Special girder bridges

Curved Bridges

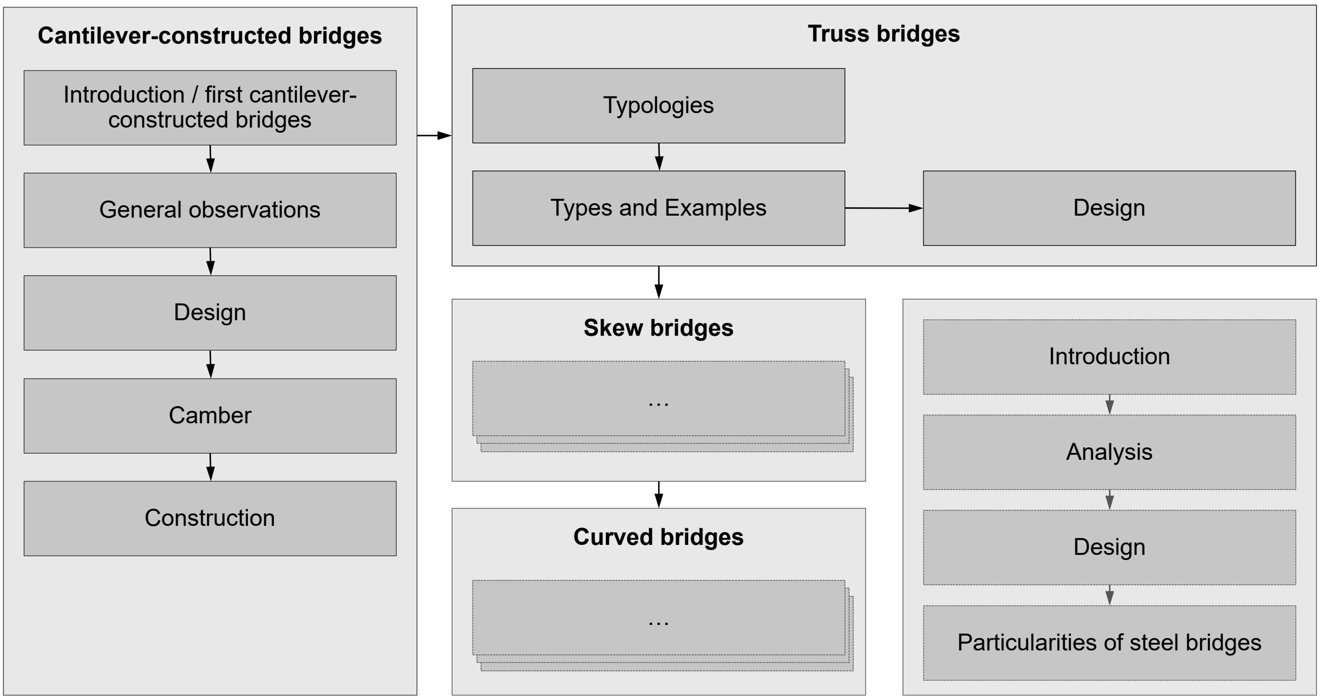
Recommended literature for further reading:

H. Bachmann, Vorlesung Spezielle Probleme der Vorspannung (Vorspannung und Querbeanspruchung exzentrisch belasteter, gekrümmter und schief gelagerter Betontragwerke), ETH Zürich, 1990.

AASHTO Guide Specifications for Horizontally Curved Highway Bridges, 2003

NCHRP Report 725 - Guidelines for Analysis Methods and Construction Engineering of Curved and Skewed Steel Girder Bridges, 2012

NSBA G13.1—Guidelines for Steel Girder Bridge Analysis, 2014



Cantilever-constructed bridges

- Introduction / first cantilever-constructed bridges
- General observations
- Design
- Camber
- Construction

Truss bridges

- Typologies
- Types and Examples
- Design

Skew bridges

- …

Curved bridges

- …

- Introduction
- Analysis
- Design
- Particularities of steel bridges

Special girder bridges

Curved Bridges
Applications

Curved Bridges – Applications

- Integration of bridge structures into the roadway and railway networks often requires the adoption of curved bridge decks.
- In the case of highway interchanges and urban expressways, they are unavoidable.
- In the case of long-span crossings, the alignment should be adjusted to place the main bridge on a tangent:
  → Simpler to design and construct
  → Easier to accommodate bridge movements at expansion joints (at the boundary between main and approach spans).
- In the case of long viaducts it is impractical and uneconomical to design the entire alignment to be straight.
- An in-plan curved alignment may favour aesthetics and view of the bridge by the users.

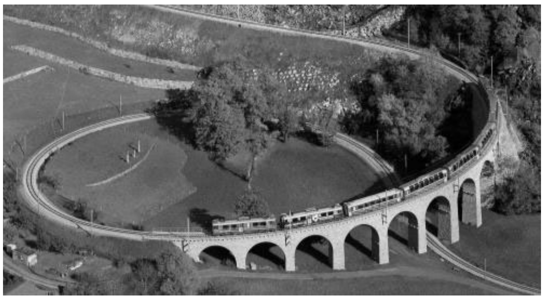


Photo Credits:

Top: Viaducto de Arbizelai, Spain © Fhecor

Bottom: Brusio spiral viaduct, Switzerland (1908), By Kabelleger / David Gubler (http://www.bahnbilder.ch) - Own work: http://www.bahnbilder.ch/picture/11543, CC BY 3.0, https://commons.wikimedia.org/w/index.php?curid=22541262

Curved Bridges – Applications

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Photo Credit:

https://www.reddit.com/r/InfrastructurePorn/comments/fa51hv/sheikh_zayed_road/

Curved Bridges – Applications

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Photo Credits:

Top: Port Mann Bridge, Vancouver, BC

https://www.reddit.com/r/vancouver/comments/6fa5aw/the_port_mann_bridge_from_a_737/

Bottom: Stonecutters Bridge, Hong Kong

https://www.arup.com/projects/stonecutters-bridge

Curved Bridges – Applications

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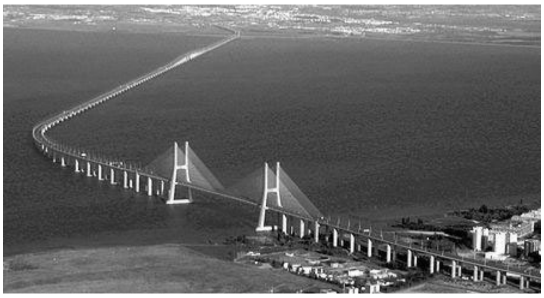


Photo Credits:

