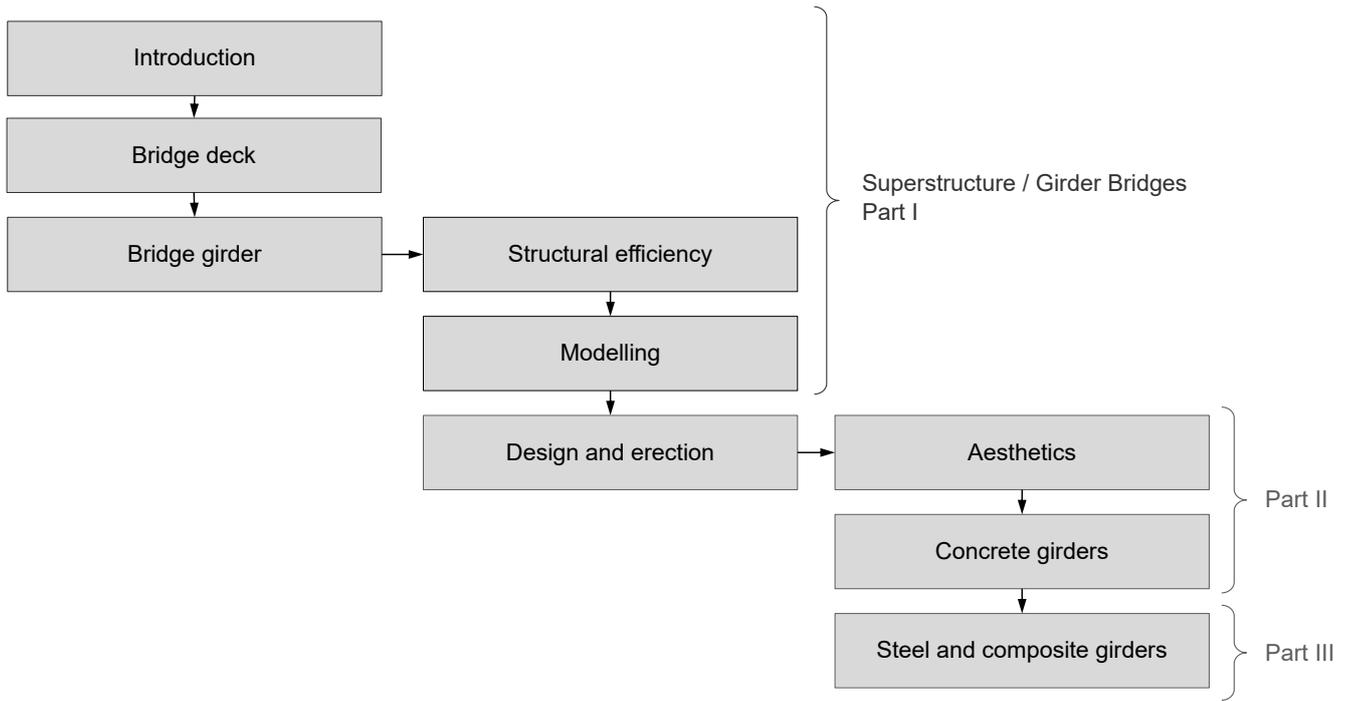


Superstructure / Girder Bridges

Überbau / Balkenbrücken



Superstructure / Girder bridges

Introduction

Introduction: Terminology and content

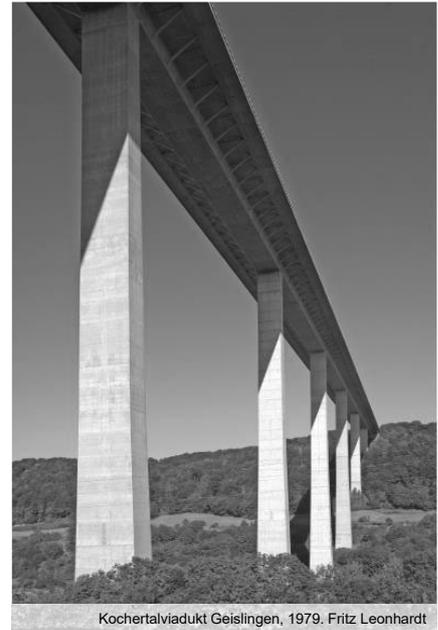
A girder bridge consists of one or several girders, that carry loads primarily by vertical shear and longitudinal bending.

The girders are supported at the bridge ends (abutments) and often also on intermediate supports (piers).

In a girder bridge, the bridge girder is equivalent to the superstructure.

In other bridge types (arches, cable-stayed bridges, ...), additional elements constitute the superstructure together with the girder, that carries the loads to these elements similar as the girder in a girder bridge.

After a brief introduction to girder bridges, this chapter therefore treats bridge girders.



Kochertalviadukt Geislingen, 1979. Fritz Leonhardt

A girder or beam can be defined as a straight, linear, horizontal (or quasi-horizontal) element supported at two or more points along its length (usually at least at both ends), that transfers loads primarily by shear and longitudinal bending.

A girder bridge consists of a single or several girders supported on abutments and piers, and primarily resists the actions through shear and longitudinal bending. Shear forces and longitudinal bending moments are not the only forces acting on a girder bridge, but they are usually dominant over the remaining forces (particularly torsion, etc.).

The stiffness of a girder, as well as its bending resistance, depend on its depth. Thus, a higher depth is required with increasing span of the bridge, as the bending moments also increase as a function of the span.

In a girder bridge, the bridge girder is equivalent to the superstructure. In other bridge types (arches, cable-stayed bridges, ...), additional elements constitute the superstructure together with the girder.

The piers are usually vertical. They can be monolithically connected to the bridge girder or provided with bridge bearings, see *bearing layout and dilatation concept*.

Note: If piers are monolithically connected to the bridge girder, or the bearings on a pier transfer horizontal forces, piers and bridge girder acts as a frame. Still, such bridges are usually referred to as "girder bridges", rather than frame bridges. The latter term is primarily used for bridges monolithically connected to the abutments, or strut frame bridges with inclined "piers".

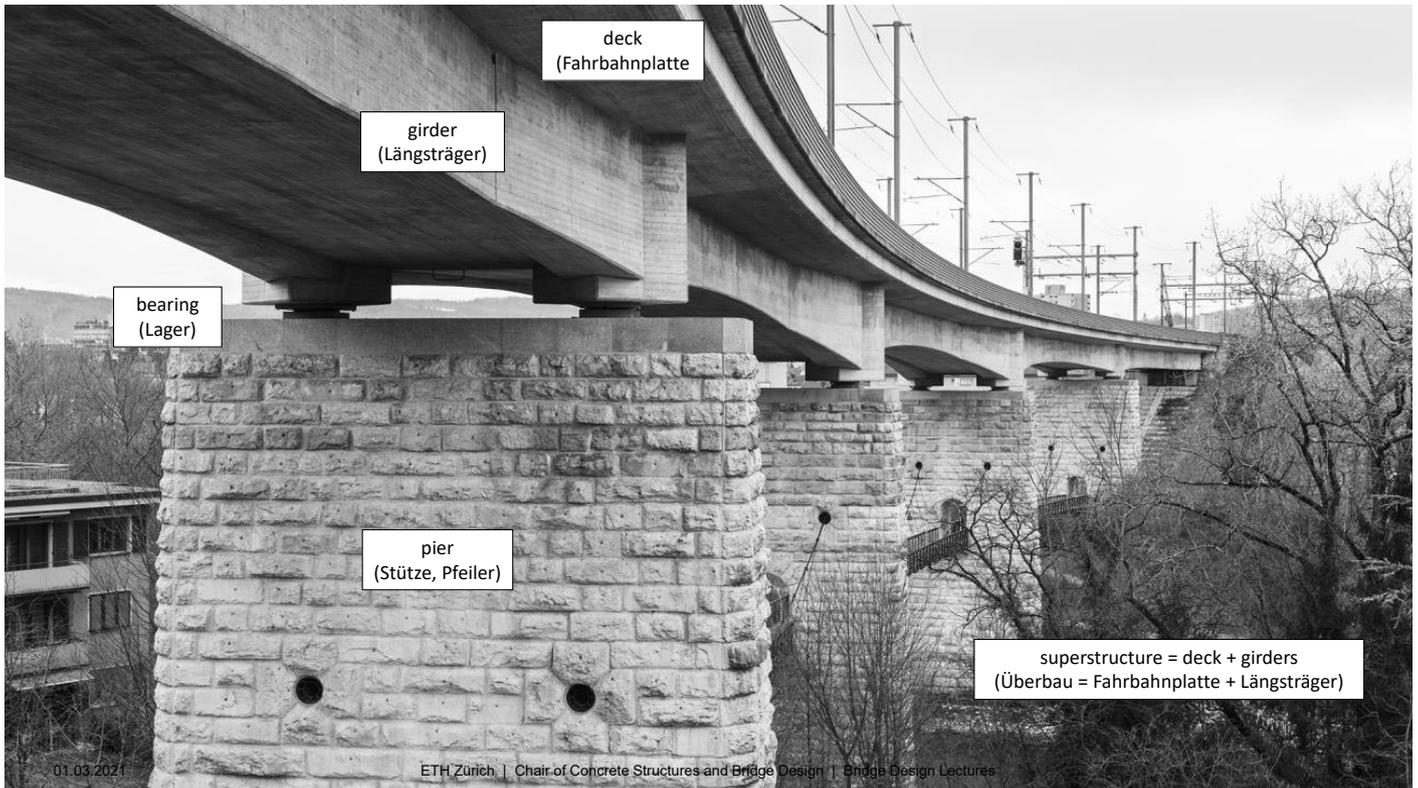


Photo: SBB Aarebrücke Brugg © Georg Aerni

Introduction: Aesthetic quality of girder bridges

Girder bridges are often seen as inelegant. Indeed, there are many dull girder bridges.

However, if carefully proportioned and detailed, they often provide good solutions in situations where a calm and unpretentious, unobtrusive bridge is appropriate.



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Photos: Left Steinbachviadukt Sihlsee, 2014 © dsp Ingenieure + Planer / right top Viaducto de Buñol © Pacadar SL / right bottom Isthmus Viaduct, Spain, 2009. Carlos Fernandez Casado, S.L.

Introduction: Aesthetic quality of girder bridges

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Photos: Left Steinbachviadukt Sihlsee, 2014 © dsp Ingenieure + Planer / Right proposals from other teams participating in the design competition © TBA Kanton Schwyz

Introduction: Advantages and drawbacks of girder bridges

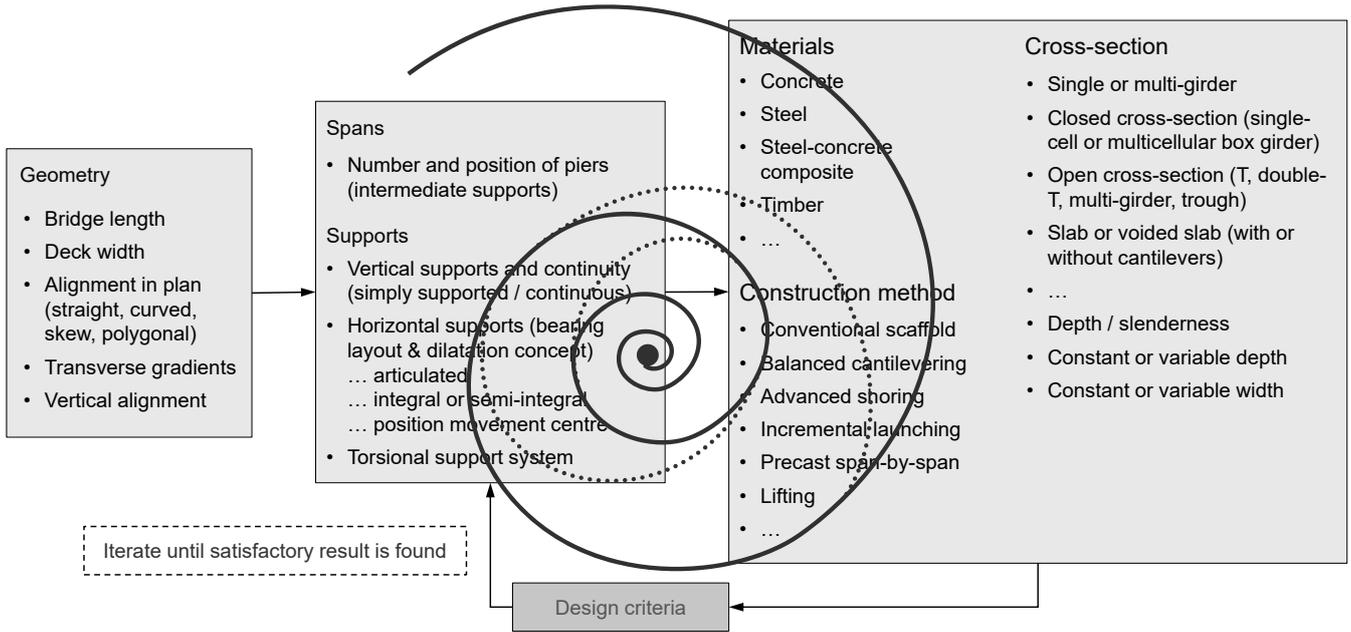
Advantages and drawbacks of girder bridges

- ✓ Economically competitive for short and medium spans (deck significantly contributes to longitudinal load transfer)
 - ✓ Repetitive, simple and efficient construction process (multiple use of formwork etc.)
 - ✓ Standard construction equipment and know-how sufficient
 - ✓ Well suited for prefabrication and fast erection (using special equipment)
 - ✓ Low level of complexity in the design phase
 - ✓ Calm and unobtrusive appearance
-
- Inefficient longitudinal structural system (bending)
 - ... limited span range, particularly for constant depth
 - ... high use of materials
 - Massive and dull appearance
 - Bridge not perceived by users crossing it (if girders are positioned underneath the deck as usual)



Photo: Nouveau tronçon Romont-Vuisternens (rendering) © dsp

Introduction: Design parameters



Note: The figure illustrates the process in the case that a girder bridge has been selected or shall be optimised to compare with other typologies. Generally, as outlined in the chapter of conceptual design, the choice of the typology is even more important than optimising within a given typology.

Introduction: Classification of girder bridges: Span

Girder bridges and bridge girders can be classified by their span, i.e., the distance between supports. This is an important parameter, as it is decisive for the choice of

- suitable / economical construction processes
- the superstructure layout (materials, cross-section, supports, etc.)

In literature, reference is frequently made to “short and medium span” or “long span” bridges. However, there is no clear limit between short, medium or long spans. Often, bridges with a span up to 50...60 m are referred to as «medium span bridges».

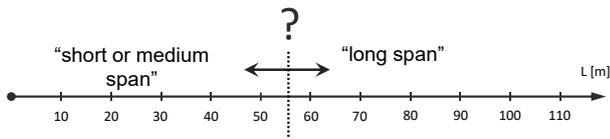


Photo: Ulla viaduct 2015 © IDEAM

Total length 1,621 metres

Spans: 50+80+3x120+225+240+225+3x120+80

Depth: 14 metres

Introduction: Classification of girder bridges: Use

As the different materials predominantly used already indicate, the use of the bridge is also an important parameter.

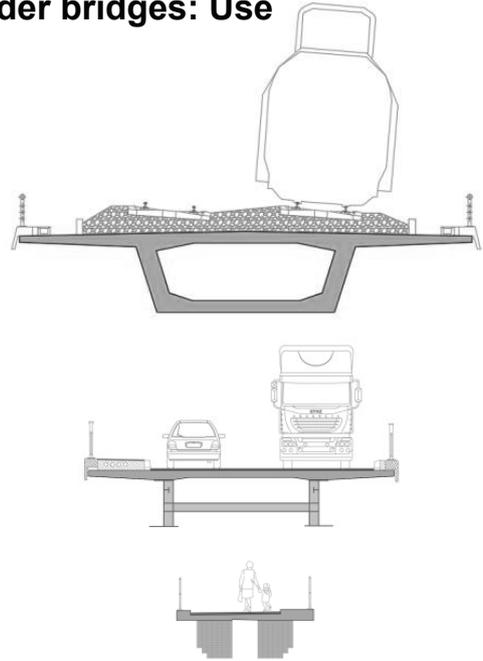
There are significant differences between

- Road bridges
- Railway bridges
- Footbridges

Important differences exist regarding

- Traffic loads (see functions of bridge deck).
- Exposure (e.g. chlorides)
- Functionality and serviceability criteria

These differences are decisive for the conception of a bridge and the bridge girder and explain why there is much more variety in the design of footbridges.

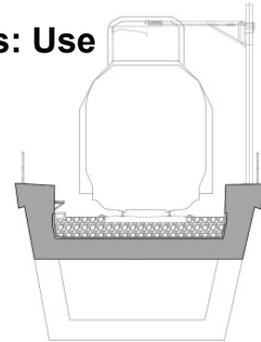
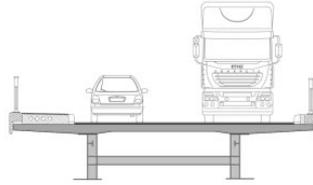


Introduction: Classification of girder bridges: Use

The loads depend heavily on the use of the bridge

→ design of “footbridges” differs significantly from “bridges”

→ focus of lecture: road and railway bridges



Bridge use	Pedestrian / Bicycle	Road ($\alpha_Q = \alpha_q = 0.9$)	Railway ($\alpha = 1.33, \Phi_{dyn} = 1.67$ for typ. deck)
Concentrated loads “ Q ”	low (service vehicles only) [CH: 10 kN]	high / var. position of vehicle axis [CH LM1: $4 \cdot \alpha_Q \cdot (150 + 100)$ kN = 900 kN]	very high / distributed by ballast [CH LM1: $4 \cdot \alpha \cdot \Phi_{dyn} \cdot 250$ kN = 2220 kN, per track]
Distributed loads “ q ”	moderate [CH: 4 kPa, full width]	moderate-high (on limited width) [CH LM1: $\alpha_q \cdot 9$ kPa = 8.1 kPa, 3 m width]	high [CH LM1: $\alpha \cdot \Phi_{dyn} \cdot 80 = 178$ kN/m, per track]
Longitudinal horizontal loads	low	moderate (braking / traction)	high (braking / traction)
Transverse horizontal loads	low	low-moderate (centrifugal)	moderate-high (centrifugal / nosing)
Fatigue	usually irrelevant	moderate (local elements)	highly relevant
Dynamic effects	slender bridges often sensitive to vibrations	included in traffic loads (most codes)	dynamic factor depending on structural element / dynamic analysis for high speed rail
Durability issues	moderate (de-icing)	high (de-icing, heavy load on joints)	low (no de-icing, joints not directly loaded)

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The loads on a bridge deck depend heavily on the use of the bridge. Accordingly, the dimensioning of bridge decks differs significantly according to its use. In particular, the low traffic loads on footbridges allow much lighter solutions for the deck than those used in road and railway bridges.

Notes on table content:

- The concentrated loads indicated are to be applied per deck on road bridges, per track on railway bridges
- The distributed loads indicated on road bridges have to be applied over a width of 3 m (fictitious traffic lane 1). On the remaining surface, a reduced load of $\alpha_q \cdot 2.5$ kPa = 2.25 kPa has to be applied.
- An overload factor has to be applied on some railway lines

Introduction: Classification of girder bridges: Material

A further parameter used for the classification of bridges and bridge girders is the material.

Usual materialisations for road / railway bridges:

- prestressed concrete girders
→ frequently used for economic reasons
- steel-concrete composite girders
→ fast erection, but usually more expensive
- steel girders (orthotropic deck on steel girders)
→ rarely used due to high cost

Timber is rarely used due to limited durability (or environmental issues if CCA-impregnated, see timber decks)

Usual materialisations for footbridges:

- steel and timber used more frequently
- new materials are gaining importance (fibre-reinforced polymers, ultra-high performance fibre-reinforced concrete)

Sir Leo Hielscher bridges, Australia, 2010.
Maunsell Group and SMEC



HS Riudellots de la Selva Viaduct, Spain, 2009.
Fhecor Ingenieros



Archidona viaduct, Spain, 2012. IDEAM



Neckartenzlingen, Germany, 2017. Ing. Miebach

Note: Reference is typically made to the bridge girder (superstructure), since the substructure is almost in all cases made from concrete.

Photos: top left: <http://www.vsl.com/>; top right: Fhecor Ingenieros; bottom left: <http://www.ideam.es/>; bottom right: <https://www.archdaily.com/>

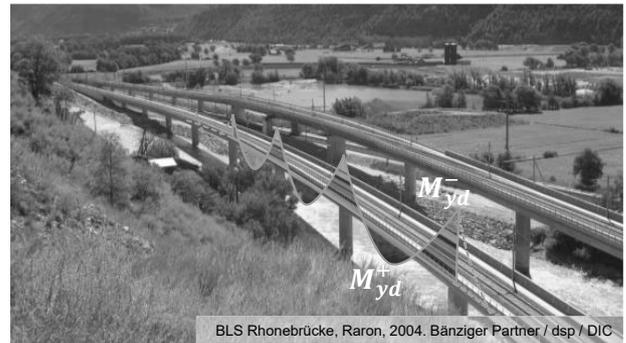
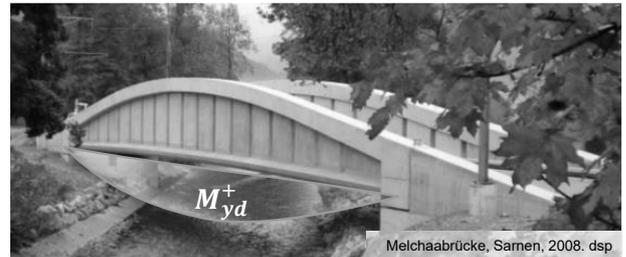
Introduction: Classification of girder bridges: Static system

Another important aspect is the longitudinal static system of the bridge girder.

Bridge girders can be simply supported or continuous over two or more spans.

In multispan bridges, continuous girders are much more efficient and durable, but their erection (if prefabricated) is more complicated.

More details see *strategies for efficient bridge girders and bearing layout and dilatation concept*.

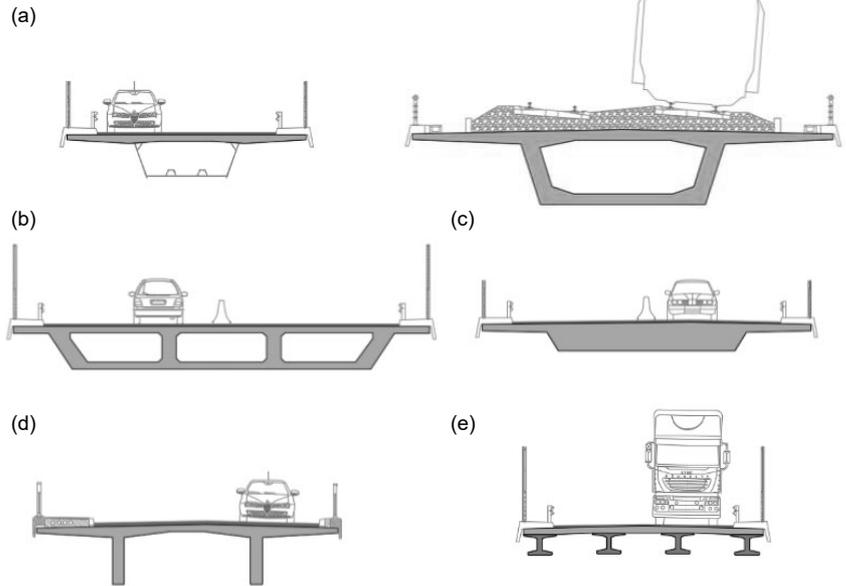


Introduction: Classification of girder bridges: Cross-section

The typology of the cross-section is also useful for classifying girder bridges and bridge girders.

Common solutions are

- (a) Box-girders (single-cell closed cross-sections, concrete, steel or composite)
- (b) Multicell box girders (multicellular closed cross-sections)
- (c) Slabs (solid cross-sections, often tapered or provided with short overhangs to save weight)
- (d) Double-T girders (open cross-sections with two girders)
- (e) Multi-girder deck (open cross sections with several girders, typically steel or prefabricated I-beams)



Note: Other possible solutions, not shown on the slide, include voided slabs or single girders, which are e.g. possible in narrow footbridges with low eccentric traffic loads.

Introduction: Classification of girder bridges: Erection method

Concrete girders are often cast in place using:

- conventional scaffold / falsework
- (balanced) cantilevering
- movable scaffold system (also referred to as advanced shoring)

Girders can also be precast in segments, which are then erected span by span or by (balanced) cantilevering. This is more frequent in concrete girders, but also possible in steel or composite bridges, see photo.

Alternatively, entire bridge girders can be launched or lifted in. The latter is usual for steel or timber girders; concrete girders are often too heavy to be transported as a whole, but can be cast behind an abutment and incrementally launched.

In composite bridges, the steel girders are often lifted in, and the concrete deck is cast on the steel girder(s), without additional scaffold.



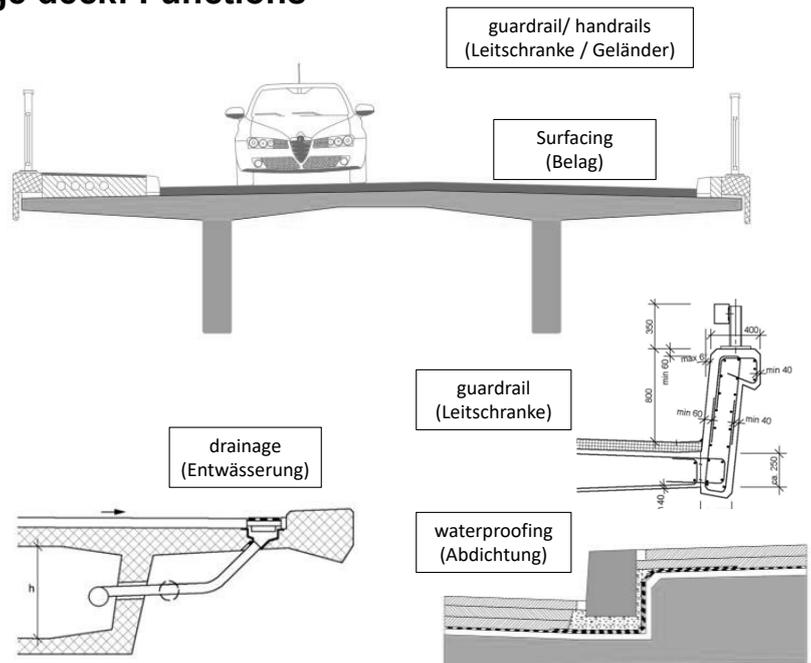
Note: For more details see chapter on erection methods.

Superstructure / Girder bridges

Bridge deck

Bridge deck: Functions

- Carry the traffic loads (and deck self-weight)
- Transfer these loads to the longitudinal girder(s)
- Contribute to the longitudinal stiffness of the girder (acting as flange)
→ consider effective widths (if transverse span is long compared to girder span)
- Integrate all elements required to comply with the functionality of the road, railway or pedestrian way it carries:
 - ... surfacing (or ballast on railway bridge)
 - ... drainage
 - ... noise protection
 - ... crash barriers and handrails
 - ... etc.



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The main function of the bridge deck is to carry the traffic loads (including the self-weight of the deck and superimposed dead loads, i.e. surfacing, waterproofing, guardrails etc.) and transfer these loads to the longitudinal girder(s).

Thereby, the deck acts as a slab, usually carrying loads primarily in the transverse direction. In composite bridges, the concrete deck may be supported not only by the longitudinal girders, but also by transverse girders (cross-beams). If the spacing of the cross-beams is small compared to the separation of the longitudinal girders, the concrete slab carries loads primarily in the longitudinal direction. Either the concrete slab alone, or the slab together with the cross-beams can be designated as "deck" in this case ("ladder deck bridge").

In an efficient cross-section, the deck contributes significantly to the longitudinal stiffness of the girder, acting as flange (usually top flange, bottom flange in trough bridges). In order to activate the deck slab, the connection between longitudinal girders and deck must be able to transfer the required longitudinal shear forces. In steel-concrete composite bridges, shear connectors are provided to this end.

Furthermore, the deck must integrate all elements required to comply with the functionality of the road, railway or pedestrian way it carries:

- ... surfacing (or ballast on railway bridge)
- ... drainage
- ... noise protection
- ... crash barriers and handrails
- ... etc.

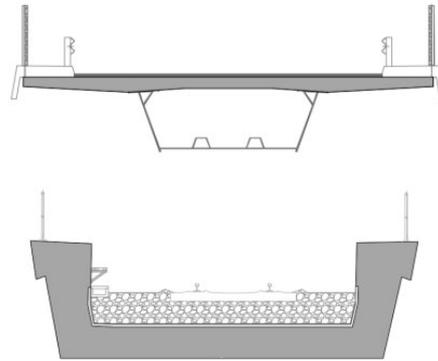
Note: In older ballastless railway bridges and early "composite" bridges with a concrete deck on steel girders, the deck was intentionally separated from the girders such that it did not contribute to the global longitudinal load transfer. However, this is inefficient (see strategies for efficient girder bridges)

Illustration adapted from ASTRA Richtlinie 12004, K04 und K06

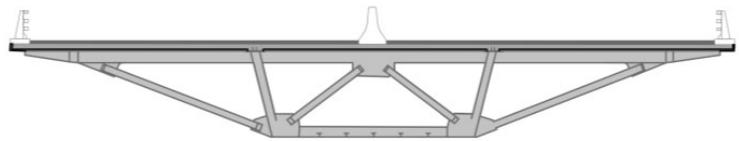
Bridge deck: Concrete deck

Concrete deck

- Slenderness ca. $L/15 \dots L/20$ (L = transverse span between webs or girders, often tapered to save weight)
- Minimum thickness $t_{min} \approx 200$ mm (4 reinforcement layers, concrete cover)
- Usually thicker ($t_m \approx 300$ mm), governed by shear strength (no shear reinforcement) and fatigue checks
- Possible options to save weight in decks with wide cantilevers and/or large internal spans:
 - ... transverse prestressing of deck
 - ... provision of transverse ribs
 - ... provision of additional supports (longitudinal ribs) supported by struts, e.g. on cantilever edge



- ✓ economical solution
- ✓ robust and durable (with proper waterproofing)
- ✓ fatigue usually not problematic
- relatively thick and heavy (7.5 kN/m^2 for $t_m = 300$ mm)



Concrete decks are the standard solution for road and railway bridges, because they are economical and robust. If a proper waterproofing is provided, they are also very durable. Furthermore, fatigue problems are a minor concern in concrete decks. On the other hand, concrete decks are relatively thick and heavy, which may require other solutions if self-weight or clearance requirements are critical.

