# 3 Steel Fibre Reinforced Concrete (SFRC)

## Fundamentals

# Learning Objectives

Within this chapter, the students are able to:

- understand and quantify the orientation effect of fibres.
- assess the load-deformation behaviour of fibre and hybrid reinforced concrete
  - elaborate on the tensile behaviour,
  - generalise the tensile behaviour to bending and shear.
- recognize the benefits and drawbacks of fibre reinforced concrete in terms of strength and ductility and judge its suitability for different applications.

# Steel Fibre Reinforced Concrete – Fundamentals

## Content

- Relevance of SFRC and current applications
- Mechanical behaviour of a single fibre in cement matrix
  - Fibre types and properties
  - Bond
  - Fibre activation and pull-out
  - Fibre stress – crack opening relationship
- Fibre content and orientation in 2D and 3D
- Mechanical behaviour of SFRC
  - Tension
  - Bending
  - Compression
  - Shear
  - Hybrid reinforcement (SFRC and conventional reinforcing bars)
- Utra High Performance Fibre Reinforced Concrete (UHPFRC)

# Relevance of SFRC and current applications

**Historical background**

- First trials to replace conventional reinforcement with fibres date back to the 1960s
- Further research led to a wider application in practice, e.g. shotcrete in tunnel linings
- Other materials (PVA, glass fibres) lead to similar behaviour, but are not treated here

- The addition of fibres enhances the structural performance of plain concrete (much higher fracture energy, not "ductility"!)
- Fibres reduce the crack spacing and crack width, thereby improving serviceability and durability
- Currently used SFRC mixes exhibit a softening behaviour in tension and cannot fully replace conventional reinforcement
- Hybrid reinforcement (fibres and conventional reinforcing bars) can be used but may affect ductility

- Several causes are preventing more widespread use of SFRC:

… Lack of standardised design procedures and material test procedures

… High fibre contents (e.g. 1.5% = 120 kg/m$^3$) as required for structural applications (and used in many experiments) are causing severe problems in terms of mixing and workability of concrete mix

… With common fibre contents (up to 0.5% = 40 kg/m$^3$), the tensile strength of concrete cannot be matched at cracking $\rightarrow$ softening behaviour

# Relevance of SFRC and current applications

**Common fields of application**

- Industrial floors
- Shotcrete linings
- Foundation slabs
- Hydraulic structures
- Bridge decks
- Explosion-resistant structures
- Façade elements

For general application in engineering practice (structures), it is necessary to include conventional reinforcement in combination with SFRC to ensure structural safety and adequate crack distribution.

The addition of steel fibres leads to a reduction of crack spacing and therefore, smaller crack widths.

Experimental investigations show that the influence of steel fibres disappears for highly reinforced concrete elements.

[ Source: Guideline for execution of steel fibre reinforced SCC
Danish Technological Intitute – SFRC Consortioum, 2013 ]

# Relevance of SFRC and current applications

**Examples (selection)**

Slab on grade

[ Source: concretefibersolutions.com ]

Shotcrete for tunnel lining

[ Source: bekaert.com ]

Thin shell structures

(with conventional reinforcement)

[ Source: ciduadfcc.com ]

# Mechanical behaviour of a single fibre in cement matrix

**Types of fibres**

Hooked ends

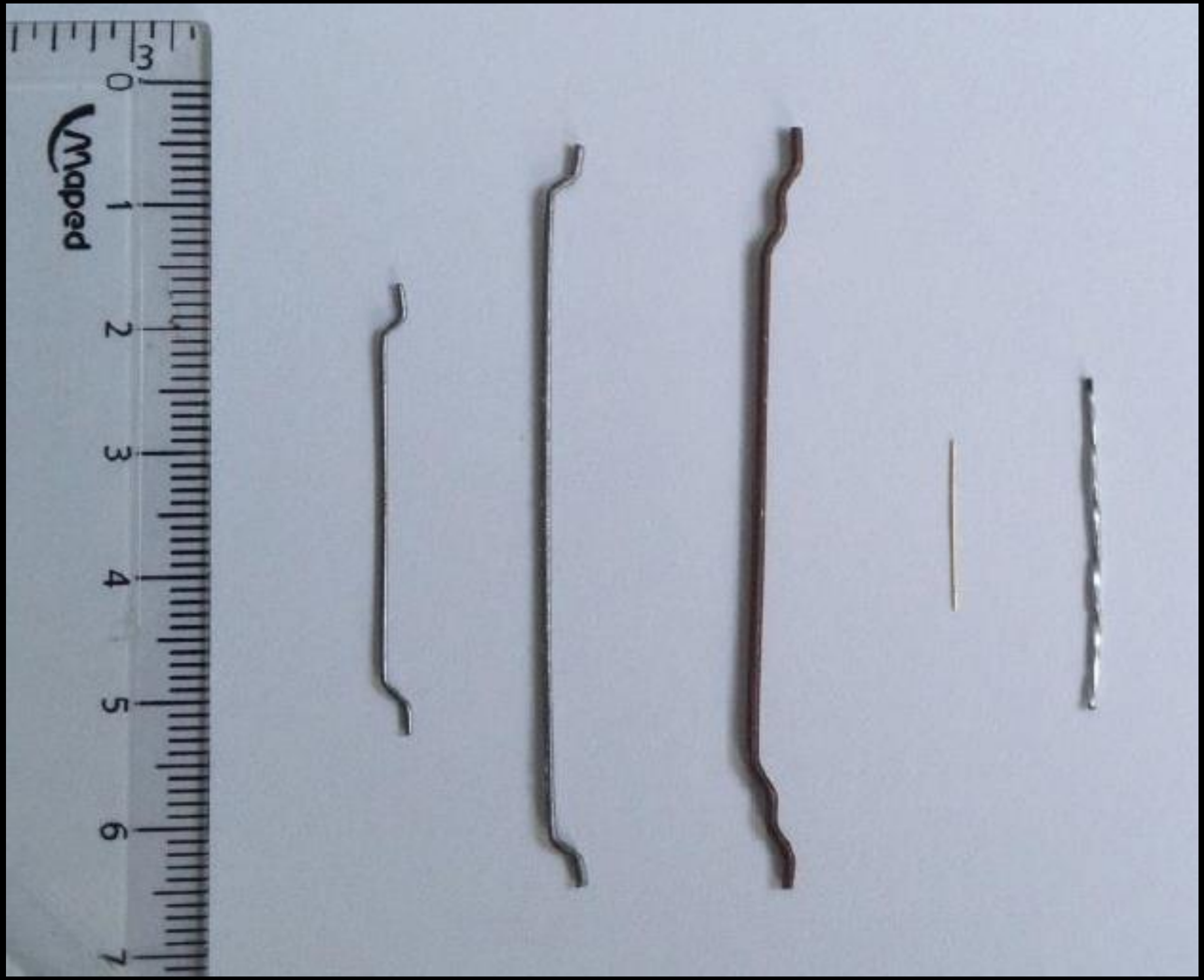
Hooked-end fibres are standard in most applications today. Other fibre types, as shown below, are also being used, or were used historically:

Crimped

Stranded (coned end)

Straight

Twisted



[ Source: Amin, 2015 ]

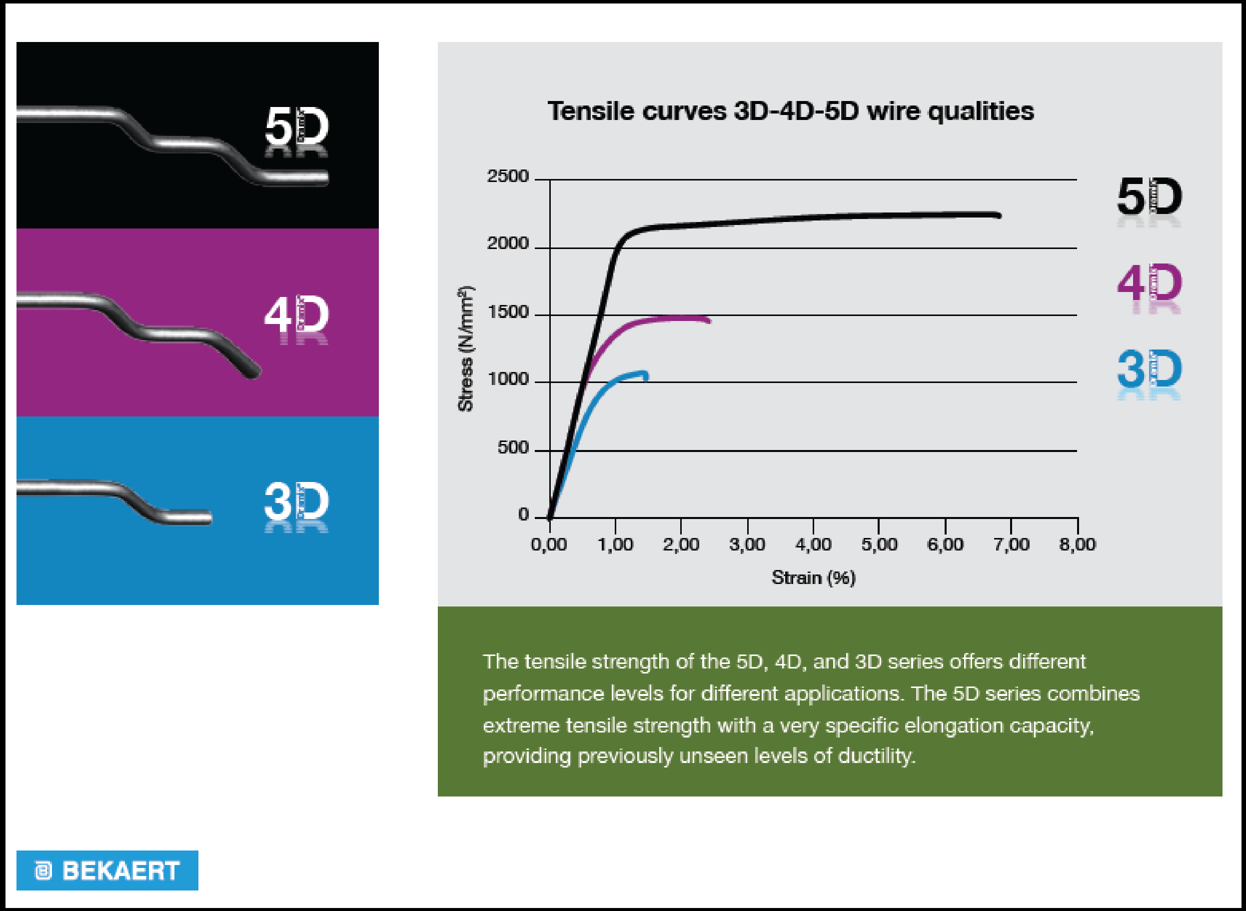
# Mechanical behaviour of single fibre in cement matrix

**Material properties of modern steel fibres**

- Steel wire with high tensile strength (usually >1,000 MPa, some >2,000 MPa)
- Typically bare (uncoated steel) or galvanized
- Typical length $l_f \approx 30 \ldots 60$ mm
- Typical slenderness (aspect ratio) $l_f / d_f \approx 55 \ldots 80$
- Usually rather low ductility of the steel (except 5D fibre)

- Fibre designation: slenderness / length:

  «80 / 60» $\rightarrow l_f / d_f = 80,\ l_f = 60$ mm
  (i.e. $d_f = 60/80 = 0.75$ mm)

