

Advanced Structural Concrete

Information Sheet: Force Method

(101-0127-00L)

The force method is used to determine the reaction forces and internal actions of statically indeterminate systems. The general approach is to release constraints until the system is statically determinate (basic system (BS)). The released constraints are replaced with redundant variables (RV), which deform (together with the actions) the system and cause compatibility conditions at the removed constraints.

Compatibility conditions for an n -times statically indeterminate system, X_j are RV

$$\delta_i = \delta_{i0} + \sum_{j=1}^n X_j \cdot \delta_{ij} = 0 \quad \text{for } i = 1..n,$$

$$\delta_{i0} = \int \frac{M_i M_0}{EI} dx + \int \frac{N_i N_0}{EA} dx + \dots \quad \text{and} \quad \delta_{ij} = \int \frac{M_i M_j}{EI} dx + \int \frac{N_i N_j}{EA} dx + \dots$$

where δ_{i0} is the deformation at i due to the external actions and δ_{ij} the deformation at i due to RV $X_j = 1$.

Procedure:

1. Determine the degree n of statically indeterminacy
2. Select a stable, statically determinate basic system (BS) by releasing n constraints and introducing n redundant variables (RV) X_i ($i = 1..n$) at the locations of the released constraints.
3. Determine the deformations δ_{i0} at the position and in the direction of X_i due to the external actions
4. Determine the deformations δ_{ij} at the position and in the direction of X_i due to RV $X_j = 1$
5. Set up and solve the compatibility conditions:

$$\delta_i = \delta_{i0} + \sum_{j=1}^n X_j \cdot \delta_{ij} = 0 \rightarrow X_j = -\delta_{ij}^{-1} \delta_{i0}$$

6. Determine the reaction forces, internal forces and deformations of the original statically indeterminate system with superposition

(R_0, S_0, w_0 from BS due to external actions and R_j, S_j, w_j from BS due to $X_j = 1$)

For reaction forces: $R = R_0 + \sum_{j=1}^n R_j \cdot X_j$

For internal actions: $S = S_0 + \sum_{j=1}^n S_j \cdot X_j$

For the deformations: $w = w_0 + \sum_{j=1}^n w_j \cdot X_j$

See Figure 1 for an example of the procedure.

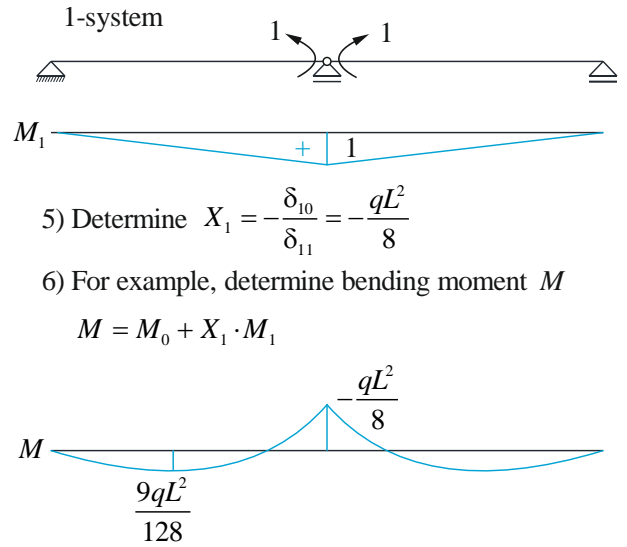
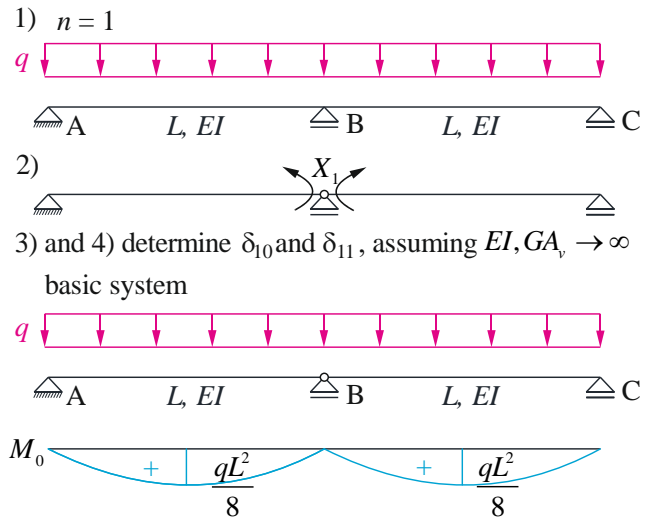


Figure 1: Example of using the force method to determine the bending moment distribution of a two-span beam with a distributed load q .