The minimum thickness of a concrete deck is about $t_{min} \approx 200$ mm. This is required to provide the usual four reinforcement layers with adequate concrete cover.

Usually, the deck is thicker, with an average thickness around $t_m \approx 300$ mm; the slenderness, referred to the transverse spacing of the longitudinal girders, is about $L/15 \dots L/20$. The deck soffit is often tapered to save weight. The deck surface is usually parallel to the roadway surface, such that the surfacing can have a constant thickness.

The main static design criterion for the deck thickness is usually the shear strength, where shear reinforcement should be avoided. Furthermore, fatigue, particularly at the cantilever supports, may be governing.

The deck constitutes a major part of the self-weight of a bridge. In order to save weight, which is highly relevant in long-span bridges, there are several options particularly in decks with wide cantilevers and/or long internal spans:

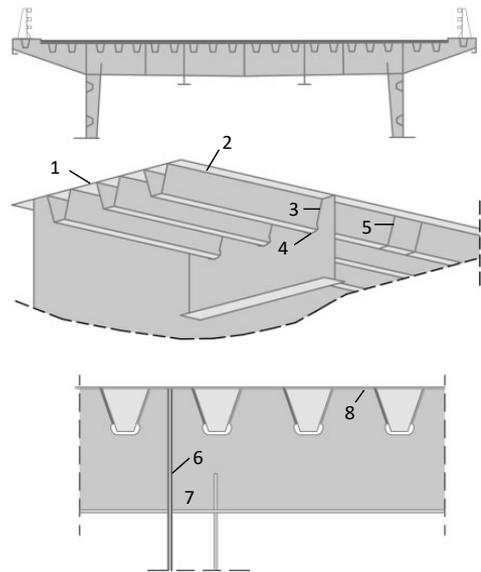
- transverse prestressing of deck
- provision of transverse ribs
- provision of additional supports (longitudinal ribs) supported by struts, e.g. on cantilever edge

Bridge deck: Steel deck

Steel deck

- Orthotropic steel deck, usual in road bridges:
 - ... deck plate $t = 12 \dots 16$ mm
 - ... trapezoidal stiffeners @ 600 mm, approx.
 $H = 300 \times b = 300/150$ mm, $t = 6 \dots 8$ mm
 - ... stiffener span (crossbeams spacing) ca. 4 m
 - Steel plate with or without flat plate stiffeners, for pedestrian and bicycle bridges (not shown)
- ✓ relatively lightweight (ca. 2.5 kN/m²)
 - ✓ thin, saves depth in case of low clearance
 - ✓ large transverse spans possible
 - expensive (high fabrication effort)
 - susceptible to fatigue problems (many welds, proper detailing essential)
 - noise emissions (particularly in railway bridges)

Orthotropic steel deck (OSD):



Legend

- 1) deck plate
- 2) welded connection of stiffener to deck plate
- 3) welded connection of stiffener to web of crossbeam
- 4) cut out in web of crossbeam
- 5) splice of stiffener
- 6) splice of crossbeam
- 7) welded connection of crossbeam to main girder or transverse frame
- 8) welded connection of the web of crossbeam to the deck plate

Orthotropic steel decks are frequently used in long-span bridges, because they are much lighter than concrete decks. They are also thinner than concrete decks (the stiffeners can be provided over the same depth as the longitudinal girders) and therefore, orthotropic steel decks are also a good solution if clearance requirements are critical, i.e., the total depth of the bridge girder, from soffit to top of surfacing, needs to be minimised. The durability depends on the quality of the waterproofing, even more than in concrete decks (if the waterproofing is damaged, the steel is directly exposed to chlorides).

On the other hand, orthotropic steel decks are much more expensive than concrete decks, primarily because of the high amount of welding labour involved. Furthermore, they are susceptible to fatigue problems; it is therefore important to carefully detail all the connections of deck plate, stiffeners, crossbeams and longitudinal beams.

Today, orthotropic decks of road bridges usually consist of a deck plate ($t = 12 \dots 16$ mm), with trapezoidal stiffeners separated axis to axis about 600 mm. The stiffeners ($t = 6 \dots 8$ mm) are about usually 300 mm deep and 300 mm (top) to 150 mm (bottom) wide. They span between the crossbeams, which are usually spaced at about 4 m. In earlier bridges, flat plate or T-shaped stiffeners were also used, but these solutions are less efficient and rarely used today.

In footbridges, deck plates with or without flat plate stiffeners are used more often, because they are easier to produce.

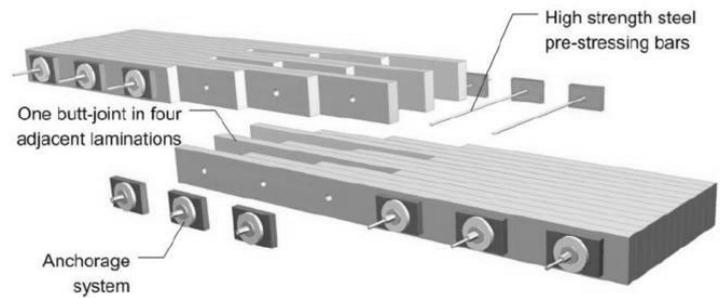
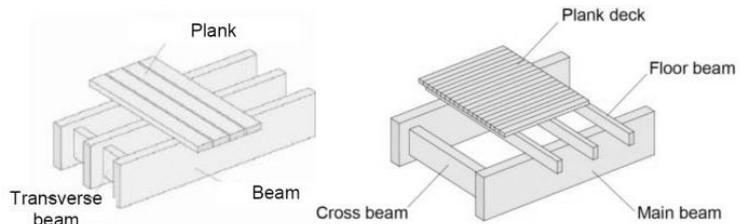
Note: Recent research has shown that the fatigue life of orthotropic decks can be substantially increased using a thin layer of ultra-high performance fibre reinforced concrete as surfacing (or protecting layer below the surfacing). This system could be very promising for long-span bridges.

Bridge deck: Timber deck

Timber deck

- Detailing depends on use (loads, exposure) and local preferences
- Possible solutions:
 - ... transverse planks (US: glulam) on longitudinal girders
 - ... longitudinal boards on transverse floor beams
- Additional wear planks (→ protection, roughness) or membrane and surfacing (road bridges)
- transverse prestressing for biaxial load transfer (account for prestress losses due to temperature and humidity variations)

- ✓ lightweight
- ✓ appealing to pedestrian use
- ✓ sustainability ... unless impregnated
- limited load capacity
- predominantly uniaxial load transfer
- limited durability (unless protected or impregnated → severe environmental issues, see notes)



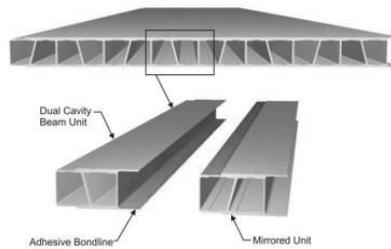
The use of CCA (chromated copper arsenate) and similar wood impregnation products like oil-tar creosotes (“carbolineum”) is generally prohibited in most countries, but allowed for selected applications due to lacking alternatives. In Switzerland, they may be used for impregnating railway sleepers and avalanche protection components; in Scandinavian countries the use of CCA in bridges is permitted. The main issue of CCA-impregnated timber is its decommissioning (toxicants particularly from arsenic).

Figures: Crocetti, R., “Timber bridges: General issues, with particular emphasis on Swedish typologies”, Internationals Holzbauforum Garmisch, 2014.

Bridge deck: GFRP deck

GFRP deck

- Pultruded GFRP profiles, assembled with adhesives and/or clamps
- Beam units for larger spans (usually transverse direction) or planks



- ✓ ultra-lightweight
- ✓ durable (no corrosion)
- lack of standardisation
- lacking long-term experience (fatigue, UV exposure)
- primarily uniaxial load transfer (usually)
- brittle material behaviour
- expensive



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Figures: top left: S. Rizkalla, M. Dawood, M. Shahawy, "FRP for Transportation and Civil Engineering Infrastructure: Reality and Vision", 50 Years of Interstate Structures: Past, Present, and Future, TRP, 2006

Others: <https://fiberline.com/cases/cases-construction/bridges/german-state-highway-agency-installs-grp-bridge/>

Bridge deck: Design

The deck slab is usually modelled as a slab supported by

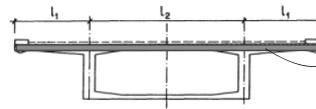
- longitudinal girders or webs
- cross-beams if they support the deck

Linear elastic FE slab analyses are standard today for the design of bridge decks. Often, rigid supports are assumed, but a refined analysis may be appropriate in special cases (e.g. thick slabs on slender cross-beams).

The rotational restraint of the supports depends on the type of girder. For concrete girders, the boundary conditions shown in the figure (adapted from Menn, 1990) may be assumed. Steel girders and cross-beams usually do not provide significant fixity (deck much stiffer than webs) as also shown in the figure.

For the investigation of transverse bending of the longitudinal girders, the support moments obtained from the deck slab analysis are applied to the box girder and the webs of open cross sections, respectively, and superimposed to transverse bending of the cross-section due to other causes (torque introduction), see *bridge girder*.

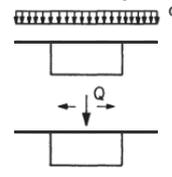
Deck model (constant depth for analysis)



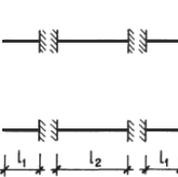
$$\frac{\partial^2 m_x}{\partial x^2} + 2 \frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} + q = 0$$

design of slabs see e.g. courses «Stahlbeton II», «Flächentragwerke», ...

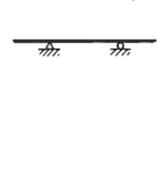
Deck on box girder



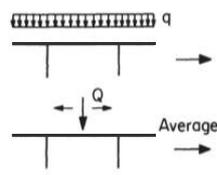
... concrete box



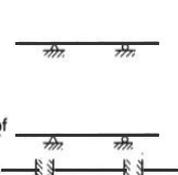
... steel box (composite)



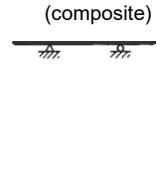
Deck on double-T beam



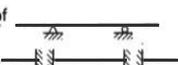
... concrete beams



... steel beams (composite)



Average of



Illustrations adapted from C. Menn, Prestressed Concrete Bridges, 1990.

Bridge deck: Design

In the analysis of the deck slab, concentrated loads are often spread as shown in the upper figure. Strictly speaking, this spreading would require reinforcement, and according to SIA 162, only a spreading in the surfacing should be considered (see AGB Report 636).

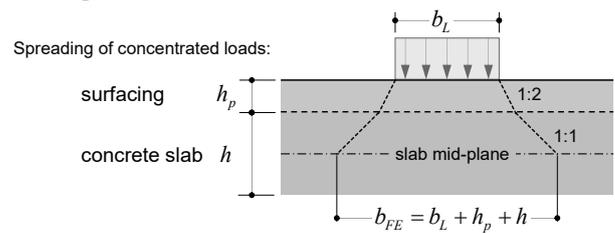
In preliminary design, bending moments in the deck may be estimated:

- assuming a spreading under 45° in-plane for concentrated loads (lower figure)
- distributed loads are transferred in the transverse direction

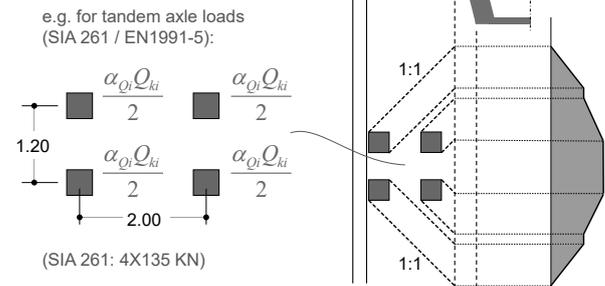
Note that this simplified treatment of concentrated loads

- presumes sufficient longitudinal resistance (usually ok)
- is not suitable for fatigue verifications
- is not suitable (potentially unconservative) for shear strength verification

According to SIA 262, the shear capacity depends on the utilisation of the bending resistance $m_d/m_{Rd} \rightarrow$ see AGB Report 636 (notes) for verification in final design (notes).



Estimate of cantilever clamping moment (transverse):



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According to SIA 262, the shear capacity depends on the utilisation of the bending resistance, i.e. the ratio m_d/m_{Rd} . Accordingly, the load-case producing the maximum shear is not necessarily the governing one.

Since concentrated loads acting closer than $2d$ from the support edge may be linearly reduced (SIA 262), the following load combinations should be checked according to AGB Report 636:

- resultant of concentrated loads acting at distance $2d$ from support edge (outer web face)
- edge of concentrated loads (without spreading in surfacing or concrete) at $2d$ from support edge

In either case, the verification is carried out in the (nominally) critical section at a distance of $d/2$ from the supported edge. A FE-analysis is recommended to determine the shear forces and bending moments.

Reference: M. Fernandez-Ruiz, R. Vas Rodrigues, A.Muttoni: "Design and verification of bridge deck slabs for highway bridges", AGB Report Nr. 636, 2009.

Bottom illustration adapted from Vogel, Lecture notes „Brückenbau“, 2019

Bridge deck: Design

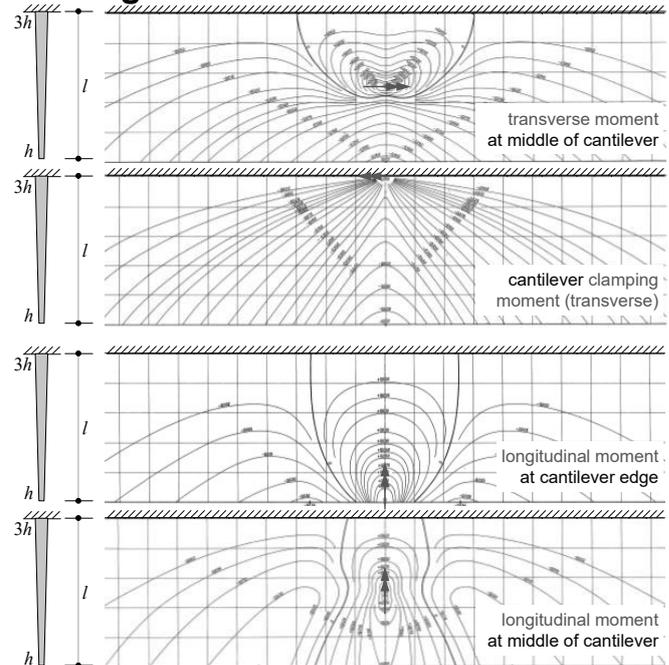
Before the advent of affordable, user-friendly FE-analyses of slabs, determining the internal actions caused by concentrated loads was challenging.

Influence surfaces (published by Homberg, Pucher and others, see notes) were used to this end until few decades ago. These show

- the bending moment (or shear force)
- at a specific point of a slab
- in a specific direction of a slab
- for a unit load (sometimes to be divided by 8π)
- assuming linear elasticity

The design actions are obtained from the influence surfaces by integration (using approximations, often by eye). Homberg's publications include evaluations for the load models used at the time of publication.

The figures on the right show influence surfaces for bending moments in an infinitely long cantilever with variable thickness (adapted from Homberg, 1965).



Influence surfaces were published as charts and tables e.g. by

- A. Pucher: Einflussfelder elastischer Platten / Influence Surfaces of Elastic Plates, 1951
- H. Homberg und W. Ropers: Fahrbahnplatten mit veränderlicher Dicke (2 Bände), 1965

In many cases (all illustrated ones, except clamping moment) the bending moment at the point of load application is infinite according to elasticity theory. Integration, however, yields a finite value (infinite bending moment at one point = infinitely small contribution to area of integration).

Illustrations: Adapted from Homberg and Ropers [1965]

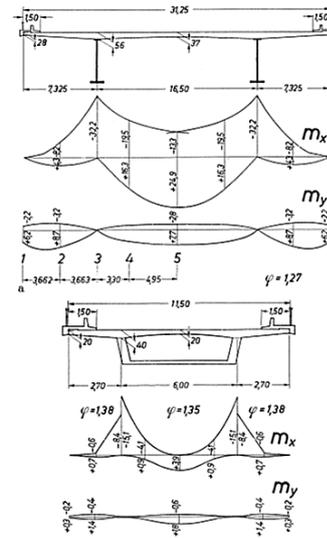
Bridge deck: Design

When designing using influence surfaces, the distribution of bending moments between the points covered in the charts need to be accounted for.

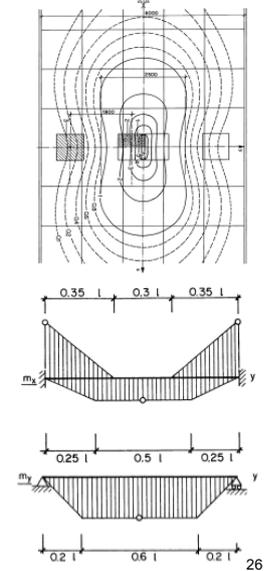
The figures on the right show possible assumptions to this end.

From today's perspective, they are obsolete for design, as FE-analyses of slabs yield this information much more efficiently. They are still useful to get an intuitive understanding, e.g. regarding the possible curtailment of reinforcement.

Transverse variation of bending moments (from Homberg+Ropers):



Influence surface for interior slab and transverse variation of bending moments (from Menn)



Illustrations: Menn[1990]

Influence surfaces were published as charts and tables e.g. by

- A. Pucher: Einflussfelder elastischer Platten / Influence Surfaces of Elastic Plates, 1951
- H. Homberg und W. Ropers: Fahrbahnplatten mit veränderlicher Dicke (2 Bände), 1965

Superstructure / Girder bridges

Bridge girder – Structural efficiency

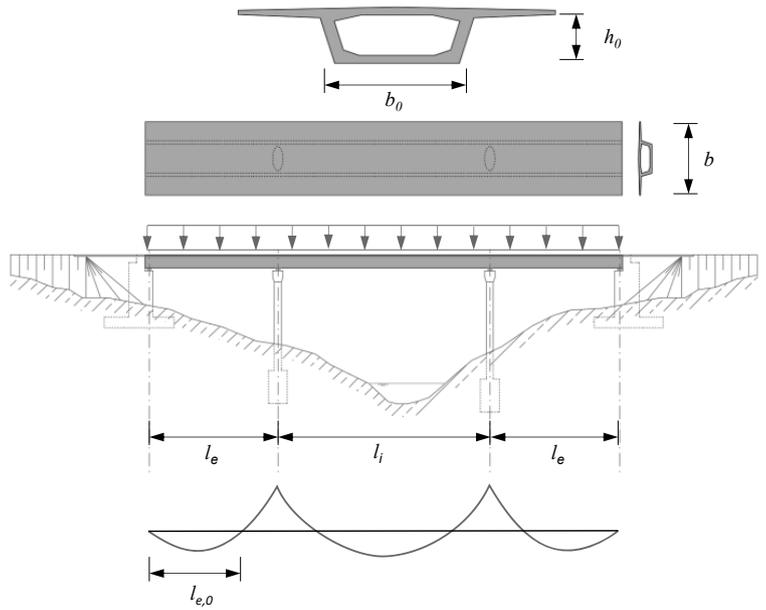
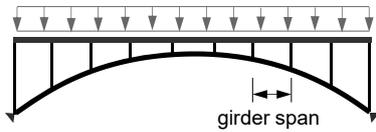
Bridge girder – Structural efficiency: Dominant internal action

The bridge girder transfers loads longitudinally to its supports (piers, abutments or elements of the superstructure supporting the girder).

In girder bridges, the spans l are significantly longer than the depth h_0 and the width b_0 of the girder. Hence, longitudinal bending is governing the design.



Note: Effective girder spans are typically much shorter in bridges types where the superstructure consists of more elements than the girder, e.g. arch bridges:



Bridge girder – Structural efficiency: Dominant load

Self-weight of the girder = large portion of the total load, bending moments due to self-weight increase with the span

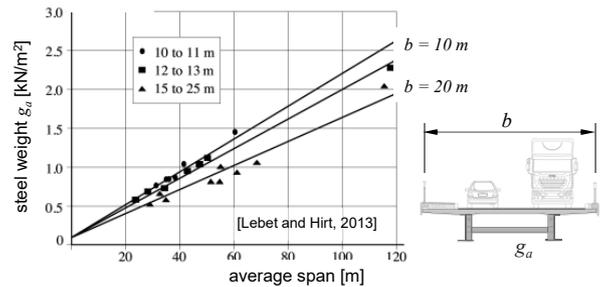
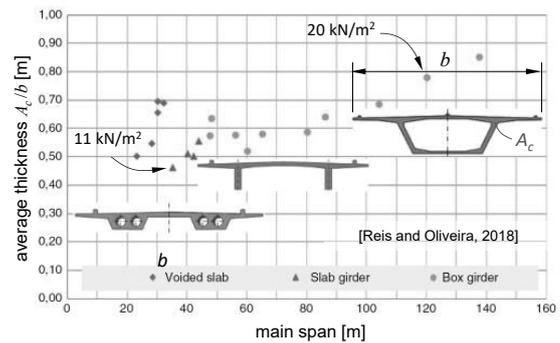
- deeper girders (= more weight) required with increasing spans
- self-weight is highly relevant

Equivalent girder thickness $t_{eq} = A_c/b$ (cross-section divided by deck width) for recent concrete girder bridges (upper figure):

- $t_{eq,min} \approx 0.45$ m at short spans → $0.45 \cdot 25 = 11$ kN/m²
- $t_{eq} > 0.80$ m for long spans → $0.80 \cdot 25 = 20$ kN/m²
- moderate increase since the deck (ca. $0.3 \cdot 25 = 7.5$ kN/m²) is always required; weight increase without deck more pronounced

Steel weight of composite girders (with concrete deck, lower figure):

- minimum ca. 0.75 kN/m² at short spans
- more than 2.2 kN/m² for long spans
- pronounced increase but steel weight = only 10...30% of the weight of the concrete deck



Except for very short spans, the self-weight of the girder accounts for a large portion of the total load in a girder bridge. As bending moments increase with the span, more depth is required to provide adequate strength and stiffness with increasing spans. The diagrams on the right illustrate the increase of the self-weight with the span:

The upper figure shows the average girder thickness A_c/b (concrete cross-section divided by the deck width) for recent prestressed concrete girder bridges. Accordingly, the self-weight of the girder increases from a minimum of about $0.45 \text{ m} \cdot 25 \text{ kN/m}^3 = 11 \text{ kN/m}^2$ at small spans to more than $0.80 \text{ m} \cdot 25 \text{ kN/m}^3 = 20 \text{ kN/m}^2$ for spans exceeding 100 m. The increase is not very pronounced since the deck is always required (ca. $0.3 \text{ m} \cdot 25 \text{ kN/m}^3 = 7.5 \text{ kN/m}^2$); without the deck, the increase is much more significant (from 3.5 to 12.5 kN/m², relevant for large spans).

The lower figure illustrates the steel weight of composite bridge girders with different deck widths. Again, the steel weight increases with the span, from about 0.75 kN/m² at short spans to 2.2 kN/m² for long spans. However, the steel weight corresponds only to about 10...30% of the weight of the concrete deck (ca. $0.3 \text{ m} \cdot 25 \text{ kN/m}^3 = 7.5 \text{ kN/m}^2$).

Bridge girder – Structural efficiency: Static system

The efficiency of a girder bridge primarily depends on

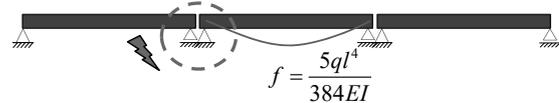
- the static system
- the cross-section and its materialisation
- the erection process

Simply supported girders can be erected very fast, particularly if prefabricated girders are used, and are often the cheapest solution (neglecting service life costs).

Therefore, despite many drawbacks (see figure), simply supported girders have been used in countless bridges, and are still popular in many countries worldwide.

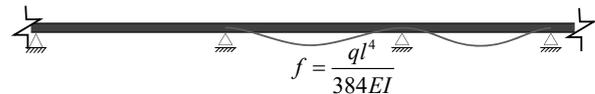
Continuous girders are statically much more efficient than simply supported girders, and have further advantages (see figure).

Simply supported girders:



- ✓ fast and simple erection (by lifting in)
- high maintenance demand
- lack of durability (mainly in road bridges)
- unsatisfactory user comfort (road bridges)
- lack of robustness

Continuous girder:



- ✓ high stiffness → higher slenderness possible
→ less material consumption
- ✓ activation of negative bending resistance
- ✓ lower maintenance demand
- ✓ higher durability
- more complicated construction

The efficiency of a girder bridge primarily depends on

- the static system
- the cross-section and its materialisation
- the erection process

Here, only the static system and the effect of varying the cross-section along the girder axis are initially considered. The erection process is treated in a separate chapter, and the cross-section is dealt with in the following slides.

The primary advantage of simply supported girders is the erection process: They can be lifted in very fast, and are therefore often economical (if maintenance costs are disregarded). Therefore, this system has been, and still is, very successful worldwide.

Obviously, continuous girders are statically much more efficient than simply supported girders. They are much stiffer, such that a higher slenderness is possible. Furthermore, continuous girders are superior regarding serviceability, durability and robustness (see bearing layout and dilatation concept).

Note: In railway bridges, providing a row of simply supported girders allows avoiding rail expansion devices. Since no de-icing salts are used on railway bridges and the bridge expansion joints are not directly driven over (protected by ballast or ballastless track), this solution may be interesting since rail expansion devices are expensive, increase the risk of derailment and require maintenance. More details are given in the chapter on bearing layout and dilatation concept.

Bridge girder – Structural efficiency: Variable depth

The depth of the girder is both

- beneficial (higher stiffness and bending resistance) as well as
 - harmful (higher self-weight and thus bending moments)
- maximise depth while minimising bending moments
 → adjust depth to required bending resistance

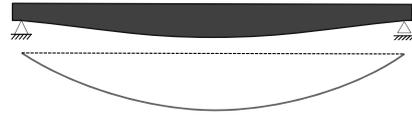
Simply supported girders

- high bending moments only in span
- reduce depth near the supports
 → limited increase in efficiency (reduced self-weight near supports has little effect on the bending moments)

Continuous girders

- highest bending moments over intermediate supports
- reduce depth at midspan
 → pronounced increase in efficiency (self-weight is reduced where it causes high bending moments)

Simply supported girder:



- ✓ maximum depth where bending moments are highest
- full weight where it causes high bending moments

Continuous girder:



- ✓ maximum depth where bending moments are highest
- ✓ reduced weight where it causes high bending moments
- positive (sagging) bending moments may become governing, particularly in end-spans (traffic loads), if depth is reduced too much
- more expensive to build, but economical for larger spans or in case of specific requirements (clearance, ...)

The depth of the girder is beneficial for the structure, as it increases its stiffness and bending resistance, but at the same time harmful, since it causes a higher self-weight and thereby higher bending moments.

A high bending resistance is not required over the supports of a simply supported girder. Therefore, reducing the depth near the supports results in an increased efficiency of the simply supported girder. The bending moments that have to be resisted are, however, only slightly reduced since the self-weight near the supports has little effect on the bending moments, as can easily be verified with an influence line.

In a continuous girder, on the other hand, the highest bending moments have to be resisted over the intermediate supports. Therefore, reducing the depth at midspan results in an increased efficiency of the continuous girder. Furthermore, since the self-weight is reduced where it causes high bending moments (both at midspan as well as over the supports, which again can be verified with an influence line), this concept is even more efficient.

Variable depth girders are more expensive (more labour, complicated formwork etc.). Therefore, variable cross sections are only economical for longer spans, or in the case of particular site conditions (low clearance etc.)

Bridge girder – Structural efficiency: Efficient cross-section

Since longitudinal bending is the dominant action and self-weight is the dominant load at large spans, efficient solutions require cross sections that combine



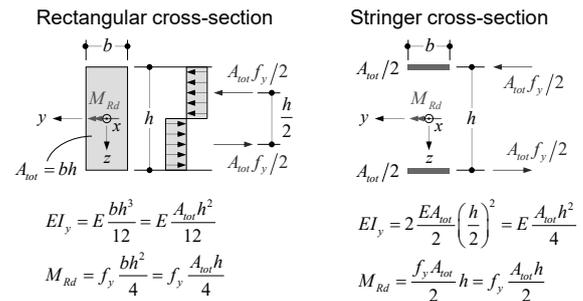
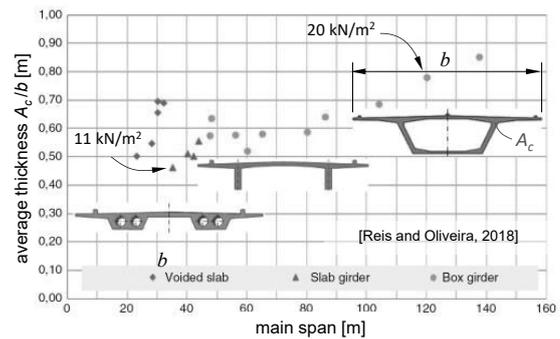
while ensuring sufficient stiffness and capacity for other loads, particularly non-symmetric traffic loads.

- use suitable material with high ratios of stiffness and strength to specific weight ($E/\gamma, f_y/\gamma$)
- optimise cross-section, i.e. maximise ratios of bending stiffness and strength to cross-section ($EI_y/A_{tot}, M_{Rd}/A_{tot}$)

Theoretically, a pure stringer cross-section would be ideal:

- 3 x stiffer
- 2 x stronger

than a rectangular cross-section (for linear elastic - ideally plastic materials)



Since longitudinal bending is the dominant action and self-weight is highly relevant, particularly at long spans, efficient solutions require cross sections that combine (i) a high bending stiffness and strength with (ii) low self-weight, while ensuring sufficient stiffness and capacity for other loads, particularly non-symmetric traffic loads.

Obviously, using a material with high ratios of stiffness and strength to specific weight, i.e., E/γ and f_y/γ , is favourable. For a given material, the optimum shape of the cross-section maximises the ratios of bending stiffness and resistance to the cross-sectional area, i.e., EI_y/A_{tot} and M_{Rd}/A_{tot} .

