arches, similar arch crown deflection:

$$\delta^c \approx \Delta l \, \frac{l}{4 \, f}$$

Three-hinged arch with horizontal support displacement ΔI





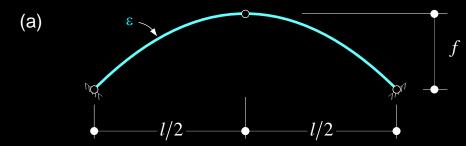


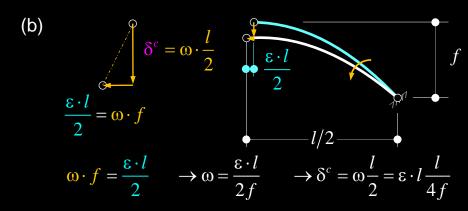
Deflections due to imposed deformations ϵ

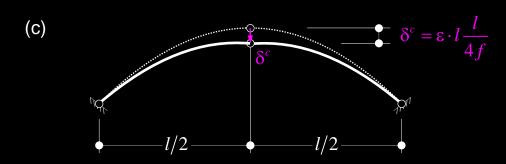
- A contraction (and, with opposite sign, expansion) of the arch rib due to imposed deformations ε (temperature change, shrinkage, ...) has a similar effect as horizontal support displacements
- With $\Delta l = \varepsilon \cdot l$, the deflection of the crown is approximately equal to: (exact for three-hinged arch)

$$\delta^c \approx \varepsilon \cdot l \frac{l}{4f}$$

Three-hinged arch with imposed strain $(\varepsilon = \alpha_T \Delta T, \varepsilon_{cs}, ... \Box 1)$







Deflections due to arch compression N/EA

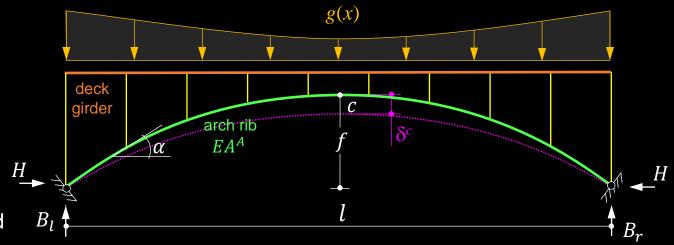
- The arch rib is axially very stiff, but not perfectly rigid
 - \rightarrow arch rib is compressed by arch normal force N
 - → deflections under permanent load even if a perfectly anti-funicular geometry has been chosen
- In preliminary design, the vertical deflection of the arch crown due to permanent loads g can be estimated based on the arch compression at the crown $\frac{H(\overline{g})}{EA^{A,c}}$ (with $A^{A,c}$ = cross-sectional area of arch at the crown) as

$$\delta^{c} \cong \frac{H(\overline{g})}{EA^{A,c}} \frac{1 + 3(f/l)^{2}}{4f/l} l \qquad \text{(for } EA^{A} = EA^{A,c} = \text{const.)}$$

if the arch rib has a constant cross-section A (i.e., normal arch compression \approx proportional to $N=H/(\cos\alpha)$, and as

$$\delta^{c} \cong \frac{H(\overline{g})}{EA^{A,c}} \frac{l}{4f} l \qquad \text{(for } EA^{A} = \frac{EA^{A,c}}{\cos \alpha} \to \frac{H(\overline{g})}{EA^{A}} \approx \text{const.)}$$

if the arch rib cross-section *A* is proportional to $1/(\cos \alpha)$ (i.e., constant arch compression as in case of imposed ϵ).



NB. The deflections due to arch compression (caused by dead load, imposed deformations or displacements) are much higher in flat arches (low ratios f/l):

$\frac{f}{l}$	$\frac{l}{4f}$	$1 + 3\left(\frac{f}{l}\right)^2$	$\frac{1+3(f/l)^2}{4f/l}$
1/2	0.5	1.750	0.875
1/4	1.0	1.188	1.188
1/6	1.5	1.083	1.625
1/8	2.0	1.047	2.094
1/10	2.5	1.030	2.575
1/12	3.0	1.021	3.063
1/14	3.5	1.025	3.554
1/16	4.0	1.012	4.047

(example: for f/I = 1/8, the crown deflects twice as much as the arch rib contracts)

(see also diagram on slide 120, case study)

Bending moments due to arch compression

- The arch rib is much stiffer axially than arch rib and deck girder in bending
 - \rightarrow deflections of arch rib (due to N/EA, ε and/or Δh) are imposed to arch rib and deck girder
 - ightarrow bending moments in arch rib and deck girder proportional to their stiffness and crown deflection δ^c
 - \rightarrow In clamped arches (with continuous deck), the bending moments can be estimated from δ^c as follows (analogous $M \leftrightarrow \delta^c = \delta_{\text{midspan}}$ in a continuous girder):

$$\delta^{c} = \frac{\overline{g}l^{4}}{384EI}$$

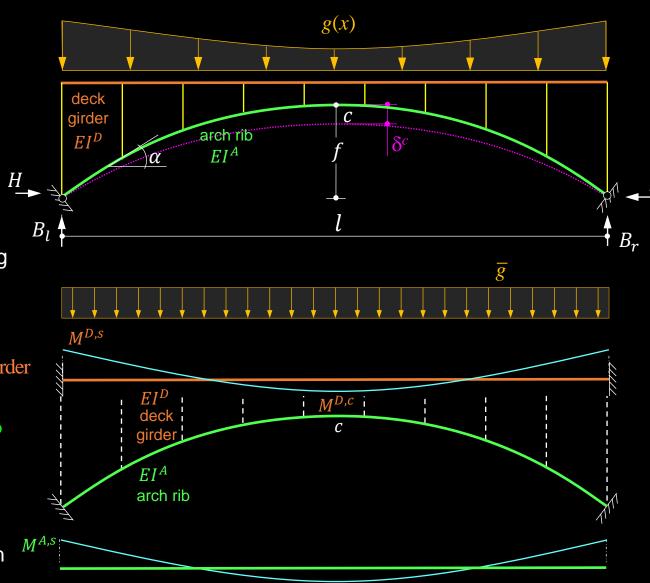
$$M^{c} = -\frac{1}{2}M^{s} = \frac{\overline{g}l^{2}}{24}$$

$$M^{A,c} = -\frac{M^{A,s}}{2} \cong \frac{16EI^{D}}{l^{2}}\delta^{c} \quad \text{deck girder}$$

$$M^{A,c} = -\frac{M^{A,s}}{2} \cong \frac{16EI^{A}}{l^{2}}\delta^{c} \quad \text{arch rib}$$

$$\left(\frac{M^{c}}{\delta^{c}} = \frac{\overline{g}l^{2}}{24} \cdot \frac{384EI}{\overline{g}l^{4}} = \frac{16EI}{l^{2}} \text{ is independent of } \overline{g}\right)$$

where $M^D =$ moment in deck girder, $M^A =$ moment in arch rib, c = crown, s = springing line (arch abutment)



 $M^{A,c}$

Bending moments due to arch compression

- Bending moments due to arch compression generally occur in arches built on conventional centrings.
- In concrete arches, the crown deflection increases with time due to creep, but the bending moments remain constant (one casting system, see Advanced Structural Concrete)
- If the arch is lifted off the formwork by opening it in the crown (with hydraulic jacks, see figure), or the arch is built by stay cantilevering, the arch rib is already compressed at closure
 - \rightarrow no crown deflection at t = 0 (time of closure), but
 - → in concrete arches, crown deflections and corresponding bending moments build up over time due to creep
 - ightharpoonup bending moments of up to 80% of the values of the arch built on centring (= one casting system, see Advanced Structural Concrete) can result at $t=\infty$
- The benefit of opening concrete arches in the crown can be increased if the jacks are kept installed, adjusting the jacking force over a long period of time (as done e.g. in the Krk Bridges during 5 years, see section *Erection*).





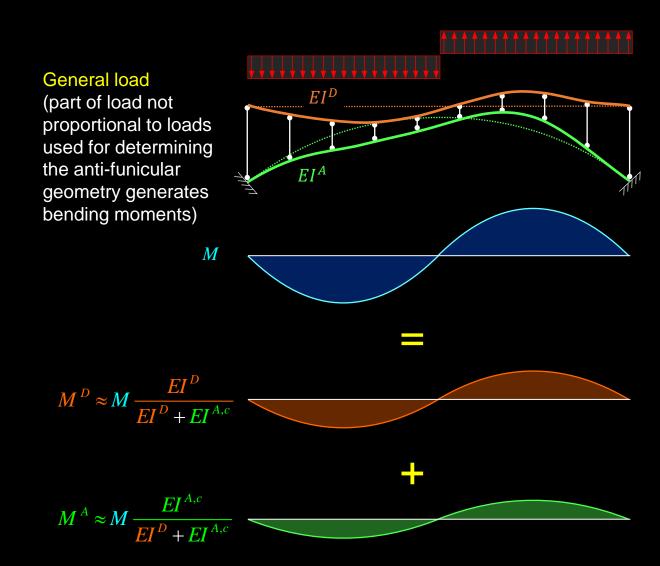


Design – Bending moments in flexible system

General behaviour – Load sharing

- Under loads causing bending moments in the arch (≠ proportional to loads used for determining the anti-funicular geometry, see Slide 22 for decomposition of load), the system acts like a flexible frame
 - → deflections of arch rib and deck girder equal (deck arch, stiff columns) or very similar (tied arch, flexible hangers)
 - → bending moments shared among deck girder and arch rib in proportion to their stiffness
- Generally, the bending stiffness of deck girder and arch rib is of similar magnitude, and both elements carry a portion of the total bending moments, see figure.

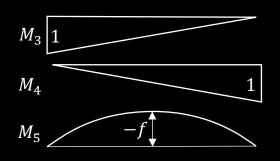
Note that this "load sharing" also applies to the bending moments due to arch compression, as these are proportional to the stiffnesses as well.



Solution using force method – General case

- Basically, the bending moments in the flexible system can be determined using the force method
 - → select isostatic basic system and introduce redundant variables
 - → determine flexibility coefficients
 - → formulate compatibility and solve for redundant variables
- However, even if the columns (hangers) are idealised as pinjointed members, the solution is tedious in the general case
 - → use frame analysis software
 - → for preliminary design, estimate bending moments using values shown on slide 63

Redundant moments:

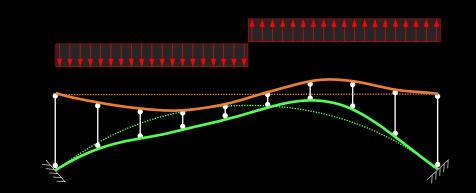


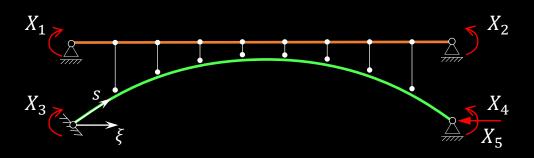
$$\delta_{ik} = \int_{0}^{l} M_{i} \frac{M_{k}}{EI^{D}} d\xi + \int_{0}^{l} M_{i} \frac{M_{k}}{EI^{A}} ds$$

 δ_{ik} : flexibility coefficients

D: deck girder

A: arch

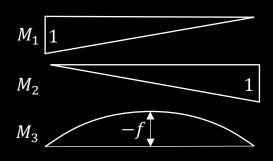




Solution using force method – Deck-stiffened arches

- If the bending stiffness of the deck girder is much higher than that of the arch rib, the latter can be neglected
 - → "deck-stiffened arch"
 - → bending moments carried (almost) by the deck girder alone
 - → reduced degree of statical indeterminacy
- If the columns (hangers) are idealised as pin-jointed members, the system is three times statically indeterminate
 - → solution using force method possible, but obsolete
 - → use frame analysis software
 - → for preliminary design, estimate bending moments using values shown on slide 63

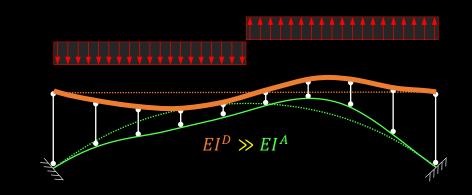
Redundant moments:



$$\delta_{ik} = \int_{0}^{l} M_{i} \frac{M_{k}}{EI^{D}} d\xi$$

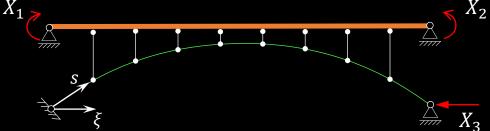
 δ_{ik} : flexibility coefficients

D: deck A: arch



→ idealisation: deck-stiffened arch



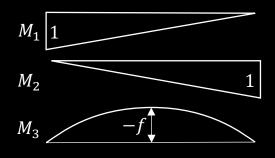


 $M^A = 0$ (but consider moments due to arch compression!)

