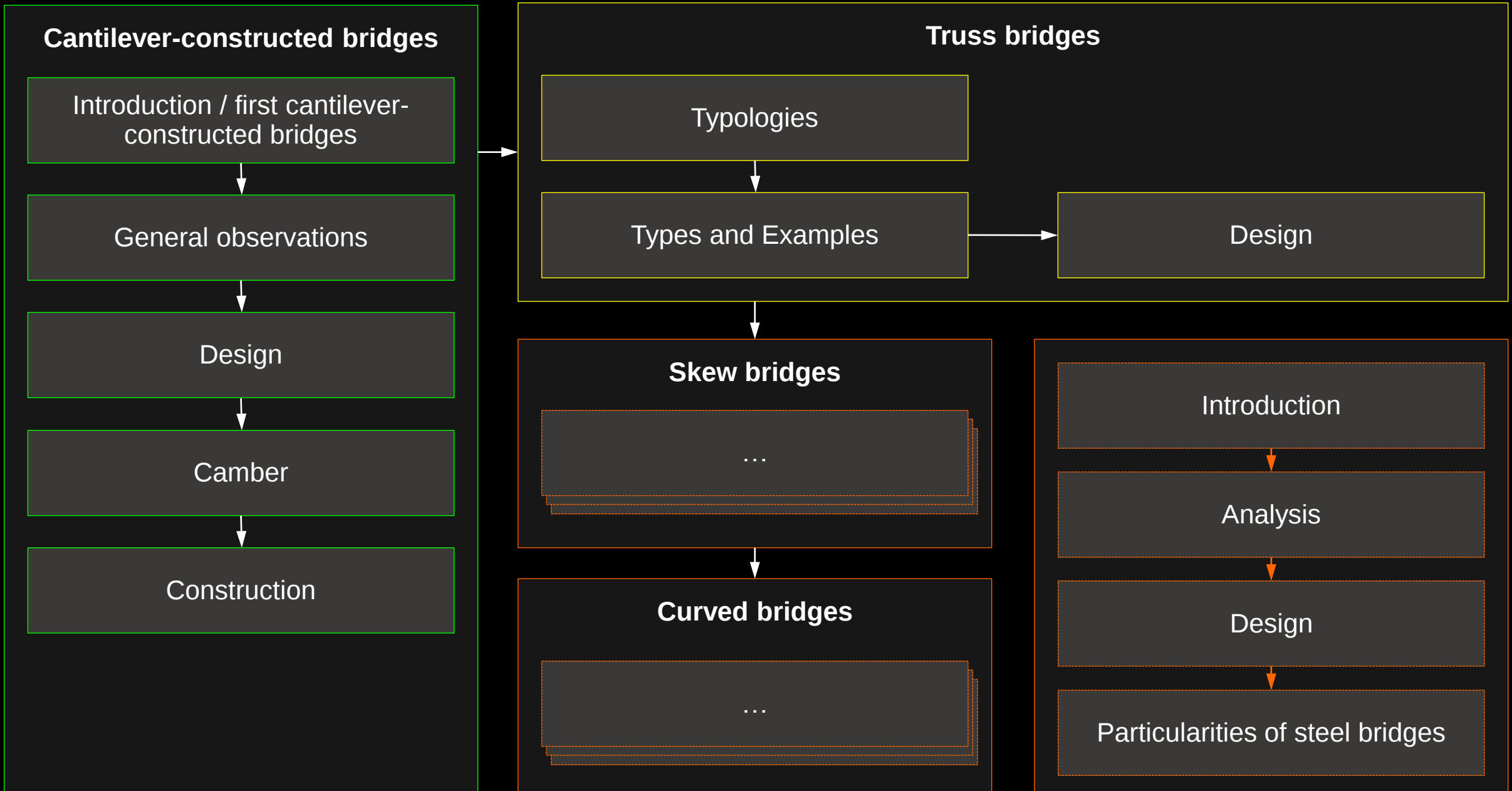


Special girder bridges

Curved Bridges



Special girder bridges

Curved Bridges Applications

Curved Bridges – Applications

- Integration of bridge structures into the **roadway and railway networks** often requires the adoption of **curved bridge decks**.
- In the case of highway interchanges and urban expressways, they are unavoidable.
- In the case of long-span crossings, the alignment should be adjusted to place the main bridge on a tangent:
 - Simpler to design and construct
 - Easier to accommodate bridge movements at expansion joints (at the boundary between main and approach spans).
- In the case of long viaducts it is impractical and uneconomical to design the entire alignment to be straight.
- An in-plan curved alignment may favour aesthetics and view of the bridge by the users.



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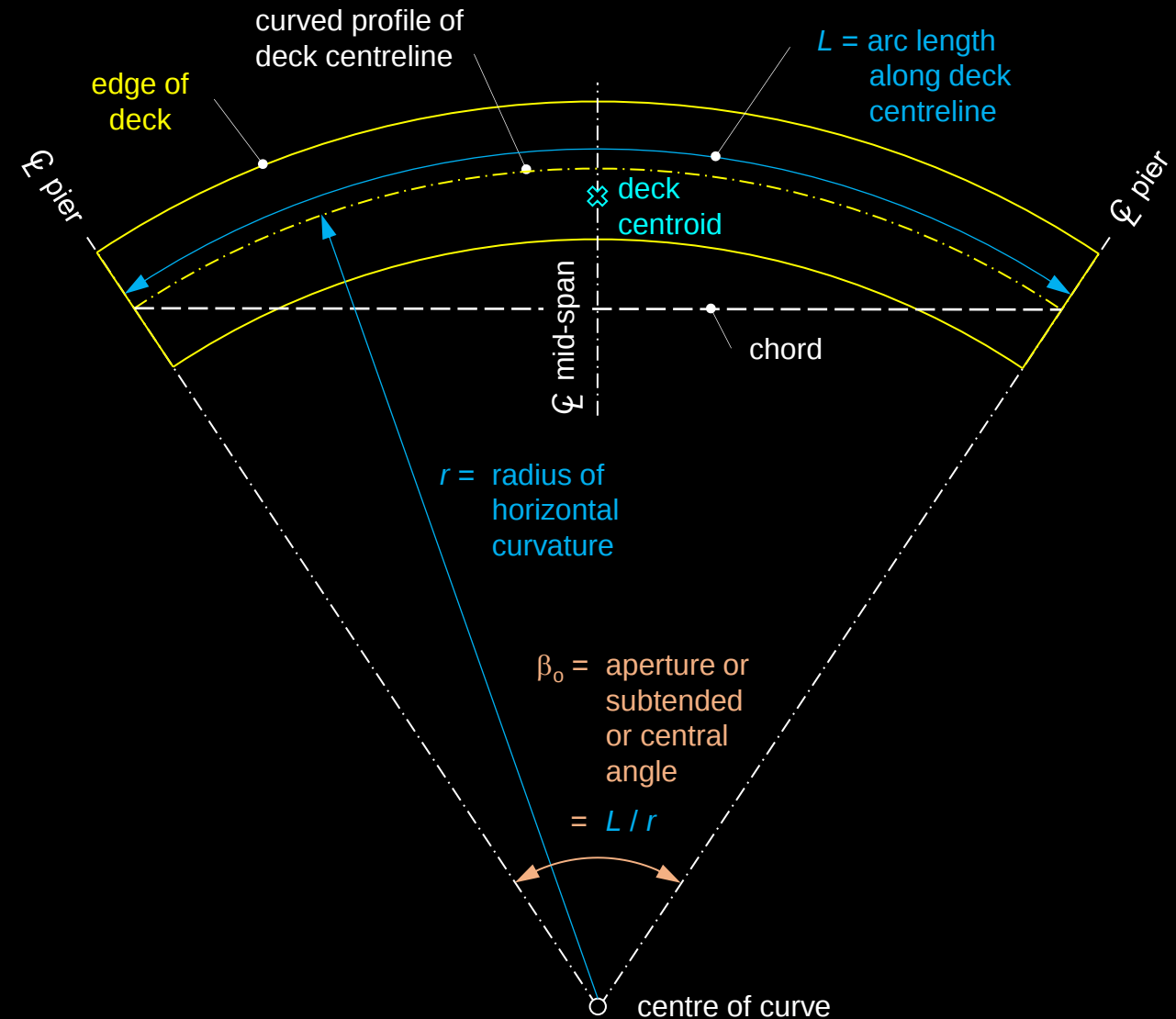


Special girder bridges

Curved Bridges Geometry & Terminology

Curved Bridges – Geometry & Terminology

- A typical segment of a **horizontally curved bridge deck** between two radially arranged piers is considered to illustrate basic geometric properties
- Note that:
 - The **radius of horizontal curvature** and the span **arc length** depend on the line of reference, i.e. deck centreline, edge of deck, roadway/railway alignment centreline, girder centreline, etc.
 - If the **radius**, **arc length** and **aperture angle** are within certain limits, the bridge is reasonably straight, and the behaviour of the girder can be approximated with an **equivalent straight girder**, having a span corresponding to the arc length of the curved girder axis.
 - Generally, the decision whether curvature needs to be considered or not is based on engineering judgement. Some design recommendations and codes provide explicit geometric limits (e.g. $\beta_o < 12^\circ$).