Top: Millau Viaduct, France

https://www.tourisme-aveyron.com/

Bottom: Vasco da Gama Bridge, Portugal

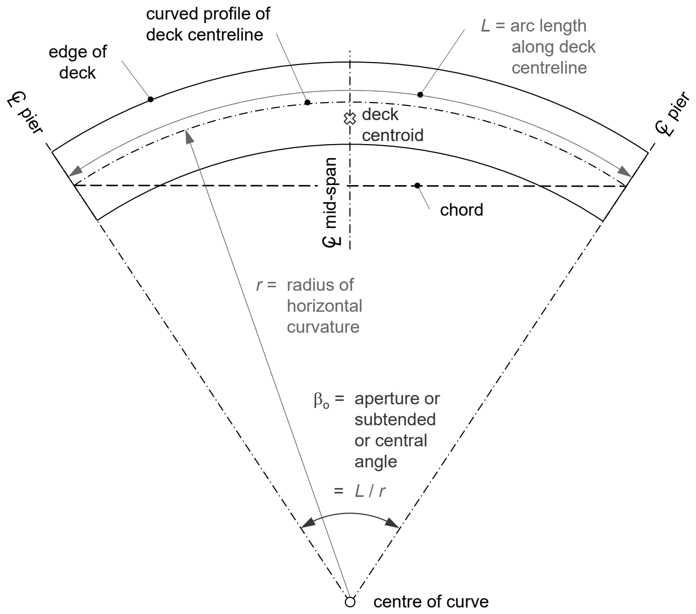
https://de.wikipedia.org/wiki/Ponte_Vasco_da_Gama

Special girder bridges

Curved Bridges
Geometry & Terminology

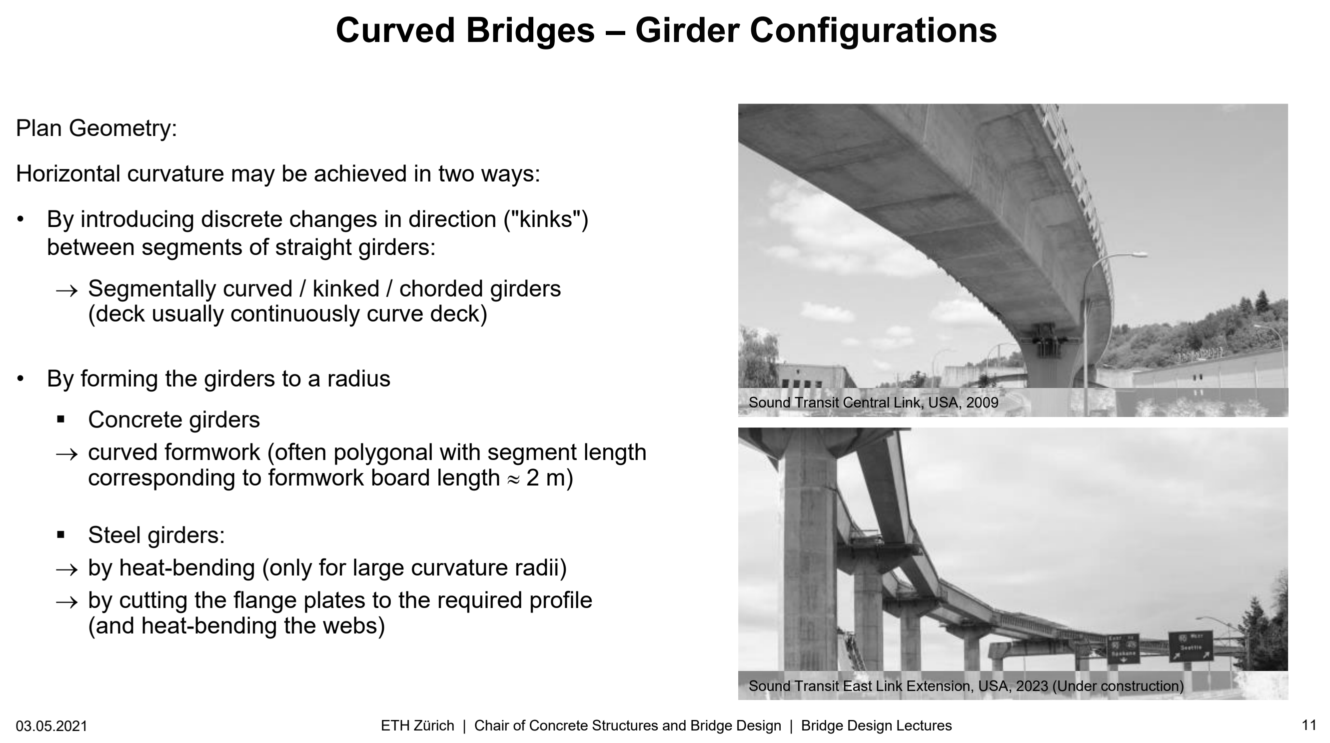
Curved Bridges – Geometry & Terminology

- A typical segment of a horizontally curved bridge deck between two radially arranged piers is considered to illustrate basic geometric properties
- Note that:
  - The radius of horizontal curvature and the span arc length depend on the line of reference, i.e. deck centreline, edge of deck, roadway/railway alignment centreline, girder centreline, etc.
  - If the radius, arc length and aperture angle are within certain limits, the bridge is reasonably straight, and the behaviour of the girder can be approximated with an equivalent straight girder, having a span corresponding to the arc length of the curved girder axis.
  - Generally, the decision whether curvature needs to be considered or not is based on engineering judgement. Some design recommendations and codes provide explicit geometric limits (e.g. $\beta_o < 12^o$).



Special girder bridges

Curved Bridges
Girder Configurations



Curved Bridges – Girder Configurations

Plan Geometry:

Horizontal curvature may be achieved in two ways:

- By introducing discrete changes in direction ("kinks") between segments of straight girders:
  - → Segmentally curved / kinked / chorded girders (deck usually continuously curve deck)
- By forming the girders to a radius
  - ▪ Concrete girders
  - → curved formwork (often polygonal with segment length corresponding to formwork board length ≈ 2 m)
  - ▪ Steel girders:
  - → by heat-bending (only for large curvature radii)
  - → by cutting the flange plates to the required profile (and heat-bending the webs)

Sound Transit Central Link, USA, 2009

Sound Transit East Link Extension, USA, 2023 (Under construction)

Both featured elevated guideways are part of the light rail rapid transit system serving the Seattle metropolitan area in Washington, USA, managed by Sound Transit. Note how different solutions were adopted for the girder configuration:

- The Central Link guideway was constructed between 2003 and 2009 using precast segmental curved girders.
- The East Link Extension is currently under construction using precast straight girders on a chorded alignment.

A number of factors may have determined the most economical typology for each section of the project. For instance, the Central Link project was more than 20 km long (combination of tunnels, elevated guideways, and surface-running sections) and therefore justified the investment for precast segmental construction. Subsequent extensions to the project may not include long enough elevated guideway sections to justify such an investment; thus the more commonly available precast straight girders are likely more economical.