Theoretically, a pure stringer cross-section (two flanges without webs) would thus be ideal. Compared to a rectangular cross-section with equal area A_{tot} , a pure stringer cross-section is three times stiffer and twice as strong in bending (for linear elastic - ideally plastic material, see formulas on the right).

Note: The strength difference between rectangular and stringer cross-section is even more pronounced if an elastic design is carried out (ultimate bending moment = onset of yielding at edge of cross-section). The strength of the stringer cross-section is three times higher than that of the rectangular cross-section in this case.

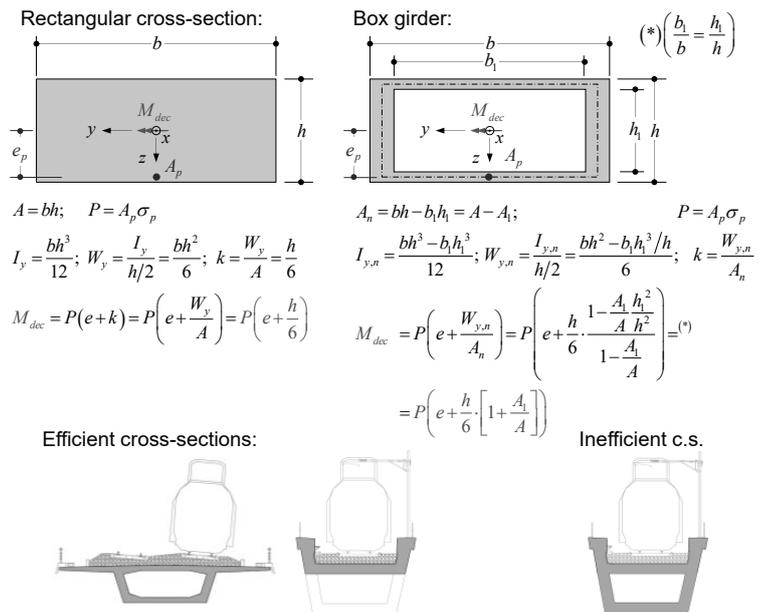
Bridge girder – Structural efficiency: Efficient cross-section

Pure stringer cross-sections are not feasible, but

- Concentrating the material far from the neutral axis is beneficial for the ratios EI_y/A_{tot} , M_{Rd}/A_{tot}
- In prestressed concrete girders, reducing the weight by doing so even increases the decompression moment (figure)

Efficient cross-sections should therefore have wide flanges but only narrow webs, and the deck should be activated as flange:

- locate deck at top or bottom of cross-section
- minimise web thickness, with limitations given by:
 - ... required shear strength
 - ... space requirement for casting of webs (particularly for internal prestressing cables)
 - ... maximum slenderness of steel plates
- use trusses instead of solid webs
 - ... only economical in long-span bridges
 - ... may be aesthetically beneficial (transparency)



Pure stringer cross-sections are not feasible in reality. Still, the comparison with the rectangular cross-section clearly shows that an efficient cross-section should concentrate the material as far away from the neutral axis as possible. This is further highlighted by investigating the effect of removing material from the middle of a rectangular cross-section (figure): While reducing self-weight (and hence the bending moments), this even increases the decompression moment of a prestressed girder.

Hence, an efficient cross-section should generally consist of wide flanges but only narrow webs, whose thickness should be limited to the strict minimum. This is often determined by reasons related to construction: space for prestressing cables in concrete, maximum plate slenderness in steel. In long span bridges, more weight may be saved by truss webs, which may also be aesthetically preferable (less massive appearance).

A deck is always required and contributes significantly to the self-weight. In an efficient cross-section, the deck should be located far away from the neutral axis of the girder and connected such that it can be activated as a flange (usually it acts as top flange, but occasionally – in trough bridges – as bottom flange).

Note: The beneficial effect of box girders on the decompression moment is also present in bridge piers (unless the compressive normal force is mainly caused by the self-weight of the pier).

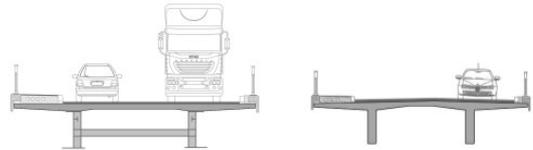
Bridge girder – Structural efficiency: Efficient cross-section

Whether an open cross-section or a box girder is appropriate depends on the static system and spans (particularly magnitude of hogging moments and torsional moments).

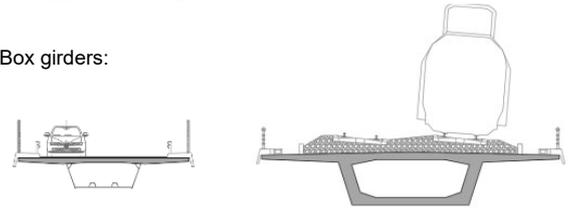
Regarding bending, the following should be considered:

- Concrete decks are particularly effective where subjected to longitudinal compression (usually sagging moments).
- Open cross-sections without a bottom slab are efficient in regions of sagging moments (compression in concrete deck, tension concentrated in bottom chord = narrow steel flange or prestressing cables at bottom of web).
- A bottom slab may be required over the supports, in order to resist the compressive forces caused by the hogging moments (particularly in concrete girders, respecting ductility criteria for the depth of the compression zone (e.g. $x/d < 0.35$)).

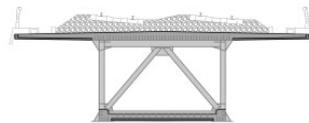
Open cross-sections:



Box girders:



Double composite action:



Usually, the deck is located at the top of the cross-section, acting as a flange. Whether a bottom flange is required (box girder = closed cross-section) or not (open cross-section) depends on the static system and the spans (magnitude of hogging moments, magnitude of torsional moments). This is outlined on this slide for bending moments, and on the next one for torsional moments.

Reinforced and prestressed concrete, frequently used for bridge girders, have non-symmetric material properties: Their strength and stiffness is much higher in compression than in tension. Concrete decks are therefore particularly effective where they are subjected to longitudinal compression (usually for sagging moments, hogging moments in trough bridges).

Unless a high bending stiffness is essential or significant torsional moments need to be resisted (strong curvature), open cross-sections without a bottom slab are effective in regions of sagging moments (midspan), since the required tensile resistance of the bottom chord can be provided by narrow flanges in steel girders, and over the web width in concrete girders since prestressing cables and reinforcement require little space.

A bottom slab may be required over the supports, in order to resist the compressive forces caused by the hogging moments (particularly in concrete girders, respecting ductility criteria for the depth of the compression zone (e.g. $x/d < 0.35$)).

Note: Concrete bottom slabs over the supports are also used in composite bridges, since they are more economical than steel flanges to resist compression (“double composite action”).

Bridge girder – Structural efficiency: Efficient cross-section

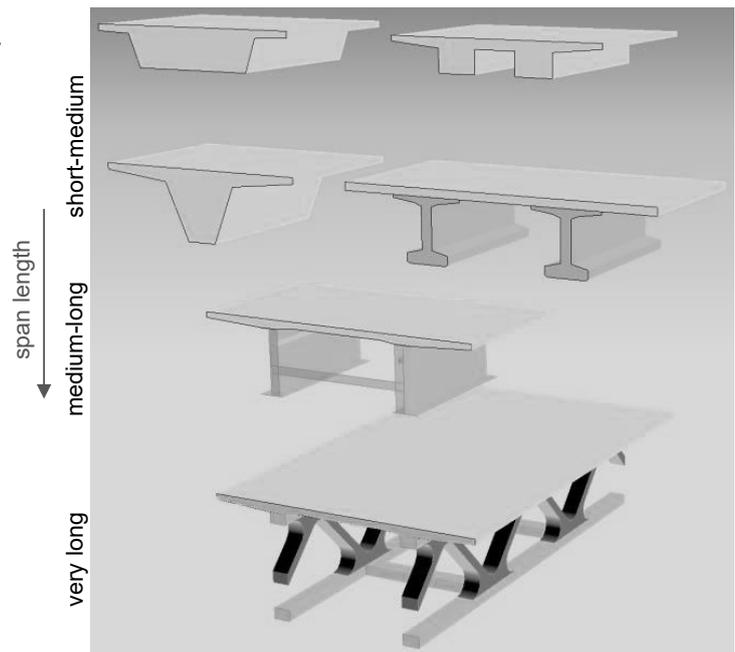
Bending is dominant, but sufficient stiffness and capacity for other loads, particularly torsional moments, is also required. Therefore, box girders (closed cross-sections) are frequently used in bridges with

- high eccentric traffic loads
- strong curvature or skew supports

Statically efficient cross-sections often require significantly more labour or more expensive materials than simpler, less efficient solutions.

With increasing spans, structural efficiency becomes more relevant and aligned with economy.

narrow / mod. wide deck	}	straight / mod. curved
		strong curvature
wide deck	}	straight / mod. curved
		strong curvature



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When optimising the cross-section for bending, sufficient stiffness and capacity for other loads, particularly torsion due to non-symmetric traffic loads, curvature or skew, needs to be guaranteed. In strongly curved or skew bridges and bridges with high eccentric traffic loads, closed cross-sections are therefore used, even if the bottom slab would not be required for bending.

The economy of a bridge not only depends on the material use and weight, but also on labour and material cost. Statically efficient cross-sections often require significantly more labour or more expensive materials than simpler, less efficient solutions. For example, box girders or variable cross-sections require much labour, and lightweight steel cross-sections are expensive (particularly trusses and orthotropic decks).

In conclusion, there is a trade-off between structural efficiency and overall economy. As illustrated in the figure on the right, structural efficiency becomes more relevant (and aligns more with economy) with longer spans.

Note: Statically inefficient solutions may also be caused by client guidelines. For example, solid cross-sections are often used where a box girder or voided slab would be much more efficient, because clients prohibit inaccessible voids. In Switzerland, ASTRA requires a minimum height of 1.50 m, meaning that up to a height of about 1.80 m, no box girders are possible and, consequently, unless an open cross-section is feasible, very massive slab bridges are built. This strict requirement is justified by negative experience with inaccessible voids in bridges from the 1950s and 1960s, that were filled with chloride congested runoff water and caused severe damage to these bridges. However, in a new bridge with proper waterproofing and drainage (ASTRA at the same time has very strict requirements for these), there is no risk of runoff water penetrating the deck (the voids could easily be monitored to prove this today). Hence, in the light of reducing global material consumption and CO2 emissions, such requirements should be re-considered.

Bridge girder – Structural efficiency: Efficient cross-section

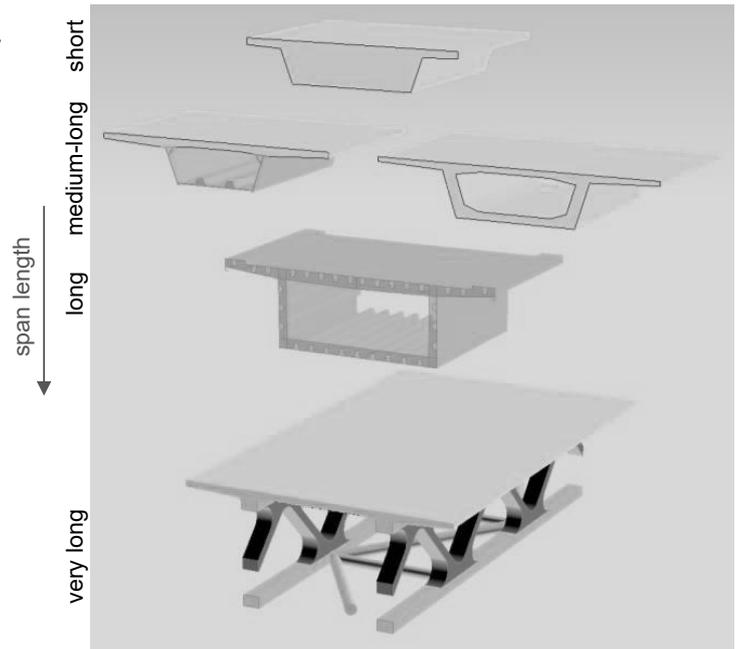
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- high eccentric traffic loads
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With increasing spans, structural efficiency becomes more relevant and aligned with economy.

narrow / mod. wide deck	{ straight / mod. curved strong curvature
wide deck	
	{ straight / mod. curved strong curvature



Bridge girder – Structural efficiency: Efficient cross-section

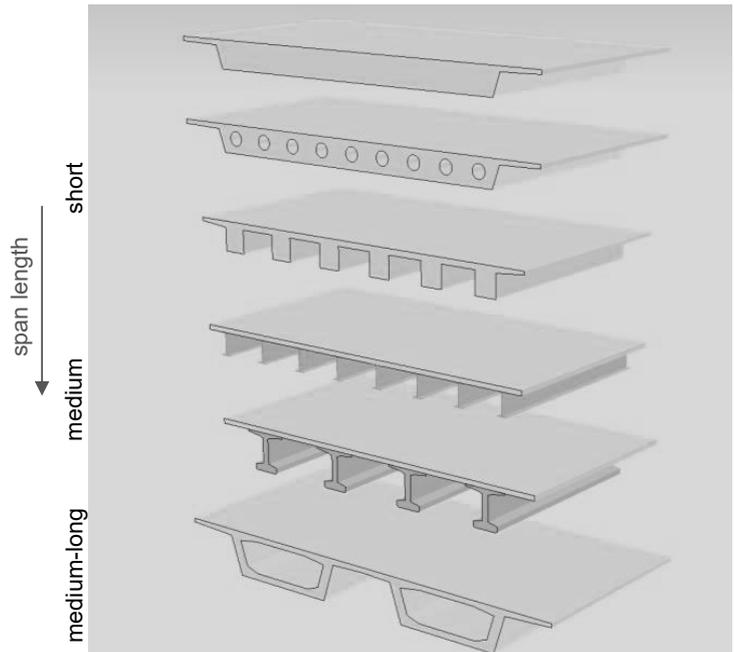
Bending is dominant, but sufficient stiffness and capacity for other loads, particularly torsional moments, is also required. Therefore, box girders (closed cross-sections) are frequently used in bridges with

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Statically efficient cross-sections often require significantly more labour or more expensive materials than simpler, less efficient solutions.

With increasing spans, structural efficiency becomes more relevant and aligned with economy.

narrow / mod. wide deck	}	straight / mod. curved
		strong curvature
wide deck	}	straight / mod. curved
		strong curvature



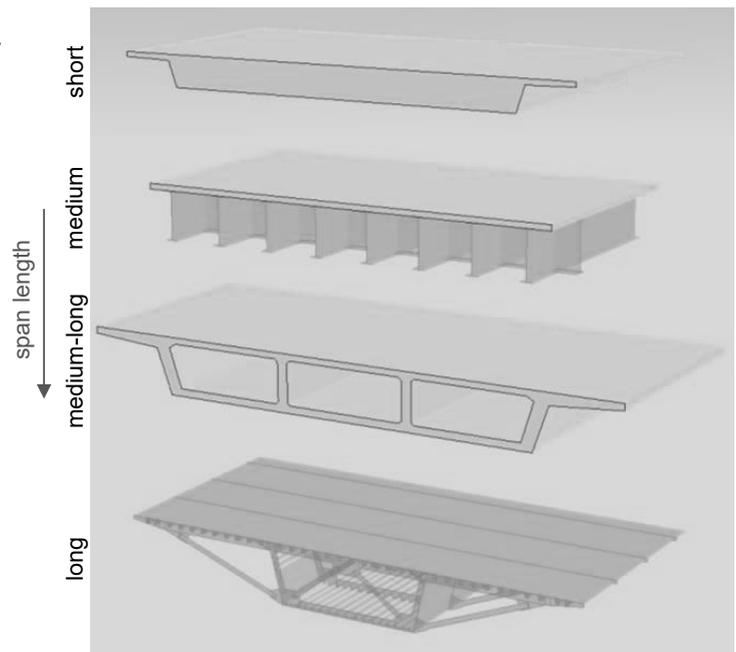
Bridge girder – Structural efficiency: Efficient cross-section

Bending is dominant, but sufficient stiffness and capacity for other loads, particularly torsional moments, is also required. Therefore, box girders (closed cross-sections) are frequently used in bridges with

- high eccentric traffic loads
- strong curvature or skew supports

Statically efficient cross-sections often require significantly more labour or more expensive materials than simpler, less efficient solutions.

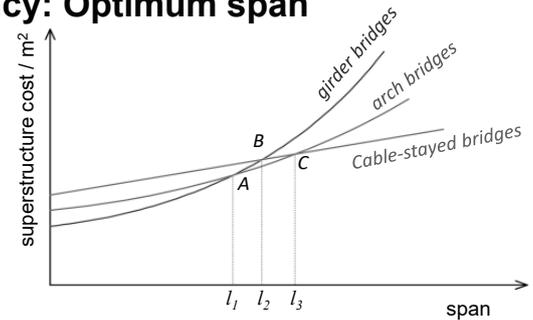
With increasing spans, structural efficiency becomes more relevant and aligned with economy.



Bridge girder – Structural efficiency: Optimum span

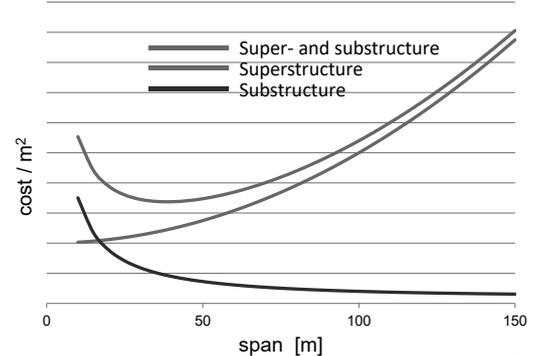
Upper figure:

- Since more depth is required at larger spans, the costs of the bridge girder increase with its span
- Girder bridges are economical at smaller spans than other, inherently more efficient typologies (since these also require a girder and are thus less efficient at small spans).



Lower figure:

- Contrary to the costs of the girder (superstructure), the substructure costs decrease with span (short spans = many piers and foundations)
- The cost of super- and substructure of a girder bridge therefore exhibit a minimum at the optimum economic span
- This optimum span is usually around 30 m
- The minimum is rather flat, leaving considerable freedom for economic solutions considering other aspects, such as aesthetics.



Even when using efficient cross-sections and an efficient static system, the girder weight increases with the span, i.e., more material is used for the same deck surface, which results in higher costs.

The upper figure illustrates the increase of superstructure cost with the span. This is more pronounced in girder bridges than in arches and cable stayed bridges, where the arch and the stay cables, respectively, are clearly more efficient than the girder: they resist by their shape. However, these systems are more expensive at small spans, where the additional elements forming the superstructure are inefficient.

The lower figure illustrates the cost of super- and substructure of a girder bridge as a function of the span. Since the superstructure costs increase, but the substructure costs decrease with span (short spans = many piers and foundations), the total costs exhibit a minimum at the optimum economic span.

The optimum span is usually around 30 m, but the minimum is rather flat. This leaves considerable freedom for economic solutions considering other aspects, such as aesthetics.

Illustrations adapted from: bottom:: Tomas Vogel; Top:: J. Manterola, Puentes I

Bridge girder – Structural efficiency: Optimum span

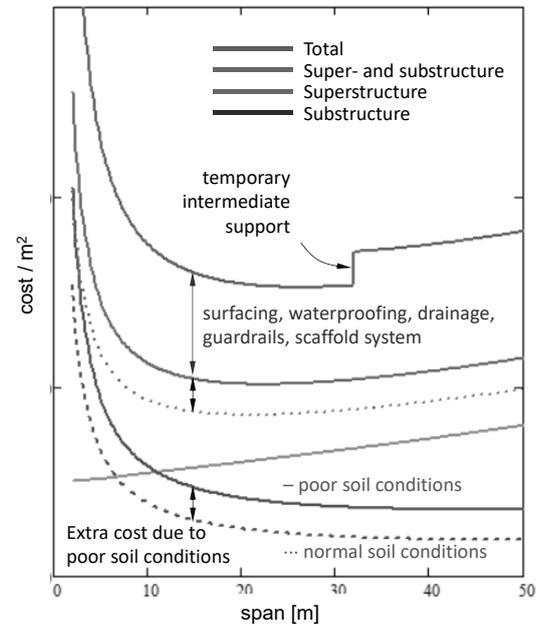
The optimum economic span of a girder bridge is rather insensitive to the soil conditions, see figure:

- Substructure costs are compared for normal (dotted) and poor soil conditions (solid), with 3x higher foundation cost
- The optimum span is only slightly increased by very poor soil conditions

Apart from superstructure and substructure, other components contribute significantly to the total cost, such as

- surfacing, waterproofing and drainage
- guardrails
- scaffold

These are largely independent of the span except for the scaffold costs. The latter decrease slightly with the span, since more scaffolding operations are required at smaller spans if the scaffold is re-used (more spans for same bridge length), up to the point where the span requires a more expensive scaffold system.



The optimum economic span of a girder bridge is rather insensitive to the soil conditions.

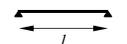
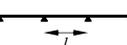
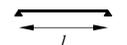
The figure on the right illustrates the cost of a continuous prestressed concrete girder bridge, with constant cross-section, in very bad soil conditions (Steinbach Viaduct). The dotted curve for the substructure cost corresponds to average soil conditions, whereas the solid line represents the specific site conditions, with three times higher costs of a pile foundation for the same load. The minimum of the cost of sub- and substructure (dotted line = normal, solid line = poor soil conditions) is only slightly shifted to the right by the poor soil conditions.

The figure further illustrates that other components contribute significantly to the total cost, which in this case include surfacing, waterproofing, drainage, guardrails and the scaffold system. Since more scaffolding operations are required at smaller spans, the latter decrease slightly with longer spans, up to the point where a change of scaffold system is required (in the specific example, temporary intermediate supports would have become necessary).

Note: The estimation of the costs of a bridge always contain a high degree of uncertainty. Among other factors, local preferences, the current workload of contractors and steel workshops, as well as the availability of specific scaffold and formwork systems at the time of bidding may have a significant impact on the contractors' bids. Therefore, the "optimum economic span" is hardly ever determined for a specific bridge in practice since it is always in the same range, and the minimum is flat.

Bridge girder – Structural efficiency: Optimum span

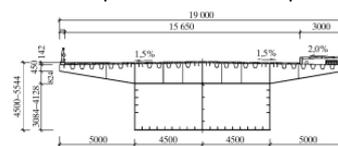
The following spans are generally considered economical for girder bridges:

		Concrete	Steel / Composite
		$l \approx 30 \dots 35 \text{ m}$	$l \approx 50 \dots 60 \text{ m}$
		$l \approx \dots 100 \text{ m}$	$l \approx \dots 120 \text{ m}$
		$l \approx 25 \dots 30 \text{ m}$	$l \approx 40 \dots 45 \text{ m}$
		$l \approx \dots 70 \text{ m}$	$l \approx \dots 100 \text{ m}$

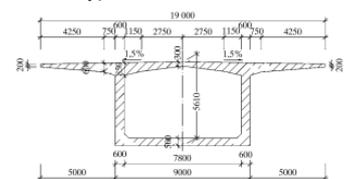
Note that these are no strict or exact limits. Rather, they depend on many site-specific aspects and are indicated here for guidance only. The bridge shown on the right, with much longer spans (max. 330 m), illustrates this.



Midspan 103 m of main span:



Typical cross-section:



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Notes [Segments, American Segmental Bridge Institute, Vol. 48, fall 2006]:

The photos show the longest box girder span in the world (in 2006), the 330m-span Shibpano Bridge in Chongqing, China. Designed by TYLI, the new Shibpano Bridge complements the original bridge, constructed in 1981, and will dramatically improve traffic flow across the Yangtze River to the central business districts. The new bridge is 19 m wide and 1'103.5 m in total length. It is positioned 25 m apart from and parallel to the old bridge. Due to the close proximity of the two structures, the new bridge main span needed to be larger and all piers had to be aligned with the piers of the old structure to facilitate smooth river traffic. This resulted in the new record-breaking main span of 330 m. (...)

Aesthetic considerations were of high importance in choosing the final design for the new bridge. Arch, cable-stayed, and suspension alternatives were considered, but ultimately, a continuous single box girder design consisting of seven spans was selected for its compatibility with the existing bridge. The entire bridge was built using a cast-in-place concrete free cantilever segmental method, with the exception of a 103 m section in the middle of the main span, which is a steel box. The steel box element was incorporated into the design to significantly reduce the bending moment and shear of the structure and make such a long span structure technically and economically efficient. This steel box section was fabricated in Wuchang, a city about 1,000 km downstream of Chongqing on the Yangtze River. The fabricated steel box section was closed on both ends with steel noses, launched as a barge and towed upstream by tug boats. Along its journey, it also went through the locks of the Three Gorge Dam. After it arrived at the site, the steel section was turned 90 degrees using cables anchored at both banks of the river. Then the 1,400-ton, 103 m-long box girder was lifted successfully from the Yangtze River to its final place with strand jacks.

In achieving the world record for longest box girder span, the new Shibpano Bridge surpasses the 301 m concrete girder of the Stolmastsunde Bridge in Norway and the 300 m steel box girder of the Ponte Costa de Silva in Brazil. The bridge was completed at a total cost of approx. \$40 million and is scheduled to be open to traffic before the end of the year.

Photo and figures (cross-sections): Man-Chung Tang (Chairman of the Board) (2010) The New Shibpano Bridge, Chongqing, China, Structural Engineering International, 20:2, 157-160, DOI: [10.2749/101686610791283533](https://doi.org/10.2749/101686610791283533)

Longitudinal view: Wikipedia https://en.wikipedia.org/wiki/Shibanpo_Yangtze_River_Bridge

Bridge girder – Structural efficiency: Span ratios

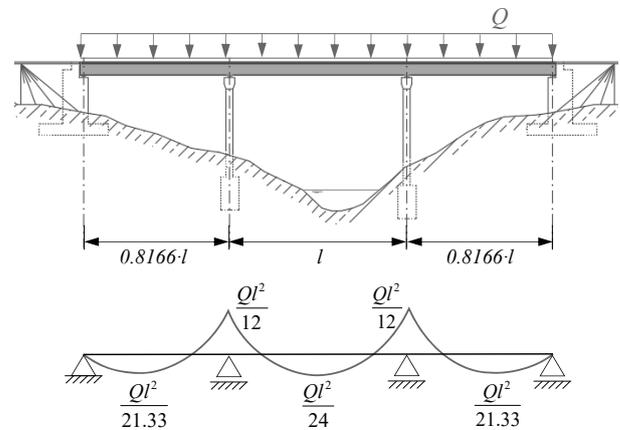
Criteria for the length of end spans:

- Ensure similar magnitude of bending moments as in interior spans → $l_{end} \approx (0.70 \dots 0.85) \cdot l_{int}$ (*)
- Prevent uplift of bearings (no negative support reactions in service conditions)
- If possible, ensure vertical support reactions at the abutments large enough to transfer horizontal forces with standard bearings (avoid separate horizontal bearings)

The governing load combination for the minimum support reaction includes a significant contribution from torsion:

- The minimum end span to prevent uplift depends on torsional behaviour (no specific value can be given; textbook recommendations often neglect torsion)
- The transverse spacing of bearings at the abutment should be as large as possible

(*) In a girder with constant EI_y subjected to uniform load, the bending moment over the intermediate supports equals that of an infinite continuous girder if $l_{end} = 0.8166 \cdot l_{int}$.



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The end spans of a girder bridge are usually chosen somewhat shorter than the interior spans, typically $l_{end} \approx (0.70 \dots 0.85) \cdot l_{int}$. By this, the bending moments in the end span are of a similar magnitude as in the interior spans. In a girder with constant EI_y subjected to a uniformly distributed load, the bending moment over the first intermediate support equals that of a continuous girder with an infinite number of spans if $l_{end} = 0.8166 \cdot l_{int}$.

On the other hand, the end spans should not be too short in abutments with bearings. Uplift of a bearing in service conditions must always be prevented. Furthermore, a minimum vertical support reaction is required to transfer horizontal support reactions via the standard bearings used for vertical support. If the minimum reaction is too small, special bearings for the horizontal loads are required.

The governing load combination for the minimum support reaction includes a significant contribution from torsional moments. Therefore, it is advisable to separate the bearings in the transverse direction at the abutments as far as possible, even if this requires a heavier end diaphragm compared to providing a direct support under the webs.

