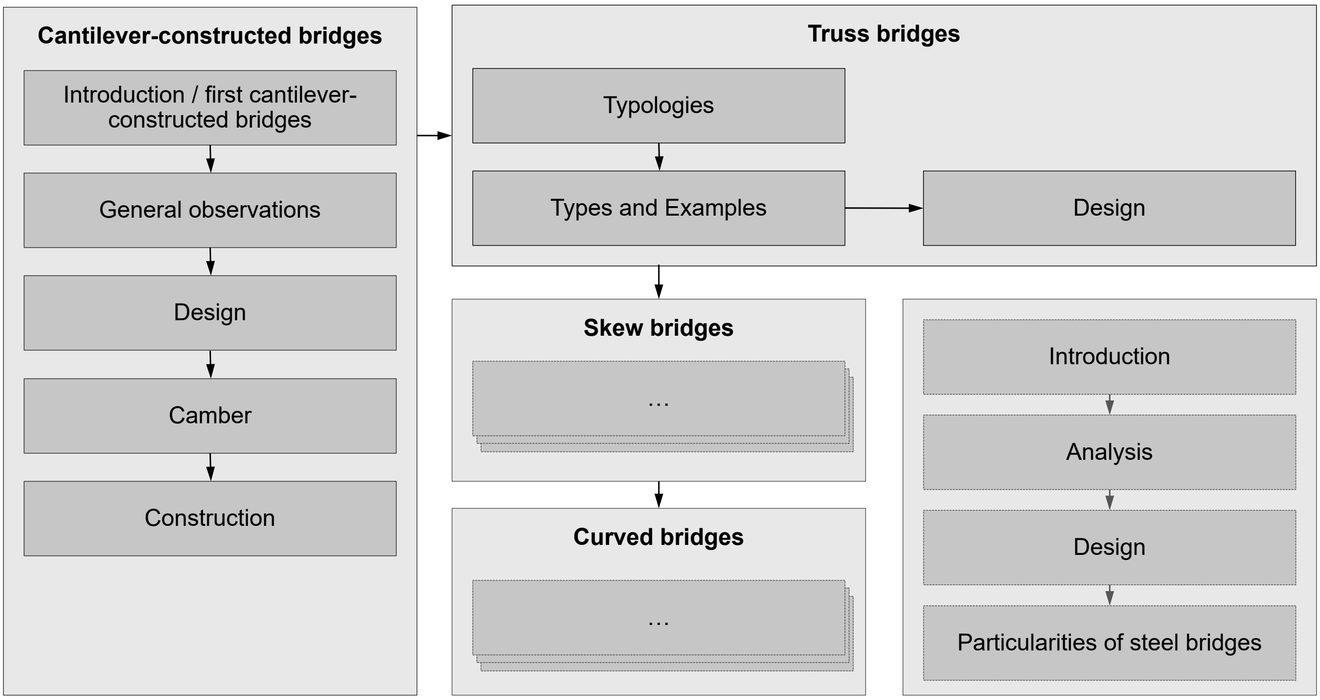
Special girder bridges


Cantilever-constructed bridges
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Special girder bridges

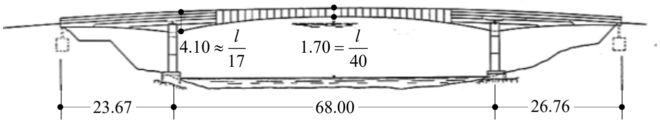
Cantilever-constructed bridges

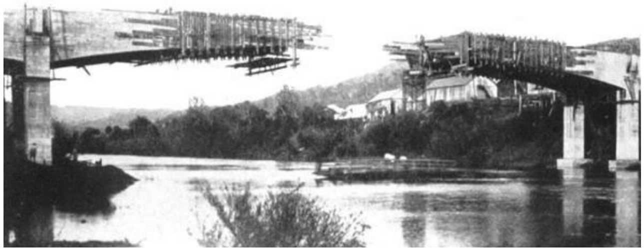
Introduction: First cantilever-constructed concrete bridges

Cantilever-constructed bridges – Introduction

Ponte Emílio Baumgart, Herval-Joaçaba, Brasil (1930-1983)

- Brazilian Engineer Emílio Baumgart conceived the world's first cantilever constructed concrete (see notes) bridge, built in 1930
- Cantilevering was chosen due to the frequent flood events at the site (Rio do Peixe rising by 10 m)
- The bridge had an open cross-section (two rectangular longitudinal beams), with depths similar to modern cantilever constructed bridges
- Passive reinforcing bars Ø38 mm were used, without prestressing
- Deformations during construction were controlled by rotations at the piers ("swing"), using counterweights at the abutments
- The bridge was destroyed in 1983 by a severe flood event





It should be noted that iron/steel cantilever constructed bridges preceded concrete bridges. Examples include the Viaduc de Garabit (wrought iron, Maurice Koechlin & Gustav Eiffel, 1882-1884) and Firth of Forth (steel, Benjamin Baker & John Fowler, 1881-1890).

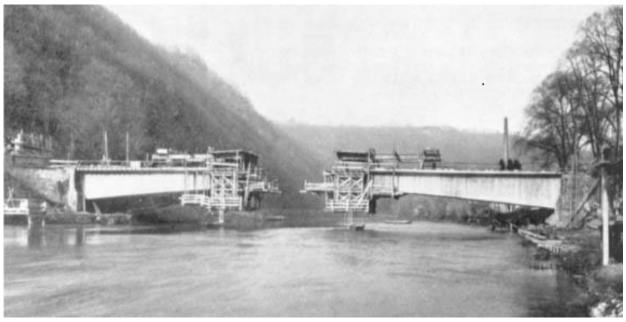
Photos and illustration: L. Girard, "Le pond du Herval au Brésil," La Technique des Travaux, No. 11, Nov. 1931, pp. 707-710

Cantilever-constructed bridges – Introduction

Lahnbrücke Balduinstein, Germany (1951) – Why prestressing?

- It took another 20 years before the first prestressed concrete cantilever-constructed bridge was built: The Lahnbrücke Balduinstein (1951) in Germany, designed by Ulrich Finsterwalder, with a span of 62 m.
- Obviously, passive reinforcement could be used for cantilever construction. However, deflections are hard to control during construction (the method used by E. Baumgart is not applicable in most cases), and long-term deflections are hard to predict. As an order of magnitude, the following displacements would be expected at midspan of the Felsenau Bridge (main span 156 m, see behind):

Midspan deflection for different creep increments (effective creep during cantilever construction)	$\Delta\varphi = 0$	$\Delta\varphi = 1$
As built (full cantilever prestressing for dead load = uncracked and bending moments partly compensated):	120	240
Without cantilever prestressing, uncracked (EI^{II}):	240	480
Without cantilever prestressing, cracked (EI^{II}):	1'200	1'400



Photos: https://www.e-periodica.ch/digbib/view?pid=cem-001:1952:20::172#26

Special girder bridges

Cantilever-constructed bridges
Recapitulation of erection method

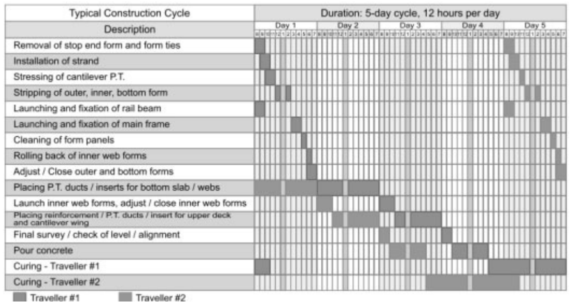
(the following 5 slides are repeated from girder bridges – design and erection)

Cantilever-constructed bridges – Construction

Free / balanced cantilevering (*Freivorbau*)

→ Cast-In-Place

- The girder is segmentally cast on a movable formwork cantilevering from the previously built segments
- Before installing the travellers, a pier table (*Grundetappe*) must be built on separate falsework
- Usually, two cantilevers are built ±symmetrically, starting from a pier (→ balanced cantilevering)
- Free cantilevering (smaller spans) is possible in other cases (e.g. right end span in example below)



(schedule assumes unbalanced moments of one element are admissible → cantilevers with ½ element offset; fully balanced construction requires casting of both cantilevers simultaneously)

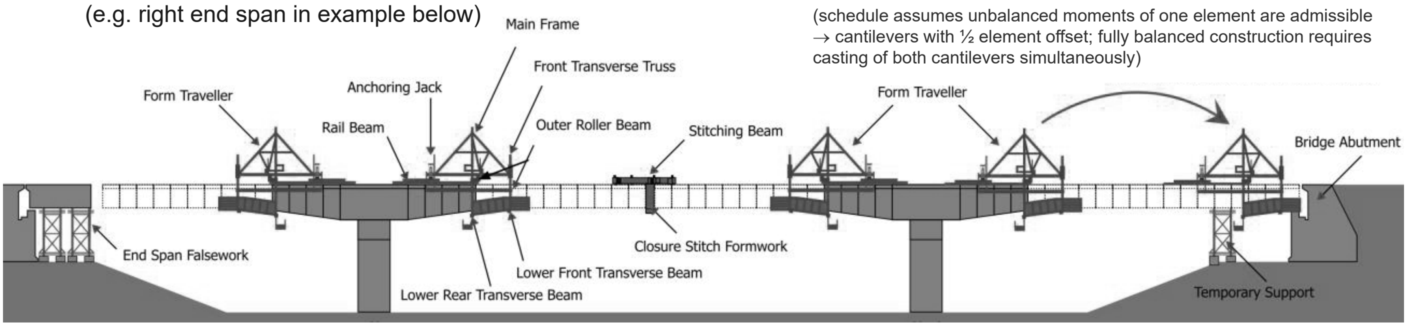


Photo: dsp, Innbrücke Vulpera

Illustration: Adapted from VSL Bridge Erection

Sources: VSL http://www.vsl.com/

Cantilever-constructed bridges – Construction

Free / balanced cantilevering (*Freivorbau*)
→ Cast-In-Place

- Cantilevers are often symmetrical (→ cast both sides simultaneously) or have ½ element offset (→ faster, but unbalanced moment)
- Economical for medium-large spans only (high initial cost for pier table and travellers)
- Suitable for high bridges crossing obstacles or soft soil, with spans $70\ \mathrm{m} \leq l \leq 160\ \mathrm{m}$ (250 m in special cases)