Special girder bridges

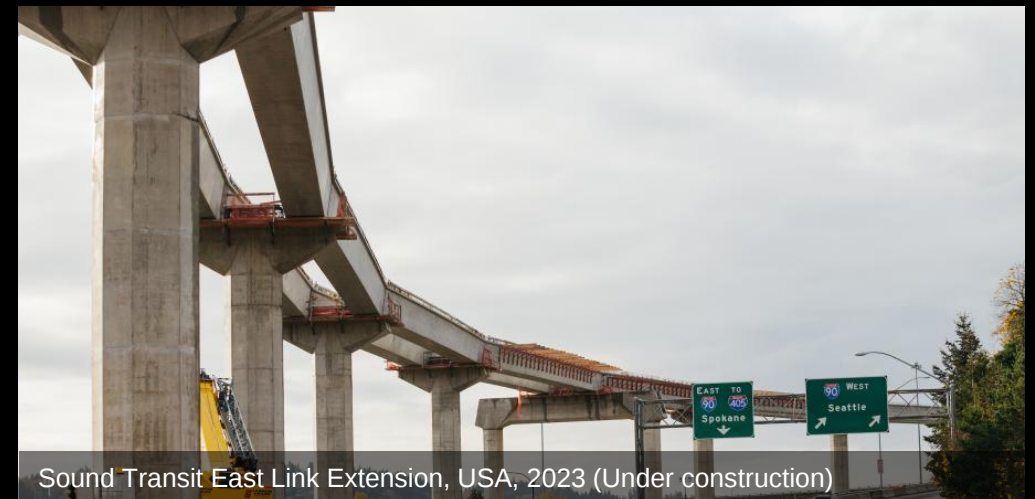
Curved Bridges Girder Configurations

Curved Bridges – Girder Configurations

Plan Geometry:

Horizontal curvature may be achieved in two ways:

- By introducing discrete changes in direction ("kinks") between segments of straight girders:
 - **Segmentally curved / kinked / chorded girders**
(deck usually continuously curve deck)
- By forming the girders to a radius
 - **Concrete girders**
 - curved formwork (often polygonal with segment length corresponding to formwork board length ≈ 2 m)
 - **Steel girders:**
 - by heat-bending (only for large curvature radii)
 - by cutting the flange plates to the required profile (and heat-bending the webs)



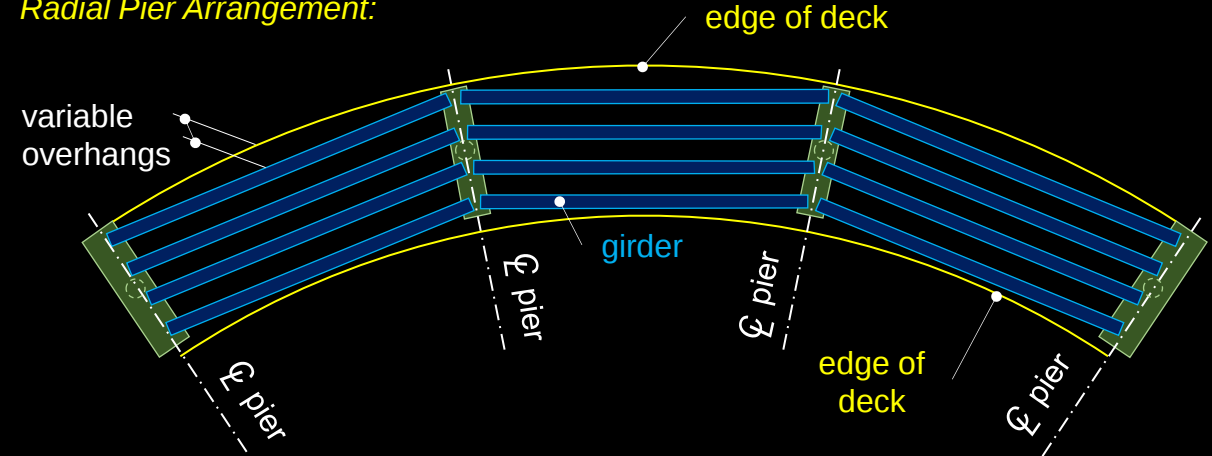
Curved Bridges – Girder Configurations

Plan Geometry:

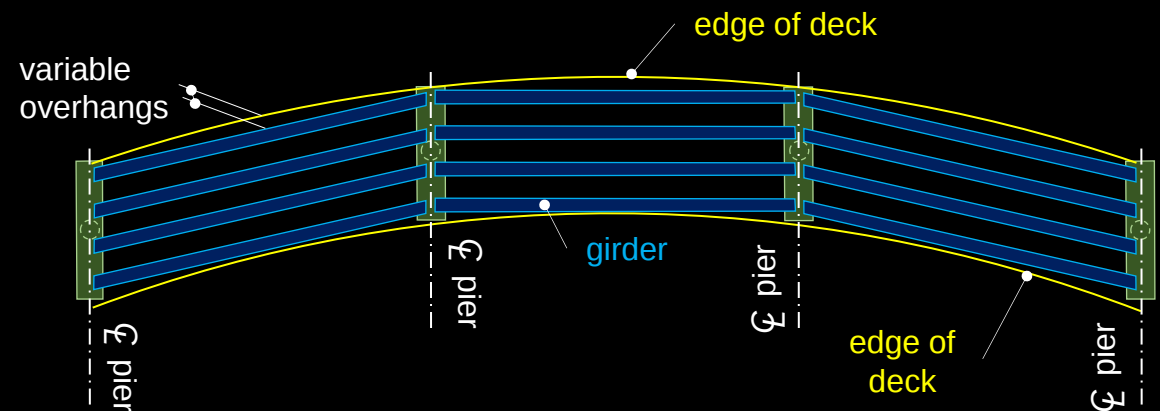
Possible girder configurations for curved bridges include:

- **Straight girders chorded from pier to pier**
 - Simple and economical:
 - ... easier fabrication and transportation
 - ... girders more stable for handling and erection
 - Variable deck overhangs (expensive formwork)
 - Radial or skew pier arrangements possible
 - Aesthetics and economics become an issue for sharply curved alignments

Radial Pier Arrangement:



Skew Pier Arrangement:



Curved Bridges – Girder Configurations

Plan Geometry:

Possible girder configurations for curved bridges include:

- Straight girders chorded from pier to pier

Example: **Cinta Costera Viaduct**, Panama City (2014)

- 2.5 km long viaduct → standardisation important
- 850 precast concrete girders
- Variable width (annulus sector “pie-shaped”) pier caps allowed for use of only 3 unique girder lengths
- Low bridge → aesthetics not affected by chorded girders (underside of bridge not visible)

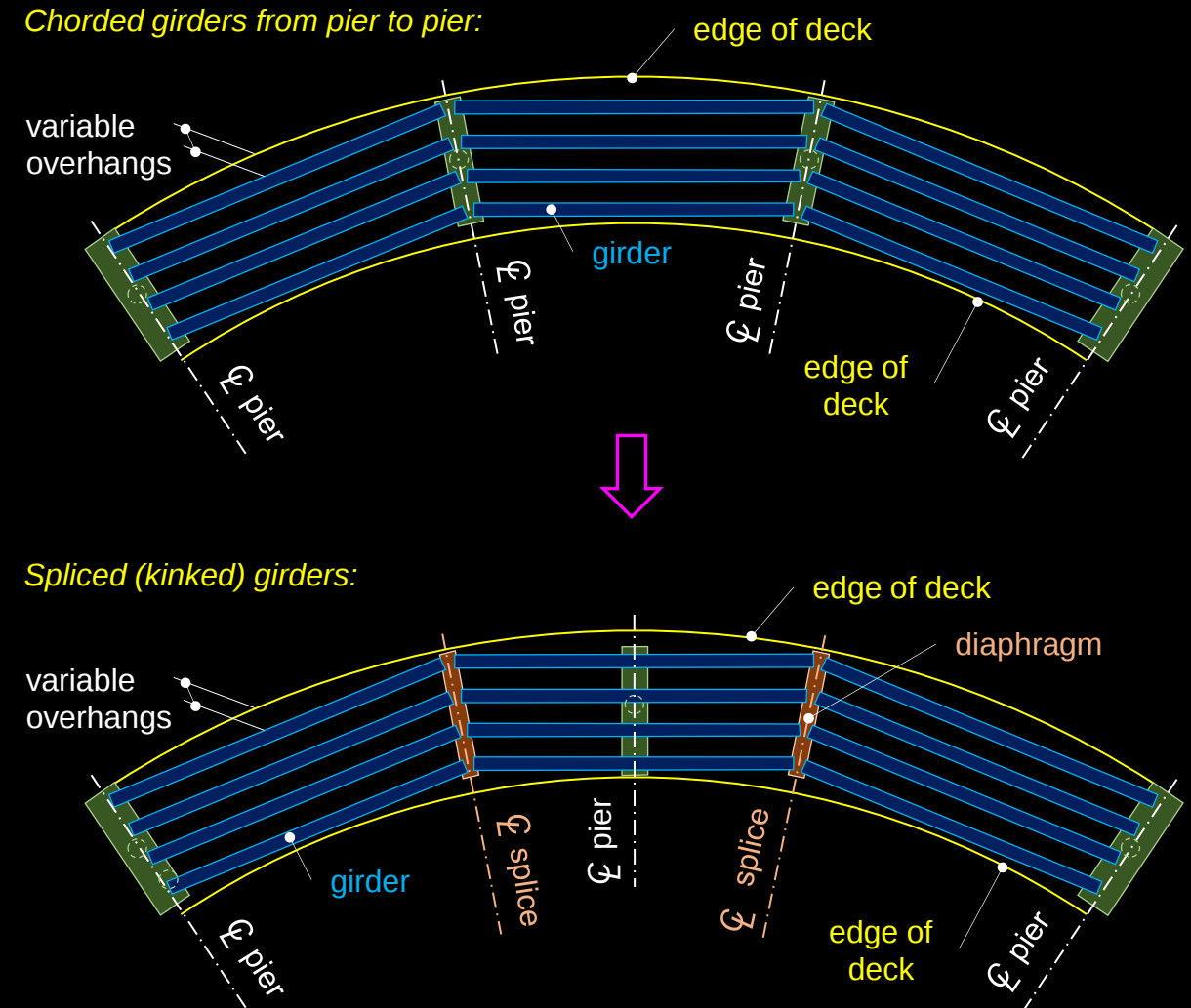


Curved Bridges – Girder Configurations

Plan Geometry:

Possible girder configurations for curved bridges include:

- Straight girders chorded from pier to pier
- **Spliced straight girders with splice (kink) points within the spans**
 - **Curved alignment is followed more closely**
 - Opportunity for multiple angular breaks per span → **longer spans and sharper curvatures possible**
 - **Aesthetics may still be an issue**
 - **Temporary shoring required for erection**
 - Change in direction of flange forces results in a horizontal radial component acting outward at a compression flange and inward at a tension flange → **Bracing must be provided at the kinks to resist these forces** (diaphragms / cross-frames)



Curved Bridges – Girder Configurations

Plan Geometry:

Possible girder configurations for curved bridges include:

- Straight girders chorded from pier to pier
- Spliced straight girders with splice (kink) points within the spans
- **Curved girders**
 - Simplified geometry
 - Better load distribution between cross-frames/diaphragms (no kinks)
 - Can be launched (if curvature is constant)
 - Higher aesthetic quality
 - Prefabrication and transportation more complicated
 - Ensuring stability during erection more complicated

→ The focus of the lecture is on curved girders



Curved Bridges – Girder Configurations

Cross-Sections for Curved Girders:

- Open Cross-Section
 - I-girders interconnected by cross-frames
 - Closely-spaced cross frames / diaphragms required
 - In bridges with pronounced curvature, a lateral bracing (horizontal truss) near the bottom flange is required (to form a quasi-closed cross-section acting in uniform torsion)
- Closed Cross-Section
 - Cast in place curved prestressed concrete girders
 - Steel box girders (single, twin, multiple)
 - Vertical or inclined webs
 - Single or multi-cell
 - Large torsional resistance in uniform torsion and reduced stresses due to warping torsion



Curved Bridges – Girder Configurations

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Special girder bridges

Curved Bridges

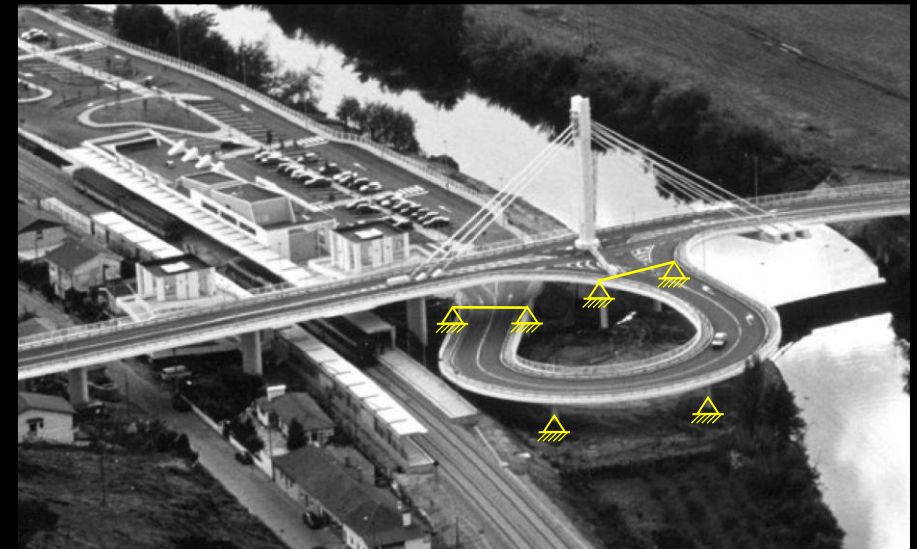
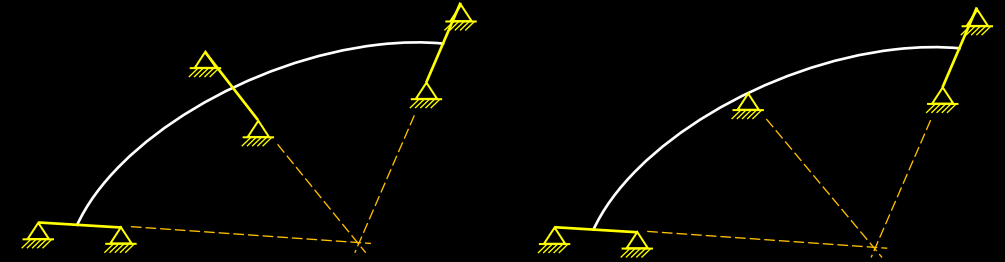
Implications / Considerations

Curved Bridges – Implications / Considerations

- Geometrical Considerations

Support & Articulation

- Placement and type of bearings need to consider the torsional response:
 - Open sections require torsional restraint at each support
 - Closed sections may require torsional restraint only at abutments (high torsional stiffness)
 - In-plane movements at bearings, due to uniform thermal actions and shrinkage effects of the concrete, are along the direction defined by the fixed in plane point and the bearing.
 - Changes in length can be accommodated through radial movements. Thus relatively long integral bridges may be designed in conjunction with slender piers.
 - If the expansion joint is not perpendicular to the direction of the movement, the expansion joint must be able to accommodate movements along its axis.
- See more detailed discussion in Chapter on Support & Articulation.



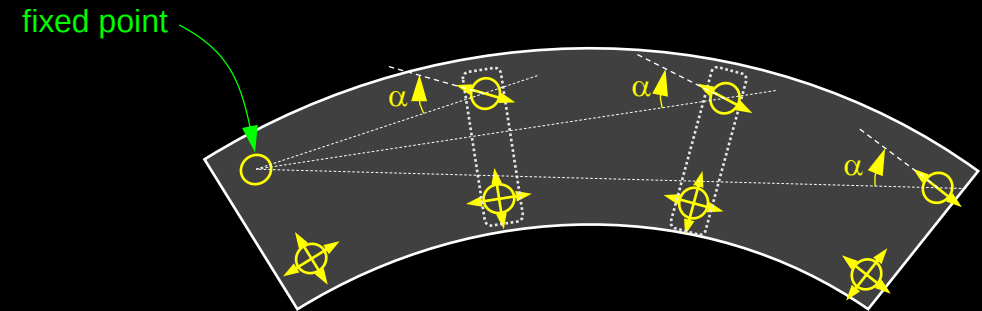
Curved Bridges – Implications / Considerations

- Geometrical Considerations

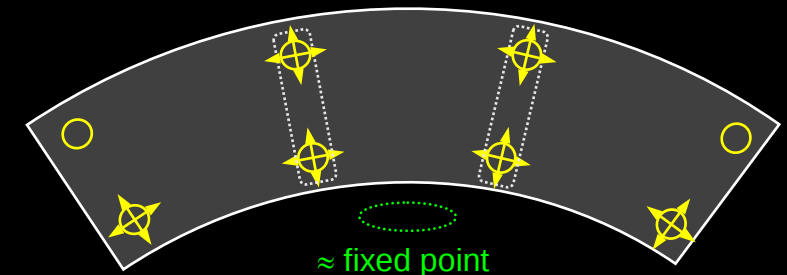
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 - Changes in length can be accommodated through radial movements. Thus relatively long curved integral bridges may be designed in conjunction with slender piers.
- If the expansion joint is not perpendicular to the direction of movement at the abutment, it must be able to accommodate transverse relative displacements.

➤ See more detailed discussion in Chapter on Support & Articulation.



Horizontal restraint-free (at piers only for temperature and shrinkage) bearing layout



Practical bearing layout causing moderate horizontal restraint (hor. fixity at both abutments)

Curved Bridges – Implications / Considerations

- Geometrical Considerations

- Need to consider **deck superelevation (Quergefälle)**. This may require a wider deck.
- The effective **centre of gravity is not at the cross-section centroid** (even without superelevation, exterior webs are longer than interior webs)

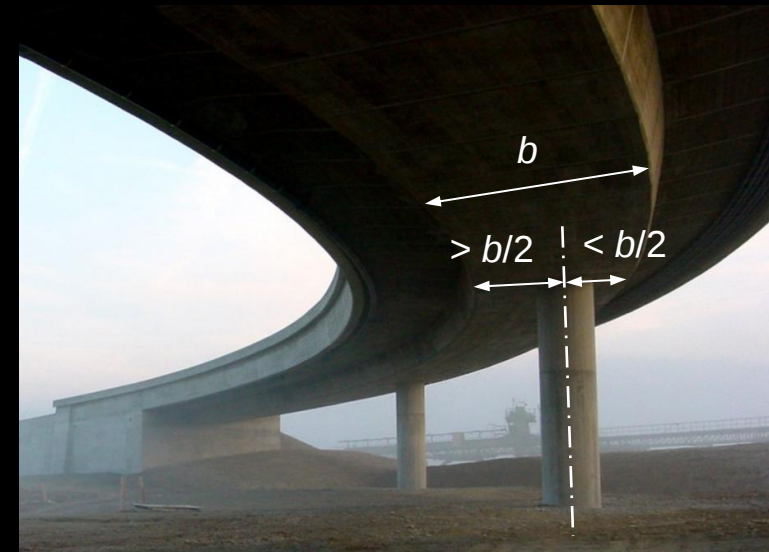
- Particularities in Analysis

- Bending – Torsion interaction (see section on Behaviour)
- Need to consider **centrifugal forces**
- Seismic analysis:
 - Static equivalent seismic analyses are **not applicable to curved bridges**

- Particularities in Design (see section on Design Aspects)

- Particularities in Construction (see section on Construction Aspects)

Superelevation Treatment Alternatives



Special girder bridges

Curved Bridges Behaviour

Curved Bridges – Behaviour

- General:

Torsion in curved girders is induced by **vertical loads**, including those that are **symmetrical about the longitudinal axis** of the bridge (e.g. self-weight).