Notes:

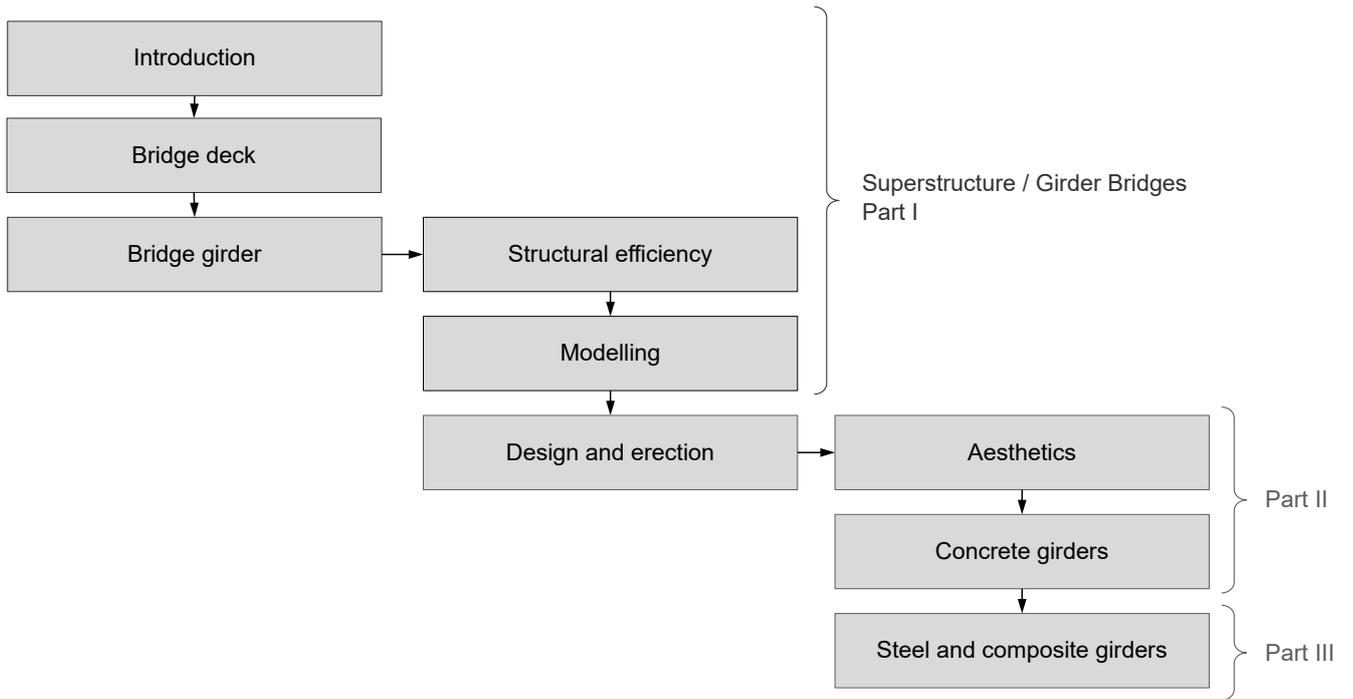
- Possible measures to prevent uplift of a bearing:

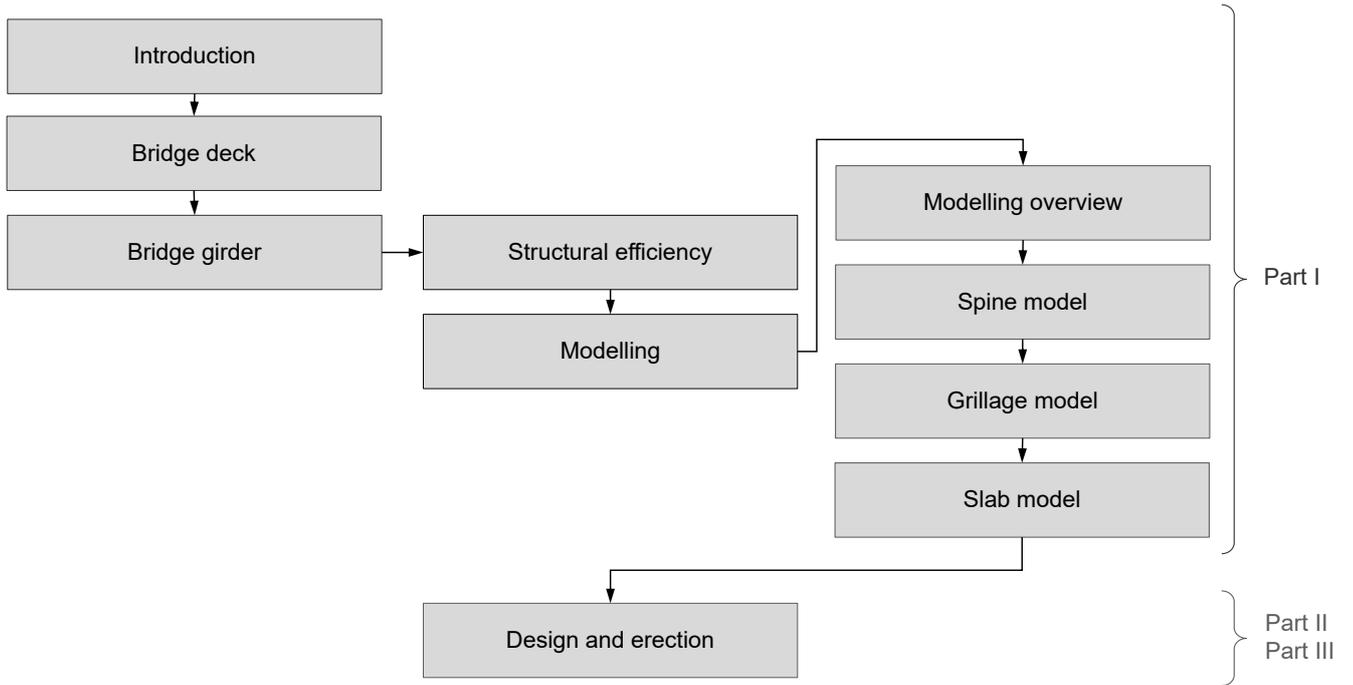
- choose a longer end span;
- increase the transverse separation of the bearings (reduce torsional contribution);
- using an integral abutment
- add weight (heavy end diaphragm or the end span with a heavier cross-section)
- add vertical prestressing
- use tension bearings

Measures (v) and (vi) should be avoided wherever possible. (If vertical prestressing is used, it can be combined with concrete hinges (that require a reasonable amount of axial compression) instead of horizontally fixed bearings (e.g. used in Bridges Nodo di Camorino, Apl Transit Gotthard)).

- Most codes and guidelines allow uplift of the bearings in ULS (at least for accidental design situations, often also for persistent and transient ones) as long as the bearings are in compression at SLS (with full, characteristic loads). If uplift is accepted, the corresponding bearing(s) must not be considered for the respective load combinations (change of static system).

Illustration: Javier Manterola, Puentes I





Superstructure / Girder bridges

Bridge Girder – Modelling overview

Bridge Girder – Modelling overview: General remarks

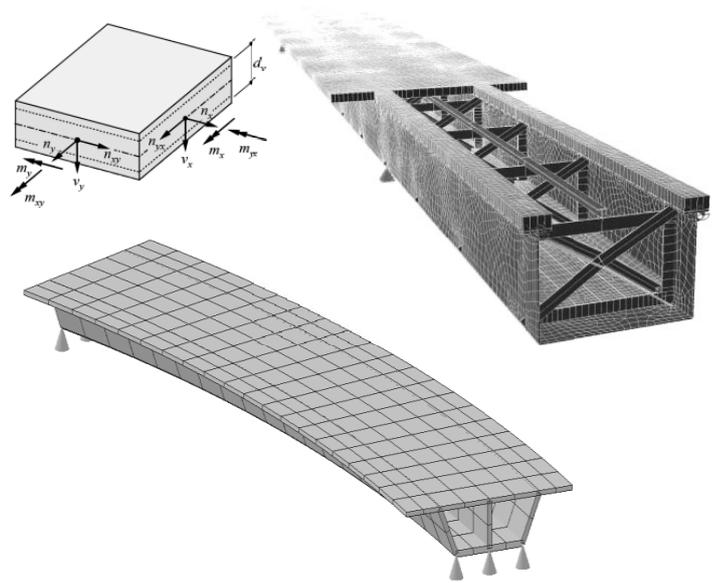
A good model is simple, yet captures the relevant phenomena and enables a safe and efficient design. Hence, a model should be

- as simple as possible, but not simpler

With today's computing power at the hands of engineers, it is tempting to use a more complex model than required.

However, it must be kept in mind that highly complex models may limit the designer's insight into the behaviour ("black box models"). If modelling errors remain undetected, overly complex models lead to worse (or even dangerous) results than simple models, which are inherently approximate but transparent. Hence, keep in mind that

- it is better to be roughly right than exactly wrong



This does of course not mean that we should recur to hand calculations. An efficient design process does involve computer models today, and in many cases, it is preferable to use a single, more complex model than several simple models.

For example, while some years ago it was common practice to use separate 1-D or 2-D beam models for

- Superstructure
- Substructure
- Foundation

it is often more efficient today to use a single 3-D frame model for the entire analysis. This more complex model may even reduce design errors, since exchanging information among separate models (reactions \leftrightarrow loads) is a frequent source of mistakes, and some interactions between superstructure and substructure neglected in separate models (e.g. transverse frame action) are implicitly accounted for.

On the other hand, it is still demanding today to use complex models (see next slide) with 3D volume or shell elements correctly, even if merely linear analyses are carried out. Such models are indeed prone to modelling and interpretation errors and should only be used by experienced designers, with an independent, simplified model for plausibility checks. Since modelling entire structures with such models is time-consuming, they are typically used for the analysis of specific details (or geometrically simple structures like on-bay frames with slab cross-section) and may yield valuable insight. For modelling entire bridges, 3-D frame models are prevailing today.

Illustration: © Autodesk (below); PhD Giraldo (right)

Quotes: Albert Einstein and Carveth Read (often attributed to John Maynard Keynes)

Bridge Girder – Modelling overview: Folded plate models (FE analyses)

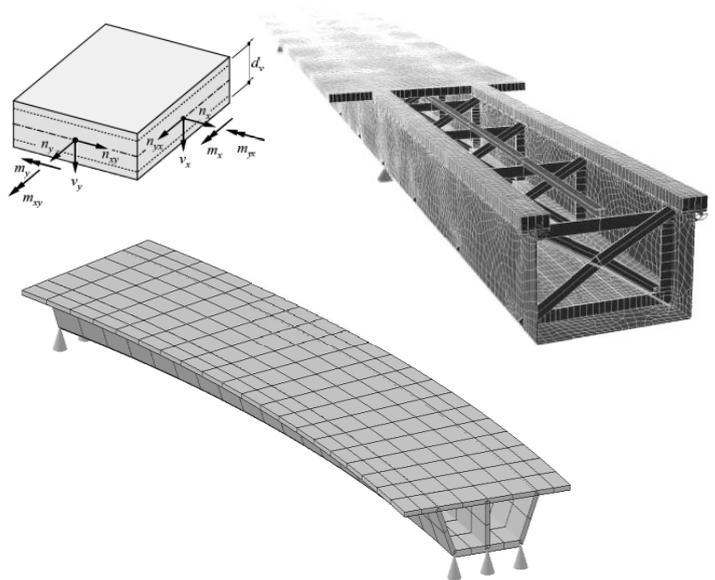
Most bridge girders consist of thin, planar elements. Hence, folded plate models (shells in the case of curved bridges) would be most “realistic”.

In spite of the progress in computational tools, such models are rarely used for design today, for the following reasons:

- highly complex models (8 stress resultants in shells)
 - very time consuming (inefficient design process)
 - lacking transparency, prone to errors
- limited use for design in spite of high computational effort
 - linear elastic analysis does not capture the real behaviour (cracking, other nonlinearities)
 - detailing based on output is not straightforward (particularly for concrete elements)

Simpler models are therefore still preferred for design purposes and presented in the lecture:

- spine models (single / line beam model = Stabmodell)
- grillage models (Trägerrostmodell)
- slab models (Plattenmodell)



Notes:

Numerically, and for modelling, volumetric 3D elements are in many cases simpler to handle than plate or shell elements. However, the interpretation of results becomes even more difficult.

Nonlinear finite element calculations are hardly ever carried out today for design purposes, since such analyses are very time-consuming. In particular, other than in linear analyses where each load case can be solved separately and linearly combined with other load cases in post-processing, in a nonlinear analysis each load combination has to be calculated separately. As there are easily several hundred combinations, a corresponding number of nonlinear analysis would have to be carried out.

Bridge Girder – Modelling overview: Simplified models

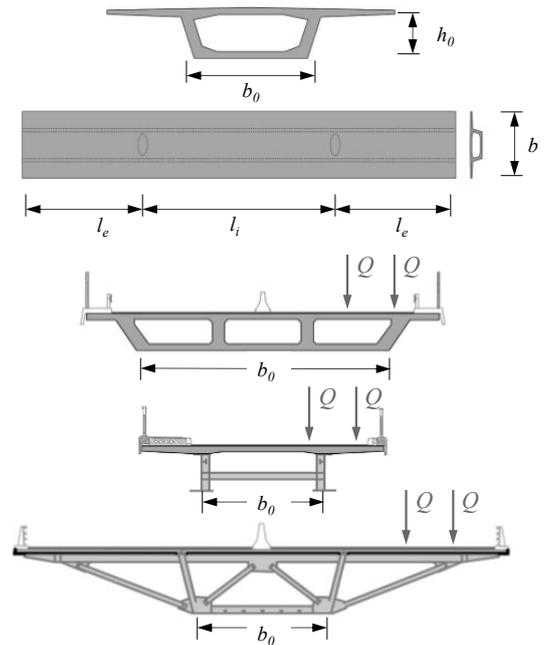
Among the simplified models (spine, grillage, slab), the simplest one that is adequate should be used. If possible, a spine model is therefore chosen.

Whether a spine model can be used depends primarily on the following criteria:

- The ratio between the width b_0 of the girder ($b_0 < b$) and the effective girder span; a spine model (single beam or line beam) is usually appropriate if

$$l_0 \geq 2(b_0 + h_0)$$

- The type of cross-section, which defines the behaviour of the girder under eccentric load; a spine model is usually appropriate for box girders



Bridge Girder – Modelling overview: Simplified models

Girders with open or closed cross-section behave fundamentally different in torsion (see *spine model for open cross-sections* for more details, including Factor κ).

Accordingly, different models are adequate:

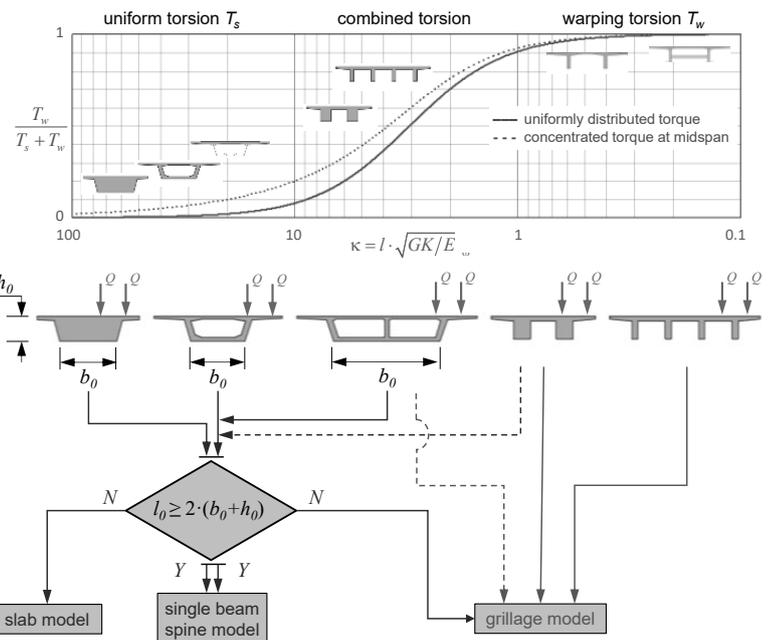
- Uniform torsion T_s prevails in girders with solid, convex cross-section and in box girders since $GK \gg EI_w/l^2$

→ spine model applicable

- Warping torsion T_w (“antisymmetric bending” with corresponding distortions) prevails in girders with an open cross-section since $GK \ll EI_w/l^2$

→ grillage model appropriate

Note: Warping torsion can be analysed analytically using a spine model as well (see Marti, Theory of Structures). However, this is tedious for general cross-sections and considering many load-cases, and yields no information on the transverse behaviour.



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Note: If warping torsion is analysed analytically using a spine model, the contributions of uniform torsion and warping torsion to the total torsional moment must be determined. However, these are not constant, but depend on the static system, the span and the position of the applied torque along the span. Hence, significant differences are obtained for concentrated loads or uniformly distributed loads, see top figure. Considering these differences for every load case or load combination, respectively, is very time-consuming (already complex for simply supported girders, and much more so for continuous girders with positive and negative bending moments).

Furthermore, in girders with open cross-sections, the transverse behaviour is even more relevant than for box-girders, and needs to be investigated in detail. However, no direct information is obtained in this respect from the spine model. Therefore, except for double-T cross-sections where reasonable simplifications are possible (see *spine model for open cross-sections*), grillage models are recommended for open cross-sections.

Figure adapted from Kollbrunner, C.F., Basler, K., *Torsion in Structures, An Engineering Approach*, Springer Verlag, 1969.

Superstructure / Girder bridges

Bridge Girder – Spine model – Global analysis
(Einstabmodell, Längsrichtung)

Spine model – Global analysis: General remarks

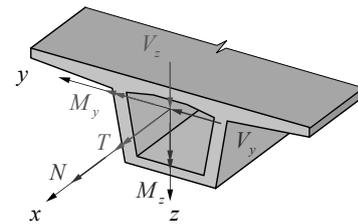
In a *spine model* (also referred to as *single beam* or *line beam model*), the girder = spine has to resist:

- Bending moments M_y and shear forces V_z caused by gravity loads (self-weight, traffic loads, ...)
- Bending moments M_z and shear forces V_y caused by transverse horizontal loads (wind, centrifugal forces, earthquake loads)
- Torsional moments T caused by the eccentricities of the applied loads (with respect to the girder axis or the shear centre), as well as by curvatures in plan.
- Axial forces N are usually small in girder bridges, even if integral abutments are used.

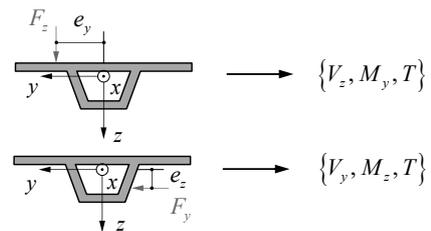
In many cases, gravity loads and the corresponding internal actions V_z , M_y and T , govern the design.

Torsion is treated much less in other courses than shear and bending, and using a spine model requires special considerations regarding the introduction of torques.

Therefore, torsion and load introduction are treated in this lecture in more detail, whereas it is assumed that students are proficient in the structural analysis and the design for shear and bending.



Internal actions (stress resultants) in a single beam model



As mentioned in the slide, torsion is treated in more detail than shear and bending since the latter receive much more attention than torsion in other courses of the BSc and MSc curriculum. However, this must not be misunderstood in the sense that torsion is the most important action in girder bridges: As stated earlier, bending is clearly dominant.

Spine model – Global analysis: General remarks

In a general cross-section with arbitrary material behaviour, internal actions (stress resultants) and deformations are related by integration or iteration (see e.g. *Stahlbeton I*).

The analysis is greatly simplified by the usual assumption of linear elastic behaviour using

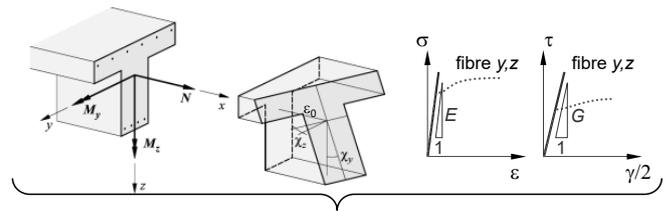
- axial stiffness EA
- bending stiffnesses EI_y and EI_z
- torsional stiffness GK ($= GI_p$ for circular cross-sections)

Shear deformations are usually neglected ($GA^* \rightarrow \infty$). However, torsional deformations are taken into account (see notes).

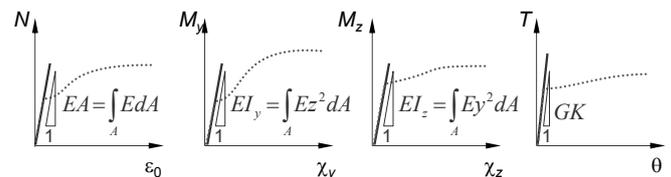
While effective flange widths are often accounted for, further simplifications are usually adopted in the structural analysis (but not in the design of the members!):

- use of uncracked stiffnesses EI for concrete members (cracking could be considered by the cracked stiffness EI^II)
- consideration of full section of slender steel plates (webs)

The determination of axial and bending stiffnesses is straightforward (see formulas in figure). The torsional stiffness GK is treated later in this lecture in more detail.



Cross-section: «real» behaviour / linear elastic idealisation



$$\begin{cases} N = \int_A \sigma_x dA \\ M_y = \int_A \sigma_x z dA \\ M_z = \int_A \sigma_x y dA \\ T = \int_A (\tau_{xy} y - \tau_{yx} z) dA \end{cases} \xleftrightarrow[\text{iterate}]{\text{integrate}} \begin{cases} \epsilon_0 \\ \chi_y \\ \chi_z \\ \theta \end{cases} \begin{cases} N = EA \cdot \epsilon_0 \\ M_y = EI_y \cdot \chi_y \\ M_z = EI_z \cdot \chi_z \\ T = GK \cdot \theta \end{cases}$$

If computer programs are used that determine the stiffnesses automatically (based on the input of the cross-sectional geometry), cracking can be accounted for by reducing the material stiffness E . While such programs reliably determine the bending stiffnesses, the automatic calculation of torsional stiffnesses should always be verified, particularly when using hollow cross-sections (many programs do not automatically detect whether a circumferential shear flow is possible or not).

Shear deformations, caused by shear strains, are usually neglected because shear forces are accompanied by bending moments, which produce much larger displacements in the same direction and reasonably proportional to the deformations caused by corresponding shear forces. Note that in many structural analysis programs, setting $GA=0$ corresponds to neglecting shear deformations («infinite» cannot be input numerically).

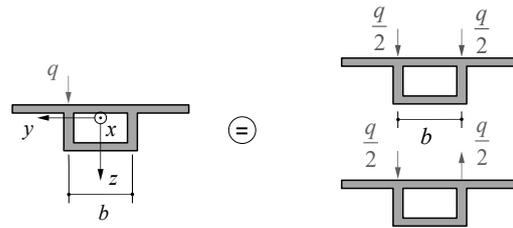
Just like shear deformations, torsional deformations are also caused by shear strains and relatively small in many cases (particularly in box girders where uniform torsion prevails). However, torsional moments are the only internal actions causing twisting of the cross-section and hence, torsional rotations of the cross-section. Therefore, they cannot be neglected.

The torsional stiffness is termed GK here, since this is usual in bridge design software and textbooks. In structural mechanics, GJ_x is often used. For circular and annular cross-sections, $K = I_x$ is equal to the polar moment of inertia $I_p = I_y + I_z$. However, this is not the case in general cross-sections.

Spine model – Global analysis: Decomposition of eccentric loads

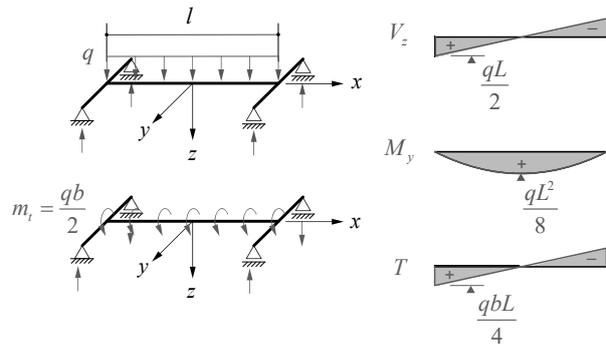
For the analysis in the spine model, eccentric loads can simply be substituted by a statically equivalent combination of

- symmetrical load causing (acting in the girder axis)
 - shear forces
 - bending moments
- and
- torque or force couple causing (“anti-symmetrical load”)
 - torsional moments



Bending and torsion can then be analysed separately, and the resulting forces (e.g. shear forces per element) superimposed for dimensioning.

Generally, eccentric loads do not act in the axis of a web. However, the decomposition in a symmetrical load and a torque is also possible. This is illustrated in the following slides for a box girder, but also applies to solid and open cross-sections (although local load introduction is different, see behind).



Spine model – Global analysis: Decomposition of eccentric loads

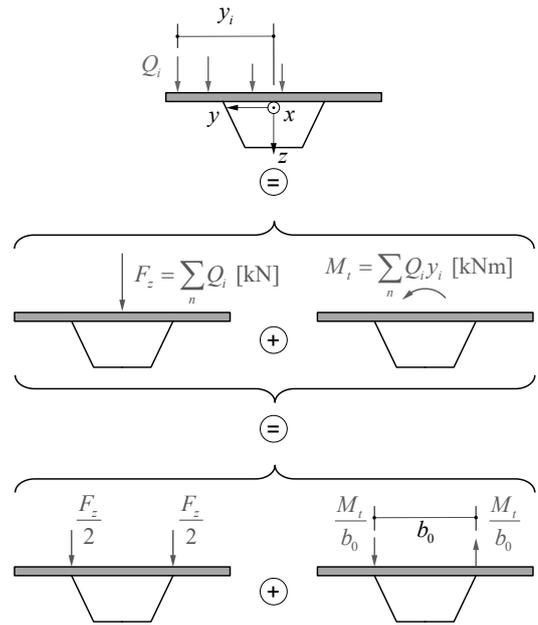
Eccentric concentrated loads [kN] are usually due to traffic loads (concentrated loads representing vehicle axle loads).

They are substituted by a statically equivalent combination of

centric concentrated load [kN] and concentrated torque [kNm] (used for global analysis)

or

two equal concentrated vertical forces and a concentrated force couple, where the forces [kN] act in the axes of the webs (used for load introduction analysis)



Spine model – Global analysis: Decomposition of eccentric loads

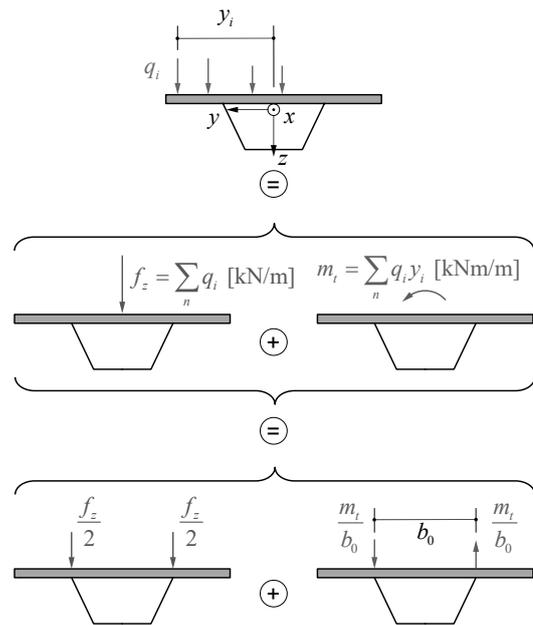
Eccentric line loads [kNm^{-1}] may be due to traffic loads (e.g. line load of ballastless track rail) or superimposed dead loads (e.g. crash barriers).

They are substituted by a statically equivalent combination (obtained by summation) of

centric line load [kNm^{-1}] and distributed torque [kN] (used for global analysis)

or

two equal line loads and a line load couple, where the forces [kNm^{-1}] act in the axes of the webs (used for load introduction analysis)



Spine model – Global analysis: Decomposition of eccentric loads

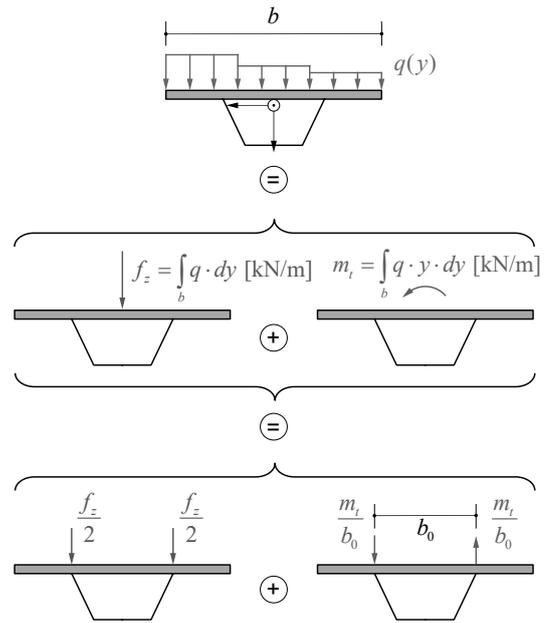
Distributed (surface) loads [kNm^{-2}] are due to self-weight, superimposed dead loads (e.g. surfacing), or distributed traffic loads.

They are substituted by a statically equivalent combination (obtained by integration) of

centric line load [kNm^{-1}] and distributed torque [kN] (used for global analysis)

or

two equal line loads and a line load couple, where the forces [kNm^{-1}] act in the axes of the webs (used for load introduction analysis)



Spine model – Global analysis: Torsion caused by curvature in plan

Torsion is not only caused by eccentric loads, but also by curvature of the girder in plan. M_y and T in curved girders are coupled \rightarrow 2nd order inhomogeneous differential equation.

For a more direct understanding of the behaviour one may determine M_y for the straight girder (developed length) and consider the torques due to the chord forces deviation:

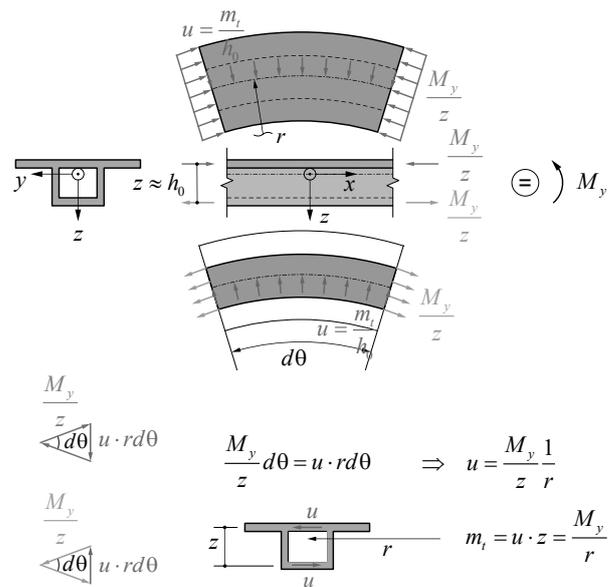
- M_y is resisted by chord forces $\pm M_y/z$, with lever arm z
- chords are curved \rightarrow deviation forces $u = \pm M_y/(r \cdot z)$

\rightarrow distributed torque $m_t = \frac{M_y}{r}$

applied to the girder by a horizontal line load couple with lever arm $z \approx h_0$

$$\pm \frac{M_y}{z \cdot r} \approx \pm \frac{M_y}{h_0 \cdot r}$$

The girder has to transfer the distributed torque (\rightarrow torsion). The cross-section (or intermediate diaphragms) must introduce the horizontal line load couple, i.e., convert it to uniform torsion (see behind and *curved bridges*).



Torsion is not only caused by eccentric loads, but also by curvature of the girder in plan. Bending and torsion in curved girders are coupled and can be analysed using a 2nd order inhomogeneous differential equation (see e.g. lecture notes Stahlbeton I).

A more direct understanding of the behaviour is obtained by first analysing the girder as if it were straight. The resulting bending moments M_y are resisted by tension and compression chord forces $\pm M_y/z$, with internal lever arm z . Since the chords are curved, these forces need to be deviated, requiring deviation forces $u = \pm M_y/(r \cdot z)$.

Hence, deviation of the chord forces requires a distributed torque $m_t = M_y/r$ [kN] applied by a horizontal line load couple u [kN/m] with lever arm $z \approx h_0$. Hence, a torque M_y/r acts on the girder, and the cross-section (or intermediate diaphragms) must ensure the introduction of the horizontal line load couple, i.e., its conversion to uniform torsion. Whereas the cross-section is usually able to introduce the line load couple concrete box girders (if curvature is moderate), the concentrated force couple at the girder end can hardly ever be resisted without an end diaphragm.