Photo Credits:

Top - https://www.flickr.com/photos/sounderbruce/17208410099/

Bottom - https://www.soundtransit.org/blog/platform/east-link-light-rail-line-already-transforming-eastside-skyline

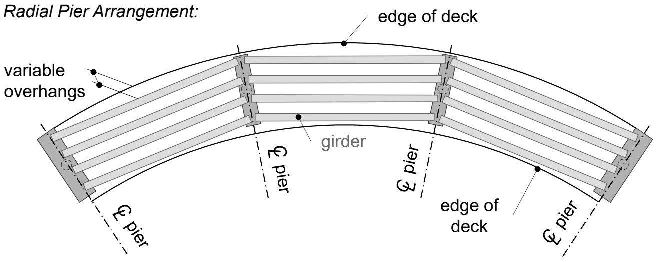
Curved Bridges – Girder Configurations

Plan Geometry:

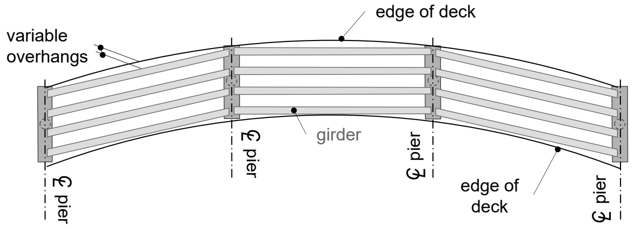
Possible girder configurations for curved bridges include:

- Straight girders chorded from pier to pier
  - ➢ Simple and economical:
    … easier fabrication and transportation
    … girders more stable for handling and erection
  - ➢ Variable deck overhangs (expensive formwork)
  - ➢ Radial or skew pier arrangements possible
  - ➢ Aesthetics and economics become an issue for sharply curved alignments

Radial Pier Arrangement:



Skew Pier Arrangement:



Further reading:

PCI (Precast/Prestressed Concrete Institute). (2012). Curved precast concrete bridges state-of-the-art report, 1st Ed., Chicago.

Curved Bridges – Girder Configurations

Plan Geometry:

Possible girder configurations for curved bridges include:

- Straight girders chorded from pier to pier

  Example: Cinta Costera Viaduct, Panama City (2014)

  - 2.5 km long viaduct → standardisation important
  - 850 precast concrete girders
  - Variable width (annulus sector "pie-shaped") pier caps allowed for use of only 3 unique girder lengths
  - Low bridge → aesthetics not affected by chorded girders (underside of bridge not visible)

Photo Credits:

Top: https://www.tylin.com/en/projects/cinta_costera_viaduct

Bottom: https://www.panamaamerica.com.pa/nacion/arquitectos-defienden-tramo-maritimo-de-cinta-costera-iii-842324

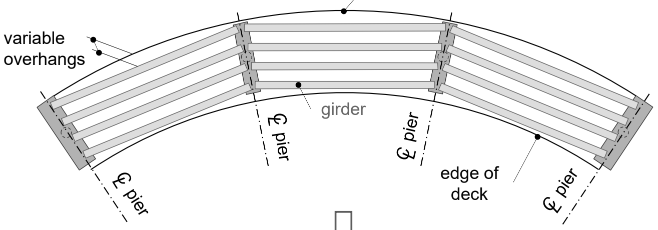
Curved Bridges – Girder Configurations

Plan Geometry:

Possible girder configurations for curved bridges include:

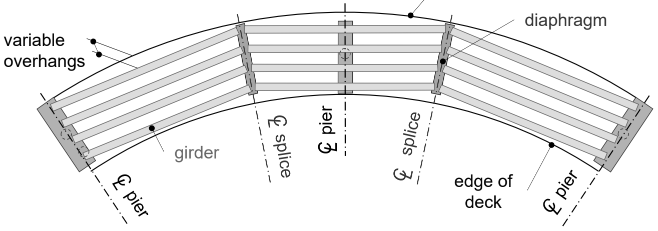
- Straight girders chorded from pier to pier
- Spliced straight girders with splice (kink) points within the spans
  - ➢ Curved alignment is followed more closely
  - ➢ Opportunity for multiple angular breaks per span → longer spans and sharper curvatures possible
  - ➢ Aesthetics may still be an issue
  - ➢ Temporary shoring required for erection
  - ➢ Change in direction of flange forces results in a horizontal radial component acting outward at a compression flange and inward at a tension flange → Bracing must be provided at the kinks to resist these forces (diaphragms / cross-frames)

Chorded girders from pier to pier:



⇩

Spliced (kinked) girders:



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PCI (Precast/Prestressed Concrete Institute). (2012). Curved precast concrete bridges state-of-the-art report, 1st Ed., Chicago.

Curved Bridges – Girder Configurations

Plan Geometry:

Possible girder configurations for curved bridges include:

- Straight girders chorded from pier to pier
- Spliced straight girders with splice (kink) points within the spans
- Curved girders
  - ➢ Simplified geometry
  - ➢ Better load distribution between cross-frames/diaphragms (no kinks)
  - ➢ Can be launched (if curvature is constant)
  - ➢ Higher aesthetic quality
  - ➢ Prefabrication and transportation more complicated
  - ➢ Ensuring stability during erection more complicated

→ The focus of the lecture is on curved girders

Photo Credits:

Top – https://www.aisc.org/globalassets/nsba/aashto-nsba-collab-docs/nsbagdc-3.pdf

Bottom – https://www.kimley-horn.com/project/i5-genesee-avenue-interchange/

Curved Bridges – Girder Configurations

Cross-Sections for Curved Girders:

- Open Cross-Section
  - I-girders interconnected by cross-frames
  - Closely-spaced cross frames / diaphragms required
  - In bridges with pronounced curvature, a lateral bracing (horizontal truss) near the bottom flange is required (to form a quasi-closed cross-section acting in uniform torsion)
- Closed Cross-Section
  - Cast in place curved prestressed concrete girders
  - Steel box girders (single, twin, multiple)
  - Vertical or inclined webs
  - Single or multi-cell
  - Large torsional resistance in uniform torsion and reduced stresses due to warping torsion

Photo Credits:

Top - http://www.stuppbridge.com/projects/the-new-i-64-design-build

Bottom - https://www.lusas.com/products/information/curved_girder_analysis.html

Curved Bridges – Girder Configurations

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Photo Credits:

https://www.kiewit.com/projects/transportation/bridge/san-francisco-oakland-bay-bridge-skyway-segment/

Special girder bridges

Curved Bridges
Implications / Considerations

Curved Bridges – Implications / Considerations

- Geometrical Considerations

  Support & Articulation

  - Placement and type of bearings need to consider the torsional response:
    - → Open sections require torsional restraint at each support
    - → Closed sections may require torsional restraint only at abutments (high torsional stiffness)
  - In-plane movements at bearings, due to uniform thermal actions and shrinkage effects of the concrete, are along the direction defined by the fixed in plane point and the bearing.
    - → Changes in length can be accommodated through radial movements. Thus relatively long integral bridges may be designed in conjunction with slender piers.
  - If the expansion joint is not perpendicular to the direction of the movement, the expansion joint must be able to accommodate movements along its axis.
  - ➢ See more detailed discussion in Chapter on Support & Articulation.

