Special girder bridges

Skew bridges

Cantilever-constructed bridges

- Introduction / first cantilever-constructed bridges
  ↓
- General observations
  ↓
- Design
  ↓
- Camber
  ↓
- Construction

Truss bridges

- Typologies
  ↓
- Types and Examples → Design

Skew bridges

...

Curved bridges

...

- Introduction
  ↓
- Analysis
  ↓
- Design
  ↓
- Particularities of steel bridges

Special girder bridges
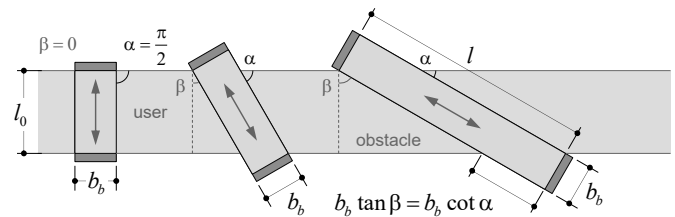
Skew bridges
Introduction

Skew bridges – Introduction

Geometry and terminology

- Bridges crossing obstacles at a right angle in plan are more economical than skew crossings (shorter bridge). Orthogonal crossings are usually also aesthetically preferable, particularly in case of river crossings

- From the perspective of the user, bridges are skewed to the left or right; torsional moments have opposite sign

- The crossing angle α is referred to as "skew" in many textbooks. However, this is counterintuitive (small α = strongly skewed) $\rightarrow$ to avoid misunderstandings, call α "crossing angle" or even indicating both: "a 30° skewed bridge (crossing angle 60°)"

- However, orthogonal crossings are not always feasible due to road and – even more so – railway alignment constraints, and providing orthogonal support to a bridge in a skew crossing requires long spans

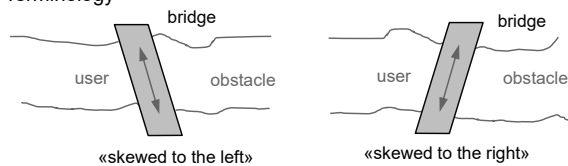$$l = \frac{l_0}{\sin \alpha} + b_b \cot \alpha = \frac{l_0}{\cos \beta} + b_b \tan \beta$$

Skew crossing – orthogonal support

Skew bridges
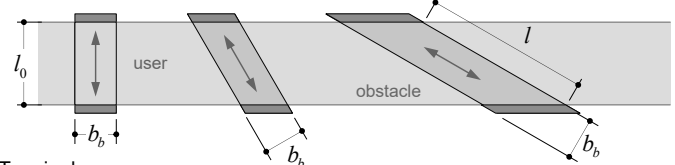
Terminology

Note that for a crossing angle $\alpha = 30°$ ($\beta = 60°$), shown on the right above, already a very narrow bridge requires more than twice the length of an orthogonal crossing. For wide bridges, this is even more pronounced.

Terminology: Also in German, "eine 30° schiefe Brücke" (used colloquially) rather refers to a $\beta = 30°$ (moderately skew), rather than a crossing angle of $\alpha = 30°$ (very skew). Indicating the crossing angle ("Kreuzungswinkel") avoids misunderstandings: "Eine 30° schiefe Brücke (Kreuzungswinkel 60°)".
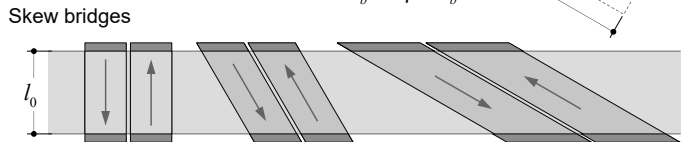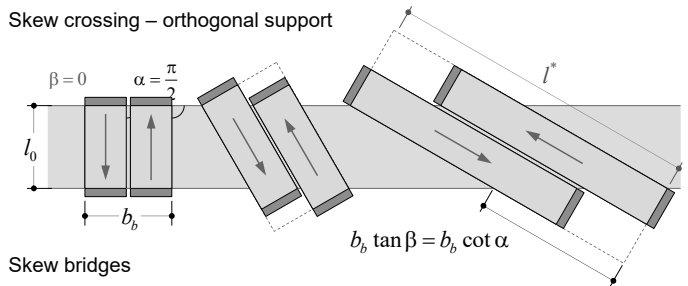
Skew bridges – Introduction
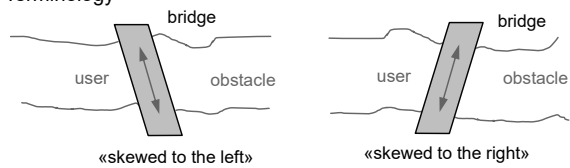
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$$l = \frac{l_0}{\sin \alpha} + b_b \cot \alpha = \frac{l_0}{\cos \beta} + b_b \tan \beta$$

- If orthogonal support is required, twin girders in skew crossings should be staggered $\rightarrow$ no excessive length l^*

Skew crossing – orthogonal support

$$b_b \tan \beta = b_b \cot \alpha$$

Skew bridges

Terminology

«skewed to the left»   «skewed to the right»

Skew bridges – Introduction

Advantages:

- Abutments and piers can be properly integrated into the landscape
- For a given skew bridge alignment, the bridge lengths and spans are minimised
- Abutments and piers of skew river crossings can be oriented parallel to the direction of flow → minimise hydraulic obstruction
- Abutments and piers of skew road or railway crossings can be oriented parallel to the direction of traffic → minimise impact risk

Disadvantages:

- Skew bridges require long and geometrically complicated abutments and embankments
- Heavy vehicles experience a twist at skew bridge ends → critical in railways (track twist), particularly in high speed lines
- If expansion joints are required, they are more complex and subject to premature damage
- The cost of superstructure falsework and formwork is higher than for non-skew bridges
- The design of skew bridges is more challenging (structural analysis, dimensioning, detailing) → see behind

Photos © dsp Ingenieure & Planer / Fhecor Ingenieros

Skew bridges – Introduction

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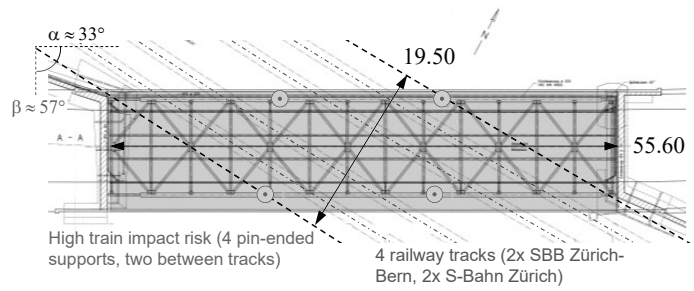
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Ponti della Circonvallazione di Roveredo. dsp Ingenieure + Planer, Spataro Petoud Partner, MAWI SA. Photos © Raoul Spataro
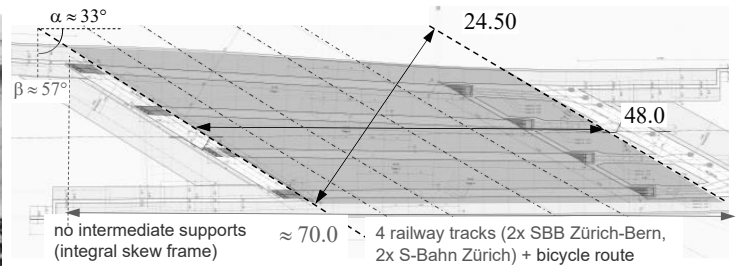
Skew bridges – Introduction

$\alpha \approx 33°$
$\beta \approx 57°$

19.50

55.60

High train impact risk (4 pin-ended supports, two between tracks)

4 railway tracks (2x SBB Zürich-Bern, 2x S-Bahn Zürich)

Photo and Illustration: Überführung Schönenwerd SBB (1936/1976, replaced 2017)

Skew bridges – Introduction

Photo and Illustration: Überführung Schönenwerd SBB, DSP Ingenieure Planer AG mit Balz Amrein, 2017. Photos © dsp Ingenieure + Planer AG