Inn Bridge Vulpera, Switzerland, 2010. dsp

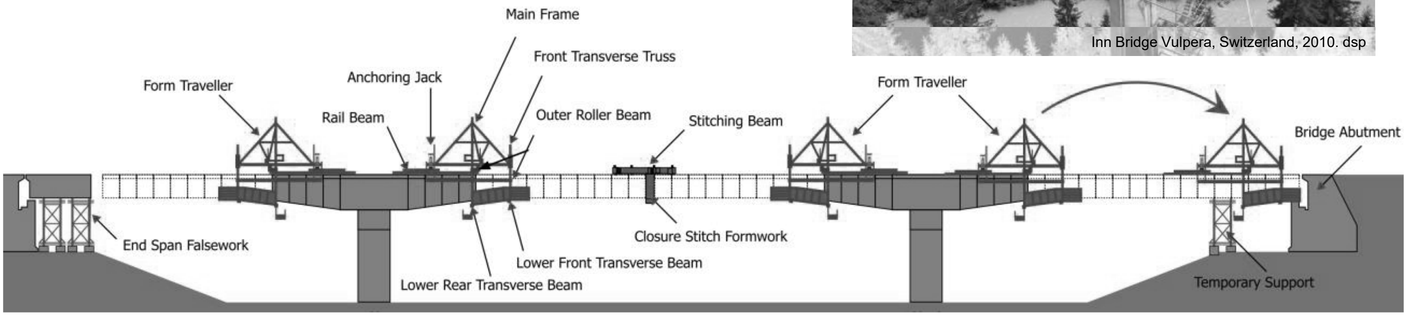


Photo: dsp, Innbrücke Vulpera

Illustration: Adapted from VSL Bridge Erection

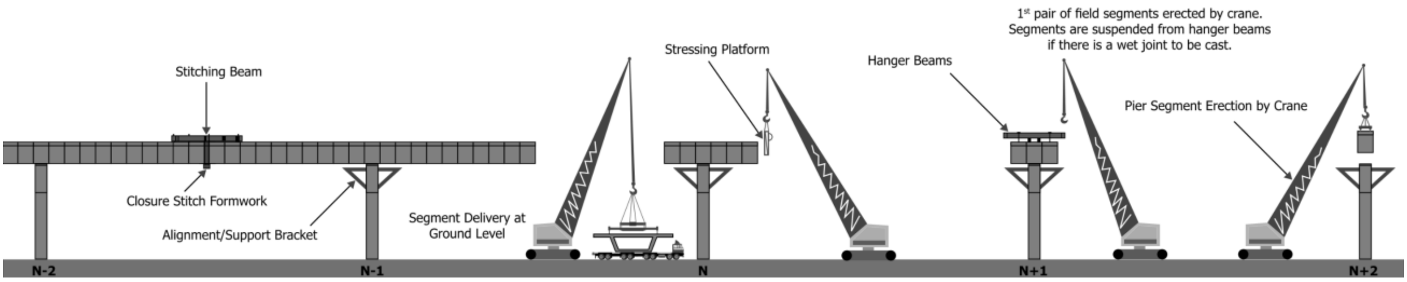
Sources: VSL http://www.vsl.com/

Cantilever-constructed bridges – Construction

Free / balanced cantilevering
→ Precast segmental with cranes

- Suitable for sites with access for trucks and cranes over entire length of bridge
- Segment weight limited by transportation and crane capacity
- Suitable for low-moderate height ($< 10\ \mathrm{m}$)
- Economic span ca. $45\ \mathrm{m} \leq l \leq 135\ \mathrm{m}$
- High flexibility for curved alignments

Typical Erection Cycle	Duration: 8 Shifts							
Description	1	2	3	4	5	6	7	8
Installation of Pier Segment Support Brackets								
Installation of Pier Segment								
Segment Erection - Pair 1-3								
Segment Erection - Pair 4-6								
Segment Erection - Pair 7-9								
Segment Erection - Pair 10-12								



Note that a great number of spans is required to be able to amortise the investments related to precasting. Among the factors that affect the minimum required bridge length are the availability and cost of skilled labour and the presence of local precasting facilities.

Further reading:

https://www.bridgetech-world.com/blogs/the-bridge-club/balanced-cantilever-construction-of-precast-segmental-bridges

https://www.bridgetech-world.com/blogs/the-bridge-club/span-by-span-construction-of-precast-segmental-bridges

Photo and illustration adapted from VSL Bridge Erection

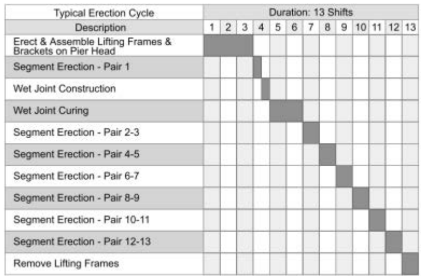
Sources: VSL http://www.vsl.com/

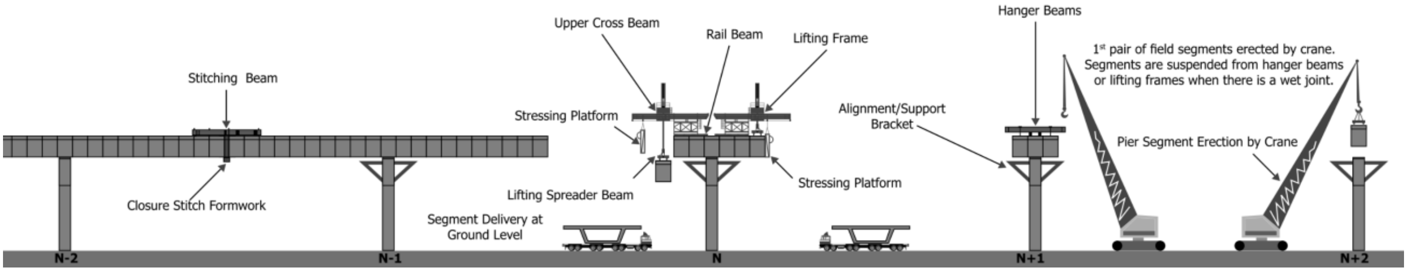
Cantilever-constructed bridges – Construction

Free / balanced cantilevering
→ Precast segmental with lifting frames

- Suitable for sites with access for trucks over entire length of bridge
- High lifting capacity of frames → large segments possible
- Economic span ca. $45\ \mathrm{m} \leq l \leq 135\ \mathrm{m}$
- High flexibility for curved alignments


Vidin – Calafat Bridge over the Danube, Romania-Bulgaria, 2012. CFCSL





Note that a great number of spans is required to be able to amortise the investments related to precasting. Among the factors that affect the minimum required bridge length are the availability and cost of skilled labour and the presence of local precasting facilities.

Photo: Vidin – Calafat Bridge over the Danube, Romania-Bulgaria, 2012 © Carlos Fernandez Casado S.L.

Illustration adapted from VSL Bridge Erection

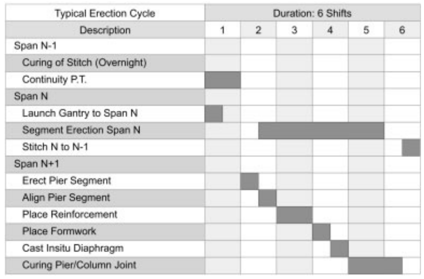
Sources: VSL http://www.vsl.com/ and BBR https://www.bbrnetwork.com/)

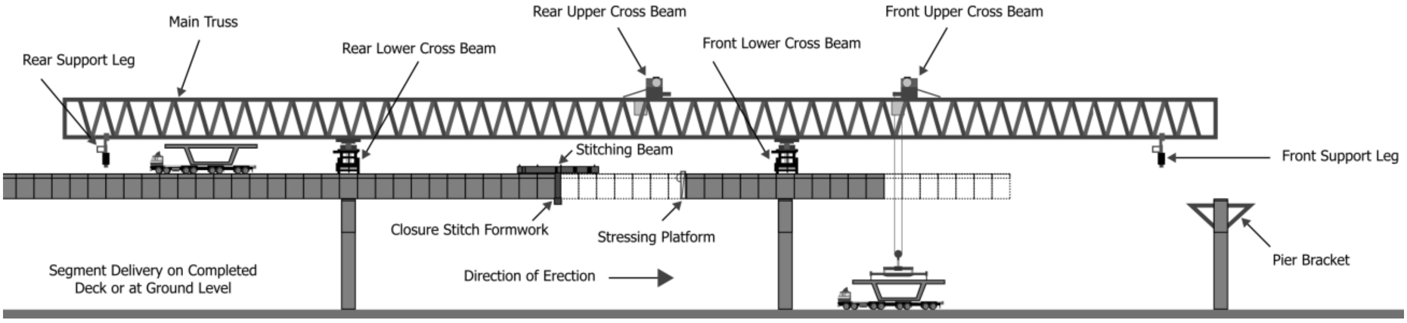
Cantilever-constructed bridges – Construction

Free / balanced cantilevering

→ Precast segmental with launching gantry

- Suitable for sites with access for trucks unless segments are delivered via bridge
- More efficient than erection on falsework, lighter gantry than for span-by-span erection
- Limited flexibility for curved alignments
- Economic span about $25\ \mathrm{m} \leq l \leq 45\ \mathrm{m}$





Note that a great number of spans is required to be able to amortise the investments related to precasting. Among the factors that affect the minimum required bridge length are the availability and cost of skilled labour and the presence of local precasting facilities.

Illustration adapted from VSL Bridge Erection. Photo © http://www.huadacrane.com/

Sources: VSL http://www.vsl.com/

Special girder bridges

Cantilever-constructed bridges
General observations

Cantilever-constructed bridges – General observations

Basic principles of cantilever construction

Classic in-situ cantilever construction – also referred to a as "balanced cantilevering" – consists of the following steps:

(i) Erection of pier and pier table (*Grundetappe*)
(ii) Installation of formwork travellers (*Vorbauwagen*)
(iii) Symmetrical cantilevering in segments ranging between 3…5 m length
(iv) Removal of travellers
(v) Midspan closure (*Fugenschluss*)

Depending on site constraints and contractor preferences, different methods are used, which differ by the demand on moment resistance at the starting pier:

- Fully balanced, simultaneous casting of segments at both cantilever ends ("1 crane bucket difference")
- Alternate casting, or installation of precast segments, at both cantilever ends, with or without cantilever offsets of half a segment length
- Unidirectional free cantilevering (typically starting from a previously erected part of the girder)

Photos: Inn Bridge Vulpera, dsp Ingenieure + Planer / ACS Partner with Dr. Vollenweider and Eduard Imhof (2010). Main span 104 m.

Cantilever-constructed bridges – General observations

Basic principles of cantilever construction

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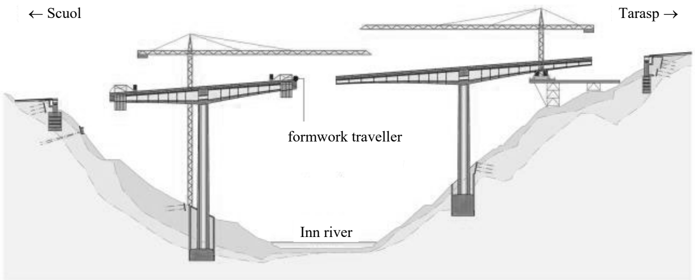
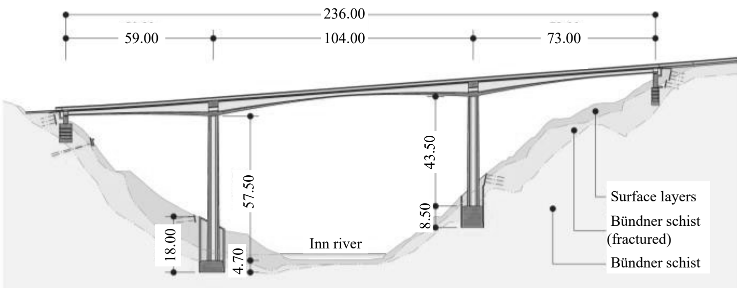
Example (photos on previous slide)





Illustrations: Inn Bridge Vulpera, dsp Ingenieure + Planer / ACS Partner with Dr. Vollenweider and Eduard Imhof (2010). Main span 104 m.