Solution using force method – Stiff arches

- If the bending stiffness of the deck girder is much lower than that of the arch rib, the former can be neglected
 - → "stiff arch"
 - → bending moments carried (almost) by the arch rib alone
 - → reduced degree of statical indeterminacy
- If the columns (hangers) are idealised as pin-jointed members, the system is three times statically indeterminate
 - → solution using force method possible, but obsolete
 - → use frame analysis software
 - → for preliminary design, estimate bending moments using values shown on slide 63

Redundant moments:



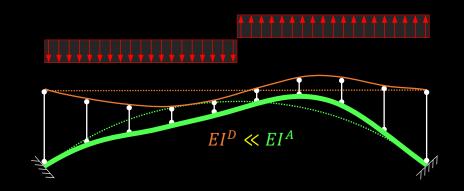
$$\delta_{ik} = \int_{0}^{l} M_{i} \frac{M_{k}}{EI^{A}} ds = \int_{0}^{l} M_{i} \frac{M_{k}}{EI^{A,c}} d\xi$$

$$(\text{for } EI^A \cos \alpha = EI^{A,c})$$

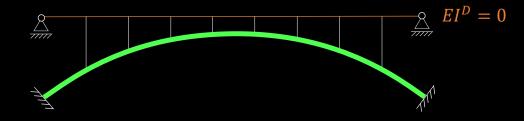
 δ_{ik} : flexibility coefficients

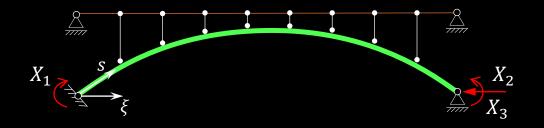
D: deck A: arch

A.c: arch at crown



→ idealisation: stiff arch





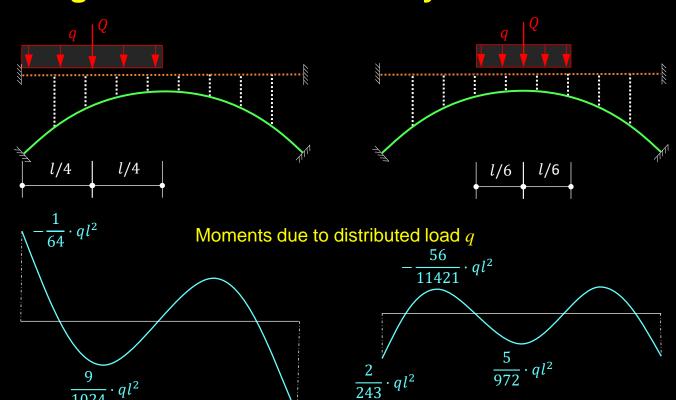
 $M^{D} = 0$ (but consider moments due to arch compression!)

Approximate values of bending moments due to traffic load (clamped arch with continuous deck girder)

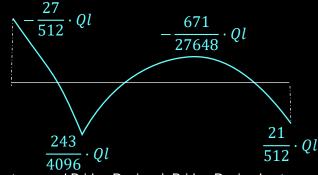
- Generally, the maximum bending moments need to be determined considering different load positions (e.g. using influence lines)
- In preliminary design, it is sufficient to check the maximum bending moments
 - → at the springing lines (arch abutments)
 - \rightarrow at the quarter-points
 - \rightarrow at the crown
- These may be estimated using the two load cases illustrated in the figure:
 - → symmetrical load over middle third of span
 - → asymmetrical load on one half span

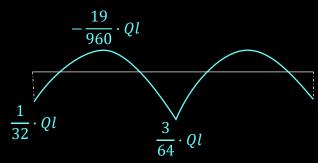
and distributed among arch rib and deck girder according to their stiffnesses

$$M^{D} \approx M \frac{EI^{D}}{EI^{D} + EI^{A,c}} \qquad M^{A} \approx M \frac{EI^{A,c}}{EI^{D} + EI^{A,c}}$$



Moments due to concentrated load Q

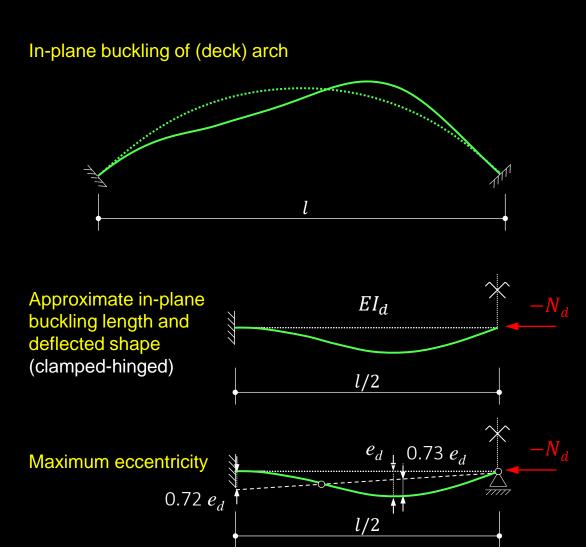




Design – Second-order bending moments

Arch bridges – Design: Second order bending moments

- Arches are compression members
 - → in addition to the (first order) moments in the flexible system, see previous slides, second order bending moments must be considered unless the arch is very stiff and they are negligible
 - → for deck arches, second order analysis can usually be limited to in-plane bending moments (deck girder provides lateral stability)
 - → for tied arches, out-of-plane stability (transverse buckling of the arch resp. corresponding 2nd order bending moments) are typically more critical
- In detailed design, a second-order analysis is carried out, assuming suitable imperfections (see substructure chapter) and the governing load positions, which typically are:
 - → in-plane stability: traffic load in one half-span
 - → out-of-plane stability: traffic load in full span
- In the preliminary design of deck arches, it is sufficient to consider anti-symmetrical in-plane buckling see figures and next slide.



Arch bridges – Design: Second order bending moments (in-plane)

- In the preliminary design of concrete deck arches, 2nd order in-plane bending moments can be determined using the curvature based method of SIA 262, see substructure chapter, considering arch rib and deck girder together as a compression member.
- If the deck is prestressed and the arch stiffness increases towards the abutments in line with the arch normal force, i.e.

$$EI^{A}(x) \approx \frac{EI^{A,c}}{\cos \alpha(x)}$$

a constant bending stiffness may be assumed:

$$EI_d \cong \frac{1}{4} (EI^{A,c} + EI^D) \longrightarrow \chi_d = 4 \frac{M_{Rd}^{A,c} + M_{Rd}^{D,c}}{EI^{A,c} + EI^D}$$

- The first-order eccentricities correspond to the bending moments for traffic load on one half span (previous slides), and the total eccentricity is as usual: $e_d = e_{0d} + e_{1d} + e_{2d}$
- The *c*-factors (superposition of actions) are given in the figure, and the resulting bending moments are resisted by arch rib and deck girder jointly, i.e.

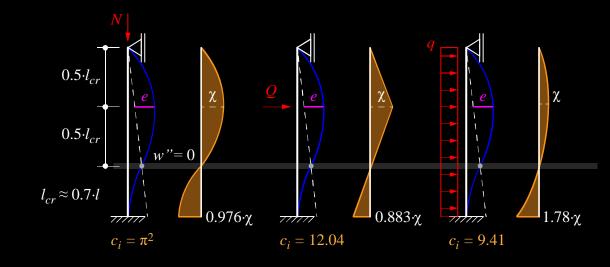
$$M^{D} \approx -N_{d}e_{d} \cdot \frac{EI^{D}}{EI^{D} + EI^{A,c}} \quad M^{A} \approx -N_{d}e_{d} \cdot \frac{EI^{A,c}}{EI^{D} + EI^{A,c}}$$

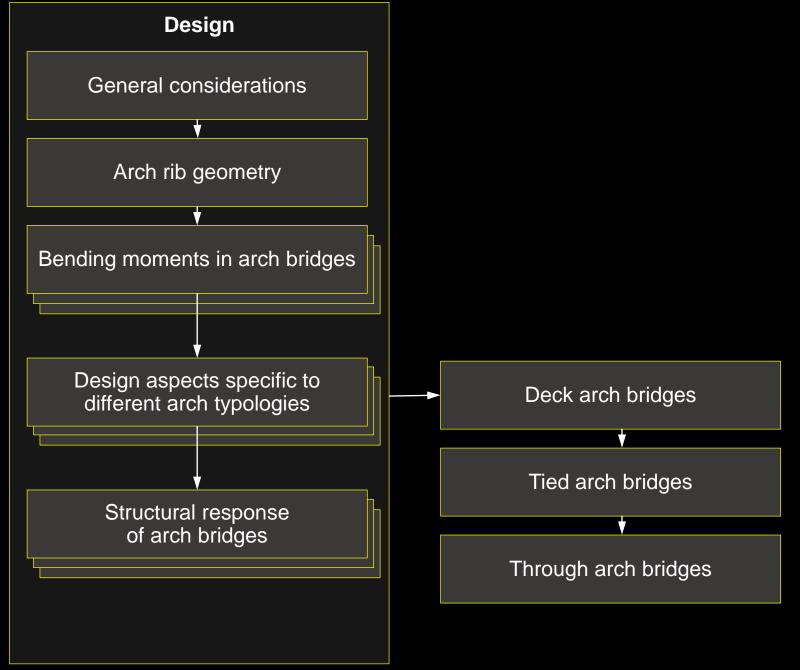
Approximate verification of in-plane 2nd order moments in deck arch

$$e_{d} = e_{0d} + e_{1d} + e_{2d}$$

$$\begin{cases} e_{0d} = \alpha_{i} \cdot \frac{l_{cr}}{2} & \text{geome} \\ e_{1d} = \frac{M_{1d}}{-N_{d}} & \text{first ord} \\ e_{1d} = \frac{l_{cr}}{2} & \text{eccent} \\ e_{2d} = \chi_{d} \frac{l_{cr}}{c} & \text{member} \end{cases}$$

eccentricity due to geometric imperfection first order eccentricity of action eccentricity due to member deformation





Design – Deck arch bridges

Arch bridge – Design: Deck arch bridges

Deck girder – General

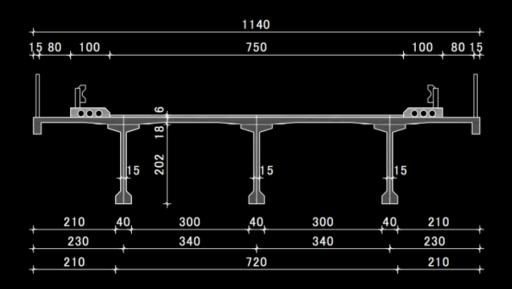
- The deck girder is supported by the arch through the axially stiff spandrel columns
 - → deck girder and arch share the same deflections
 - → the cross-sections of girder and arch must be chosen in consideration of their interaction:
 - \dots stiff arch \leftrightarrow slender deck girder
 - ... slender arch ↔ stiff deck girder
 - \rightarrow the stiffness ratio of deck and arch EI^D/EI^A is highly relevant for the structural response
- The girder depth is usually kept constant over the entire length of the bridge, and the girder needs to resist additional bending moments due to frame action (crown deflects due to arch compression, see structural response)
 - → less slender than in girder bridges
 - \rightarrow for prestressed concrete $1/15 \le h/l \le 1/12$

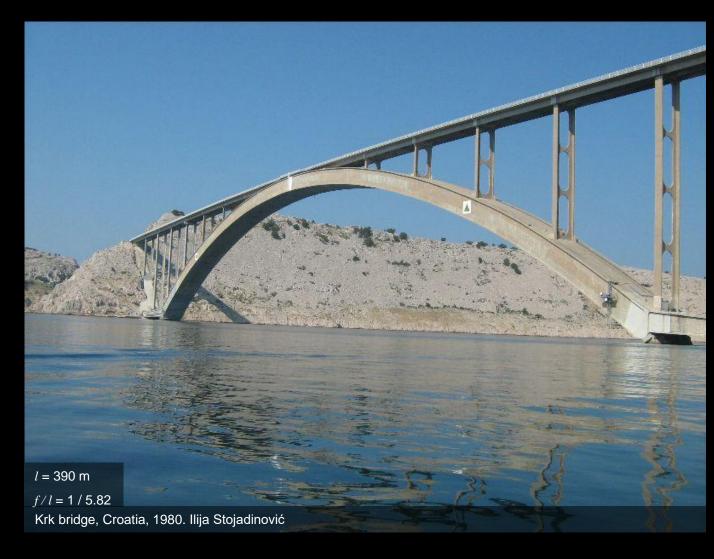


Arch bridge – Design: Deck arch bridges

Deck girder – Cross-section

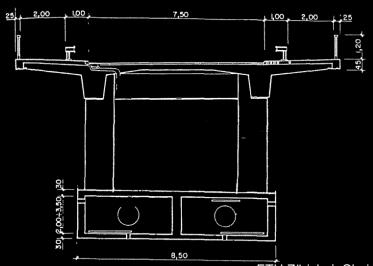
- For reasonably stiff arches (EI^D << EI^A), double-T or solid slab deck girders can be used, regardless of the arch span
 - → frame moments primarily resisted by arch
 - → bending moments in the girder depend mainly on the spandrel column span
 - → behaviour similar to continuous girder bridges (hogging moments ≈ 2·sagging moments)





Deck girder – Cross-section

- For reasonably stiff arches (EI^D << EI^A), double-T or solid slab deck girders can be used, regardless of the arch span
 - → bending moments in flexible system primarily resisted by arch rib
 - → bending moments in the deck girder depend mainly on the spandrel column span
 - → behaviour of deck girder similar to continuous girder bridges (hogging moments ≈ 2·sagging)





Deck girder – Cross-section

• For stiff arches ($EI^D << EI^A$), slender steel-concrete composite decks are also possible, regardless of the arch span.





Deck girder – Cross-section

- In flexible arches ($EI^D \approx EI^A$) or even $EI^D > EI^A$), the stiffness of the deck girder has a significant influence on the behaviour of the frame system
 - → significant part of frame moments resisted by deck girder
 - → higher deck girder stiffness required
 - → box girder cross-sections for deck of long-span arches
 - → sagging and hogging moments in the girder of similar magnitude over the entire length of the arch



NB. Aesthetics (arch abutments in river)?