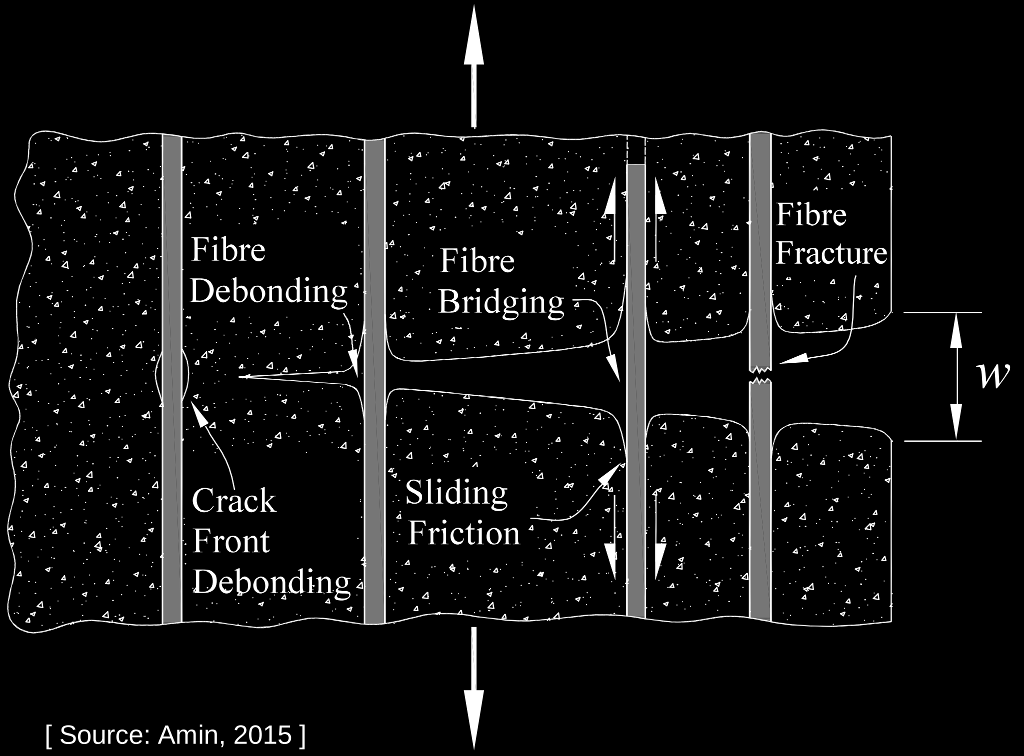



[ Source: bekaert.com ]

# Mechanical behaviour of single fibre in cement matrix

**Fibre-matrix failure mechanisms**

- Typically, fibres are not fully activated, i.e. they are pulled out of the cement matrix before the fibre breaks.
- Unless long fibres with high ductility (e.g. Dramix 5D) are used, fibre pullout is desirable since fibre fracture would lead to a very low ductility.
- The pull-out of the fibres is softening, i.e. load decreases with increasing crack opening since the bonded length is reduced in proportion with the crack opening.

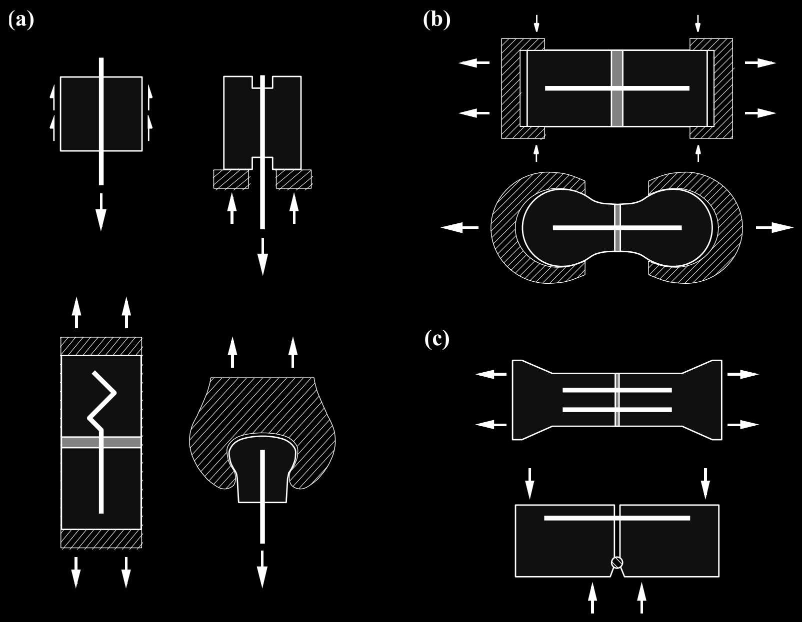



[ Source: Amin, 2015 ]

# Mechanical behaviour of single fibre in cement matrix

**Bond-slip relationship and pull-out behaviour**

- Bond is caused by adhesion, friction, and end hooks (fibre deformation)
- The anchorage effect of hooked-ends is typically considered as contribution to bond (higher nominal bond stresses)
- Usual assumption: Constant bond shear stresses over fibre length, rigid-plastic bond shear stress-slip relationship
- Differential equation for bond shear stress - slip relationship assuming linear elastic behaviour of fibre and matrix




Faserausziehversuche – schematische Versuchsanordnungen nach Bartos [16] und Gray [39]: (a) Einzelfasern mit einseitigem Verbund; (b) Einzelfasern mit beidseitigem Verbund; (c) Fasergruppen mit beidseitigem Verbund.

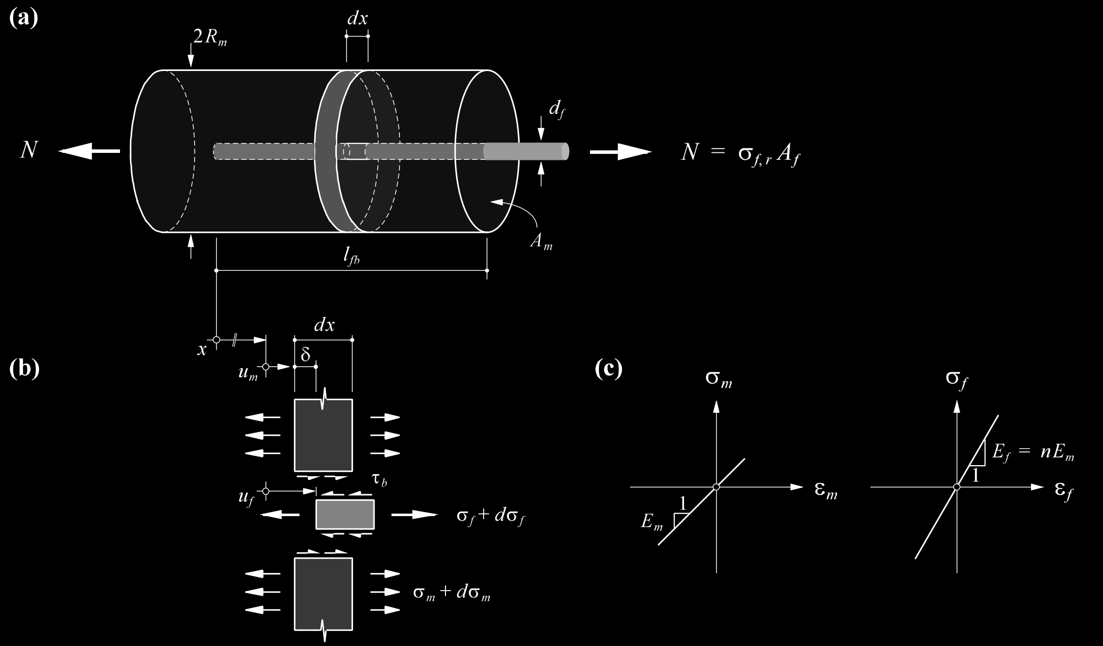


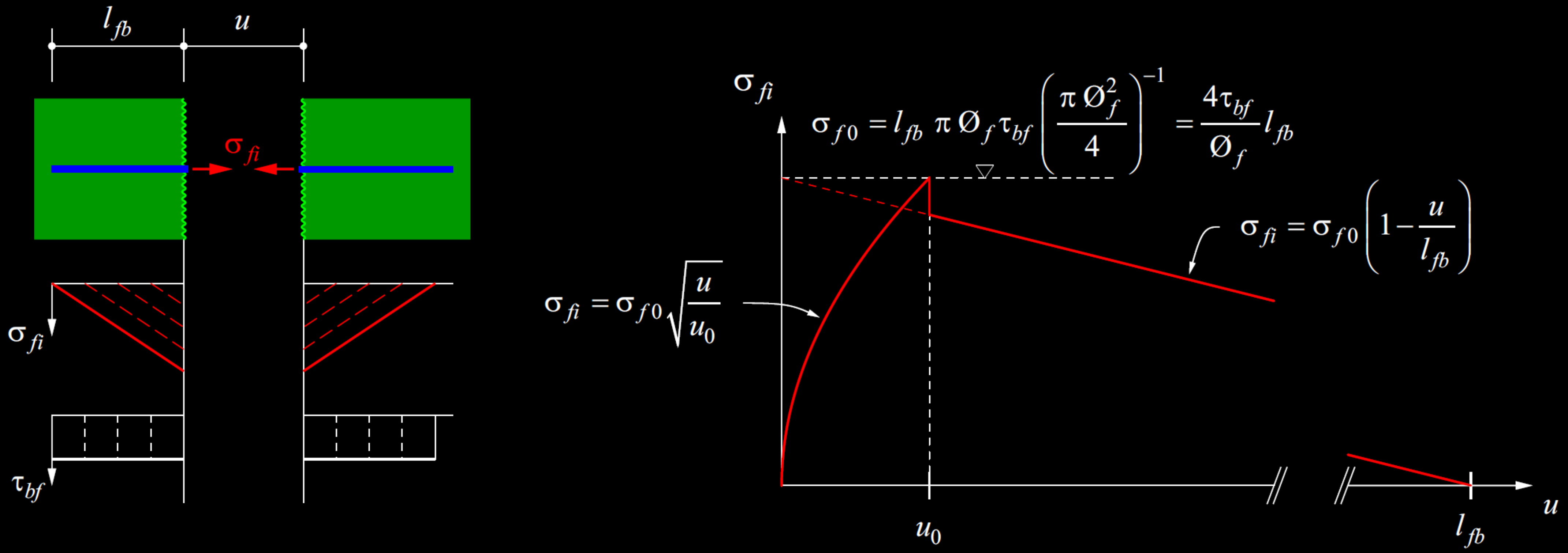

**Bild 2.3 –** Faserausziehversuch: (a) Prinzipskizze; (b) Verschiebungen und Spannungen am differentiellen Element; (c) Spannungs-Dehnungsbeziehungen.

[ Source: Pfyl, 2003 ]

# Mechanical behaviour of single fibre in cement matrix

**Marti and Pfyl's simplified model for fibre activation and pull-out**

- Rigid bond shear stress-slip relationship between fibre and matrix over embedment length $l_{fb}$
- Once the bond shear stresses are fully activated, the fibre is pulled out of the matrix (on the shorter embedded side)
- Simplification: Only the slip contributes to the crack width
- Linear softening due to decreasing bond length of fibre