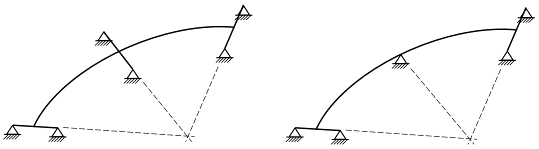

Photo Credit:

A.J. Reis & J.J. Oliveira Pedro (2019), Bridge Design - Concepts and Analysis, Wiley, Hoboken, NJ

Bridge over River Ave at St Tirso, Portugal © GRID, SA

Curved Bridges – Implications / Considerations

- Geometrical Considerations

  Support & Articulation

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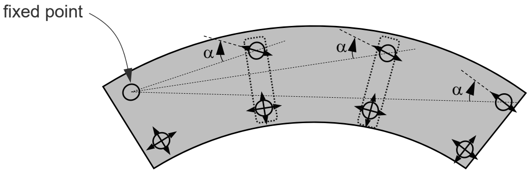
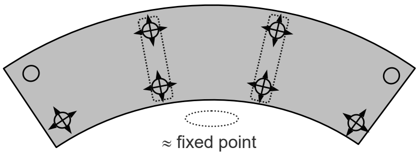
    - → Changes in length can be accommodated through radial movements. Thus relatively long curved integral bridges may be designed in conjunction with slender piers.
  - If the expansion joint is not perpendicular to the direction of movement at the abutment, it must be able to accommodate transverse relative displacements.

  ➢ See more detailed discussion in Chapter on Support & Articulation.



Horizontal restraint-free (at piers only for temperature and shrinkage) bearing layout

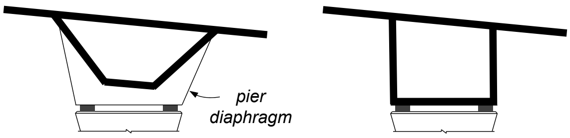


Practical bearing layout causing moderate horizontal restraint (hor. fixity at both abutments)

Curved Bridges – Implications / Considerations

- Geometrical Considerations
  - Need to consider deck superelevation (Quergefälle). This may require a wider deck.
  - The effective centre of gravity is not at the cross-section centroid (even without superelevation, exterior webs are longer than interior webs)
- Particularities in Analysis
  - Bending – Torsion interaction (see section on Behaviour)
  - Need to consider centrifugal forces
  - Seismic analysis:
    - Static equivalent seismic analyses are not applicable to curved bridges
- Particularities in Design (see section on Design Aspects)
- Particularities in Construction (see section on Construction Aspects)

Superelevation Treatment Alternatives



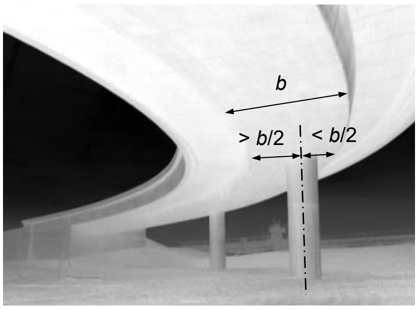


Photo: Einfahrtsrampe BW714, Dreieck Zürich West, length 120 m, spans 4x29 m (side span = interior span), dsp Ingenieure + Planer AG, 2004 © W. Kaufmann

Special girder bridges

Curved Bridges
Behaviour

Curved Bridges – Behaviour

- General:

  Torsion in curved girders is induced by vertical loads, including those that are symmetrical about the longitudinal axis of the bridge (e.g. self-weight).

  Bending and torsional moments are coupled due to the curved geometry and their relationship depends mainly on:

  - radius of curvature
  - bending to torsional stiffness ratio *EI* / *GK*
  - boundary conditions

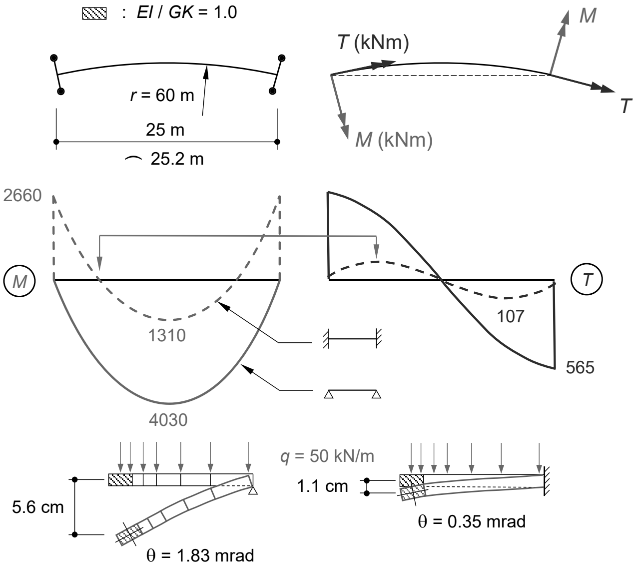
- Example:

  Compare the response of a curved girder under uniformly distributed vertical load for:

  (a) pinned end supports for flexure

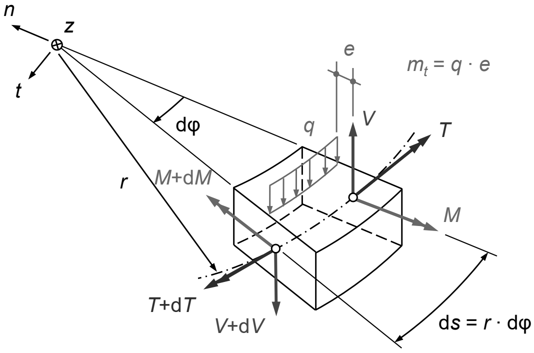
  (b) fixed end supports for flexure

  (twist at the ends is prevented for both cases)



Example reference: Manterola, J., “*Puentes – Apuntes para su diseño, cálculo y construcción*“, Madrid, 2005

Curved Bridges – Behaviour



Curved Beam Theory

- Consider an element of a curved beam with infinitesimal length under uniform vertical load and torque
- Equilibrium equations of the free body diagram yield:

$$\sum F_z = 0 = -V + V + dV + q \cdot ds$$

$$q \cdot ds + dV = q \cdot r \cdot d\varphi + dV = 0 \rightarrow q = -\frac{dV}{rd\varphi}$$

$$\sum M_n = 0 = -M + \left(M + dM\right)\overbrace{\cos d\varphi}^{\approx 1} + \left(T + dT\right)\overbrace{\sin d\varphi}^{\approx d\varphi} - \left(V + dV\right)ds - q \cdot ds \cdot \frac{ds}{2} + m_t \cdot ds \cdot \sin d\varphi$$

$$-V \cdot ds + dM + Td\varphi = -V \cdot r \cdot d\varphi + dM + Td\varphi = 0 \qquad \rightarrow \frac{dM}{rd\varphi} = V - \frac{T}{r}$$

$$\sum M_t = 0 = -T + \left(T + dT\right)\cos d\varphi - \left(M + dM\right)\sin d\varphi + \left(V + dV\right) \cdot ds \cdot \sin d\varphi + q \cdot ds \cdot \frac{ds}{2}\sin d\varphi + m_t \cdot ds \cdot \cos d\varphi$$

$$m_t \cdot ds - Md\varphi + dT = m_t \cdot r \cdot d\varphi - Md\varphi + dT = 0 \qquad \rightarrow \frac{dT}{rd\varphi} = \frac{M}{r} - m_t$$

