Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Dr. Severin Haefliger
Prof. Dr. Walter Kaufmann
Institute of Structural Engineering
D-BAUG, Master Civil Engineering
Autumn Semester 2025

Advanced Structural Concrete
Information Sheet: Force Method

(101-0127-00L)

The force method is used to determine the reaction forces and internal actions of statically indeterminate systems. The method consists in (i) releasing constraints until the system is statically determinate (basic system (BS)) and (ii) introducing redundant variables (RV) corresponding to each released constraint.
The RV are subsequently determined by formulating compatibility conditions in the BS: The deformations due to the actions and all RV must vanish at each released constraint.
Compatibility conditions for an *n*-times statically indeterminate system (X_j and δ_i are the RV and the deformations corresponding to the released constraint *i*, respectively):

$$\delta_i = \delta_{i0} + \sum_{j=1}^{n} X_j \cdot \delta_{ij} = 0 \quad \text{for } i = 1..n,$$

$$\delta_{i0} = \int \frac{M_i M_0}{EI} dx + \int \frac{N_i N_0}{EA} dx + .. \text{ and } \delta_{ij} = \int \frac{M_i M_j}{EI} dx + \int \frac{N_i N_j}{EA} dx + ..$$

with δ_{i0} = deformation at *i* due to the external actions and δ_{ij} = deformation at *i* due to RV $X_j = 1$.

Procedure:

1. Determine the degree *n* of statically indeterminacy
2. Select a stable, statically determinate basic system (BS) by releasing *n* constraints and introducing *n* redundant variables (RV) X_i (i = 1..n) at the locations of the released constraints.
3. Determine the deformations δ_{i0} at the position and in the direction of X_i due to the external actions.
4. Determine the deformations δ_{ij} at the position and in the direction of X_i due to RV $X_j = 1$.
5. Set up and solve the compatibility conditions:
$$\delta_i = \delta_{i0} + \sum_{j=1}^{n} X_j \cdot \delta_{ij} = 0 \rightarrow X_j = -\delta_{ij}^{-1} \delta_{i0}$$
6. Determine the reaction forces, internal forces and deformations of the original statically indeterminate system with superposition
(R_0, S_0, w_0 from BS due to external actions and R_j, S_j, w_j from BS due to $X_j = 1$)
For reaction forces: $R = R_0 + \sum_{j=1}^{n} R_j \cdot X_j$
For internal actions: $S = S_0 + \sum_{j=1}^{n} S_j \cdot X_j$
For the deformations: $w = w_0 + \sum_{j=1}^{n} w_j \cdot X_j$

See Figure 1 for an example of the procedure.

Dr. Severin Haefliger
Prof. Dr. Walter Kaufmann
Institute of Structural Engineering
D-BAUG, Master Civil Engineering
Autumn Semester 2025

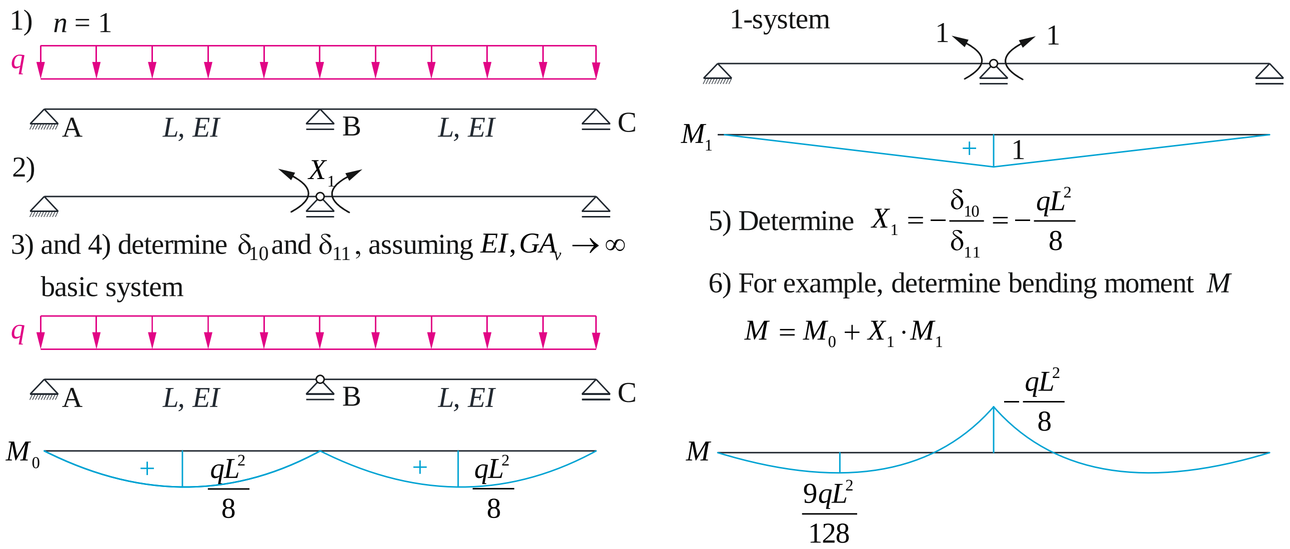

Figure 1: Example of using the force method to determine the bending moment distribution of a two-span beam with a distributed load q.