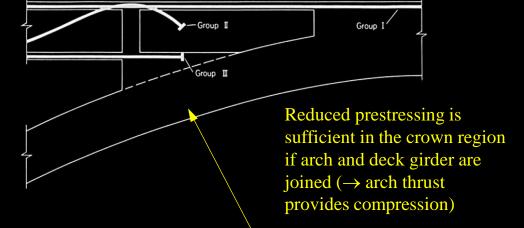


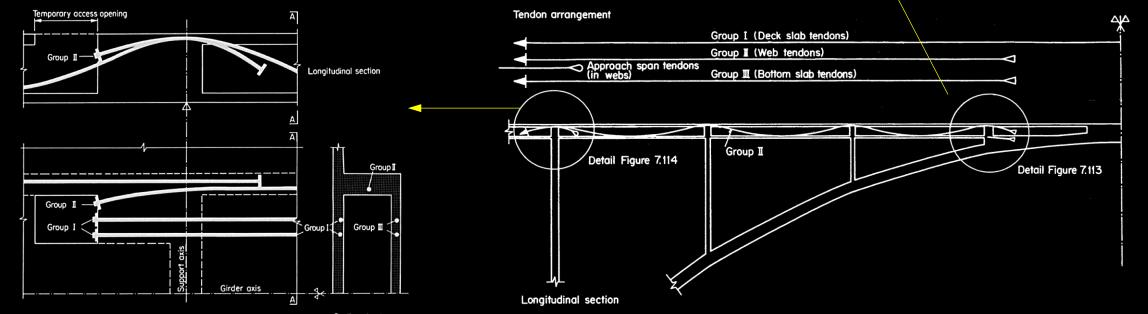
NB. Aesthetics (arch abutments on shore)!



Deck girder – Prestressing

- Concrete deck girders are commonly fully prestressed for permanent loads
 - → higher, uncracked stiffness improves global stability of the frame system (cracked-elastic second-order analysis is subjected to many uncertainties)
 - → enhanced durability





Arch rib

The arch transmits a significant horizontal reaction to the supports \rightarrow a strong soil is ideal

The structural response of the arch depends strongly of the ratio of rise-span f / ℓ



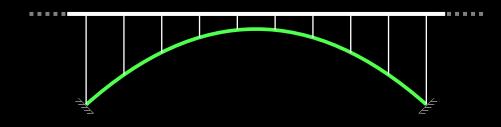
The structural response of the arch depends strongly on the supports and hinge arrangement:

- clamped arch
- two-hinged arch
- three-hinged arch



Arch rib

Clamped arch:

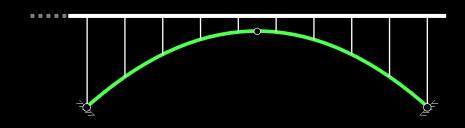


- → robust (specially during construction)
- → superior for non-symmetric actions
- → low clamped arches f / l < 1/10 are sensitive to imposed deformations and movements of the foundation (see structural response)
- → high arches are more economical (but low arches often aesthetically more satisfactory).



Arch rib

Two and three hinged arches:



- → hinges should basically be avoided (maintenance), but
- → if substantial movements of the foundations are expected, hinges at the springing lines may be beneficial (avoid high bending moments in the arch rib, see structural response)
- → hinges at the crown should be avoided where possible (durability, construction process)

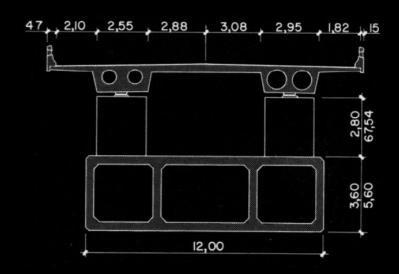


Arch rib

Usual cross sections of large-span arch ribs are:

- Hollow sections (single- or multi-cell)
 - ... low weight
 - ... high stiffness (radius of gyration I/A)
- Trusses (in steel bridges)

For shorter spans l < 150 m, solid cross sections or U-shaped cross sections are suitable





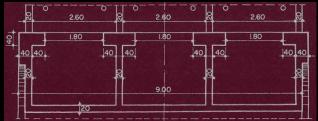
Arch rib

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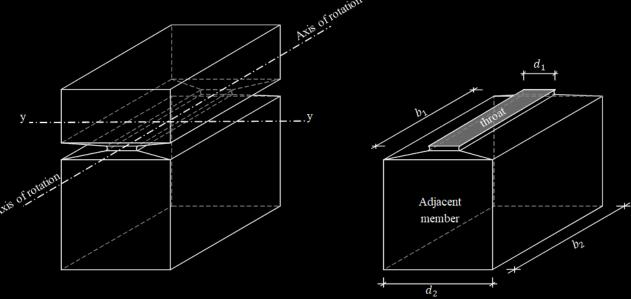


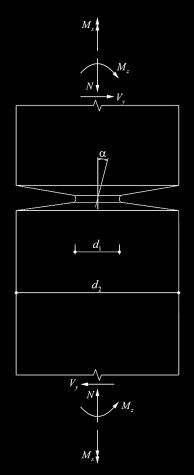


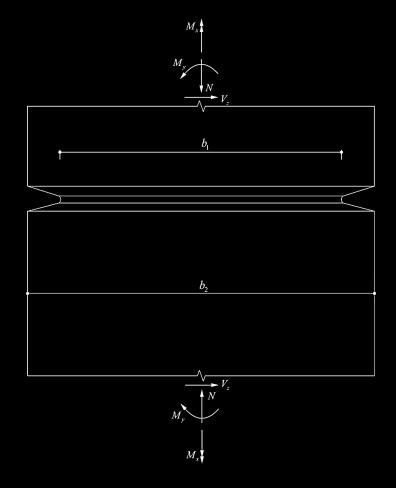


Spandrel columns

- Spandrel columns should be monolithically connected to deck girder and arch where possible, e.g. using slender columns
 - → enhanced durability
 - → simpler construction
 - → higher stiffness (frame) under non-anti-funicular load
- If hinged connections are required, concrete hinges are preferred (durability, maintenance) to bearings







Spandrel columns

- Spandrel columns should be monolithically connected to deck girder and arch where possible, e.g. using slender columns
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 - → higher stiffness (frame) under non-anti-funicular load
- If hinged connections are required, concrete hinges are preferred (durability, maintenance) to bearings

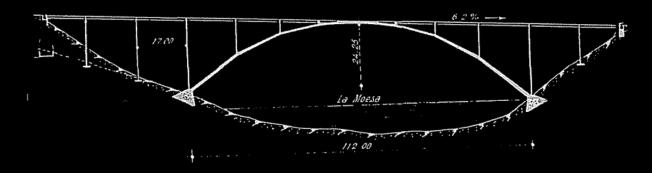


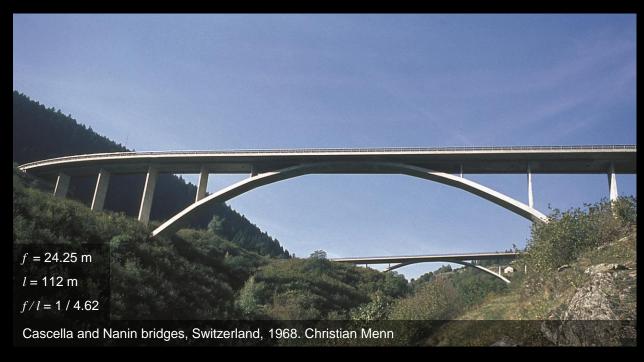




Stiffness of deck girder vs arch rib

- Basically, the required bending stiffness (stability, flexible system moments) can be arbitrarily allocated to the arch rib or the deck girder
- Concrete arch ribs have a high moment capacity
 without extra cost due to the compressive normal force,
 and a high stiffness EI^A of the arch rib is also favourable
 during construction
 - → for structural efficiency, the concrete arch rib should be stiffer in bending than the deck (such that it will carry most of the moments)
- On the other hand, the deck girder always provides a minimum stiffness
 - → very slender arches possible if built on centring
 - → "secret" of the elegance of arch bridges designed by Christian Menn
 - → however, arches built on centring are uneconomical (even if still built occasionally, if economy is of little importance)

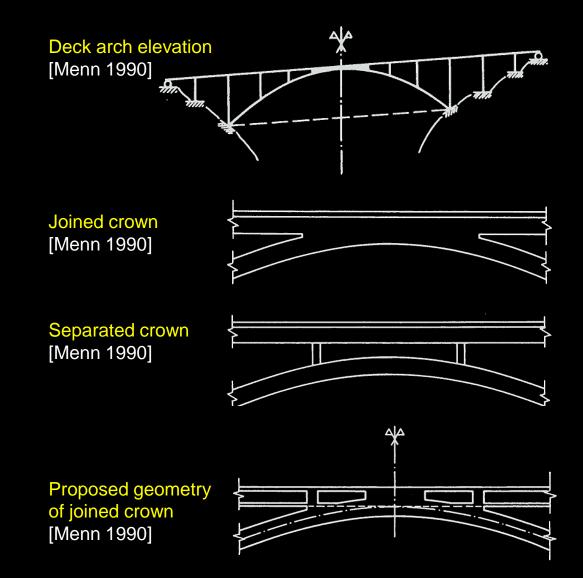




Aesthetics

When designing a deck arch bridge, the following points – mostly proposed by Ch. Menn – should be considered; note that these are no rules, but merely points of orientation:

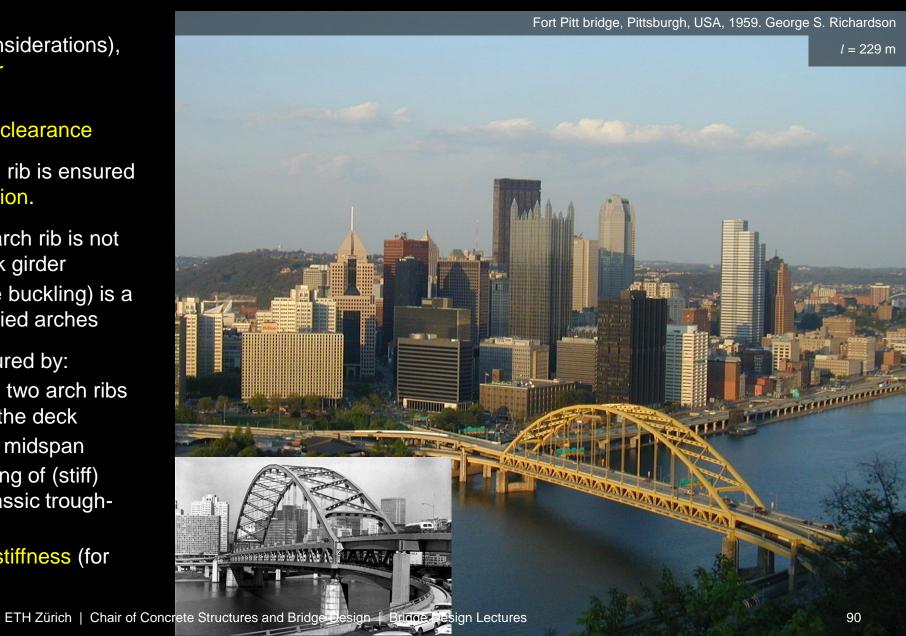
- The connecting line of the arch abutments (springing line) resp. the arch intersection with the ground should be parallel to the girder (top figure).
- Providing at least 4-6 spandrel columns at equal distance (5-7 equal parts) is preferable (if less spandrel columns are required, check feasibility of strut-frame bridge, see frame bridges, and if not possible, provide polygonal arch).
- If arch and deck (stiffening girder) are separated, no column should be provided at midspan.
- If arch and deck (stiffening girder) are joined monolithically, a satisfactory appearance is obtained by using the same depth for girder and arch and making sure that the arch axis is tangent to the (extended) girder soffit line (intrados), see bottom figure).



Arch bridges

Design – Tied arch bridges

- As already outlined (general considerations), tied arch bridges are suitable for
 - → locations with soft soil
 - → single-span bridges with low clearance
- The in-plane stability of the arch rib is ensured by the deck girder acting in tension.
- Other than in deck arches, the arch rib is not commonly stabilised by the deck girder
- → out-of-plane stability (transverse buckling) is a governing design parameter of tied arches
- Transverse stability can be ensured by:
 - → transverse bracings between two arch ribs running along the outside of the deck
 - → inclined arches connected at midspan
 - → transverse U-frames consisting of (stiff) "hangers" and deck (as in classic troughsection girder bridges)
 - → arches with high transverse stiffness (for short spans)



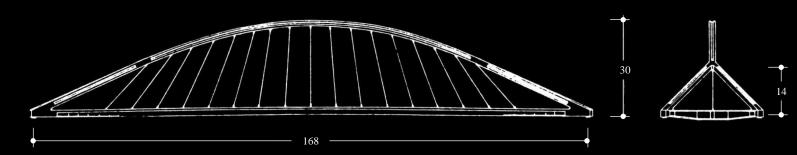
Tied arches are often steel bridges. The Rheinhauser Brücke in Duisburg is the longest tied arch in Germany (since 1988 "Brücke der Solidarität").

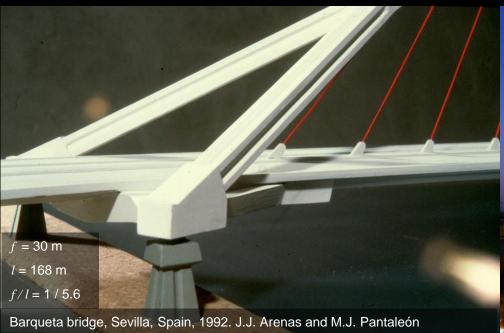






The Barqueta Bridge was the first tied arch with one central arch rib above the roadway (rather than joining two continuous arch ribs).







Concrete tied arch bridges are less frequent. The Puente del Tercer Milenio in Zaragoza is one of few large-span concrete tied arches.







- Arches with transverse bracings or connected arch ribs (previous two examples) require a minimum height of the to provide sufficient clearance on the bridge.
- In smaller span arches, such bracings can be eliminated if the "hangers" act as frames, stabilising the arch ribs
 - → provide hangers with transverse stiffness
 - → transverse frame action of deck-hanger-arch
- Such arches can be very slender, and are attractive to cross as they generate a «curtain effect» to the user (bottom photos).