Since the torsional moments cause bending moments, these have to be adjusted to account for the coupled behaviour (iterative procedure), to obtain the exact result. However, for usual curvatures of road and railway bridges, and also in most footbridges, the results are accurate enough without iterating. More details see lecture notes Stahlbeton I and curved girders.

Spine model – Global analysis: Torsion caused by skew supports

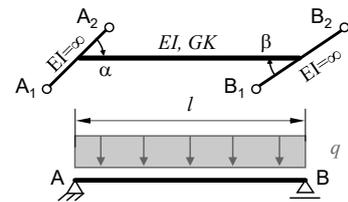
Torsion is also caused by skew supports, since eccentric vertical support reactions are applied.

If stiff diaphragms and articulated bearings are provided, the behaviour can be analysed using models as shown on the right for a simply supported girder:

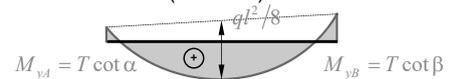
- diaphragms rigid ($EI=\infty$), simply supported (no torsion in diaphragms, can rotate around their axis!)
- determine internal actions analytically or using force method (see Stahlbeton I) or frame analysis software
- skew supports provide a partial fixity, where M_y and T are coupled geometrically
- supports on side of acute angles (A2, B1) receive higher reactions than those on side of obtuse angles (A1, B2)

The girder has to transfer the concentrated torque (\rightarrow torsion). Support diaphragms introduce the concentrated vertical force couple applied by the support reactions, i.e., convert it to uniform torsion (see behind and *skew bridges*).

Static system and loading (plan):



Internal actions (elevation):



$$T = -\frac{ql^2}{8} \cdot \frac{\cot \alpha + \cot \beta}{\cot^2 \alpha + \cot \alpha \cot \beta + \cot^2 \beta + \frac{3EI}{GK}}$$

Note: Whereas the cross-section is usually able to introduce the line load couple to concrete box girders (if curvature is moderate), the concentrated force couple at the girder end can hardly ever be resisted without an end diaphragm.

Spine model – Global analysis: Torsion in box girders (shear flow)

Box girders can be treated as thin-walled hollow cross sections. Torsional moments T are primarily resisted by uniform torsion (“St.-Venant torsion”), i.e., a circumferential shear flow of constant magnitude $\tau \cdot t$ (Bredt):

$$\tau \cdot t = \frac{T}{2A_0} \quad \text{with } A_0 = b_0 \cdot h_0 \quad (\tau \cdot t = \tau_i \cdot t_i \nabla i)$$

→ shear force per element of the cross-section, with thickness t_i and length l_i : $V_i = \tau \cdot t \cdot l_i$

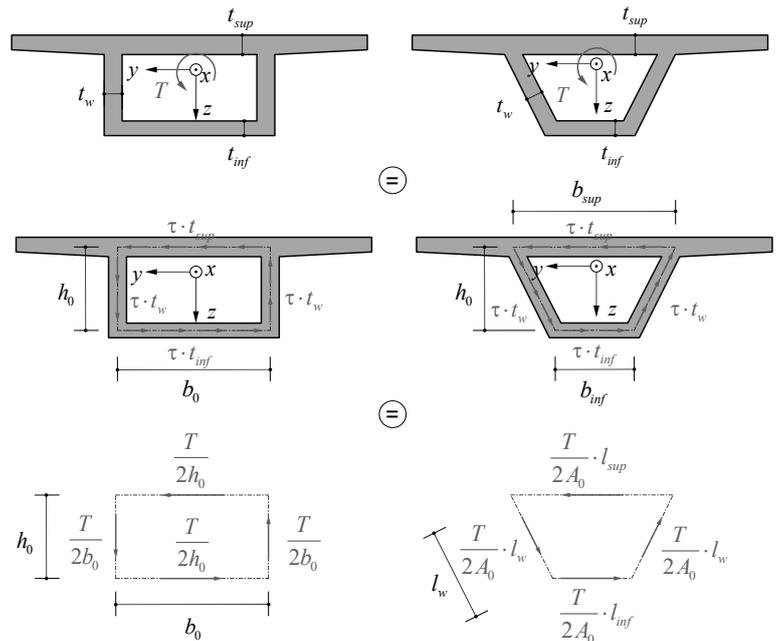
→ shear forces in webs and top / bottom slab of an orthogonal box girder:

$$V_w = \pm \tau \cdot t \cdot h_0 = \pm \frac{T}{2b_0} \quad V_{\text{sup,inf}} = \pm \tau \cdot t \cdot b_0 = \pm \frac{T}{2h_0}$$

→ ditto, for box girder with inclined webs:

$$\tau \cdot t = \frac{T}{2A_0} \quad \text{with } A_0 = \frac{b_{\text{sup}} + b_{\text{inf}}}{2} h_0$$

$$V_i = \tau \cdot t \cdot l_i \quad \text{with } l_w = \sqrt{h_0^2 + \left(\frac{b_{\text{sup}} - b_{\text{inf}}}{2}\right)^2}$$



Note that in the case of an orthogonal box girder, the force couples in the webs and in the flanges, respectively, contribute 50% to the torsional moment each.

In multi-cell box girders, the inner cells contribute little to the torsional resistance; usually, it is sufficient to neglect the interior webs for torsion, using the formulas given above.

Spine model – Global analysis: Torsion in box girders (stiffness)

The torsional stiffness for thin-walled, homogeneous hollow cross-sections (steel “a” or uncracked concrete “c”) is

$$GK = \frac{4A_0^2 G}{\oint \frac{ds}{t}} = \frac{4A_0^2 G}{\sum \frac{l_i}{t_i}} \quad \left(G = \frac{E}{2 \cdot (1 + \nu)} \right)$$

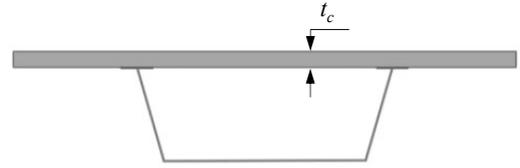
In composite cross-sections, using the steel as reference material (E_a), accordingly

$$GK = \frac{4A_0^2 G_a}{\sum \frac{n_i \cdot l_i}{t_i}} \quad \left(G_a = \frac{E_a}{2 \cdot (1 + \nu)}, n_i = \frac{E_a}{E_i} \right)$$

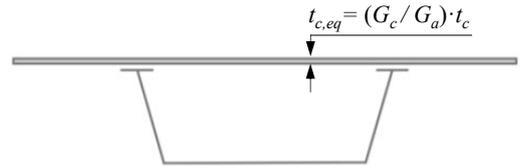
For cracked concrete, the determination of GK is more complicated. For a concrete box girder with constant wall thickness, having a uniformly distributed stirrup reinforcement ρ_w and longitudinal reinforcement ρ_l :

$$GK'' = \frac{4A_0^2 E_s}{\frac{\cot^2 \alpha}{\rho_l} + \frac{\tan^2 \alpha}{\rho_w} + n \cdot (\tan \alpha + \cot \alpha)^2} \frac{t}{\sum l_i} \quad \left(\tan \alpha = \sqrt[4]{\frac{\rho_l^{-1} + n}{\rho_w^{-1} + n}} \right)$$

see lecture notes Stahlbeton I (E_s = stiffness of reinforcement).



real section



equivalent section

The slide shows the torsional stiffness of a box concrete girder for a given reinforcement based on the compression field model (for derivation see Advanced Structural Concrete lecture, stiffness of cracked-elastic disk elements).

In multi-cell box girders, the inner cells hardly contribute to the torsional stiffness; it is sufficient to neglect the interior webs for torsion, using the formulas given above.

In composite bridges with narrow or shallow box girders, the torsional stiffness of the deck slab may contribute significantly to the overall torsional stiffness. Some textbooks recommend to add the torsional stiffness of the deck slab (approximately $GK_c = b_c t_c^3 / 3$) to the stiffness of the box girder. Strictly speaking, this is not correct, since the area of the deck slab between the webs is activated twice.

For a derivation of the torsional stiffness of the cracked concrete box girder, see lecture notes Stahlbeton I.

Spine model – Global analysis: Torsion in box girders (stiffness)

If the bottom slab is replaced by trusses, being part of a closed cross-section, the torsional stiffness may be calculated using an effective thickness.

The corresponding values of the equivalent thicknesses may be obtained e.g. using the work method.

The table on the right gives values for usual truss typologies (from Lebet and Hirt, 2013).

Trussed webs may be treated similarly.

Equivalent thicknesses of other truss layouts are obtained by applying the virtual work equation (for a unit shear deformation) and equating the deformation of the solid plate to that of the truss.

t_{eq}	Plan Bracing Geometry
$\frac{E}{G} \frac{as}{\frac{d^3}{A_{dia}} + \frac{a^3}{3} \left(\frac{1}{A_g} + \frac{1}{A_d} \right)}$	
$\frac{E}{G} \frac{as}{\frac{2d^3}{A_{dia}} + \frac{s^3}{4A_t} + \frac{a^3}{12} \left(\frac{1}{A_g} + \frac{1}{A_d} \right)}$	
$\frac{E}{G} \frac{as}{\frac{d^3}{2A_{dia}} + \frac{a^3}{12} \left(\frac{1}{A_g} + \frac{1}{A_d} \right)}$	
$\frac{E}{G} \frac{as}{\frac{d^3}{A_{dia}} + \frac{s^3}{A_t} + \frac{a^3}{12} \left(\frac{1}{A_g} + \frac{1}{A_d} \right)}$	

Superstructure / Girder bridges

Bridge Girder – Spine model – Transverse analysis
(Einstabmodell, Querrichtung)

Spine model – Transverse analysis: Limitations of spine model

In the spine model, the girder is idealised as a beam:
 → results of the global analysis are the internal actions
 = stress-resultants acting on the entire cross-section.

In reality, the girder is not a beam that merely transfers loads applied to its axis longitudinally. Rather

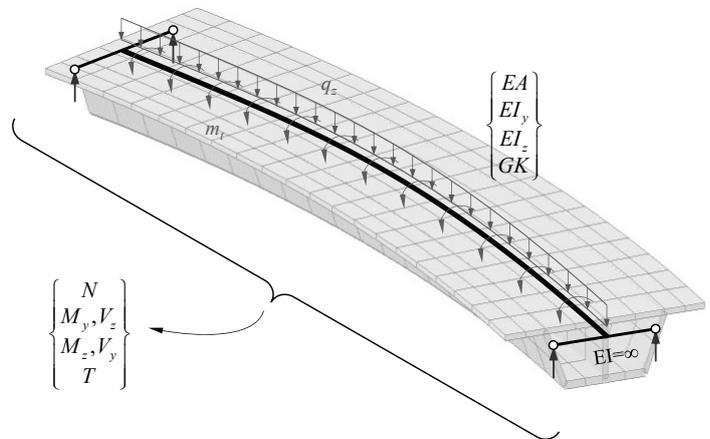
- loads also need to be carried in transverse direction
- The cross-section is not rigid but may be distorted

The spine model does not yield direct information on this transverse behaviour, particularly regarding:

- local bending of the deck
- introduction of torques
- warping torsion

Hence, these effects need to be investigated separately. This is feasible with reasonable effort and accuracy for box girders and solid cross-sections, see following slides.

For girders with open cross-sections, this does not apply, and a spine model is therefore usually inappropriate (see *spine model for open cross-sections*).



Illustrations: Menn [1990]

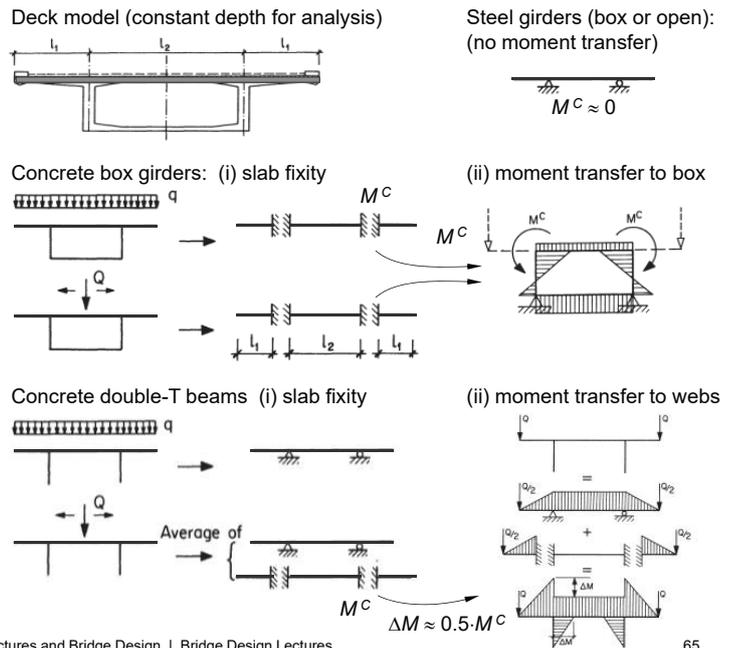
Spine model – Transverse analysis: Transverse bending

Local bending of the deck has been dealt with in *bridge deck*. The bottom slab of box girders can be modelled accordingly (primarily carries self-weight).

The support moments obtained from the deck slab analysis (usually only in concrete girders) need to be applied to the girder to ensure equilibrium. Usually, primarily the cantilever moment M^C is relevant.

These moments cause transverse bending of the longitudinal girders as illustrated in the figure for symmetrical load on the cantilevers.

In box girders, more general load combinations can be analysed using the frame model shown in the figure. For open cross-sections, this is more complicated, see e.g. [Menn 1990, 5.3.1].



Illustrations: Menn [1990]

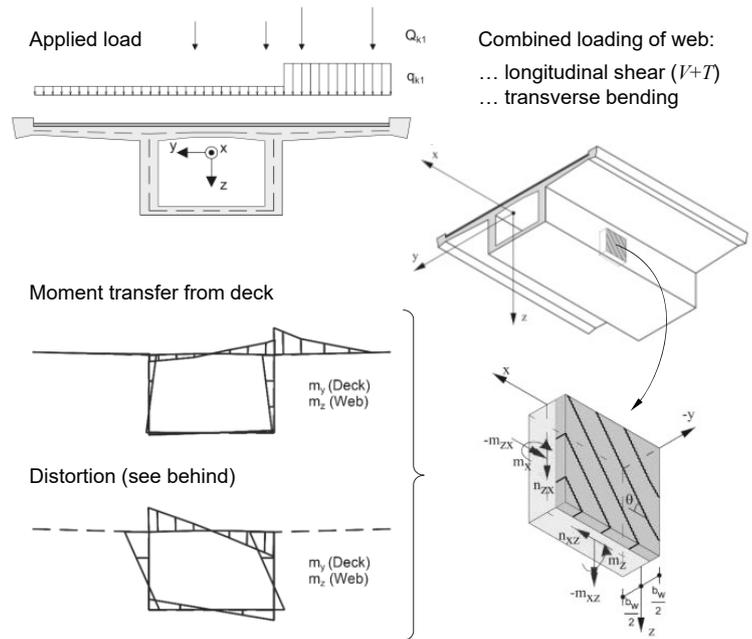
Spine model – Transverse analysis: Transverse bending

The web of concrete box girders is typically much thicker, and therefore stiffer than the deck:

- most of the cantilever moments are transferred to the web
- further transverse bending moments are caused by torque introduction, see behind
- webs of concrete box girders need to be designed for the combination of longitudinal shear and transverse bending

When widening existing bridges by increasing the deck cantilevers, neglecting moment transfer from the deck to the webs may be unsafe even if the deck is designed to resist the full bending moments. It should always be checked if the webs have

- sufficient capacity to resist higher transverse bending moments due to widening (combined with the longitudinal shear), or
- sufficient deformation capacity to justify neglecting transverse bending moments in the webs



Note that the web is generally also subjected to general loading, including axial normal forces, longitudinal bending moments and torsional moments. These additional loads are usually considered as generalised reactions, rather than generalised stresses (i.e. they may assume any value, maximising the resistance in transverse bending and in-plane shear). This may be justified for ductile elements, based on the lower-bound theorem of plasticity theory.

Current research at the Chair of Concrete Structures and Bridge Design (D. Karagiannis) investigates the applicability of this assumption to existing bridges with potentially lacking ductility.

Spine model – Transverse analysis: Transverse bending

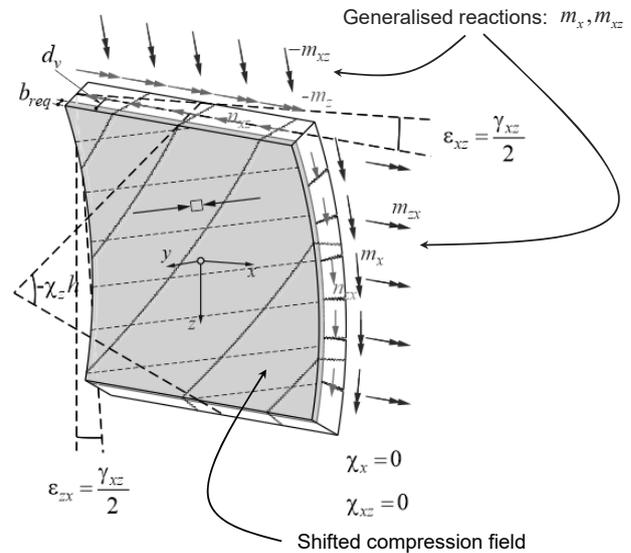
The combined application of transverse bending and in-plane shear leads to a simultaneous:

- shift of the compression field towards the flexural compressive side of the web, which in turn is facilitated by / requires...
- generalised reactions (the shift of the compression field corresponds to twisting moments m_{zx} and bending moments m_x)

These generalised reactions are able to develop due to the web being restrained against twisting and longitudinal bending by the deck and bottom flange.

Note that generally, the principal compressive direction varies throughout the thickness of the web. In the following, a simpler equilibrium model, with a compression field of constant inclination, but shifted to the flexural compression side of the web, is considered (see notes for additional remarks).

Web element loaded in in-plane shear and transverse bending



Generally, the principal compressive direction varies throughout the thickness of the web, which complicates the analysis; for details see Marti, Zur Plastischen berechnung von Stahlbeton, IBK Bericht 104, 1980. In the lecture, a compression field of constant inclination, but shifted to the flexural compression side of the web, is considered.

Alternatively, the design of the web could be based on a sandwich model (see Advanced Structural Concrete) with both covers transferring half of the shear (current recommendation in the Eurocode). However, this neglects the favourable effect of the generalised reactions (and the resulting shift of the compression field) hence leading to considerably lower capacities and is therefore not considered further in the lecture.

Spine model – Transverse analysis: Transverse bending

The minimum required width to transfer the shear force is:

$$b_{req} = \frac{n_{xz}}{f_{c,eff} \cos(\theta_c) \sin(\theta_c)}$$

Equilibrium (compression field shifted as much as possible to the flexural compression side) requires:

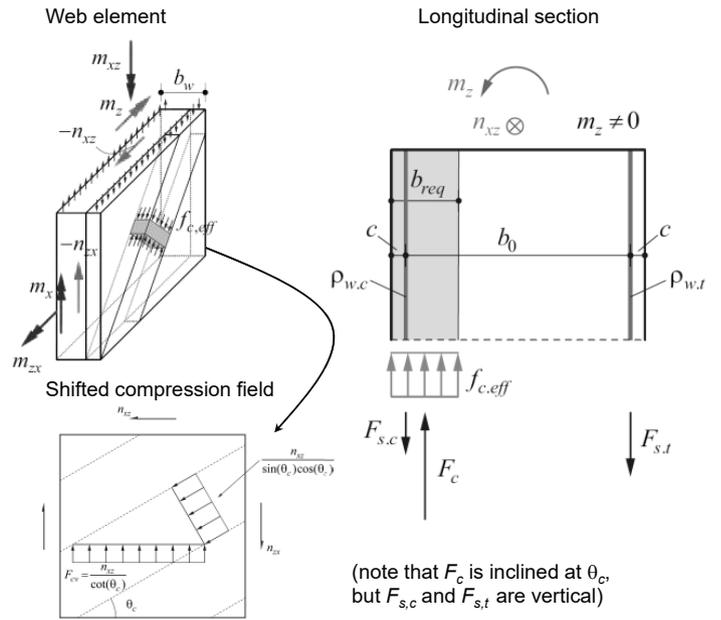
$$\frac{n_{xz}}{\cot(\theta_c)} - F_{s,c} - F_{s,t} = 0, \quad F_{s,t} b_0 - \frac{n_{xz}}{\cot(\theta_c)} \left(\frac{b_{req}}{2} - c \right) - m_z = 0$$

which can be solved for the stirrup forces:

$$F_{s,c} = \frac{n_{xz}}{b_0 \cot(\theta_c)} \left(b_0 - \frac{b_{req}}{2} + c \right) - \frac{m_z}{b_0},$$

$$F_{s,t} = \frac{n_{xz}}{b_0 \cot(\theta_c)} \left(\frac{b_{req}}{2} - c \right) + \frac{m_z}{b_0}$$

The above equations are valid for the case of predominant shear force.



Spine model – Transverse analysis: Transverse bending

In the case of predominant transverse moment, the force in the stirrups on the compressive side is assumed to be zero, $F_{s,c} = 0$. $F_{c,m}$ is the bending compression force acting on a width equal to:

$$b_m = \frac{F_{c,m}}{f_c}$$

The two equilibrium equations are thus:

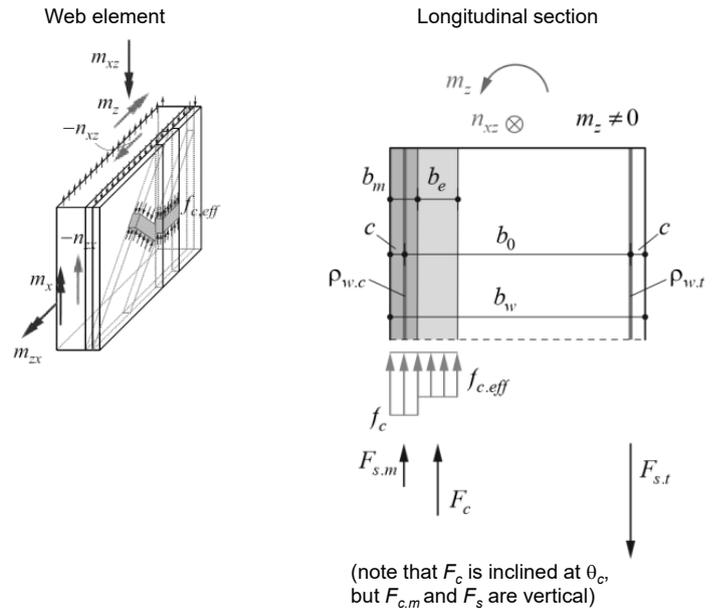
$$\frac{n_{xz}}{\cot(\theta_c)} - F_{s,t} + F_{c,m} = 0,$$

$$m_z - F_{s,t} \left(b_w - c - \frac{b_m}{2} \right) + \frac{n_{xz}}{\cot(\theta_c)} \left(\frac{b_e}{2} + \frac{b_m}{2} \right) = 0$$

and the stirrup force on the tensile side is given by:

$$F_{s,t} = \frac{m_z + \frac{n_{xz}}{\cot(\theta_c)} \left(\frac{b_e}{2} + \frac{b_m}{2} \right)}{b_w - c - \frac{b_m}{2}}$$

Interaction diagrams based on these equations, suitable for design purposes can be found in: [Menn 1990, 5.3.2].



In the case of predominant transverse bending, the tensile force in the stirrups on the compression side vanishes and an additional concrete compressive force is introduced to maintain equilibrium. The concrete compression due to shear is shifted towards the centreline of the of the web.

Spine model – Transverse analysis: Torsion in box girders (general)

Box girders resist torsion primarily by uniform torsion but torques are typically applied by eccentric vertical or horizontal forces (rather than circumferential loads). Hence

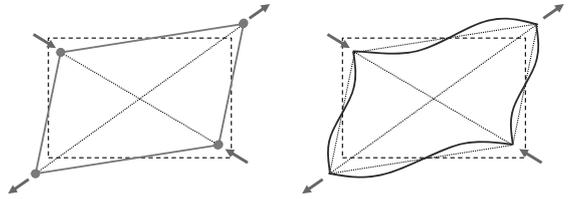
- introduction of torques tends to distort the cross-section (see upper figures and next slides), causing
- significant warping torsion and corresponding longitudinal stresses unless distortion of the cross-section is impeded

Longitudinal stresses due to distortion of box girders are difficult to quantify (complex analysis required)

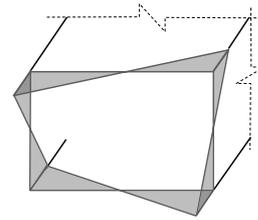
- box girders are usually designed to avoid significant distortion, which can be achieved
 - ... by a transversely stiff cross-section acting as frame (upper right figure)
 - ... by an adequate number of sufficiently stiff diaphragms if the girder lacks transverse stiffness (upper left figure)

Note: Even without distortional loading, the cross-section of box girders generally warps, see bottom figure. However, this does not cause significant stresses (see notes for details).

Distortion of a rectangular cross-section with hinged connections (left) and stiff corners (right): displacements in the transverse direction



Warping of a rectangular cross-section: longitudinal stress-free displacements (unless warping is restrained)



Warping is a longitudinal deformation of the cross-section, occurring if the longitudinal displacements $u(y,z)$ are not planar (i.e. plane sections not remaining plane). *Distortion* designates a transverse change of shape of the cross-section in the (y,z) -plane, caused by transverse bending, as illustrated on the next slides. *Warping torsion* is linked to the distortion of the cross-section and involves longitudinal stresses.

Even in uniform torsion, the cross-section of a box girder will generally warp, unless the thickness of all elements constituting the thin hollow section is inversely proportional to their distance from the shear centre – which would ensure proportionality of shear stresses (and hence, shear strains) due to *uniform torsion without warping* to the distance from the shear centre (measured perpendicularly to the stresses considered). This requirement for uniform torsion without warping is hardly ever satisfied: Typical box girders are wider than deep, but the webs are thicker than the slabs.

Unless warping is restrained, the cross-section of a girder acting in uniform torsion will thus usually warp (to ensure proportionality of the shear strains with the distance from the shear centre). However, no stresses are caused by this warping. For a rectangular box girder, the warping function (longitudinal deformation of the cross-section due to warping) is a hyperbolic paraboloid aligned with the principal axes of the cross-section, as illustrated in the upper figure above. For further details, see Marti, Theory of Structures. If warping is restrained, longitudinal stresses develop; however, these are of minor importance in box girders, as their uniform torsional stiffness is much higher than the warping torsional stiffness (which is activated by restraining warping of the cross-section).

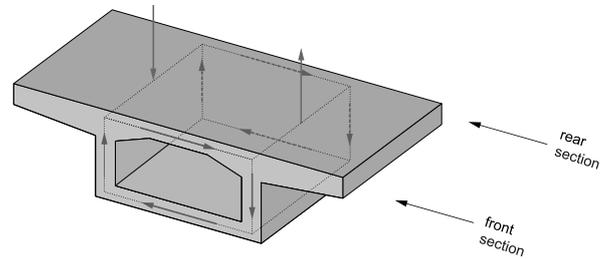
Spine model – Transverse analysis: Introduction of eccentric loads

In the following slides, the introduction of torques in box-girders due to different types of load (concentrated, distributed, horizontal, vertical) is outlined. In all cases,

- applied torques and circumferential shear flow are statically equivalent (= in equilibrium)
- the load introduction (the transformation of torques to a circumferential shear flow) causes a self-equilibrated set of distortional forces

Depending on static system and load position along girder

- the percentage of the applied torque transferred in positive and negative x -direction varies, but
- the change of the torsional moments (circumferential shear flows) in two sections in the span is always statically equivalent to the torque applied between these sections.



Spine model – Transverse analysis: Introduction of eccentric loads

Concentrated torques due to vertical force couples are usually caused by traffic loads (concentrated loads representing vehicle axle loads).

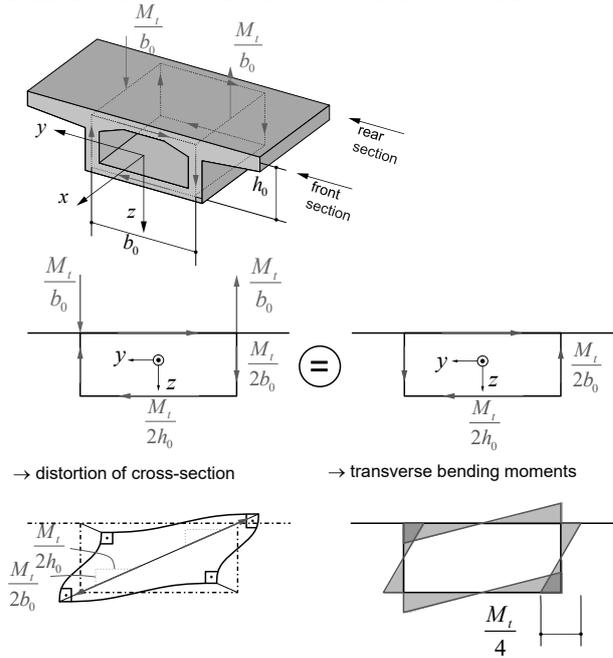
The figure illustrates the forces acting on the free body (girder between front and rear sections):

- applied loads
- circumferential shear flow

The sum of these forces (per side of the cross-section) are the distortional forces, which can alternatively be represented by two equal diagonal distortional forces of opposite sign (passing through the corners since loads are applied in the web axes).

The cross-section tends to distort rhombically due to the distortional forces. If it has a transverse bending resistance, distortion is restrained by transverse bending.