Bending and torsional moments are coupled due to the curved geometry and their relationship depends mainly on:

- radius of curvature
- bending to torsional stiffness ratio EI / GK
- boundary conditions

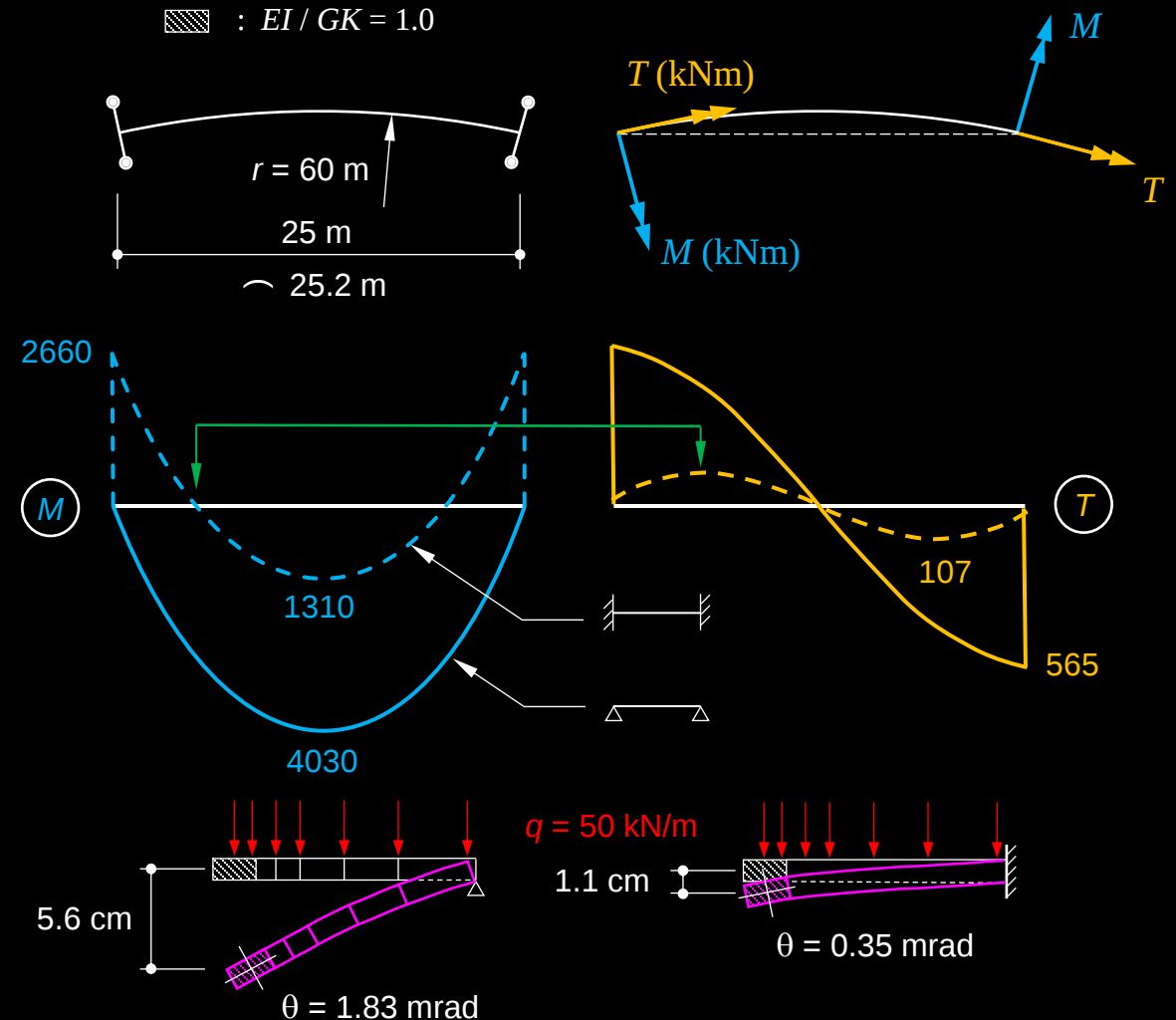
- Example:

Compare the response of a curved girder under uniformly distributed vertical load for:

(a) **pinned** end supports **for flexure**

(b) **fixed** end supports **for flexure**

(twist at the ends is prevented for both cases)



Curved Bridges – Behaviour

Curved Beam Theory

- Consider an **element of a curved beam** with infinitesimal length under uniform vertical load and torque
- Equilibrium** equations of the free body diagram yield:

$$\sum F_z = 0 = -V + V + dV + q \cdot ds$$

$$q \cdot ds + dV = q \cdot r \cdot d\phi + dV = 0 \rightarrow q = -\frac{dV}{rd\phi}$$

$$\sum M_n = 0 = -M + \underbrace{(M + dM)}_{\approx 1} \cos d\phi + \underbrace{(T + dT)}_{\approx d\phi} \sin d\phi - (V + dV) ds - q \cdot ds \cdot \frac{ds}{2} + m_t \cdot ds \cdot \sin d\phi$$

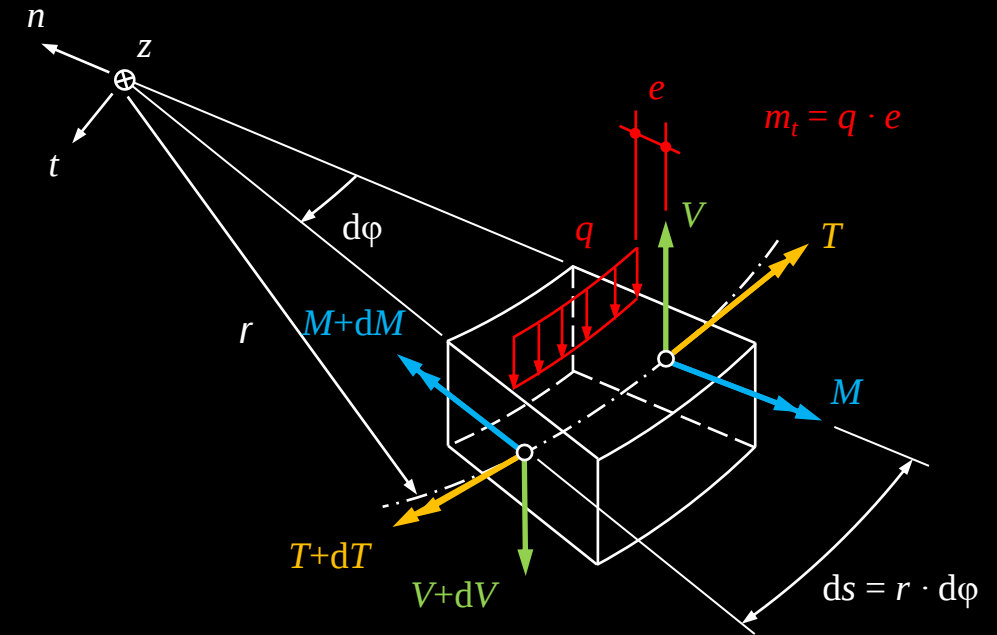
$$-V \cdot ds + dM + Td\phi = -V \cdot r \cdot d\phi + dM + Td\phi = 0$$

$$\sum M_t = 0 = -T + (T + dT) \cos d\phi - (M + dM) \sin d\phi + (V + dV) \cdot ds \cdot \sin d\phi + q \cdot ds \cdot \frac{ds}{2} \sin d\phi + m_t \cdot ds \cdot \cos d\phi$$

$$m_t \cdot ds - Md\phi + dT = m_t \cdot r \cdot d\phi - Md\phi + dT = 0$$

$$\rightarrow \frac{dM}{rd\phi} = V - \frac{T}{r}$$

$$\rightarrow \frac{dT}{rd\phi} = \frac{M}{r} - m_t$$



Curved Bridges – Behaviour

Curved Beam Theory

- Consider an **element of a curved beam** with infinitesimal length under uniform vertical load and torque
- Equilibrium** equations of the free body diagram yield:

$$q = -\frac{dV}{r \cdot d\varphi} \quad \frac{dM}{r \cdot d\varphi} = V - \frac{T}{r} \quad \frac{dT}{r \cdot d\varphi} = \frac{M}{r} - m_t$$

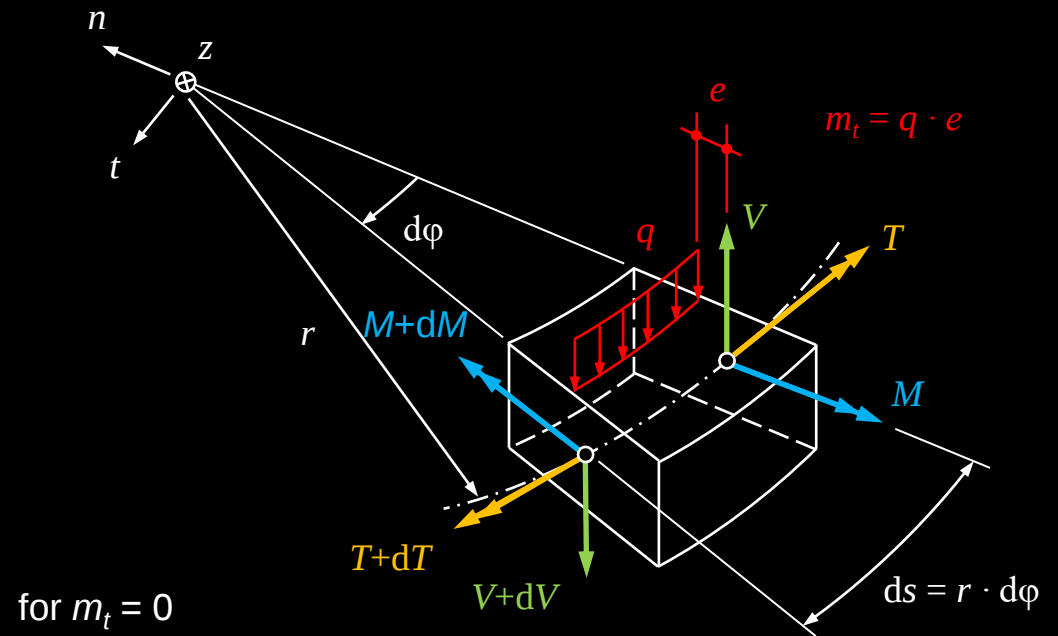
$$q \cdot r = -\frac{dV}{d\varphi} \quad \frac{dM}{d\varphi} = V \cdot r - T \quad \frac{dT}{d\varphi} = M - m_t \cdot r$$