Cantilever-constructed bridges – General observations

Economy of cantilever-constructed bridges

Cantilever-constructed bridges are suitable for sites where conventional falsework is not feasible or would cause high cost due to
- height above ground
- access restrictions (rivers, soft soil, traffic)

and if the spans
- exceed the economical span range of other girder bridge erection methods not requiring falsework (MSS, precast girders, …)
- but are below the economical span of cable stayed bridges

Cantilever-constructed bridges are economical since
- only short, inexpensive, reusable formwork is needed, using the previously cast portions of the superstructure as support
- Identical tasks are repeated many times, enhancing productivity

For short spans, these advantages are less pronounced, and cantilever construction is less economical also due to the high initial cost of the pier table and travellers, see erection.

Usually, the economical span range of cantilever-constructed bridges is thus in the range of ca. 70…160 m.

Viadotto della Biaschina, Ingenieurbüro Guzzi / Ch. Menn (1983). Length 645 m, spans 58+85+140+160+140+62 m and 78+140+160+140+62 m, width 12.20 / 13.90 m, maximum pier height 100 m. Photo © P. Marti, Ingenieur-Betonbau (O. Monsch)

Cantilever-constructed bridges – General observations

Economy of cantilever-constructed bridges

The design of cantilever-constructed bridges is governed by the construction process, which is decisive e.g. for

- span layout
- girder geometry (variable depth)
- prestressing layout

If side spans are built by balanced cantilevering, they will be relatively short (side spans > 50% of the interior span require special measures).

Typically, a strongly variable girder depth is adopted for structural efficiency and elegance. For prestressed concrete cantilever-constructed girders, the following span/depth ratios are typical:

- above piers: $h/L \approx 1/17$ (large, limit cantilever deformations)
- at midspan: $h/L \approx 1/50$

Constant depth girders can also be cantilevered, but are structurally inefficient due to the excessive weight at midspan, where the large depth required to limit deformations during construction is not needed. Furthermore, they are subject to larger moment redistributions and lack a beneficial contribution of the bottom slab to the shear resistance, see dimensioning.

Gateway bridges (Sir Leo Hielscher Bridges), Brisbane, length 1627 m, main span 260 m. VSL (P. Marti / B. Ramsey 1986, duplicated 2011). Photo © P. Marti, Ingenieur-Betonbau

Special girder bridges

Cantilever-constructed bridges
Design

Cantilever-constructed bridges – Design

Particularities in design – Overview

The design of cantilever-constructed bridges needs to account for their following particularities

- Change of static system from cantilever to continuous frame
  → moment redistribution, affecting:
  … prestressing concept / tendon layouts
  … midspan moment
- Strongly variable girder depth
  → choose statically optimised girder profile
  → account for inclined chord forces in dimensioning

These particularities are further outlined on the following slides.

Gateway bridges (Sir Leo Hielscher Bridges), Brisbane, length 1627 m, main span 260 m. VSL (P. Marti / B. Ramsey 1986, duplicated 2011). Photo © magebagroup.com / structurae.net / https://www.reddit.com/r/bridgeporn/comments/1ag40f/gateway_bridge_at_night_brisbane_australia/

Cantilever-constructed bridges – Design

Particularities in design – Change of static system

The static system of cantilever-constructed bridges changes fundamentally when establishing continuity at midspan

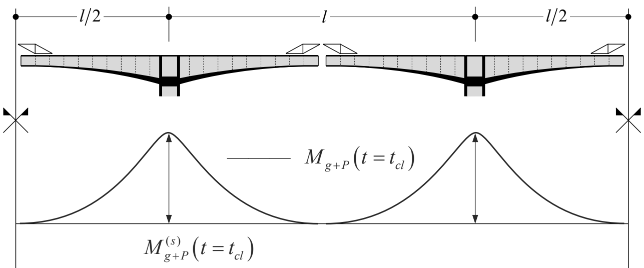
- before midspan closure: cantilevers (hogging moments only)
- after midspan closure: continuous frame system

If – as strongly recommended, see next slide – no hinges are provided at midspan, the change of the static system thus causes a moment redistribution due to long-term effects (concrete creep and shrinkage, prestressing steel relaxation).

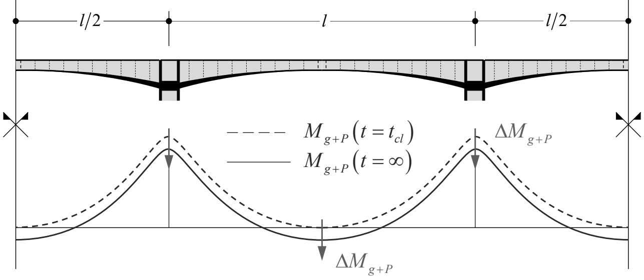
The redistribution is schematically illustrated in the figure:

- same difference in bending moments ΔM_{g+P} along the entire girder (or very similar in non-symmetrical cases)
- slightly favourable over piers (reducing the hogging moments by a small fraction of the initial value)
- very unfavourable in the span (causing a large portion of the moments at midspan, even if permanent loads applied after closure and traffic loads are considered)

System and moment line before midspan closure



System and moment line before midspan closure



Cantilever-constructed bridges – Design

Particularities in design – Change of static system

Resisting the same bending moment ΔM_{g+P} at the weak midspan section requires much more reinforcement or prestressing than the corresponding moment reduction saves in the strong the support region.

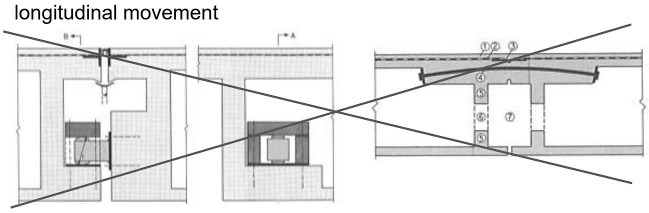
Historically, hinges were therefore provided at midspan to avoid moment redistribution (→ hinges permitting rotation). Hinges were sometimes also provided to prevent frame action (→ hinges permitting rotation and longitudinal movements), i.e., provide horizontally statically determinate support.

However, such hinges cause many problems (durability, excessive deflections) and must be avoided:

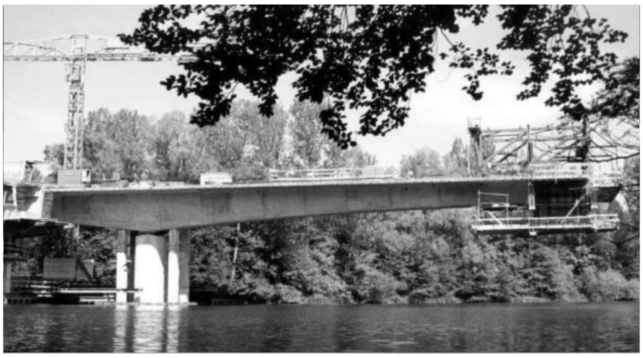
- → Bending moments at midspan can be covered in design, see next slides.
- → Longitudinal restraint may be problematic in case of short, stiff piers, but rather than hinges, bearings may be provided on the piers (with temporary measures for stability in construction, see photo).

Hinge permitting rotation and longitudinal movement

Hinge permitting rotation only



Balanced cantilevering from pier with bearings (temporary supports)



Figures: CH. Menn, Prestressed Concrete Structures (1990).

Photo. Aarebrücken A3, Schinznach, Bänziger Partner (1995). Length 1225 m / 1209 m, main span 90 m, low piers, additional support during balanced cantilevering.

Animated photo: Koror-Babelthuap Bridge, Palau (1977-1996)

Cantilever-constructed bridges – Design

Particularities in design – Change of static system

Moment redistribution is caused by long-term stresses and deformations, i.e., stresses due to all long-term actions:

- permanent load (self weight, superimposed dead load)
- prestressing

The moment redistribution ΔM_{g+P} can be determined using the time dependent force method and Trost's approximation (ageing factor $\mu \approx 0.85$, see Advanced Structural Concrete lecture):

- one-casting system (subscript "OC"), compatibility:

$$\theta_m = \theta_{10} + M_{g+P,OC} \cdot \theta_{11} = 0 \rightarrow M_{g+P,OC} = -\theta_{10}/\theta_{11}$$

- with system change, compatibility at $t = t_{cl}$ (midspan closure):

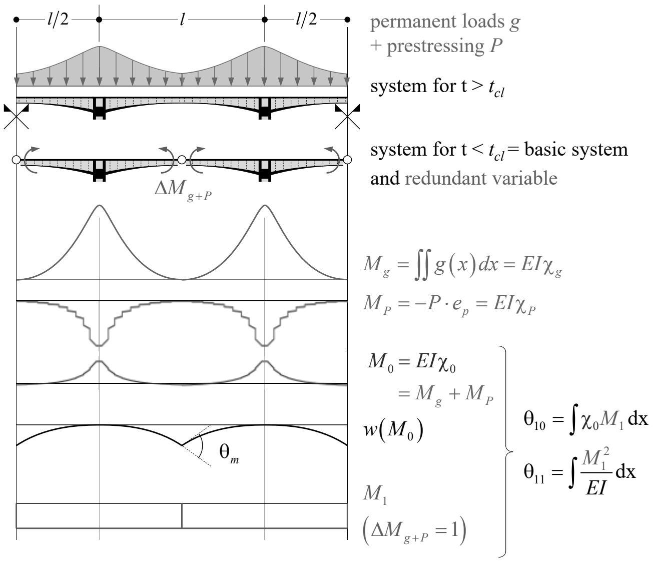
$$\theta_m(t_{cl}) = \theta_{10} \qquad \Delta M_{g+P}(t_{cl}) = 0$$

- with system change, compatibility for $t > t_{cl}$:

$$\Delta\theta_m(t) = \theta_{10} \cdot \varphi(t) + \Delta M_{g+P}(t) \cdot \theta_{11} \cdot \left(1 + \mu\varphi(t)\right) = 0$$

$$\rightarrow \Delta M_{g+P}(t) = -\frac{\theta_{10}}{\theta_{11}} \cdot \frac{\varphi(t)}{1+\mu\varphi(t)} = \frac{\varphi(t)}{1+\mu\varphi(t)} \cdot M_{g+P,OC}$$

Application of time-dependent force method to determine ΔM_{g+P}



Cantilever-constructed bridges – Design

Particularities in design – Change of static system

Even if the system has already crept at midspan closure, such that a reduced creep coefficient can be used for determining ΔM_{g+P} , a pronounced moment redistribution occurs, which is non-negligible particularly at midspan.

