Arch bridges
Structural reponse / Parametric study

Arch bridges

Structural response – Parametric study

Arch-deck girder interaction

Arch bridges – Structural response: Arch-deck girder interaction

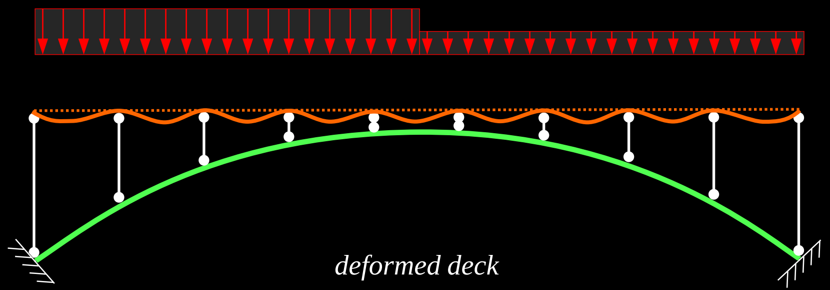
If an anti-funicular arch geometry is chosen, usually for permanent loads, arch bridges carry the corresponding loads efficiently.

However, arch compression and non-anti-funicular loads need to be accounted for in design. Under such loads, the arch rib, deck girder and spandrel columns or hangers generally act as a frame system, whose behaviour depends on

→ the stiffness ratio of arch rib and deck girder

→ the type of connection between arch rib and deck girder (clamped or pin-jointed spandrel columns / "hangers")

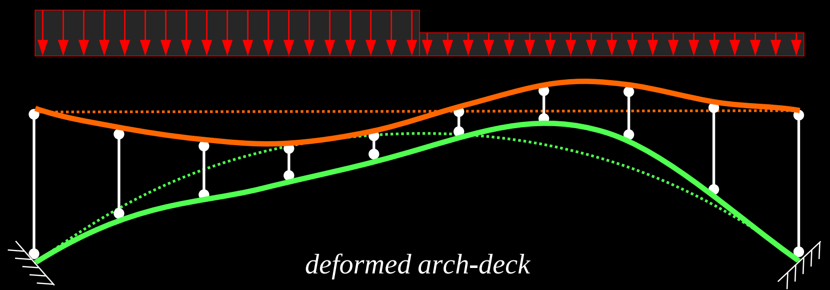
fixed system



In a first step, the bending moments in the frame system can be subdivided into two components:

- fixed system
- flexible system

flexible system

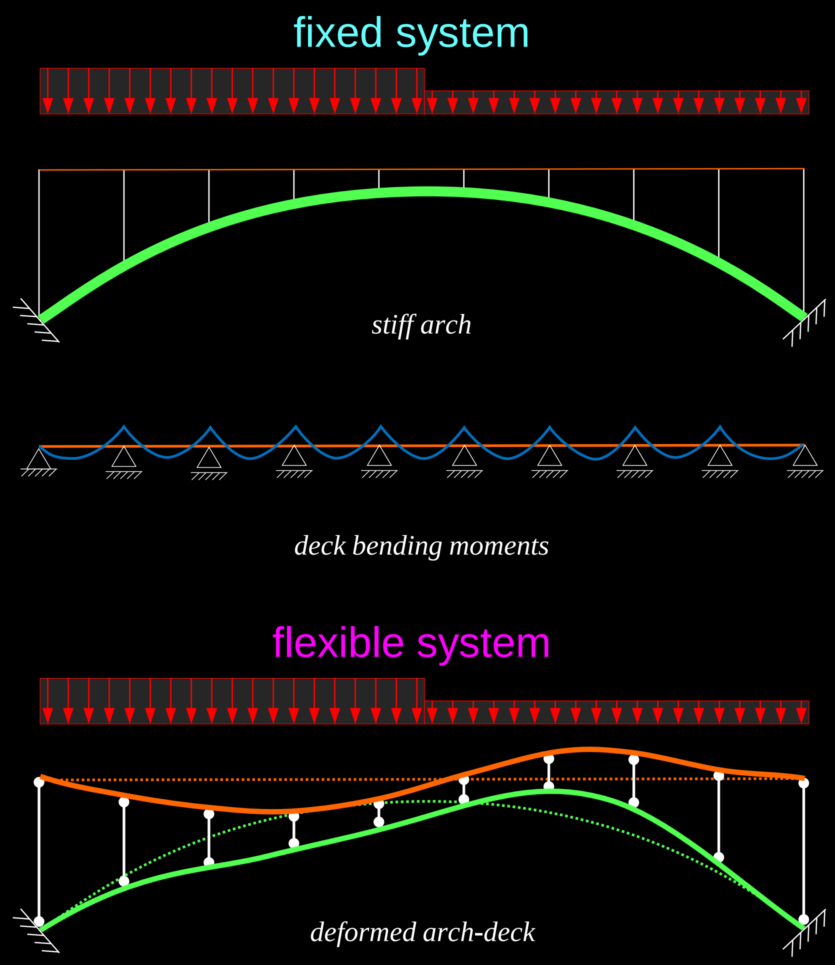


NB. The bending moments due to arch compression (strictly also acting on the frame system) and second order moments must be superimposed to obtain the total moments.

Arch bridges – Structural response: Arch-deck girder interaction

The following points have essentially been outlined in the *Design* section. Here, they are repeated and a case-study is presented to highlight some specific aspects.

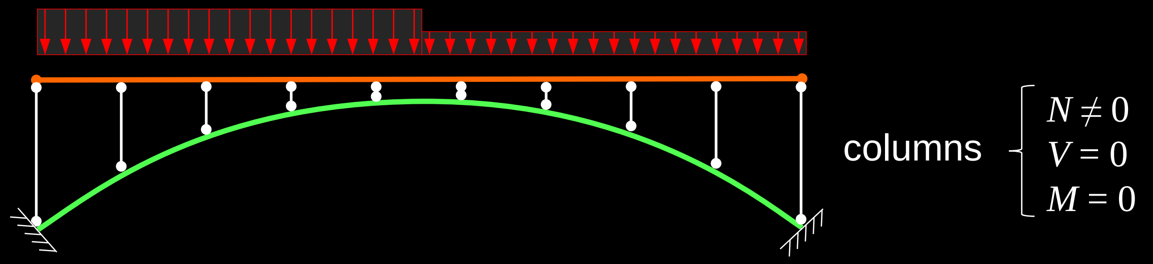
- fixed system
  - → assume a perfectly rigid arch
  - → bending moments in deck girder corresponding to those in a continuous beam (replacing spandrel columns by supports).
- flexible system
  - → bending moments in the flexible system involve arch deflections due to non-anti-funicular loads
  - → generally, these bending moments are shared by arch rib and deck girder in proportion to their bending stiffnesses
  - → two ideal limiting cases can be considered:
    - → *deck-stiffened arches* (*"versteifter Stabbogen"*), where the entire flexible system moments are resisted by the deck girder ("Versteifungsträger")
    - → *stiff arches* resisting the entire flexible system moments alone

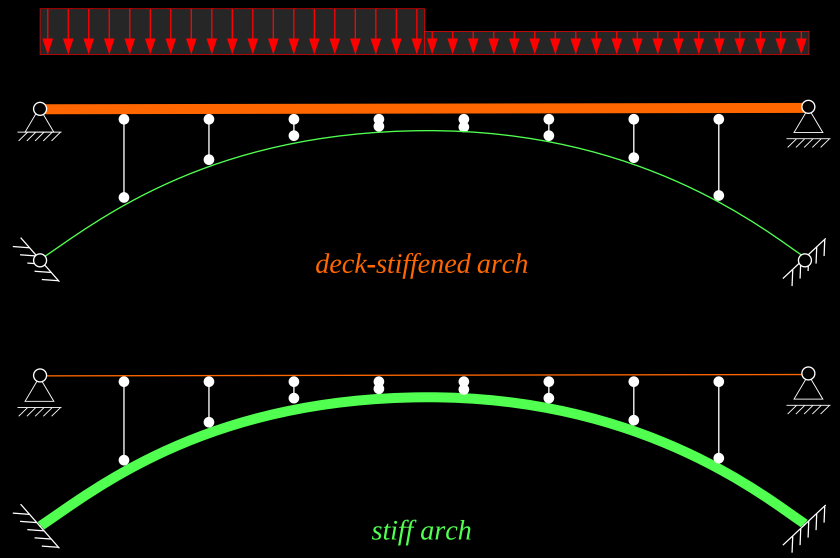


Arch bridges – Structural response: Arch-deck girder interaction

In this study, a clamped deck-arch bridge, with expansion joints of the deck above the arch abutments (intersection of springing line with arch axis) is considered (unlike Slide 63: deck continuous).

In the first part, pin-jointed spandrel columns are assumed.





Two limiting cases:

- deck-stiffened arch
  $\rightarrow$ flexural deck girder stiffness EI^D >> flexural arch rib stiffness EI^A
- stiff arch
  $\rightarrow$ flexural arch rib stiffness EI^A >> flexural deck girder stiffness EI^D

In these limiting cases, either the stiffening girder or the stiff arch resists (almost) the entire bending moments.

Arch bridges – Structural response: Arch-deck girder interaction

In this study, a clamped deck-arch bridge, with expansion joints of the deck above the arch abutments (intersection of springing line with arch axis) is considered (unlike Slide 63: deck continuous).

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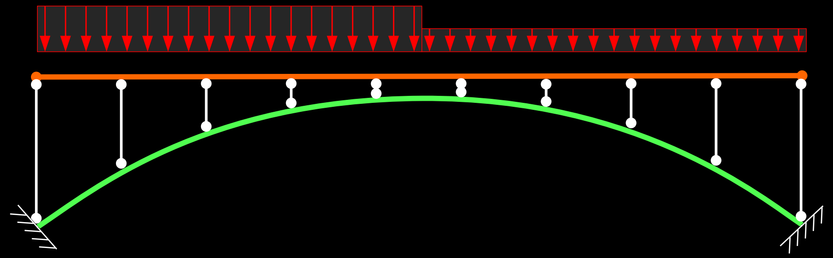

columns $\begin{cases} N \neq 0 \\ V = 0 \\ M = 0 \end{cases}$

Two limiting cases:

- deck-stiffened arch
  $\rightarrow$ flexural deck girder stiffness EI^D >> flexural arch rib stiffness EI^A
- stiff arch
  $\rightarrow$ flexural arch rib stiffness EI^A >> flexural deck girder stiffness EI^D

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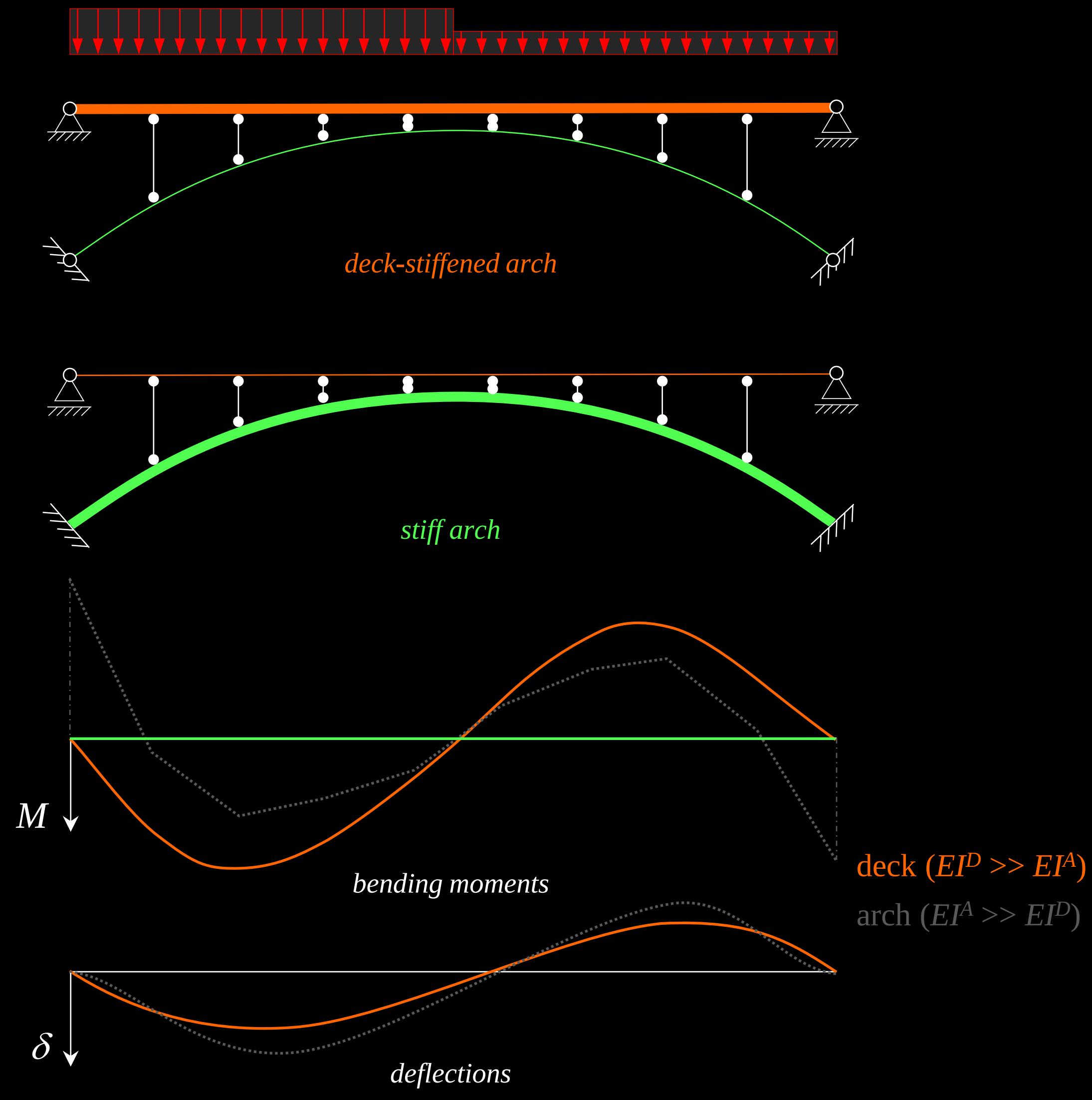

Arch bridges – Structural response: Arch-deck girder interaction

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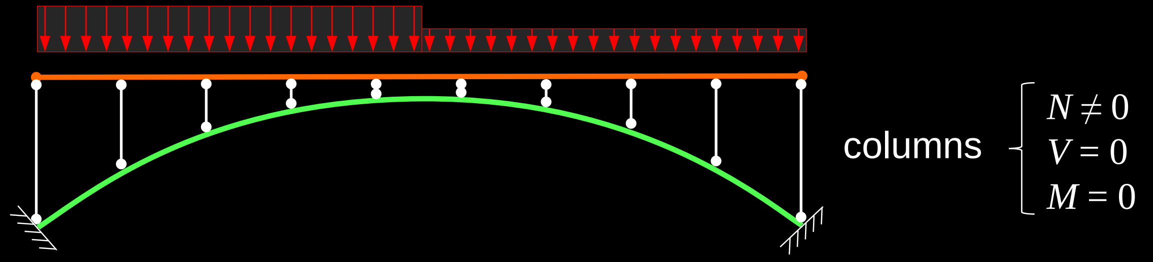