Skew bridges – Introduction

General behaviour of skew bridges

- In a slab with skew supports, the loads are transferred in the most direct way, i.e., they tend to follow the shortest path to the nearest support
  - → Supports in obtuse corners receive higher reactions than those in acute corners
- The outer edges, parallel to the bridge axis, deflect similarly to a simply supported beam each. Cross-sections perpendicular to the longitudinal axis therefore rotate (most obvious for cross-sections through corners: One side has zero deflection)
- The rotation of the cross-sections varies along the span (changing sign at midspan in symmetrical cases)
  - → Slab is twisted, causing torsional moments depending on the stiffness ratio GK/EI_y
  - → Track twist particularly at bridge ends
- Torsional moments at the slab ends induce a force couple (difference in support reactions) and longitudinal bending moments (see next slides)
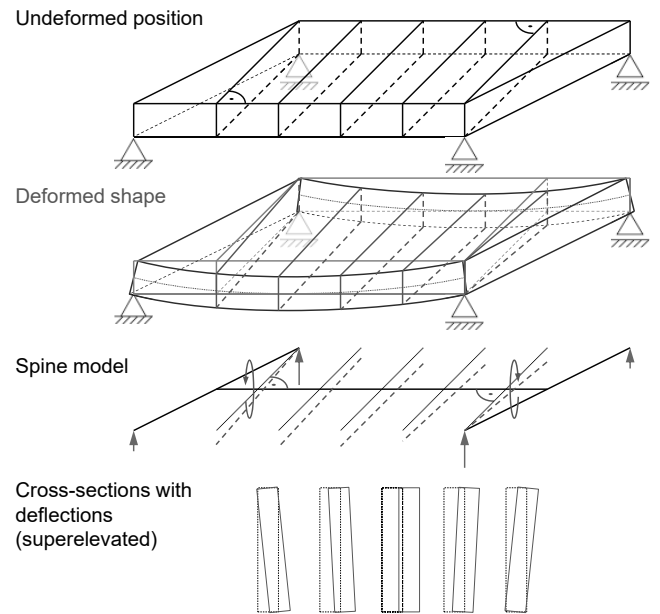
Undeformed position

Deformed shape

Spine model

Cross-sections with deflections (superelevated)

Illustration adapted from C. Menn, Prestressed Concrete Bridges (1990)

Skew bridges – Introduction

An intuitive understanding of the behaviour at skew end supports can also be obtained by
→ first considering a simple support in the girder axis, and
→ then superimposing a force couple at the girder ends to establish compatibility at the supports
(see notes for details)
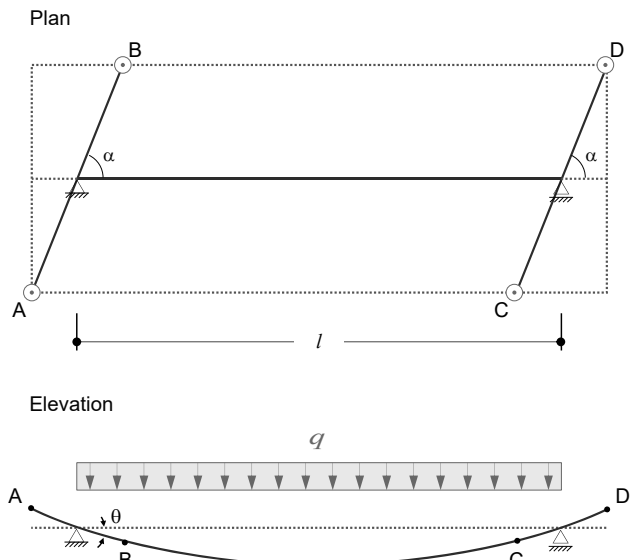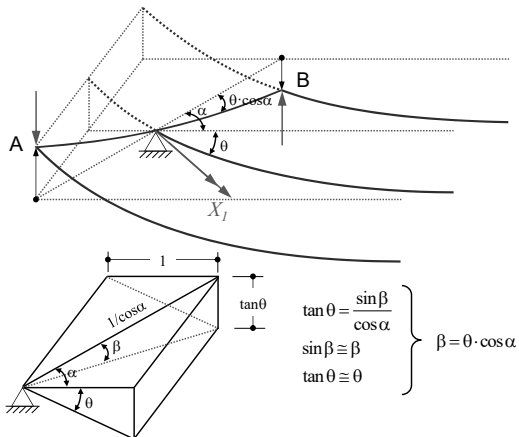
Plan

Elevation

$$\tan\theta = \frac{\sin\beta}{\cos\alpha}$$

$$\sin\beta \cong \beta$$

$$\tan\theta \cong \theta$$

$$\Big\} \quad \beta = \theta \cdot \cos\alpha$$

Consider the following steps (for simplicity, uniform load q and equal support angles α at both ends are assumed):

1. The four corner supports (A-D) are removed and replaced by two fictitious supports in the girder axis (midway between A-B and C-D, respectively)
   → simply supported girder, deflects vertically without rotation around its axis
   → all points on any line perpendicular to the girder axis undergo the same deflection
   → Points B and C move downward (as the corresponding points on the axis are in the span), Points A and D move upward (corresponding points on axis are on overhang)
   → all four reactions A-D are zero

2. To establish compatibility at the supports ($w_A = w_B = w_C = w_D = 0$), a force couple (B=-A, D=-C) is applied at each girder end, corresponding to a bending moment X_1 perpendicular to the supports. The fictitious supports are removed (assuming an infinitely stiff end diaphragm, the support reactions are shared equally among the two supports at each girder end, to be superimposed with the force couple)

3. The bending moment X_1 corresponds to a torsional moment and a bending moment acting at the girder ends, see next slides.

Special girder bridges

Skew bridges
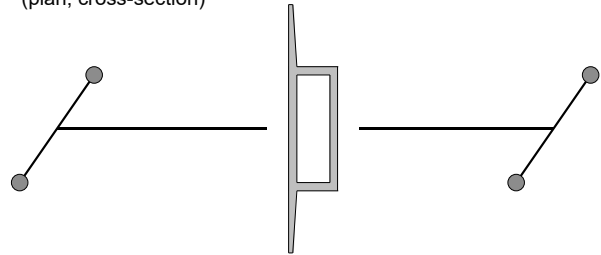## Analysis

Skew bridges – Design

General remarks: Modelling

- Regarding models for global structural analysis, basically, the same observations as for orthogonally supported bridges apply to skew bridges as well
  - $\rightarrow$ uniform torsion dominant in box girders, warping torsion in girders with open cross-section
  - $\rightarrow$ spine models appropriate for box girders
  - $\rightarrow$ grillage models appropriate for girders with open cross-section
- In skew bridges, the difference between open and closed cross-sections is particularly pronounced at the end supports, since
  - $\rightarrow$ torsion caused by skew end supports directly depends on the stiffness ratio GK/EI_y (see general behaviour)
  - $\rightarrow$ ratio GK/EI_y is orders of magnitude lower in girders with open cross-section than in box girders
  - $\rightarrow$ Therefore, the following slides primarily address box girders (unless indicated otherwise)

Skew girder with open cross-section: Grillage model
(plan, cross-section)
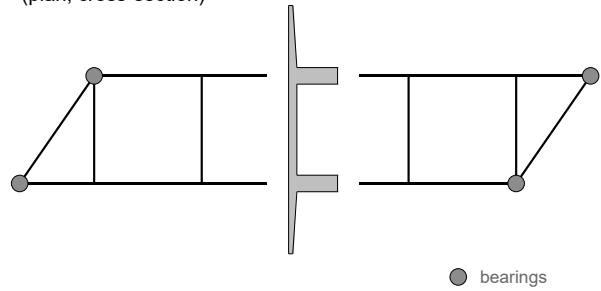
● bearings

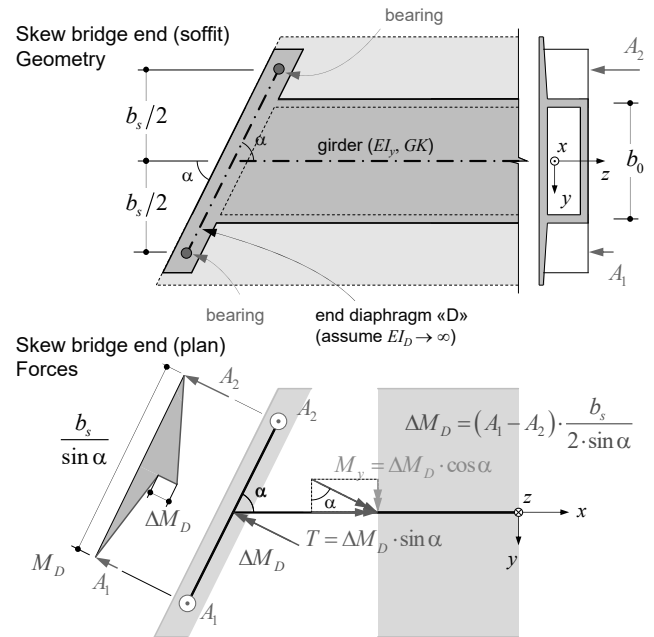Skew box girder: Spine model
(plan, cross-section)

Skew bridges – Analysis

Torsion and bending at skew bridge ends

- Consider a skew bridge end, with support angle α and an end diaphragm supported on two bearings (figure)
- The end diaphragm can rotate freely around its axis, and is loaded at its ends by the support reactions
  - → zero torsion in end diaphragm, $T_D = 0$
  - → bending moments in end diaphragm M_D differ by ΔM_D, at intersection with girder, unless support reactions are equal (they are not)
  - → ΔM_D causes bending and torsion in the girder, which by equilibrium are:
    $$M_y = \Delta M_D \cdot \cos\alpha \qquad T = \Delta M_D \cdot \sin\alpha$$