- Aesthetically, the elevated arch ribs of tied arch bridges should be slender and are thus flexible
 - → stiffness (for non-anti-funicular loads) of tied arch bridges must be ensured by other elements
- Conventionally, stiff deck girders were used to ensure sufficient stiffness (previous examples)
- Alternatively, the hangers can be used to this end, with the following options
 - → Hangers inclined in elevation forming a truss together with arch rib and deck girder
 - ... hangers forming a Warren truss (Strebenzug) without intersections = Nielsen arch
 - ... hangers intersecting = Network tied arch
 - → Stiff "hangers" forming a Vierendeel girder together with arch rib and = Vierendeel arch
- Network tied arches have gained increasing popularity in the recent years due to their high structural efficiency (photo and next slides).





l = 71 mf/l = 1/4.73

The Fehmarnsund bridge was the first long-span network tied arch bridge (conversion to local traffic only planned for 2028, new tunnel across Fehmarnsund connecting to Fehmarnbelt).







- Network tied arch bridges are very efficient and can thus be used for very long spans.
- They are aesthetically attractive and very economical if an efficient erection method can be used.
- The slide shows the currently longest span network arch bridge.





- Thanks to their efficiency, network tied arch bridges can be designed extremely slender and lightweight (photos).
- However, they are challenging for analysis and detailing
 - → sign reversals in the hanger forces, resulting in sagging hangers, must be avoided (critical for high live load to dead load ratio and flat hangers)
 - → hangers are prestressed, analysis needs to account for hanger preload (similar as in cable-stayed bridges)
 - → steep hangers are prone to fatigue (high load variation due to slender deck)
 - → hanger arrangement requires complicated details (no standard connections)
- For these reasons, designers were reluctant using this efficient bridge typology for many decades.
- However, with modern analysis, drafting and fabrication methods, these challenges can be mastered.

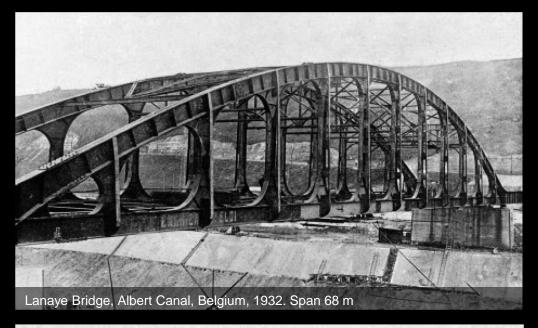


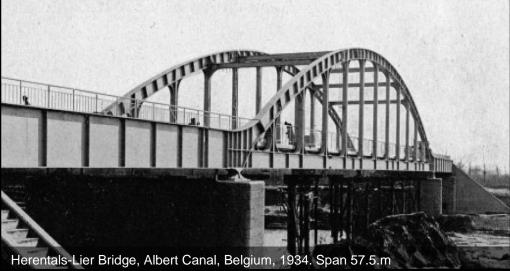


Brandanger Bridge, Norway, 2010. Aas-Jakobsen.

Vierendeel arches have only been used in few bridges, despite a large number of such bridges being built in Belgium in the 1930s over the Albert Canal.







Arch bridges

Design – Through arch bridges

- The logic of form is a strong positive point of deck arches – which are obviously true arches.
- At least to structural engineers, the force flow is equally clear in tied arches (laymen often think they are true arches).
- In through arches, however, it is often impossible to tell whether they act as true or tied arches, even to experienced bridge designers, without closely inspecting the bridge ends or even consulting drawings.
- As an example, consider the Castelmoron Bridge:
 - → well-known bridge (as it is one of the few original Nielsen arch bridges) in the bridge community
 - → arch exhibits no kink at the hinges at deck level: indicates that any tie force in the deck girder would be continuous (equal in main span and adjoining part of the bridge)
 - → but the adjoining part of the bridge might act as Vstruts



- Few bridge designers would thus bet much on how this bridge carries the loads without knowing more.
- Only a virtual visit to the bridge reveals that
 - → it is (most likely) acting as true arch, as there is no element that could transfer the arch thrust from the springing line back up to the girder
 - → its soffit is also worth having a closer look





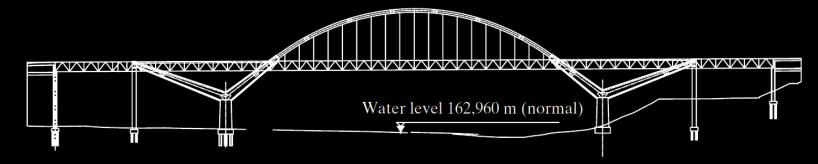


- When designing a through arch, which makes sense in many cases (clearances vs road alignment), it should be ensured that the force flow is legible.
- This slide shows a clear example of a through arch acting as true arch.

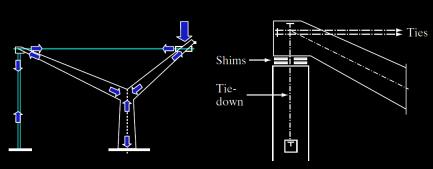




 This slide shows an equally clear example of a through arch acting as tied arch on V-struts.



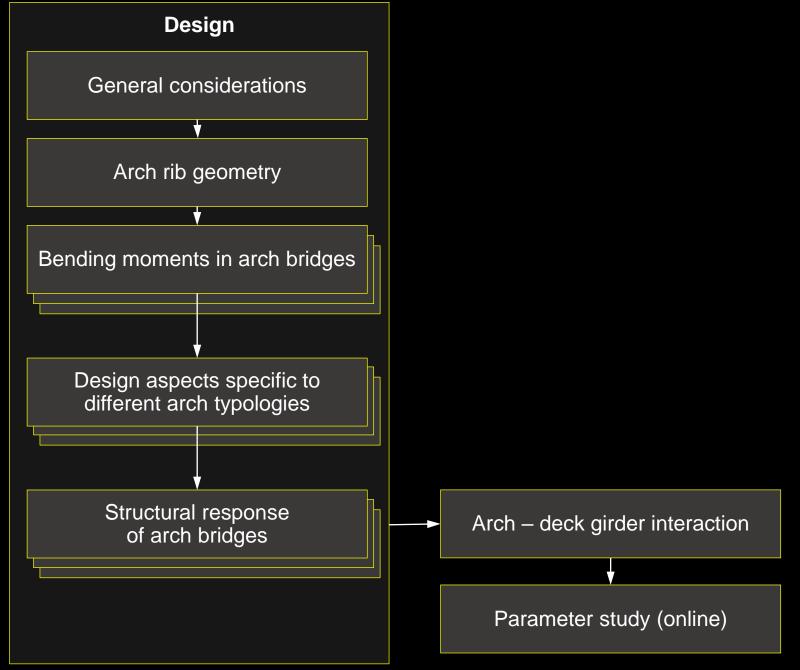






Arch bridges

Structural response



Arch bridges

Structural response – Arch-deck girder interaction

Arch bridges – Structural response: Arch-deck girder interaction

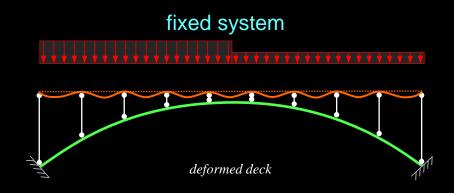
If an anti-funicular arch geometry is chosen, usually for permanent loads, arch bridges carry the corresponding loads efficiently.

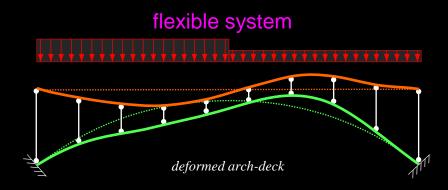
However, non-anti-funicular loads need to be accounted for in design. Under such loads, the arch – arch rib, deck girder and spandrel columns or hangers – act as a frame system, whose behaviour depends on

- → the stiffness ratio of arch rib and deck girder
- → the type of connection between arch rib and deck girder (clamped or pin-jointed spandrel columns resp. "hangers" – see notes)

To better understand the behaviour, the bending moments in the frame system can be subdivided into two components:

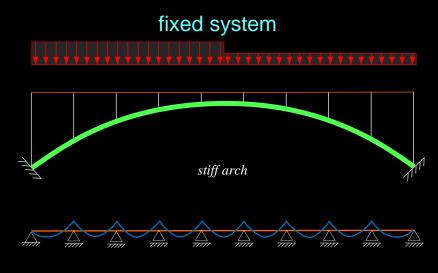
- fixed system
- flexible system





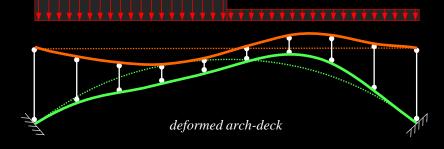
The following points have essentially been outlined in the *Design* section. Here, they are repeated and a case-study is presented to highlight some specific aspects.

- fixed system
 - → assume a perfectly rigid arch
 - → bending moments in deck girder corresponding to those in a continuous beam (replacing spandrel columns by supports).
- flexible system
 - → bending moments in the flexible system involve arch deflections due to non-anti-funicular loads
 - → generally, these bending moments are shared by arch rib and deck girder in proportion to their bending stiffnesses
 - → two ideal limiting cases can be considered:
 - → deck-stiffened arches ("versteifter Stabbogen"), where the entire flexible system moments are resisted by the deck girder ("Versteifungsträger")
 - → *stiff arches* resisting the entire flexible system moments alone



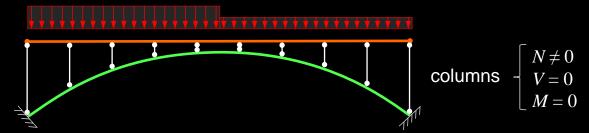
deck bending moments

flexible system



In this study, a clamped deck-arch bridge, with expansion joints of the deck above the arch abutments (intersection of springing line with arch axis) is considered (unlike Slide 63: deck continuous).

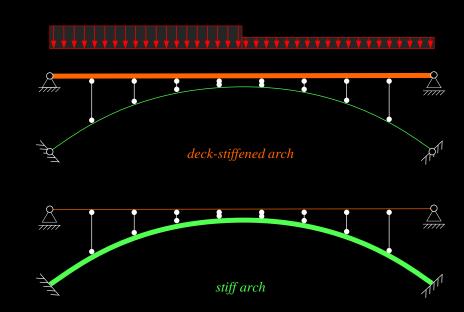
In the first part, pin-jointed spandrel columns are assumed.



Two limiting cases:

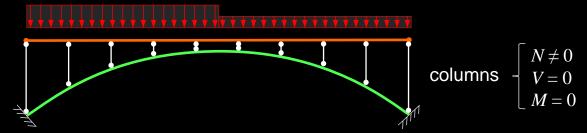
- deck-stiffened arch
 - \rightarrow flexural deck girder stiffness $EI^{D} >>$ flexural arch rib stiffness EI^{A}
- stiff arch
 - \rightarrow flexural arch rib stiffness $EI^A >>$ flexural deck girder stiffness EI^D

In these limiting cases, either the stiffening girder or the stiff arch resists (almost) the entire bending moments.



In this study, a clamped deck-arch bridge, with expansion joints of the deck above the arch abutments (intersection of springing line with arch axis) is considered (unlike Slide 63: deck continuous).

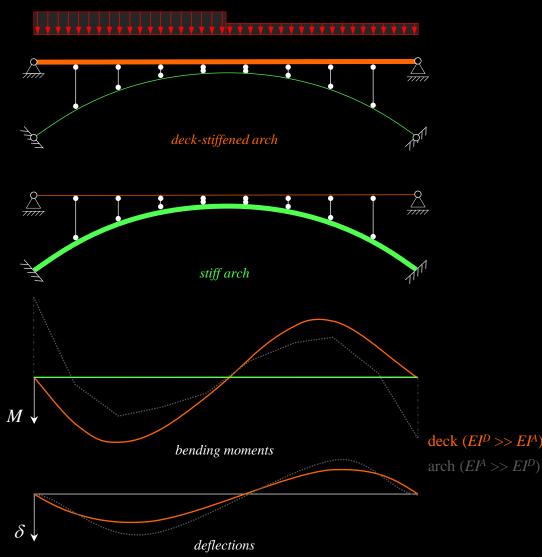
In the first part, pin-jointed spandrel columns are assumed.



Two limiting cases:

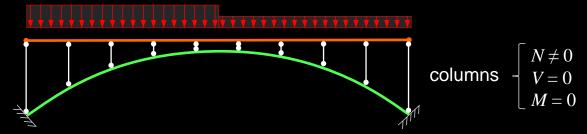
- deck-stiffened arch
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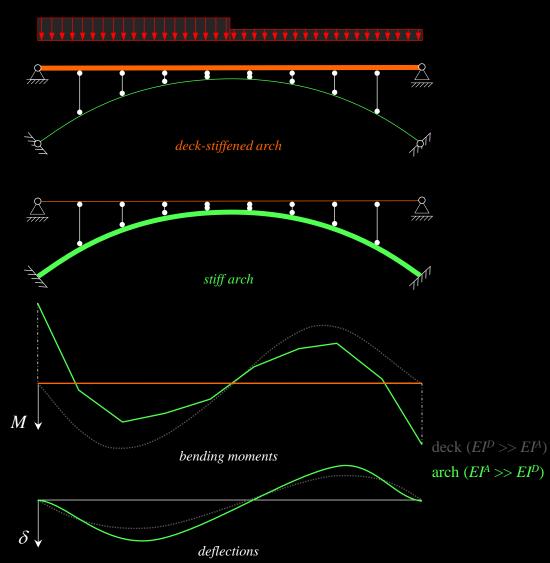
In the first part, pin-jointed spandrel columns are assumed.