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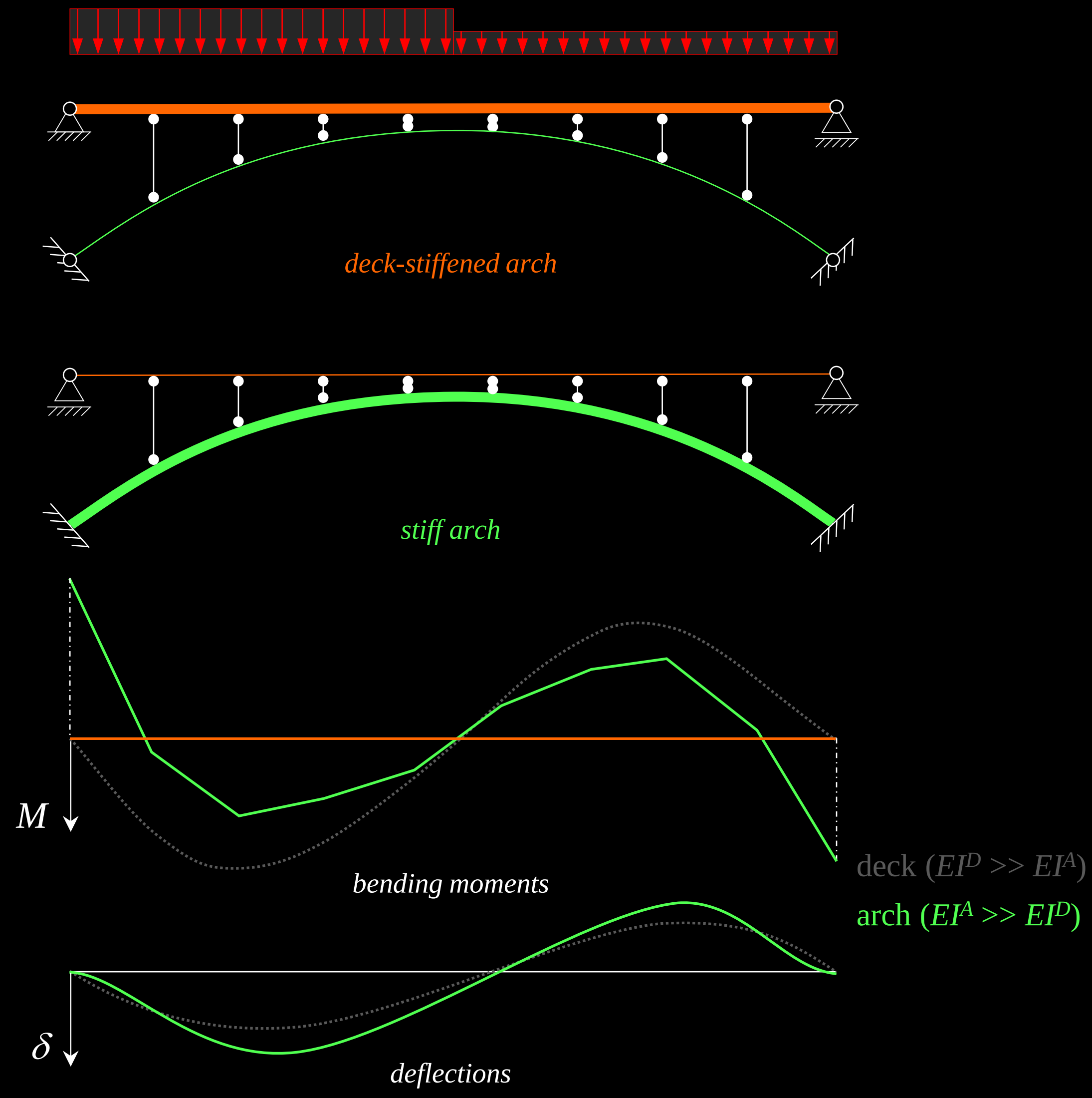

Arch bridges – Structural response: Arch-deck girder interaction

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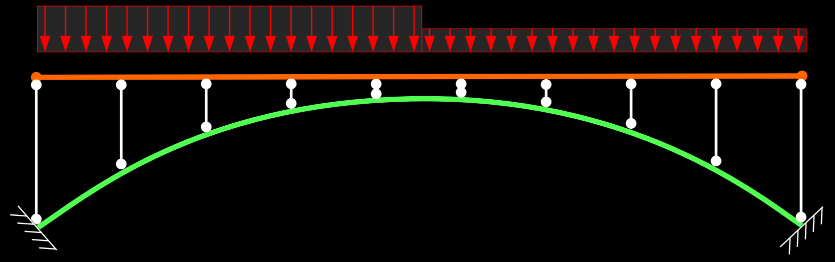

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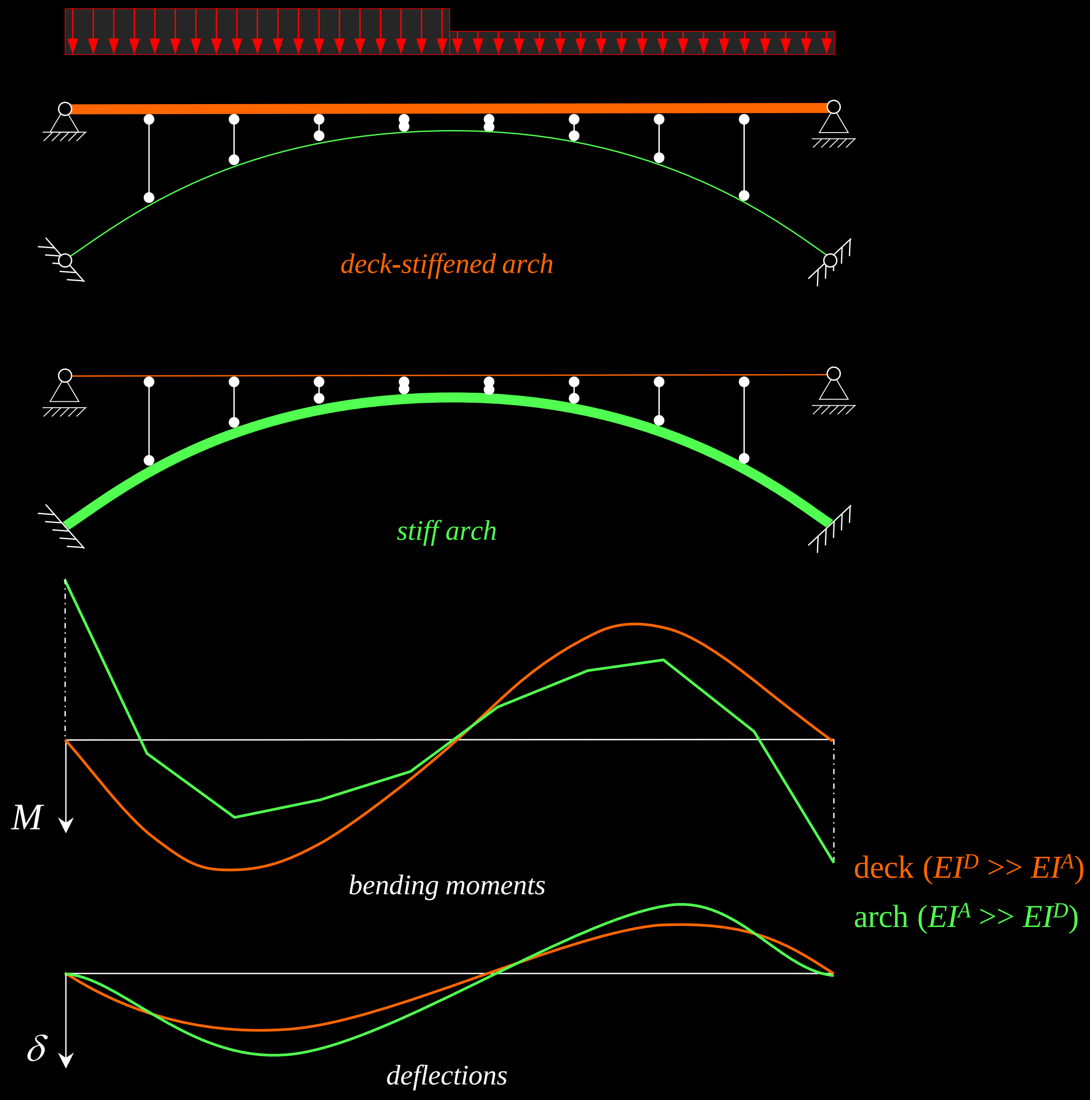
- stiff arch
  → flexural arch rib stiffness EI^A >> flexural deck girder stiffness EI^D

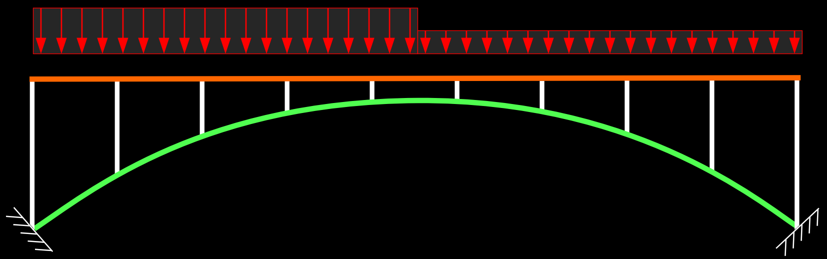
In these limiting cases, either the stiffening girder or the stiff arch resists (almost) the entire bending moments.

The differences of bending moments and deflections between arch rib and deck girder are due to the different support conditions assumed here (clamped vs. simply supported). In the design section (Slide 63), both are assumed to be continuous.



Arch bridges – Structural response: Arch-deck girder interaction

If the spandrel columns are clamped, rather than pin-jointed, arch rib and deck are not only coupled in terms of vertical deformations, but act as frame system.



Two limiting cases:

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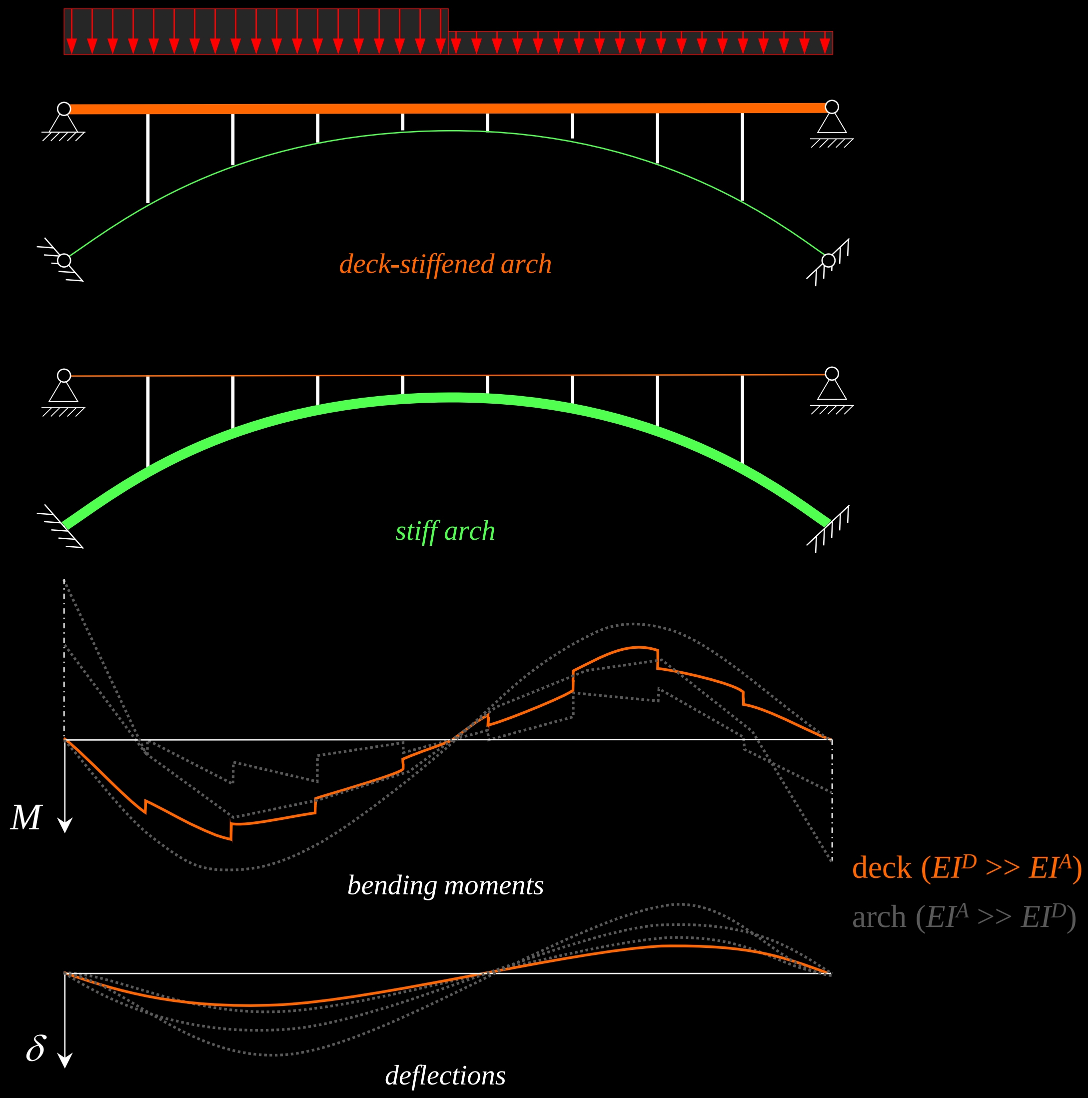

Arch bridges – Structural response: Arch-deck girder interaction

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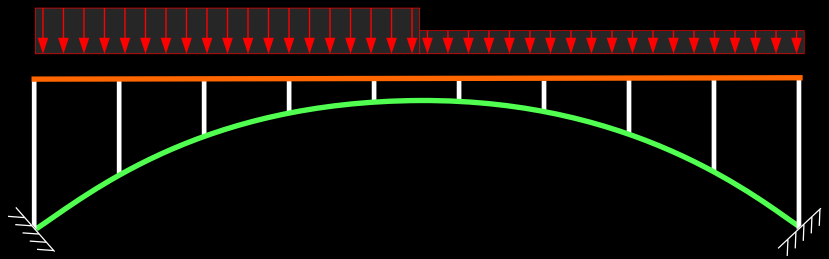

Two limiting cases:

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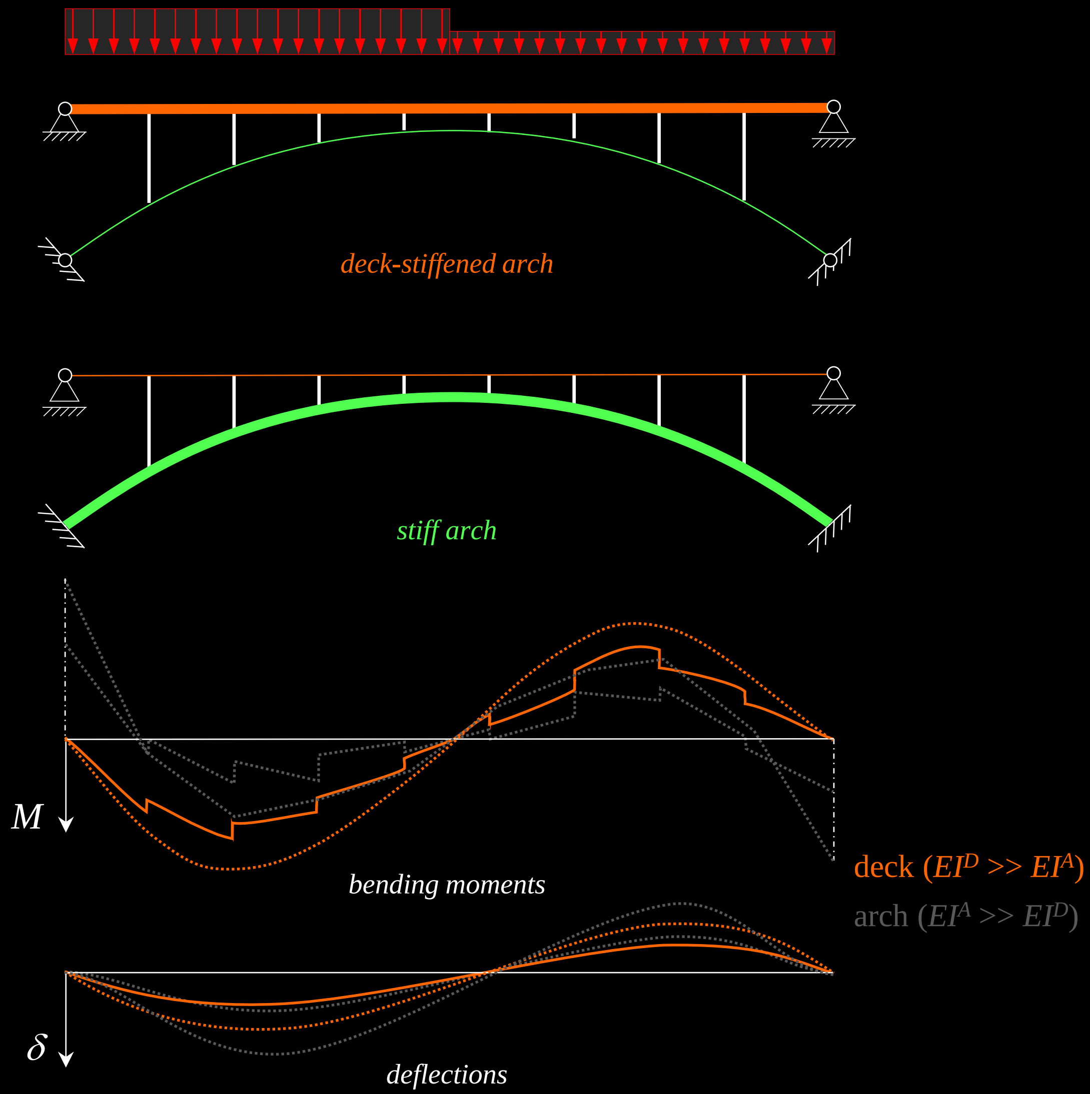

Arch bridges – Structural response: Arch-deck girder interaction

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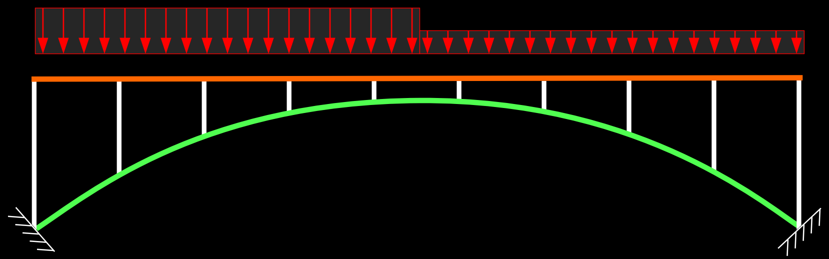