- The reaction in the obtuse angle is larger, $A_2 > A_1$ ($\alpha < \pi/2$), hence the difference ΔM_D is negative
  - → negative bending moments in girder $M_D < 0$ (partial moment restraint)
  - → for $\alpha > \pi/2$, torsional moments in the girder change sign (switch of acute and obtuse angle, $A_2 < A_1$) but bending moments remain negative ($\cos\alpha$ also changes sign)
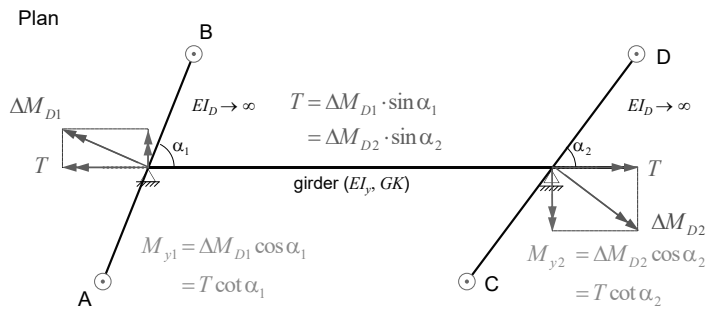
Skew bridge end (soffit)
Geometry

Skew bridge end (plan)
Forces

$$\Delta M_D = \left(A_1 - A_2 \right) \cdot \frac{b_s}{2 \cdot \sin\alpha}$$

Note: The vertical shear force V_z is not shown in the figure on the right, as it simply corresponds to the sum of the support reactions; see behind (diaphragm design).

The moment arrows are shown in direction they act; since $\Delta M_D < 0 \rightarrow M_y < 0$, $T_D < 0$ (for sign convention used in Baustatik lectures).

Skew bridges – Analysis

Torsion and bending at skew bridge ends

- A single-span girder with skew supports at both ends is once statically indeterminate, and can easily be analysed e.g. using the force method (see Stahlbeton I, Torsion, use e.g. T as redundant variable)

- For vertical loads and infinitely stiff diaphragms, the equations shown to the right are obtained:
  - $\rightarrow$ torsional moment is constant
  - $\rightarrow$ negative bending moments at girder ends $\rightarrow$ if modelled as a beam, the girder is partially clamped

- The partial clamping caused by skew supports in girders with high torsional stiffness is favourable regarding stiffness (deflections) and strength. It may, however, cause problems if not considered properly:
  - $\rightarrow$ check uplift (negative support reactions) at supports in acute corners
  - $\rightarrow$ ensure ductile behaviour and account for torsional moments in design
  - $\rightarrow$ design end diaphragms for torque introduction

Plan

$$T = \Delta M_{D1} \cdot \sin\alpha_1$$
$$= \Delta M_{D2} \cdot \sin\alpha_2$$

$$M_{y1} = \Delta M_{D1}\cos\alpha_1$$
$$= T\cot\alpha_1$$

$$M_{y2} = \Delta M_{D2}\cos\alpha_2$$
$$= T\cot\alpha_2$$

$$T = -\frac{ql^2}{12}\frac{\dfrac{\cot\alpha_1 + \cot\alpha_2}{2}}{\dfrac{\cot^2\alpha_1 + \cot\alpha_1\cot\alpha_2 + \cot^2\alpha_2}{3} + \dfrac{EI}{GK}}$$

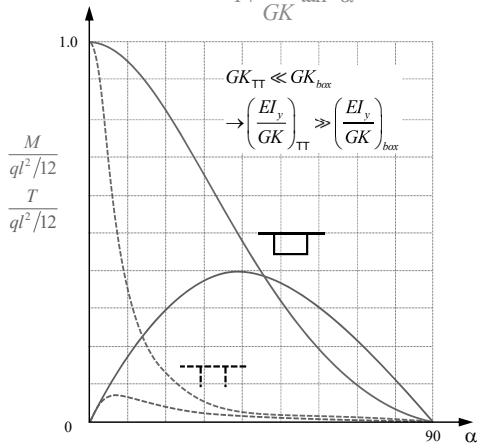$$\rightarrow \Delta M_{D1} = \frac{T}{\sin\alpha_1}, \quad \Delta M_{D2} = \frac{T}{\sin\alpha_2}$$

$$\rightarrow M_{y1} = T\cot\alpha_1, \quad M_{y2} = T\cot\alpha_2$$

Skew bridges – Analysis
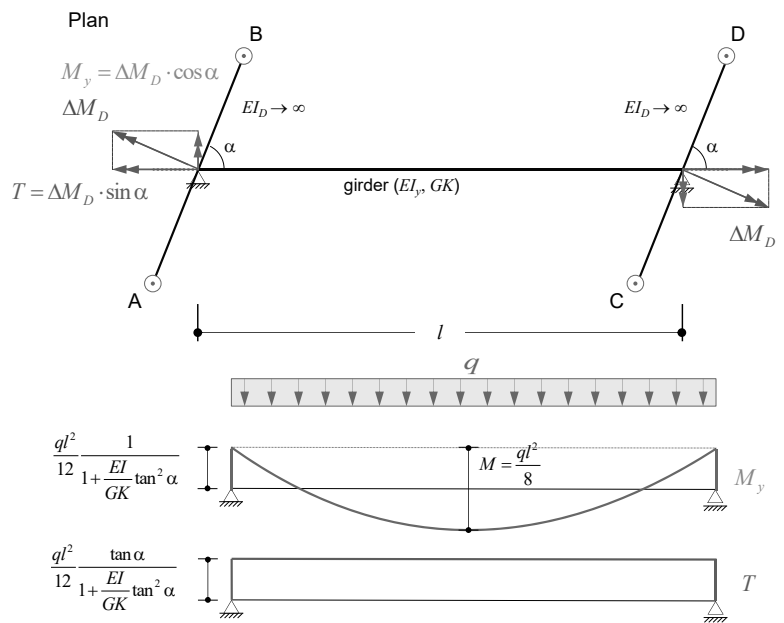
Special case of equal skew at both girder ends.

$$T = -\frac{ql^2}{12}\frac{\cot\alpha}{\cot^2\alpha + \dfrac{EI}{GK}} = -\frac{ql^2}{12}\frac{\tan\alpha}{1+\dfrac{EI}{GK}\tan^2\alpha}$$

$$M_y = T\cot\alpha = -\frac{ql^2}{12}\frac{1}{1+\dfrac{EI}{GK}\tan^2\alpha}$$

$GK_{TT} \ll GK_{box}$

$$\rightarrow \left(\frac{EI_y}{GK}\right)_{TT} \gg \left(\frac{EI_y}{GK}\right)_{box}$$

$\dfrac{M}{ql^2/12}$
$\dfrac{T}{ql^2/12}$

Plan

$M_y = \Delta M_D \cdot \cos\alpha$

ΔM_D

$EI_D \rightarrow \infty$ $EI_D \rightarrow \infty$

$T = \Delta M_D \cdot \sin\alpha$ girder (EI_y, GK)

ΔM_D

l

q

$$\frac{ql^2}{12}\frac{1}{1+\dfrac{EI}{GK}\tan^2\alpha}$$ $M = \dfrac{ql^2}{8}$ M_y

$$\frac{ql^2}{12}\frac{\tan\alpha}{1+\dfrac{EI}{GK}\tan^2\alpha}$$ T

Obviously, and as confirmed by evaluating the equations, the structural behaviour strongly depends on the ratio bending to torsional stiffness EI/GK and the support angle α.

If modelled as a single beam (spine model), a simply supported girder can be considered, from the bending point of view, as a girder with elastically clamped ends. The stiffness of the clamped ends depends on the ratio of torsional stiffness to bending stiffness and the support angle α.

If GK is small, i.e. the ratio EI/GK is large (open cross-sections), the torsional moments are small, and the bending moments correspond closely to those in a girder with orthogonal supports. In box girders, GK is much higher and consequently, EI/GK is small. High torsional moments result at skew girder supports.
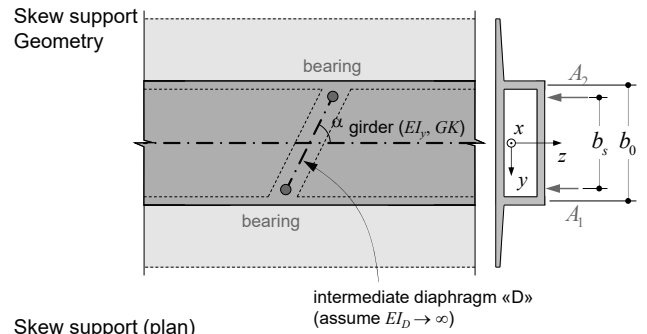
Skew bridges – Analysis
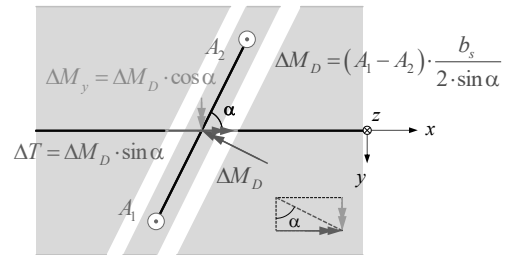
Torsion and bending at skew intermediate supports (piers)