Moment redistribution is caused by the total permanent curvatures, i.e., only by the part of the permanent loads not compensated by prestressing (using long-term values of prestressing forces). If prestressing was neglected, ΔM would be severely overestimated.

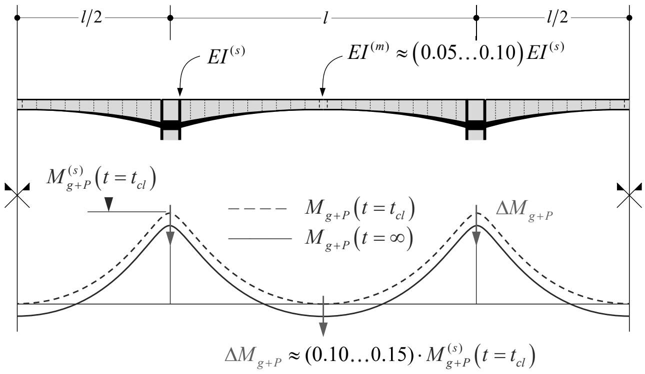
For usual stiffness ratios $EI^{(m)} \approx (0.05\ldots0.10)\cdot EI^{(s)}$ (correponding to common slendernesses h/l), ΔM_{g+P} can be estimated as:

$$\Delta M_{g+P} \approx (0.10\ldots0.15)\cdot M_{g+P}^{(s)}(t = t_{cl})$$

If furthermore, the cantilever tendons are designed to avoid decompression during cantilevering as usual (see prestressing concept), i.e., they compensate about 80% of the permanent loads, ΔM_{g+P} is approximately:

$$\Delta M_{g+P} \approx (0.02\ldots0.03)\cdot M_{g}^{(s)}(t = t_{cl})$$

Estimation of moment redistribution for preliminary design



Cantilever-constructed bridges – Design

Particularities in design – Prestressing concept

During cantilever construction, cracking must be avoided since it would lead to

- large deflections hard to predict (camber =?) due to large scatter of deflections (section might crack or not depending on the concrete tensile strength)
  - → Typically, the cantilever tendons are designed to avoid decompression during cantilevering

Moment redistribution could be reduced (or even eliminated) by providing more cantilever prestressing. However, this is not economical since there are usually reserve capacities for ULS over piers anyways, due to

- minimum passive reinforcement
- low ratio of traffic loads to self-weight

Furthermore, space requirements limit the number of cantilever tendons, see figure: At the pier table, all tendons must be accommodated.

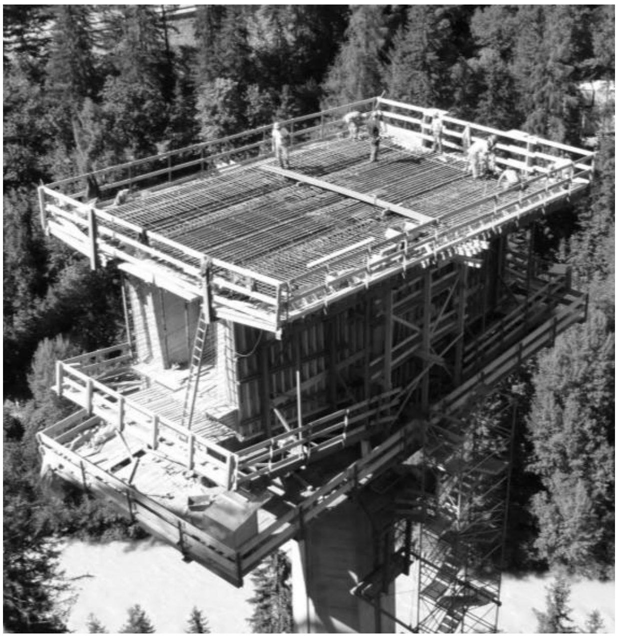


Photo: Pier table of Inn Bridge Vulpera, dsp Ingenieure + Planer / ACS Partner with Dr. Vollenweider and Eduard Imhof (2010). Main span 104 m.

Cantilever-constructed bridges – Design

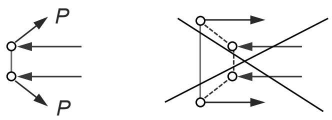
Particularities in design – Prestressing concept

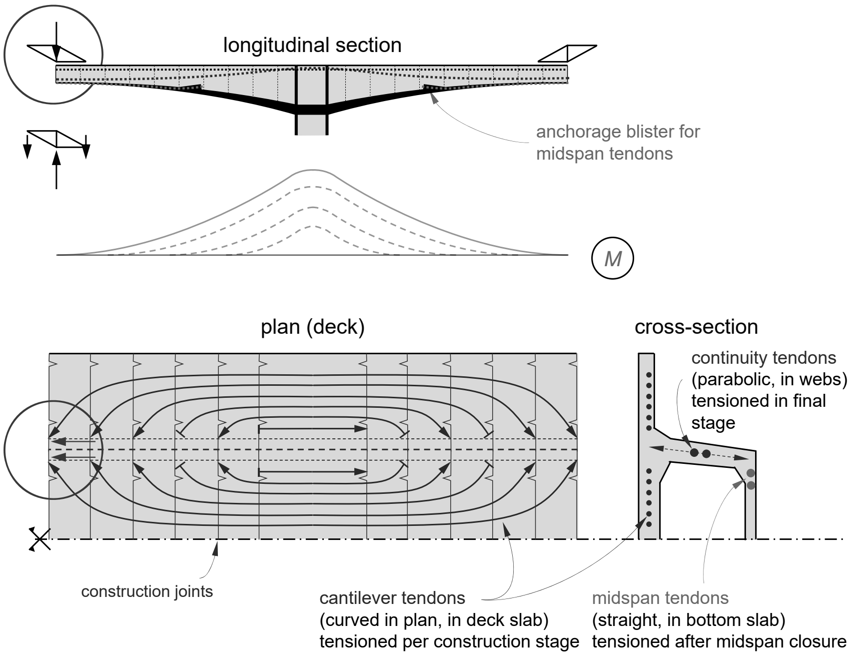
Rather, additional tendons with different layouts are usually tensioned after midspan closure (see cast in place girder erection methods and tendon layouts):

… cantilever tendons (essential)
… midspan tendons (usual today)
… continuity tendons (optional)

As also mentioned there already, cantilever tendons are anchored near the webs
→ space for anchorages
→ longitudinal shear flow

The deck acts as tension chord, but the horizontal shear transferred to the deck cannot be spread via compressive forces:





Note that in spite that the cantilever tendons are straight in elevation, they are rather acting as parabolic tendons due to the variable depth: With respect to the girder axis (centroid), they are indeed curved.

Cantilever-constructed bridges – Design

Photos: Inn Bridge Vulpera, dsp Ingenieure + Planer / ACS Partner with Dr. Vollenweider and Eduard Imhof (2010). Main span 104 m.

Cantilever-constructed bridges – Design

Particularities in design – Midspan moment

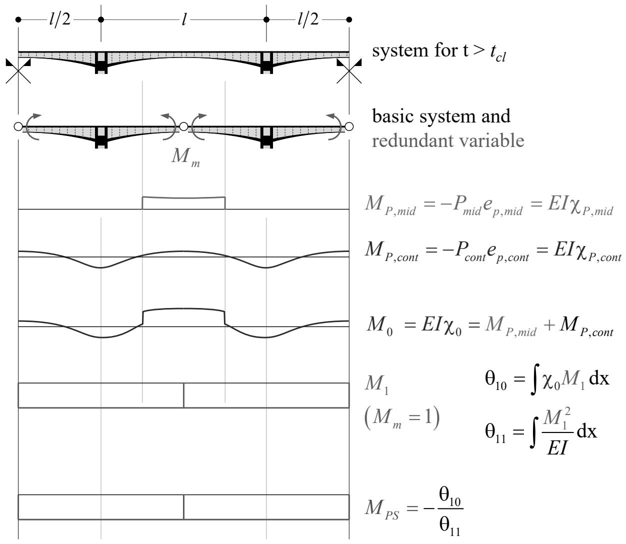
Midspan (P_{mid}) and continuity (P_{cont}) tendons cause significant secondary moments, which need to be accounted for in the design of the midspan section in addition to ΔM_{g+P} (unless significant moment redistributions are taken into account, which is unusual).

Hence, the midspan cross-section needs to be designed for the sum of the following bending moments:

- moment redistribution ΔM_{g+P} (long-term effects)
- secondary moment M_{PS} due to midspan tendons P_{mid} and continuity tendons P_{cont}
- midspan moment due to permanent loads applied after midspan closure
- midspan moment due to traffic loads (envelope)

Due to long-term losses of prestressing force, ΔM_{g+P} increases with time (resp. has a larger value), but M_{PS} decreases. If a strong continuity and midspan prestressing is provided, the permanent bending moment at midspan (ΔM_{g+P} + M_{PS}) may thus even slightly decrease with time.

Secondary moments due to continuity and midspan tendons



Cantilever-constructed bridges – Design

Particularities in design – Strongly variable depth

- Usually, cantilever-constructed girders have a strongly variable depth
  - → girder axis (centroid) substantially inclined even if deck is horizontal in elevation
- However, segment joints and stirrups are usually vertical
  - → internal actions obtained form global structural analysis using a 2D or 3D frame model need to be transformed (see figures)
  - → the inclination δ of the girder axis (centroid) is relevant here (inclinations δ_{sup} and δ_{inf} of top and bottom slab affect δ via variation of section properties)

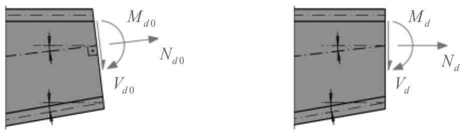
Internal actions obtained from structural analysis

$$N_{d0} = N_d \cos\delta - V_d \sin\delta$$
$$V_{d0} = N_d \sin\delta + V_d \cos\delta$$
$$M_{d0} = M_d$$

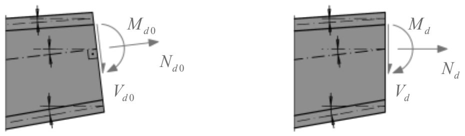
Internal actions used in stress-field design

$$N_d = N_{d0} \cos\delta + V_{d0} \sin\delta$$
$$V_d = -N_{d0} \sin\delta + V_{d0} \cos\delta$$
$$M_d = M_{d0}$$

Deck horizontal (no longitudinal gradient)



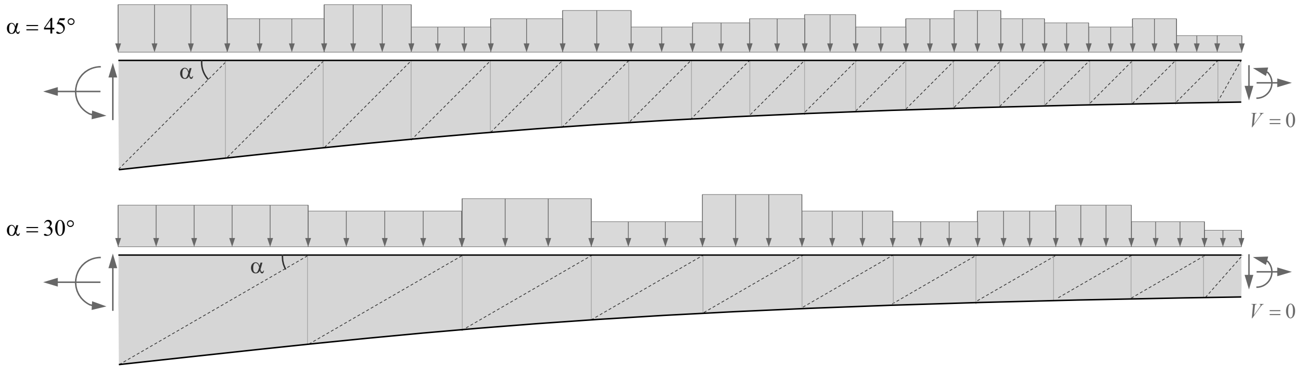
Deck with longitudinal gradient



Cantilever-constructed bridges – Design

Particularities in design – Strongly variable depth

- Once the internal actions have been determined, dimensioning can be carried out using strut-and-tie models or stress-fields (see lecture Stahlbeton I), as illustrated below for two different inclinations of the web compression field and arbitrary loads.