Two limiting cases:

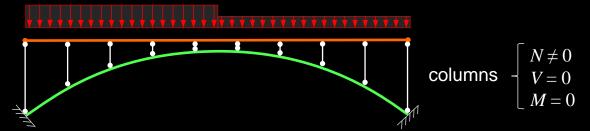
- deck-stiffened arch
 - \rightarrow flexural deck girder stiffness $EI^D >>$ flexural arch rib stiffness EI^A
- stiff arch
 - \rightarrow flexural arch rib stiffness $EI^A >>$ flexural deck girder stiffness EI^D

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In the first part, pin-jointed spandrel columns are assumed.

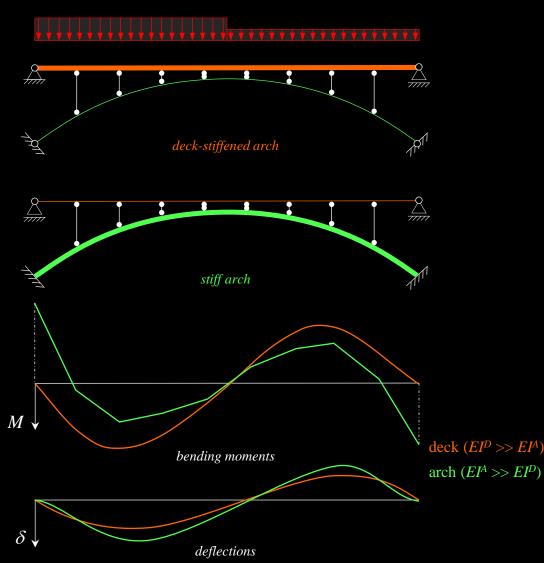


Two limiting cases:

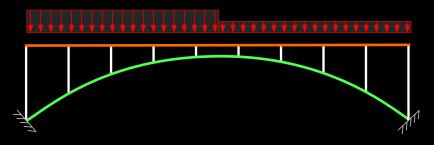
- deck-stiffened arch
 - \rightarrow flexural deck girder stiffness $EI^{D} >>$ flexural arch rib stiffness EI^{A}
- stiff arch
 - \rightarrow flexural arch rib stiffness $EI^A >>$ flexural deck girder stiffness EI^D

In these limiting cases, either the stiffening girder or the stiff arch resists (almost) the entire bending moments.

The differences of bending moments and deflections between arch rib and deck girder are due to the different support conditions assumed here (clamped vs. simply supported). In the design section (Slide 63), both are assumed to be continuous.



If the spandrel columns are clamped, rather than pin-jointed, arch rib and deck are not only coupled in terms of vertical deformations, but act as frame system.

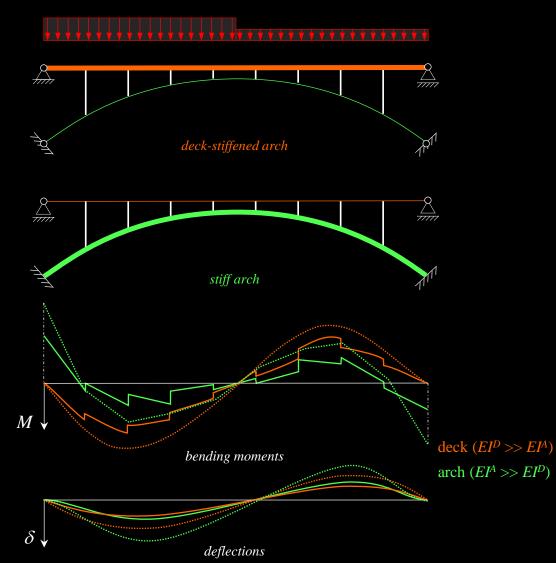


Clamped spandrel columns, together with deck girder and arch rib, act as Vierendeel girder

- → significantly stiffer than sum of deck girder and arch stiffness
- → deflections significantly reduced

The short clamped spandrel columns close to the crown have a high flexural stiffness and transfer the axial normal force from the arch rib to the deck.

In some cases, shear forces and bending moments in such spandrel columns may be excessive → (concrete) hinges may be provided to reduce these actions (e.g. Tamina bridge)



Arch bridges – Structural response: Arch support conditions / hinges

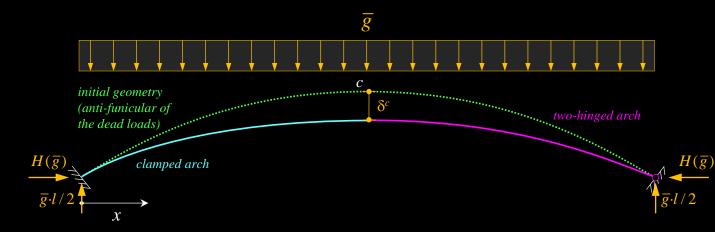
Effect of rise-to-span ratio f/l on bending moments

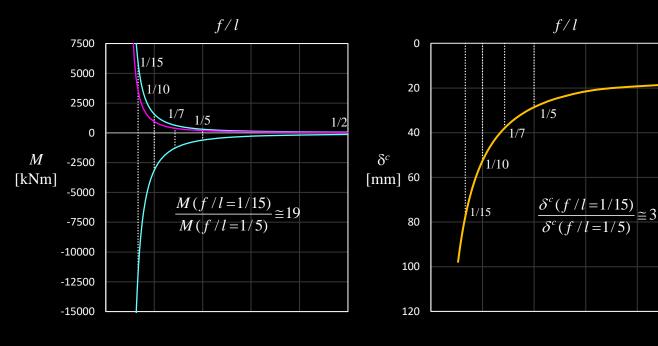
Here, a uniform permanent load g and a linear analysis is used. The arches considered are:

- two-hinged arch
- clamped arch

Using these assumptions and equations in the numerical example (l=100 m; h=1.20 m; DL=140 kN/m), the following results are obtained (see graphs):

- The rise-span ratio f/l is highly relevant, having a strong impact on structural behaviour, particularly for small values of f/l (low arches)
- Bending moments increase exponentially with smaller values of f/l, particularly pronounced for f/l < 1/10. For f/l = 1/15, bending moments are up to 15 times higher than for f/l = 1/5.
- The crown displacement also grows progressively as f/l decreases, especially for f/l < 1/10
- Clamped and two-hinged arches show similar tendencies.





Arch bridges – Structural response: Arch support conditions / hinges

Effect of rise-to-span ratio f/l on bending moments

Here, a uniform permanent load g and a linear analysis is used. The arches considered are:

- two-hinged arch
- clamped arch

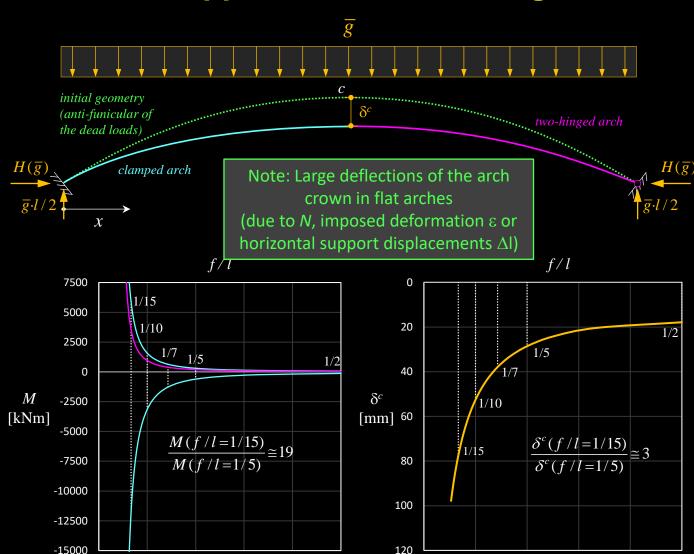
Note that similar results are obtained when the arches are subjected to horizontal displacements of the supports.

The resulting bending moments, for a low arch (risespan ratio lower than 1/10), may exceed the moments produced by the gravity loads.

Conversely, the influence of imposed deformations are relatively small in arches which rise-span ratios > 1/7.

The numerical results correspond closely to the approximation (slide 55) for *EA*=const., i.e.

$$\delta^c \cong \frac{H(\bar{g})}{EA^{A,c}} \cdot l \cdot \frac{1+3(f/l)^2}{4f/l}$$
 is a good approximation.



Arch bridges – Structural response: Arch support conditions / hinges

Permanent load + imposed deformation 1st and 2nd order analysis

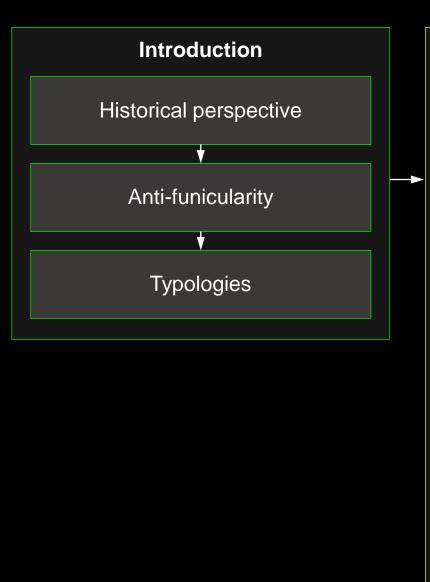
- reinforced concrete
- three-hinged arches → two-hinged arches
- central span: 72.5 m
- f/l = 1/15

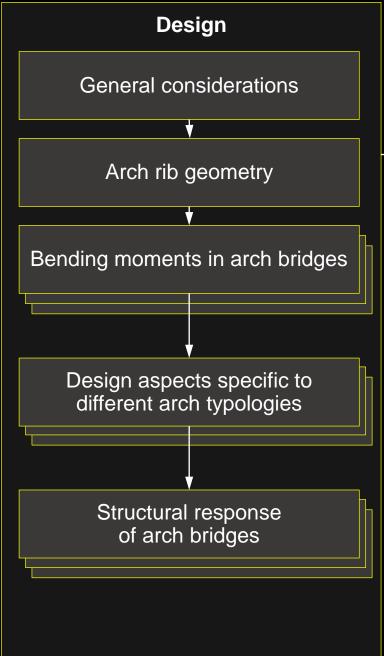


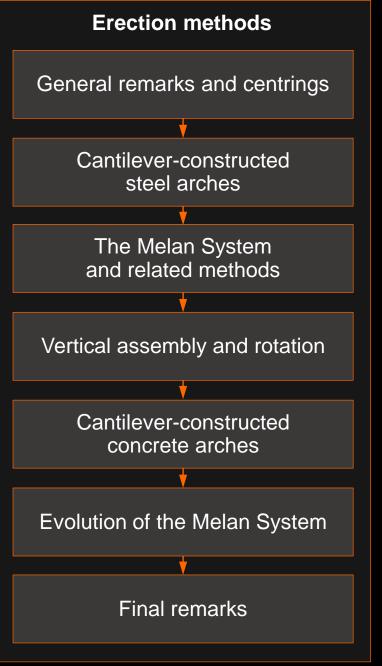


Arch bridges

Erection methods





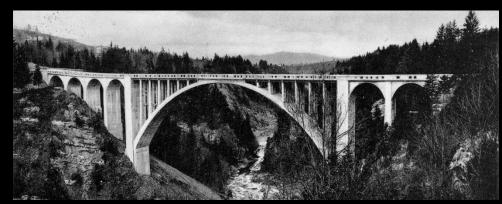


Arch bridges

Erection methods – General remarks and centrings

High relevance of erection method in arch bridges

- The construction process is an essential part of the conceptual design of any bridge
- While arches are very efficient structures in the final configuration
 - → the efficient arch action is only activated once the arch is able to transfer the arch thrust, i.e., after closure (Bogenschluss)
 - → the load transfer during construction differs strongly from that in the final configuration, undermining economy (see Conceptual Design)
 - → the construction process is particularly relevant for the economy of arch bridges



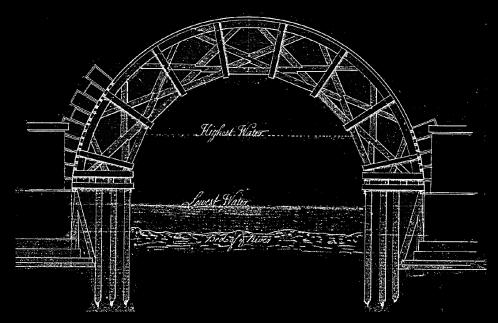


Stone arches

- For centuries, stone arches have been erected on timber centrings (= arch or dome falsework)
- Information on Roman arch bridges, and more so their centrings, is scarce (Vitruvius gives some information)
- The practice of building stone bridges died out in Europe with the collapse of the Roman empire and only reappeared in the middle age (see notes)

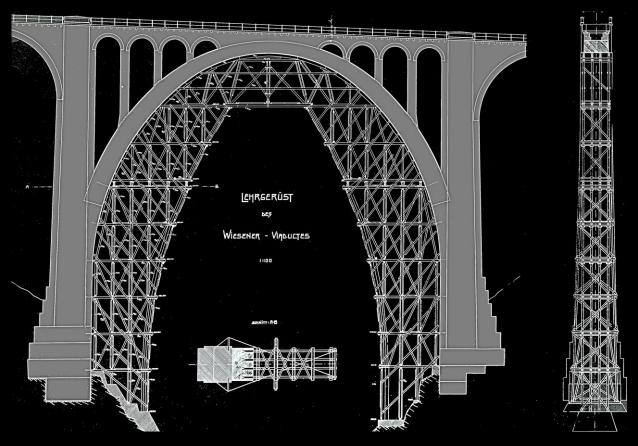








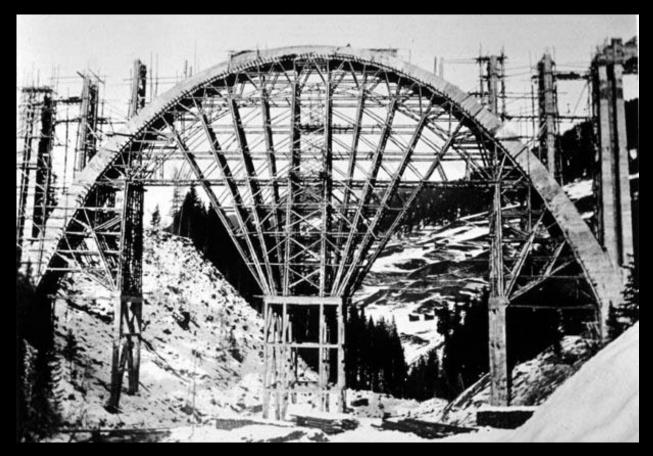
 Though inherently inefficient for larges spans (see previous slides), this method was used for the first concrete arch bridges, that were indeed "concrete stone" arch bridges, using concrete as inexpensive stone surrogate.