- At a skew intermediate support with two vertical bearings (support angle α, figure), the girder can rotate around the axis of the intermediate diaphragm, which is again loaded at its ends by the support reactions

  $\rightarrow$ zero torsion in intermediate diaphragm, $T_D = 0$

  $\rightarrow$ bending moments in intermediate diaphragm M_D differ by ΔM_D, at intersection with girder, unless support reactions are equal (generally, they are not)

  $\rightarrow$ ΔM_D causes jumps of the bending and torsion in the girder, which by equilibrium are:

$$\Delta M_y = \Delta M_D \cdot \cos\alpha \quad \Delta T = \Delta M_D \cdot \sin\alpha$$

- The bearing reactions at skew intermediate supports generally differ less than at end supports (if adjacent spans are similar)

- Still, the jumps in bending and torsional moment need to be considered in the design of the intermediate diaphragm
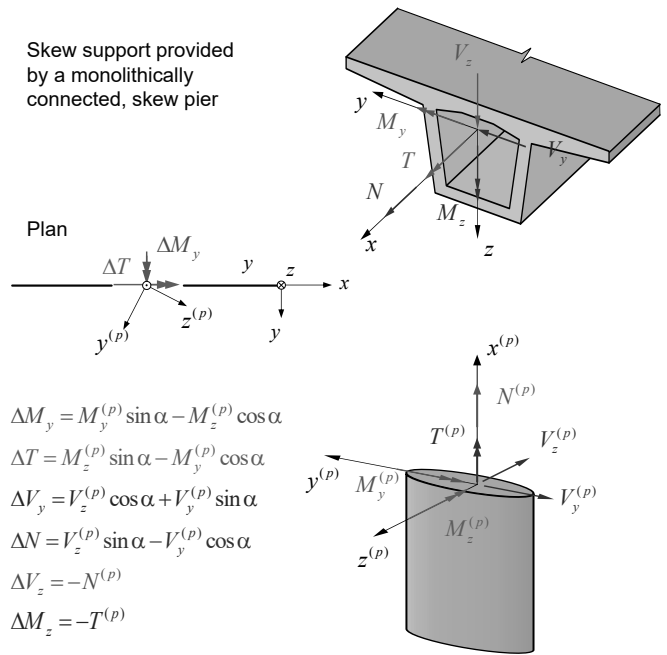
Skew support
Geometry

intermediate diaphragm «D»
(assume $EI_D \rightarrow \infty$)

Skew support (plan)
Forces

$$\Delta M_y = \Delta M_D \cdot \cos\alpha \qquad \Delta M_D = (A_1 - A_2) \cdot \frac{b_s}{2 \cdot \sin\alpha}$$

$$\Delta T = \Delta M_D \cdot \sin\alpha$$

Skew bridges – Analysis

Torsion and bending at skew intermediate supports (piers)

Skew support provided by a monolithically connected, skew pier

- In skew piers are monolithically connected to the girder, $T_D \neq 0$. Rather, all stress resultants of the pier and girder, respectively, need to be considered (see substructure for orthogonal piers), as illustrated in the figure

- As for piers with bearings, the jumps in bending and torsional moment need to be considered in the design of the intermediate diaphragm (ΔM_D = vector sum of $M_z^{(p)}$ and $M_y^{(p)}$)

- Piers are usually much wider in the transverse direction of the bridge (y in figure) $\rightarrow M_z^{(p)} >> M_y^{(p)}$, i.e., ΔM_D is approximately parallel to $M_z^{(p)}$ as in skew piers with bearings

- The design of skew diaphragms with monolithically connected piers is challenging. Envelopes of internal actions in the girder are of limited use; using internal actions at the pier top is more straightforward

- Note that the signs of the individual components depend on the orientation of coordinate axes (pier!) $\rightarrow$ formulae on slide need to be adjusted accordingly

Plan

$$\Delta M_y = M_y^{(p)} \sin\alpha - M_z^{(p)} \cos\alpha$$
$$\Delta T = M_z^{(p)} \sin\alpha - M_y^{(p)} \cos\alpha$$
$$\Delta V_y = V_z^{(p)} \cos\alpha + V_y^{(p)} \sin\alpha$$
$$\Delta N = V_z^{(p)} \sin\alpha - V_y^{(p)} \cos\alpha$$
$$\Delta V_z = -N^{(p)}$$
$$\Delta M_z = -T^{(p)}$$

Special girder bridges

Skew bridges
Design

Skew bridges – Design

General remarks: Stiffness ratio GK/EI_y

- The stiffness of concrete bridges, and of concrete bridge decks in composite bridges, is significantly reduced by cracking
- Usually, the reduction of the torsional stiffness GK by cracking is much more pronounced than that of the bending stiffness EI_y
  - → in statically indeterminate systems where the magnitude of torsional and bending moments depends on the ratio GK/EI_y (compatibility torsion, see lecture Stahlbeton I), cracking causes moment redistributions
- The ratio GK/EI_y is significantly reduced in the ULS of structural safety (ULS STR), when considering pure bending or pure torsion. Under combined bending and torsion (compression zone remains uncracked) and serviceability, particularly in prestressed concrete bridges, this effect is much less pronounced
  - → Consider reduction of ratio GK/EI_y in ULS STR for fully cracked behaviour (in preliminary design, reduce e.g. by a factor of 3)
  - → Use uncracked or moderately reduced ratio GK/EI_y for serviceability and fatigue
  - → Ensure ductile behaviour in bending and torsion to avoid brittle failures in case of over- or underestimation of ratio GK/EI_y

Viadukt Glattzentrum. dsp Ingenieure + Planer, Höltschi + Schurter, Feddersen Klosterman. Photo © dsp Ingenieure & Planer
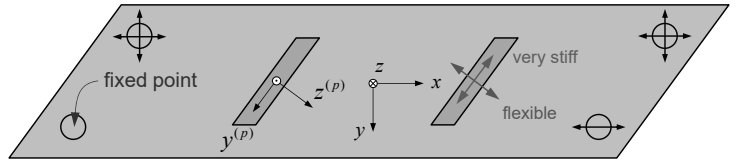
Skew bridges – Design

General remarks: Bearing layout

- Piers of orthogonally supported bridges are usually wide (=stiff) in the transverse direction of the bridge and hence, resist a large portion of transverse horizontal forces F_y (wind, nosing etc.)

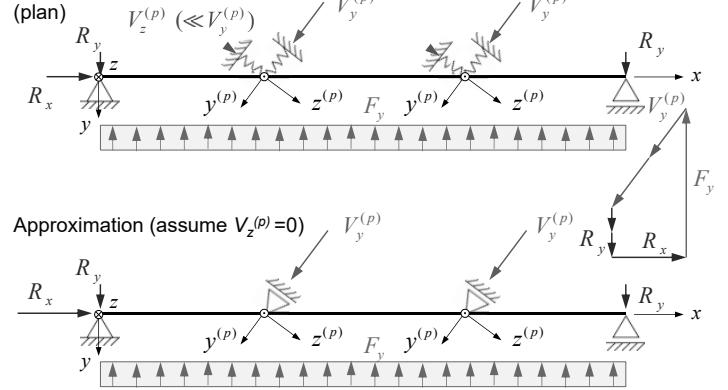- Skew piers resist F_y in different ways, depending on the longitudinal support system:

    → bridge longitudinally fixed at abutment: Piers resist large portion of F_y (longitudinal component of $V_y^{(p)}$ primarily resisted by R_x at fixed support)

    (= shown in figures on this slide)

Skew continuous girder, longitudinally fixed at abutment

Horizontal static system (plan)

Approximation (assume $V_z^{(p)} = 0$)

Skew bridges – Design

General remarks: Bearing layout

- Piers of orthogonally supported bridges are usually wide (=stiff) in the transverse direction of the bridge and hence, resist a large portion of transverse horizontal forces F_y (wind, nosing etc.)