Curved Bridges – Behaviour

Curved Beam Theory

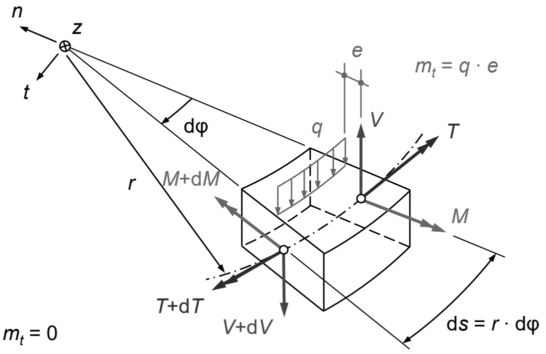
- Consider an element of a curved beam with infinitesimal length under uniform vertical load and torque
- Equilibrium equations of the free body diagram yield:

$$q = -\frac{dV}{r \cdot d\varphi} \qquad \frac{dM}{r \cdot d\varphi} = V - \frac{T}{r} \qquad \frac{dT}{r \cdot d\varphi} = \frac{M}{r} - m_t$$

$$q \cdot r = -\frac{dV}{d\varphi} \qquad \underbrace{\frac{dM}{d\varphi} = V \cdot r - T \qquad \frac{dT}{d\varphi} = M - m_t \cdot r}_{M \text{ and } T \text{ are coupled}}$$

For a straight beam ($r \rightarrow \infty$) above equations reduce to the familiar form:

$$q = -\frac{dV}{ds} \qquad \underbrace{\frac{dM}{ds} = V \qquad \frac{dT}{ds} = -m_t}_{M \text{ and } T \text{ are decoupled}}$$



for $m_t = 0$

$$\frac{dT}{d\varphi} = M \quad \rightarrow \quad \Delta T = \int M \, d\varphi$$

i.e., the torque variation between two sections is equal to the area of the bending moment diagram (integrated over φ) between those two sections

→ Important for continuous girders where the sign of the moment diagram changes along the girder

Curved Bridges – Behaviour

Curved Beam Theory

- Solve system of equilibrium equations:

$$\underbrace{q\cdot r=-\frac{dV}{d\varphi}\qquad \frac{dM}{d\varphi}=V\cdot r-T\qquad \frac{dT}{d\varphi}=M-m_t\cdot r}$$

$$\frac{d^2M}{d\varphi^2}=\frac{d}{d\varphi}\left(V\cdot r-T\right)=-q\cdot r^2-M+m_t\cdot r$$

for circular beam (r = const.)

$$\boxed{\rightarrow\frac{d^2M}{d\varphi^2}+M=m_t\cdot r-q\cdot r^2}$$

→ 2nd order inhomogeneous differential equation; for constant parameters, i.e.

$m_t(\varphi) = m_t$ = const. and $q(\varphi) = q$ = const.,

the general solution is:

$$\boxed{M(\varphi)=c_1\sin\varphi+c_2\cos\varphi+m_t r-qr^2}$$

Alternatively, the differential equation can be solved iteratively:

1. First approximation for M:

   Straight beam with span length $s = r\cdot\varphi$ under loading $q-\frac{m_t}{r}$

   $$\frac{d^2M}{d\varphi^2}+M=m_t\cdot r-q\cdot r^2\quad\rightarrow\quad\frac{d^2M}{ds^2}=\frac{m_t}{r}-\frac{M}{r^2}-q\approx\frac{m_t}{r}-q$$

2. Iteration of (1) with loading $q+\frac{M}{r^2}-\frac{m_t}{r}$ until convergence is achieved

   (often unnecessary because $\left|\frac{M}{r^2}\right|\ll|q|$)

3. Determination of T:

   Straight beam with span length $s = r\cdot\varphi$ under torque $\frac{M}{r}-m_t$:

   $$\frac{dT}{d\varphi}=M-m_t\cdot r\qquad\rightarrow\qquad\frac{dT}{ds}=\frac{M}{r}-m_t$$

For most practical bridge applications, the effect of curvature on the calculation of bending moments may be neglected; i.e. the transformation of the curved girder to an equivalent straight girder of span length equal to the arc length results in small errors in the bending moments.

For example, in the case of a single simply-supported span with constant in-plan radius, under uniformly distributed load, the error for the bending moment at mid-span is less than 1% for aperture angles of less than 17 degrees (0.3 rad) [for derivation, see Reis & Oliveira (2019), Bridge Design – Concepts and Analysis, Wiley]

Curved Bridges – Behaviour

- Approximate Methods

From the static equilibrium equations:

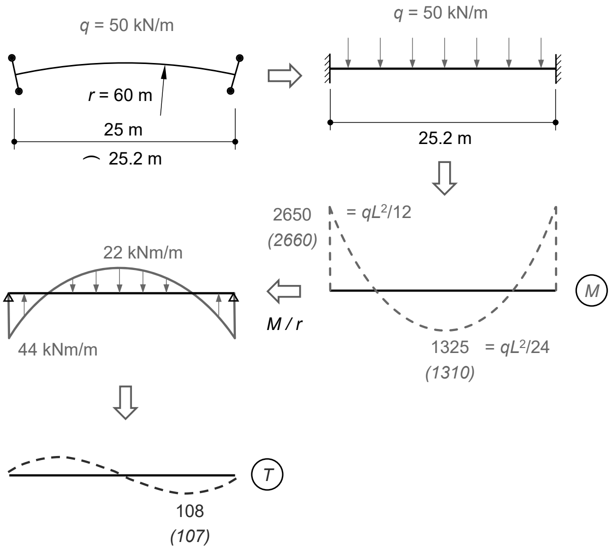
$$\frac{dT}{ds} = \frac{M}{r} - m_t \qquad \text{or when } m_t = 0, \qquad \frac{dT}{ds} = \frac{M}{r}$$

This is analogous to the equilibrium equation for shear forces in the case of a straight beam:

$$\frac{dV}{dx} = -q$$

Thus, if the bending moments along the girder are known, e.g. by analysing the equivalent straight girder, the torsional moments may be obtained by loading the equivalent straight girder with a distributed moment equal to ($M/r - m_t$).

This is known as the "*M* over *r* method" and corresponds to the iterative solution outlined on the previous slide. Its applicable for single span and continuous girders (good approximation if the radius is reasonably > than the span).



In the above moment diagrams, the values shown in parentheses correspond to the curved beam theory solution. Note that using the approximate method results in less than 1% error.

Curved Bridges – Behaviour

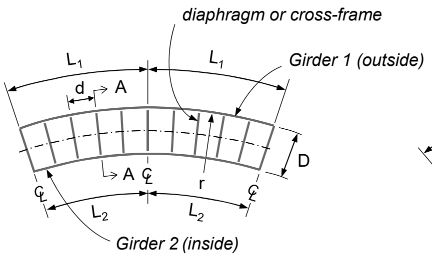
- Approximate Methods

  The extension of the *M/r* method for open cross-sections with multiple girders is known as the V-Load method.

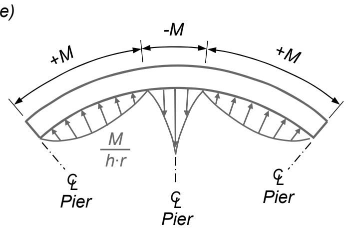
  Two-step process:

  1. Determine (primary) bending moments under vertical loads applied on straightened girders → consider each girder separately
  2. Apply additional fictitious, self-equilibrating forces (V-Loads) to the straightened structure so that the resulting internal forces are the same as those in the curved structure.