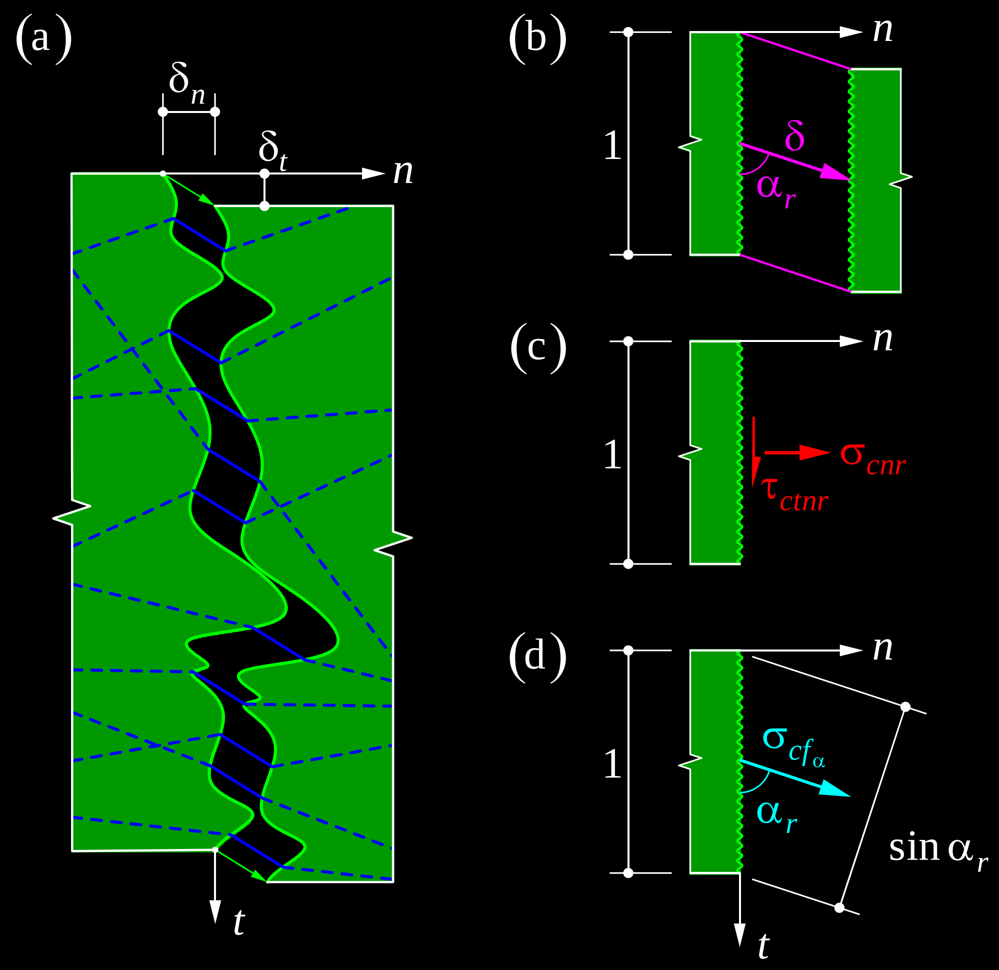


# Fibre content and orientation factor

**Cement matrix with randomly distributed fibres**

- The fibre content of SFRC is measured by the weight of the fibres per volume of the concrete mix [kg/m³] or the fibre volume fraction $V_f$ (78.5 kg/m³ $\leftrightarrow$ $V_f$ = 1%)
- Higher fibre dosages lead to difficulties in the workability and applicability of the concrete mix.
- In the mixing process, fibres theoretically distribute equally and with random directions in the cement matrix.
- However, due to the casting process, fibres are usually unevenly distributed and oriented in practice
- Fibres are inclined to the crack face at arbitrary angles
- Fibre stresses at cracks are assumed to be aligned with the direction of the crack face displacement ($EI_f \rightarrow 0$)

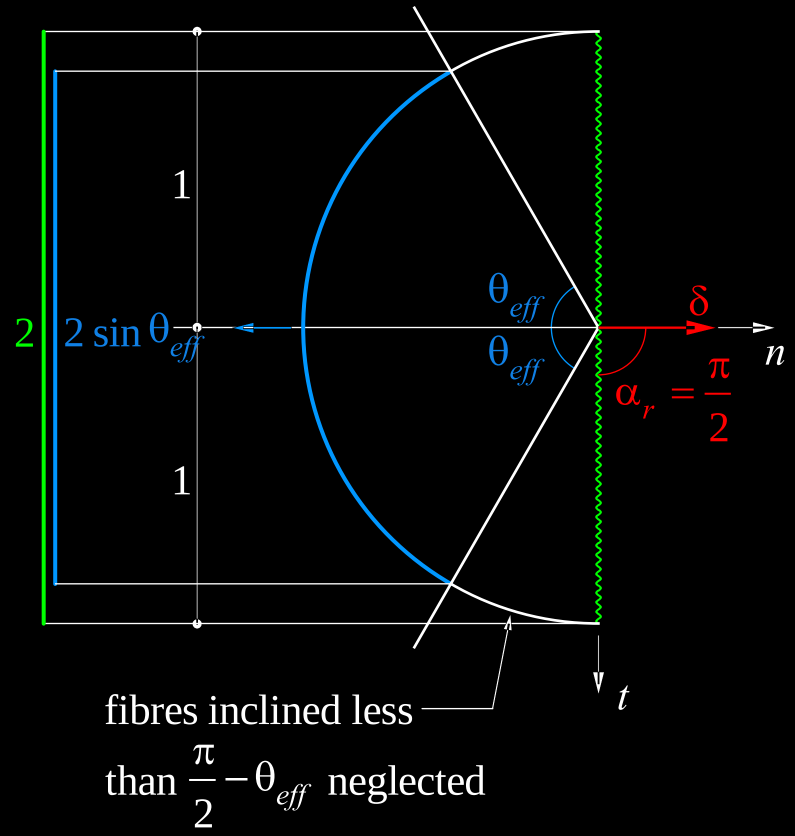
| Typical fibre contents [ kg / m³ ] | |
|---|---|
| < 20 | uneconomic, ineffective |
| 20-50 | Most commonly used fibre content |
| 50-100 | Highly fibre reinforced, expensive |
| > 100 | Problematic due to limited workability |

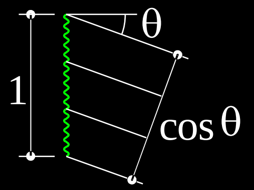


# Fibre content and orientation factor

**Fibre orientation factor in 2D (thin elements)**

- All fibres randomly orientated in 2D-plane. All directions have equal probability of occurrence.
- Fibres with very low inclination to the normal plane are assumed to be ineffective (also in 3D, next slide)
- Number of fibres crossing the crack per unit length (effective fibres) = cos$\theta$ → projection of fibre end loci on crack (also in 3D)




Semi-circle = loci of fibre ends with equal probability: length $\pi$ (for crack length with $r$ = 1)

→ Fibre orientation factor = length of sector, projected on crack (or equivalent integral), divided by length of semi-circle:

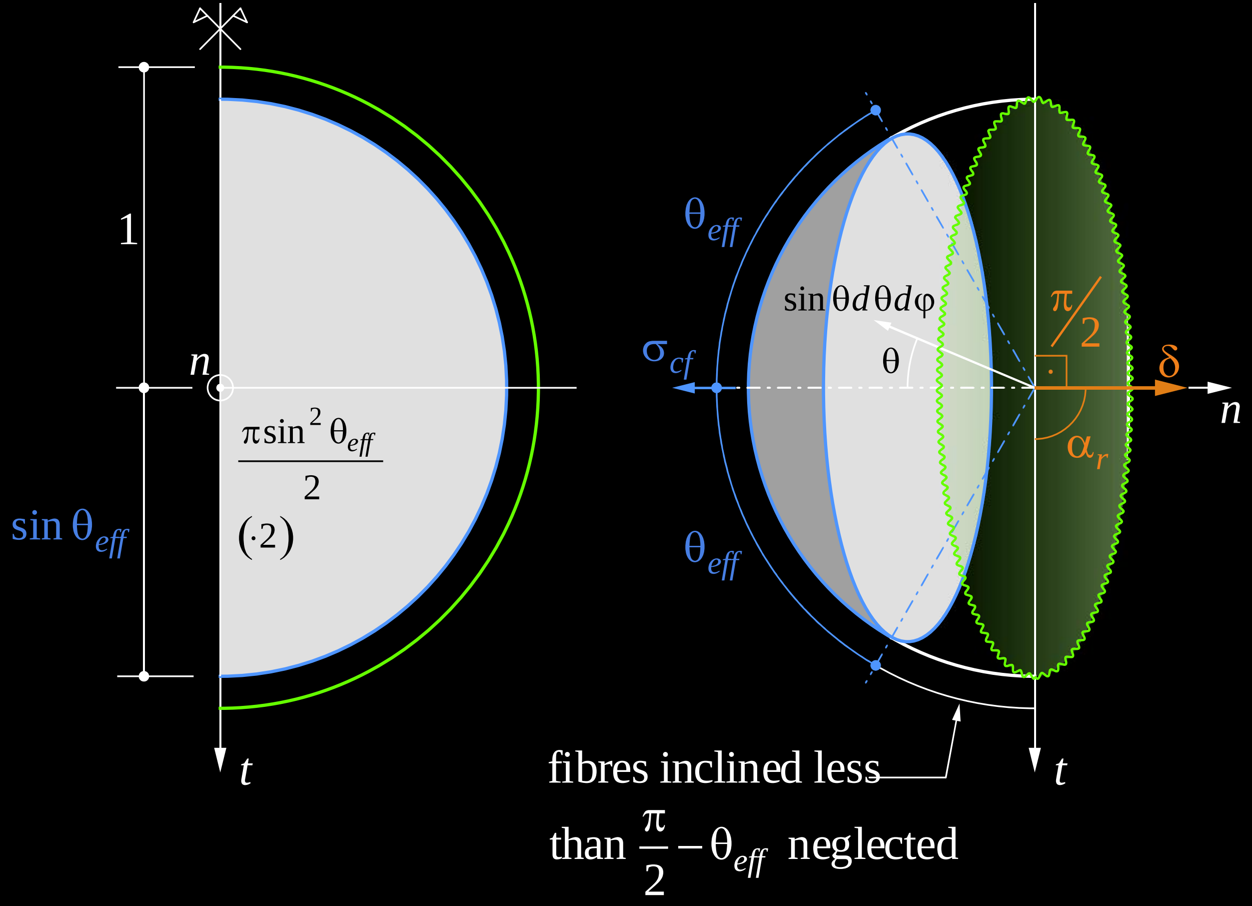
$$K_f = \frac{1}{\pi}\int_{-\theta_{eff}}^{\theta_{eff}} \cos\theta \, d\theta = \frac{2\sin\theta_{eff}}{\pi}$$

$$\theta_{eff} = \frac{\pi}{2}: \quad K_f = \frac{2}{\pi} \qquad \theta_{eff} = \frac{\pi}{3}: \quad K_f = \frac{\sqrt{3}}{\pi}$$

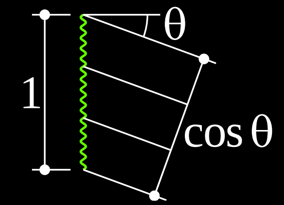
# Fibre content and orientation factor

## Fibre orientation factor in 3D

- Fibres randomly orientated in 3D-plane. All directions have equal probability of occurrence.
- Consideration of semi-sphere and projection on crack plane




- Semi-sphere = loci of fibre ends with equal probability, $A = 2\pi$ (for crack surface with $r = 1$)
- Number of fibres with inclination $\theta$ crossing crack plane




→ Fibre orientation factor = surface of spherical sector, projected to crack plane $n$ (or equivalent integral), divided by surface of semi-sphere:

$$K_f = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\theta_{eff}} \cos\theta \sin\theta \, d\theta \, d\varphi = \frac{\sin^2\theta_{eff}}{2}$$

$$\theta_{eff} = \frac{\pi}{2}: \quad K_f = \frac{1}{2}; \qquad \theta_{eff} = \frac{\pi}{3} = 60°: \quad K_f = \frac{3}{8}$$

# Mechanical behaviour of SFRC

**SFRC members in tension**

- Pre-cracking behaviour is not (marginally) influenced by fibres, stiffness of matrix is governing
- After cracking, the fibres transfer stresses across the cracks.
- Tensile stresses after cracking → superposition of fibres and matrix (note: the softening of plain concrete in tension is much more pronounced than the pull-out of the fibres → matrix only relevant initially, at very small crack openings)

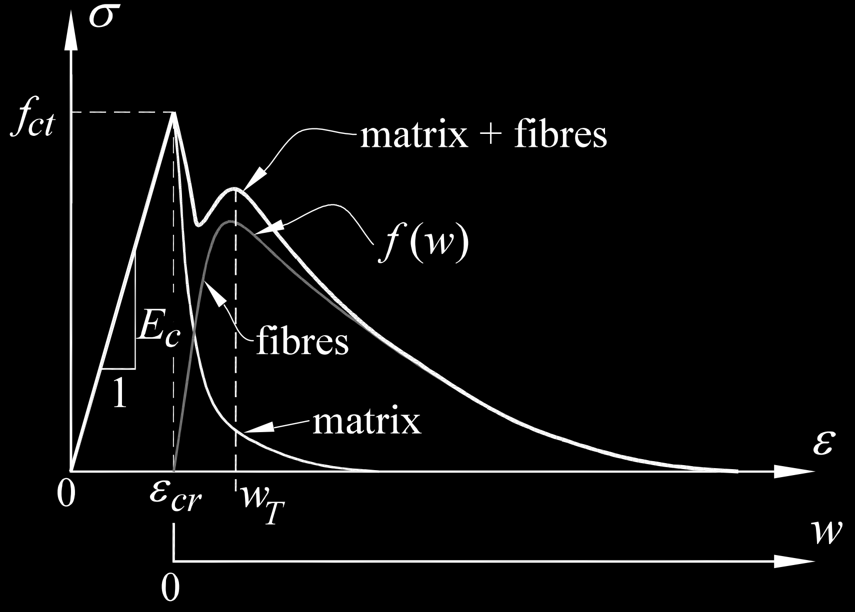



Figure 3.1. Stress versus crack COD ($w$) for SFRC.