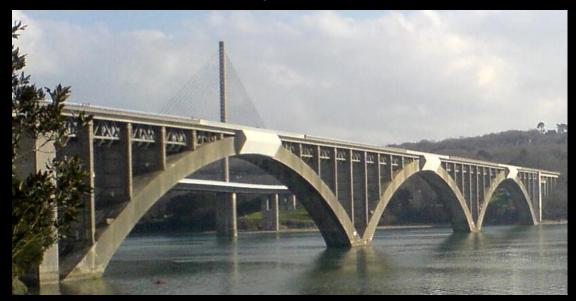
- Centrings, often using timber, were also used for the first reinforced concrete arch bridges.
- In many cases, the falsework was as attractive, if not even more appealing, than the final structure



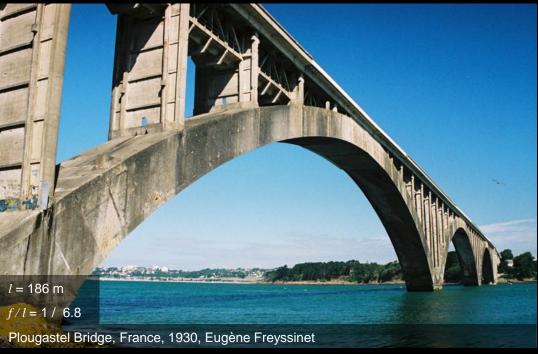




- Timber centrings can also be used for arch bridges crossing water. They can be assembled on shore (where the centring can easily be supported) and then floated in as tied arch.
- The most prominent example is Freyssinet's Pont de Plougastel (Pont Albert-Louppe) crossing the bay of Brest, see photos.
- "Wind deviation" devices (see photo below) were mounted on Freyssinet's iconic bridge after the construction of a modern cable-stayed bridge nearby, to protect the latter from turbulence – a disgrace.

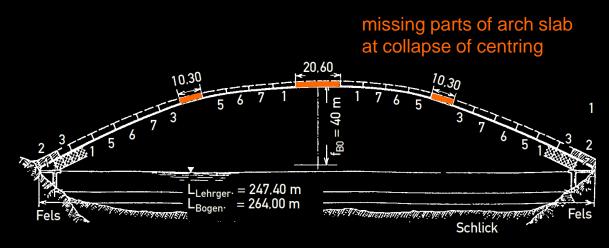






- Large-span timber falsework arches need to be designed and detailed as meticulously as final structures.
- While there were no problems in the Plougastel bridge, the similar falsework of the Sandö Bridge – though with a substantially longer span (record concrete arch span at the time) – collapsed on the 31.8.1939. Eighteen construction workers died.
- The bridge was then finally built on a massive falsework with intermediate shoring.

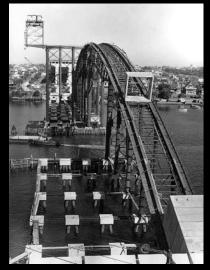






- The Gladesville Bridge in Sydney (main span 305) succeeded The Sandö Bride in 1964 as longest span concrete arch bridge.
- While looking similar, the Gladesville Bridge is more slender and featured several innovative construction methods, with a high degree of prefabrication, resulting in a highly efficient construction process (see notes).
- The bridge was designed mainly by T. Gee, a young British engineer (born 1934), and was the last major project in which E. Freyssinet was personally involved.





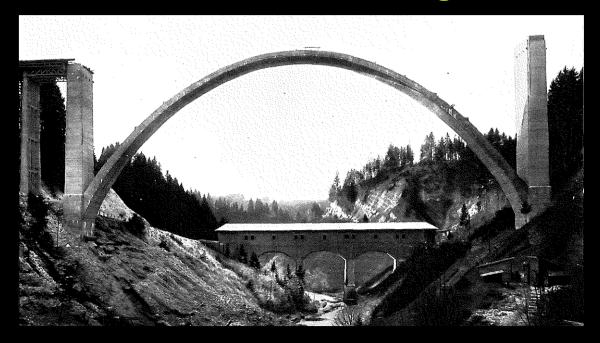






- Most of the prominent early concrete arch bridges were built using remarkable timber centrings.
- This and the following slides show three further extraordinary Swiss examples (Hundwilertobel Bridge, Salginatobel Bridge, Gueroz Bridge).

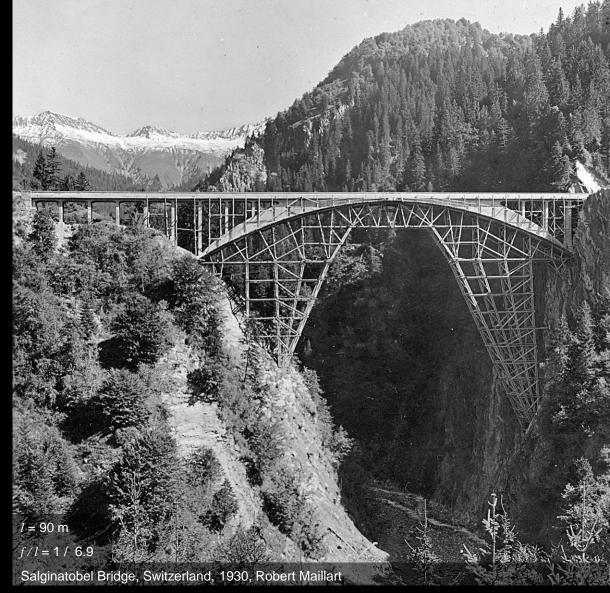






- These centrings were already expensive at the time, but still competitive with other construction methods due to the relatively low labour cost compared to materials, particularly steel.
- Richard Coray designed many of these centrings, whose erection was often challenging.





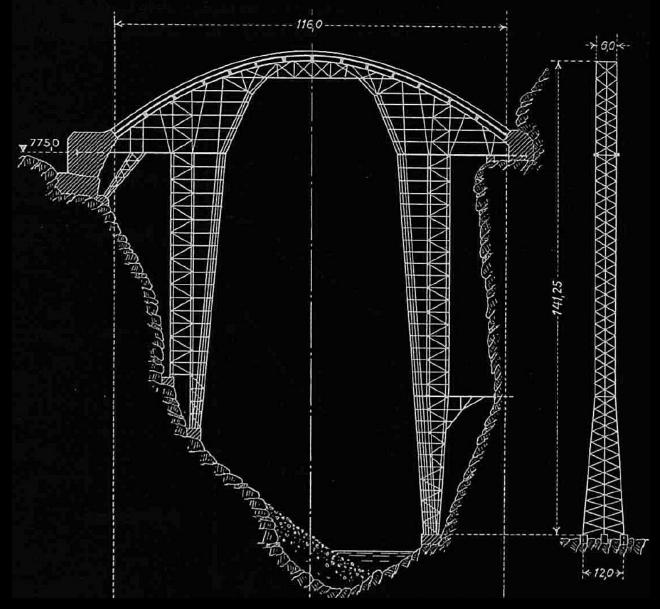
- Centrings remained the preferred construction method of many designers for reinforced concrete arches until the 1940s, although alternative erection methods existed already (Melan system, see behind).
 - → Consequently, only few reinforced concrete arches with spans above 80, and only a handful above 100 m were built (see notes).





- Economy remained a problem of concrete arches cast on centrings, despite progress in construction and analysis methods such as casting of the arches in rings (similar to the erection of stone arches in rings, e.g. in the Soliser Viadukt).
- In the Tara bridge (aka Đurđevića-Tara Bridge), the arch was cast in three rings, enabling a lighter centring (centring by R. Coray).





Arch bridges

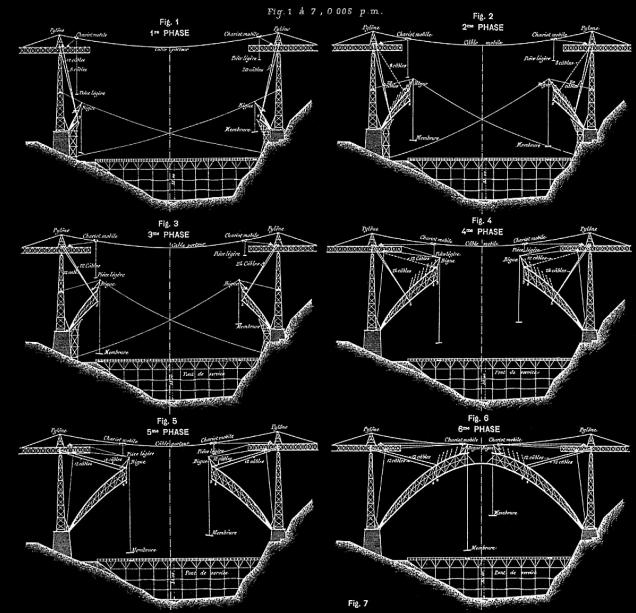
Erection methods – Cantilever-constructed steel arches

- On the other hand, for almost 150 years, steel truss arch bridges have been built by cantilevering, either with or without temporary supports or stays.
- Typically, they were designed as two- or three-hinged arches to minimise restraint
- The first, prominent example is the Eads Bridge across the Mississippi, built by cantilevering (with temporary towers and stays) as early as 1874, with three spans above 150 m.





- In truss arches built by cantilevering without backstays, the arch abutment is clamped during construction, and only free to rotate after closure of the arch. Hence, the upper truss chords
 - are fully utilised in construction (tension chords)
 - receive little load in the final configuration (they primarily help stabilizing the arch from buckling)
- This is most obvious at the bridge ends, where forces during cantilevering are highest, but once the arch is closed, the top chords are virtually (completely in twoor three-hinged arches) stress free
 - → such arches are inherently uneconomical and, in this respect, lack logic of form.
- Using temporary towers and stays during cantilevering as in the Garabit Viaduct, the arch can be hinged at its abutments from the beginning, yielding a much more consistent design in the final configuration.



- If the arch abutments are hinged from the beginning of construction, the structural safety during erection fully relies on the temporary towers and stays
- While this appears more economical, it is certainly less robust. In the following years, steel truss arches were thus frequently cantilevered starting with a clamped arch (converted to a hinge after closure) combined with temporary towers and stays.

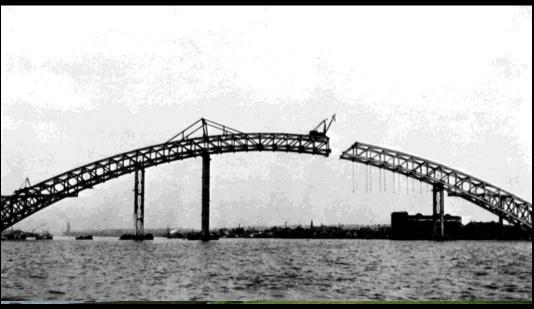






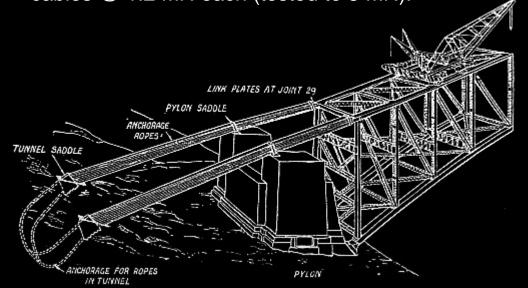
- Among the many cantilever-constructed truss arch bridges worldwide, there are several iconic structures.
- The Bayonne Bridge, across the Kill van Kull strait, designed by Swiss engineer Othmar Ammann and his team, set a new arch span record of 511 m when it opened in 1930 (top right photo), that held until 1977.
- In order to increase navigational clearance, the deck was raised by about 20 m in 2017, under full traffic during construction – an extraordinary achievement (bottom photos)







- The Sydney Harbour Bridge, with a slightly smaller span of 503 m, is another, perhaps even more iconic steel truss arch bridge.
- While temporary supports in the Kill van Kulll were used in the former, the Sydney Harbour Bridge was built by cantilevering without temporary towers nor stays.
- In turn, massive temporary steel support cables running in tunnels were used during construction (128 cables @ 1.2 MN each (tested to 5 MN).

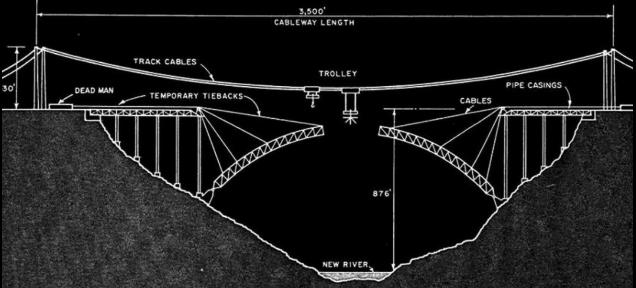




- The New River Gorge Bridge, West Virginia, set a new arch span record (518 m) in 1977, that held until 2012.
- While the Bayonne Bridge and the Sydney Harbour Bridge are through arches – though the full arch thrust is resisted by the foundations –, the New River Gorge Bridge is a deck arch.
- This enabled using stays extending from the deck above the abutment (figure), similar as in the Garabit viaduct, and building the arch hinged at abutments.
- Other than in Garabit, the arch segments were transported via a cableway system (Seilkran).