- Skew piers resist F_y in different ways, depending on the longitudinal support system:

  → bridge longitudinally fixed at abutment: Piers resist large portion of F_y (longitudinal component of $V_y^{(p)}$ primarily resisted by R_x at fixed support)

  → bridge longitudinally stabilised by piers: Piers contribute much less to F_y (longitudinal component of $V_y^{(p)}$ must be resisted by respective component of $V_z^{(p)}$ (very flexible)

  → much larger transverse reactions at abutments R_y if no longitudinal support is provided there (may require separate guide bearings)

- Therefore, longitudinal fixity at an abutment is preferred in skew continuous girders

Skew continuous girder, longitudinally stabilised by piers
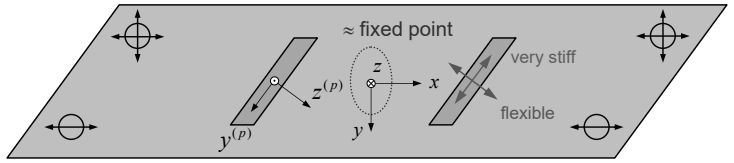
Horizontal static system (plan)
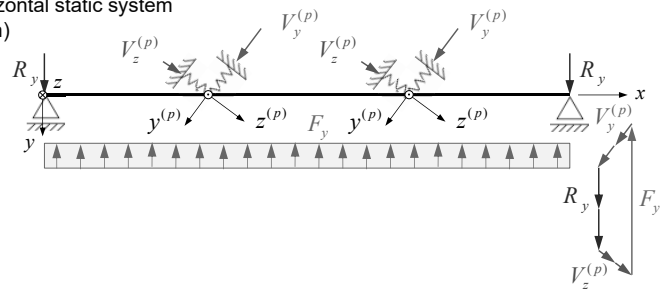
Skew bridges – Design

General remarks: Detailing

- Typically, skew supports significantly complicate detailing, particularly of the diaphragms (photos), where reinforcement in three (or even four) in-plan directions is typically required
- Monolithically connected skew piers with skew diaphragms in box girders are particularly demanding for detailing

- In all cases, observe the following:
    - → carefully detail the reinforcement
    - → avoid providing excessive amounts of reinforcement to cover uncertainties in design: enough space to cast and compact the concrete, ensuring a proper concrete quality, is equally important
   - → using T-headed bars to anchor pier reinforcement helps reducing reinforcement congestion
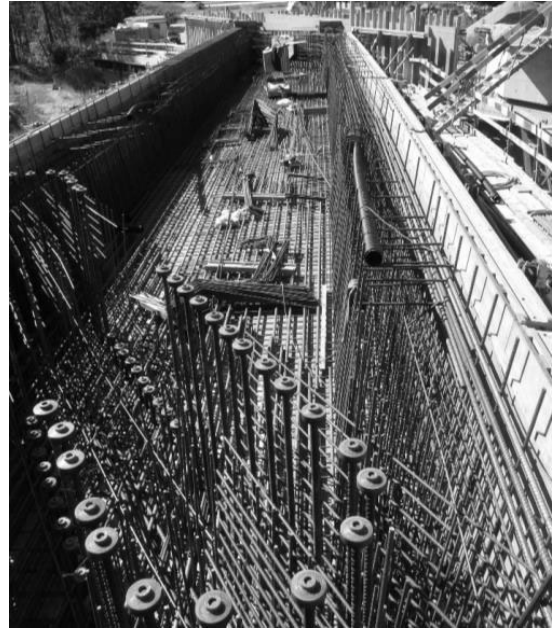
Reinforcement at skew intermediate support being fixed at Ponte Calancasca, Circonvallazione di Roveredo. dsp Ingenieure + Planer, Spataro Petoud Partner, MAWI SA. Photo © Raoul Spataro

Skew bridges – Design

Design of skew end diaphragms and bridge ends

- As outlined under analysis, skew end supports provide an elastic clamping to the bridge girder, particularly to box girders with a high torsional stiffness

- On the previous slides, this has been dealt with using a spine model for the girder. However, the load introduction cannot be examined using this approach (the bridge is not a line beam)

- The introduction of torsion, bending moments and shear forces at skew girder ends is outlined on the following slides, using equilibrium models ($\rightarrow$ provide minimum reinforcement in all elements to ensure a ductile behaviour)
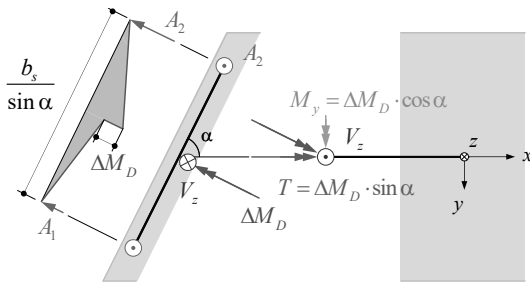
$$M_y = \Delta M_D \cdot \cos \alpha$$

$$T = \Delta M_D \cdot \sin \alpha$$

Skew abutment with end diaphragm, Ponte Calancasca, Circonvallazione di Roveredo. dsp Ingenieure + Planer, Spataro Petoud Partner, MAWI SA. Photo © Raoul Spataro

Skew bridges – Design
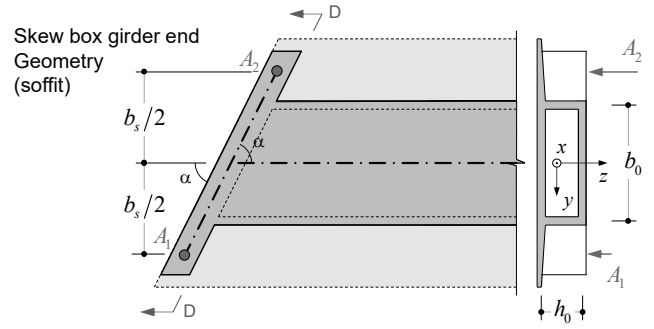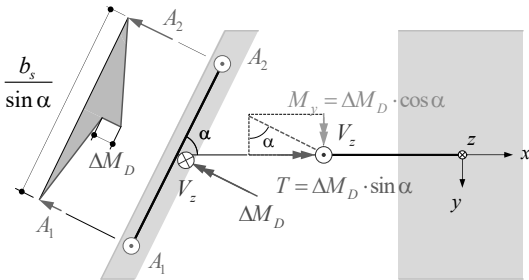
Design of skew end diaphragms – box girders

- The end diaphragm is loaded by the vertical shear force V_z and the moment ΔM_D (see analysis), causing a vertical flow $0.5 \cdot V_z / h_0$ in the webs and a circumferential shear flow $\tau \cdot t(\Delta M_D)$, respectively, where:

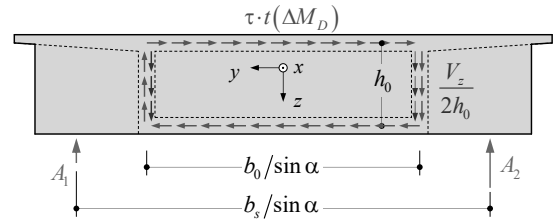$$\tau \cdot t\left(\Delta M_D\right) = \frac{\Delta M_D}{2h_0 b_0 / \sin\alpha} = \frac{T}{2h_0 b_0}$$

- The support reactions are:

$$A_{1,2} = \frac{V_z}{2} \mp \frac{\Delta M_D}{b_s / \sin\alpha} = \frac{V_z}{2} \mp \frac{T}{b_s}$$

$$\left(\begin{array}{l} M_y = \Delta M_D \cos\alpha \\ T = \Delta M_D \sin\alpha \\ \rightarrow M_y = T \cot\alpha \end{array}\right)$$
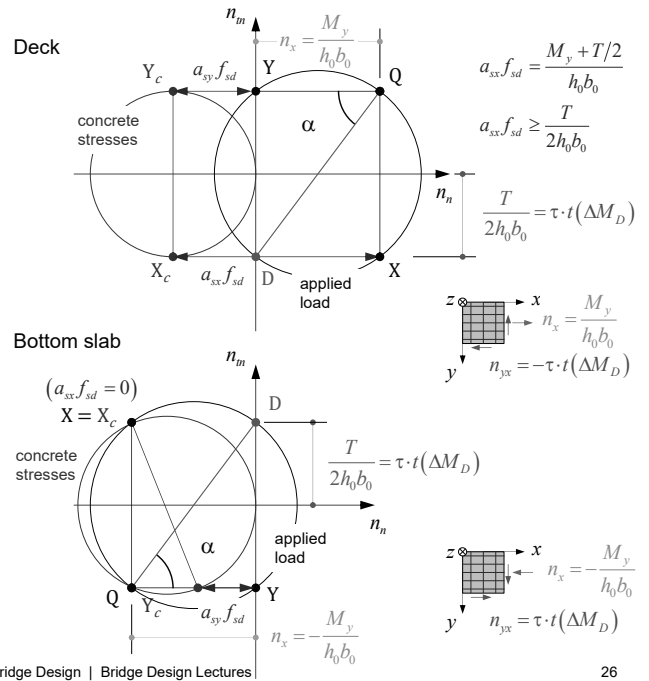
Skew box girder end
Geometry
(soffit)

Skew end diaphragm (section D-D, ca. 2·scale of above)
Forces acting on end diaphragm = free body cut off along D-D

Skew bridges – Design

Design of skew girder ends – box girders

- The dimensioning of skew end diaphragms is thus similar as in the case of orthogonal support. Unless the bearings are separated much more than the webs, the diaphragm is primarily loaded in in-plane shear

- In the girder, the following states of stress result at the girder end (M_y carried by force couple with lever arm h_0):
  - → webs: pure shear
  - → deck: shear and longitudinal tension
  - → bottom slab: shear and longitudinal compression

- The top and bottom slab reinforcement can be dimensioned using the parametric yield conditions for membrane elements (to ensure shear flow, proper detailing at diaphragm is required), see Stahlbeton I and Advanced Structural Concrete