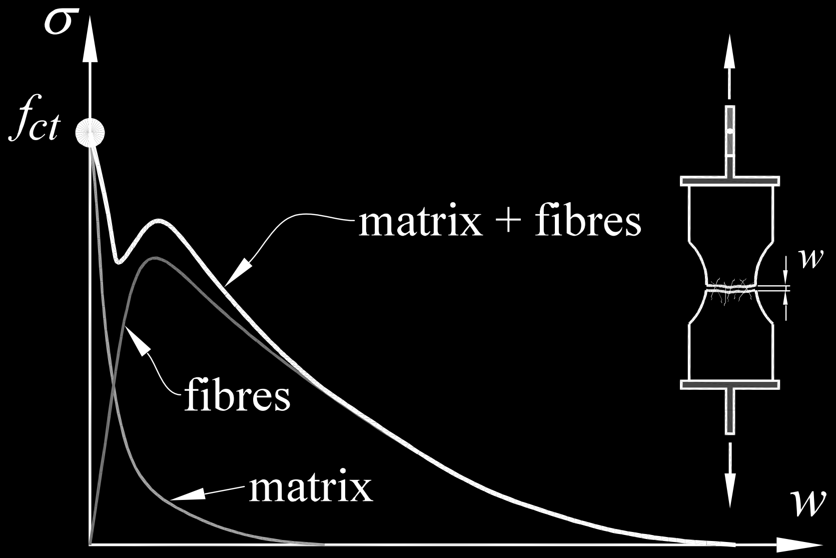



Figure 2.1. Stress versus Crack Opening Displacement (COD), $w$ for SFRC.

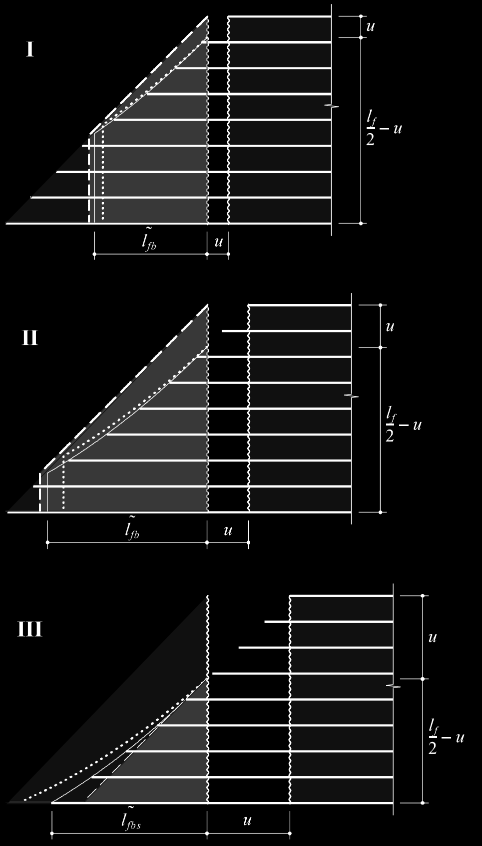
[ Source: Amin, 2015 ]

# Mechanical behaviour of SFRC

**Marti and Pfyl's simplified model for fibre activation and pull-out in tension → «fibre effectiveness» $\sigma_{cf0}$**

- Simplified but sufficiently accurate assumptions for activation and pull-out
- Slip is neglected until all fibres in the cross section are fully activated (→ elliptic curve)
- After full activation of the fibres, only pull-out contributes to crack opening (→ hyperbolic)

Note: Unlike the fibre stress $\sigma_f$, $\sigma_{cf}$ and $\sigma_{cf0}$ are referred to the concrete surface (→vol. fibre content $\rho_f$, fibre orientation factor)




$\rho_f$ : fibre content (volume)

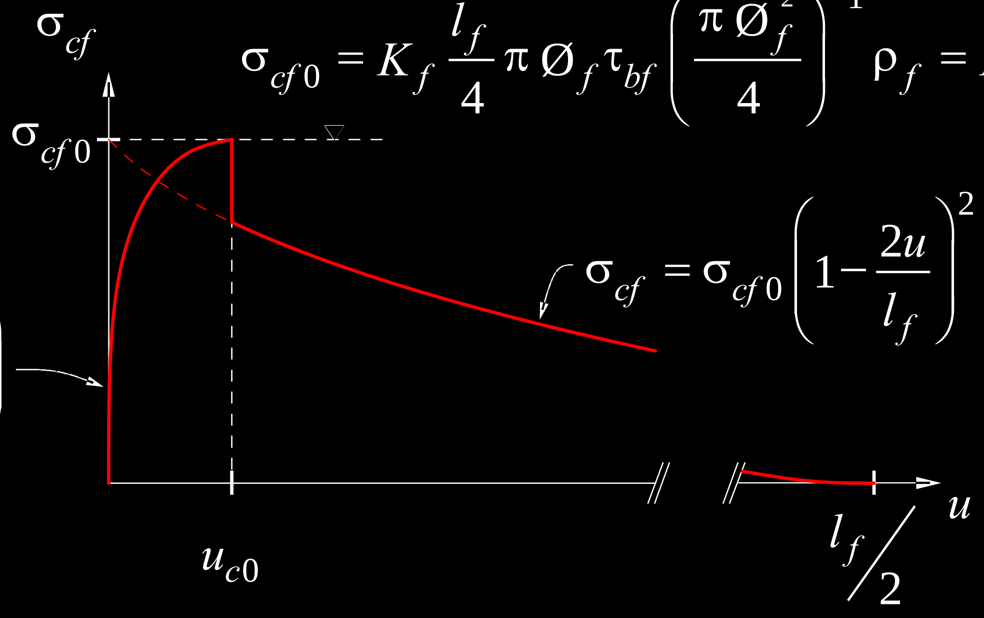
$K_f$ : fibre orientation factor

$$\text{av. embedment: } \left.\begin{matrix}\max & \frac{l_f}{2} \\ \min & 0\end{matrix}\right\} \rightarrow \frac{l_f}{4}$$

$$\sigma_{cf0} = K_f \frac{l_f}{4} \pi \varnothing_f \tau_{bf} \left(\frac{\pi \varnothing_f^2}{4}\right)^{-1} \rho_f = K_f \rho_f \frac{\tau_{bf}}{\varnothing_f} \cdot l_f$$

$$\sigma_{cf} = \sigma_{cf0} \left(2\sqrt{\frac{u}{u_{c0}}} - \frac{u}{u_{c0}}\right)$$

$$\sigma_{cf} = \sigma_{cf0} \left(1 - \frac{2u}{l_f}\right)^2$$




[ Source: Pfyl, 2003 ]

# Mechanical behaviour of SFRC

**Strain softening and damage localization in SFRC**

- The softening behaviour of fibres being pulled out of the cement matrix results in the concentration of deformations = localisation in one single crack after exceeding the cracking load.
- Depending on the amount of fibres (very high dosages) and the fibre activation mechanism, tension chords under uniaxial loading can also show a hardening post-cracking behaviour, with multiple cracks before reaching the peak load where localisation starts (typical in some ultra-high performance fibre reinforced concrete mixes).