- Alternatively, a sectional design approach can be used as for parallel chord girders (see Stahlbeton I), as illustrated on the following slides. This is often more practical, particularly since envelopes of traffic loads need to be considered.

The “segments” in the stress-field models, depending on the chosen compression field inclination for dimensioning, are of course in no way related to the segments built (cast-in-place or precast segment lengths).

Cantilever-constructed bridges – Design

Particularities in design – Strongly variable depth

- The strong influence of variable depth and draped prestressing tendons (prestressing force $F_p = P_\infty$) can also be accounted for using the sectional design approach illustrated in the figure
- Formulating equilibrium on the free body one gets

$$N_d = F_t \cos\delta_{sup} - F_{cw}\cos\alpha + F_p\cos\delta_p - F_c\cos\delta_{inf}$$
$$V_d = -F_t \sin\delta_{sup} + F_{cw}\sin\alpha + F_p\sin\delta_p + F_c\sin\delta_{inf}$$
$$M_d = F_t\cos\delta_{sup}\cdot\left(\frac{d_v}{2} - e\right) + F_{cw}\cos\alpha\cdot e - F_p\cos\delta_p\cdot\left(e + e_p\right) + F_c\cos\delta_{inf}\cdot\left(\frac{d_v}{2} + e\right)$$

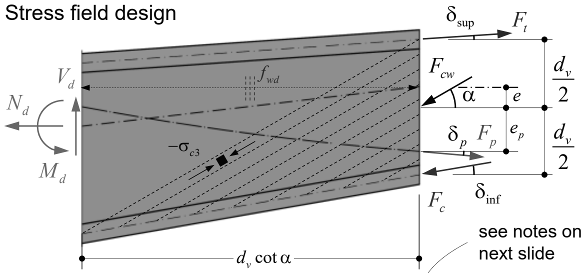
and solving for the unknown forces:

$$F_t = \frac{2k_{inf}\dfrac{M_d}{d_v} + N_d\left(1 + \dfrac{2k_{inf}e}{d_v}\right) + \dfrac{V_d - F_p\sin\delta_p}{\tan\alpha} - F_p\cos\delta_p\left(1 - \dfrac{2k_{inf}e_p}{d_v}\right)}{\left(k_{sup} + k_{inf}\right)\cos\delta_{sup}} \qquad F_c = \frac{2k_{sup}\dfrac{M_d}{d_v} - N_d\left(1 - \dfrac{2k_{sup}e}{d_v}\right) - \dfrac{V_d - F_p\sin\delta_p}{\tan\alpha} + F_p\cos\delta_p\left(1 + \dfrac{2k_{sup}e_p}{d_v}\right)}{\left(k_{sup} + k_{inf}\right)\cos\delta_{inf}}$$

$$F_{cw} = \frac{2\left(V_d - F_p\sin\delta_p\right) + \left(\tan\delta_{sup} + \tan\delta_{inf}\right)\left(N_d - F_p\cos\delta_p\right) + 2\left(\tan\delta_{sup} - \tan\delta_{inf}\right)\left(\dfrac{M_d + N_d e + F_p\cos\delta_p e_p}{d_v}\right)}{\left(k_{sup} + k_{inf}\right)\sin\alpha} \quad \text{where} \quad \begin{cases} k_{sup} = 1 - \dfrac{\tan\delta_{sup}}{\tan\alpha} \\ k_{inf} = 1 - \dfrac{\tan\delta_{inf}}{\tan\alpha} \end{cases}$$

- These forces are to be superimposed with the shear flow due to torsion, as in prismatic girders.

Stress field design



$$-\sigma_{c3} = \frac{F_{cw}}{b_w\cdot d_v\cdot\cos\alpha} \qquad f_{wd} = a_{sw}f_{sd} \approx \frac{F_{cw}\cdot\sin\alpha}{d_v\cdot\cot\alpha}$$

Source (equations for horizontal alignment): Marti, P., „Schubbemessung von Voutenträgern mit geneigten Spanngliedern / Shear design of variable-depth girders with draped prestressing tendons", Vorgespannter Beton in der Schweiz, FIP Schweizer-Gruppe, Zürich 1994, pp. 16-19.

Cantilever-constructed bridges – Design

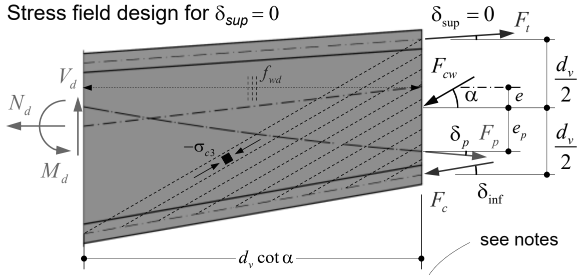
Particularities in design – Strongly variable depth

- Since δ_{sup} is small for typical road alignments, it may usually be neglected, which yields simpler equations:

$$N_d = F_t - F_{cw}\cos\alpha + F_p\cos\delta_p - F_c\cos\delta_{\inf}$$
$$V_d = F_{cw}\sin\alpha + F_p\sin\delta_p + F_c\sin\delta_{\inf}$$
$$M_d = F_t\cdot\left(\frac{d_v}{2}-e\right) + F_{cw}\cos\alpha\cdot e - F_p\cos\delta_p\cdot\left(e+e_p\right) + F_c\cos\delta_{\inf}\cdot\left(\frac{d_v}{2}+e\right)$$

- Solving for the unknown forces:

Stress field design for $\delta_{sup} = 0$



$$-\sigma_{c3} = \frac{F_{cw}}{b_w\cdot d_v\cdot\cos\alpha} \qquad f_{wd} = a_{sw} f_{sd} \approx \frac{F_{cw}\cdot\sin\alpha}{d_v\cdot\cot\alpha}$$

$$F_t = \frac{2k\dfrac{M_d}{d_v} + N_d\left(1+\dfrac{2ke}{d_v}\right) + \dfrac{V_d - F_p\sin\delta_p}{\tan\alpha} - F_p\cos\delta_p\left(1-\dfrac{2ke_p}{d_v}\right)}{1+k}$$

$$F_c = \frac{2\dfrac{M_d}{d_v} - N_d\left(1-\dfrac{2e}{d_v}\right) - \dfrac{V_d - F_p\sin\delta_p}{\tan\alpha} + F_p\cos\delta_p\left(1+\dfrac{2e_p}{d_v}\right)}{\left(1+k\right)\cos\delta_{\inf}}$$

$$F_{cw} = \frac{2\left(V_d - F_p\sin\delta_p\right) - \tan\delta_{\inf}\left[2\dfrac{M_d}{d_v} - N_d\left(1-\dfrac{2e}{d_v}\right) + F_p\cos\delta_p\left(1+\dfrac{2e_p}{d_v}\right)\right]}{\left(1+k\right)\sin\alpha} \quad \text{where} \quad k = 1 - \frac{\tan\delta_{\inf}}{\tan\alpha}$$

- These forces are to be superimposed with the shear flow due to torsion, as in prismatic girders.

The vertical component of the web compression force F_{cw} is resisted by the stirrups and the vertical deviation forces due to F_c over the length $d_v \cdot \cot\alpha$. Hence, the formula on the slide applies exactly for hypothetical polygonal girder soffits. Differences are, however, small and neglecting the upward deviation forces due to F_c provides a safe design. Furthermore, for sagging tendon segments (tendon profile concave from above), stirrup forces between the draped tendon and the top chord may be reduced due to the deviation forces generated by the tendon curvature. Likewise, if the tendon segment is hogging (convex from above), additional stirrup forces are required to transfer the downward deviation forces to the top chord.

Source (equations for horizontal alignment): Marti, P., „Schubbemessung von Voutenträgern mit geneigten Spanngliedern / Shear design of variable-depth girders with draped prestressing tendons", Vorgespannter Beton in der Schweiz, FIP Schweizer-Gruppe, Zürich 1994, pp. 16-19.

Cantilever-constructed bridges – Design

Particularities in design – Strongly variable depth

On the next slides, using the Felsenau viaduct as example, the effect of the following parameters on the design is studied:

- girder geometry = shape of soffit (reference: second order parabola)
- inclination of the web compression field (reference: α = 45°)
- continuity prestressing (reference: F_p = 0)
- midspan moment = moment redistribution (reference: M_y = 0)

One parameter is varied at a time, keeping the others at the reference values.

Longitudinal section (entire viaduct, L = 1'116 m; main span, l = 144m)

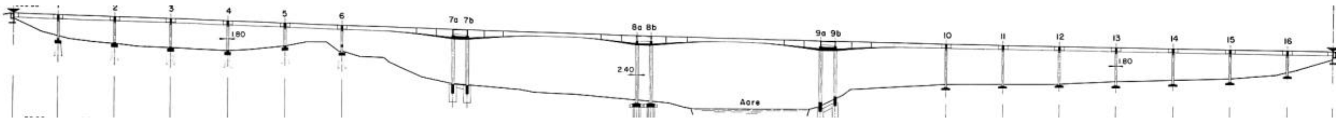
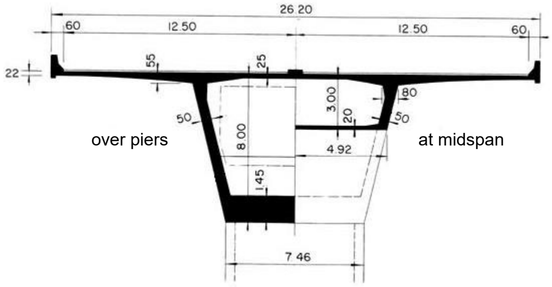
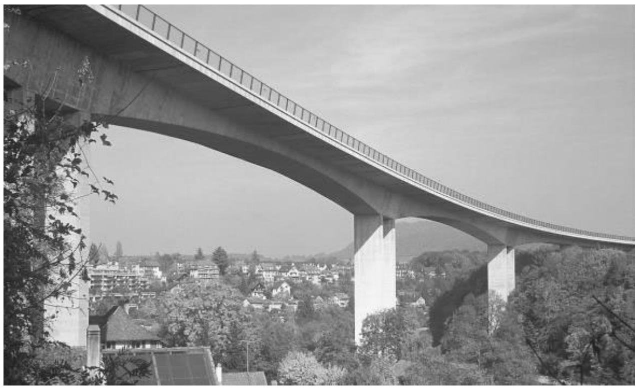


Photo and figure: Felsenauviadukt Bern, Ch. Menn (1974). Length 116 m, spans 38+5x48+(94+6)+2x(144+12)+(94+6)+6x48+38 m, width 26.20 m. © P. Marti, Ingenieur-Betonbau.