M and T are coupled

For a straight beam ($r \rightarrow \infty$) above equations reduce to the familiar form:

$$q = -\frac{dV}{ds} \quad \frac{dM}{ds} = V \quad \frac{dT}{ds} = -m_t$$

M and T are decoupled



for $m_t = 0$

$$\frac{dT}{d\varphi} = M \rightarrow \Delta T = \int M d\varphi$$

i.e., the variation of torsional moments between two sections is equal to the area of the bending moment diagram (integrated over φ) between those two sections

→ **Important for continuous girders where the sign of the moment diagram changes along the girder**

Curved Bridges – Behaviour

Curved Beam Theory

- Solve system of equilibrium equations:

$$q \cdot r = -\frac{dV}{d\phi} \quad \frac{dM}{d\phi} = V \cdot r - T \quad \frac{dT}{d\phi} = M - m_t \cdot r$$

$$\frac{d^2 M}{d\phi^2} = \frac{d}{d\phi} (V \cdot r - T) = -q \cdot r^2 - M + m_t \cdot r$$

for circular beam ($r = \text{const.}$)

$$\rightarrow \frac{d^2 M}{d\phi^2} + M = m_t \cdot r - q \cdot r^2$$

→ 2nd order inhomogeneous differential equation;
for constant parameters, i.e.

$m_t(\phi) = m_t = \text{const.}$ and $q(\phi) = q = \text{const.}$,
the general solution is:

$$M(\phi) = c_1 \sin \phi + c_2 \cos \phi + m_t r - q r^2$$

Alternatively, the differential equation can be solved iteratively:

1. First approximation for M :

Straight beam with span length $s = r \cdot \phi$ under loading $q - \frac{m_t}{r}$

$$\frac{d^2 M}{ds^2} + M = m_t \cdot r - q \cdot r^2 \rightarrow \frac{d^2 M}{ds^2} = \frac{m_t}{r} - \frac{M}{r^2} - q \approx \frac{m_t}{r} - q$$

2. Iteration of (1) with loading $q + \frac{M}{r^2} - \frac{m_t}{r}$ until convergence is achieved

(often unnecessary because $\left| \frac{M}{r^2} \right| \ll |q|$)

3. Determination of T :

Straight beam with span length $s = r \cdot \phi$ under torque $\frac{M}{r} - m_t$:

$$\frac{dT}{d\phi} = M - m_t \cdot r \rightarrow \frac{dT}{ds} = \frac{M}{r} - m_t$$

Curved Bridges – Behaviour

- Approximate Methods

From the static equilibrium equations:

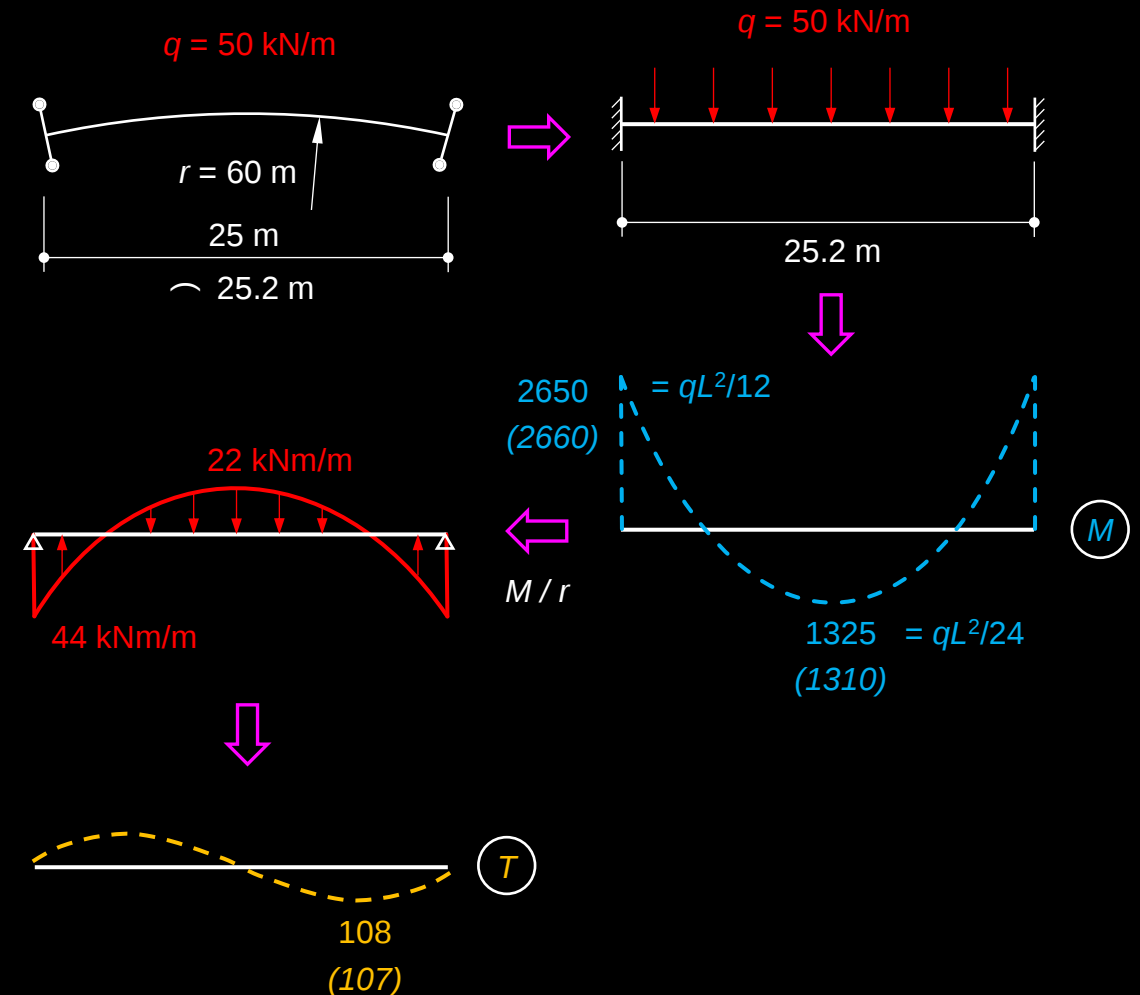
$$\frac{dT}{ds} = \frac{M}{r} - m_t \quad \text{or when } m_t = 0, \quad \frac{dT}{ds} = \frac{M}{r}$$

This is **analogous** to the equilibrium equation for **shear forces** in the case of a **straight beam**:

$$\frac{dV}{dx} = -q$$

Thus, if the **bending moments** along the girder are known, e.g. by analysing the equivalent straight girder, the **torsional moments** may be obtained by loading the equivalent straight girder with a **distributed moment** equal to $(M/r - m_t)$.

This is known as the “**M over r method**” and corresponds to the **iterative solution outlined on the previous slide**. Its applicable for single span and continuous girders (good approximation if the radius is reasonably > than the span).



Curved Bridges – Behaviour

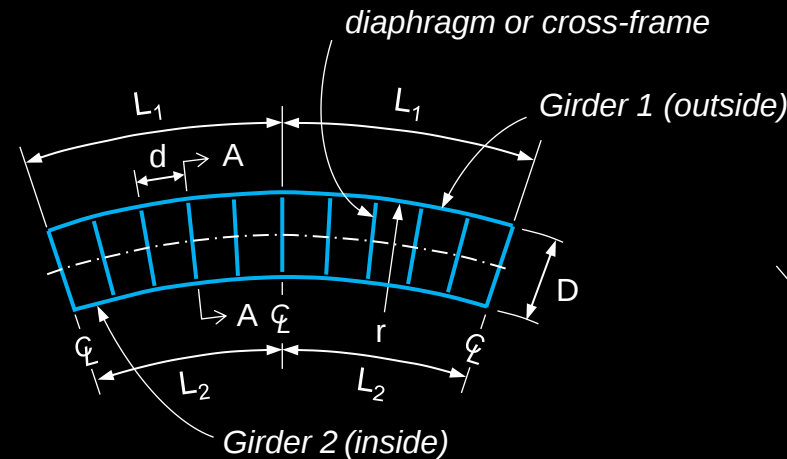
- Approximate Methods

The extension of the M/r method for **open cross-sections** with **multiple girders** is known as the **V-Load method**.

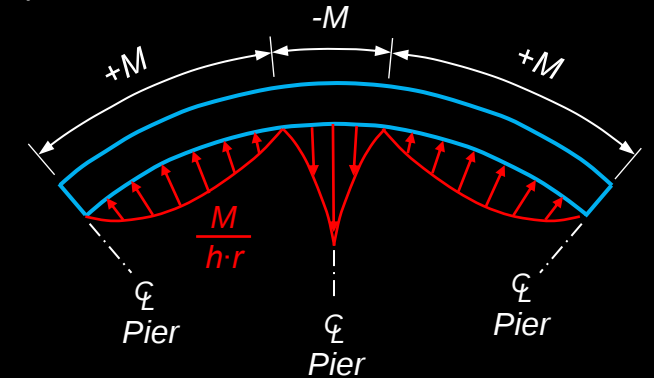
Two-step process:

1. Determine (primary) bending moments under vertical loads applied on straightened girders → consider each girder separately
2. Apply additional fictitious, self-equilibrating forces (**V-Loads**) to the straightened structure so that the resulting internal forces are the same as those in the curved structure.