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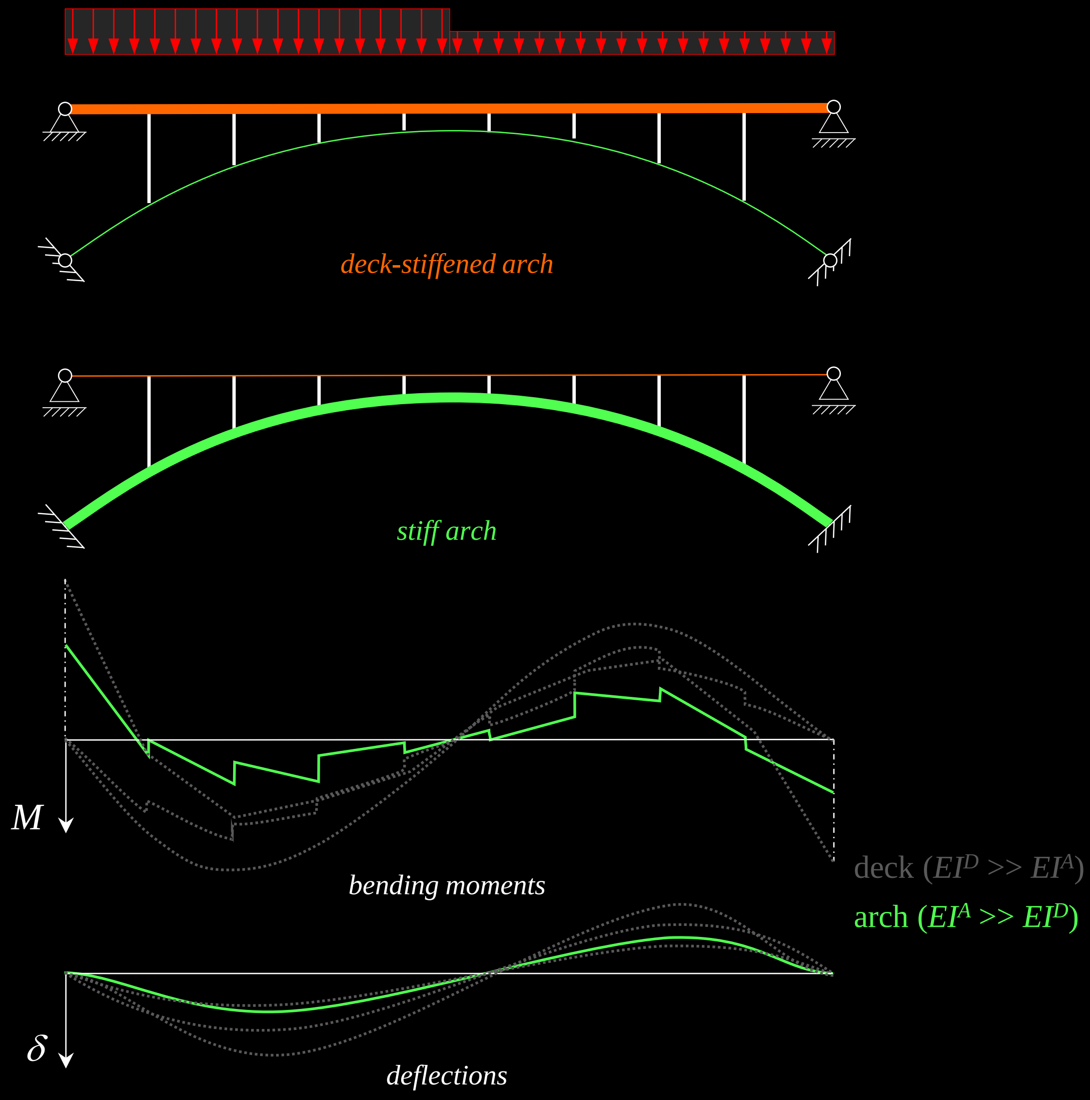

Arch bridges – Structural response: Arch-deck girder interaction

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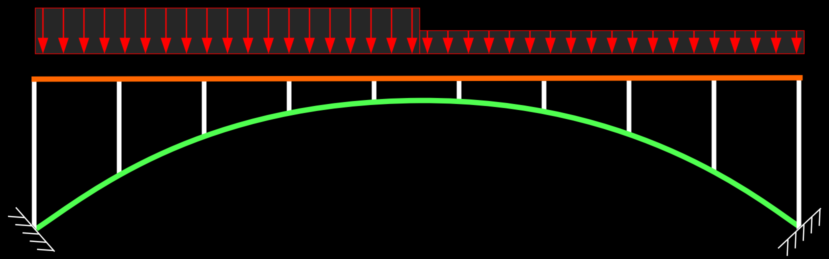

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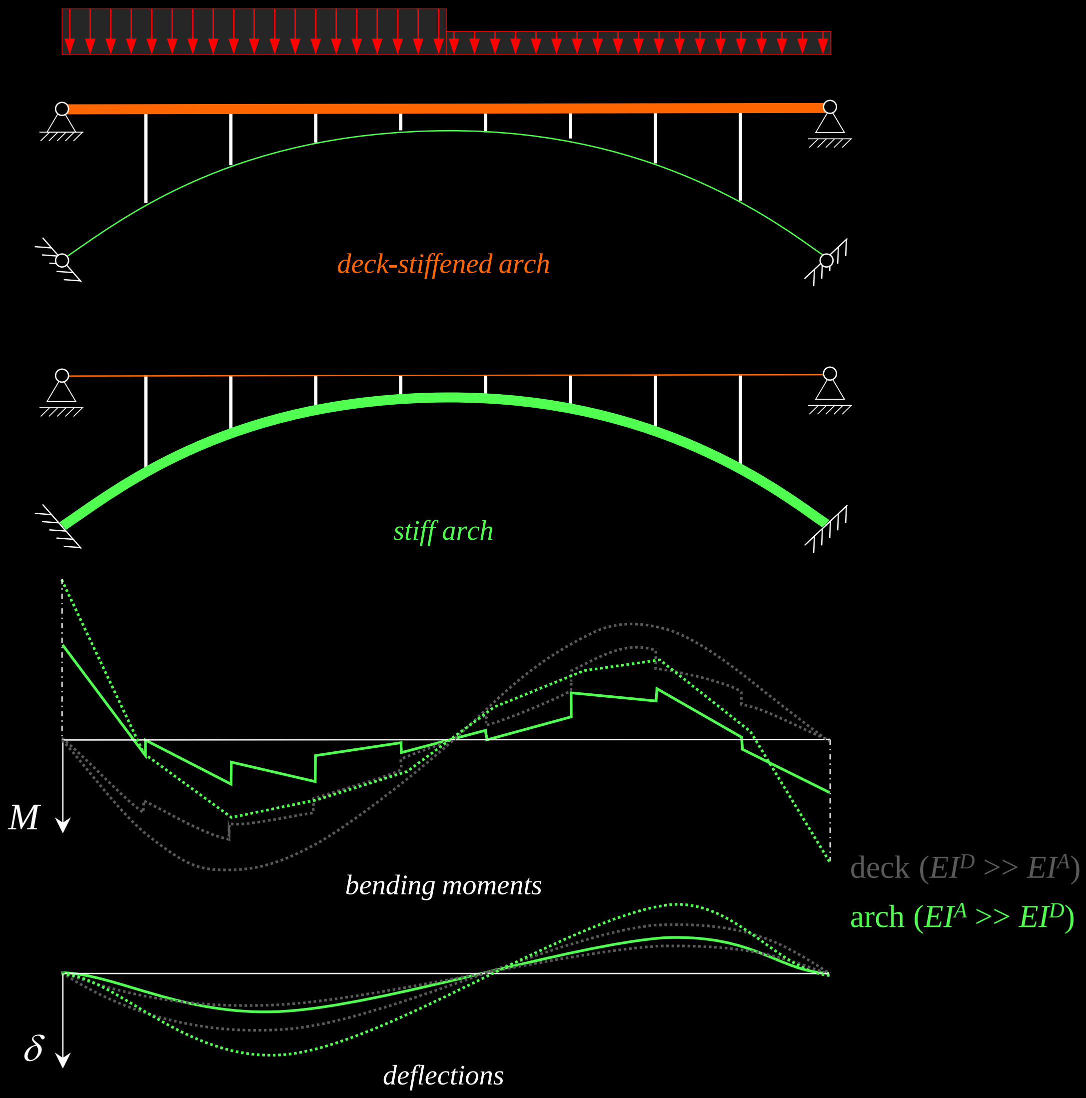

Arch bridges – Structural response: Arch-deck girder interaction

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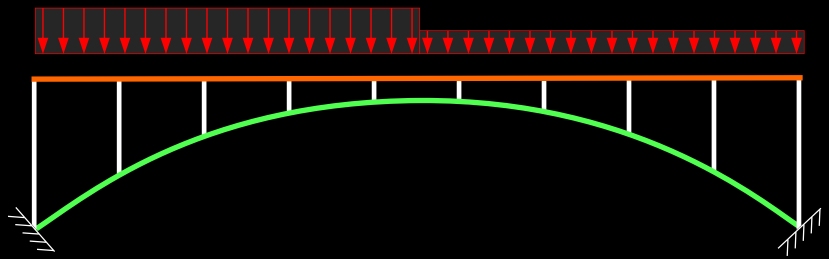


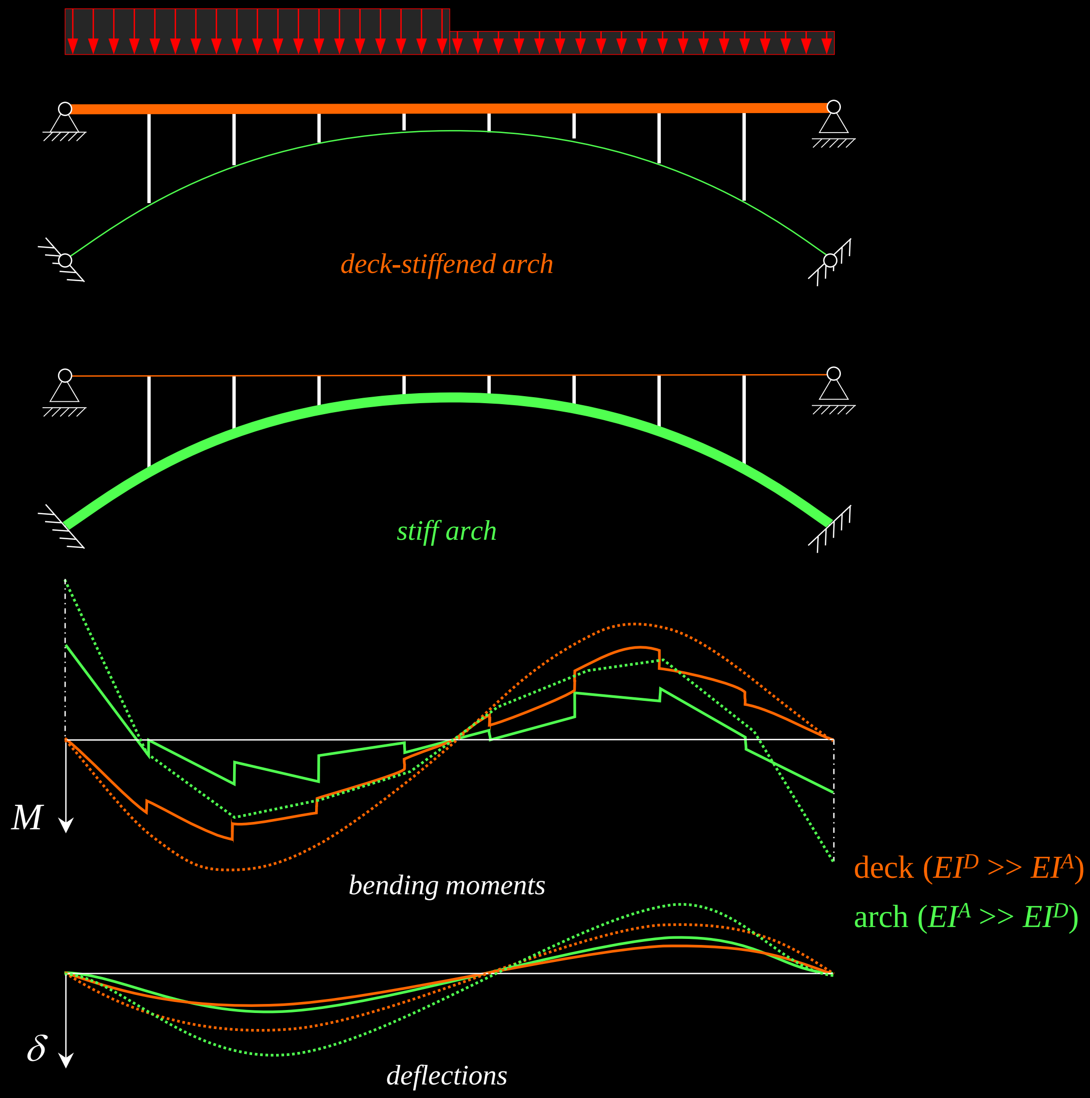
Clamped spandrel columns, together with deck girder and arch rib, act as Vierendeel girder

→ significantly stiffer than sum of deck girder and arch stiffness

→ deflections significantly reduced

The short clamped spandrel columns close to the crown have a high flexural stiffness and transfer the axial normal force from the arch rib to the deck.

In some cases, shear forces and bending moments in such spandrel columns may be excessive → (concrete) hinges may be provided to reduce these actions (e.g. Tamina bridge)



Arch bridges

Structural response – Parametric study

Arch support conditions / hinges

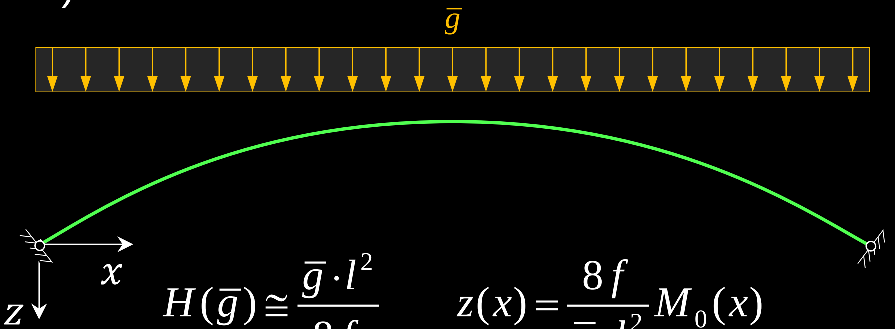
Arch bridges – Structural response: Arch support conditions / hinges

Basic assumptions

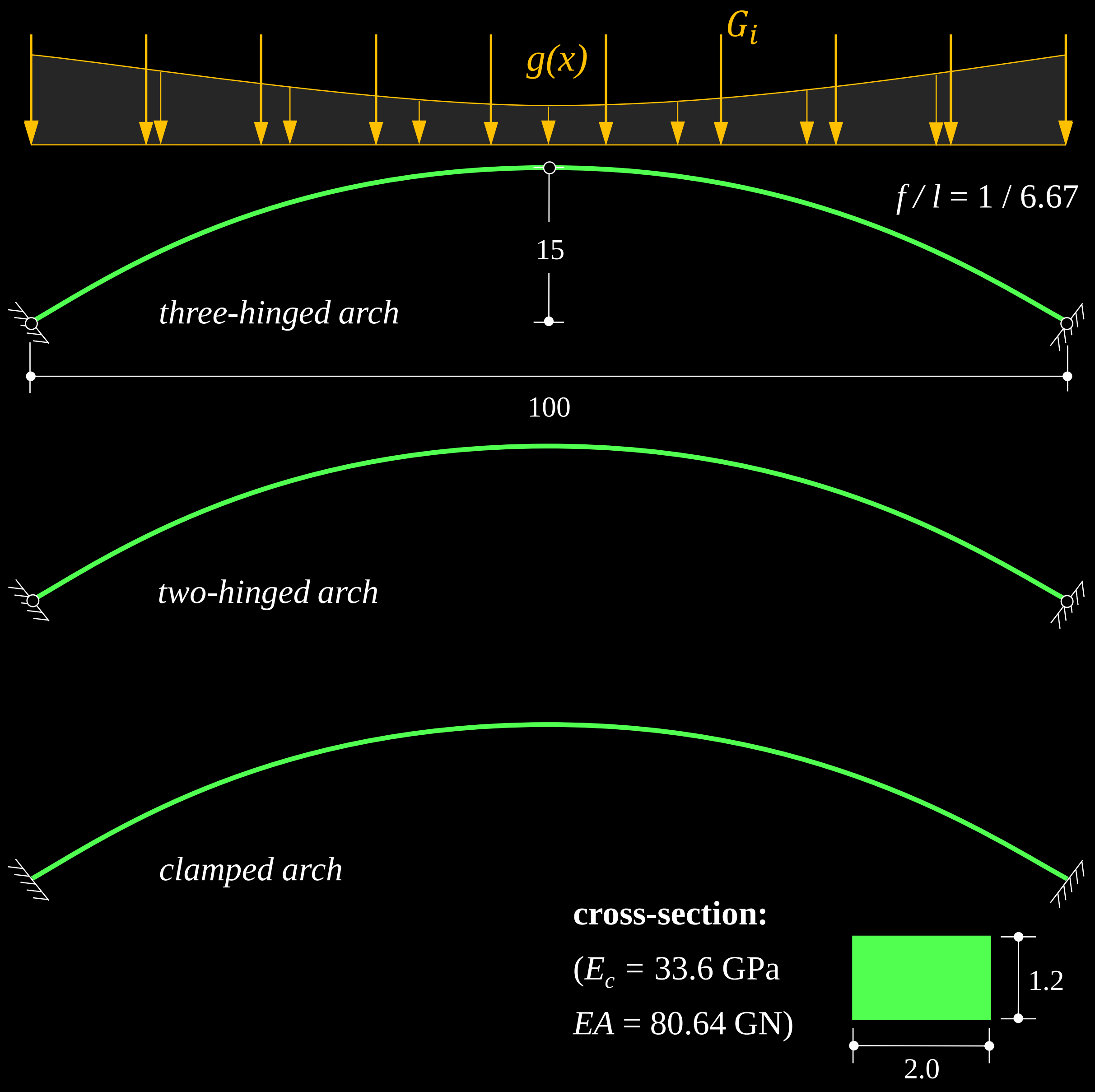
This and the next slides compare the structural behaviour of arches with three common (in the past) support / hinge conditions:

- three-hinged arch (hinges at springing line and crown)
- two-hinged arch (hinges at springing line)
- clamped arch ("zero-hinge" arch)

The response is compared numerically for a concrete arch with 100 m span and 15 m rise
→ rise-span ratio f/l = 1/6.67
→ solid concrete cross-section = constant over span
→ geometry of arch: anti-funicular curve of the average permanent loads (simplified method, see "*Design*" section):



$$H(\bar{g}) \cong \frac{\bar{g} \cdot l^2}{8f} \qquad z(x) = \frac{8f}{\bar{g} \cdot l^2} M_0(x)$$



Arch bridges – Structural response: Arch support conditions / hinges

Permanent loads / linear analysis (1st order)

Considering a uniform permanent load of 200 kN/m, a linear analysis yields the following results for:

- three-hinged arch
- two-hinged arch
- clamped arch

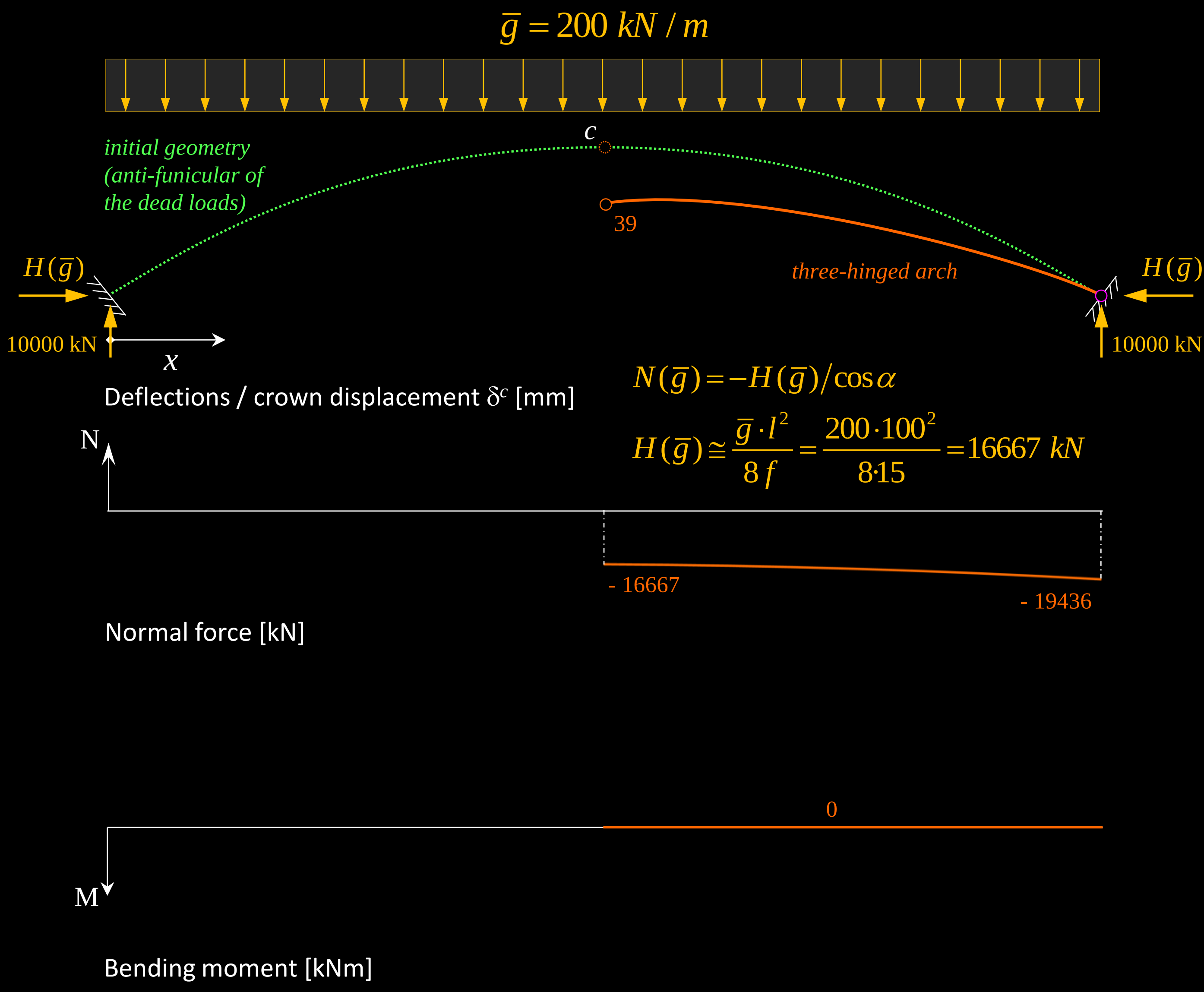
The arch compression causes vertical deflections → these depend only (three-hinged arch) on the axial stiffness EA.

However, as the arch is isostatic, the internal actions and the reactions are independent of the stiffnesses (EA, EI, …)

→ constant arch thrust H = 16'667 kN

→ bending moment along the arch $M(x)$ = 0

→ displacement compatibility is not needed to obtain the internal forces



Arch bridges – Structural response: Arch support conditions / hinges

Permanent loads / linear analysis (1st order)

Considering a uniform permanent load of 200 kN/m, a linear analysis yields the following results for:

- three-hinged arch
- two-hinged arch
- clamped arch

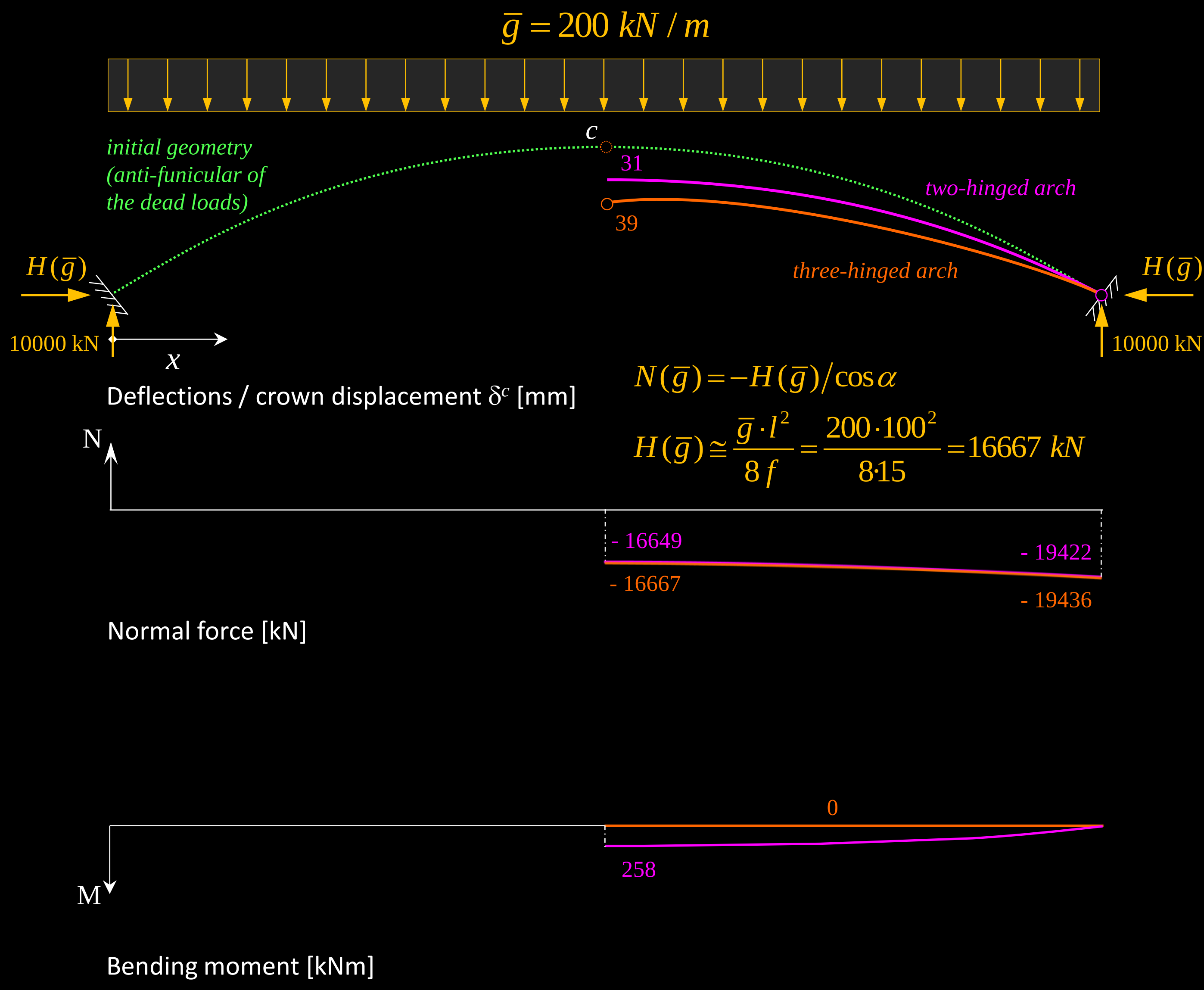
The arch compression causes vertical deflections → these depend on the axial stiffness EA and (slightly) on the bending stiffness EI ($M(x) \neq 0$).

The arch is hyperstatic → internal actions and reactions depend on the stiffnesses (EA, EI)

→ constant arch thrust $H \cong$ 16'667 kN

→ positive moments in the arch $M(x) \neq 0$

→ displacement compatibility is required to obtain the internal forces



Arch bridges – Structural response: Arch support conditions / hinges

Permanent loads / linear analysis (1st order)

Considering a uniform permanent load of 200 kN/m, a linear analysis yields the following results for:

- three-hinged arch
- two-hinged arch
- clamped arch

The arch compression causes vertical deflections → these depend on the axial stiffness EA and (slightly) on the bending stiffness EI ($M(x) \neq 0$).

The arch is hyperstatic → internal actions and reactions depend on the stiffnesses (EA, EI)

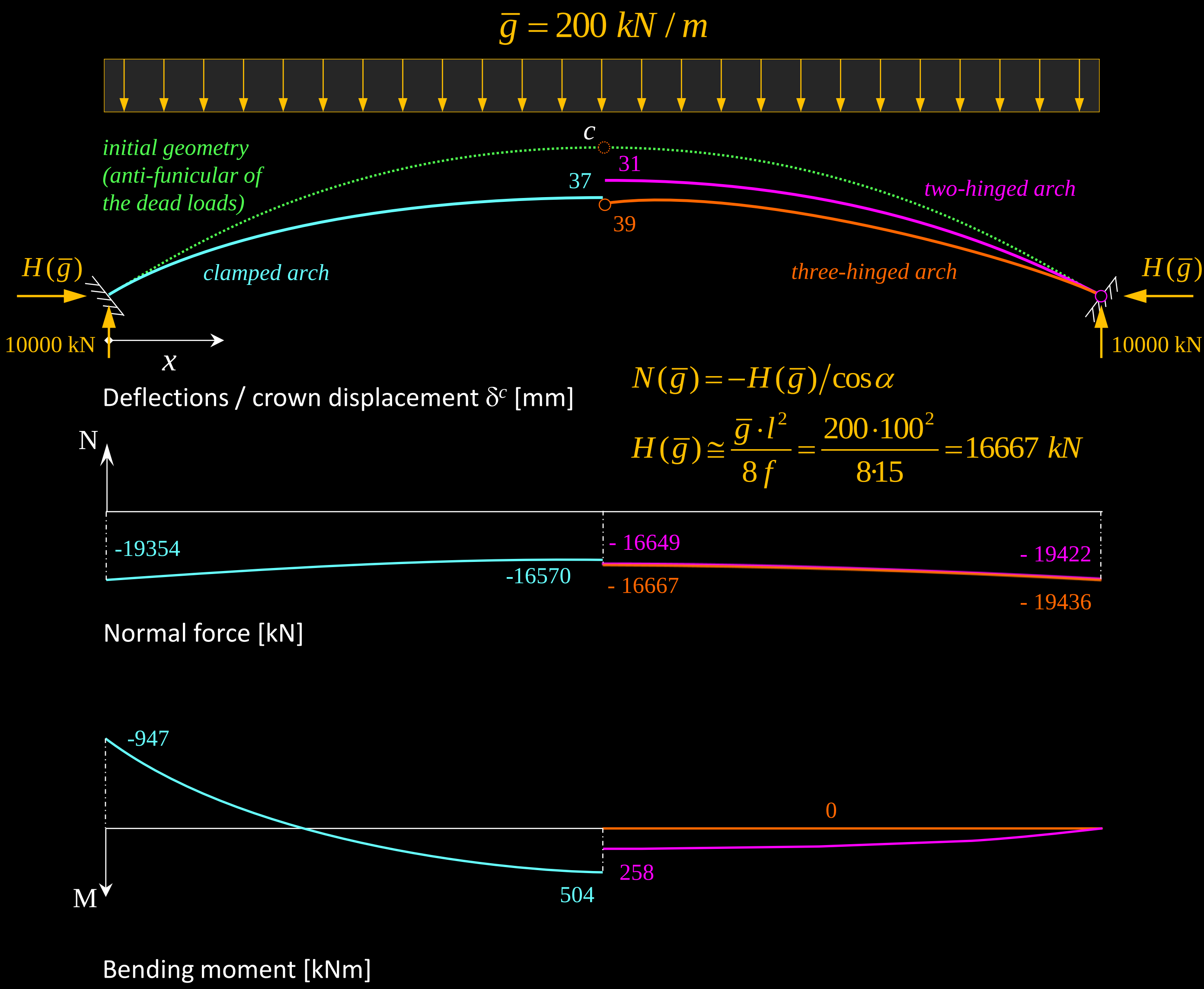
→ constant arch thrust $H \cong$ 16’667 kN

→ positive and negative moments in the arch $M(x) \neq 0$

→ displacement compatibility is required to obtain the internal forces

NB. Approximation: $\delta^c \cong \frac{H(\bar{g})}{EA^{A,c}} \cdot l \cdot \frac{1+3(f/l)^2}{4f/l} = 37\ \mathrm{mm}$

(Slide 55, $EA^A = EA^{A,c}$ = const.)



Arch bridges – Structural response: Arch support conditions / hinges

Permanent loads / linear analysis (1st order)

Considering a uniform permanent load of 200 kN/m, a linear analysis yields the following results for:

- three-hinged arch
- two-hinged arch
- clamped arch

The axial force N is almost identical in the three cases.