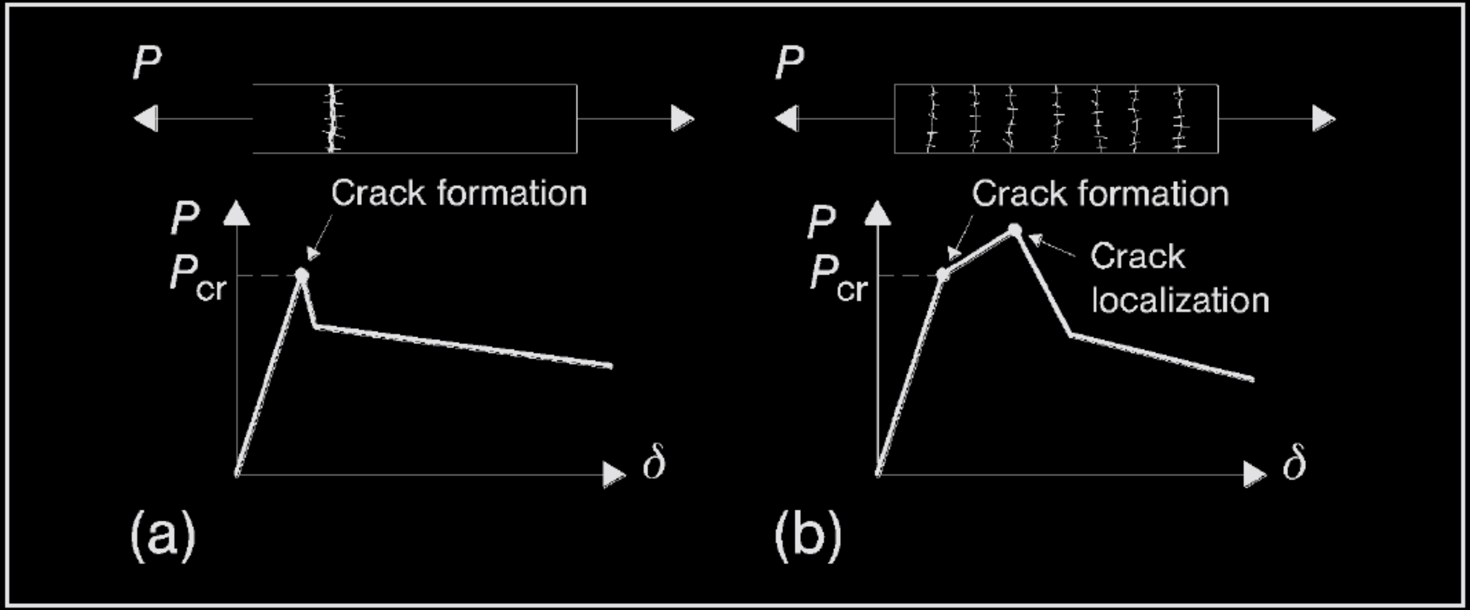


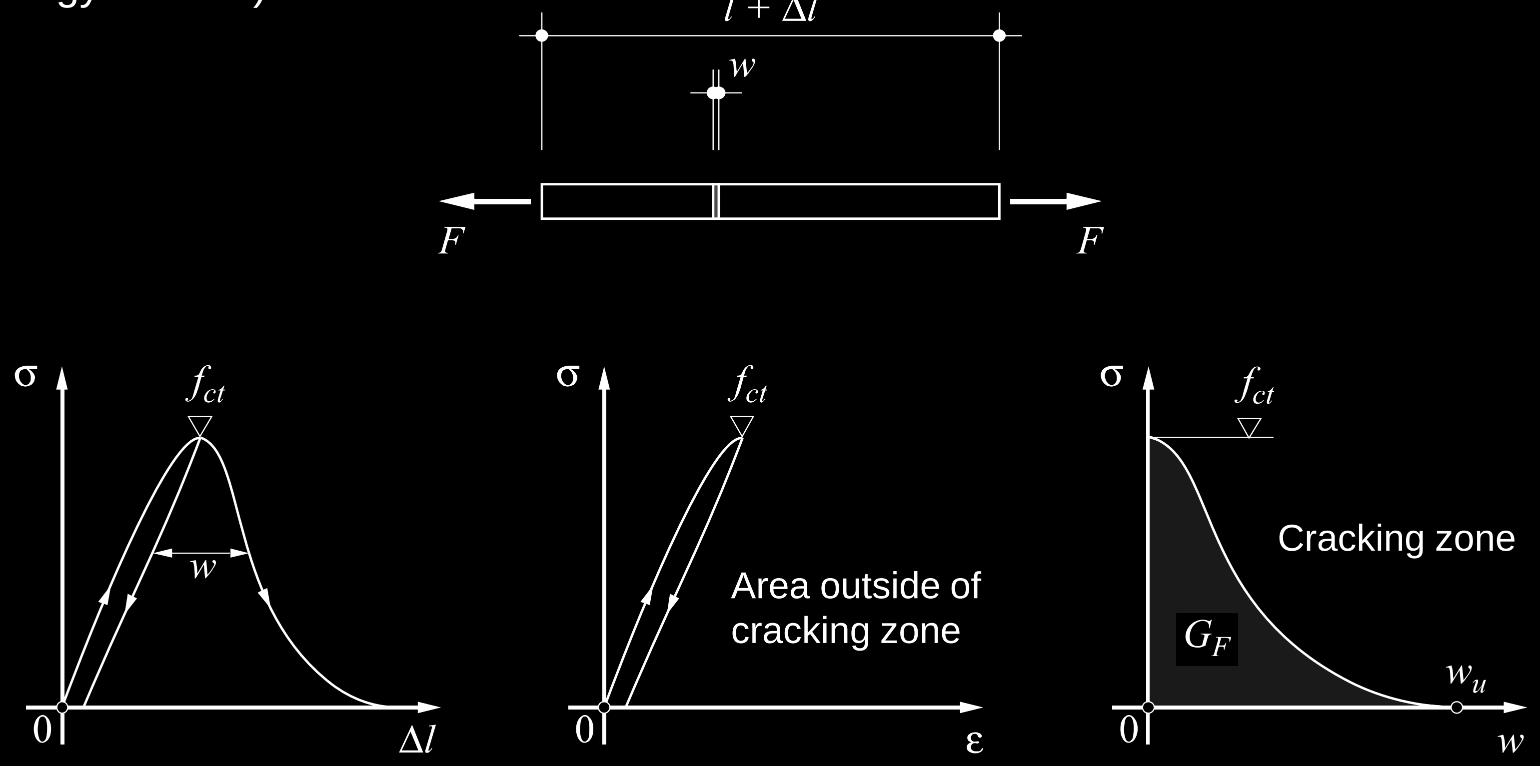

**Figure 5.6-2 :** Softening (a) and hardening (b) behaviour in axial tension

[ Source: fib Model Code, 2010 ]

# Mechanical behaviour of SFRC

**Mechanical model for softening behaviour / strain localisation: Fictitious crack model (Hillerborg)**

1. Linear elastic σ-ε-relationship in elastic phase up to limiting strain $\varepsilon_l$
2. With increasing deformation, a fracture zone develops and the stress σ decreases (cracking zone: σ-*w* relationship = energy dissipation)
3. Any additional elongation is concentrated in the fracture zone (= localisation); stress and strain decrease in adjacent unloading parts (energy release)

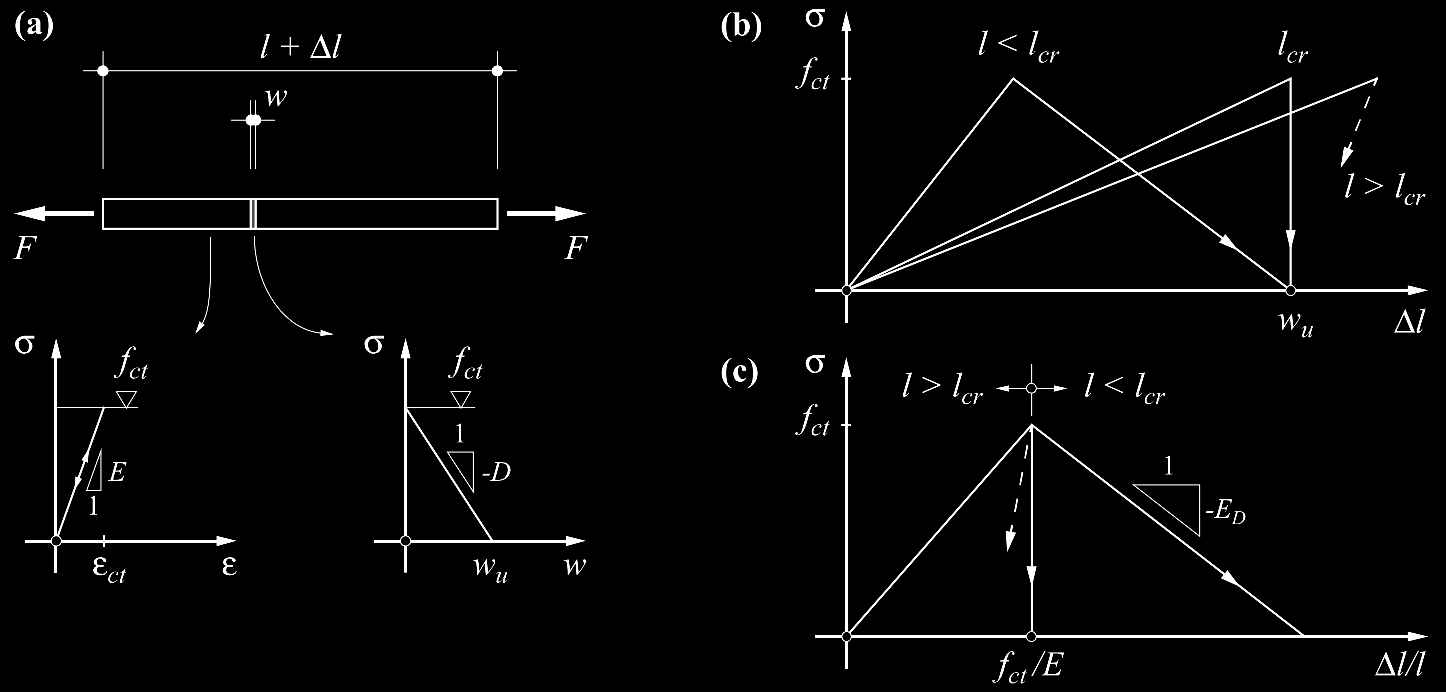



[ Source: Sigrist, 1995 ]

# Mechanical behaviour of SFRC

**Mechanical model for softening behaviour / strain localisation: Fictitious crack model (Hillerborg)**

- Hillerborg's fictitious crack model can be used to analyse materials with strain-softening behaviour such as SFRC
- It provides a direct explanation of the size effect observed in experiments: Fracture energy $G_f$ is considered constant, but elastic energy in unloading parts, released at fracture, increases with specimen size
  $\rightarrow$ the unloading branch can be observed in short specimens ($l < l_{cr}$) in deformation controlled tests
  $\rightarrow$ long specimens ($l > l_{cr}$) fail in a brittle manner at the peak load even in deformation control
- Alternatively, smeared «crack band» models may be used (assumed crack band width $\rightarrow$ mesh dependency in FE analyses)




[ Source: Sigrist, 1995 ]

# Mechanical behaviour of SFRC

**Strain softening and deformation hardening**

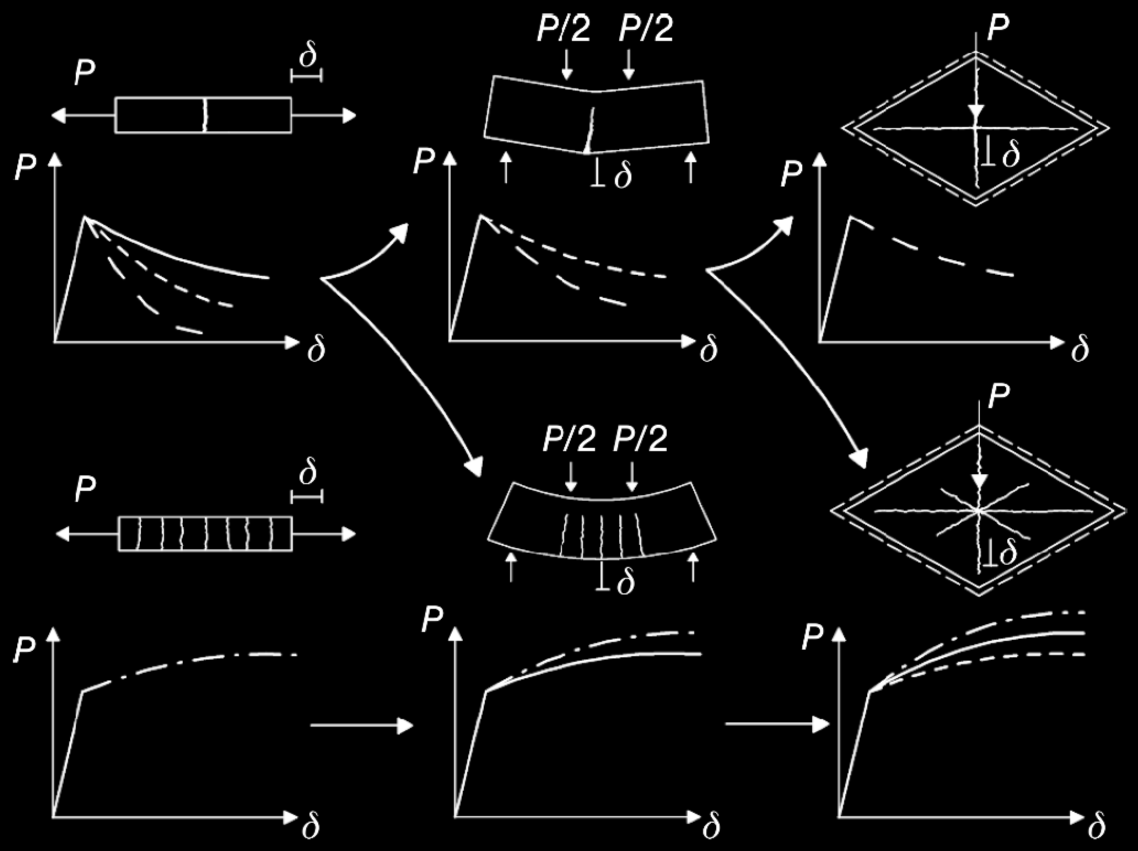
- Structures can have different responses under different loading conditions (depending on size and structural configuration)
- Even if a softening response is observed in tension, using the same SFRC mix strain hardening may be achieved in bending (particularly if biaxial load transfer is possible)

Note:

Other than in most laboratory tests, real structures are not loaded displacement-controlled, i.e., the load will not drop if the structure «softens». Hence, isostatic «softening SFRC» structures WILL COLLAPSE at cracking.

In such cases, the length of the softening branch (often erroneously called «ductility») essentially does not matter – the failure is brittle.

However, if alternate load paths are possible, i.e. in hyperstatic structures (internally or externally), softening structural elements (with long softening branch) may significantly contribute to the load carrying mechanism when softening.