Cantilever-constructed bridges – Design

Cross-section





Longitudinal section (main spans)

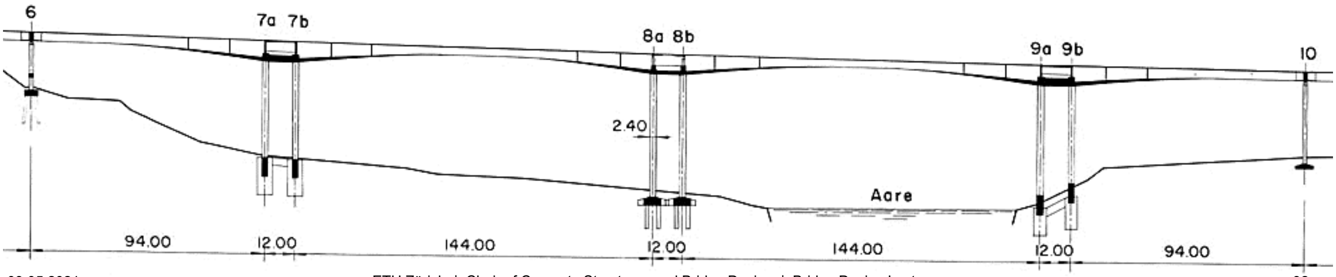


Photo and figures: Felsenauviadukt Bern, Ch. Menn (1974). Length 116 m, spans 38+5x48+(94+6)+2x(144+12)+(94+6)+6x48+38 m, width 26.20 m. © P. Marti, Ingenieur-Betonbau.

Cantilever-constructed bridges – Design

Effect of girder geometry on internal actions

As a first parameter, the variation of girder depth is studied, comparing two exponential geometries of the bottom slab, both with vertex at midspan:

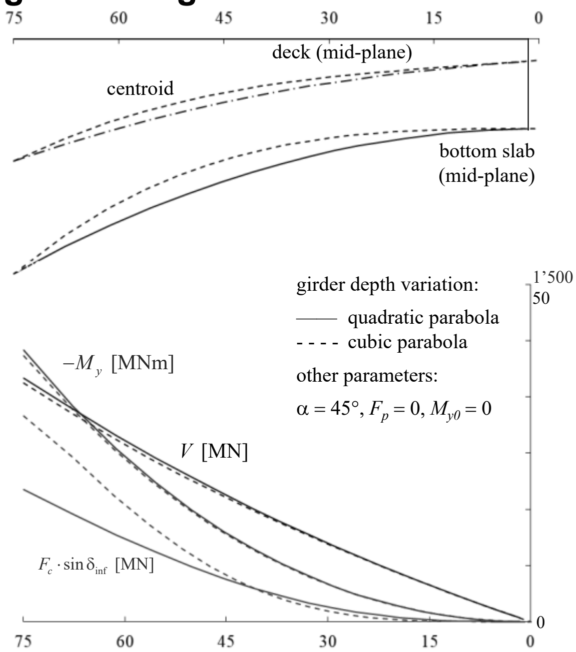
- quadratic parabola (exponent 2)
- cubic parabola (exponent 3)

while all the remaining parameters are kept constant.

On this slide, the effect of girder geometry on the internal actions is studied.

It is seen that the geometry of the bottom slab (≈ soffit) has a small effect on the bending moment and shear forces.

However, it does affect the contribution of the inclined bottom chord force F_c to the shear resistance (vertical component $F_c \cdot \sin\delta_p$). Near the piers, the bottom chord contributes more than 50% to the shear resistance in the case of the quadratic soffit, and even more for the cubic geometry.



In the calculations underlying the figures on this and the following slides, a linear variation of the bottom slab thickness from piers to midspan has been assumed for simplification.

Cantilever-constructed bridges – Design

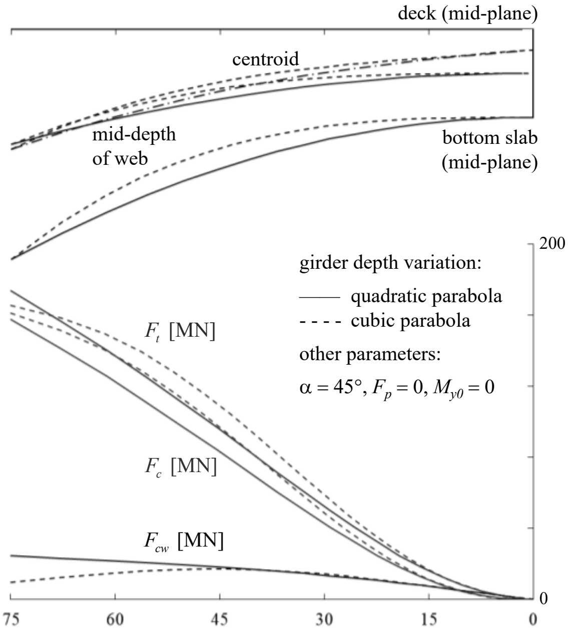
Effect of girder geometry on chord and web forces

On this slide, the effect of girder geometry on the chord forces F_t and F_c, as well as the web compression force F_{cw} is studied.

The geometry of the bottom slab (≈ soffit) has a relevant effect:

- Top and bottom chord forces are significantly higher for the cubic parabola over large parts of the span (similar bending moment, smaller static depth)
- The web compression force is smaller for the cubic parabola near the pier

“Straighter” geometries (third order parabola) thus require significantly more reinforcement in the top chord, and thicker bottom slabs (quarter span region).

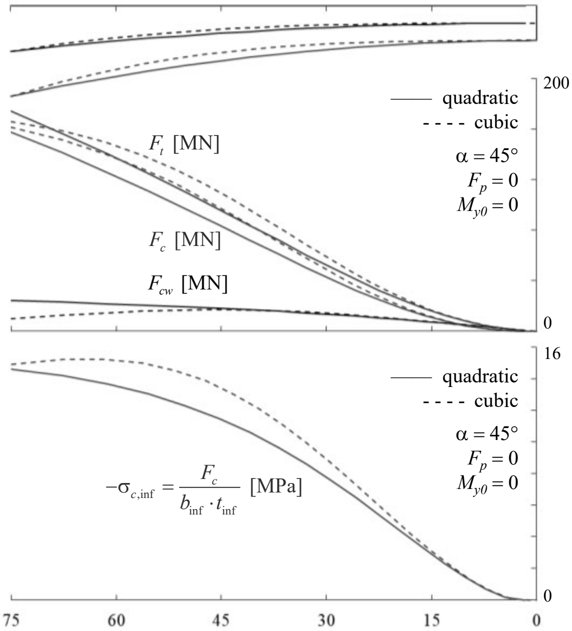


Cantilever-constructed bridges – Design

Effect of girder geometry on chord and web forces

This slide again shows the effect of girder geometry on the chord forces F_t and F_c, as well as the web compression force F_{cw}.

The bottom diagram compares the compressive stresses in the bottom slab, which are significantly higher for the cubic parabola as expected, given the higher compression chord force.



Cantilever-constructed bridges – Design

Effect of girder geometry on shear design

This slide shows the effect of girder geometry on the shear design:

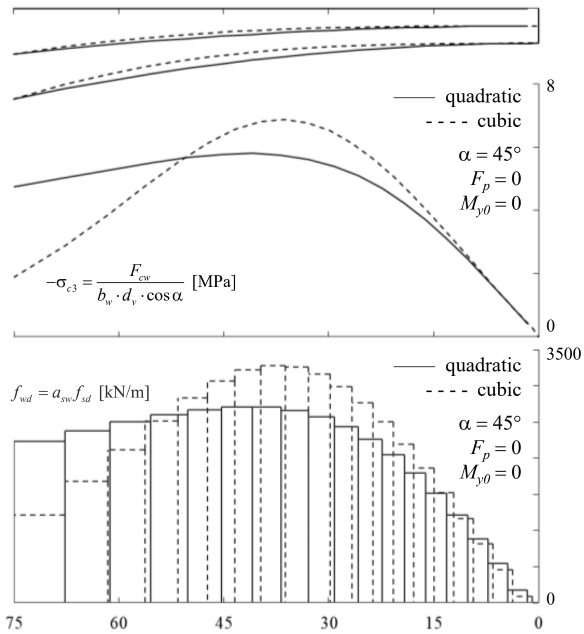
- principal compressive stresses in the web
- required resistance of vertical stirrups

Here, the geometry of the bottom slab (≈ soffit) has a pronounced effect.

Both, the principal compressive stresses in the web as well as the stirrup forces, vary much stronger over the span for the cubic parabola.

Since varying the web thickness complicates cantilever construction, and high stirrup forces cause reinforcement congestions, uniform values over the entire span are preferred, i.e.

→ quadratic parabola is superior to cubic parabola
→ more uniform distributions are possible (optimum exponent ≈ 1.7), but “straighter” soffits than the quadratic parabola are aesthetically challenging



Cantilever-constructed bridges – Design

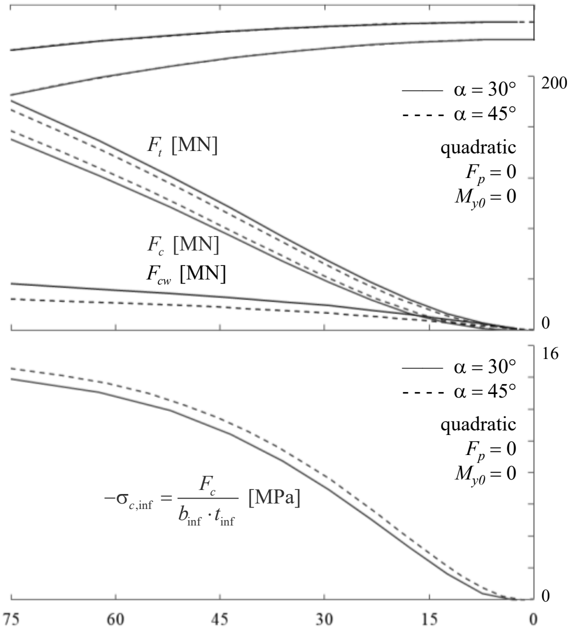
Effect of compression field inclination on chord and web forces

This slide shows the effect of the web compression field inclination on the chord forces F_t and F_c, as well as the web compression force F_{cw}.