- The figure illustrates the forces and dimensioning graphically (Mohr's circles); no longitudinal reinforcement is required in the bottom slab for $-M_y \geq T/2$ (i.e. $\tan\alpha \leq 2$), as in the illustrated case with $\tan\alpha = 4/3$

- Note that pure shear in direction of end diaphragm D

Deck

$$n_x = \frac{M_y}{h_0 b_0}$$

$$a_{sx} f_{sd} = \frac{M_y + T/2}{h_0 b_0}$$

$$a_{sx} f_{sd} \geq \frac{T}{2 h_0 b_0}$$

$$\frac{T}{2 h_0 b_0} = \tau \cdot t \left(\Delta M_D \right)$$

$$n_x = \frac{M_y}{h_0 b_0}$$

$$n_{yx} = -\tau \cdot t \left(\Delta M_D \right)$$

Bottom slab

$$\frac{T}{2 h_0 b_0} = \tau \cdot t \left(\Delta M_D \right)$$

$$n_x = -\frac{M_y}{h_0 b_0}$$

$$n_{yx} = \tau \cdot t \left(\Delta M_D \right)$$

Skew bridges – Design

Design of skew end diaphragms– open cross-sections

- In open cross-sections, GK/EI_y is small
  - $\rightarrow$ small T and M_y (hence ΔM_D) at girder ends
  - $\rightarrow$ almost equal support reactions (under symmetrical load)
- The small T and M_y (hence ΔM_D) may be attributed to the webs (50% per web $\rightarrow$ force flow shown in figure
- As illustrated in the figure, skew end diaphragms of girders with open cross-section are primarily loaded in bending (as opposed to box girders, where the skew end diaphragms are primarily loaded in shear)
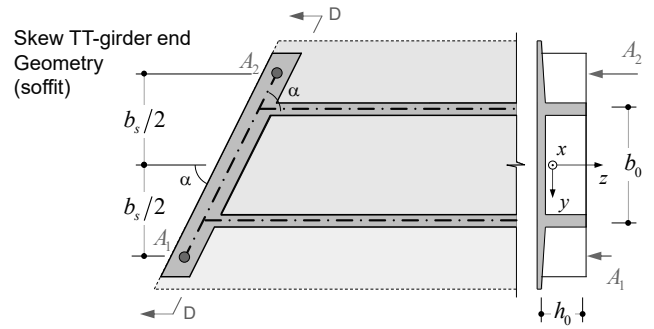
Skew TT-girder end
Geometry
(soffit)

Skew end diaphragm (section D-D, ca. 2·scale of above)
Forces acting on end diaphragm = free body cut off along D-D

$$\begin{pmatrix} M_y = \Delta M_D \cos\alpha \\ T = \Delta M_D \sin\alpha \\ \rightarrow M_y = T\cot\alpha \end{pmatrix}$$
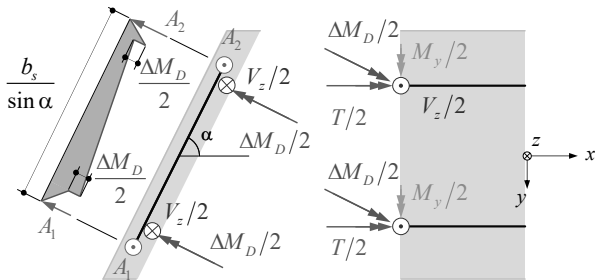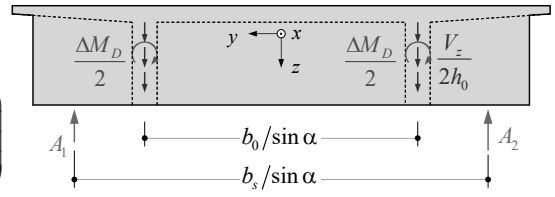
Only uniform torsion is relevant here, warping torsion does not cause a clamping effect at skew bridge ends.
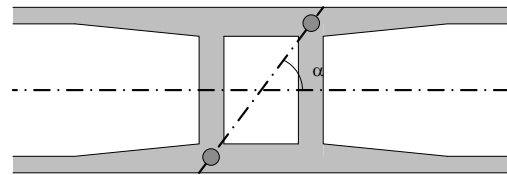
Skew bridges – Design

Intermediate diaphragms at skew supports (piers)

- Two different layouts are common for intermediate diaphragms at skew supports (over piers):
    - → skew intermediate diaphragms (top figure)
    - → pair of diaphragms perpendicular to the bridge axis (bottom figure)
- Skew intermediate support diaphragms may be dimensioned like skew end diaphragms. Unless adjacent spans vary strongly, support reactions are similar, i.e. ΔM_D is small
    - → small discontinuity in bending moments
    - → neglect skew in preliminary design
- In a spine model, diaphragm pairs perpendicular to the bridge axis can be modelled as rigid members extending out from the axis to the bearing centreline (next slide), but
    - → model only yields sectional forces of the entire cross-section (e.g. difference in forces in the two webs not considered)
    - → better use grillage model for box girders with skew intermediate supports and perpendicular diaphragms

Skew support – skew support diaphragm
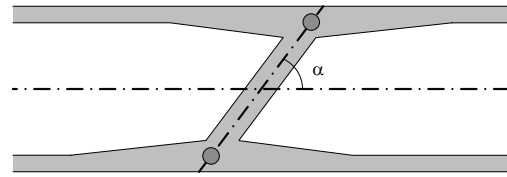
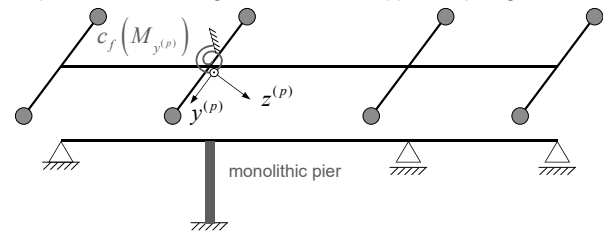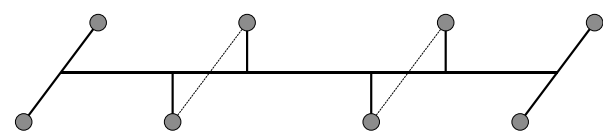Skew support – pair of perpendicular diaphragms

Skew bridges – Design

Models for continuous box girders with skew supports

- If skew support diaphragms are used, a spine model is appropriate for single-cell box girders

- If only the superstructure is modelled, rigid vertical supports provide full fixity against rotations around the pier axes $z^{(p)}$ ($M_z^{(p)}$), which is appropriate for wide = very stiff piers in direction $z^{(p)}$. Skew slender piers should be included in the global analysis model.

- Skew Piers monolithically connected to the girder should also be included in the global analysis model. In preliminary design, the model shown in the figure may be used (full fixity for $M_z^{(p)}$, elastic spring for $M_y^{(p)}$).

- For single-cell box girders with perpendicular support diaphragms, the spine model shown is of limited use (see previous slide). Rather, a grillage model (bottom figure) should be used.
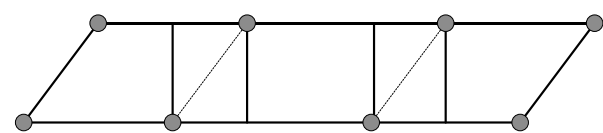
Spine model for box girder with skew support diaphragms

$c_f \left(M_{y^{(p)}} \right)$ $z^{(p)}$ $y^{(p)}$ monolithic pier

Spine model for box girder with perpendicular support diaphragms


Grillage model for box girder with perpendicular support diaphragms

Skew bridges – Design

Comparison of continuous box girders with skew supports

- The figure (taken from Menn (1990)) illustrates the differences in a two-span girder between
    - → skew and perpendicular support diaphragms
    - → spine and grillage model for perpendicular support diaphragms

- Considered load cases:
    - → uniform load (left column)
    - → traffic load in left span only (right column)

- It can be seen that
    - → differences in global behaviour (total internal actions) are small
    - → relevant differences are obtained in the intermediate support region
    - → there, only the grillage model captures the differences in web shear forces caused by the perpendicular diaphragms

skew diaphragms
spine model

perpendicular diaphragms
spine model

perpendicular diaphragms
2D grillage
—— rear web / half box
- - - front web / half box
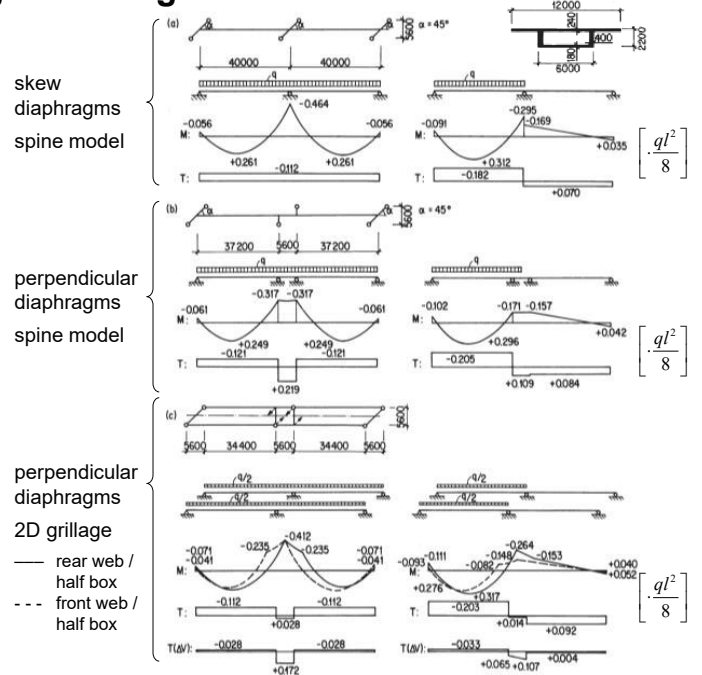
Skew bridges – Design