[ Source: fib Model Code, 2010 ]

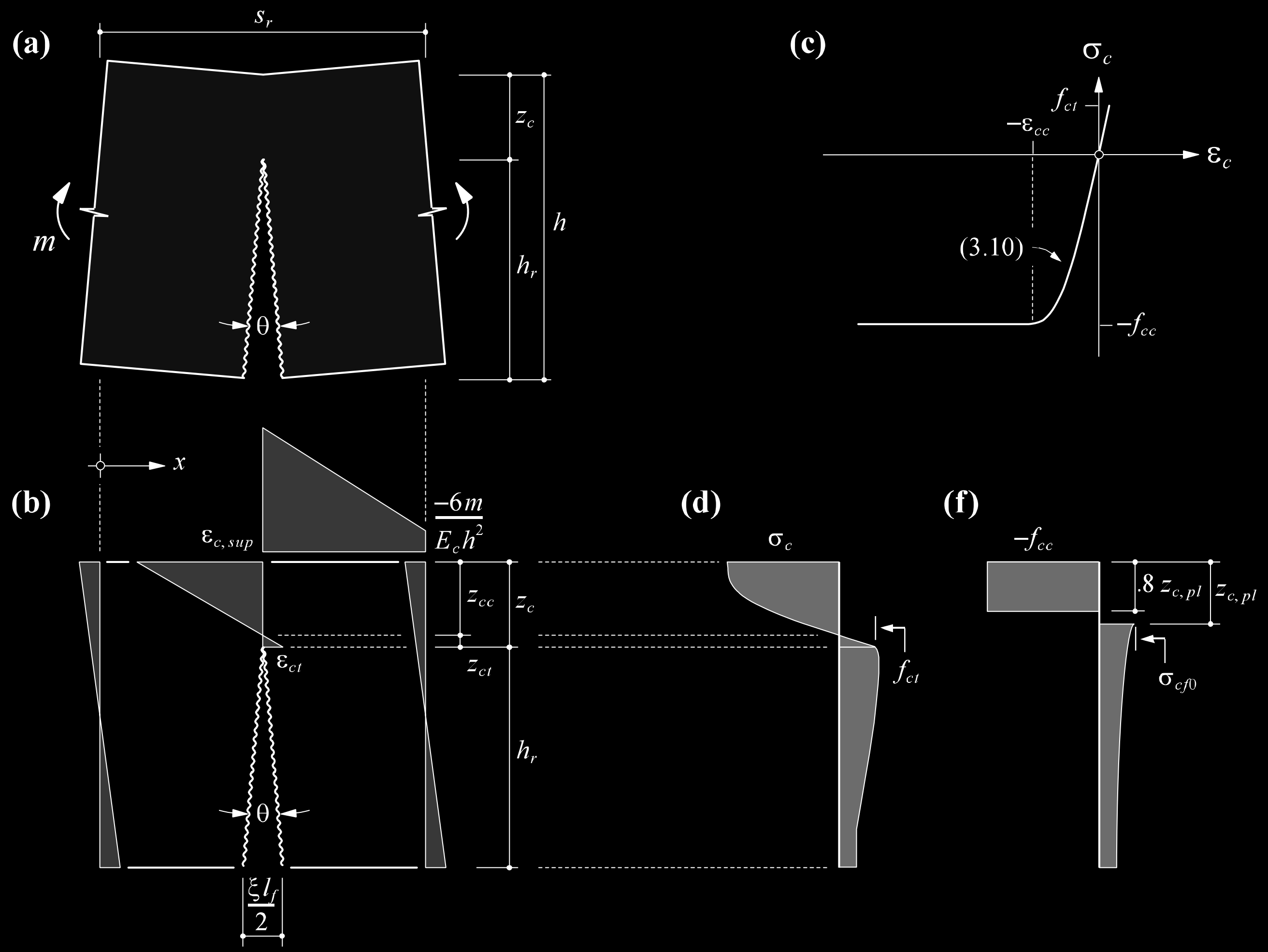
# Mechanical behaviour of SFRC

## SFRC members in bending

- After cracking, the stress distribution in the cracked section depends on the crack opening
- It is assumed that the crack opening varies linearly over the cracked depth (rotation $\theta$)
- A linear strain distribution is assumed
  … in the uncracked cross sections
  (at distances $\pm s_r/2$ from crack)
  … in the uncracked part of the cracked section
  … along the compression face
- The value of $s_r$ (crack element length / "characteristic length") varies strongly in experiments. It can be estimated as $s_r \approx d$.
- Crack opening parameter $\xi$ ($\xi = 1$: all fibres at bottom of cross-section pulled out):

$$\xi = \frac{2 \cdot \theta \cdot (h - z_c)}{l_f}$$

(at crack opening $l_f/2 \rightarrow$ hardly ever achieved)

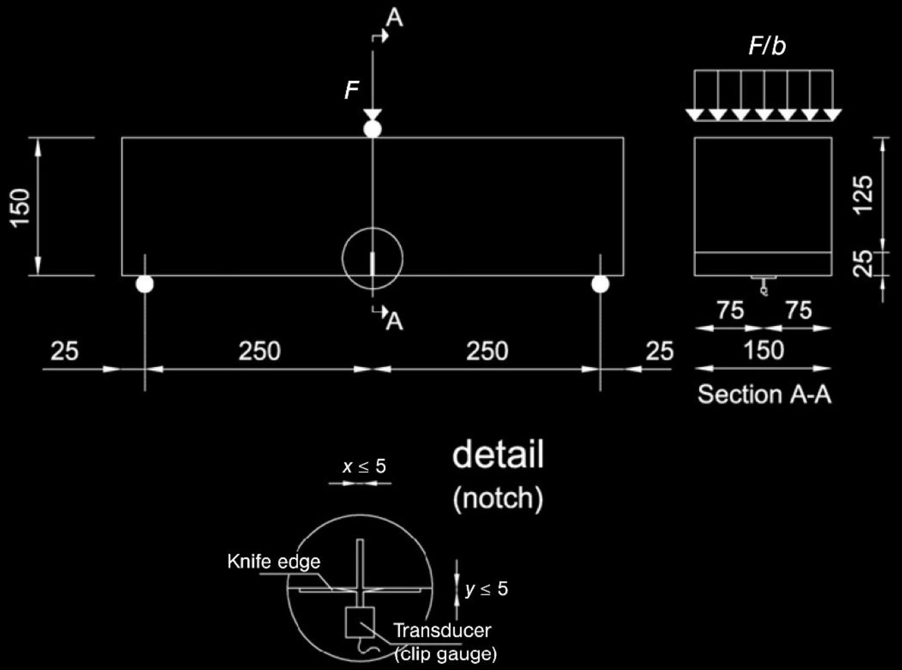



[ Source: Pfyl, 2003 ]

# Mechanical behaviour of SFRC

**SFRC members in bending**

- The fib Model Code [3] proposes 3- or 4-point-bending-tests for the inverse analysis of the fibre stress - pull-out behaviour.
- A notch in the prism pre-determines the location of the crack and simplifies the measurement of the crack width.
- Modern measurement technologies – e.g. digital image correlation – allow the measurement of the crack kinematics for continuous SFRC beams. This is especially useful for members with deformation hardening, where multiple cracks occur.



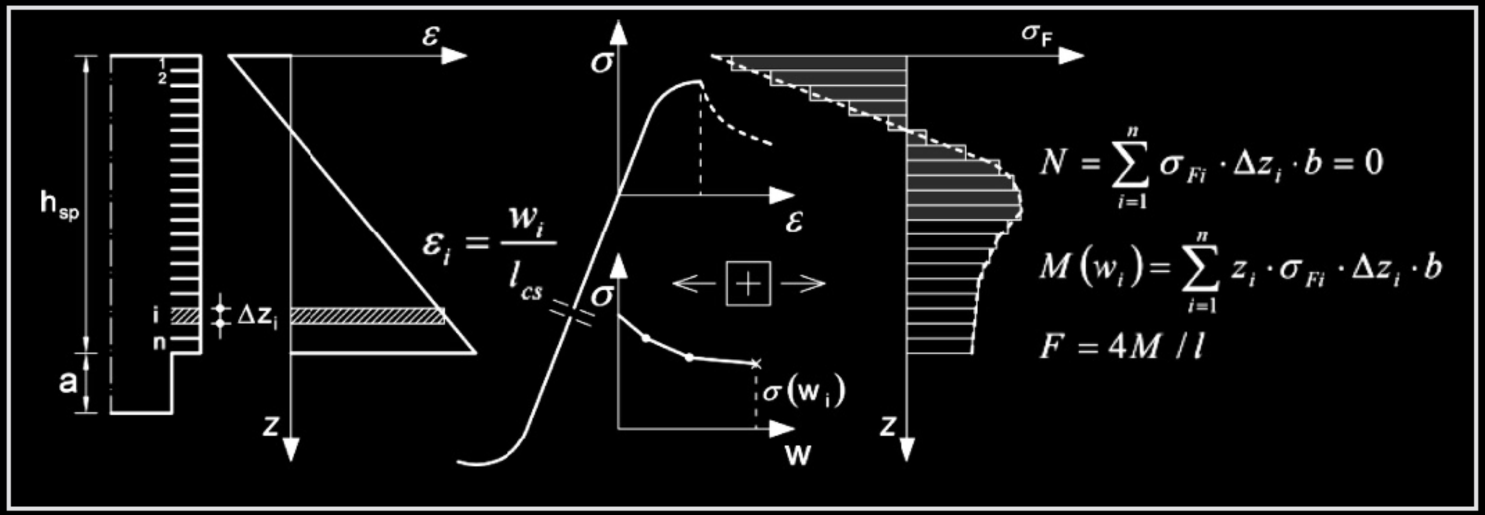




**Figure 5.6-4:** Inverse analysis of beam in bending performed to obtain stress-crack opening relation

[ Source: fib Model Code, 2010 ]

# Mechanical behaviour of SFRC

**SFRC members in compression**

- Steel fibres do not significantly affect the compressive strength
- Ductility is improved in post-peak behaviour
- Fibres prevent “explosive” failure and excessive spalling (may be useful / relevant in high strength concrete)

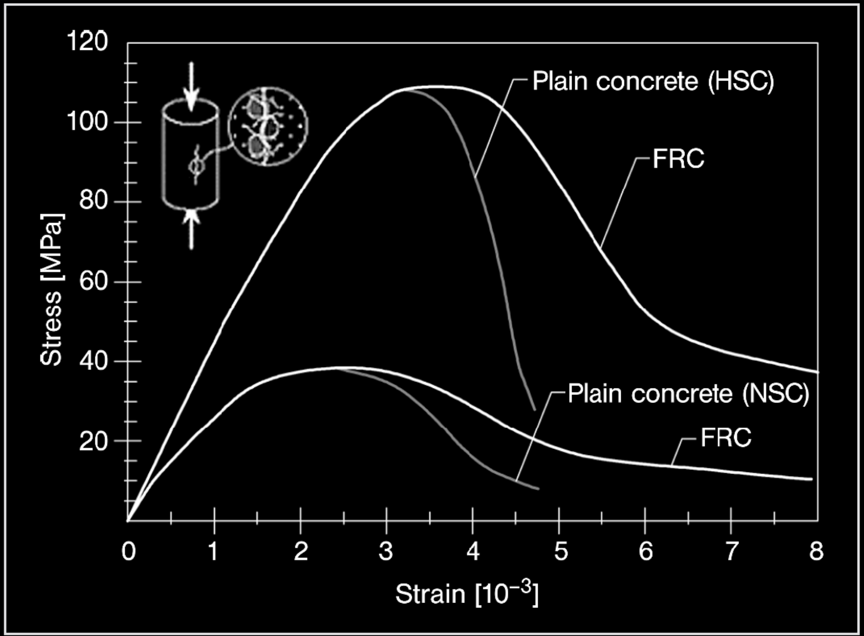



**Figure 5.6-3:** Main differences between plain and fibre reinforced concrete having both normal and high strength under uniaxial compression

[ Source: fib Model Code, 2010 ]

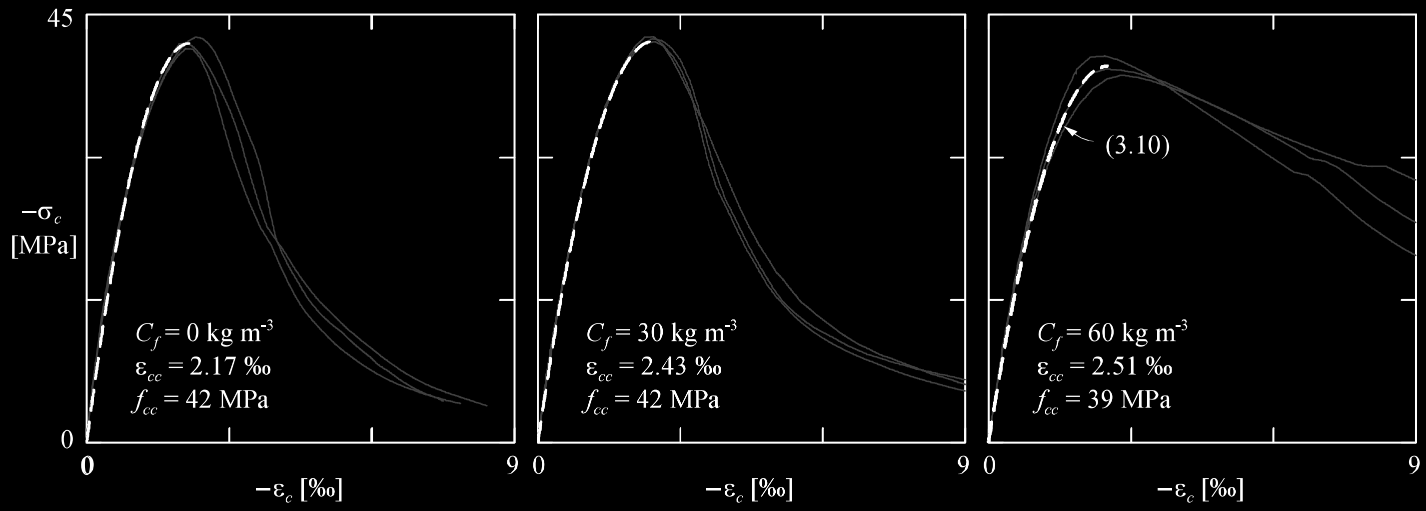



**Bild 3.5 –** Spannungs-Stauchungsdiagramme für Betone mit unterschiedlichem Fasergehalt $C_f$ aus [105] (Stahlfaser mit Endhaken, $l_f = 35$ mm, $l_f/d_f = 65$).