- Steel truss arches are still being built today, see e.g. the bridge illustrated on this slide (New Burro Creek Bridge, 2007), cantilevered using temporary diagonals similar as in recent concrete arches, see behind).
- However, mainly due to the relatively high cost of steel as a compression member, they have become less competitive compared to other typologies:
 - → Cable-stayed bridges are more economical than tied or through arches in most cases, particularly for very large spans
 - → Concrete arch bridges have become more economical for medium-large spans by the development of erection methods that are much more efficient than centrings
 - → Recently, steel-concrete arch bridges have become economical for even longer spans and are frequently used, particularly in China





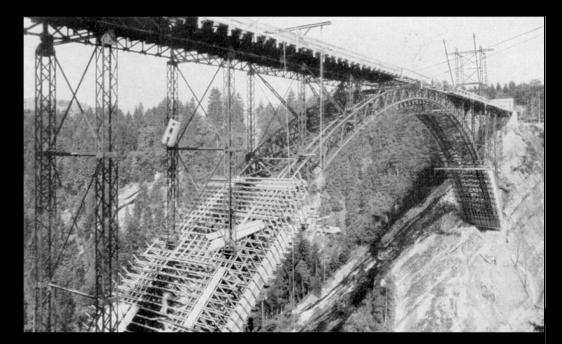
New Burro Creek Canyon Bridge, USA, 2006, Arizona Department of Transportation Bridge Group

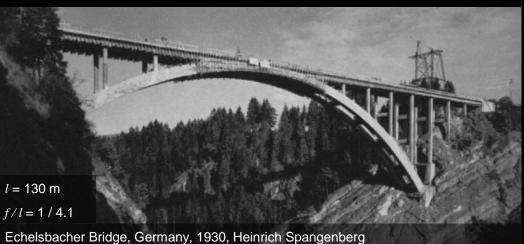
Arch bridges

Erection methods – The Melan System and related methods

Arch bridges – Erection methods: The Melan System and related methods

- The erection of efficient arch bridges can be greatly facilitated by the combined use of steel and concrete.
- Already in 1892, Josef Melan patented his Melan System, which used steel profiles as "rigid reinforcement" – essentially a composite construction system (see notes). Applying this system to arch bridges consists of the following:
 - → erecting a steel arch (steel truss, bracings provided to ensure stability against buckling)
 - → fixing a (timber) formwork to the steel arch
 - → casting the concrete around the steel profiles
- Melan himself did not design many structures, and many engineers at the time had concerns about the combined action of steel profiles and concrete. Composite action was not well understood, and shear connectors unknown.
- Heinrich Spangenberg resolved the concerns regarding different stress states in steel and concrete by ballasting the steel arch with gravel and removing the latter in the sections where the concrete was cast (System Melan-Spangenberg).
- The Echelsbacher Brücke (illustrations) was the longest span arch built using this system.





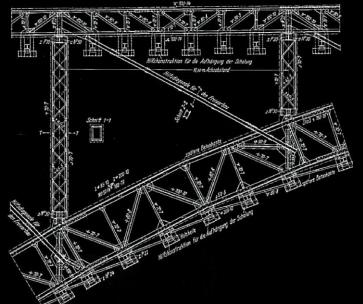
Arch bridges – Erection methods: The Melan System and related methods

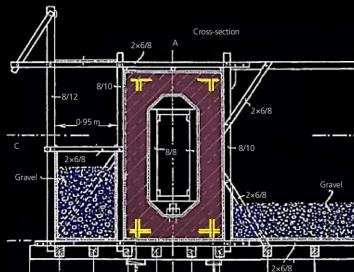
- However, the Melan-Spangenberg system complicated erection and undermined the economical advantages of the Melan System
 - → many, if not most "Melan arch bridges" were built using conventional falsework
 - → Often, Melan System trusses were supported on towers / shoring, see photos of Pont des Planches

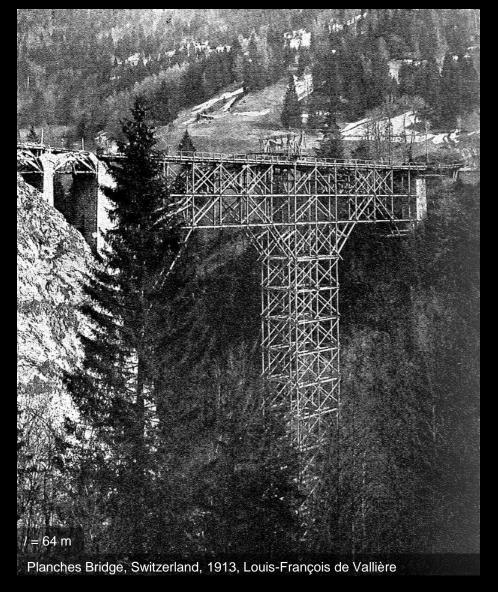
Echelsbacher Brücke: Steel truss

Echelsbacher Brücke: Arch section and formwork

yellow: steel truss / blue: gravel

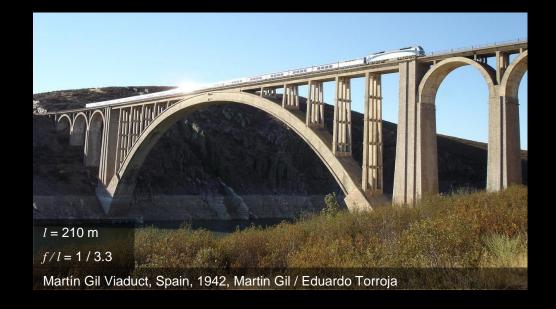




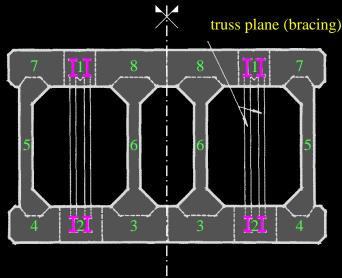


Arch bridges – Erection methods: The Melan System and related methods

- The Spanish engineer and entrepreneur José Eugenio Ribera optimised the Melan System (double trusses providing more stiffness) and patented the modified system himself in 1902.
- Ribera was very successful with this system in Spain, building several hundred bridges his patent.
- Fritz von Emperger, a scholar of Melan, was similarly successive with the original Melan System in the U.S. (Melan Arch. Constr. Company).
- The Melan-Ribera System was refined to perfection by Eduardo Torroja in the Viaducto Martín Gil (Río Esla, embalse de Ricobayo), by subdividing the concrete section in several parts, successively increasing the strength and adding more weight. This way, the arch with 210 m span could be built using a surprisingly light steel truss (less than 500 kg/m, according to L.M. Viartola, for the 4.5 m deep concrete arch, see notes).

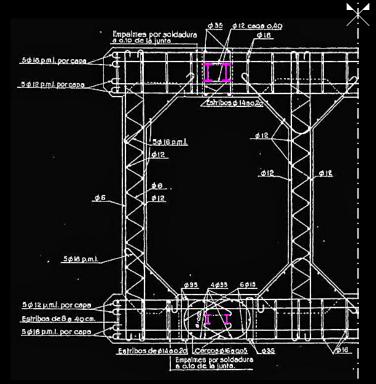


Arch section (casting sequence)



- By subdividing the cross-section in several stages, both in cross-section as well as along the arch axis, bending moments during erection and buckling risk could be minimised.
- While this was economical at the time, such a refined subdivision of the section would be excessively expensive today (high labour cost)

Half arch section with reinforcement







- In spite of the success of Ribera and Emperger, designers like Maillart did thus not use these systems, partly due to the mentioned concerns about the bond between steel and concrete (in fact, delamination has been observed in some early Melan arches), partly due to other reasons (rivalry, nationalism, ...).
- During and after World War II, due to the scarcity of steel, the building systems with rigid reinforcement (Melan, Ribera, and others) disappeared.
- For example, the elegant arches of Ch. Menn (Tamina, Nanin e Casciella) were built on timber falsework, just like arches centuries earlier.
- Due to the increasing Labour cost, this was already very costly at the time, and would be excessively expensive today.





- Actually, it appears that the Melan System, and its potential economical benefits, had faded into oblivion (or it was still regarded as inferior due to the concerns about steel-concrete connection).
- For example, in his seminal book Prestressed Concrete Bridges, Ch. Menn – doubtlessly a leading arch bridge designer of his time – briefly mentioned the Melan system and Emperger's applications to arches in the historical overview, but
 - → throughout the entire section of arch bridges implicitly presumed casting on centring
 - → merely referred to different ways of casting the arch to minimise the load to be carried by the centring
- In slender slab arches, which are very elegant in the final configuration (and therefore preferred by Ch. Menn), the centring needs to carry not only the weight of the arch, but – to avoid instability – also a significant portion of the column and deck girder weights, requiring heavy and expensive centrings.

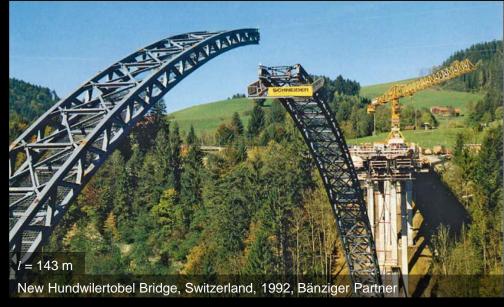




- Some designers did, however, use the Melan System. This and the
 next slide show two examples of Swiss and Austrian applications,
 where the steel trusses were assembled upright and rotated
 subsequently around the arch abutments (as previously used in
 erecting large arch centrings, e.g. for the Pont de Longeray, 1943).
- Although the clients and engineers involved in the projects shown on this slide were convinced that the system had many advantages and anticipated a more frequent use in the future, very few arch bridges were built in Europe using the Melan System over the past decades.

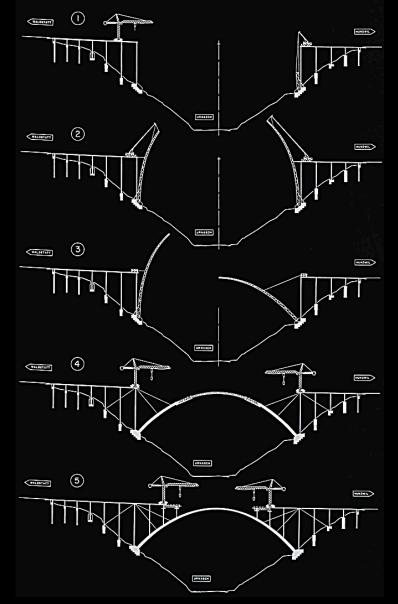


Rotation of centering for the Longeray arch bridge









Arch bridges

Erection methods – Vertical assembly and rotation

Arch bridges – Erection methods: Vertical assembly and rotation

- The vertical assembly of arches, with subsequent rotation to closure, has also been used but for entire arch halves.
- In steel arches, tieback forces are moderate thanks to the reduced weight, as in the Viaducto de Alconétar (2006).









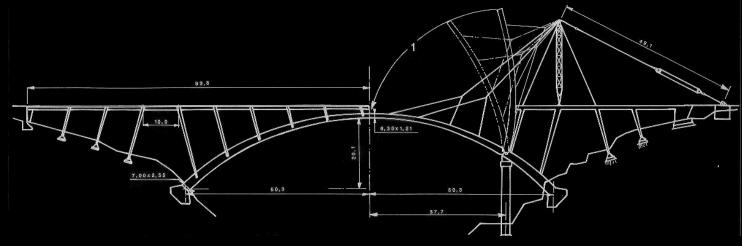






Arch bridges – Erection methods: Vertical assembly and rotation

- Much higher tieback forces are required in concrete arches, due to the higher weight.
- Nonetheless, Riccardo Morandi used this method already in the 1950s, first in a footbridge (Vagli Sotto, Garfagnana) and then in the Paul Sauer Bridge over the Storms River, South Africa (1956, span 120 m, rotated arch halves 37 m each).





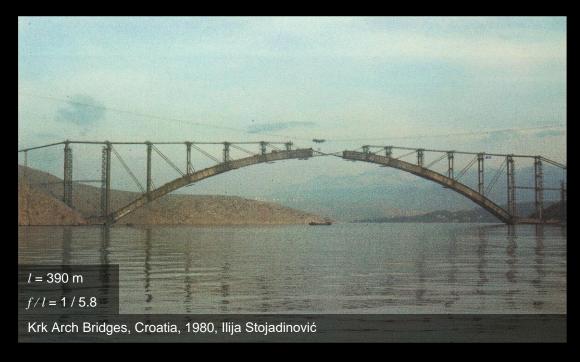


Arch bridges

Erection methods – Cantilever-constructed concrete arches

- Rather than Melan System arches, the following construction methods have been frequently used for medium-large span concrete arches in Europe in the last decades
 - → cantilevering using temporary stays and, in longer spans, towers ("stayed arch cantilevering")
 - → cantilevering of deck and arch as a truss, with temporary diagonals ("deck-and-arch truss cantilevering", sometimes using temporary cables running parallel to the deck and temporary spandrel columns)
- In the following, deck and arch truss cantilevering is described first. Stayed arch cantilevering was used earlier and is more frequently used today. It is also used in the modern CFST method, and therefore outlined afterwards.
- The first large-span deck-and-arch truss cantilevered concrete arches known to the authors are the Krk bridges (spans of 244 and 390 m), designed by Ilija Stojadinović.
- The longer of the two bridges was the record span for concrete arch bridges until 1997; accounting for the underwater part, it would have held this record even longer.