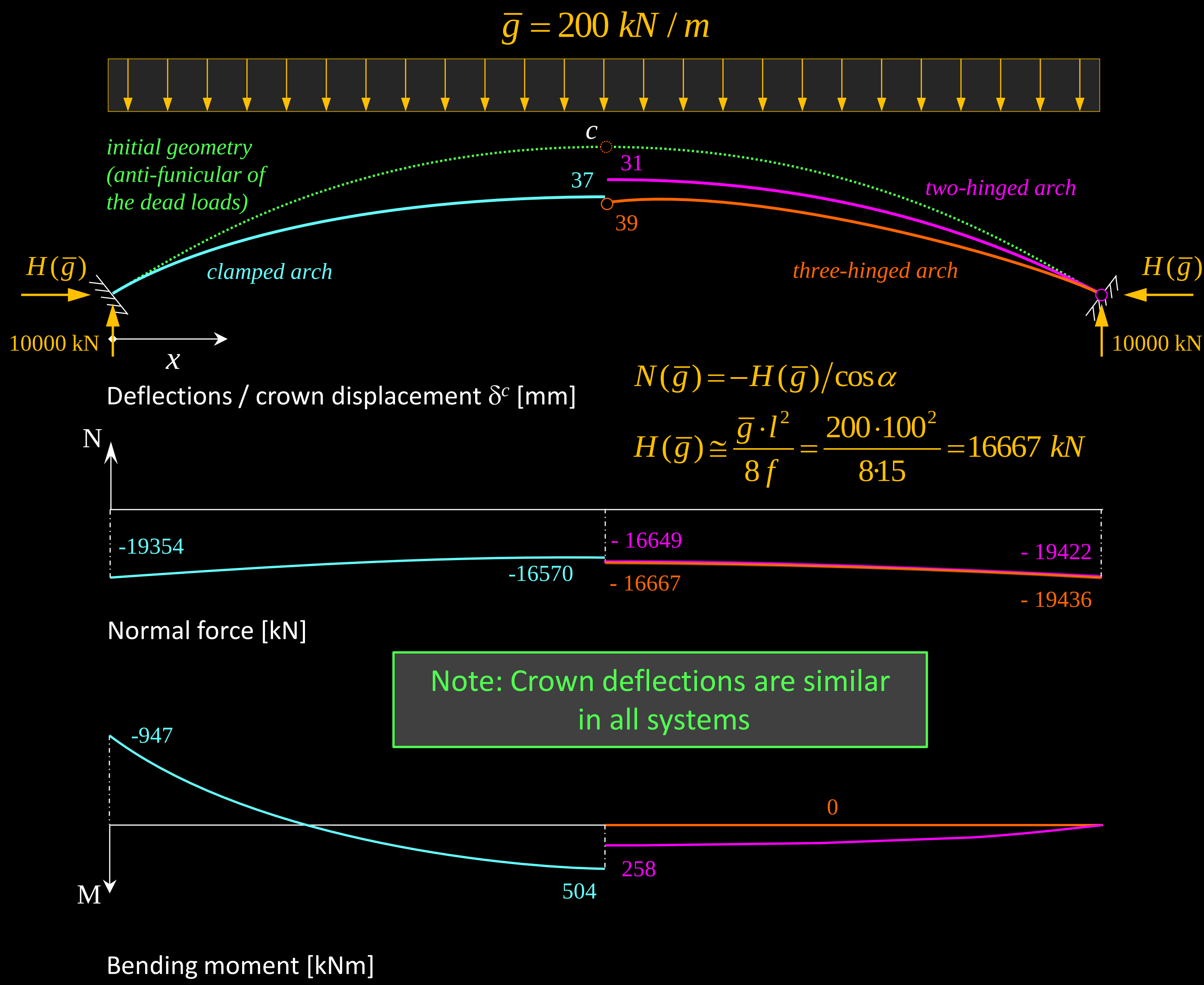
The vertical displacements of the crown δ_c, due to the arch compression, are almost identical for the three arches (see notes), as they depend mainly on the axial force N and the axial stiffness $EA \rightarrow \varepsilon = N / EA$

$\rightarrow$ if $EA \rightarrow \infty$, $\delta^c = 0$ (rigid arch)

$\rightarrow$ if the f / l ratio decreases, N and δ_c will increase

Since the axial stiffness of the arch is much higher than the bending stiffness, the vertical displacements due to arch compression are essentially imposed to the arches.

The bending moments M in the stiffer clamped arch are thus considerably higher than those in the other cases.



Arch bridges – Structural response: Arch support conditions / hinges

Permanent loads / nonlinear analysis (2nd order)

Considering a uniform permanent load of 200 kN/m, a nonlinear (2nd order) analysis yields the following results for:

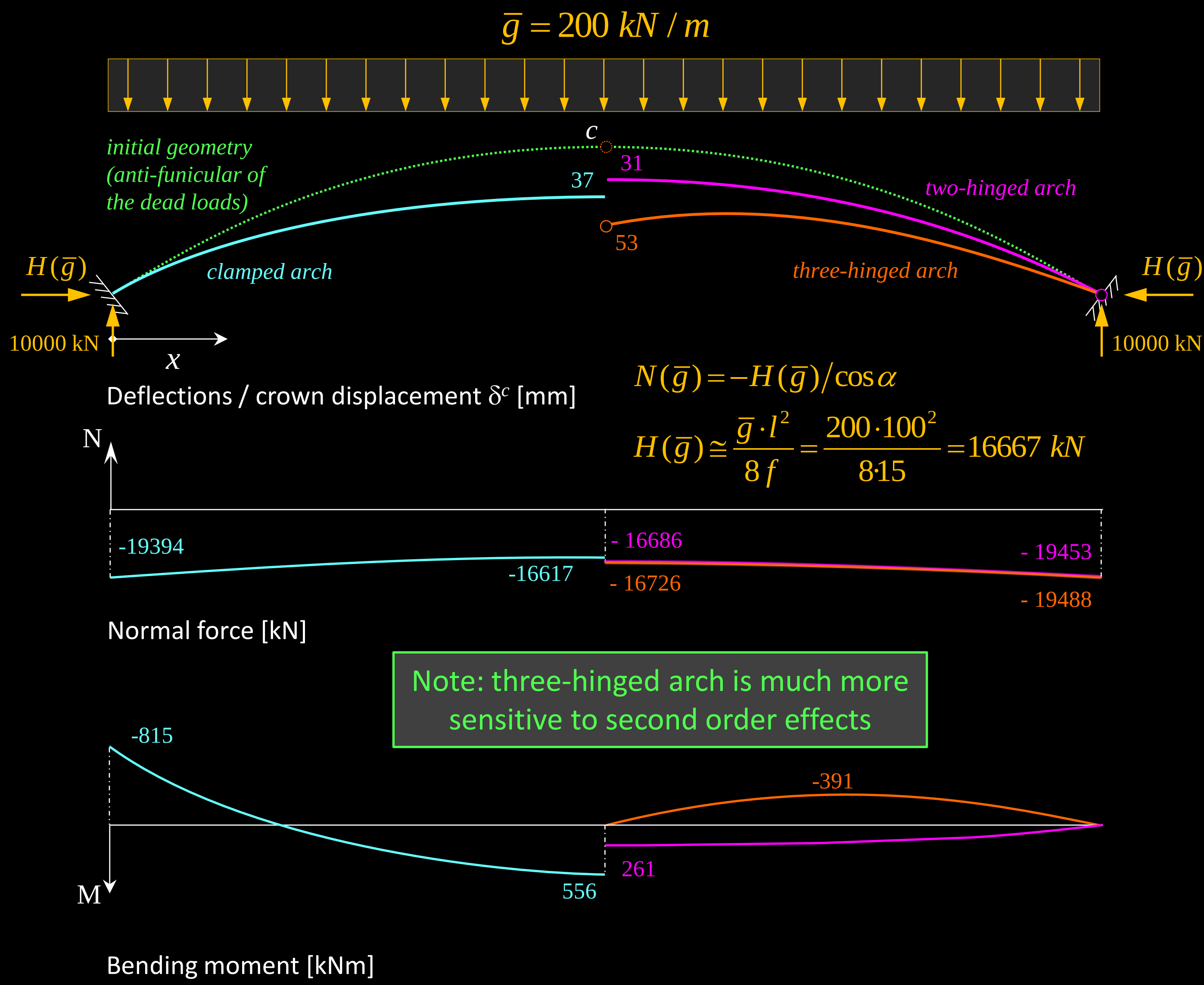
- three-hinged arch
- two-hinged arch
- clamped arch

Geometric nonlinearity has a minor impact on the clamped and two-hinged arches → reduced second order effects in these hyperstatic arches (for f / l = 1/6.67).

However, geometric nonlinearity strongly affects the three-hinged arch:

→ significant negative bending moments (rather than zero)

→ strong increase of the displacements: δ^c increased by 36%



Arch bridges – Structural response: Arch support conditions / hinges

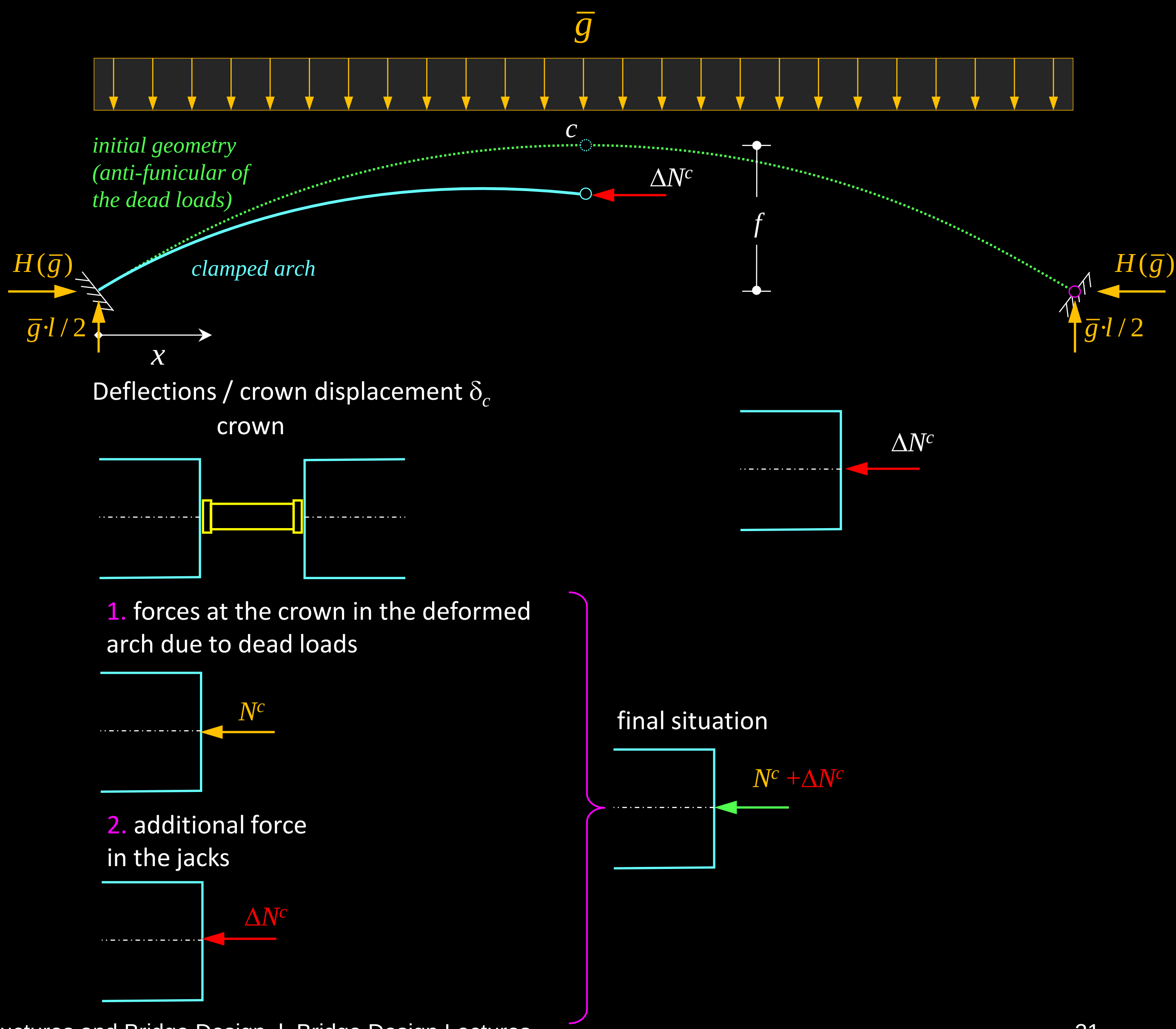
Opening the crown with jacks (to lift arch off falsework)

The bending moments and deflections due to arch compression can be reduced – at the time of closure, see next slide – by opening the crown with jacks (first done by E. Freyssinet, usual today in some countries).

Jacks align with the centre of gravity: no bending moments are produced in the crown until it is closed → the two-hinged and clamped arches are composed for two system:

→ hinged arch at the crown (dead loads + part of the creep)

→ closed arch at the crown (all other loads)



Arch bridges – Structural response: Arch support conditions / hinges

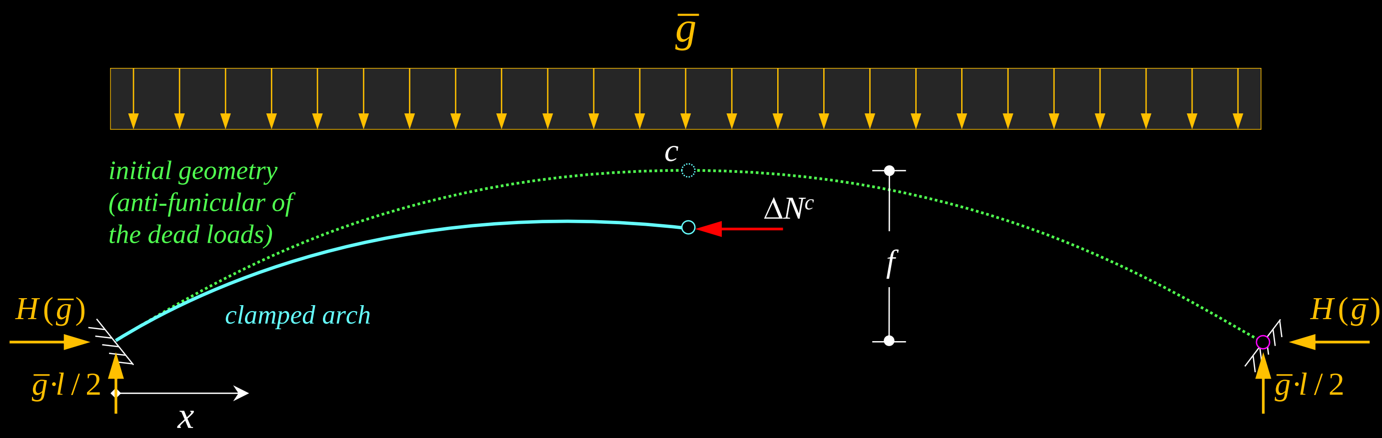
Opening the crown with jacks (to lift arch off falsework)

The bending moments and deflections due to arch compression can be reduced – at the time of closure, see next slide – by opening the crown with jacks (first done by E. Freyssinet, usual today in some countries).

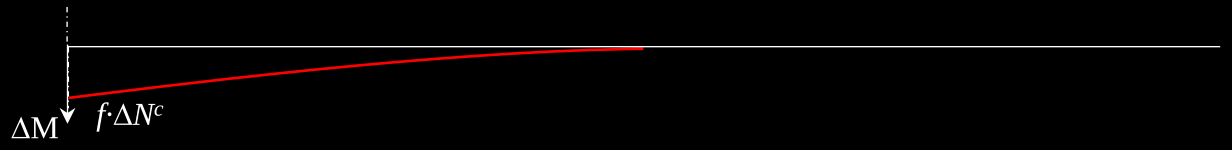
The additional normal force ΔN^c is introduced at the crown (by means of a centric normal force ΔN^c, applied by jacks) to reduce the total eccentricity $e = M / N$.