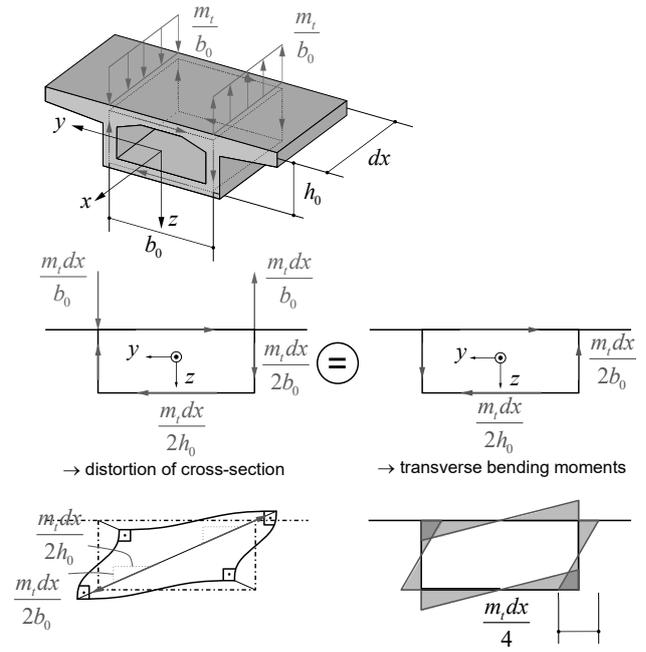
Otherwise, furthermore, distortion of the cross-section is hindered only by longitudinal bending of its elements, i.e., warping torsion, over the distance to the next intermediate diaphragm impeding distortion.



Spine model – Transverse analysis: Introduction of eccentric loads

Distributed torques due to vertical line load couples may be due to traffic loads (e.g. line load of ballastless track rail) or superimposed dead loads (e.g. crash barriers).

(further comments see previous slide)



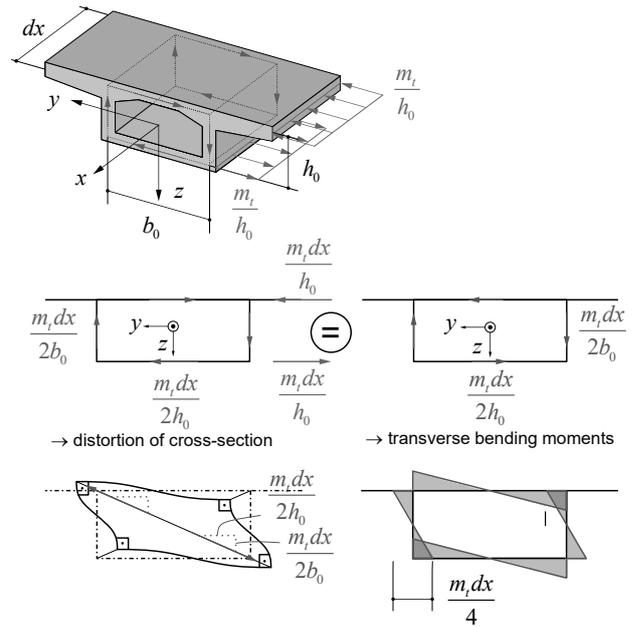
Spine model – Transverse analysis: Introduction of eccentric loads

Distributed torques due to horizontal line load couples may be due to wind or girder curvature in plan.

Torques applied by horizontal forces couples are particularly relevant in curved bridges, as commented on slide on torsion in curved bridges (general).

Distortional forces caused by a torque applied through a horizontal force couple have opposite signs compared to those caused by a torque of equal sign applied through a vertical force couple.

(further comments see previous slide)



Spine model – Transverse analysis: Introduction of eccentric loads

The distortional forces obtained by applying vertical force couples in the web axes (as in the previous slides) are usually on the safe side.

If the loads are applied on the cantilever, a smaller distortional force results (see figure on the right, noting that R is aligned to the diagonal of the section with its vertical component corresponding to the distortional force).

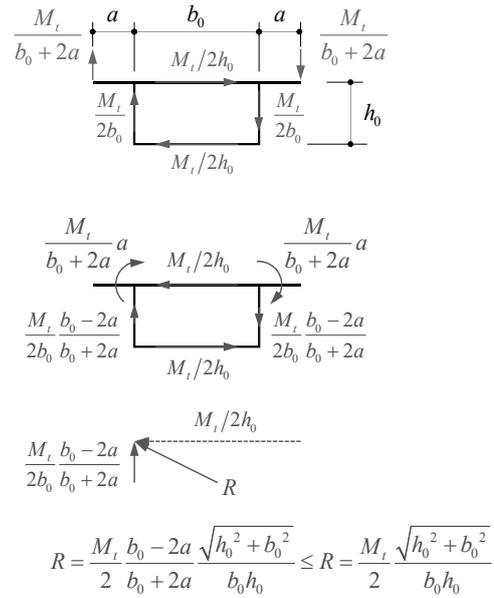


Figure: Adapted from Menn, Prestressed concrete bridges (1990).

Spine model – Transverse analysis: Torsion design of box girders

Concrete box girders are significantly stiffer in the transverse direction than steel and composite box girders.

Straight or slightly curved concrete box girders usually have

- sufficient strength to introduce torques applied in the span
- sufficient stiffness to prevent significant distortion of the cross-section without intermediate diaphragms

→ intermediate diaphragms are only required in strongly curved concrete box girders.

Contrary to concrete box girders, steel or composite box girders are usually unable to resist significant torques applied in the span, nor to provide adequate restraint to distortion of the cross-section, without intermediate diaphragms.

→ several intermediate diaphragms (usually about 5) per span are therefore provided even in straight steel and composite box girders

Hence, there are considerable differences in the torsion design of concrete and steel or composite box girders, see next slide.



More details and justification see e.g. [Menn 1990].

Photos: Arrollo de las Piedras viaduct, Spain, 2006. IDEAM

Spine model – Transverse analysis: Torsion design of box girders

The design of box girders for torsion avoiding significant distortion thus usually involves the following:

- dimension the box girder to resist the full applied torsional moment in uniform torsion
- account for the higher shear forces caused by eccentric loads in the longitudinal shear design i.e. design for higher shear forces over distance to next diaphragm (or length required to convert torques to uniform shear), see next slide.
- provide support diaphragms to introduce concentrated torques

Additionally, only for steel and composite box girders:

- dimension intermediate diaphragms to introduce torques applied in the span
- provide intermediate diaphragms with adequate stiffness to prevent significant warping of the cross section

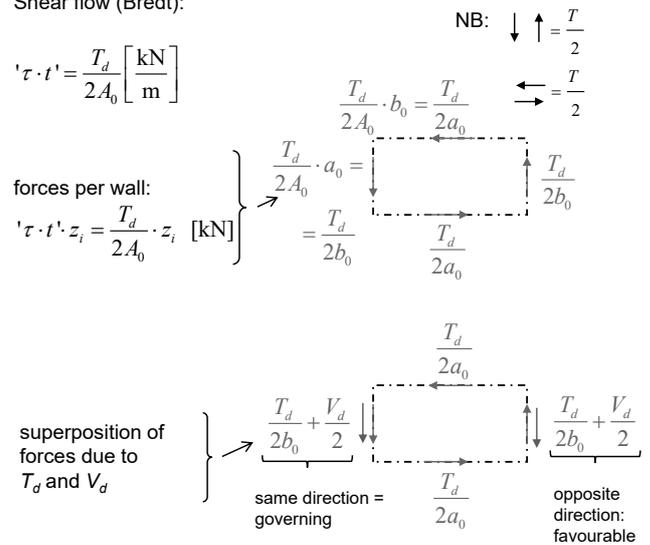
Additionally, only for concrete box girders:

- dimension the cross-section for transverse bending caused by the introduction of torques applied in the span (to be superimposed with transverse bending due to moment transfer from deck, and longitudinal shear)

Shear flow (Bredt):

$$' \tau \cdot t ' = \frac{T_d}{2A_0} \left[\frac{\text{kN}}{\text{m}} \right]$$

forces per wall: $' \tau \cdot t ' \cdot z_i = \frac{T_d}{2A_0} \cdot z_i$ [kN]



Neglecting warping torsion in ULS design can be justified by the lower bound theorem of plasticity theory, even if significant warping torsion is expected to occur in the serviceability state, as long as sufficient ductility is provided. This usually applies in concrete box girders, but, since an economic design of steel girders usually involves slender elements susceptible to instabilities, is not applicable to steel and composite box girders.

Therefore, a check of the sufficient stiffness of the cross-section to prevent relevant distortion is hardly ever required in concrete box girders, but checking the stiffness of the intermediate diaphragms of steel and composite box girders is necessary.

Spine model – Transverse analysis: Torsion design of box girders

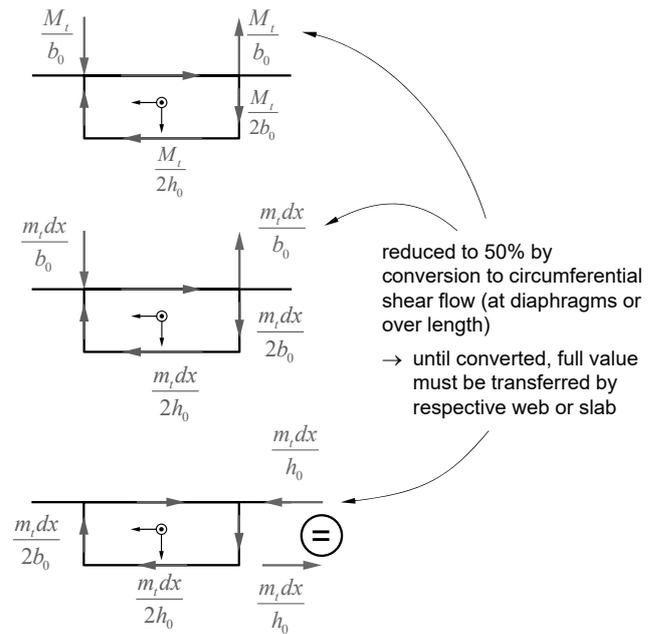
Since the applied torques are only converted to a circumferential shear flow

- by intermediate diaphragms
 - by transverse bending of the cross-section, which requires a certain length, or
- higher shear forces than obtained assuming a circumferential shear flow need to be accounted for in longitudinal shear design:

in girders with intermediate diaphragms:
 ... for concentrated and distributed torques
 ... over the distance to the next intermediate diaphragm

in concrete box girders without intermediate diaphragms
 ... for concentrated torques (*)
 ... over the distance required to introduce torques by transverse bending

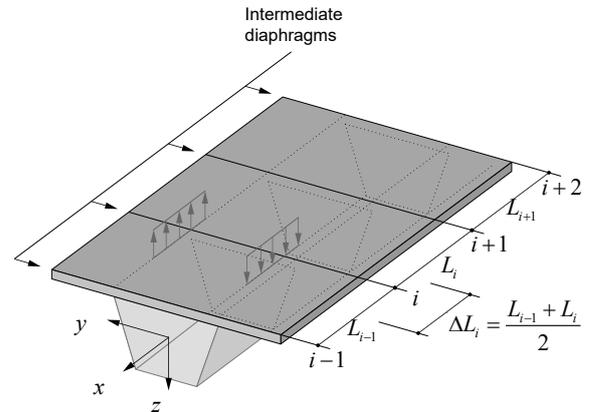
(*) If transverse bending moments due to distributed torque introduction exceed the shear+transverse bending capacity of a concrete girder, intermediate diaphragms are required.



Spine model – Transverse analysis: Design of intermediate diaphragms

Intermediate diaphragms are designed to

- introduce torques applied in the span
 - each diaphragm needs to resist the distortional forces over its respective share of the span ΔL_i (see figure)
 - neglecting contributions from the cross-section between the diaphragms (even in concrete girders)
- provide adequate stiffness to prevent significant distortion of the cross section of steel and composite box girders; commonly accepted criteria (based on numerical studies) to achieve this are:
 - minimum stiffness shall limit normal stresses due to warping torsion (caused by distortion) to $\leq 5\%$ of the normal stresses due to global bending, which is in turn
 - deemed to be satisfied if the following is provided
 - ... 5 solid steel plate diaphragms per span or
 - ... 5 cross-bracings per span, each with a distortional stiffness of $\geq 20\%$ of a 20 mm steel plate diaphragm (see e.g. Lebet and Hirt, 2013 for more details)



Spine model – Transverse analysis: Design of intermediate diaphragms

In summary, the design of the intermediate diaphragms is determined by:

- Minimum stiffness to control longitudinal stresses due to distortion
- the table shows the distortional stiffnesses of the most used cross bracings in a steel or steel-concrete composite box section
- Resistance required for torque introduction (and bending if used as support for deck)

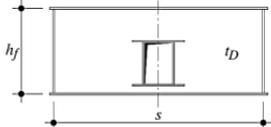
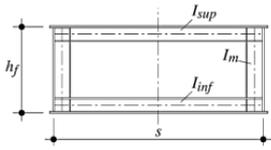
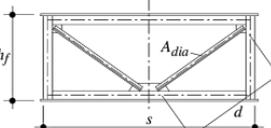
	<p>Diaphragm cross bracing</p> $K_D = G t_D s h_f$
	<p>Frame cross bracing</p> $K_D = \frac{24 E I_m}{\alpha_0 h_f}$ $\alpha_0 = 1 + \frac{(2s)/h_f + 3(I_{inf} + I_{sup})/I_m}{(I_{inf} + I_{sup})/I_m + (6h_f/s) \cdot (I_{inf} I_{sup}/I_m^2)}$
	<p>V Truss cross bracing</p> $K_D = \frac{E A_{dia} s^2 h_f^2}{2 d^3}$

Table adapted from J.P. Lebet and M.A. Hirt, Steel bridges

Spine model – Transverse analysis: Design of intermediate diaphragms

The minimum stiffness requirement ($\geq 20\%$ of a 20 mm steel plate diaphragm) given on the previous slide is simple, but strict and arbitrary.

Alternatively, the minimum stiffness of intermediate diaphragms to comply with the “ $\leq 5\%$ normal stress” criterion can be determined by modelling the box girder as illustrated schematically in the figure on the right:

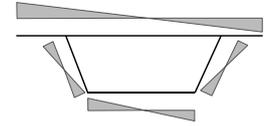
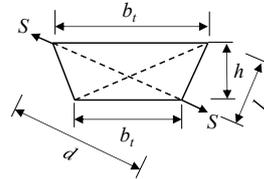
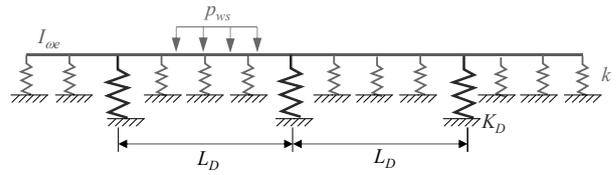
→ the distortion of a box girder, elastically restrained by the distortional stiffness of the cross-section (transverse frame) and cross-bracings

$$EI_{oe} \frac{d^4 w}{dx^4} + kw = p_{ws}$$

I_{oe} = warping moment of inertia
 w = web movement contained in its plane
 k = distortional stiffness

→ is analogous to a beam on elastic foundation

$$EI \frac{d^4 w}{dx^4} + kw = q$$



K_D = cross-bracing distortional stiffness
 k = box distortional stiffness
 L_D = diaphragm spacing
 M_Q = concentrated torsion moment
 m_q = distributed torsion moment
 M_f = bending moment
 R = radius in plan

$$S = p_{ws} \frac{I_w}{d}$$

$$p_{ws} = \frac{M_t}{(b_t + b_b)} \frac{b_b I_w}{b_t h}$$

$$p_{ws} = \frac{M_t}{2b} \text{ (for rectangle)}$$

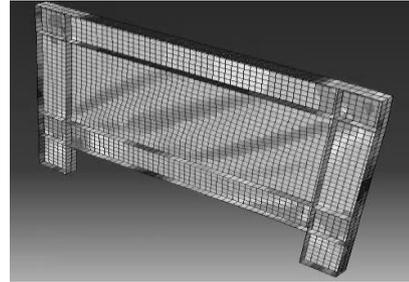
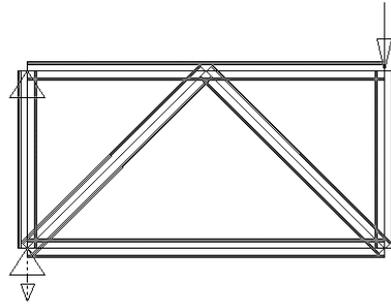
$$M_t = M_Q + m_q L_D + \frac{M_f}{R} L_D$$

For more details, see e.g. Viñuela Rueda, L.; Martínez Salcedo, J., Proyecto y Construcción de Puentes Metálicos y Mixtos. Publicaciones APTA

Spine model – Transverse analysis: Design of intermediate diaphragms

To design an intermediate diaphragm by resistance, the structural element is isolated and all actions acting on it are applied (ensuring that all forces are auto-equilibrated):

- torsion due to eccentricity of external loads and geometry in curved bridges (see previous slides)
 - loads acting directly on the diaphragm
 - forces due to its function as transverse stiffener (steel and steel-concrete composite cross-section)
- Truss, frame or stiffened diaphragm cross bracing: Truss analysis (usually using commercial frame analysis software)
- Solid diaphragm: Strut-and-tie model / stress field, or FE analysis (membrane element, linear elastic for steel diaphragms, nonlinear analysis for concrete diaphragms, see Advanced Structural Concrete)



Illustrations: top-right: <https://www.eatherton.cee.vt.edu/>; bottom: analysis of diaphragm using CSFM (Idea Statica Detail) © Fürst Laffranchi GmbH

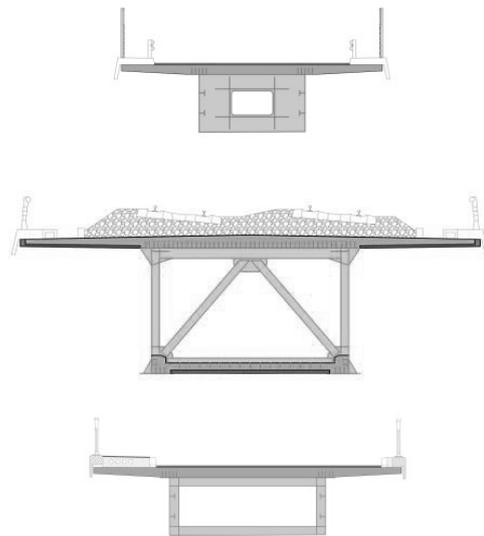
Spine model – Transverse analysis: Intermediate diaphragm types (steel)

Intermediate diaphragms should

- be lightweight (minimise self-weight)
- allow access (passage) for inspection

The following are used in steel and composite bridges:

- Solid diaphragm (steel plate)
 - + high stiffness
 - high weight → cost
 - usually inefficient (minimum thicknesses)
 - limited access (manholes reduce stiffness)
- V-truss cross-bracing
 - ± moderate stiffness
 - ± moderate weight
 - + efficient
 - + good access
 - many connections
- Frame cross-bracing
 - low stiffness
 - ± moderate weight
 - + good access

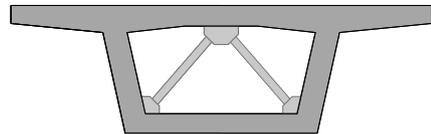
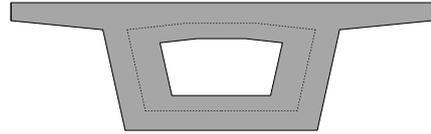
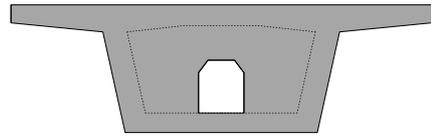


Spine model – Transverse analysis: Intermediate diaphragm types (concrete)

Intermediate diaphragms in concrete box girders should be avoided. If required, complication of the construction process should be minimised (moving internal formwork).

The following solutions are used in concrete bridges:

- Solid with manhole
 - + high stiffness
 - high weight
 - completely obstructs moving of internal formwork
 - complicated removal of diaphragm formwork
- Concrete frame
 - ± moderate stiffness
 - ± moderate weight
 - ± easier moving of internal formwork
 - complicated diaphragm formwork
- Steel bracing (post-installed)
 - low stiffness
 - + low weight
 - + perfect solution for moving internal formwork
 - complicated connections



Spine model – Transverse analysis: Support diaphragms

Piers and abutments provide:

- vertical support (virtually always) ...
- torsional restraint (abutments always, piers often) ...
- transverse horizontal fixity (usually) ...
- longitudinal horizontal fixity (in some cases) ...

to the girder, see *bearing layout and dilatation concept*.

The support reactions need to be transferred to the girder (converted to forces acting in the planes of the webs and slabs of the cross-section)

→ Support diaphragms

Note: Since the vertical reactions are smaller at the abutments (end support of continuous girder) than at intermediate supports, the transverse distance between the bearings b_R should be as large as possible to avoid uplift (despite the transverse bending caused by the eccentricity of vertical supports to the web axes).



Photos: Viaducto de Arbizelej, Spain © Fhecor Ingenieros (above); Viaducto sobre la Presa Mularroya, Spain © IDEAM (below)

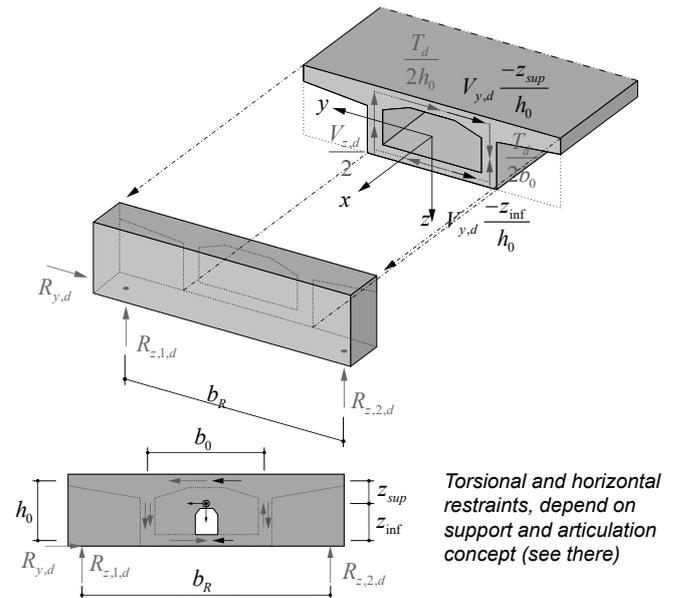
Spine model – Transverse analysis: Design of support diaphragms

Torsional restraint is usually provided by vertical support reactions, hence support diaphragms need to resist

- distortion due to torque introduction (analogous to intermediate diaphragms) and
- significant transverse bending (resisted by cross-section in the span) unless bearings are located in the web axes

The support diaphragms have to resist much higher forces than intermediate diaphragms, since

- support torques correspond to the integral of torques applied over half the torsion span
- support reactions correspond to the integral of loads applied over the distance to the point of zero shear.
- support diaphragms required also in straight concrete girders



Piers and abutments provide:

- vertical support (virtually always) ...
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to the girder, see *bearing layout and dilatation concept*. The support reactions need to be transferred to the girder (converted to forces acting in the planes of the webs and slabs of the cross-section).

Since the vertical reactions are smaller at the abutments (end support of continuous girder) than at intermediate supports, the transverse distance between the bearings b_R should be as large as possible to avoid uplift (negative support reactions).

Spine model – Transverse analysis: Design of support diaphragms

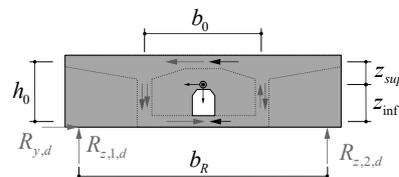
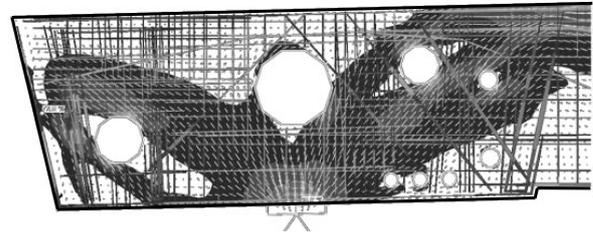
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 - support reactions correspond to the integral of loads applied over the distance to the point of zero shear.
- support diaphragms required also in straight concrete girders

Solid end diaphragms are therefore often required. These are usually designed based on a plane stress analysis (concrete diaphragms → stress fields by hand or CSFM, see advanced structural concrete, steel diaphragms → FEM).



Torsional and horizontal restraints, depend on support and articulation concept (see there)

Piers and abutments provide:

- vertical support (virtually always) ...
- torsional restraint (abutments always, piers often) ...
- transverse horizontal fixity (usually) ...
- longitudinal horizontal fixity (in some cases) ...

to the girder, see *bearing layout and dilatation concept*. The support reactions need to be transferred to the girder (converted to forces acting in the planes of the webs and slabs of the cross-section).

Since the vertical reactions are smaller at the abutments (end support of continuous girder) than at intermediate supports, the transverse distance between the bearings b_R should be as large as possible to avoid uplift (negative support reactions).

Superstructure / Girder bridges

Bridge Girder – Spine model for open cross-sections

Spine model for open cross-sections: General remarks

Using a spine model for girders with open cross-section is inefficient, because (as outlined on the following slides):

- the contributions of uniform torsion and warping torsion to the total torsional moment vary along the span and depend
 - ... on the static system and
 - ... the position of applied torques
- design for several load-cases tedious
- analysis cannot be carried out efficiently (using e.g. structural analysis software for 2D or 3D frames)

Furthermore, investigating the transverse behaviour of girders with open cross-section based on the results of a spine model is even more demanding than for box girders (which is already demanding, twice as many slides as for global analysis ...):

- transfer of a significant part of torsional moments by warping torsion results in
 - substantial distortion of the cross-section (by torsion, not only by torque introduction as in box girders)
 - significant longitudinal stresses due to torsion
 - high transverse bending moments due to torsion



Photo: Mac Arthur Causeway East bridge, Florida, USA. Pdelta (<http://pdelta.com>)

Spine model for open cross-sections: General remarks

In spite of these inconveniences, spine models were frequently used in the past for the analysis of girders with open cross-section, since more complex 2D or 3D-models required a much higher computational effort (which was critical before the advent of modern, user-friendly structural analysis software and affordable personal computers).

Today, running a grillage analysis (see *grillage model*), or even using a folded plate model, is

- more efficient and
- yields more detailed insight into the structural behaviour, particularly regarding transverse load transfer

→ Use of grillage models is recommended for girders with open cross-section

The application of spine models to girders with open cross-section is treated here only to the extent required for understanding the basic concepts of older design recommendations and codes, and because it is still useful for preliminary design of double-T girders, as illustrated on the following slides.



Photos: Top Rehabilitation of the Viaducto de Santurtzi / Santurce, Bilbao © Arenas&Asociados..

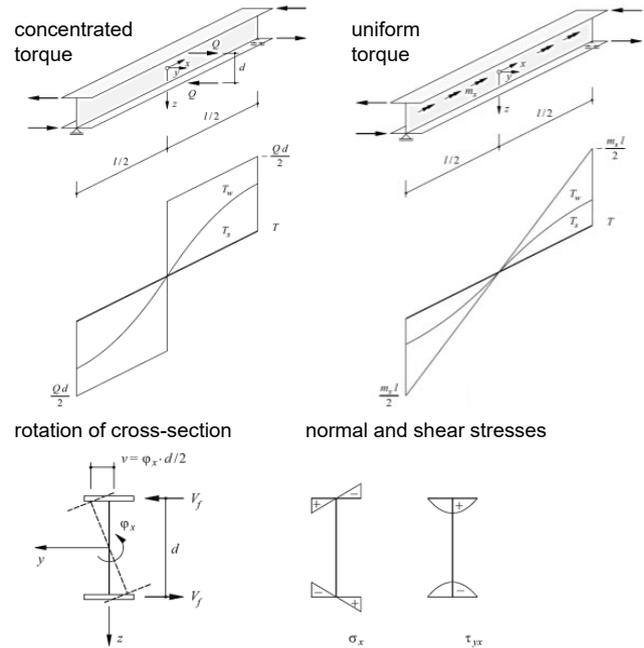
Spine model for open cross-sections: General remarks

Girders with open cross-section transfer eccentric loads primarily by warping torsion (antisymmetric bending), rather than uniform torsion

- cross-section is significantly distorted by torsional moments
- share of torque transferred by warping torsion T_w and uniform torsion T_s , respectively, varies ...
 - ... depending on position of applied torque
 - ... along the span
- complicated analysis, particularly in the case of wide bridges with more than two webs (idealisation as spine not reasonable!)

In simple cases the longitudinal behaviour of girders with open cross-section can though be analysed with a spine model.

As an example, see figure on the right (from P. Marti, Theory of Structures, Section 13.4.3). The behaviour of girders with two webs will be treated in the following as the I-beam in this example, but rotated by 90° .



Illustrations: P. Marti, Theory of Structures, Section 13.4.3

Spine model for open cross-sections: Equilibrium model

Generally, eccentric loads acting on girders with open cross-section can be decomposed analogously as in box girders. For example (figure), distributed loads are decomposed in a symmetrical force f_z and a torque m_t .

In symmetric girders (with respect to the z-axis), carrying torsion by a combination of uniform and warping torsion

$$T = T_s + T_w \quad m_t = m_{t,s} + m_{t,w}$$

→ equivalent design loads applied to half-girders:

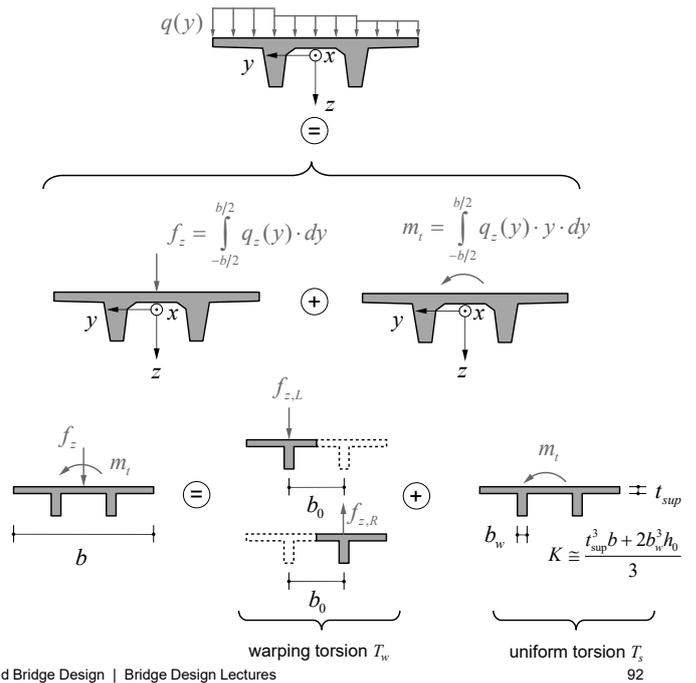
- half the applied vertical load f_z and an additional vertical load corresponding to the torques transferred by warping torsion T_w

$$\tilde{f}_{z,L,R} = \frac{f_z}{2} \pm \frac{m_t}{b_0} \cdot \frac{m_{t,w}}{m_{t,w} + m_{t,s}} = \frac{f_z}{2} \pm \frac{m_{t,w}}{b_0}$$

- half of the torques transferred by uniform torsion T_s

$$\tilde{m}_t = m_t \cdot \frac{m_{t,s}}{m_{t,w} + m_{t,s}} = m_{t,s}$$

the latter being carried by the web and the part of the deck belonging to each half girder (by uniform torsion of the components constituting the cross-section).