Skew frame bridges

- In skew frame bridges, the abutment walls provide a higher degree of fixity to the girder than in orthogonal frames, due to
    - → the high in-plane stiffness of the wide walls
    - → restraint to horizontal movement provided by the backfill
- Nonetheless, the abutment walls are usually stiffened by vertical ribs, particularly if the girder is prestressed (transfer of clamping moment); haunching the ribs as shown in the figure reduces restraint to girder expansion and contraction
- The design of skew frame bridges is demanding, particularly regarding the frame corners. The figure (taken from Menn (1990)) illustrates a truss model for a skew frame corner
- Providing full moment continuity would usually require prestressing the abutment walls, which complicates execution
    - → allow cracking of abutment walls at the top
    - → account for reduced stiffness due to cracking in analysis
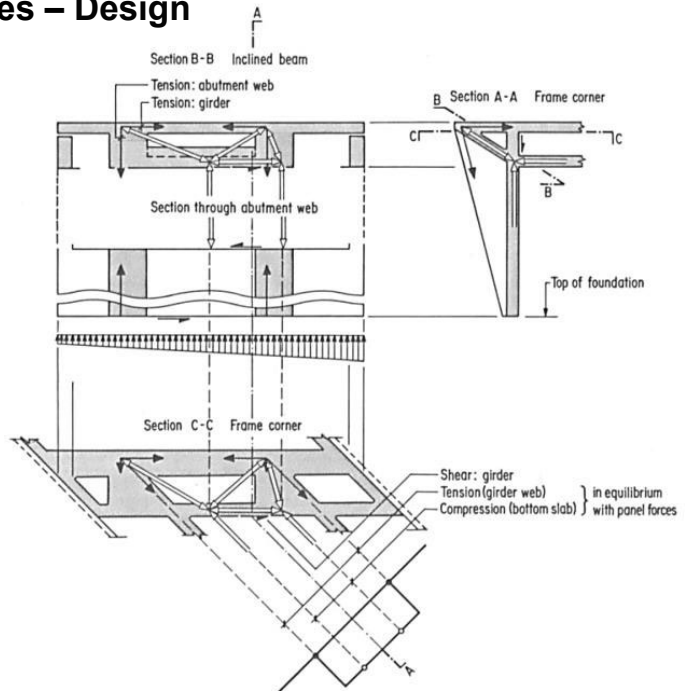
Skew bridges – Design

Movements of skew frame bridges due to girder deformations

- Since the abutment walls are very stiff in their plane, expansion and contraction of skew girder bridges causes a rotation in plan

$$\Delta = \varepsilon_x \frac{l/2}{\sin \alpha}$$

$$\varphi = \frac{\Delta \cos \alpha}{l/2} = \varepsilon_x \cot \alpha$$

- To minimise restraint in the girder (which has to be accounted for in design, relevant for contraction causing tension):
  - → use flexible abutment walls (out of plane)
  - → separate wing walls from abutment wall, or use short cantilevered wings

- Even with flexible abutment walls, girder expansion is resisted by the backfill (flexible restraint). In long frame bridges, account for strain ratcheting (see integral bridges)

Contraction of bridge deck
(sliding bearings at abutments)
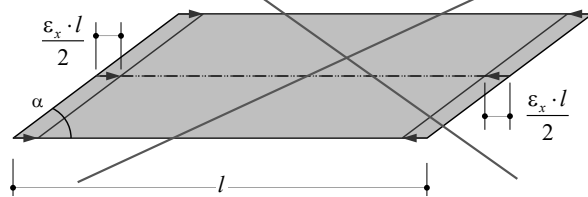
Abutment walls are much stiffer in the direction parallel to bridge end than perpendicular to this direction → no displacements perpendicular to bridge end!


Movements of bridge deck
(top view))
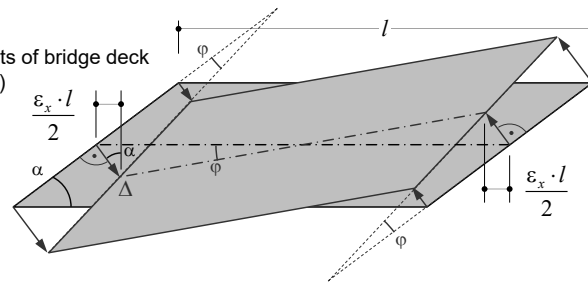
The equations are valid for small deformations only (the ones shown are large, for better illustration).

Skew bridges – Design

Prestressing layouts for skew girder bridges

- Tendon layouts in skew girder bridges are similar to those in orthogonally supported bridges.

- At skew end supports, a high tendon anchorage is preferred (corresponding to the negative bending moment caused by the flexible clamping by the skew support) (if present, check space requirements of expansion joint and protect anchorage from leaking de-icing salt)

- The figure (taken from Menn (1990)) illustrates the tendon layout for skew intermediate supports of a continuous girder

- If skew varies along the bridge, the webs have different spans → adjust prestressing layout accordingly (higher force in longer web)

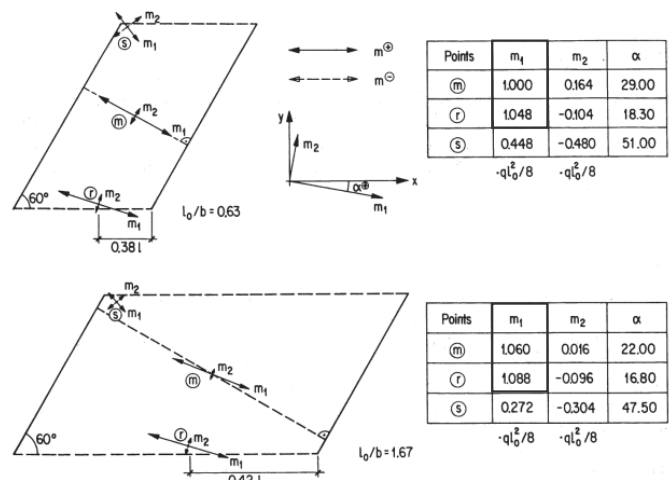Tendon layout for skew continuous girders
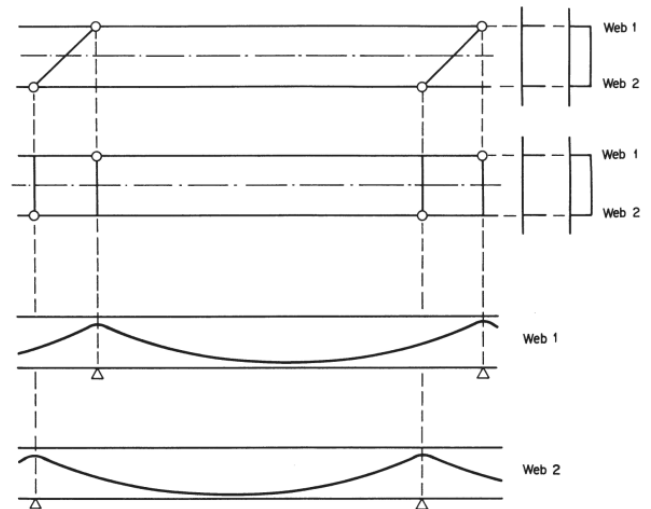
Skew bridges – Design

Bending moments in skew slab bridges

- Slabs are very ductile elements, as long as (punching) shear is not governing the behaviour
  - → provide shear reinforcement in thick slabs and near supports
- In the design of slabs for bending, significant moment redistributions may then be assumed, which is particularly useful in skew slabs (e.g. to concentrate reinforcement / tendons in bands along edges)
- Reinforcement parallel to the slab edges (skew reinforcement) is often practical. However, the bending resistance in the direction between the obtuse angles is strongly reduced → account for correct resistances in design (see Advanced Structural Concrete)
- The direction of principal moments in skew, simply supported slabs differs only slightly from that of lines perpendicular to the support axes, particularly in wide slabs (see figure, taken from Menn (1990)) → in preliminary design, a single-span, orthogonally supported slab may be assumed

Principal bending moments in skew slabs

Points	m_1	m_2	α
ⓜ	1.000	0.164	29.00
ⓡ	1.048	−0.104	18.30
ⓢ	0.448	−0.480	51.00
	$-ql_0^2/8$	$-ql_0^2/8$	

Points	m_1	m_2	α
ⓜ	1.060	0.016	22.00
ⓡ	1.088	−0.096	16.80
ⓢ	0.272	−0.304	47.50
	$-ql_0^2/8$	$-ql_0^2/8$	

The direction of the principal bending moments due to dead load deviates only slightly from lines drawn perpendicular to the support axes.