The optimum values of the jacking forces can be determined by imposing the condition that the total bending moments at the abutments (springing line) vanish:

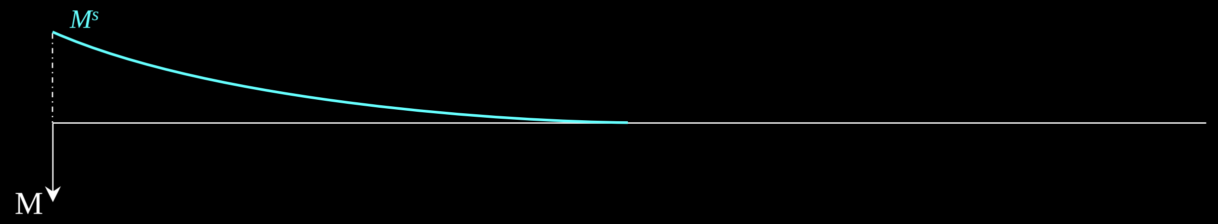
$$\sum M^s = 0 = -M^s + f \cdot \Delta N^c \rightarrow \Delta N^c = -\frac{M^s}{f}$$



Deflections / crown displacement δ_c



Bending moments due to ΔN^c



Bending moment due to dead and imposed deformations

Arch bridges – Structural response: Arch support conditions / hinges

Opening the crown with jacks (to lift arch off falsework)

Using the parameters of the numerical example on the previous slides (including a hinge at the crown), the additional normal force ΔN^c at the crown in the clamped arch is:

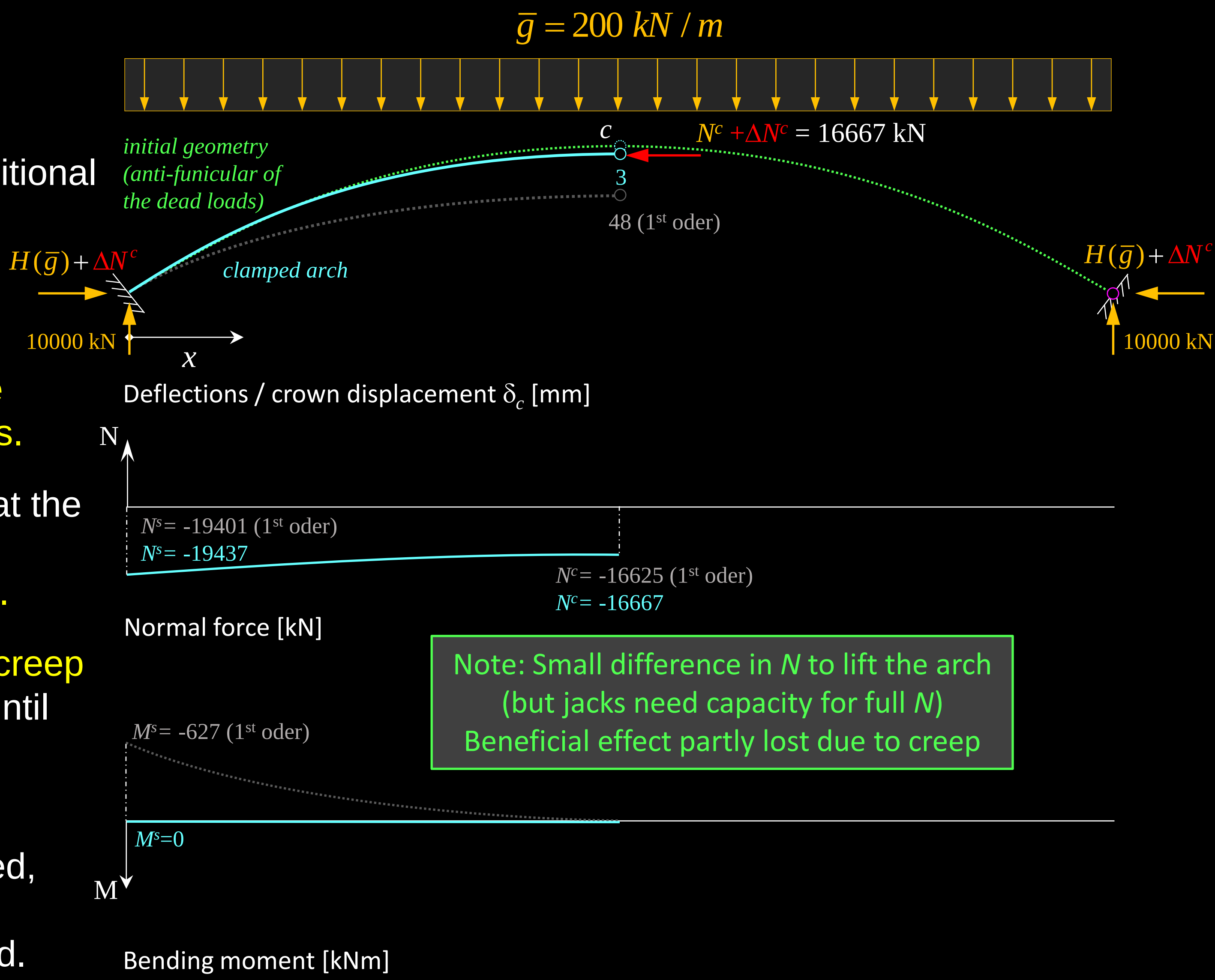
$$\Delta N^c = -\frac{M^s}{f} = \frac{627}{15} = 41.8 \text{ kN}$$

Physically, the jacks have to apply the total normal force $N^c + \Delta N^c = 16625 + 42 = 16667$ kN, acting in the arch rib axis.

Thereby, the total bending moment obviously vanishes at the springing lines (higher normal force in the arch chosen accordingly) → bending moments have been eliminated.

However, the beneficial effect will largely be lost due to creep unless the jacks are kept installed and are re-adjusted until creep has decayed (as e.g. done for 5 years in the Krk bridges, see *Design* section).

The clamped arch hinged at the crown, before it is closed, has certain sensitivity to 2nd effects (similar to the three-hinged arch) → 2nd order effects should to be considered.



Note: Small difference in *N* to lift the arch (but jacks need capacity for full *N*) Beneficial effect partly lost due to creep

Arch bridges – Structural response: Arch support conditions / hinges

Point load at crown / linear analysis (1st order)

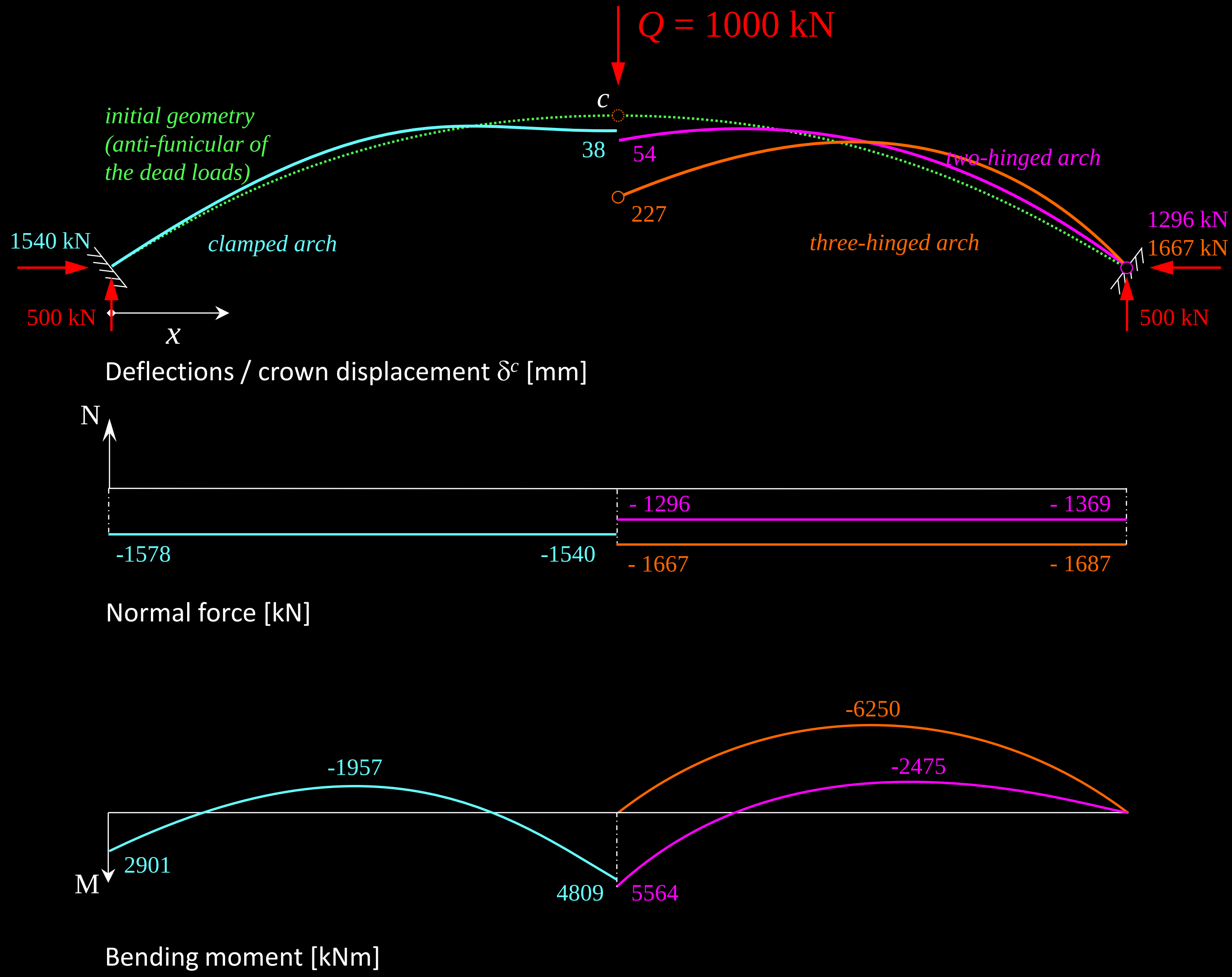
Considering a point load of 1000 kN at the crown, a linear analysis yields the following results:

- three-hinged arch
- two-hinged arch
- clamped arch

The differences between the axial forces in the three cases are moderate.

The bending moments and the deflections of the three-hinged arch are markedly higher than in the other two cases: The vertical displacement at the crown δ^c is ca. 4…6 times greater.

The bending moments and deflections of two-hinged and clamped arches are very similar, except at the springing lines (obviously).



Arch bridges – Structural response: Arch support conditions / hinges

Point load at crown / nonlinear analysis (2nd order)

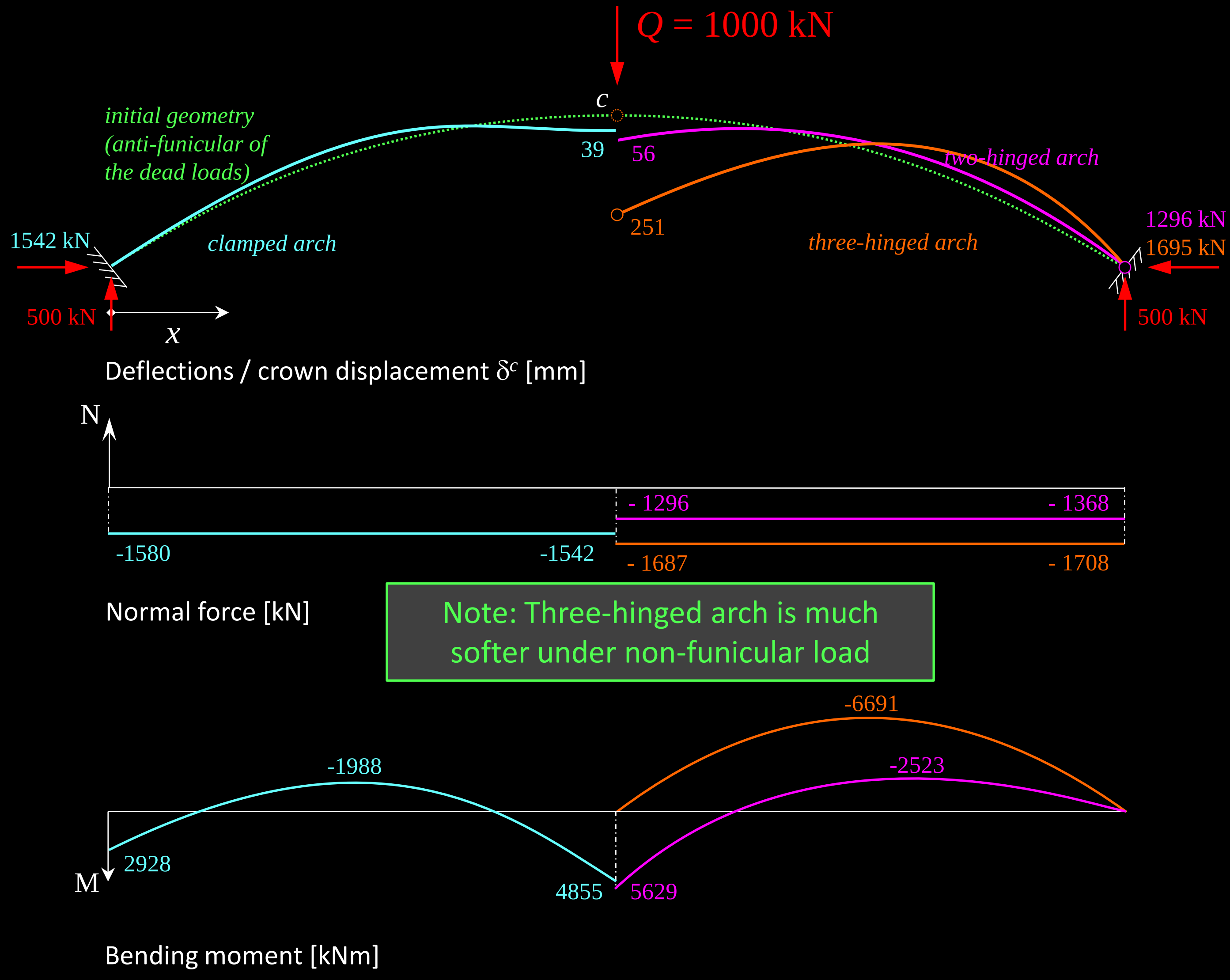
Considering a point load of 1000 kN at the crown, a nonlinear (2nd order) analysis yields the following results:

- three-hinged arch
- two-hinged arch
- clamped arch

Geometrical nonlinearity has a relevant effect (in this example) only in the three-hinged arch:

→ the maximum bending moment is increased by 7%

→ the vertical displacement at the crown is increased by 10%



Arch bridges – Structural response: Arch support conditions / hinges

Point load at quarter points / linear analysis (1st order)

Considering a point load of 1000 kN at the quarter point, a linear analysis yields the following results:

- three-hinged arch
- two-hinged arch
- clamped arch

The axial forces are similar for the three cases.

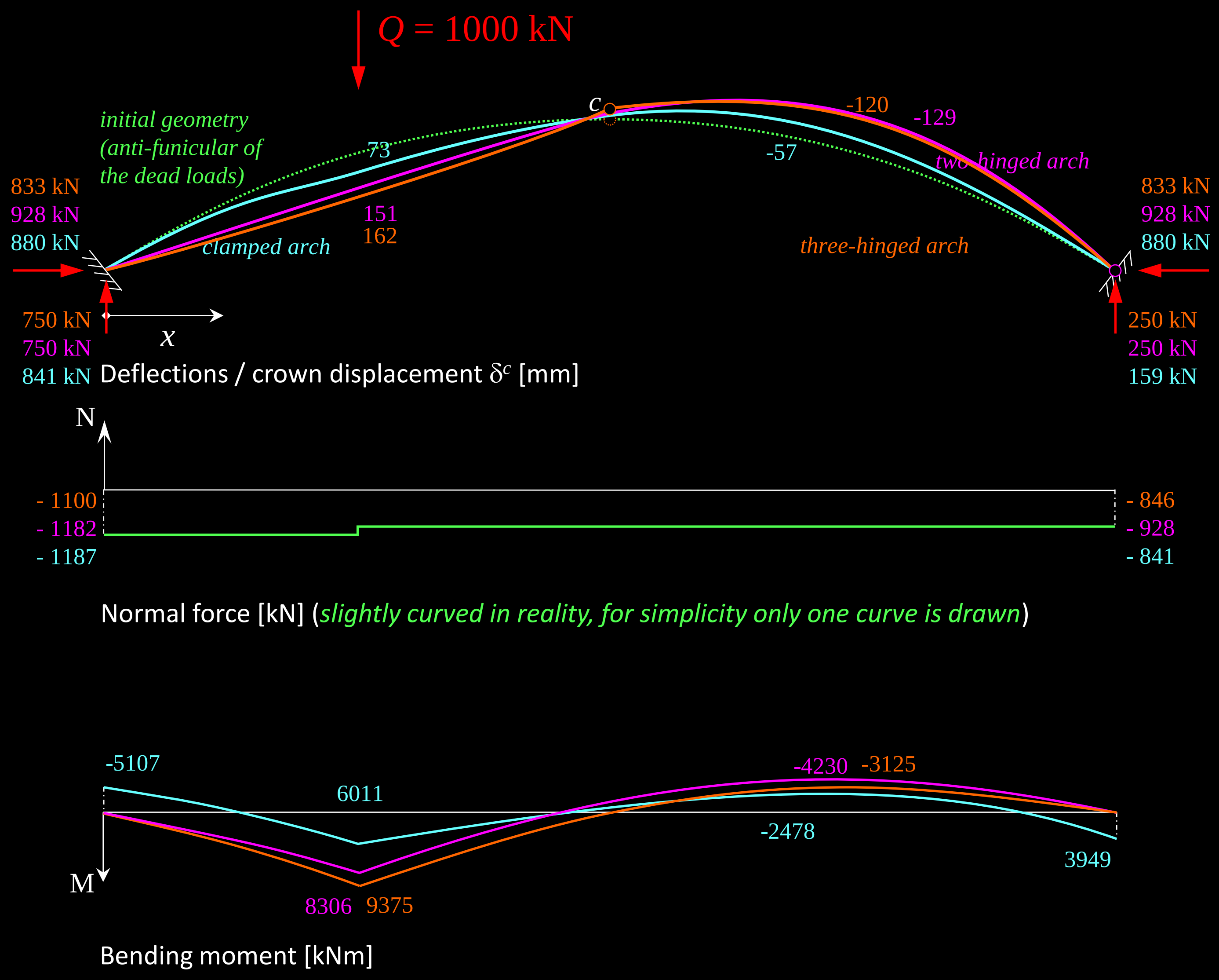
The two-hinged and three-hinged arches have a similar response (internal forces and deflections).

The clamped arch is clearly superior under asymmetric loads. For this example:

→ the maximum bending moment is approximately 30% smaller than in the other two cases.

→ the maximum vertical displacement is approximately 50% smaller than in the other two cases.

Note: the 2nd order effects have no significant influence in this example for this load case.