Spine model for open cross-sections: Equilibrium model

As mentioned above, the ratio $m_{t,s}/m_{t,w}$ varies along the span and depends on the position of applied loads.

The distribution $m_{t,s}/m_{t,w}$ can theoretically be determined by the condition that the rotations of the cross-section caused by $m_{t,s}$ and $m_{t,w}$ be equal along the entire span:

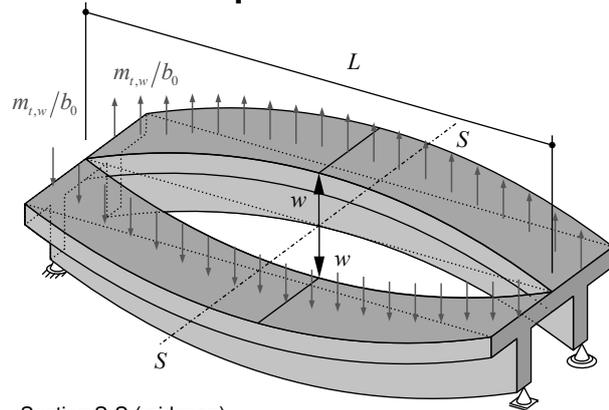
$$\forall x: \varphi_s(x) = \varphi_w(x) = \frac{w_L - w_R}{b_0}$$

Nevertheless, these calculations are complicated and time-consuming, and "accurate" results are hardly ever required (nor obtained, linear elasticity \neq reality).

Therefore, in concrete girders

- a constant ratio $m_{t,s}/m_{t,w}$ over the entire girder length is usually assumed
- which may be determined by compatibility at midspan (see figure) or using the chart on the next slide
- or simply estimated using typical values
 - ... $m_{t,s}/m_{t,w} \approx 0.5$ for long spans
 - ... $m_{t,s}/m_{t,w} \approx 0.25$ for short spans

In steel and composite girders, refined calculations may be required (limited ductility due to stability issues).

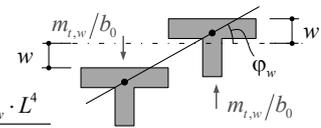


Section S-S (midspan)

→ simple supported girder and uniformly distributed torsion

$$w = \frac{5 \cdot m_{t,w}/b_0 \cdot L^4}{384EI^{(T)}}$$

$$\varphi_w = \frac{2w}{b_0} = \frac{5 \cdot m_{t,w} \cdot L^4}{192EI^{(T)} \cdot b_0^2} = \frac{5 \cdot m_{t,w} \cdot L^4}{96EI^{(TT)} \cdot b_0^2}$$



For refined analyses, see P. Marti, Theory of Structures (2014), and Lebet and Hirt, Steel bridges (2013)

Spine model for open cross-sections: Equilibrium model

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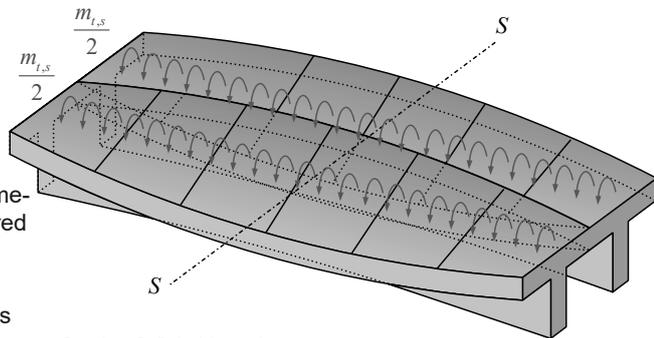
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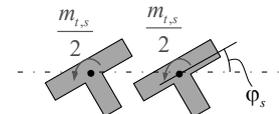
In steel and composite girders, refined calculations may be required (limited ductility due to stability issues).



Section S-S (midspan)

→ simple supported girder and uniformly distributed torsion

$$GK^{(TT)} \cong G \frac{t_{\text{sup}}^3 b + 2b_w^3 h_0}{3}$$



$$\varphi_s = \int_0^{L/2} \frac{T_s \cdot \bar{T}}{GK^{(T)}} dx = \frac{2}{GK^{(T)}} \cdot \frac{L}{2} \cdot \frac{(m_{t,s}/2)L}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{(m_{t,s}/2)L^2}{8GK^{(T)}} = \frac{m_{t,s}L^2}{8GK^{(TT)}}$$

For refined analyses, see P. Marti, Theory of Structures (2014), and Lebet and Hirt, Steel bridges (2013)

Spine model for open cross-sections: Equilibrium model

As mentioned above, the ratio $m_{t,s}/m_{t,w}$ varies along the span and depends on the position of applied loads.

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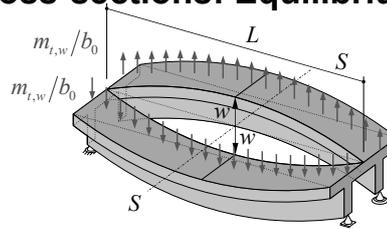
$$\forall x: \varphi_s(x) = \varphi_w(x) = \frac{W_L - W_R}{b_0}$$

Nevertheless, these calculations are complicated and time-consuming, and "accurate" results are hardly ever required (nor obtained, linear elasticity \neq reality).

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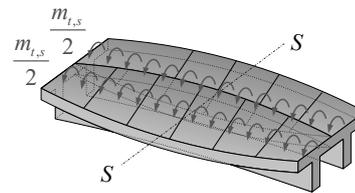
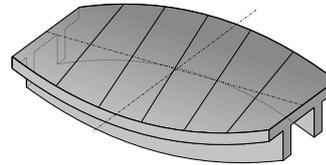
Section S-S (midspan)

→ simple supported girder and uniformly distributed torsion

$$\varphi_w = \frac{2w}{b_0} = \frac{5 \cdot m_{t,w} \cdot L^4}{96EI^{(TT)} \cdot b_0^2}$$

$$\varphi_w = \varphi_s \rightarrow \frac{m_{t,s} L^2}{8GK^{(TT)}} = \frac{5 \cdot m_{t,w} \cdot L^4}{96EI^{(TT)} b_0^2}$$

$$\rightarrow \frac{m_{t,s}}{m_{t,w}} = \frac{5}{12} L^2 \frac{GK^{(TT)}}{EI^{(TT)} b_0^2}$$



$$\varphi_s = \int_0^{L/2} \frac{T_s \cdot \bar{T}}{GK^{(T)}} dx = \frac{m_{t,s} L^2}{8GK^{(TT)}}$$

For refined analyses, see P. Marti, Theory of Structures (2014), and Lebet and Hirt, Steel bridges (2013)

Spine model for open cross-sections: Equilibrium model

On the previous slide, the $m_{t,s}/m_{t,w}$ was estimated as

$$\frac{m_{t,s}}{m_{t,w}} = \frac{5}{12} L^2 \frac{GK^{(TT)}}{EI^{(TT)}b_0^2} \left(= \frac{T_s}{T_w} \text{ if } \frac{m_{t,s}}{m_{t,w}} = \text{const } \forall x \right)$$

where $EI^{(TT)}$ = bending stiffness of full section and

$$GK^{(TT)} \cong G \frac{t_{\text{sup}}^3 b + 2b_w^3 h_0}{3}$$

is the uniform torsional stiffness of the entire cross-section. The warping constant of the cross-section [m⁶] is approximately

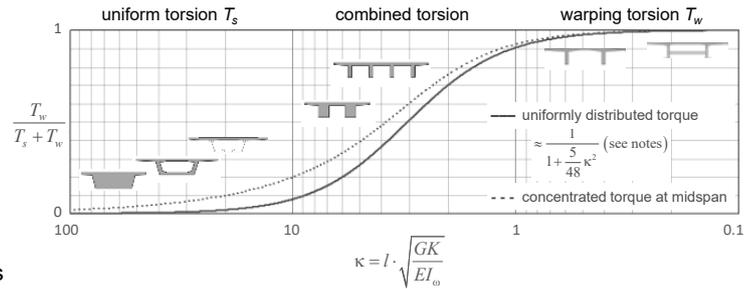
$$I_{\omega} \approx 2 \cdot I^{(T)} \cdot \frac{b_0^2}{4} \approx \frac{I^{(TT)} b_0^2}{4} \quad \left(I^{(T)} \approx \frac{I^{(TT)}}{2} \right)$$

and hence, the ratio m_s/m_w is equal to:

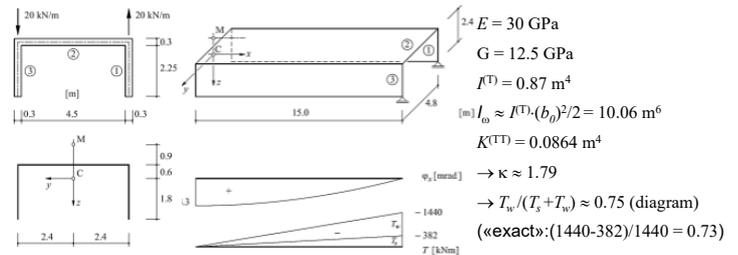
$$\frac{m_{t,s}}{m_{t,w}} = \frac{5}{48} L^2 \frac{GK}{EI_{\omega}} = \frac{5}{48} \kappa^2; \quad \frac{m_{t,w}}{m_{t,w} + m_{t,s}} = \frac{1}{1 + \frac{5}{48} \kappa^2} \quad \kappa = L \sqrt{\frac{GK}{EI_{\omega}}}$$

The parameter κ (used before) is thus indeed a measure for the ratio of uniform to warping torsion.

Note: The equations and the diagram apply to a simply supported girder under uniform torque. For other configurations, similar results are obtained.



Example (figures and exact result see Marti, Theory of structures)



Reference: Kollbrunner, C.F., Basler, K., *Torsion in Structures, An Engineering Approach*, Springer Verlag, 1969.

Note that Kollbrunner and Basler define the curves using three different equations, which differ slightly from the formula derived by formulating rotation compatibility at midspan. Note also that Kollbrunner and Basler define the coefficient plotted in the graph by comparing the Bimoment at midspan obtained for a given value of κ to the Bimoment at midspan for $\kappa = 0$; in the “exact” solution, torques and torsional moments vary along the girder axis and are only approximately proportional to the parameter κ .

Approximation for distributed torques, obtained by formulating compatibility at midspan (see slides):

$$\frac{m_{t,w}}{m_{t,w} + m_{t,s}} \approx \frac{1}{1 + \frac{5}{48} \kappa^2} \quad (\text{for full range})$$

Kollbrunner and Basler's equations:

$$\frac{m_{t,w}}{m_{t,w} + m_{t,s}} \approx \frac{48 + \kappa^2}{48 + 6\kappa^2} \quad (\text{for } \kappa \leq 2)$$

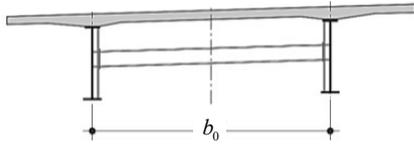
$$\frac{m_{t,w}}{m_{t,w} + m_{t,s}} \approx \frac{8}{\kappa^2} \left(1 - \frac{1}{\cosh(\kappa/2)} \right) \quad (\text{for } 2 < \kappa < 5)$$

$$\frac{m_{t,w}}{m_{t,w} + m_{t,s}} \approx \frac{8}{\kappa^2} \quad (\text{for } \kappa > 5)$$

Finally, note that for a concentrated torque at midspan, a similar approximation of Kollbrunner and Basler's set of equations is possible (substituting the term $5/48$ in the approximate equation by $1/12$). However, this approximation deviates more from the exact solution than in the case of distributed torques, underestimating the contribution of warping torsion for $k > 2$ (box girders).

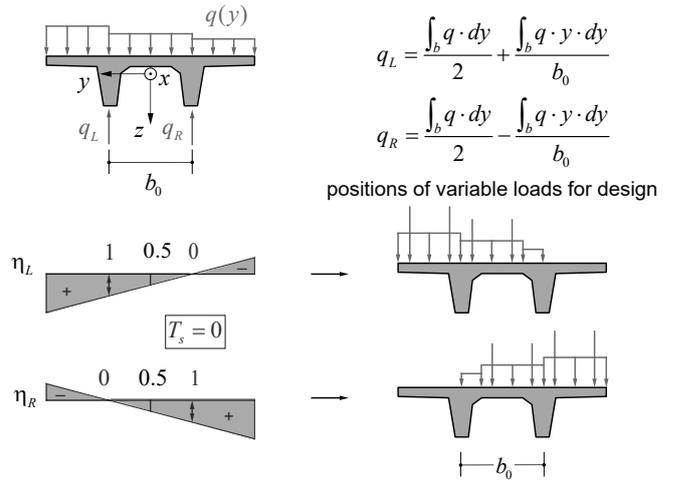
Spine model for open cross-sections: Equilibrium model

The assumption $T_s = 0$ (pure warping torsion) is particularly appropriate for the design of steel and steel-concrete composite bridges with two plate girders, since the torsional stiffness of the latter is indeed negligible.



The simplified model assuming $T_s = 0$ is on the safe side for the design of the longitudinal girders, and thus often sufficient for their ULS and SLS design in straight bridges with such cross-sections.

However, in skew or curved steel and composite bridges, determining camber requires more refined models to avoid fit-up issues, see respective chapters (final slides in skew / curved bridge presentations).

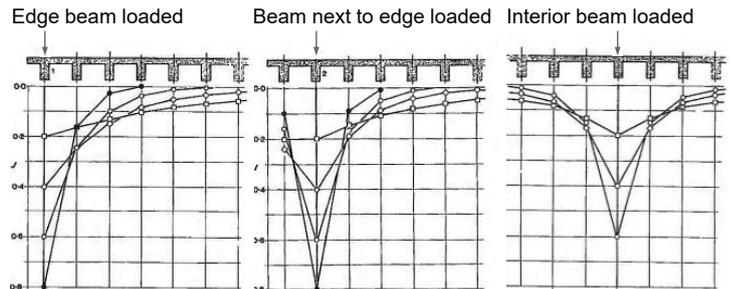
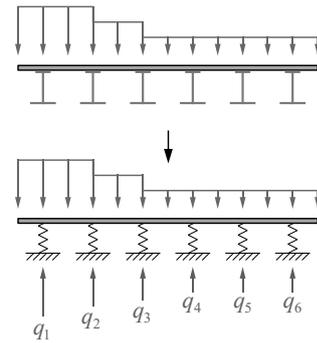


Spine model for open cross-sections: Multi-girder bridges

In multi-girder bridges (open cross-section with more than two webs/beams):

- determination of $m_{t,s}/m_{t,w}$ is further complicated since the deck is statically indeterminate in the transverse direction (even if $GK = 0$ is assumed for individual webs/beams, see top figure)
- loads carried by each web cannot be determined by equilibrium even for $T_s = 0$
- determination of the loads q_i carried by each web requires several assumptions, but remains complicated
- still no direct information on transverse behaviour needs to be analysed
- grillage models should be used for multi-girder bridges

Older textbooks and design recommendations, and several existing bridge design codes, contain detailed information on the analysis of multi-girder bridges. These are outlined on the following slide without entering into details.



Illustrations. Adapted from E.C. Hambly, *Bridge Deck Behaviour*, 1976 (second edition, 1991)

Spine model for open cross-sections: Multi-girder bridges

Design charts (bottom figure) show load distribution factors that may be used to determine the loads acting on each single web/beam of a multi-girder bridge.

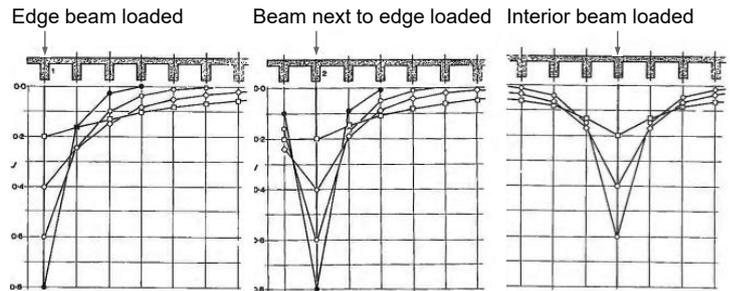
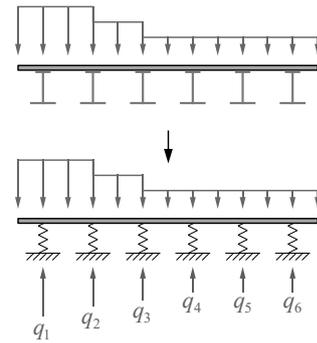
These factors may be used in design for determining e.g.

- longitudinal shear and bending moments
- damage factor λ_4 for fatigue verifications (bending moments due to fatigue load in different positions)

The values given by the design charts

- essentially correspond to transverse influence lines
- show that, depending on the deck configuration (cantilevers, beam spacings) the edge beams and adjoining interior beams receive significantly higher load than the standard interior beams.

Note that the peak values of the design charts (influence lines) depend on the flexural and torsional stiffness ratios in the longitudinal and transverse directions. Separate charts exist for determining these peak values.



Illustrations. Adapted from E.C. Hambly, *Bridge Deck Behaviour*, 1976 (second edition, 1991)

Superstructure / Girder bridges

Bridge Girder – Grillage model (Trägerrostmodell)

Grillage model – General aspects

Girders with open cross-section, as well as multi-cell box girders, can be analysed with grillage models.

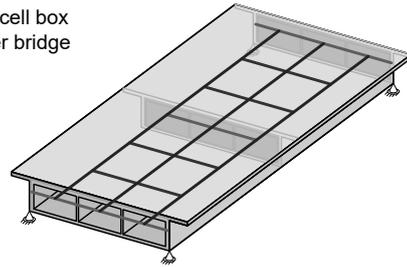
In a grillage model, the girder is idealised as a grid of longitudinal and transverse beams, where

- longitudinal beams “ LB ”
 - represent webs (concrete), beams (steel) or cells of box girders
- transverse beams (usually no more than 3 to 5 per span)
 - represent diaphragms or transverse ribs “ D ”
 - simulate the stiffness of the deck and (if applicable) the bottom slab (“virtual diaphragms”) “ TB ”

Usually, an orthogonal grid is chosen, and consideration of a plane (two-dimensional) grillage (upper figure) is sufficient.

In specific cases, three-dimensional analysis (lower figure) may be useful, particularly to account for membrane action of the deck slab in girders with open cross-section.

Multicell box girder bridge



Multi-girder bridge



Historically, grillage models were also used for slab bridges. Today, FE slab analyses are much more efficient and standard for such bridges.

Grillage model – General aspects

The stiffnesses of the longitudinal and transverse members should reasonably represent the real bridge girder.

To this end, member stiffnesses are essentially determined as for the girder of a spine model, accounting for

- cracking (in non-prestressed members)
- long-term effects
- composite action in composite members

Even the most complex model will not be able to represent the "true" behaviour, particularly due to

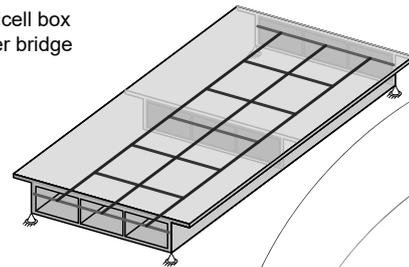
- nonlinearities due to cracking
- time dependent effects

→ grillage models should be as simple as possible to capture the dominant phenomena

→ in preliminary design and ULS design of concrete girders, a torsionless grillage ($GK = 0$ for all members) is often sufficient

(this can be justified by the lower bound theorem of plasticity theory if ductile behaviour is guaranteed, see *spine model for open cross-section – equilibrium model*)

Multicell box girder bridge



webs = longitudinal members
webs = transverse members
deck slab = transverse member

Multi-girder bridge



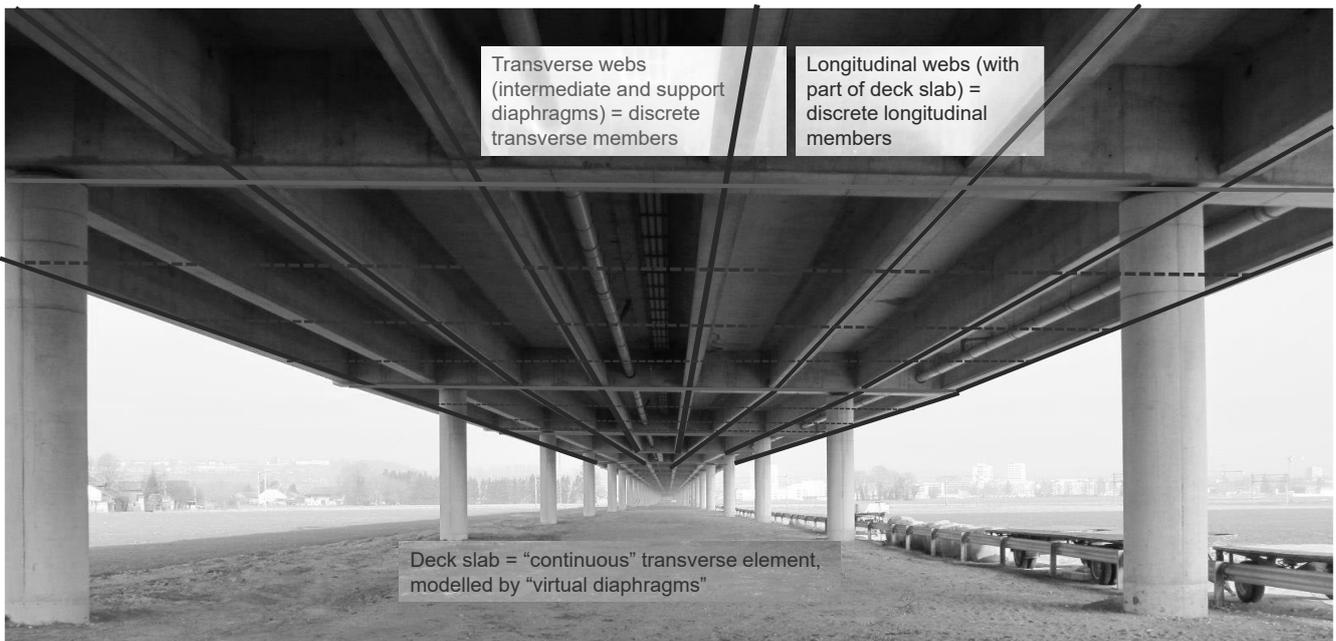
The individual longitudinal and transverse members need to be designed for the internal actions obtained in them from the grillage analysis.

The longitudinal beams are thereby treated in the same way as the girder of a spine model, and the remarks made in the respective chapters (effective widths, distortion of cross-section, shear connectors) also apply to longitudinal beams of grillages. In some cases, particularly regarding torsional moments, additional considerations are required, see following slides.

Usually, the deck slab cannot be dimensioned based on the grillage model alone, particularly if significant concentrated loads need to be considered. Rather, additional considerations, similar as outlined in the chapters on the *bridge deck* and the *spine model*, are required.

In concrete girders, a minimum reinforcement preventing brittle (torsional) failures due to torsion (closed stirrups in webs, appropriate detailing of deck reinforcement) is always required, even in torsionless members ($GK=0$), unless torsion in these members is excluded conceptually (e.g. hinged connection between deck and girder in GFK or timber decks).

Grillage model – General aspects



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Viaduc d'Yverdon, Perret-Gentil, Rey & Associés (1984). Longest viaduct in CH, length 3'155m.
Photo © Opan Concept (rehabilitation 2018)

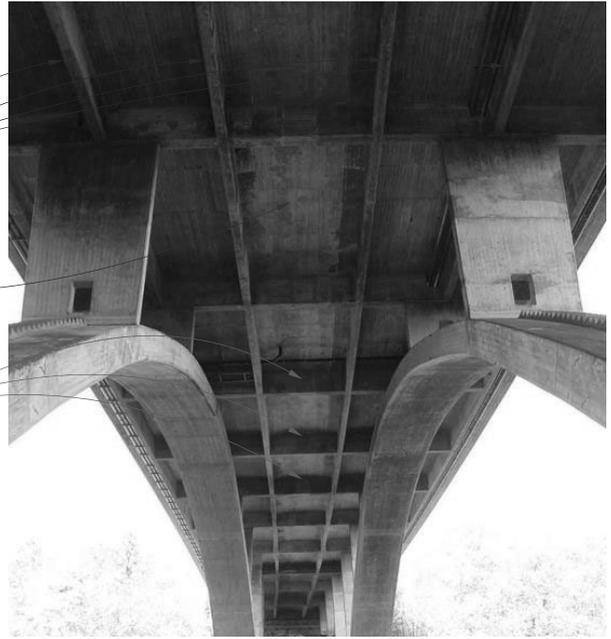
Grillage model – General aspects

Grillage models can also be used for analysing bridge girders of other bridge types

Longitudinal webs (with part of deck slab) = discrete longitudinal members

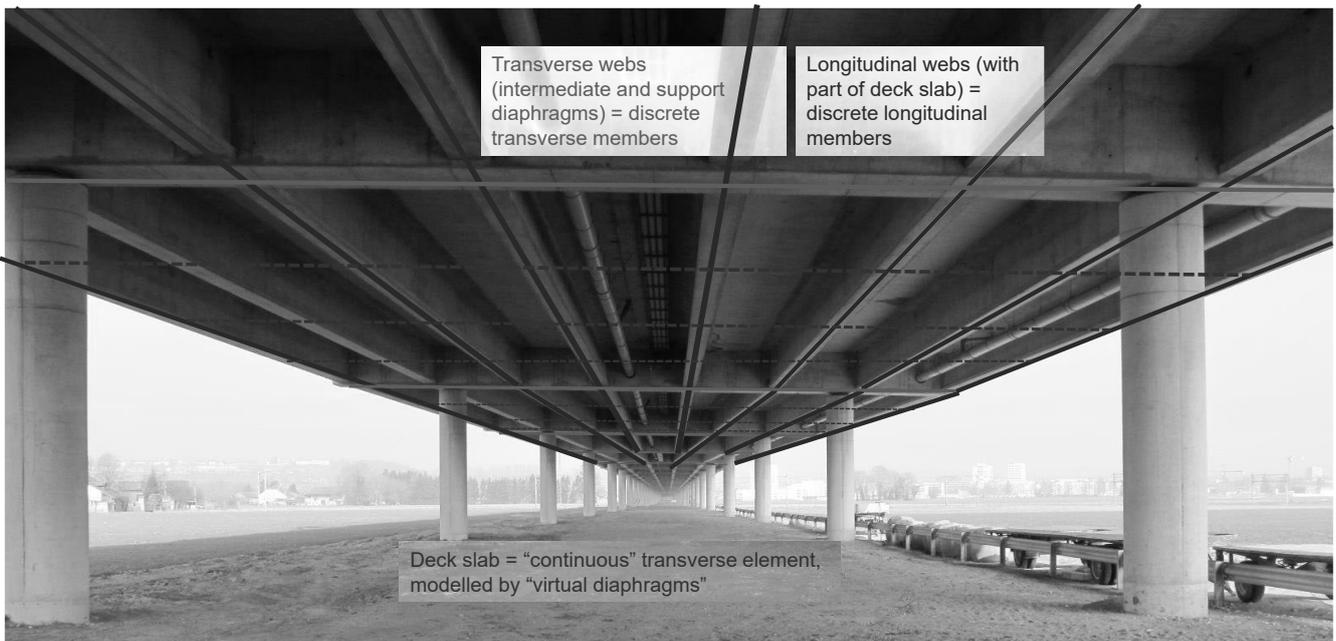
Deck slab = “continuous” transverse element, modelled by “virtual diaphragms”

Transverse webs (intermediate and support diaphragms) = discrete transverse members



Teufelsschluchtbrücke (Objekt X1), A2 Belchen Südrampe. Photo © W. Kaufmann

Grillage model – General aspects



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Viaduc d'Yverdon, Perret-Gentil, Rey & Associés (1984). Longest viaduct in CH, length 3'155m.
Photo © Opan Concept (rehabilitation 2018)

Grillage model – General aspects

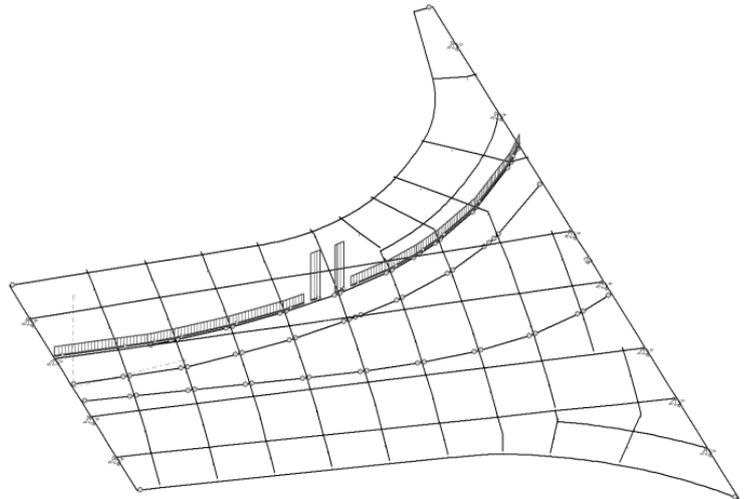
The definition of loads (particularly traffic loads) in grillage models may be quite time-consuming since loads have to be defined with respect to the grillage members

- introduce additional, virtual beams along traffic lanes (connected to grillage) and apply loads to these
- some software programs offer the possibility to define a virtual surface simulating the deck, to which the loads can be applied in their actual position (internally, a slab calculation is run)

In all cases, it must be ensured that the self-weight of the girder is correctly modelled: Avoid that the deck weight is accounted for twice

- assign weight to longitudinal beams and diaphragms
- model transverse beams representing deck and bottom slab (“virtual diaphragms”) as weightless

If cross-sections are defined in a frame analysis software, stiffnesses and weights are assigned automatically. They need to be partially overwritten (stiffnesses) or deleted (weight assigned to the transverse beams).