- In the Krk bridges, temporary cables running parallel to the deck were used, rather than activating the deck in tension, and temporary spandrel columns were also used during cantilevering
- The arch was built in stages, connecting the precast elements by in-situ joint casting:

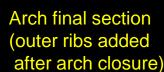
arch cantilevering (temporary cables, diagonals + columns)

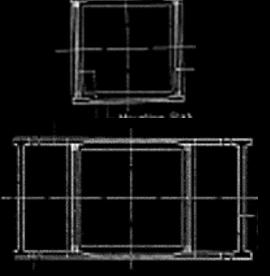


midspan closure (jacks regulating arch thrust = geometry installed until 1985)



Arch basic section (cantilevered)



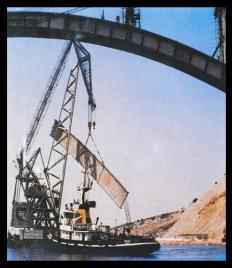


assembly of arch basic section (cantilevered part)

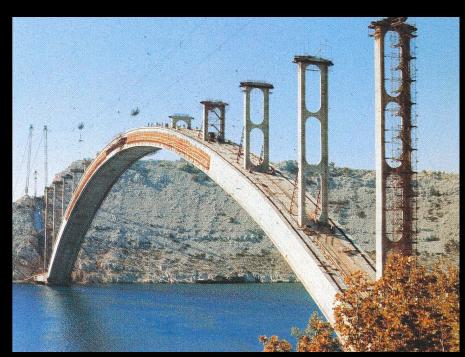




hoisting of outer arch ribs



- In the Krk bridges, temporary cables running parallel to the deck were used, rather than activating the deck in tension, and temporary spandrel columns were also used during cantilevering
- Precast elements were also used for the deck girder, resulting in a very efficient erection



spandrel column



spandrel column with temporary diagonal



deck erection (precast girders)



- The Arco de la Regenta (Puente Pintor Fierros)
 was also built using deck-and-arch cantilevering, in
 this case using the steel-concrete composite deck
 as tension chord.
- This bridge was opened in 1996 and widened from two to four lanes (12 → 22 m width) in 2008, under traffic, without substantial strengthening need on arch nor foundations: These had already been designed in 1996 to enable a later widening.







- The Arco de los Tilos is one of the longest span concrete arches built in the past decades by deckand-arch truss cantilevering.
- As in the Arco de la Regenta, the steel-concrete composite deck was used as tension chord.











- Stayed arch cantilevering was used earlier than deck-and-arch cantilevering and is more frequently used today. It is also used in the modern CFST method described at the end of this chapter.
- While stayed arch cantilevering had been used in steel bridges much earlier, the first known application of stayed concrete arch cantilevering are the three arch viaducts of the Caracas-La Guaira motorway in Venezuela, designed by E. Freyssinet / J. Muller and built by Campenon Bernard.
- Rather than cantilevering the entire arch, the middle part was built on an 80 m long falsework suspended from the arch cantilevers. This has the advantage that flat, inefficient stays can be avoided without the need for towers.





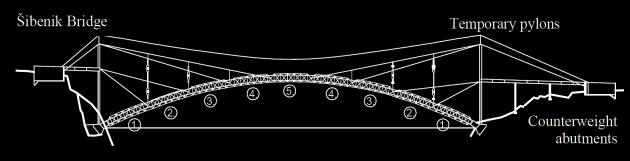
- A similar erection method as in the Caracas-La Guaira arches was used for the outer parts of the falsework of the Ponte da Arrábida (span 270 m), see photos on right side.
- Today, stayed cantilevering of the entire arch is more frequent, see bottom photo (Ponte Val Crotta in Ticino, span 90 m).
- Commonly, the arch is cast in situ, using formwork travellers similar to those used for cantilever-constructed concrete girders
- Alternatively, precast segmental cantilevering is also used.





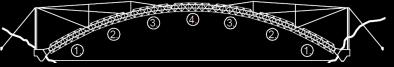


- In the deck-and-arch truss cantilevering method, high tieback forces are required, limiting the field of application in terms of span and soil conditions for anchorage of temporary backstays.
- In stayed arch cantilevering, equally high stayback tie forces result if no towers are used. The tieback forces can be substantially reduced by using temporary towers, similar as used when cantilevering large span steel truss arches
- If temporary towers are used in stayed arch cantilevering is an economical decision: The extra cost for the towers needs to be compensated by the reduced stay forces and backstay anchorage cost. Usually, towers are economical for large span arches.
- The slide shows different choices for tower heights adopted in two arch bridges designed by Ilija Stojadinovic: The Šibenik arch bridge (span 246 m, high towers) and the Pag arch bridge (span 193 m, low towers).





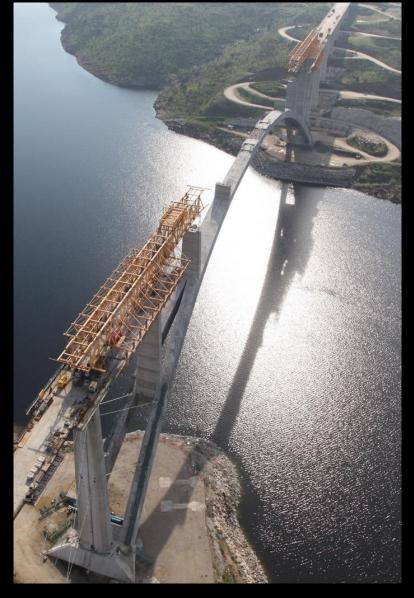






- The Viaducto de Almonte, whose arch was built cantilevered using towers and stays, is one of the world's longest and most elegant concrete arches, and the longest span high speed train arch bridge worldwide.
- More details, see presentation of guest speaker Guillermo Capellán.







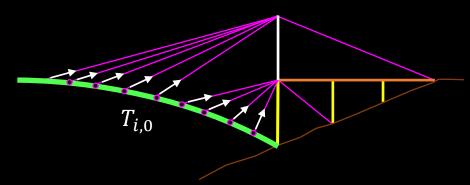
 The Tamina Bridge is another recent example of a large span concrete arch cantilevered using towers and stays



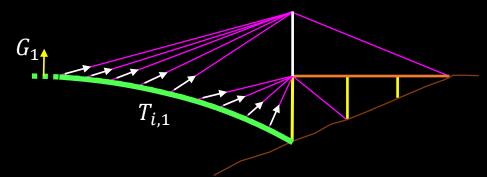




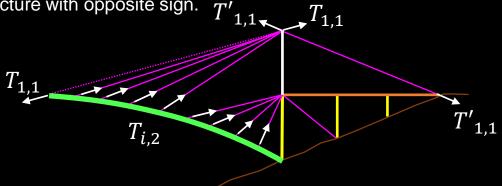
0. Final stage of half arch: Half arch modelled with hinges at all intersection points (arch-ties) except in the last tie close to the crown. This arrangement gives the tension forces $T_{i,0}$ of the ties in the last construction phase. Cable preload is chosen such that the correct arch geometry is obtained.



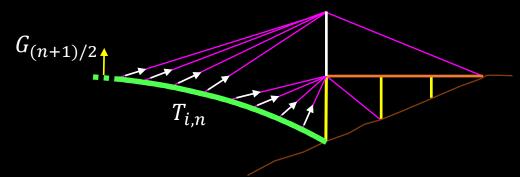
1. Disassembling the structure from the final stage of the half arch: Half arch without hinges. The last segment is removed and its self-weight is applied to the remaining structure with opposite sign.



2. Disassembling the structure from the previous step: Half arch without last segment, without hinges. The last stay cable is removed and the tension forces $T_{I,I}$ and $T'_{I,I}$ (cable forces in corresponding cables after applying the negative self-weight G_1 in stage 1) are applied to the structure with opposite sign. $T'_{1,1} = T_{1,1}$

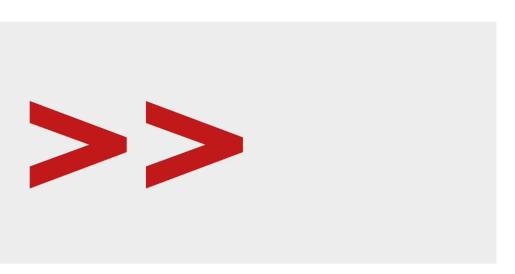


...n. Disassembling the structure from an intermediate stage of the half arch: Gradually shorter part of half arch without hinges. The same procedure (steps 1-2) is used to obtain the forces in each stage until the half arch is completely disassembled.









Tamina bridge: erection

Arch bridges

Erection methods – Evolution of the Melan System

- In Asia, arch erection methods inspired by the Melan System have been much more successful ofer the past decades.
- In Japan, more than 20 arch bridges have been built since 1970 using partial Melan System solutions (according to Eggemann and Kurrer, see notes and photos on this slide: Kashirajima Bridge, span 218 m).
 - → erecting parts near abutments conventionally by arch cantilevering (stayed or trussed)
 - → lifting in steel girders for the Melan System midspan part
- In Japan and particularly in China, many long span arch bridges have been, and are being erected using hollow steel profiles, filled with concrete after closure. The steel profiles thus serve as combined falsework and reinforcement.
- This method, evidently similar to the Melan System (though only recognised by the Japanese), is known as "concrete lapping with pre-erected composite" (CLCA) in Japan, and as "Concrete filled steel tube arches" (CFST) in China.
- The CFST Method is described on the following slides.





- In CFST arch bridges, hollow section steel arches are erected by stayed cantilevering and subsequently grouted with concrete, forming a steel-concrete composite arch.
- In China, more than 400 CFST arch bridges have been built (≥ 12 with L > 300 m, ≥ 4 with L > 400 m). This slide shows a recent example (Xiangxi Yangtze River Bridge, span 508 m (2019).

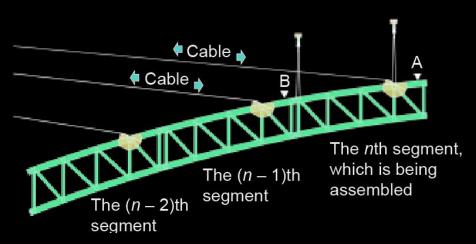






- Currently, the maximum span of a CFST arch bridge is 530 m (First Hejiang Yangtze River Bridge, aka Bosideng Bridge, 2013, see photos).
- Much research has been carried out in China to optimise this type of structures, e.g.
 - ... adjusting stay forces during grouting to minimise bending moments
 - ... grout properties and vacuum grouting etc.
 - ... composite action of tubes and concrete
 - ... etc.











- A further development of CFST bridges consists in arches made of a CFST composite steel skeleton encased by concrete

 even closer to the concept of the original Melan System – are being built, mainly also in China ("CFST reinforced concrete arches").
- A recent example is the Yunnan–Guangxi Railway Nanpan River Bridge (aka Nanpanjiang Railway Bridge Qiubei, see photos).

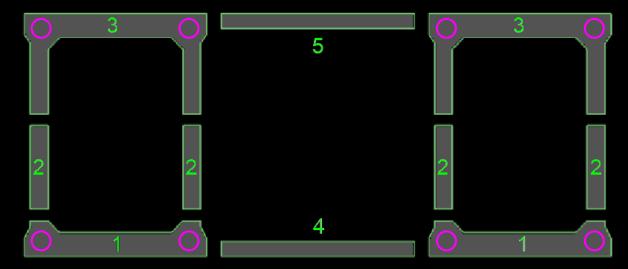




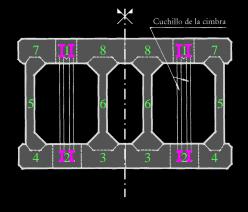


- The similarity of the cross-section of the Yunnan–Guangxi Railway Nanpan River Bridge to Torroja's solution for the Viaducto Martín Gil is striking– albeit at a much larger scale (figures on right side):
 - → Viaducto Martín Gil: Span 192 m, f/L = 1/3.3, $h_{arch} = 4.5$ m
 - → Nanpan River Bridge: Span 416 m, f/L = 1/4.2, $h_{arch} \approx 9$ m (steel tubes = 8 m)
- CFST reinforced concrete arches have clear advantages in terms of durability and maintenance (no coating)
- Furthermore, they are very efficient and economical:
 - → high contribution of inexpensive concrete
 - → avoidance of buckling issues by gradually increasing inertia and load carried by the arch
 - → minimisation of bending moments during casting by optimising casting sequence along arch span, and actively controlling stay forces
- Spans up to 700...800 m appear economically feasible in China according to Zheng and Wang (source see note).

Yunnan–Guangxi Railway Nanpan River Bridge:
Cross-section and casting sequence (size steel tubes approximate)



Comparison with Viaducto Martín Gil: Cross-section and casting sequence (≈ same scale as above)







Arch bridges

Erection methods – Final remarks

- In many arch bridges, the deflection caused by the axial deformation of the arch – causing significant bending moments, see structural behaviour – is compensated at closure by applying a controlled axial force at the crown by means of hydraulic jacks.
- Throughout the history of arch bridges, there has been a debate whether such an "opening of the crown" is useful or even required, as there are pros and cons:
 - → helps actively controlling the geometry
 - → helps removing the formwork and falsework in concrete arches (if the jacking force corresponds to the arch thrust under dead load, the arch lifts off the formwork)
 - → in tied arches, it may eliminate the need for hanger retensioning
 - → causes extra cost and complicates the erection process
 - → in concrete arches, most of the effect is lost due to creep
- Essentially, whether such an operation is carried out is a decision of the designer. In any case, the design has to consider the corresponding internal actions.







- Short span steel arch bridges are usually lifted in, where possible with temporary shoring.
- The slides show two examples, with and without shoring.







- Tied arch bridges, being "externally" simply supported, can be launched longitudinally or transversally like girder bridges.
- The Brücke Bernstrasse (Fürst Laffranchi / IUB) in Oftringen was first launched longitudinally over the SBB tracks, subsequently carrying the traffic in this position while the old bridge was demolished. Finally, it was launched transversely into its final position.







- Tied arch bridges crossing water can be built on shore, using conventional construction methods (shoring, access for cranes, ...) and floated into their final position (similar to the Plougastel and Sandö bridge falsework commented earlier).
- The Barqueta Bridge in Sevilla was built on one riverbank and rotated 90° in plan across the river Guadalquivir into its final position.