When the width of the slab, b, is large relative to l_0, the sectional forces in its middle region will be identical to the sectional forces in a slab of infinite width. A special investigation will thus only be required at the free edges.

Skew bridges – Design

Prestressing layouts for skew slab bridges

- The figure (taken from Menn (1990)) illustrates practical tendon layouts for skew, single-span slabs
- Concentrating tendons in bands simplifies placement and execution
  - → The required moment redistributions to fully activate the tendons in ULS are usually not critical
  - → Spreading of the prestressing force (beneficial compression) over the width of the slab may be accounted for in SLS and ULS (for punching shear verifications, use a cautious value, see Advanced Structural Concrete)
- At skew end supports, a high tendon anchorage is preferred (see skew girders), but slab thickness usually limits the possible eccentricity.

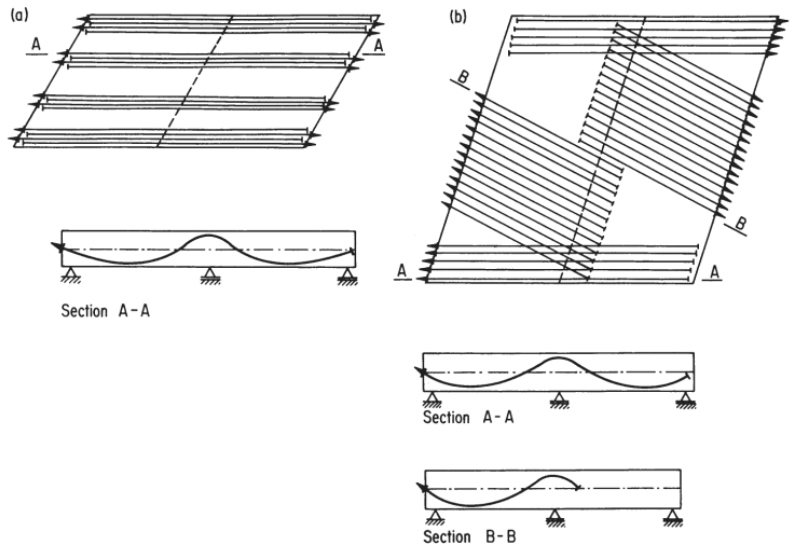Tendon layout for skew simply supported slabs (and frames with flexible abutments)
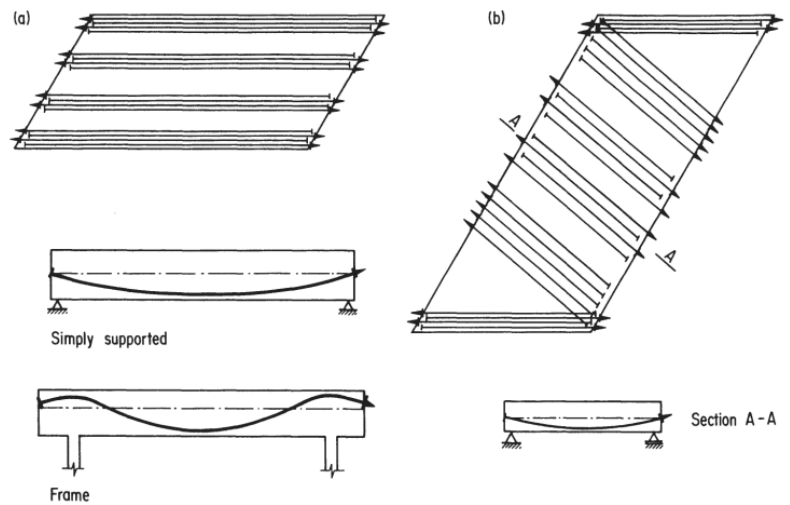
Skew bridges – Design

Prestressing layouts for skew slab bridges

- The figure (taken from Menn (1990)) illustrates practical tendon layouts for skew, multi-span slabs

- Remarks see previous slide

Tendon layout for skew multi-span slabs

Section A–A

Section A–A

Section B–B

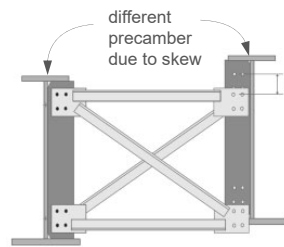# Special girder bridges

Skew bridges
Particularities of steel bridges
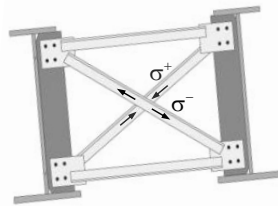
Skew bridges – Design

Orthogonal cross-frames

- Steel girders are commonly fabricated with camber but plumb (no "twisting camber" of individual beams), see Figure (1), and erected by either
  - (i) connecting steel beams and cross-frames under zero load and lifting them in together, or
  - (ii) lifting in the beams separately = installing the cross-frames after the application of (steel / total) dead load
- In the case (ii), the analysis must account for the fact that the cross-frames are stress-free under steel or total dead load (Fig. 4), but not under zero load
  - → activate cross-frames in the analysis model only after application of dead load (= staged construction model)
  - → alternatively, consider locked-in stresses determined by following the steps illustrated in Figures (2)-(3):
    … apply fictitious strains ε to fit fabricated geometry (beams blocked in this stage)
    … releasing beams causes twist
    … locked-in stresses in cross-frames = $\varepsilon \cdot E_a$
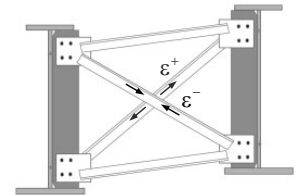- Further details, see reference given in notes.

(1) Fabricated girders: cambered (for steel or total dead load), but plumb
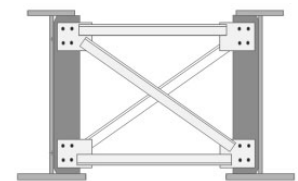

(2) Virtual Strains applied to cross-beams to fit cambered but plumb beams (virtually blocked)

(3) Virtual geometry after releasing beams ≈ geometry when removing dead load in system with installed cross-frames (and locked-in stresses)

(4) Geometry after application of dead load = installation of stress-free cross-beams
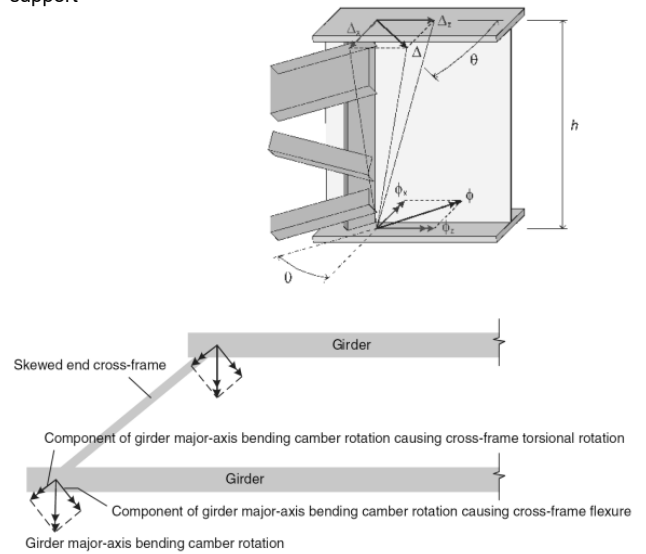
NCHRP Report 725 - Guidelines for Analysis Methods and Construction Engineering of Curved and Skewed Steel Girder Bridges, 2012

Skew bridges – Design

Skew cross-frames at supports

- At skew support lines, cross-frames essentially act like end diaphragms in skew concrete girders, i.e., the blue cross-frame rotates around the bearing line (below cross-frame, parallel to its axis), forcing the beam top flanges to move in direction Δ (top figure)

- Like intermediate cross-frames, if the steel girders are lifted in separately, the cross frames at skew supports are installed only after the dead load (steel or total) has been applied, they are stress-free under this load, but not under zero load

  $\rightarrow$ activate skew cross-frames in the analysis model only after application of dead load (= staged construction model)

  $\rightarrow$ Alternatively, consider locked-in stresses determined similarly as for orthogonal cross-frames (previous slide)

- Further particularities of skew steel bridges, see reference given in notes.

Skew cross-frame at support

Skewed end cross-frame

Component of girder major-axis bending camber rotation causing cross-frame torsional rotation

Girder

Component of girder major-axis bending camber rotation causing cross-frame flexure

Girder major-axis bending camber rotation

NCHRP Report 725 - Guidelines for Analysis Methods and Construction Engineering of Curved and Skewed Steel Girder Bridges, 2012