  The process is illustrated by considering the case of a system consisting of two curved girders continuous over one interior pier connected by uniformly-spaced, full-depth cross-frames under uniformly distributed vertical loading.

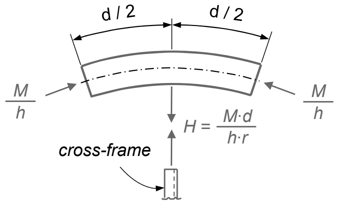


Curved Bridge – Plan View



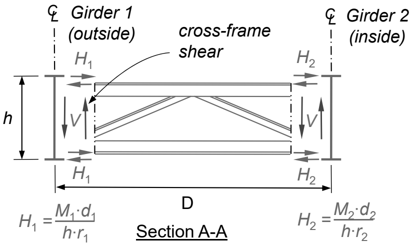
Radial Components of Top Flange Forces

(Bottom Flange Opposite)



Curved Segment of Top Flange

(Bottom Flange Opposite)



Section A-A

The considered curved bridge system consists of two prismatic girders continuous over one interior support with full-depth cross-frames uniformly spaced at a distance “d” along Girder 1 (outside). Cross-frames provide the primary resistance to the torsional loads caused by the bridge curvature. In so doing, the cross-frames prevent the warping stresses from becoming too large by restricting the lateral bending of the girder flanges (more on this on Slide 36).

Initially, the curved girders are analysed as isolated straight girders, using developed span lengths equal to their respective arc lengths, L_1 and L_2. Assuming that the flanges resist the full bending moment, the longitudinal force in the flange at any point is equal to *M*/*h*, where *h* is the depth between flanges. Because of the bridge curvature, these axial forces are not collinear along any given segment of the flange; thus to maintain equilibrium distributed radial forces $M/(h \cdot r)$ must be developed. These forces follow the shape of the bending moment diagram and act in opposite directions on the top and bottom flanges, thus causing twisting of the girder.

The radial forces and resulting torsional moments are equilibrated by the cross-frames via horizontal force couples. The magnitude of the horizontal forces, *H*, may be approximated by multiplying the radial forces with the cross-frame spacing: $(M \cdot d)/(h \cdot r)$, where M is the value of the bending moment at the cross-frame location. Considering the free body diagram of a cross-frame, a vertical force couple (V) is required to equilibrate the horizontal force couple (H). These vertical forces (V-Loads) then react on the girders, resulting in a state of self-equilibrating shears. These shears tend to increase the moments on the outside girder (this is true for positive and negative moment regions). Applying the V-Loads to the girders creates internal forces in the straight structure that are similar as those that exist in the curved structure under applied vertical loads.

Reference: AISC Marketing Inc. (1984) "V-Load Analysis" for curved steel bridges

Curved Bridges – Behaviour

- Approximate Methods
  - The *M*/*r* and *V*-Load methods were widely used to analyse curved girders in the past century, but are nowadays essentially replaced by grillage or 3D finite element models.
  - There are still relevant though to gain insight into the system behaviour and for preliminary calculations.
- Spine Models (see also superstructure, spine models)
  - Suitable for box and solid cross-section girders.
- Grillage Models (see also superstructure, grillage models)
  - Sufficient for predicting the response of open cross-section curved girders for most cases.
  - The cross-frames must be modelled through equivalent beam elements capturing the behaviour of the cross-frames (including shear deformations).
  - In the case of I-girders, the restraint of warping must be taken into account in the estimation of the torsional stiffness (see notes).
  - Locked-in forces due to the lack of fit between cross-frames and girders must be considered (*see Skew Bridges*).

Note on effective torsional stiffness of I-girders:

The use of just the St. Venant term (GK/L) in characterising the torsional stiffness of I-girders results in a dramatic underestimation of the true girder torsional stiffness. This is due to the neglect of the contributions from flange lateral bending, that is, warping of the flanges, to the torsional properties. Even for intermediate steel erection stages where some of the cross-frames are not yet installed, the typical torsional contribution from the girder warping rigidity (EI_w) is substantial compared to the contribution from the St. Venant torsional rigidity (GK). Conventional 2D-grillage methods commonly neglect the warping torsion contribution.

NCHRP Report 725 describes an approximate method to account for the warping torsion by calculating an equivalent St. Venant torsional rigidity (GK_{eq}), assuming warping fixity at intermediate cross-frame locations and warping-free conditions at the simply-supported ends of the girder.

Further reading: NCHRP Report 725 - Guidelines for Analysis Methods and Construction Engineering of Curved and Skewed Steel Girder Bridges, 2012

Photo Credit: FHWA (2015), Steel Bridge Design Handbook: Redundancy, FHWA-HIF-16-002 - Vol. 9

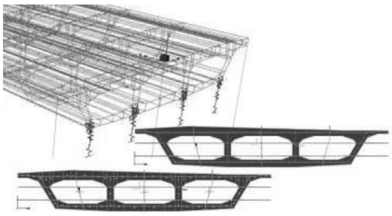
Curved Bridges – Behaviour

- 3D Finite Element Methods

  The 3D finite element method is the most general and comprehensive method:

  - Allows for modelling of complex geometries

    (sharp and/or variable curvature, skew supports, complex cross-sections)
  - Provides detailed information on displacement and stresses
  - Provides an accurate analysis for live load distribution
  - Provides displacement and stress information at various construction stages
  - Provides forces/stresses in cross frames / diaphragms
  - Permits accurate fatigue design
  - The detailed analysis gives designers confidence in going to the limits of the code so that the most economical solution is achieved (unless artefacts of the model cause stress peaks)
  - Generally more time consuming than grillage methods
  - Response depends on modelling choices and model tends to be intransparent → critical review and verification with approximate methods



Figures: https://www.csiamerica.com/products/csibridge/releases

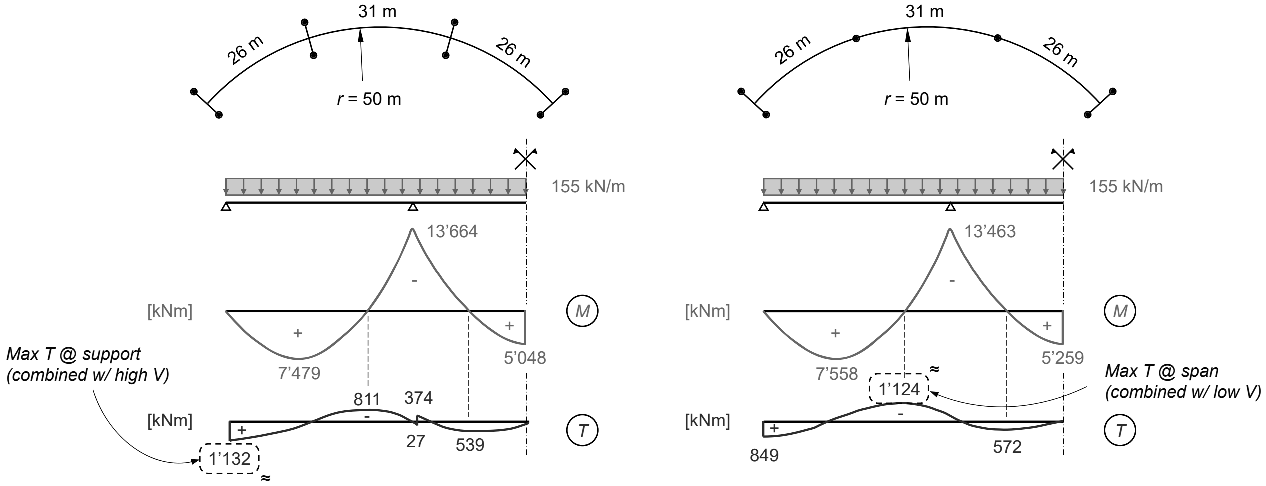
Special girder bridges

Curved Bridges
Design Aspects

Curved Bridges – Design Aspects

- Support and Articulation

  Due to the interaction between bending and torsional moments, torsional support is not required at all piers, provided that the cross-section has adequate torsional resistance and stiffness → closed section