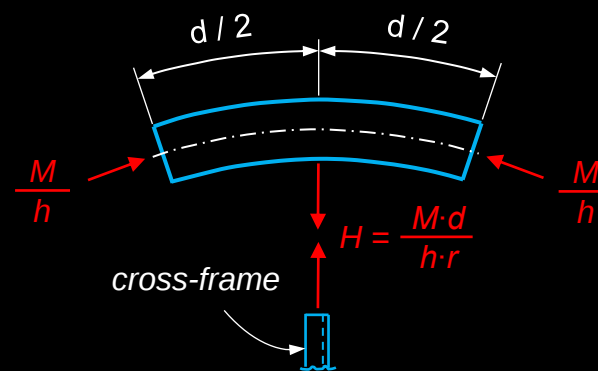
The process is illustrated by considering the case of a system consisting of **two curved girders** continuous over one interior pier connected by **uniformly-spaced, full-depth cross-frames** under uniformly distributed vertical loading.



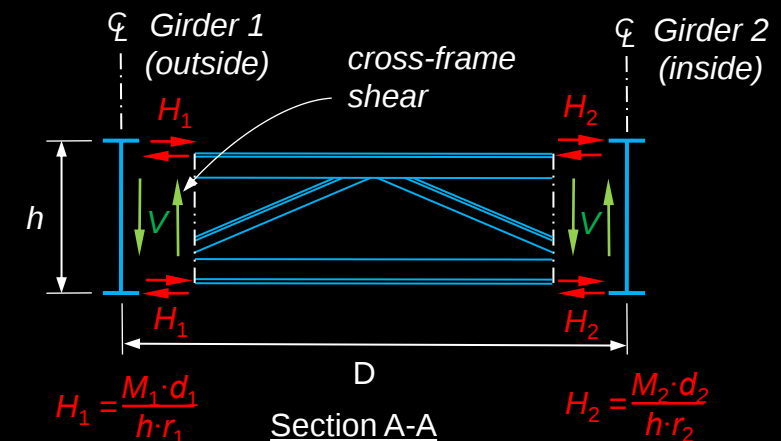
Curved Bridge – Plan View



Radial Components of Top Flange Forces
(Bottom Flange Opposite)



Curved Segment of Top Flange
(Bottom Flange Opposite)



Section A-A

Curved Bridges – Behaviour

- **Approximate Methods**
 - The **M/r and V-Load methods** were widely used to analyse curved girders in the past century, but are nowadays essentially replaced by **grillage or 3D finite element models**.
 - There are still relevant though to **gain insight into the system behaviour** and for preliminary calculations.
- **Spine Models** (see also superstructure, spine models)
 - Suitable for **box and solid cross-section** girders.
- **Grillage Models** (see also superstructure, grillage models)
 - Sufficient for predicting the response of **open cross-section curved girders** for most cases.
 - The **cross-frames** must be modelled through **equivalent beam elements** capturing the behaviour of the cross-frames (including shear deformations).
 - In the case of I-girders, the **restraint of warping** must be taken into account in the estimation of the torsional stiffness (see notes).
 - Locked-in forces due to the **lack of fit between cross-frames and girders** must be considered (see *Skew Bridges*).

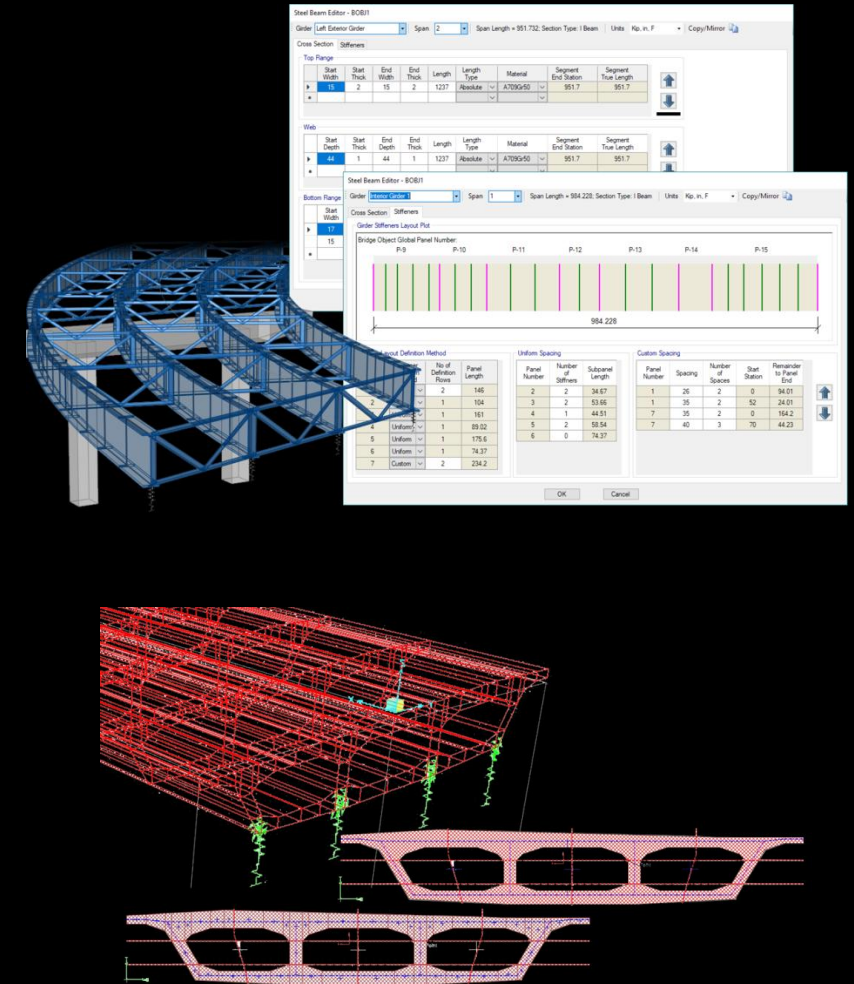


Curved Bridges – Behaviour

- 3D Finite Element Methods

The 3D finite element method is the most general and comprehensive method:

- Allows for modelling of **complex geometries** (sharp and/or variable curvature, skew supports, complex cross-sections)
- Provides **detailed information** on displacement and stresses
- Provides an **accurate analysis** for live load distribution
- Provides displacement and stress information at various **construction stages**
- Provides forces/stresses in **cross frames / diaphragms**
- Permits accurate **fatigue** design
- The detailed analysis gives designers confidence in going to the limits of the code so that the most **economical solution** is achieved (unless **artefacts of the model** cause stress peaks)
- Generally more time consuming than grillage methods
- Response depends on modelling choices and model tends to be **intransparent** → critical review and verification with approximate methods



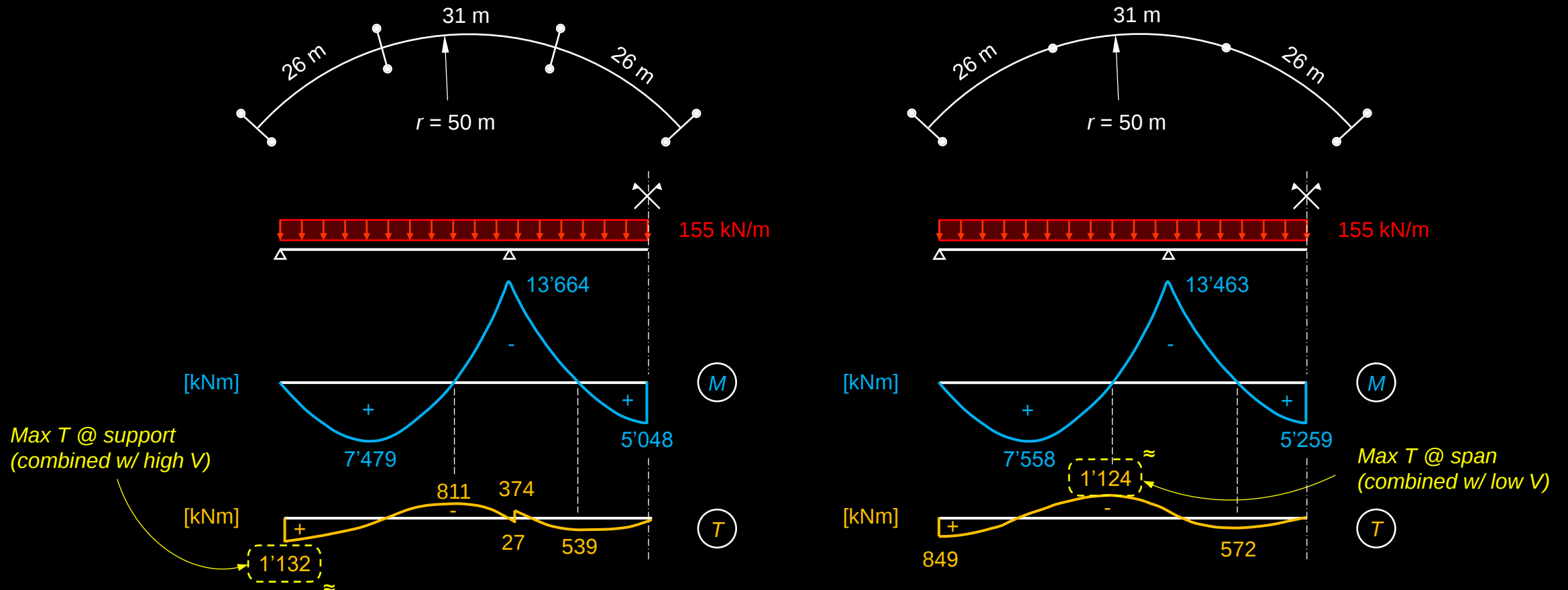
Special girder bridges

Curved Bridges Design Aspects

Curved Bridges – Design Aspects

- Support and Articulation

Due to the interaction between bending and torsional moments, torsional support is not required at all piers, provided that the cross-section has adequate **torsional resistance and stiffness** → closed section