The compression field inclination has a similar effect as in parallel chord girders (tension shift), i.e., with flatter inclinations of the compression field:

- the tension chord force F_t increases
- the compression chord force F_c (compression+) decreases and consequently, the compressive stresses in the bottom slab are reduced

Flatter inclinations of the compression field in the web thus require more reinforcement in the top chord (but less stirrups, see next slide).



Cantilever-constructed bridges – Design

Effect of compression field inclination on shear design

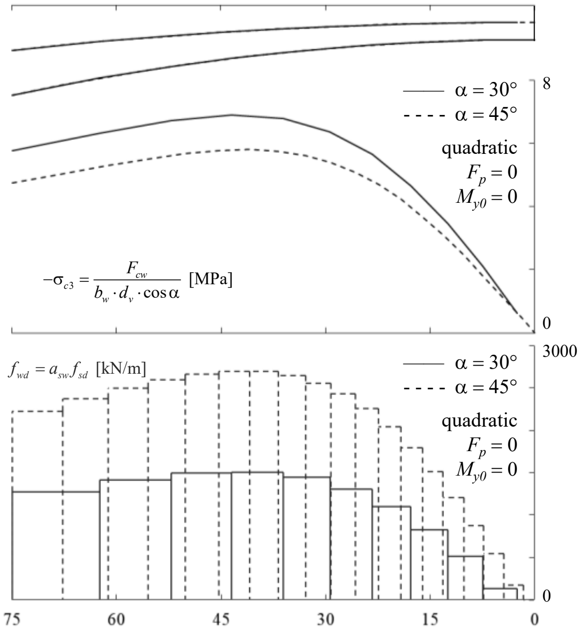
This slide shows the effect of the web compression field inclination on the shear design:

- principal compressive stresses in the web
- required resistance of vertical stirrups

Again, the compression field inclination has a similar effect as in parallel chord girders (tension shift), i.e., with flatter inclinations of the compression field:

- the required stirrup resistance f_{wd} decreases
- the web compression force, and consequently the principal compressive stresses in the web, increase

Flatter inclinations of the compression field in the web thus require more reinforcement in the top chord (see previous slide), but significantly less stirrups. Since stirrups are more complicated to fix, and the top chord reinforcement has adequate capacity (if moment redistributions take place before relevant traffic loads are applied), flatter inclinations are usually preferred in cantilever-constructed bridges.



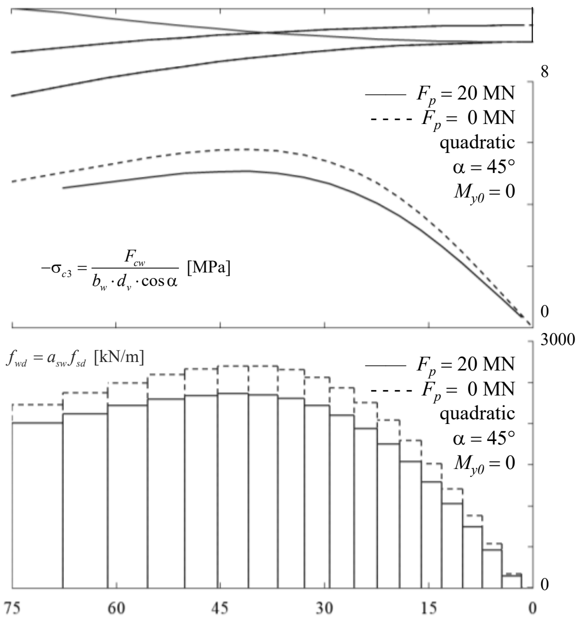
Cantilever-constructed bridges – Design

Effect of continuity prestressing on shear design

This slide shows the effect of continuity prestressing on on the shear design:

- principal compressive stresses in the web
- required resistance of vertical stirrups

Continuity prestressing is favourable for both, web compressive stresses as well as stirrup forces, since the vertical component of the tendons resists part of the applied shear force.



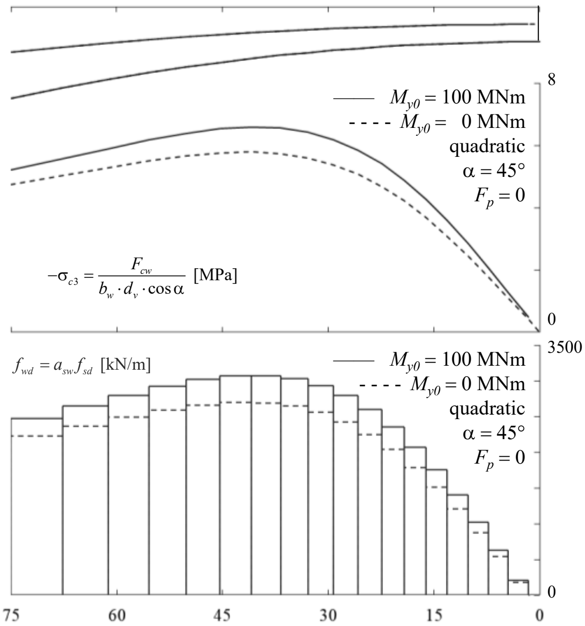
Cantilever-constructed bridges – Design

Effect of midspan moment (moment redistribution) on shear design

This slide shows the effect of a midspan moment (due to moment redistribution or loads applied after midspan closure) on the shear design:

- principal compressive stresses in the web
- required resistance of vertical stirrups

A midspan moment is unfavourable for both, web compressive stresses as well as stirrup forces, since the positive bending moment reduces the beneficial effect of the inclined compression chord force that resists part of the applied shear force.



Special girder bridges

Cantilever-constructed bridges
Camber

Cantilever-constructed bridges – Design

Relevance of camber

Even if cantilever prestressing is designed to avoid cracking during construction (see prestressing concept), deflections in cantilever-constructed girders are relatively large

→ To achieve the desired profile grade line of the bridge, significant camber needs to be provided

→ There is no “safe side” in determining camber

→ Accurate calculations, accounting for time-dependent effects and friction losses of prestressing forces, are essential

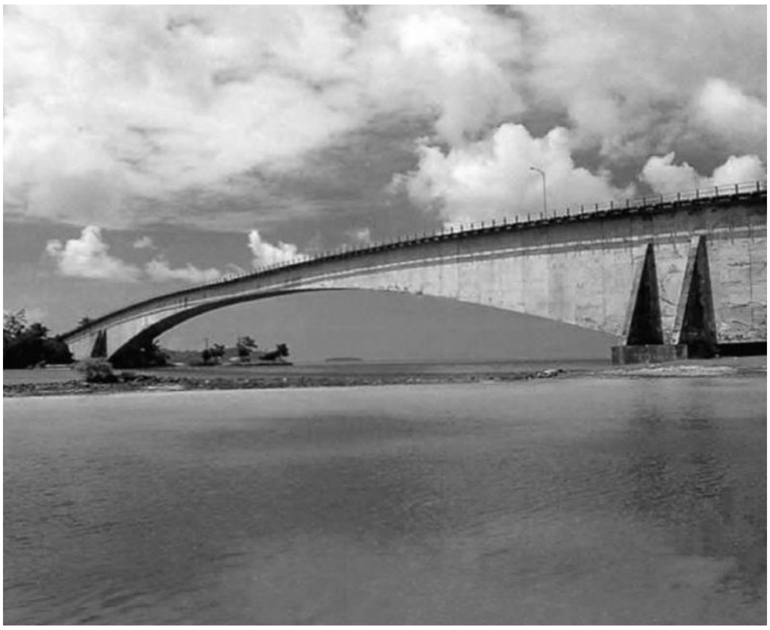


Photo: Koror-Babalthaup bridge, from C. Burgoyne, R. Scantlebury. « Lessons learned from the bridge collapse in Palau». Civil Engineering 161, November 2008, pp. 28–34

Cantilever-constructed bridges – Design

Principle and contributions to camber (cast-in-place girders)

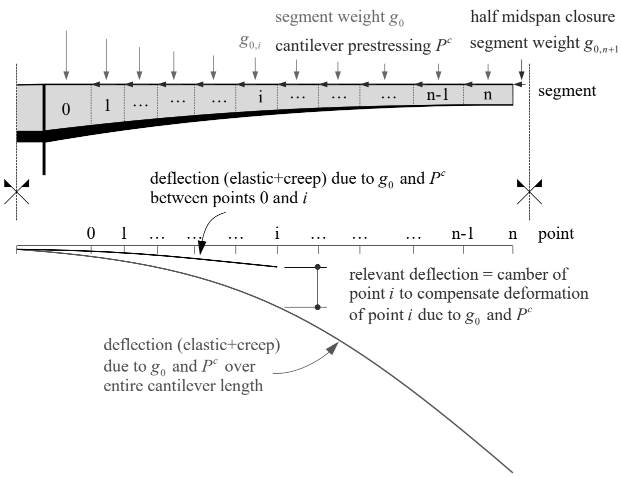
Principle: Camber at any point *i* of the girder must compensate the deflections occurring after its construction

→ camber (positive upward) = total deflection of point *i* minus deflection at point *i* at time of its construction (see figure)

Deflections of cantilever-constructed bridges are caused by the following (including creep where appropriate):

- w^F: deformations of traveller and formwork (form camber)
- w^{BC}: deflections of the cantilever system before closure, due to
  … segment weights $g_{0,0\ldots n}$ and cantilever prestressing $P^c_{0\ldots n}$
  … midspan closure segment weight $g_{0,n+1}$
  … weight of traveller G_T
- w^{AC}: deflections of the continuous system after closure, due to
  … residual creep deformations due to g_0 and P^c (including residual prestressing losses)
  … midspan and continuity prestressing including losses
  … superimposed dead load applied in continuous system
- deformations of piers and foundations (settlements) (in the appropriate system)

Cantilever system deflections for selected loads



Cantilever-constructed bridges – Design

Camber due to cantilever deflections (cast-in-place girders)

The camber w^{BC} due to deflections in the cantilever system before closure can be expressed as:

$$w_i^{BC} = \underbrace{\sum_{j=0}^{n} w_i\left(g_{0,j}, P_j^c\right)\cdot\left(1+\Delta\varphi\left(t_j \ldots t_{cl}\right)\right)}_{\substack{\text{deflection (elastic+creep) of point } i \\ \text{due to } g_{0j}+P^c \text{ on entire cantilever} \\ \text{(= deflection of point } i \text{ at } t=t_{cl}\text{)}}} - \underbrace{\sum_{j=0}^{i} w_i\left(g_{0,j}, P_j^c\right)\cdot\left(1+\Delta\varphi\left(t_j \ldots t_i\right)\right)}_{\substack{\text{deflection (elastic+creep) of point } i \\ \text{due to } g_{0j}+P^c \text{ between 0 and } i \\ \text{(= deflection of point } i \text{ at } t=t_i\text{)}}} + \underbrace{\sum_{j=1}^{n} w_i\left(G_{T,j}\right)\cdot\Delta\varphi\left(t_j \ldots t_{j+1}\right)}_{\substack{\text{creep deflection of point } i \\ \text{due to traveller weight} \\ \text{during casting of segments}}} - \underbrace{w_i\left(G_{T,i}\right)}_{\substack{\text{elastic deflection of} \\ \text{point } i \text{ due to traveller} \\ \text{weight in } i \text{ (removed)}}} + \underbrace{w_i\left(\frac{g_{0,n+1}}{2}\right)}_{\substack{\text{deflection of point } i \\ \text{due to midspan} \\ \text{closure segment}}}$$

Note that using hand calculations, the evaluation of the creep increments is tedious (t_{0k} is different for each segment, i.e., when calculating deflections, $\Delta\varphi$ varies along the girder axis, being different for each segment).