Reference: Vorlesung – Spezielle Probleme der Vorspannung, Vorspannung und Querbelastung exzentrisch belasteter, gekrümmter und schief gelagerter Betontragwerke, Prof. Dr. Hugo Bachmann, ETHZ, 1990.

Curved Bridges – Design Aspects

- Concrete girders

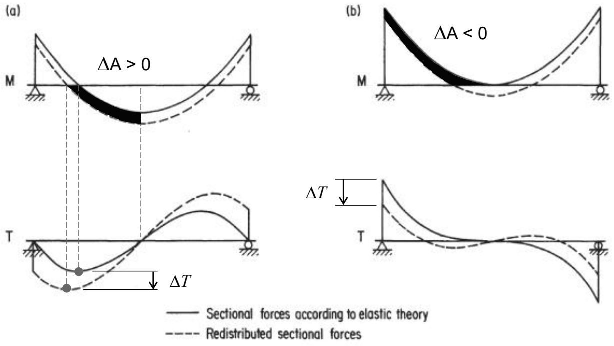
  Redistribution of bending moments

  - When bending moment redistributions are considered, e.g. due to cracking or creep, the associated torsional moments must be redistributed in a corresponding manner (related by equilibrium).
  - The effect of bending moments redistribution on the torsional moments depends on the initial shape of the bending moment diagram, i.e. construction method.
  - The change in the area of the moment diagram will result in a corresponding increase/decrease to the torsional moments.

$$\Delta T = \int M \, d\varphi$$

Internal span of continuous girder erected on falsework (constant depth)

Internal span of continuous cantilever-constructed girder (variable depth)

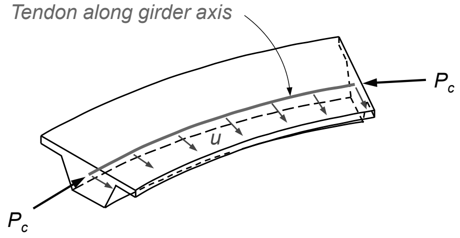


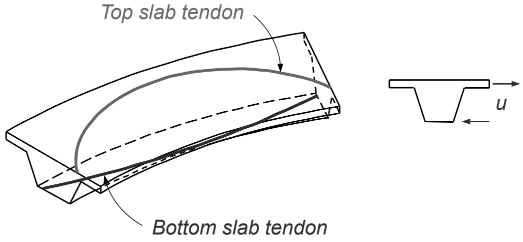
Curved Bridges – Design Aspects

- Concrete girders

  Prestressing concepts & tendon layout

  - Due to the horizontal curvature of the girder, tendon deviation forces are produced in the horizontal plane, normal to the axis of the girder. These act in addition to those due to the (vertical and horizontal) profile of the tendon relative to the axis of the girder
  - The horizontal deviation forces are globally equilibrated by the concrete section (deviation of compression), but not locally → restrain tendons against pullout, see next slide.
  - Tendons can be arranged to balance a given torsional moment diagram:
    - ➢ By adjusting the profile of slab tendons in the horizontal plane (without altering the effect of the prestressing in bending), see figure
    - ➢ By adjusting the profile of web tendons in the vertical plane (reducing the sag of the inner web tendons, i.e., reducing the effect of prestressing in bending)





Curved Bridges – Design Aspects

- Concrete girders

  Prestressing concepts & tendon layout

  - Due to the horizontal curvature of the girder, tendon deviation forces are produced in the horizontal plane, normal to the axis of the girder. These act in addition to those due to the (vertical and horizontal) profile of the tendon relative to the axis of the girder
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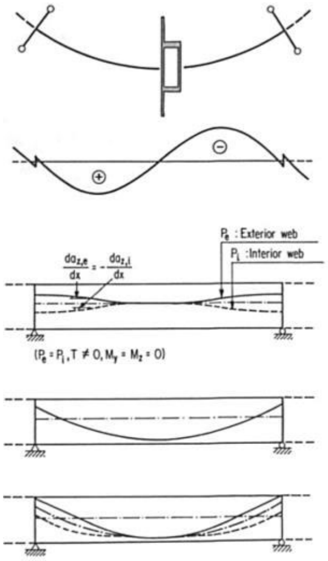
  - Tendons can be arranged to balance a given torsional moment diagram:
    - ➢ By adjusting the profile of slab tendons in the horizontal plane (without altering the effect of the prestressing in bending), see figure
    - ➢ By adjusting the profile of web tendons in the vertical plane (reducing the sag of the inner web tendons, i.e., reducing the effect of prestressing in bending)



Continuous curved girder

Torsional Moments due to dead load

Tendon profiles to balance torsional moments

⊕

Tendon profile to balance bending moments

⊜

Tendon profiles to balance bending and torsional moments

In the example shown, starting from a tendon geometry that does not utilise the full cross-section depth, moving the tendons symmetrically in opposite directions does not alter the bending moment effect. However, compared to a tendon geometry that utilises the full cross-section depth in both webs (as is common in straight girders), the bending effect is of course reduced if torsional moments are compensated by web tendons, since the potential eccentricities of the inner tendons are not fully utilised.

Illustrations from Menn, C., “Prestressed Concrete Bridges“, Birkhäuser Basel, 1990

Curved Bridges – Design Aspects

- Concrete girders

  Reinforcement and tendon detailing

  - Prestressing tendons and mild reinforcement must be restrained against pull-out along the concave surfaces of the webs

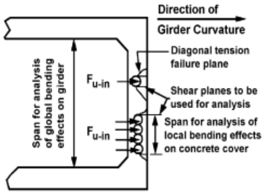


Figure C5.10.4.3.1a-2—In-Plane Force Effects in Curved Girders Due to Horizontally Curved Tendons

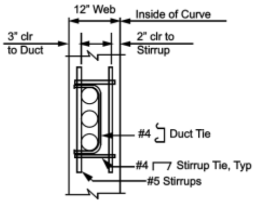
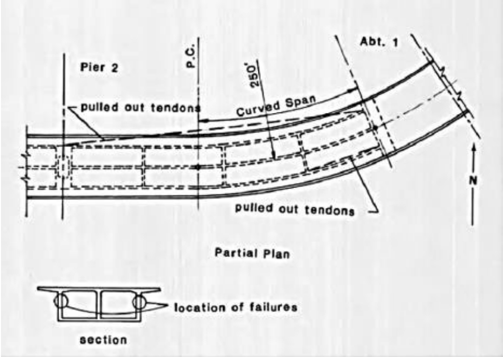
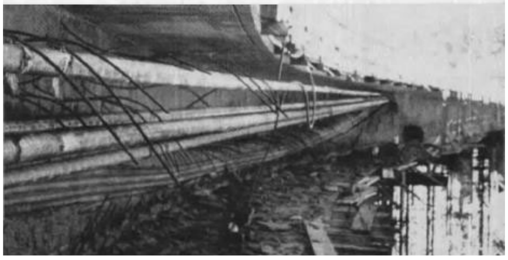


Figure C5.10.4.3.1b-1—Typical Stirrup and Duct Tie Detail

Case study: Las Lomas Bridge, Hawaii (1978):

- All four tendons broke away from the web during stressing of the last tendon.