Arch bridges – Structural response: Arch support conditions / hinges

Horizontal support displacements

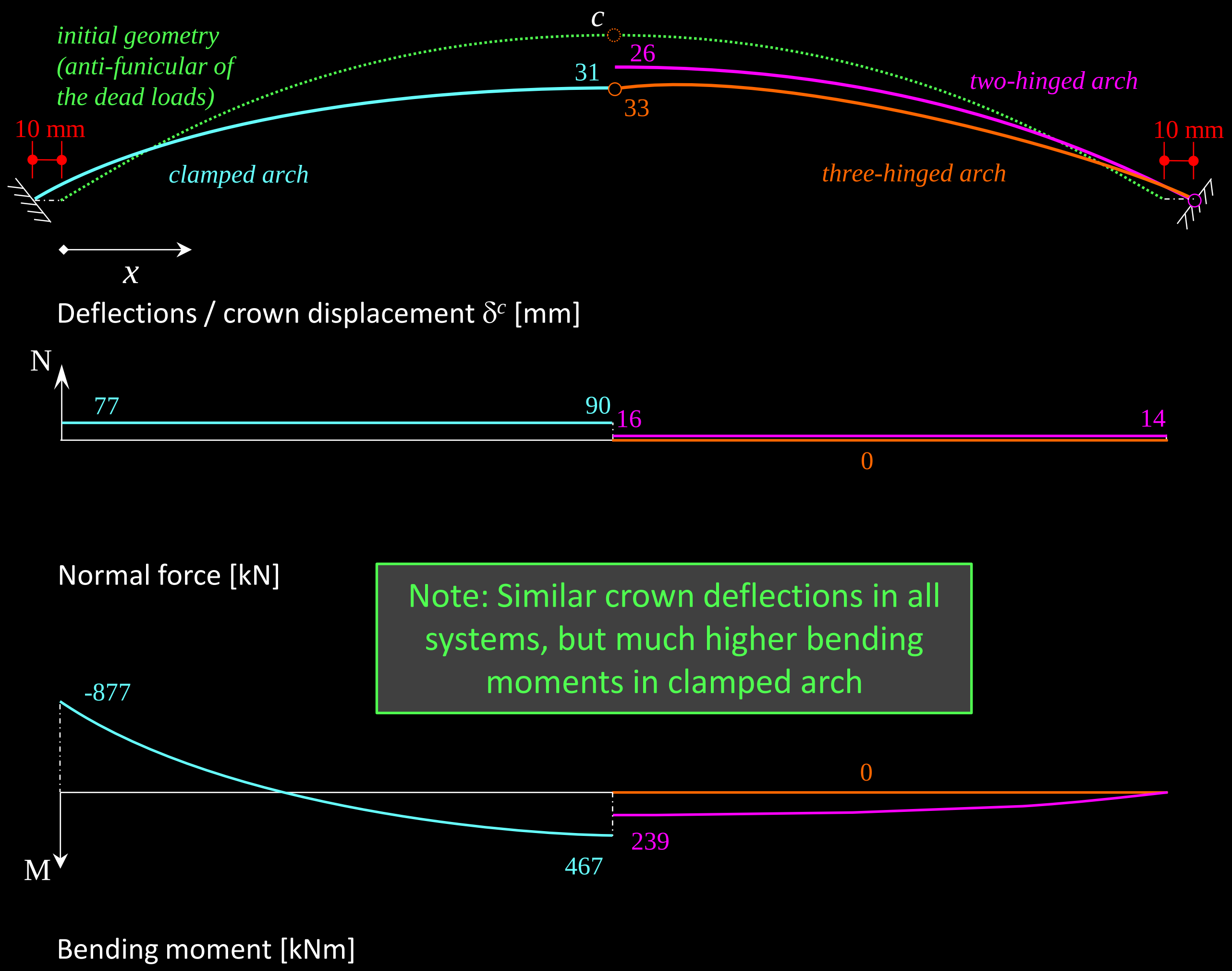
To analyse the influence of imposed deformations, horizontal displacements of 10 mm are imposed to the supports. The following results are obtained:

- three-hinged arch
- two-hinged arch
- clamped arch

The bending moments increase with the degree of statical indeterminacy:

→ the internal actions in the three-hinged arch are zero (isostatic system)

→ the bending moments are much higher for the clamped arch than the two-hinged arch



Note: Similar crown deflections in all systems, but much higher bending moments in clamped arch

NB1: The same conclusion applies for other imposed deformations (temperature, creep,…).

NB2: Approximation: $\delta^c \cong \frac{H(\bar{g})}{EA^{A,c}} \cdot l \cdot \frac{1+3(f/l)^2}{4f/l} = 37\ \text{mm}$

(Slide 55, horizontal displacement)

Arch bridges – Structural response: Arch support conditions / hinges

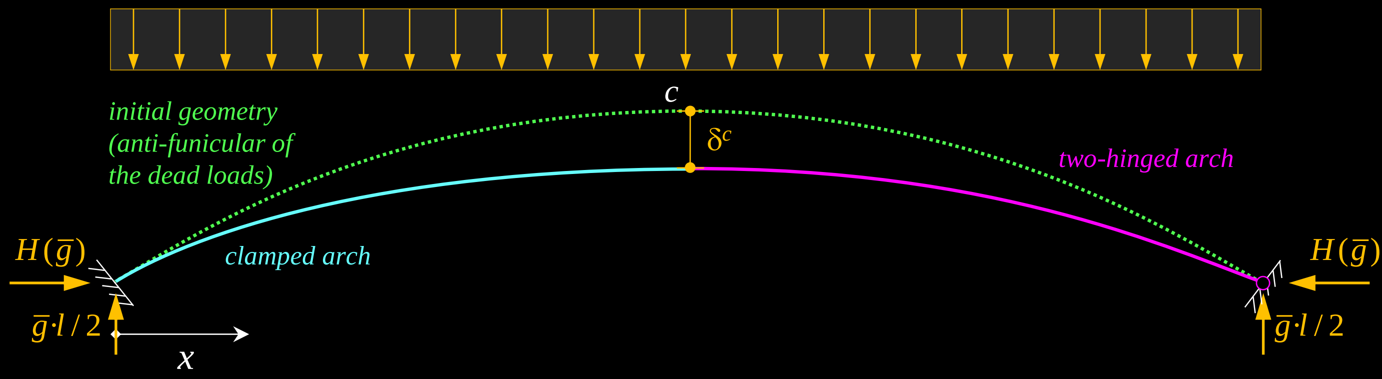
Effect of rise-to-span ratio $f\,/\,l$ on bending moments

Here, a uniform permanent load g and a linear analysis is used. The arches considered are:

- two-hinged arch
- clamped arch

As outlined in the *Design* section and in the permanent loads analysis, the arch compression causes vertical deflections δ^c. These deflections produce bending moments $M(x)$, and the maximum and minimum bending moments can be expressed in terms of the vertical deflection.

As the normal force N depends on the rise-to-span ratio $f\,/l$, the latter has a strong influence on the vertical deflections and the bending moments.



$EA \;\square\; N$ $(A = const$ along the arch$)$

$$N(\bar{g}) = -\frac{H(\bar{g})}{\cos\alpha}$$

$$H(\bar{g}) \cong \frac{\bar{g}\cdot l^2}{8f}$$

$$\delta^c \cong \frac{H(\bar{g})}{EA}\cdot\frac{l}{4}\cdot\frac{1+3\left(f\,/\,l\right)^2}{f\,/\,l}$$

simply supported beam

$$\delta_{midspan} = \frac{5}{384}\frac{\bar{g}l^4}{EI}$$

$$M_{midspan} = \frac{\bar{g}l^2}{8}$$

two-hinged arch

$$M^c \cong \frac{48}{5}\frac{EI}{l^2}\delta^c$$

clamped beam

$$\delta_{midspan} = \frac{\bar{g}l^4}{384EI}$$

$$M_{midspan} = \frac{\bar{g}l^2}{24}$$

clamped arch

$$M^c = -\frac{1}{2}M^s \cong \frac{16EI}{l^2}\delta^c$$

c : crown

s : springing line = arch abutments

Arch bridges – Structural response: Arch support conditions / hinges

Effect of rise-to-span ratio f / l on bending moments

Here, a uniform permanent load g and a linear analysis is used. The arches considered are:

- two-hinged arch
- clamped arch

To isolate the effect of the rise-to-span ratio f / l, the following assumptions are made:

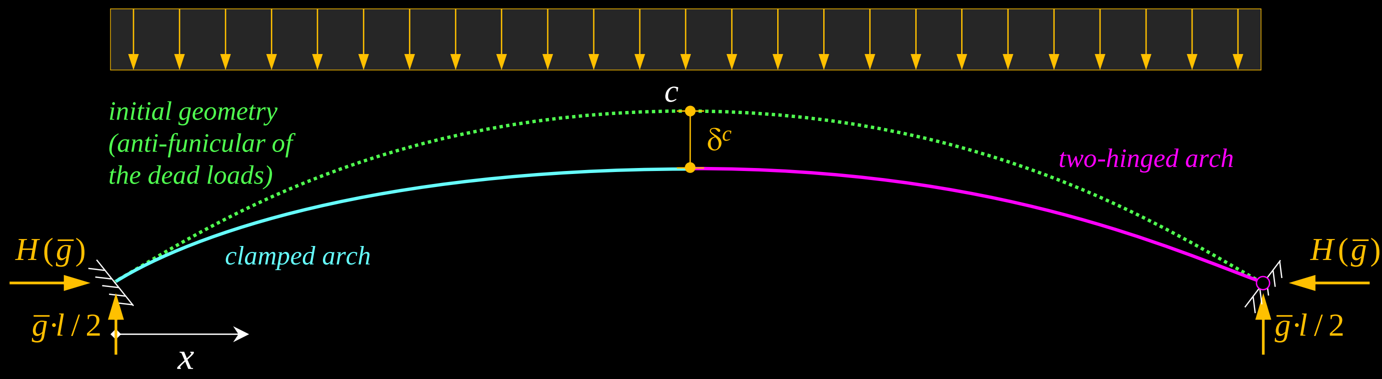
→ $H/(EA)$ = const. ∀ f/l, i.e., similar axial deformation $\varepsilon = N/(EA)$ due to arch compression for all f / l ratios

→ radius of gyration $i^2 = I / A$ = const, i.e.

  → constant arch height h, arch width $b(f/l)$ determined such that $H/(E{\cdot}h{\cdot}b)$ = const. ∀ f/l

→ variable self-weight as function of the arch width b

$\bar{g} = \gamma_c \cdot h \cdot b + DL$ (DL: permanent loads)

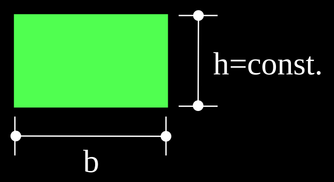


$EA \gg N$ ($A = const$ along the arch)

$$N(\bar{g}) = -\frac{H(\bar{g})}{\cos\alpha}$$

$$H(\bar{g}) \cong \frac{\bar{g}\cdot l^2}{8f}$$

$$\delta^c \cong \underbrace{\frac{H(\bar{g})}{EA}}_{const.} \cdot \frac{l}{4} \cdot \frac{1+3(f/l)^2}{f/l}$$



simply supported beam

$$\delta_{midspan} = \frac{5}{384}\frac{\bar{g}l^4}{EI}$$

$$M_{midspan} = \frac{\bar{g}l^2}{8}$$

two-hinged arch

$$M^c \cong \frac{48}{5}\frac{EI}{l^2}\delta^c$$

clamped beam

$$\delta_{midspan} = \frac{\bar{g}l^4}{384EI}$$

$$M_{midspan} = \frac{\bar{g}l^2}{24}$$

clamped arch

$$M^c = -\frac{1}{2}M^s \cong \frac{16EI}{l^2}\delta^c$$

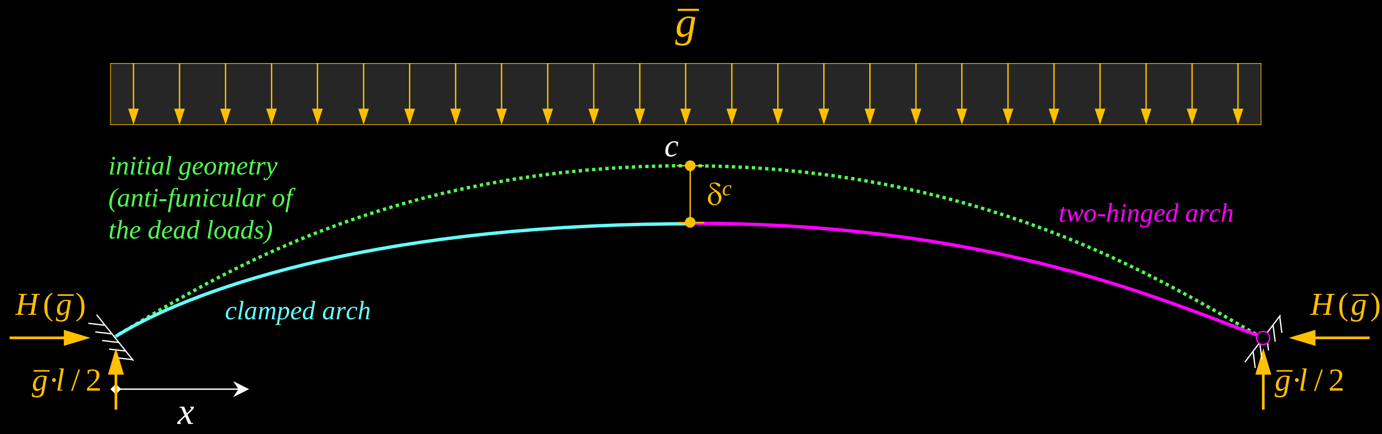
c : crown
s : springing line = arch abutments

Arch bridges – Structural response: Arch support conditions / hinges

Effect of rise-to-span ratio f / l on bending moments

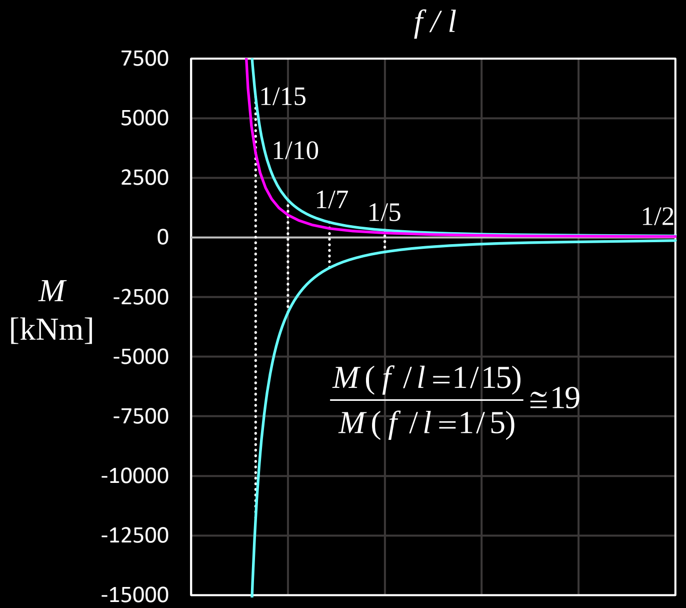
Here, a uniform permanent load g and a linear analysis is used. The arches considered are:

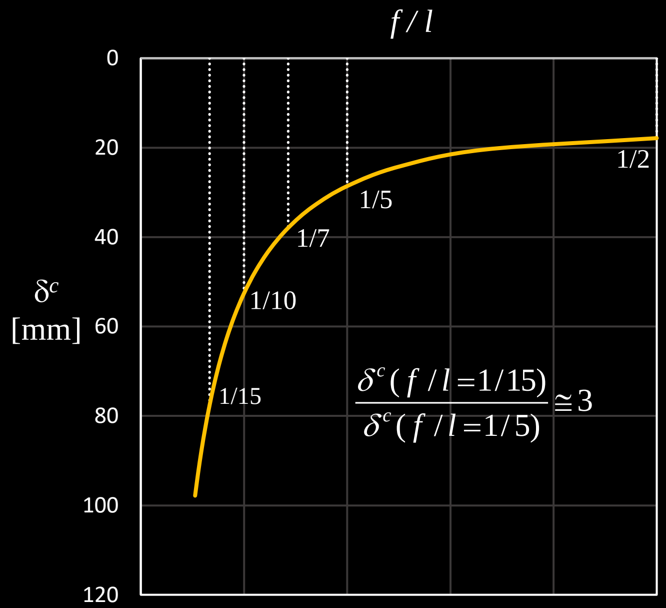
- two-hinged arch
- clamped arch



Using these assumptions and equations in the numerical example (l=100 m; h=1.20 m; DL = 140 kN/m), the following results are obtained (see graphs):

- The rise-span ratio f / l is highly relevant, having a strong impact on structural behaviour, particularly for small values of f / l (low arches)
- Bending moments increase exponentially with smaller values of f / l, particularly pronounced for f / l < 1/10. For f / l =1/15, bending moments are up to 15 times higher than for f / l = 1/5.
- The crown displacement also grows progressively as f / l decreases, especially for f / l < 1/10
- Clamped and two-hinged arches show similar tendencies.