Loads and times ("absolute", i.e., counting from casting of segment 0)

g_{0j}	concrete weight of segment j ($n+1$: midspan closure)
P_j^c	cantilever prestressing of segment j
G_T	traveller weight
t_j	time of casting of segment j
$t_{cl} = t_{n+1}$	time of midspan closure
t_0	concrete age at start of exposure (similar for all segments)

Creep increments $\Delta\varphi$

$\Delta\varphi\left(t_a \ldots t_b\right) = \varphi\left(t_b - \left[t_k + t_{0k}\right], t_{0k}\right) - \varphi\left(t_a - \left[t_k + t_{0k}\right], t_{0k}\right) =$ creep between t_a and t_b

Cantilever-constructed bridges – Design

Camber due to deflections in continuous system

The camber w^{AC} due to deflections of the continuous system after closure is determined for the final continuous system, with the exception of the deformations due to g_0 and P^c (including residual prestressing losses). These are obtained in the cantilever system, accounting for moment redistribution.

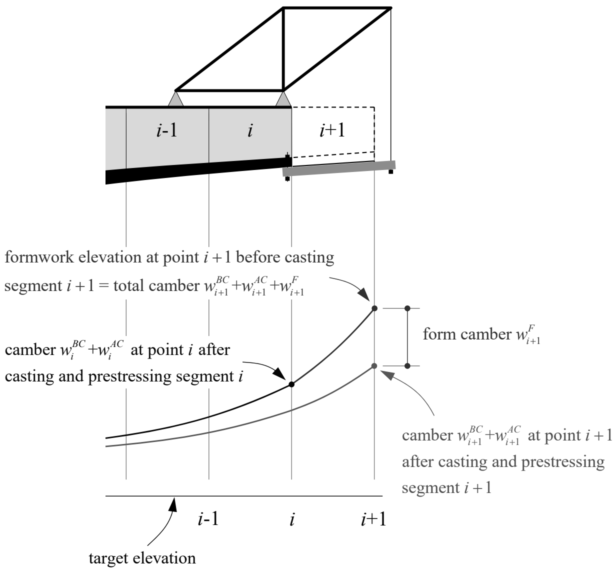
Form camber (cast-in-place girders)

In addition to the camber due to deflections in the cantilever and continuous systems w^{BC} + w^{AC}, form camber w^F needs to be considered when aligning the formwork before casting a segment, see figure. The form camber compensates:

- the deformations of the traveller and formwork under the weight $g_{0,i+1}$ of segment $i+1$
- the deformations of the previously cast cantilever (segments 0… i) under the weight $g_{0,i+1}$ and prestressing P^c_{i+1} of segment i+1

Thereby, after casting segment $i+1$, the desired camber at point $i+1$ is obtained.

Bridge and formwork profile before casting segment i+1



Cantilever-constructed bridges – Design

Camber profile (cast-in-place girders)

The camber profile w^{BC} + w^{AC} can be determined by interpolating between few points; it will schematically look as illustrated (without form camber w^{F}) in the figure.

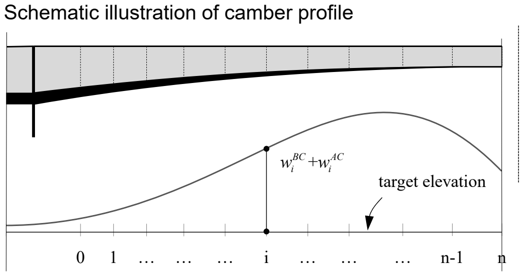
Camber for precast segmental cantilever-constructed girders

Determining camber for precast segmental girders is simpler. Essentially, the following contributions of deflections need to be combined:

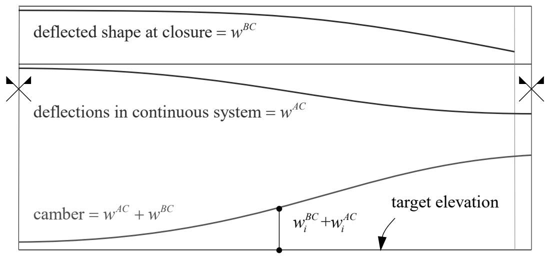
- w^{BC}: deflections of the cantilever system before closure
- w^{AC} : deflections of the continuous system after closure

The total camber w^{BC} + w^{AC} must then be built into each segment at precasting, requiring very precise alignment, particularly of the pier segments.

Schematic illustration of camber profile



Camber for precast segmental construction



Special girder bridges

Cantilever-constructed bridges
Construction

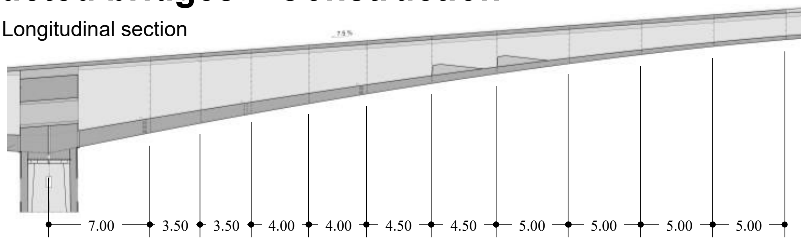
Cantilever-constructed bridges – Construction

Design for efficient construction

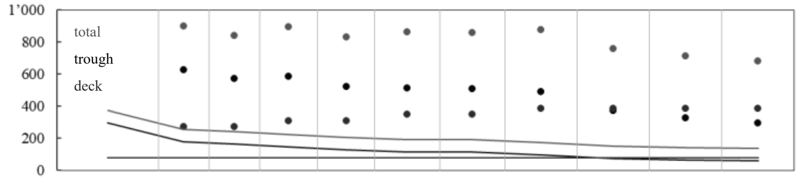
The following aspects should be considered to facilitate an efficient cantilever construction:

- Minimise the length of the pier table (Grundetappe): two travellers must fit
- Select segment length variation to ensure similar load on travellers for all segments (figure, example Inn Bridge Vulpera)
- In case of alternating casting or lifting of segments at the two cantilevers in balanced cantilevering:
  - → check admissible difference in bending moments on pier (higher cost for pier and foundation may be justified by more efficient cantilevering)
  - → shift segment joints by half a segment if required

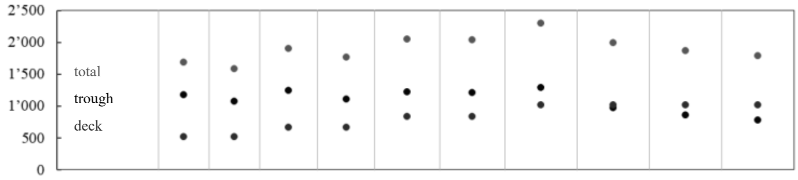
Longitudinal section



Inn Bridge Vulpera, weight/length (lines, [kN/m]) and per segment (dots, [kN])



Inn Bridge Vulpera, Traveller bending moment per segment [kNm]



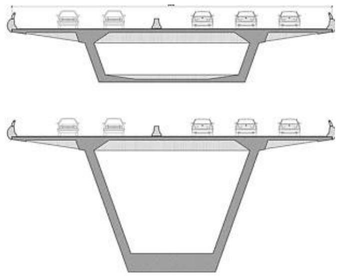
Figures: Inn Bridge Vulpera, dsp Ingenieure + Planer / ACS Partner with Dr. Vollenweider and Eduard Imhof (2010). Main span 104 m.

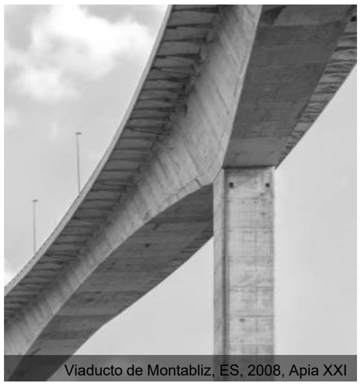
Cantilever-constructed bridges – Construction

Design for efficient construction

(continued)

- Girder geometry should minimise formwork adjustments between segments; this does however not mean that dull rectangular geometries are mandatory
  - → inclined webs combined with variable depth result in attractive soffit geometry





Viaducto de Montabliz, ES, 2008, Apia XXI

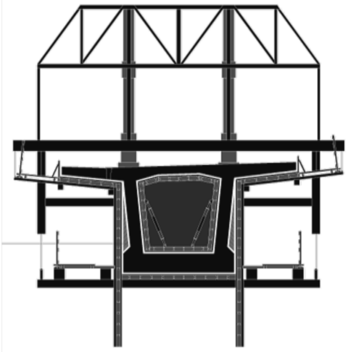
Viaducto de Montabliz, Cantabria, Spain, 2008, Apia XXI Ingenieros, Photo © Ferrovial Agroman

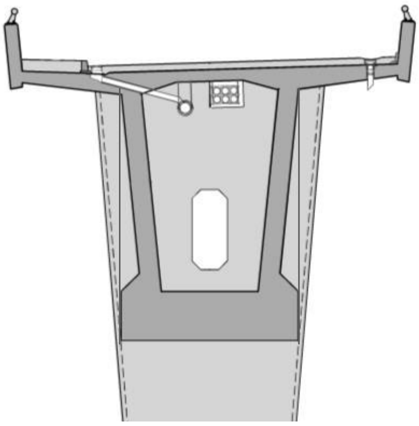
Cantilever-constructed bridges – Construction

Design for efficient construction

(continued)

- Use girder geometry minimising formwork adjustments between segments; this does however not mean that dull rectangular geometries are mandatory
  - → alternative solutions are possible





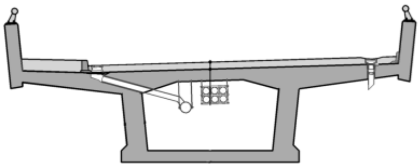


Photo and figures: Inn Bridge Vulpera, dsp Ingenieure + Planer / ACS Partner with Dr. Vollenweider and Eduard Imhof (2010). Main span 104 m.