If loads are applied to the members directly, the results of the grillage model (internal actions in transverse members) may be used for the design of the deck. Still, in order to account for concentrated loads correctly, it is recommended to run a slab analysis for deck design in general cases.

If loads are applied using software programs running a hidden slab calculation, or to virtual beams:

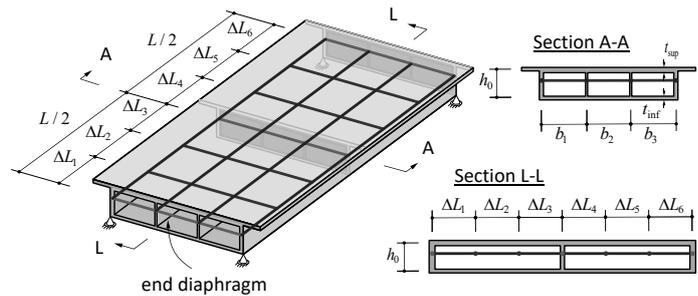
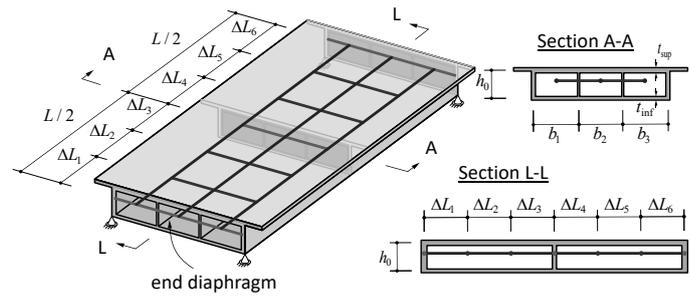
- check that the (virtual) slab is supported correctly by the grillage members (all or selection of user)
- check that the stiffness of the slab is reasonable (too stiff slabs are unsafe, particularly in case of concentrated loads)
- design the deck slab using a slab model

Grillage model – Multi-cell box girders and voided slabs

In multi-cell box girders and voided slabs, there are two options for defining the longitudinal beams "LB" of the grillage.

- Option A (prioritise longitudinal beams):
 - one beam per cell → $n_{LB} = n_{cells}$
 - full torsional stiffness of cross-section GK_{tot} assigned to (distributed among) longitudinal beams
- Option B (treat torsion as in a slab):
 - one beam per web → $n_{LB} = n_{cells} + 1$
 - torsional stiffness of the cross-section GK_{tot} shared
 - $GK_{tot}/2$ → distributed among longitudinal beams
 - $GK_{tot}/2$ → assigned to transverse beams

Similar results are obtained using both options. Option A appears more appropriate for box girders with few cells, and option B for voided slabs.



Grillage model – Multi-cell box girders and voided slabs

Bending and shear stiffnesses of longitudinal beams

- In grillage option A and B, each longitudinal beam is assigned its share of the total bending stiffness $EI_{y,tot}$ of the entire girder:

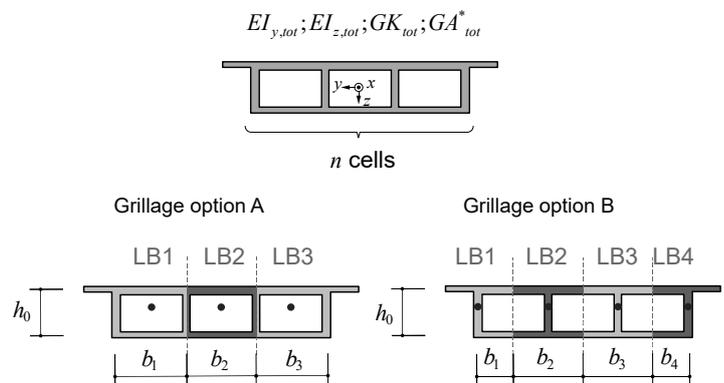
$$EI_{y,LBi} \cong \frac{b_i}{\sum b_i} EI_{y,tot}$$

and each longitudinal beam is assigned the bending and axial stiffness corresponding to its cross-section (see notes)

$$EI_{z,LBi} \cong \int_{A_{LBi}} Ey^2 dA_{LBi}, \quad EA_{LBi} \cong \int_{A_{LBi}} EdA_{LBi}$$

- In grillage option A and B, each longitudinal beam is assigned its share of the total shear stiffness GA^*_{tot} of the entire girder, usually neglecting shear deformations in both directions, i.e.

$$GA^*_{LBi} = \frac{b_i}{\sum b_i} GA^*_{tot} \rightarrow \infty$$



Assigning to each longitudinal beam its share of the total bending stiffness $EI_{y,tot}$ corresponds well to assigning to each girder the bending stiffness $EI_{y,LBi}$ obtained when considering part of the cross-section belonging to each individual longitudinal beam; this is often preferred in design, since computer programs then automatically evaluate the self-weight.

In the case of EI_z , it might appear adequate – as in the case of a spine model for open cross-sections, see there – to assign each longitudinal beam its share of the total transverse stiffness $EI_{z,tot}$, to ensure that $\sum EI_{z,LBi} \approx EI_{z,tot}$ (i.e., increase $EI_{z,LBi}$ of each longitudinal beam by a factor of roughly n^2). However, the longitudinal beams are coupled by the transverse beams (whose stiffnesses are increased, see behind), i.e., the higher transverse stiffness of the total cross-section is implicitly accounted for by the contribution of the axial stiffnesses of the individual longitudinal beams (multiplied by the horizontal spacing, according to the parallel axis theorem = “Satz von Steiner”). Therefore, using the stiffnesses $EI_{z,LBi}$ of the individual longitudinal beams is appropriate here.

Grillage model – Multi-cell box girders and voided slabs

Torsional stiffness of longitudinal beams

- In grillage option A, each longitudinal beam is assigned its share of the full total torsional stiffness GK_{tot} of the entire girder

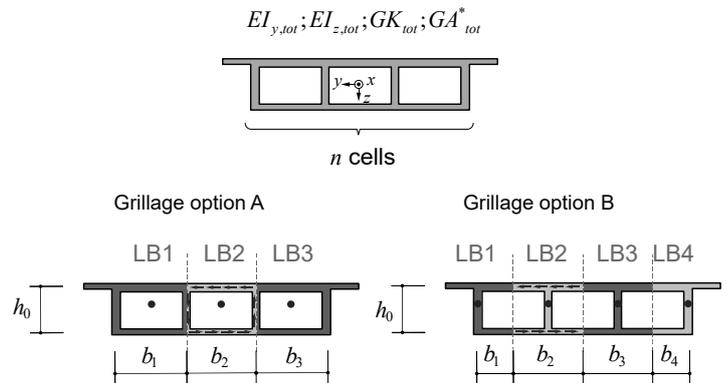
$$GK_{LBi}^A = \frac{b_i}{\sum b_i} GK_{tot} \quad (\text{but } GK_{TBi}^A = 0, \text{ see behind})$$

and the resulting torsional moments are assigned to the box section of each longitudinal beam as in a single cell box girder (see notes)

- In grillage option B, each longitudinal beam is assigned only the total torsional stiffness corresponding to the deck and bottom slab, which roughly corresponds to half the total torsional stiffness, i.e.

$$GK_{LBi}^B = \frac{b_i}{\sum b_i} \frac{GK_{tot}}{2} \quad (\text{but } GK_{TBi}^B \approx GK_{LBi}^B \frac{\Delta L_j}{b_i}, \text{ see behind})$$

and consequently, the resulting torsional moments are assigned to the deck and bottom slab of each longitudinal beam (see notes)



In option A, the shear flows due to torsion obtained in the interior webs of adjoining boxes have opposite signs and approximately cancel out (exactly for equal box widths) and hence, the torsional shear flow essentially corresponds to that of the outermost box, as expected.

In option B, only horizontal shear flows result due to torsion. The vertical shear flow corresponding to the uniform torsion of the entire cross-section is part of the vertical shear forces obtained in the outermost longitudinal girders. These shear forces are higher than in option A since the torsional stiffness of the transverse beams is accounted for in model B only.

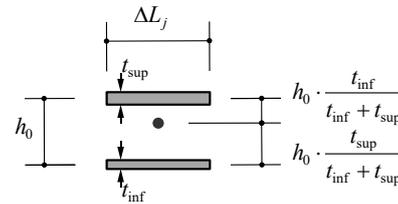
The results of both models are very similar, see example at the end of the chapter.

Grillage model – Multi-cell box girders and voided slabs

Bending stiffnesses of transverse beams

- In grillage option A and B, each transverse beam is assigned the bending stiffness EI_y , corresponding to the stringer cross-section of deck and bottom slab over the length $\Delta L =$ transverse beam spacing):

$$EI_{y,TBi} \cong E\Delta L_j h_0^2 \frac{t_{inf} t_{sup}}{t_{inf} + t_{sup}}$$



- In grillage option A and B, each transverse beam is assigned its share of the bending stiffness $EI_{z,tot}$ of the entire girder (deck and bottom slab over full span length):

$$EI_{z,TBi} \cong E \frac{\Delta L_j}{L} (t_{sup} + t_{inf}) \frac{L^3}{12} \quad (\approx \infty)$$

$$EI_{y,TBi} \cong E\Delta L_j \left(\frac{t_{inf}^3 + t_{sup}^3}{12} + t_{sup} \frac{t_{inf}^2 h_0^2}{(t_{inf} + t_{sup})^2} + t_{inf} \frac{t_{sup}^2 h_0^2}{(t_{inf} + t_{sup})^2} \right)$$

which is much larger than the sums of the stiffnesses EI_z of the individual beams. This high transverse stiffness ensures that the axial stiffness of the longitudinal beams, and the corresponding higher effective transverse bending stiffness of the entire deck, can be activated (see notes on EI_z of longitudinal beams).

Grillage model – Multi-cell box girders and voided slabs

Shear stiffness of transverse beams

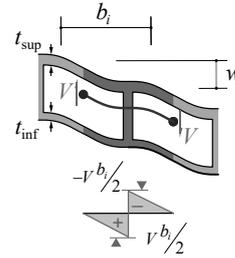
- In grillage option A and B, the transverse beams consist only of the deck and bottom slab, without web
- assumption $GA_{tot}^* \rightarrow \infty$ is inappropriate for vertical shear
- act vertically as Vierendeel girders with stiff posts; neglecting deformations of webs GA^* is:

$$GA_{z,TB}^* = E \frac{\Delta L_j}{b_i^2} (t_{sup}^3 + t_{inf}^3) \quad (\text{but } GA_{y,TB}^* \rightarrow \infty)$$

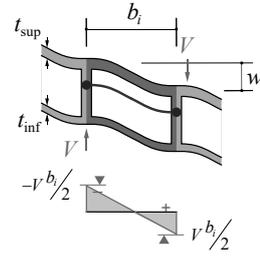
- Despite neglecting deformations of the web, the shear stiffness GA^* of transverse beams is underestimated if the webs are wide or the slabs tapered towards the webs
- better approximation: replace b_i by clear span of slabs between webs
- use tapered section in virtual work equation

- In voided slabs, the shear stiffness GA_z^* of transverse beams can be estimated by replacing the circular voids by square ones of equal area.

Grillage option A



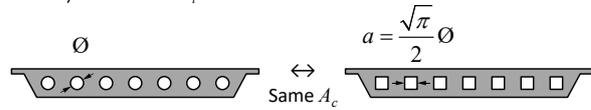
Grillage option B



$$V_{sup} = V \frac{EI_{sup}}{EI_{sup} + EI_{inf}}, \text{ with } I_{sup} = \frac{\Delta L_j \cdot t_{sup}^3}{12}, I_{inf} = \frac{\Delta L_j \cdot t_{inf}^3}{12}$$

$$w = \int \frac{M\bar{M}}{EI} dx = V_{sup} \frac{b_i}{2} \cdot \frac{b_i}{2} \cdot \frac{b_i}{2} \cdot \frac{1}{3EI_{sup}} = \frac{V \cdot b_i^3}{12 \cdot (EI_{sup} + EI_{inf})}$$

$$GA^* = \frac{V}{\gamma} = \frac{V \cdot b_i}{w} = E \frac{\Delta L_j}{b_i^2} (t_{sup}^3 + t_{inf}^3)$$



Grillage model – Multi-cell box girders and voided slabs

Torsional stiffness of transverse beams

- In grillage option A, the entire torsional stiffness GK_{tot} of the girder is assigned to the longitudinal beams, i.e.

$$GK_{TB}^A = 0$$

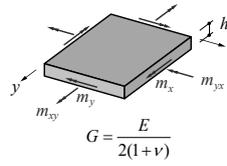
- In grillage option B, about half of the torsional stiffness GK_{tot} is assigned to longitudinal and transverse beams each, similar as in a slab (whose torsional stiffness per direction is half that of a uniaxial beam, see top figure).

→ Transverse beams are assigned the same torsional stiffness per unit length as longitudinal beams, i.e.

$$GK_{TBj}^B = GK_{LBi}^B \frac{\Delta L_j}{b_i} = \frac{b_i}{\sum b_i} \frac{GK_{tot}}{2} \frac{\Delta L_j}{b_i} = \frac{\Delta L_j}{\sum b_i} \frac{GK_{tot}}{2}$$

- A more refined approach (applicable e.g. if e.g. slab thicknesses vary strongly over the width) consists in using the torsional stiffness of the deck and bottom slab, i.e. (see lower figure)

$$GK_{TBj}^B \approx G \cdot \Delta L_j \cdot h_0^2 \frac{2 \cdot t_{inf} \cdot t_{sup}}{t_{inf} + t_{sup}} \left(\approx EI_{y,TBj} \text{ since } G \approx \frac{E}{2} \right)$$

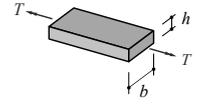


$$m_x = \frac{Eh^3}{12(1-\nu^2)} (\chi_x + \nu\chi_y)$$

$$m_y = \frac{Eh^3}{12(1-\nu^2)} (\chi_y + \nu\chi_x)$$

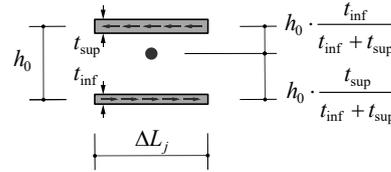
$$m_{xy} = \frac{Eh^3}{12(1+\nu)} \chi_{xy} = G \frac{h^3}{6} \chi_{xy}$$

$$\rightarrow "K" = \frac{h^3}{6} \text{ per unit width}$$



$$GK \triangleq \frac{bh^3}{3} \text{ for } h \ll b$$

$$\rightarrow K = \frac{h^3}{3} \text{ per unit width}$$



Grillage model – Multi-cell box girders and voided slabs

Stiffnesses of diaphragms

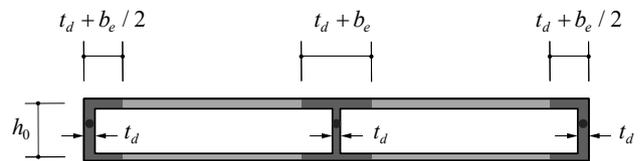
- Diaphragms are modelled as beams, with an effective width of the deck and bottom slab
- Stiffnesses determined accordingly, as for the girder in a spine model, usually neglecting shear deformations:

$$EI_{y,D} = \int_A Ez^2 dA$$

$$EI_{z,D} = \int_A Ey^2 dA$$

$$GA_D^* \rightarrow \infty$$

$$GK_D \approx G \left(\frac{h_0 t_D^3}{3} + \frac{(t_D + b_{e,\text{sup}}) t_{\text{sup}}^3}{3} + \frac{(t_D + b_{e,\text{inf}}) t_{\text{inf}}^3}{3} \right)$$



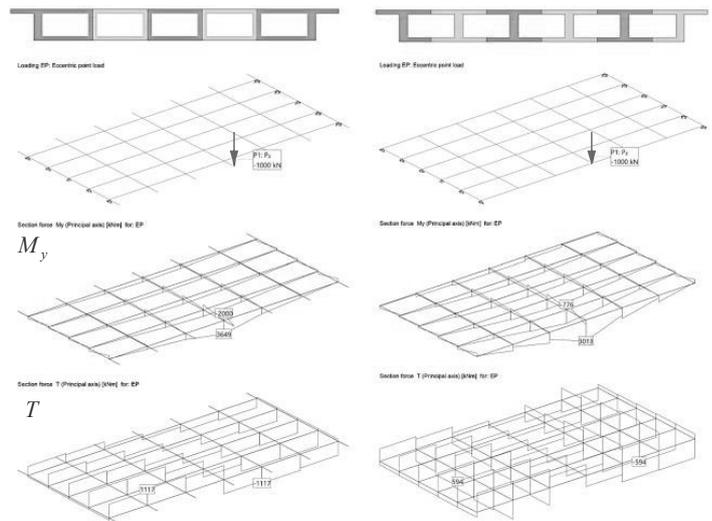
Grillage model – Multi-cell box girders and voided slabs

The figure compares the results of grillage analyses using the options A (left) and B (right) for a single-span girder with a multi-cell box cross-section, loaded by an eccentric concentrated load at midspan.

The results are as expected:

- Deformations are approximately equal in both models (difference < 10%)
- Bending moments are approximately equal in both models (sum over 5 and 6 longitudinal beams)
- Torsional moments result only in longitudinal beams in Model A, but also in transverse beams in Model B
- Torsional moments in the longitudinal beams of Model B are roughly 50% of those in Model A
- Torsional moments in longitudinal and transverse beams of Model B are approximately equal at intersections

→ Both models yield the same results

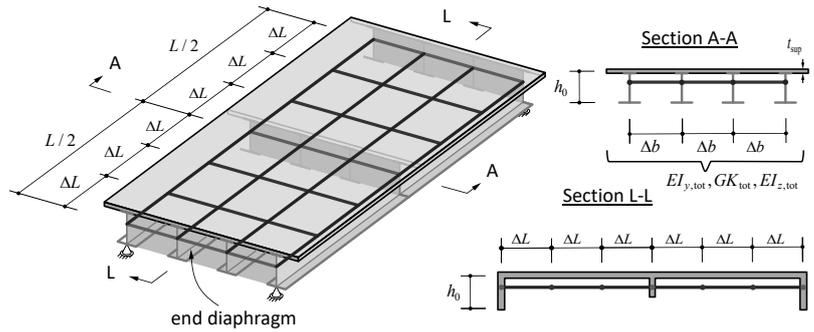


Note that the "saw-tooth" shaped discontinuities in the moment diagrams of the longitudinal girders of Model B are caused by the torsional moments in the (virtual) transverse beams and therefore an artefact of the model; in reality, the longitudinal bending moments will vary gradually, and the longitudinal beams therefore may be dimensioned for the average bending moments at the intersections with the transverse beams.

Grillage model – Open cross-sections (plane grid)

In girders with open cross-sections, the determination of the stiffnesses of longitudinal and transverse beams is much simpler than for multi-cell box girders:

- Longitudinal beams = webs (concrete) / steel beams
→ one beam per web → $n_{LB} = n_{web}$
- Transverse beams (virtual diaphragms)
→ Simulate the deck stiffness
- Diaphragms = “physical” transverse beams
→ Similar as multi-cell box girder



Longitudinal beams
Each beam is assigned its corresponding EI_y and GK of the deck slab, i.e. approximately:

$$EI_{y, LB} \approx \frac{EI_{y, tot}}{n}$$

$$EI_{z, LBi} \cong \int_{A_{LBi}} E y^2 dA_{LBi}, \quad EA_{LBi} \cong \int_{A_{LBi}} E dA_{LBi}$$

$$GK_{LB} \approx \frac{GK_{tot}}{n}, \quad G\bar{A}_{LB} \approx \infty$$

Transverse beams

$$EI_{y, TB} \approx E \cdot \Delta L \cdot \frac{t_{sup}^3}{12}$$

$$EI_{z, TB} \approx E \cdot \frac{\Delta L}{L} \cdot t_{sup} \cdot \frac{L^3}{12}$$

$$G\bar{A}_{TB} \approx \infty$$

$$GK_{TB} \approx \frac{\Delta L \cdot t_{sup}^3}{3} \approx 0$$

As for closed cross-sections (Slide 109), in the case of EI_z , it might appear adequate – as in the case of a spine model for open cross-sections, see there – to assign each longitudinal beam its share of the total transverse stiffness $EI_{z, tot}$, to ensure that $\sum EI_{z, LBi} \approx EI_{z, tot}$ (i.e., increase $EI_{z, LBi}$ of each longitudinal beam by a factor of roughly n^2). However, the longitudinal beams are coupled by the transverse beams (whose stiffnesses are increased), i.e., the higher transverse stiffness of the total cross-section is implicitly accounted for by the contribution of the axial stiffnesses of the individual longitudinal beams (multiplied by the horizontal spacing, according to the parallel axis theorem = “Satz von Steiner”). Therefore, using the stiffnesses $EI_{z, LBi}$ of the individual longitudinal beams is appropriate here.

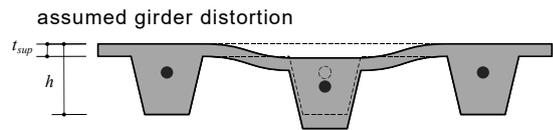
Grillage model – Open cross-sections (plane grid)

In the case of wide webs or beams (e.g. separated box sections)

→ transverse stiffness of the deck is significantly underestimated by the formulas given on the previous slide

Example: three-web girder

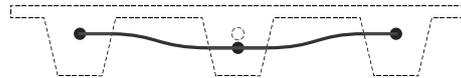
- middle longitudinal beam is displaced downwards
 - edge beams remain in their original, unrotated position
- to match real behaviour, transverse beam stiffness needs to be corrected over the length corresponding to the width of the webs
- Use higher average value, or tapered section with stiff part over longitudinal beam (usual in computer programs)



grillage deformations with transverse beams having a constant stiffness → underestimates deck stiffness



grillage deformations with stiff transverse beams over the width of the webs



$$\begin{aligned}
 - EI_{y,TB} &\cong E\Delta L \frac{h^3}{12} \\
 &\Downarrow \\
 - EI_{y,TB} &= E\Delta L \frac{t_{sup}^3}{12}
 \end{aligned}$$

Grillage model – Open cross-sections (membrane action of deck / 3D grid)

Membrane action of deck slab

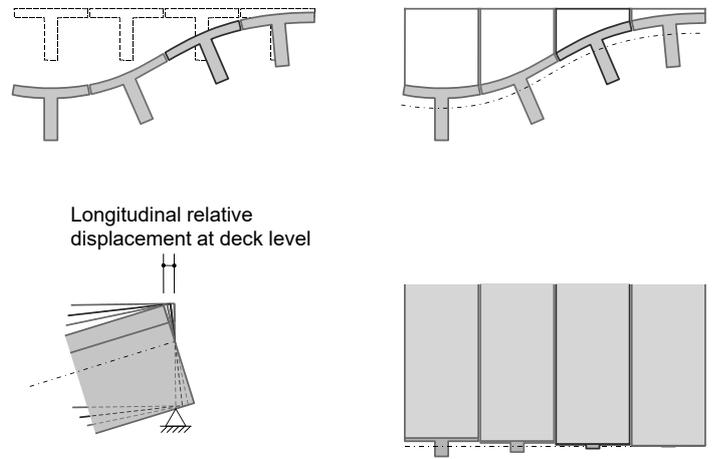
Plane grillages cannot reproduce in plane shear transfer between the parts of the deck assigned to each longitudinal beam. However

- such membrane forces are however required to avoid longitudinal relative displacements in the “longitudinal joints” between the beams
- which occur in plane grillages despite that the distortions of the girder are well reproduced

This is illustrated by the figure:

- distortion of the cross-section (a) is correctly represented by the plane grillage model and its individual longitudinal beams (b), since the transverse beams ensure compatibility
- However, longitudinal relative displacements at the level of the deck result, as shown in elevation (c) and plan (d).

→ 2D grillage underestimates stiffness of the girder.



Illustrations. Adapted from E.C. Hambly, Bridge Deck Behaviour, 1976 (second edition, 1991)

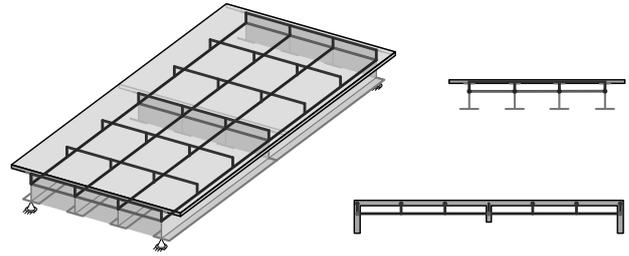
Grillage model – Open cross-sections (membrane action of deck / 3D grid)

Membrane action of deck slab

The underestimation of girder stiffness due to neglecting the compatibility between adjacent longitudinal beams is often accepted, as it gives results on the safe side.

If required, the membrane action of the deck slab can be accounted for by using a 3D grillage model, where

- longitudinal and transverse beams are positioned at the levels of their centres of gravity (→ transverse beams are positioned above the longitudinal beams, which causes membrane action) and
- connected by means of vertical rigid link elements
- stiffnesses of the longitudinal and transverse beams are essentially the same as in the plane grid but
- if transverse beams are introduced at locations of diaphragms, the stiffness of the diaphragms is defined by their cross-section without deck slab (effective width = 0, avoid accounting for deck slab stiffness twice)



| rigid connections:

$$EI_y \rightarrow \infty$$

$$EI_z \rightarrow \infty$$

$$GK \rightarrow \infty$$

$$GA \rightarrow \infty$$

Superstructure / Girder bridges

Bridge Girders – Slab model (slab bridges)

Bridge Girders – Slab model (slab bridges): Modelling

Modelling of slab bridges

In slab bridges, deck and bridge girder are combined, i.e., loads are carried in two directions (slab):

$$\frac{\partial^2 m_x}{\partial x^2} + 2 \frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} + q = 0$$

For the design of slabs, see e.g. courses «Stahlbeton II», «Flächentragwerke».

Linear elastic FE analyses are standard today for slab bridges:

- Spreading of concentrated loads see section on bridge deck analysis
- Support conditions corresponding to bearing layout

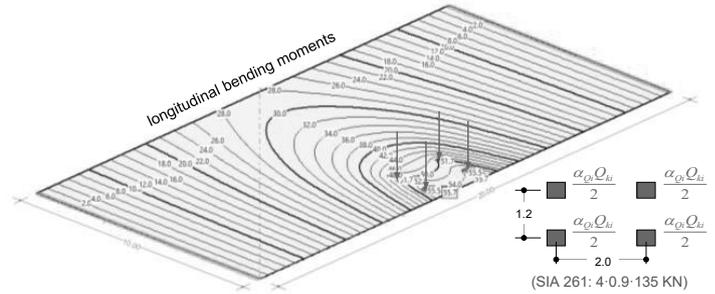
Before the advent of user-friendly, affordable FE slab analysis programs, grillage models were used to analyse slab bridges (using similar stiffnesses as in grillage option B for multi-cell box girders). Today, this is obsolete and therefore not further outlined here.

Slab dimensioning

$$\begin{aligned} m_{x,Rd} &\geq m_{x,d} + k \cdot |m_{xy,d}| \\ m_{y,Rd} &\geq m_{y,d} + \frac{1}{k} \cdot |m_{xy,d}| \end{aligned}$$

$$\begin{aligned} m'_{x,Rd} &\geq -m_{x,d} + k' \cdot |m_{xy,d}| \\ m'_{y,Rd} &\geq -m_{y,d} + \frac{1}{k'} \cdot |m_{xy,d}| \end{aligned}$$

$$v_{0d} \leq v_{Rd} = k_d \cdot \tau_{cd} \cdot d_v$$

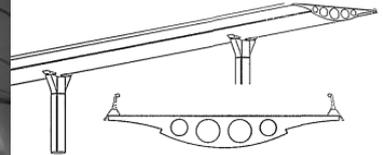
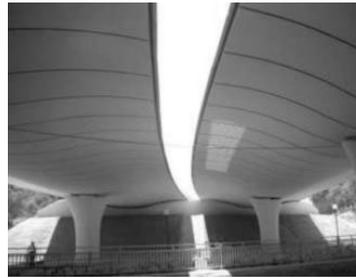
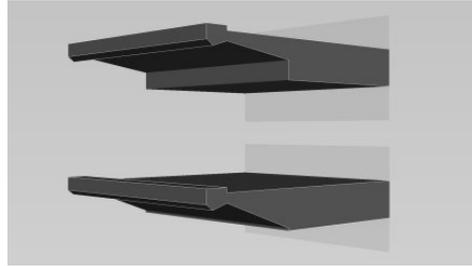


Illustrations: Adapted from Menn [1990]

Bridge Girders – Slab model (slab bridges): Selected aspects

Specific aspects of slab bridges / slab models

- It is recommended to treat prestressing in slabs as anchor, deviation and friction forces, acting on the subsystem "reinforced concrete structure without prestressing", see lectures "Stahlbeton II", "Advanced structural concrete" and notes.
- Slab bridges are often supported on several bearings per abutment ("line support")
... make sure the intended distribution of support reactions is reasonably achieved
... particularly if using precast elements (tolerances!)
- Uplift may be a problem at supports near acute corners of skew slabs
- To enhance visual slenderness, it is recommended to reduce the thickness along the free edges.



Reyes de Aragón overpass, Spain, 2005. CFCSL

Reminder (from Advanced Structural Concrete):

The treatment of prestressing as a residual stress state in the total system is deemed to fail in two-dimensional or three-dimensional structures because the residual stress state due to prestressing cannot be uniquely determined (internal static indeterminacy, unknown spreading of compressive force, reference cross-section unclear, etc.).

The treatment of the prestressing as anchorage, deviation and friction forces on the subsystem "reinforced concrete structure without prestressing", on the other hand, is possible without any problems.

In design practice, the anchorage, deviation and friction forces are usually determined considering the prestressing force without any increase. The increase in the prestressing force at ULS could theoretically be investigated with suitable considerations (e.g. stress fields), but the effort is not worthwhile usually (small influence, since the initial preload $0.7f_{pk}$ is only slightly (approx. 3-7%) lower than the design value of the yield stress $f_{p0.1k} / 1.15$). It is more relevant to estimate the influence of long-term losses on the prestressing force.

The photo shows twin voided slab bridges with a curved cross-section, which favours the aesthetic aspect. However, the ratio of bending resistance to weight efficiency is less optimal, requiring somewhat more prestressing and making the formwork more expensive.

Illustrations: bottom: J. Manterola, Puentes I.

Photo: © Carlos Fernandez Casado S.L.