[ Source: Pfyl, 2003 ]

# Mechanical behaviour of SFRC

**SFRC members in shear**

- The addition of steel fibres generally has similar effects on the structural behaviour as in tension and in bending.
- Combined with stirrups, steel fibres contribute to the shear resistance. Unfortunately, current design rules for beams with SFRC reinforcement are typically semi-empirical, using additive terms («$V_{Rd}=V_c+V_s+V_f$»).
- These semi-empirical approaches postulate that the peak resistances of stirrups and steel fibres are reached at different crack openings. Therefore, the maximum total shear resistance is lower than the sum of the individual peak resistances.
- Tests indicate that fibres may be used as only shear reinforcement (without stirrups), and compression field analyses indicate that a hardening behaviour may be achieved with SFRC mixes softening in tension (beneficial effect of crack reorientation, i.e. flatter cracks activating more fibres); however, experimental evidence (practical fibre dosages) is sca.rce

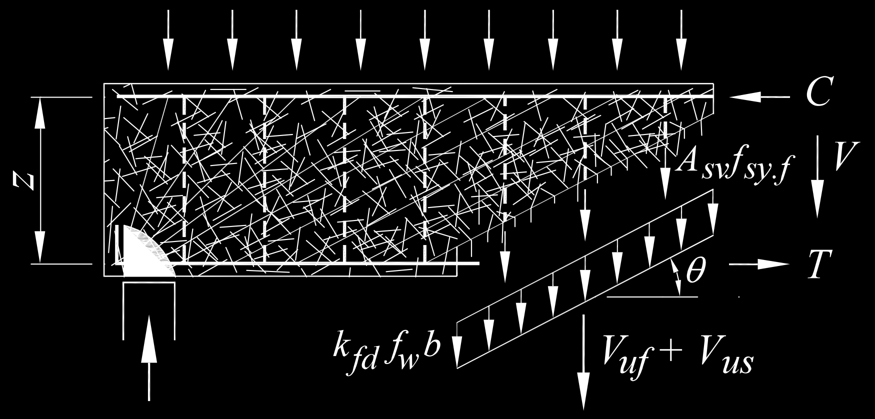
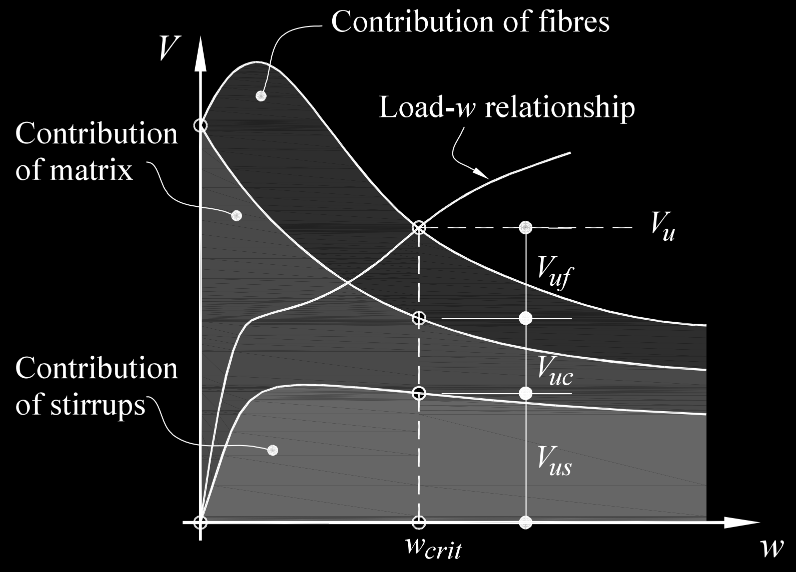



Figure 5.53. Transverse and fibre reinforcing components of SFRC beams with stirrups failing in shear (Foster, 2014).

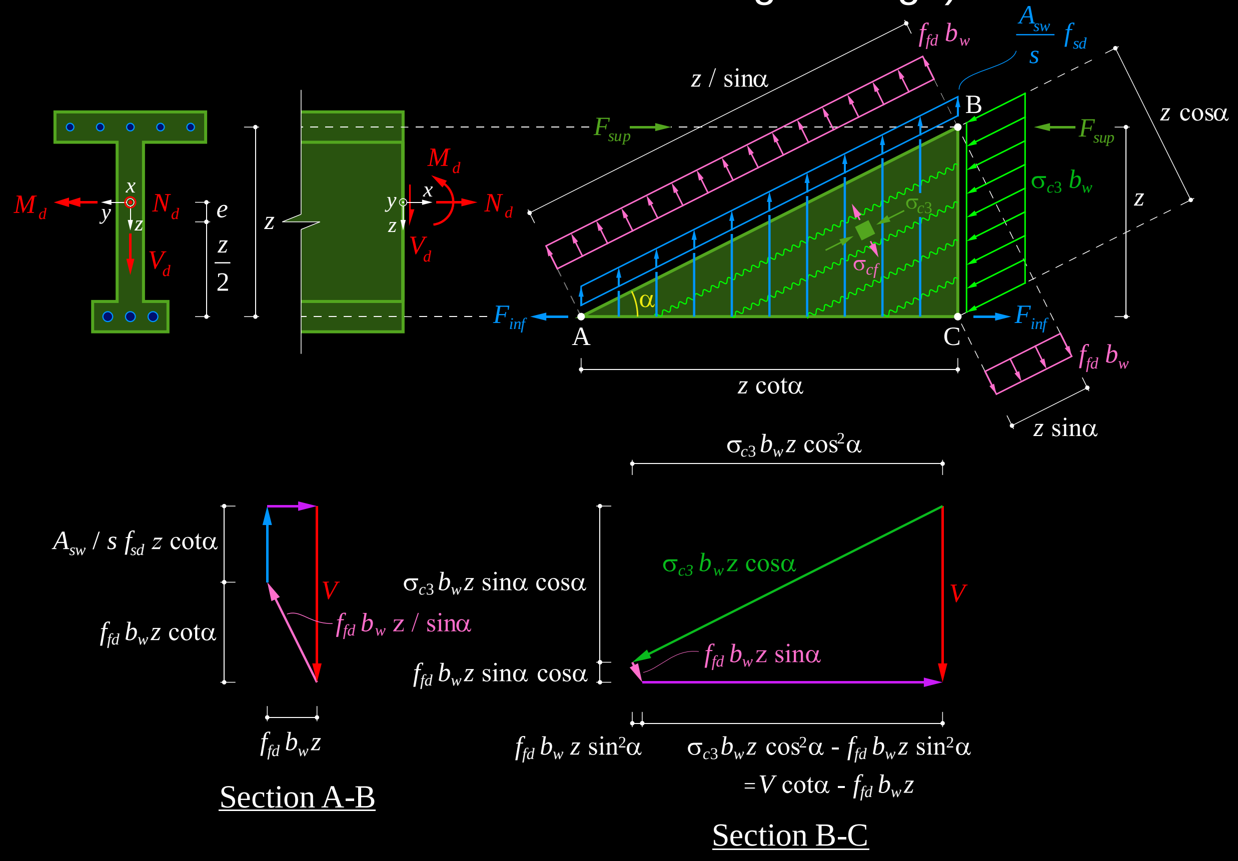



[ Source: Amin, 2015 ]

# Mechanical behaviour of SFRC

## SFRC members in shear

- Idealising the fibre reinforcement as rigid-perfectly plastic material (assuming a conservative residual fibre stress $f_{fd}$ as "yield stress"), the shear strength can be determined as for conventionally reinforced girders by superposition.
- However, this model has not been experimentally validated sufficiently to be applied in design (ongoing research at the Chair of Concrete Structures and Bridge Design).




- Resistance of reinforcement $$V_{Rd,s} = \left(\frac{A_{sw}}{s} f_{sd} + f_{fd}\, b_w\right) z \cot\alpha$$
- Resistance of concrete compression field $$V_{Rd,c} = b_w \left(k_c f_{cd} + f_{fd}\right) z \sin\alpha \cos\alpha$$

$$V_{Rd} = \min\left\{V_{Rd,s}, V_{Rd,c}\right\}$$

- Longitudinal tensile force resulting from the shear force $F_t(V_d)$ is resisted equally by the compression and tension chord

$$F_{sup} = \frac{M_d - N_d e}{z} - \frac{N_d}{2} - \frac{V_d \cot\alpha - f_{fd}\, b_w z}{2}$$

$$F_{inf} = \frac{M_d - N_d e}{z} + \frac{N_d}{2} + \frac{V_d \cot\alpha - f_{fd}\, b_w z}{2}$$

# Hybrid reinforcement (SFRC and conventional reinforcing bars)

## Modified tension chord model

- Equilibrium at crack with residual tensile strength $\sigma_{cf}$

$$f_{ct}\cdot(1-\rho_s+n\cdot\rho_s)=\sigma_{cf}\cdot(1-\rho_s)+\sigma_{sr}\cdot\rho_s$$