References:

Left: AASHTO LRFD Bridge Design Specifications (2014)

Right: Podolny, W., „The Cause of Cracking in Post-Tensioned Concrete Box Girder Bridges and Retrofit Procedures,“ Journal of Prestressed Concrete Institute, vol. 30, no. 2, March/April 1985.

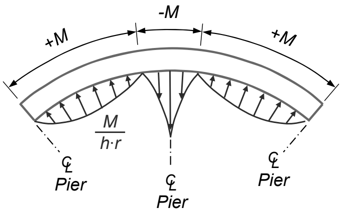
Curved Bridges – Design Aspects

- Steel girders

  Girder design:

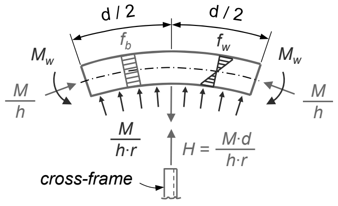
  - As discussed, in the case of curved girders, the internal forces from major axis bending are non-collinear
  - The non-collinearity results in virtual radial pressures which cause lateral bending or warping stresses in the flanges
  - Unless the analysis is performed using 3D finite element methods, where the curved geometry of the flanges is explicitly modelled, these lateral stresses need to be calculated separately
  - The magnitude of the warping stresses is controlled through the cross-frame spacing (*d*)
  - In the case of sharp horizontal curvature, the flanges may need to be supported by a lateral truss system (see Slide 15 and notes); alternatively, box girders should be utilised

[Repeated from Slide 27]



Radial Components of Top Flange Forces

(Bottom Flange Opposite)



Curved Segment of Top Flange

(Bottom Flange Opposite)

Primary flange stresses due to bending:

$$f_b = \frac{M}{h} \Big/ \left(b_f \cdot t_f\right)$$

where b_f and t_f are the width and thickness of the flange, resp. and h is the depth of the girder

Lateral flange stresses due to warping:

$$f_w = M_w / S_f = \left(\frac{M}{h \cdot r} \cdot \frac{d^2}{10}\right) \Big/ \left(\frac{b_f^2 \cdot t_f}{6}\right)$$

Approximation (8 … 12)

During the erection phase, when the bare steel girders support the weight of the wet concrete, the flanges are discretely braced by the cross-frames. Once the deck hardens, the top flange is continuously supported, so the warping stresses are limited to those locked-in during the erection phase. However, the bottom flange remains discretely supported even during the service phase. Therefore, to control the warping stresses, a lateral truss system may be required, essentially forming quasi-closed cross-sections between adjacent girders (see bottom photo on Slide 15 – exterior girders). At that point, substituting I-girders with box girders may become more economical.

Curved Bridges – Design Aspects

- Steel girders

  Bracing design:
  - Cross-frames
    - should be treated as main structural members (primary tension members)
    - should be full depth and be provided between all I girders
    - spacing of cross frames is partially an economical consideration; the closer the cross frames, the lighter the flanges and vice-versa (note though that cross-frames are relatively high-cost elements)

      A cross frame spacing of 4…6 m is common for curved bridges
  - Diaphragms
    - Full depth solid plate diaphragms should be considered at supports points (particularly at deck joints) and should extend continuously across the full width of the bridge
  - Lateral Bracing
    - should be provided when required for wind and stability (see also erection)
    - should be designed for other forces they may attract in maintaining compatibility with the girders

Photo Credit: http://www.twm-inc.com/portfolio-items/curved-girder-erection-design/

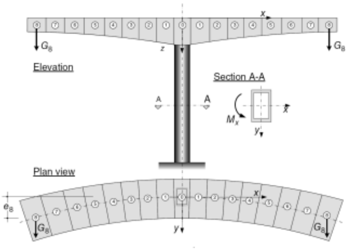
Special girder bridges

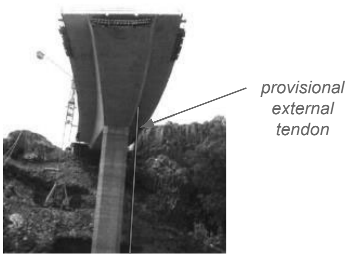
Curved Bridges
Construction Aspects

Curved Bridges – Construction Aspects

- Construction methods:
  - A horizontal alignment with constant radius of curvature is required if the bridge is to be erected by the incremental launching method.
  - Variable radii of curvature are possible for a concrete bridge built by the balanced cantilever or span-by-span method

  - During the balanced cantilever scheme of a curved box girder bridge, the dead load of the deck induces transverse bending moments in the piers, unless some provisional prestressing, internal or external, is adopted





Top photos:

Section 5 Palmetto SR826/836 Interchange (Bridges 9, 11, 15, and 19) - Miami, Florida

http://www.asbi-assoc.org/projects/project.cfm?articleID=B2D820E5-0B74-13B8-36C70F9B618BB9F7&categoryIDs=44A57F74-F1F6-B13E-81940961706FF0A9&searchString=&mainPageNumber=2&resultsPerPage=20

Bottom illustration and photo:

A.J. Reis & J.J. Oliveira Pedro (2019), Bridge Design - Concepts and Analysis, Wiley, Hoboken, NJ

Curved Bridges – Construction Aspects

- Steel girders
  - Transportation
    - Horizontally curved I girders are typically shipped on their sides → consider stresses and vibrations (may lead to fatigue issues with long distance transports)
  - Erection
    - Bracing is required to control stresses and deformations (vertical and lateral) under gravity loads, and to provide stability (lateral torsional buckling)
    - Effects of wind during construction may require lateral bracing to control displacements
    - (Temporary) lateral bracing is usually required for an erection scheme where strongly curved I-girders are lifted in pairs
  - Control of deformations
    - Fit-up issues between girders and cross-frames are not uncommon due to deflections and rotations of curved girders (determining camber is more demanding than in straight girders)

Further reading:

FHWA (2005), Engineering for Structural Stability in Bridge Construction, Publication No. FHWA-NHI-15-044

https://www.fhwa.dot.gov/bridge/pubs/nhi15044.pdf

Photo:

https://www.aisc.org/globalassets/nsba/aashto-nsba-collab-docs/g-13.1-guidelines-for-steel-girder-bridge-analysis.pdf