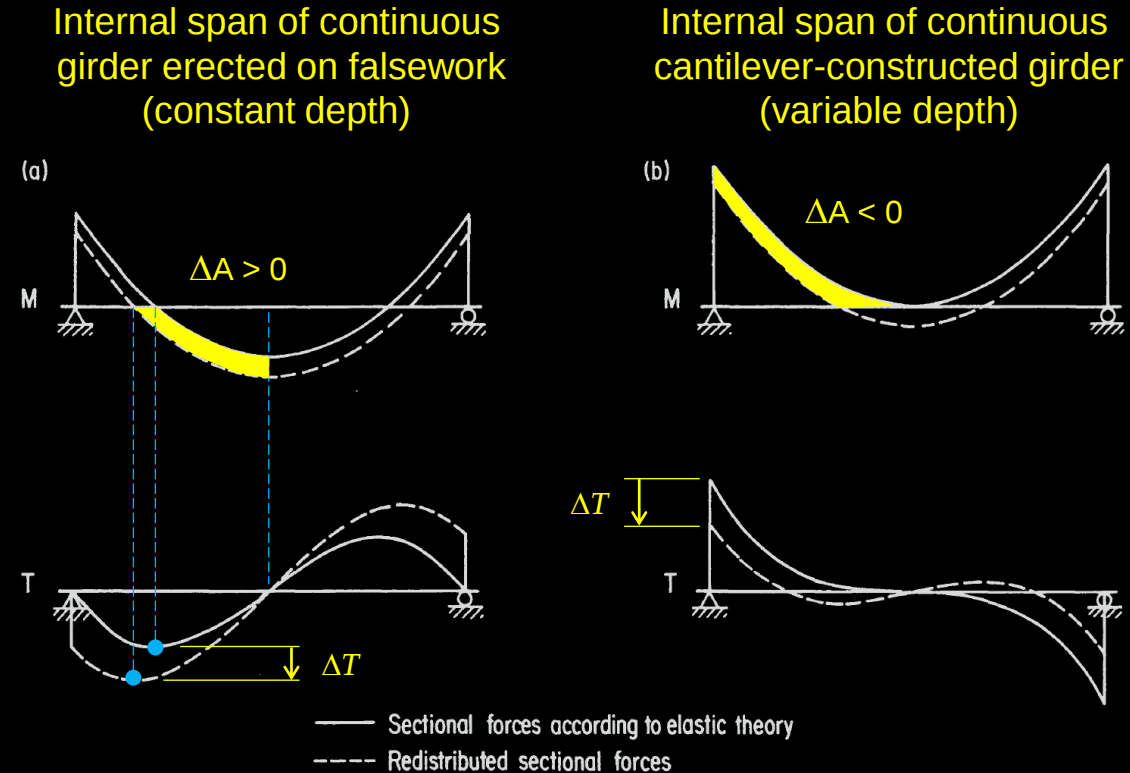
Curved Bridges – Design Aspects

- Concrete girders

Redistribution of bending moments

- When **bending moment redistributions** are considered, e.g. due to **cracking or creep**, the associated **torsional moments must be redistributed** in a corresponding manner (related by equilibrium).
- The effect of bending moments redistribution on the torsional moments depends on the **initial shape of the bending moment diagram**, i.e. construction method.
- The change in the area of the moment diagram will result in a corresponding increase/decrease to the torsional moments.

$$\Delta T = \int M d\phi$$

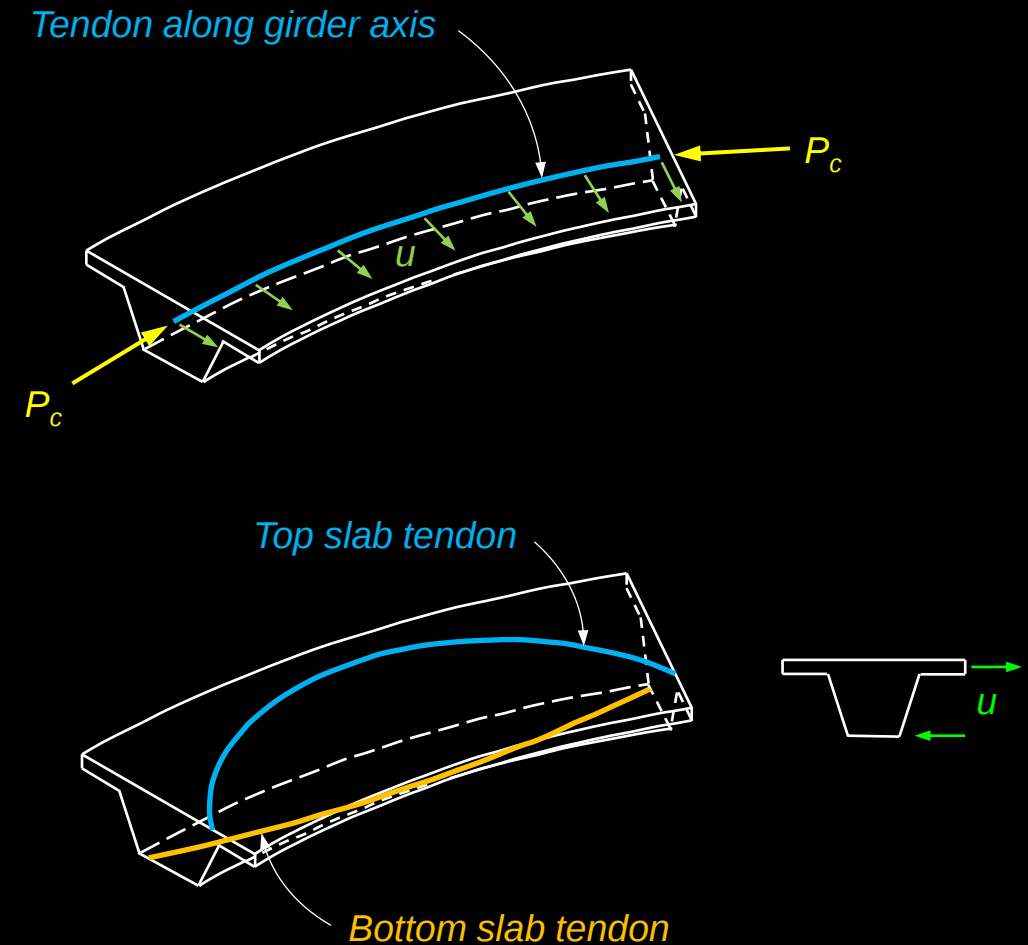


Curved Bridges – Design Aspects

- Concrete girders

Prestressing concepts & tendon layout

- Due to the horizontal curvature of the girder, tendon **deviation forces** are produced in the **horizontal plane**, normal to the axis of the girder. These act **in addition to those due to the** (vertical and horizontal) **profile of the tendon** relative to the axis of the girder
- The horizontal deviation forces are **globally equilibrated by the concrete section (deviation of compression)**, but not locally → restrain tendons against pullout, see next slide.
- Tendons can be arranged to **balance a given torsional moment diagram**:
 - By adjusting the profile of **slab tendons in the horizontal plane** (without altering the effect of the prestressing in bending), see figure
 - By adjusting the profile of web tendons in the vertical plane (reducing the sag of the inner web tendons, i.e., reducing the effect of prestressing in bending)

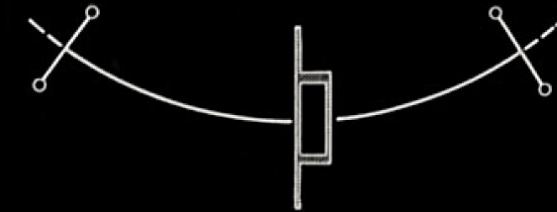


Curved Bridges – Design Aspects

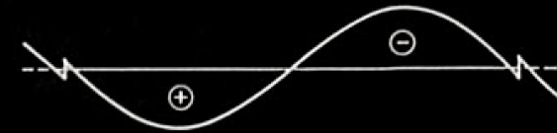
- Concrete girders

Prestressing concepts & tendon layout

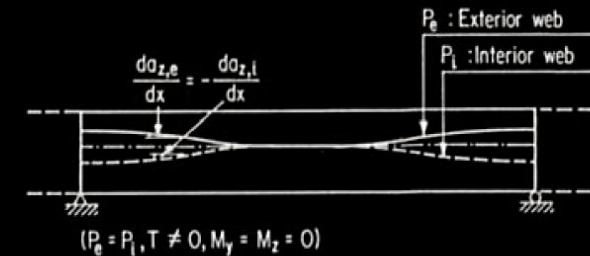
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Continuous curved girder



Torsional Moments due to dead load



Tendon profiles to balance torsional moments



Tendon profile to balance bending moments



Tendon profiles to balance bending and torsional moments

Curved Bridges – Design Aspects

- Concrete girders

Reinforcement and tendon detailing

- Prestressing tendons and mild reinforcement must be **restrained against pull-out** along the concave surfaces of the webs

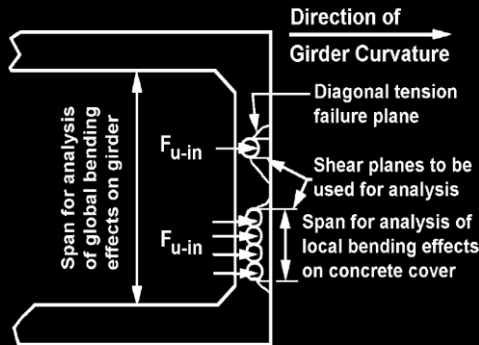


Figure C5.10.4.3.1a-2—In-Plane Force Effects in Curved Girders Due to Horizontally Curved Tendons