Arch bridges – Structural response: Arch support conditions / hinges

Effect of rise-to-span ratio $f\,/\,l$ on bending moments

Here, a uniform permanent load g and a linear analysis is used. The arches considered are:

- two-hinged arch
- clamped arch

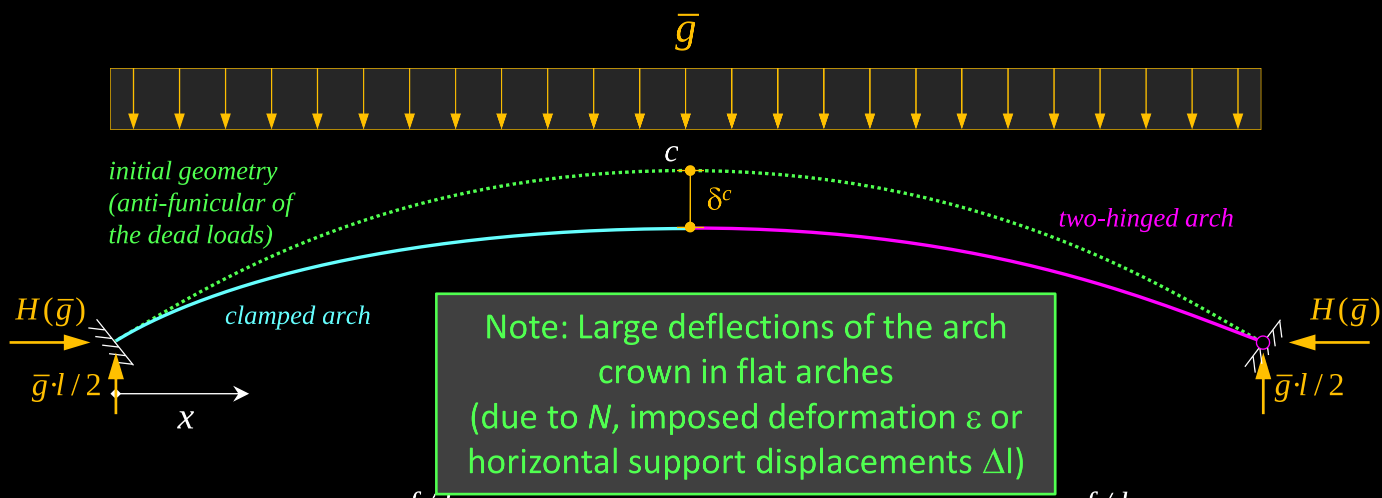
Note that similar results are obtained when the arches are subjected to horizontal displacements of the supports.

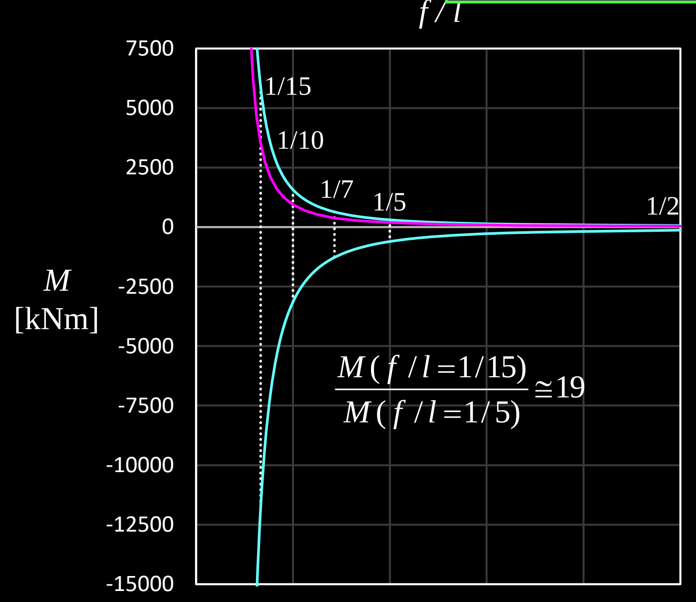
The resulting bending moments, for a low arch (rise-span ratio lower than 1/10), may exceed the moments produced by the gravity loads.

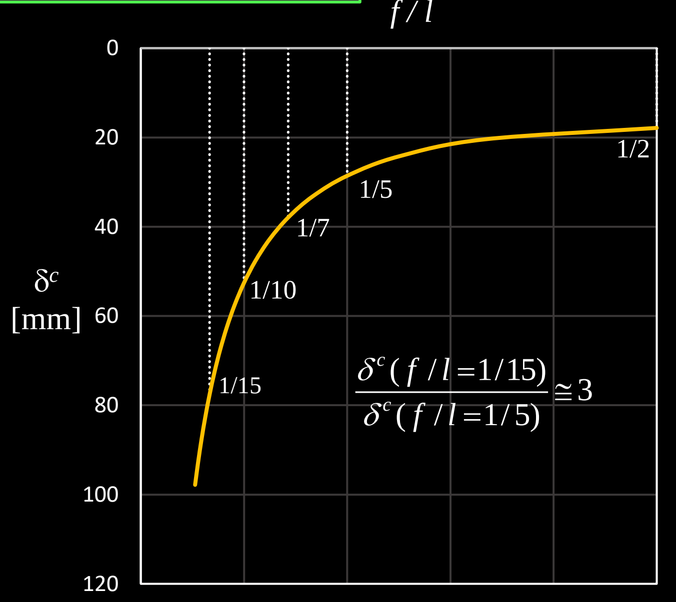
Conversely, the influence of imposed deformations are relatively small in arches which rise-span ratios > 1/7.

The numerical results correspond closely to the approximation (slide 55) for EA=const., i.e.

$\delta^c \cong \frac{H(\bar{g})}{EA^{A,c}} \cdot l \cdot \frac{1+3(f/l)^2}{4f/l}$ is a good approximation.







Arch bridges – Structural response: Arch support conditions / hinges

Permanent load + imposed deformation
1st and 2nd order analysis

- three-hinged arch
- two-hinged arch
- clamped arch

Imposed deformations never act alone. Rather, other actions are present, e.g. permanent loads or traffic loads. Consequently, the deformations caused by imposed deformations (change of geometry) produce an increase of the internal actions (bending moments).

For this study, aa low arch (f/l =1/15) is chosen in order to accentuate the nonlinearity effects.

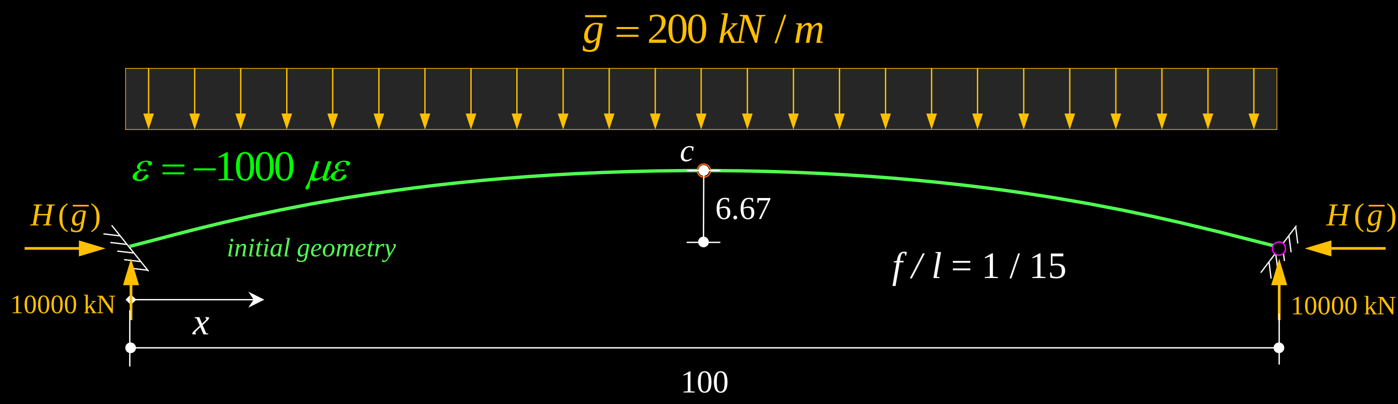
Arch geometry and loads :

→ l = 100 m; f = 6.67 m; f/l =1/15

→ cross-section: h x b = 1.2 m x 5.4 m

→ uniform permanent load: g = 200 kN/m

→ imposed deformation: ε = -1000 με (temp. + creep)



$N(\bar{g}) = -H(\bar{g})/\cos\alpha$

$H(\bar{g}) \cong \frac{\bar{g}\cdot l^2}{8f} = 16667\ kN$

Arch bridges – Structural response: Arch support conditions / hinges

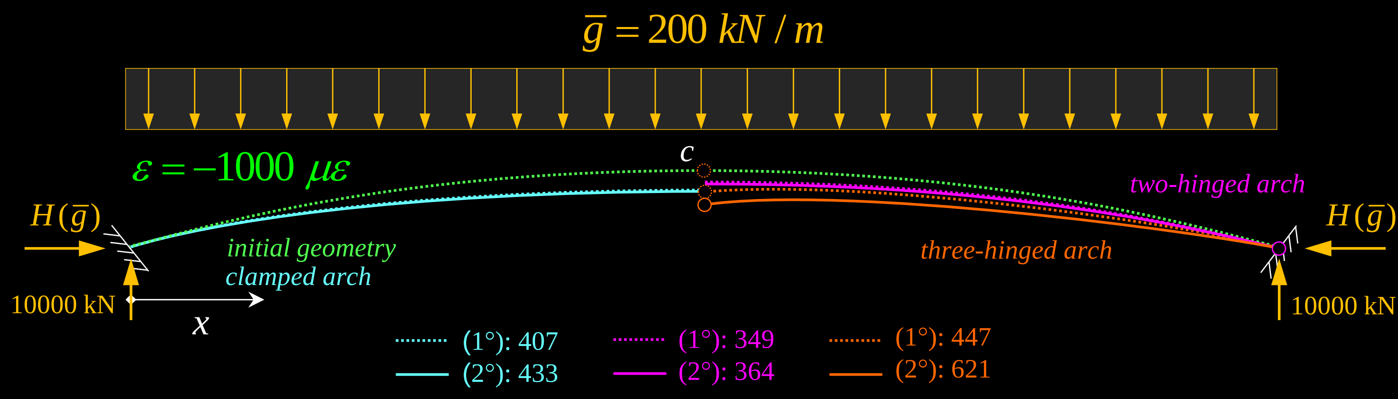
Permanent load + imposed deformation
1st and 2nd order analysis

- three-hinged arch
- two-hinged arch
- clamped arch

The figures compare the deflections and bending moments of the arches.

The three-hinged arch is the most flexible of all arches. The crown deflection (2nd order) is roughly 1.4 and 1.7 times larger than in the clamped and two-hinged arches, respectively. It is more sensitive to geometrical nonlinearity and, therefore, has a greater risk of instability.

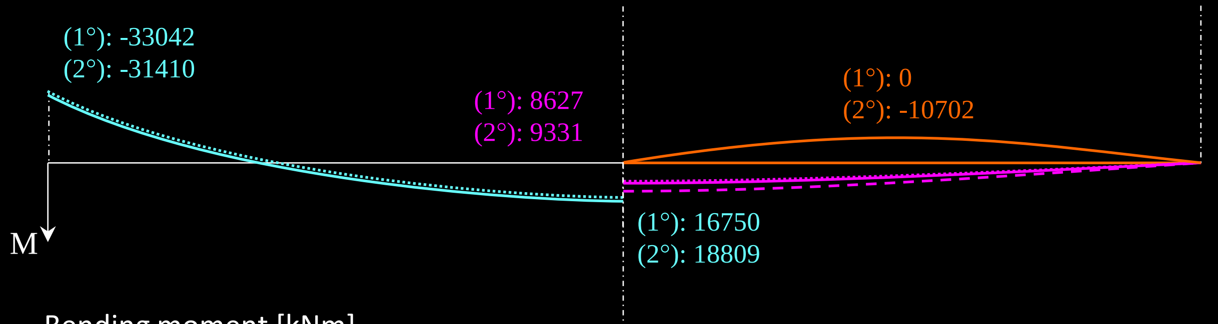
The bending moments in the clamped arch are higher than the other arches and, similar to the two-hinged arch, there is no significant difference between 1st order and 2nd order results.



Deflections / crown displacement δ^c [mm]

$$H(\bar{g}) \cong \frac{\bar{g} \cdot l^2}{8f} = 16667\ kN$$

$(1°): H(\bar{g}) = 30031\ kN$	$(1°): H(\bar{g}) = 36206\ kN$	$(1°): H(\bar{g}) = 37500\ kN$
$(2°): H(\bar{g}) = 32067\ kN$	$(2°): H(\bar{g}) = 38207\ kN$	$(2°): H(\bar{g}) = 41356\ kN$



Bending moment [kNm]

Arch bridges – Structural response: Arch support conditions / hinges

Permanent load + imposed deformation
1st and 2nd order analysis

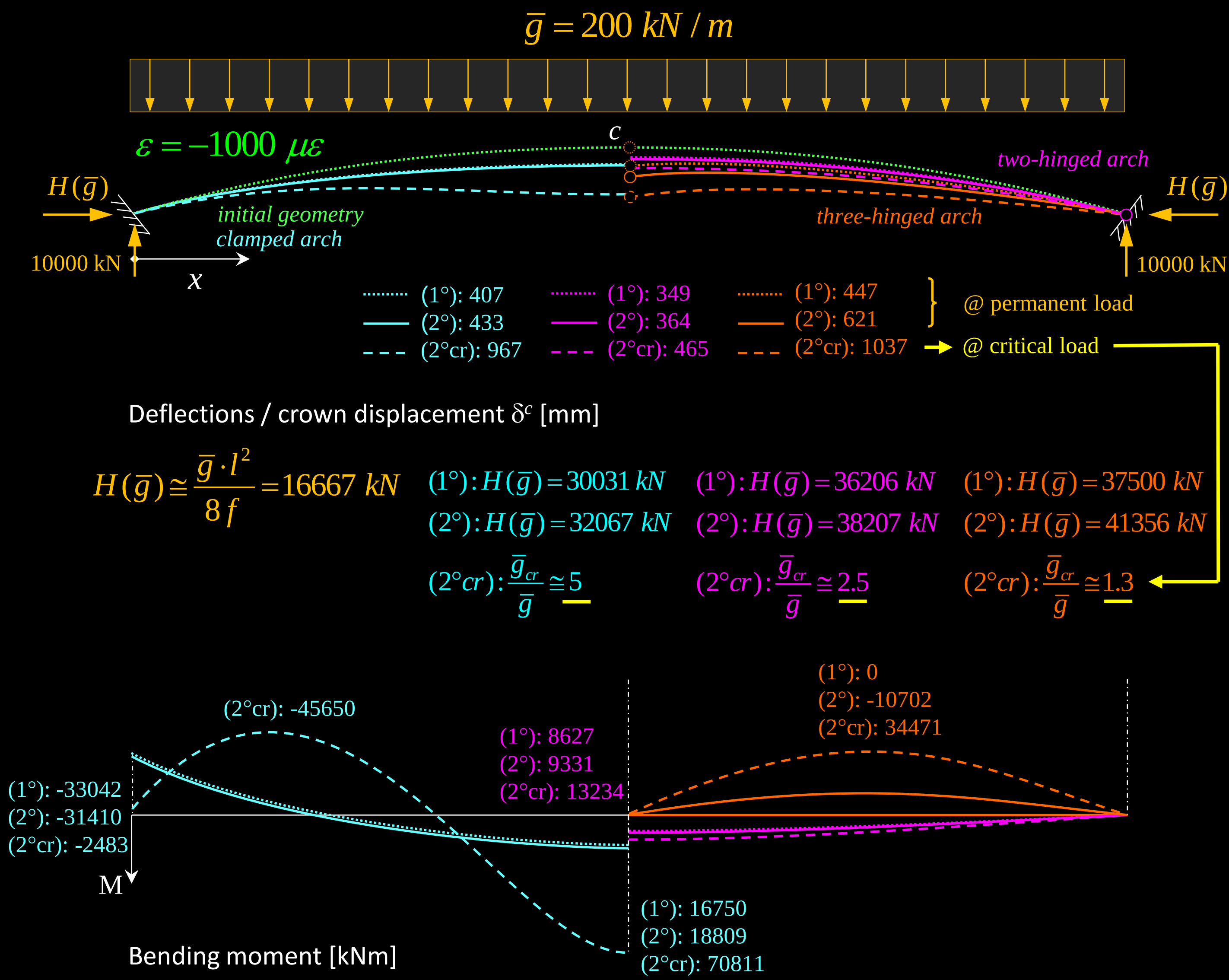
- three-hinged arch
- two-hinged arch
- clamped arch

Instability and critical load g_{cr}:

Instability is reached quickly in the three-hinged arch. The critical load g_{cr} is only 1.3 times higher than the permanent load g.

The clamped arch is the most stable → instability is reached at a critical load g_{cr} 5 times higher than the permanent load g.

The two-hinged arch is in an intermediate position → instability is reached at a critical load g_{cr} 2.5 times greater than the permanent load g.



Arch bridges – Structural response: Arch support conditions / hinges

Permanent load + imposed deformation
1st and 2nd order analysis

- reinforced concrete
- three-hinged arches $\rightarrow$ two-hinged arches
- central span: 72.5 m
- $f / l =$ 1 / 15


Le Veurdre bridge, France, 1910. Eugène Freyssinet