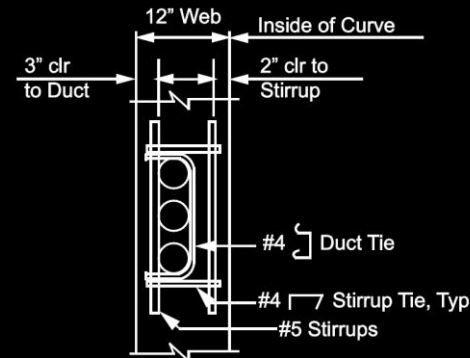
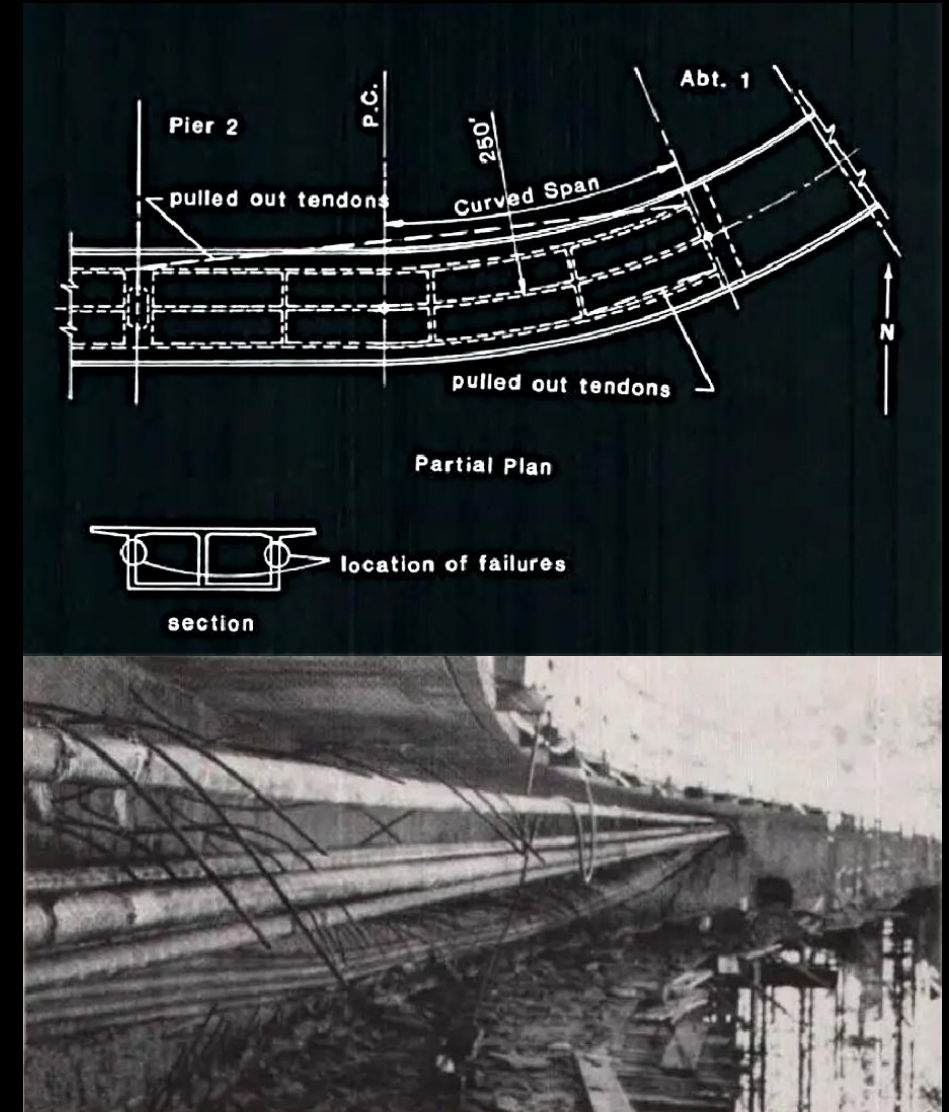


Figure C5.10.4.3.1b-1—Typical Stirrup and Duct Tie Detail



Case study: Las Lomas Bridge, Hawaii (1978):

- All four tendons broke away from the web during stressing of the last tendon.

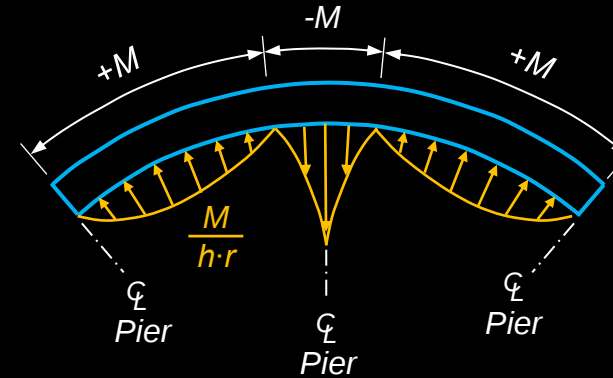
Curved Bridges – Design Aspects

- Steel girders

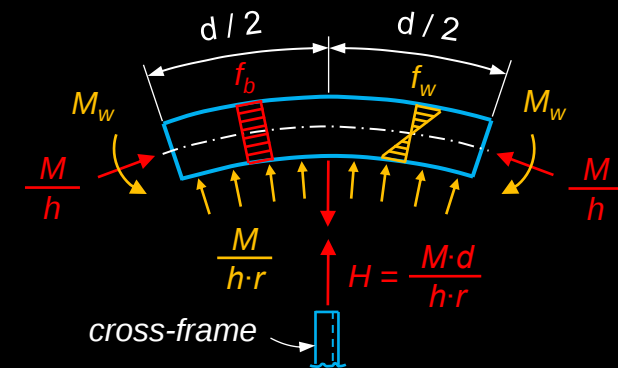
Girder design:

- As discussed, in the case of curved girders, the internal forces from major axis bending are non-collinear
- The non-collinearity results in **virtual radial pressures** which cause **lateral bending** or **warping stresses** in the flanges
- Unless the analysis is performed using 3D finite element methods, where the curved geometry of the flanges is explicitly modelled, these lateral stresses need to be calculated separately
- The magnitude of the warping stresses is controlled through the cross-frame spacing (d)
- In the case of sharp horizontal curvature, the flanges may need to be supported by a lateral truss system (see Slide 15 and notes); alternatively, box girders should be utilised

[Repeated from Slide 27]



Radial Components of Top Flange Forces
(Bottom Flange Opposite)



Curved Segment of Top Flange
(Bottom Flange Opposite)

Primary flange stresses due to **bending**:

$$f_b = \frac{M}{h} \bigg/ (b_f \cdot t_f)$$

where b_f and t_f are the width and thickness of the flange, resp. and h is the depth of the girder

Lateral flange stresses due to **warping**:

$$f_w = M_w / W_f = \left(\frac{M}{h \cdot r} \cdot \frac{d^2}{10} \right) \bigg/ \left(\frac{b_f^2 \cdot t_f}{6} \right)$$

Approximation
(8 ... 12)

Curved Bridges – Design Aspects

- Steel girders

Bracing design:

- Cross-frames

- should be treated as **main structural members** (primary tension members)
- should be **full depth** and be provided between **all** I girders
- **spacing** of cross frames is partially an economical consideration; the closer the cross frames, the lighter the flanges and vice-versa (note though that cross-frames are **relatively high-cost elements**)

A cross frame spacing of 4...6 m is common for curved bridges

- Diaphragms

- **Full depth** solid plate diaphragms should be considered at **supports points (particularly at deck joints)** and should extend continuously across the full width of the bridge

- Lateral Bracing

- should be provided when required for **wind** and **stability** (see also **erection**)
- should be designed for other forces they may attract in maintaining **compatibility** with the girders



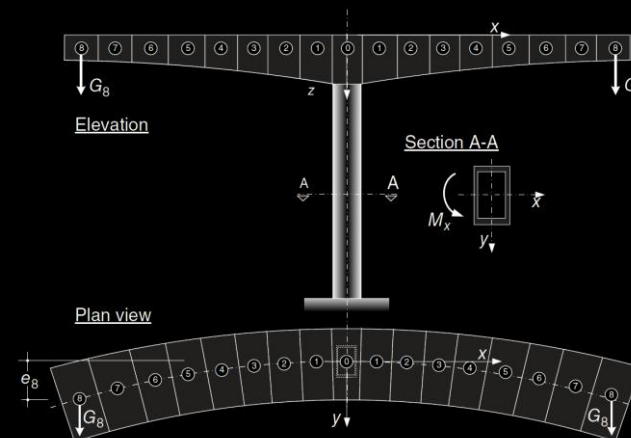
Special girder bridges

Curved Bridges Construction Aspects

Curved Bridges – Construction Aspects

- Construction methods:

- A horizontal alignment with **constant radius** of curvature is required if the bridge is to be erected by the **incremental launching** method.
- Variable radii** of curvature are possible for a concrete bridge built by the **balanced cantilever** or **span-by-span** method
- During the balanced cantilever scheme of a curved box girder bridge, the dead load of the deck induces **transverse bending moments in the piers**, unless some provisional prestressing, internal or external, is adopted



provisional
external
tendon

Curved Bridges – Construction Aspects

- Steel girders
 - Transportation
 - Horizontally curved I girders are typically shipped on their sides → consider **stresses** and **vibrations** (may lead to fatigue issues with long distance transports)
 - Erection
 - **Bracing** is required to control **stresses** and **deformations** (vertical and lateral) under gravity loads, and to provide **stability** (lateral torsional buckling)
 - Effects of **wind** during construction may require **lateral bracing** to control displacements
 - **(Temporary) lateral bracing** is usually required for an erection scheme where strongly curved I-girders are lifted in pairs
 - Control of deformations
 - **Fit-up issues** between girders and cross-frames are not uncommon due to deflections and rotations of curved girders (determining camber is more demanding than in straight girders)