- Maximum crack spacing

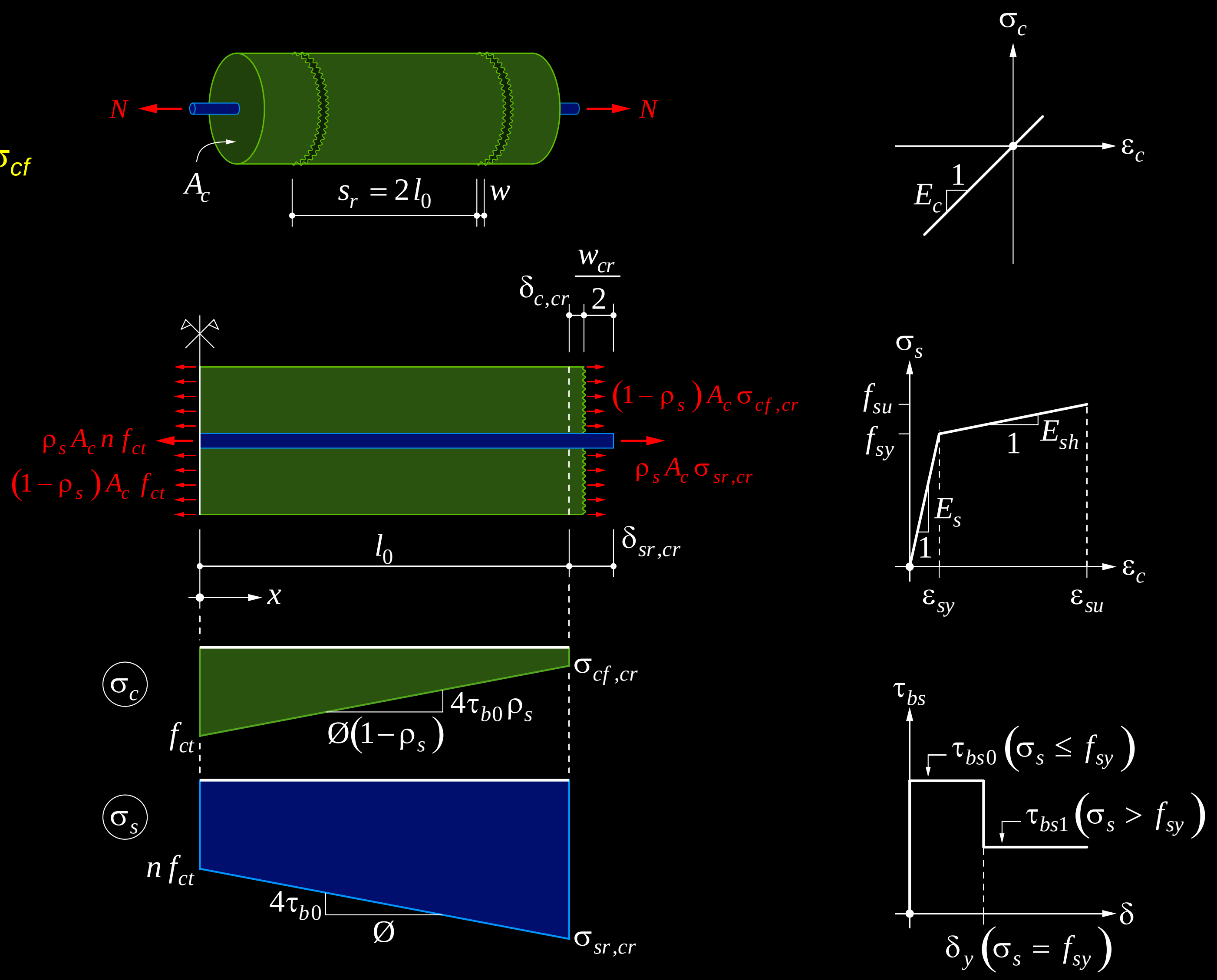
$$s_{r0}=\frac{Ø\cdot f_{ct}\cdot(1-\rho_s)}{2\cdot\tau_{bs}\cdot\rho_s}\cdot\left(1-\frac{\sigma_{cf}(w)}{f_{ct}}\right)$$

- Crack width

$$w=s_r\cdot(\varepsilon_{sm}-\varepsilon_{cm})=\frac{s_r^2\cdot\tau_{bs}}{Ø\cdot E_s}\cdot\left(1+n\cdot\frac{\rho_s}{1-\rho_s}\right)$$

- Minimum reinforcement ratio

$$\rho_{s,\min}=\frac{f_{ct}-\sigma_{cf}(w)}{f_{sy}-\sigma_{cf}-f_{ct}\cdot(n-1)}$$

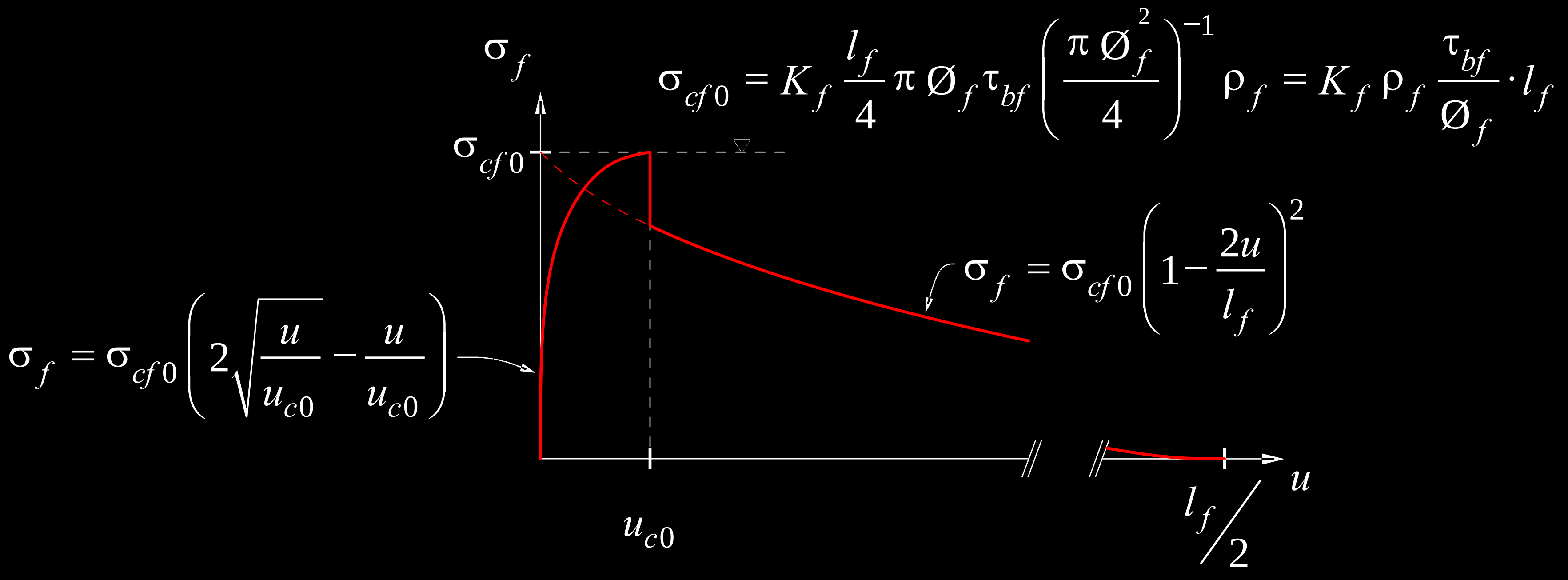


# Hybrid reinforcement (SFRC and conventional reinforcing bars)

**Modified tension chord model**

- Crack width and crack spacing are interdependent ($\sigma_{cf}$ depends on crack opening) → iterative solution procedure.
- As an approximation, the residual tensile strength $\sigma_{cf}$ (at a chosen crack opening) or even the fibre effectiveness $\sigma_{cf0}$ can be used, which normally leads to reasonable results.

$$s_{r0} = \frac{Ø \cdot f_{ct} \cdot (1-\rho_s)}{2 \cdot \tau_{bs} \cdot \rho_s} \cdot \left(1 - \frac{\sigma_{cf0}}{f_{ct}}\right) \qquad \sigma_{cf0} = K_f \cdot \rho_f \cdot \frac{\tau_{bf}}{Ø_f} \cdot l_f$$



# Hybrid reinforcement (SFRC and conventional reinforcing bars)

**Steel stress and average strains according to tension chord model**

- Neglecting the deformation of concrete between cracks, the crack widths can be determined from the average steel strains, which are obtained according to the tension chord model.

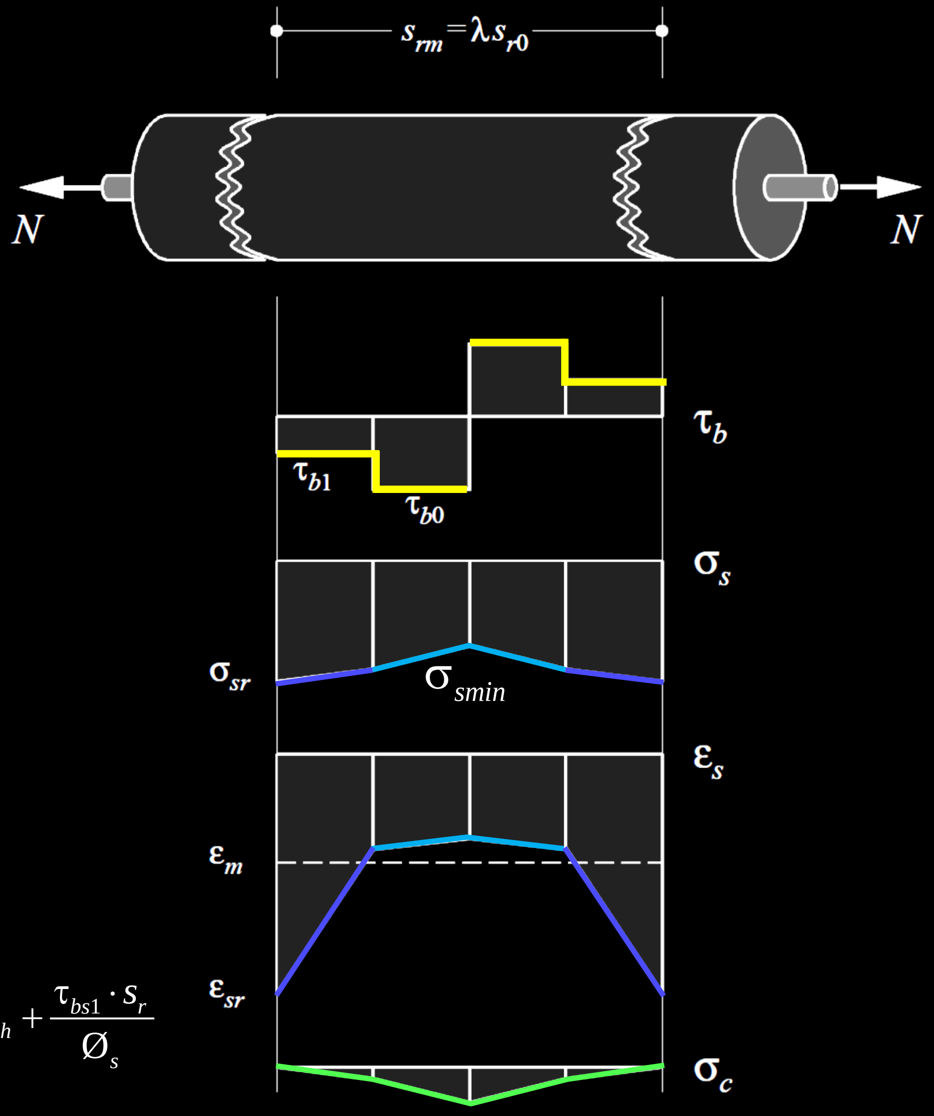
Reinforcement is partially elastic and partially plastic: $f_s \le \sigma_{sr} \le \left( f_s + \frac{2\tau_{b1} s_r}{\varnothing} \right)$

$$\varepsilon_{sm} = \frac{(\sigma_{sr} - f_s)^2 \varnothing}{4 E_{sh} \tau_{bs1} s_r} \left( 1 - \frac{E_{sh}\tau_{bs0}}{E_s \tau_{bs1}} \right) + \frac{(\sigma_{sr} - f_s)}{E_s} \frac{\tau_{bs0}}{\tau_{bs1}} + \left( \varepsilon_{sy} - \frac{\tau_{bs0} s_r}{E_s \varnothing} \right)$$

$$\sigma_{sr} = f_{sy} + 2 \cdot \frac{\frac{\tau_{bs0} \cdot s_r}{\varnothing_s} - \sqrt{\left( f_{sy} - \varepsilon_{sm} \cdot E_s \right) \cdot \frac{\tau_{bs1} \cdot s_r}{\varnothing_s} \cdot \left( \frac{\tau_{bs0}}{\tau_{bs1}} - \frac{E_s}{E_{sh}} \right) + \frac{\tau_{bs0} \cdot \tau_{bs1} \cdot s_r^2 \cdot E_s}{\varnothing_s^2 \cdot E_{sh}}}}{\frac{\tau_{bs0}}{\tau_{bs1}} - \frac{E_s}{E_{sh}}}$$

Reinforcement is fully plastic: $\left( f_s + \frac{2\tau_{b1} s_r}{\varnothing} \right) \le \sigma_{sr} \le f_t$

$$\varepsilon_{sm} = \varepsilon_{sy} + \frac{(\sigma_{sr} - f_s)}{E_{sh}} - \frac{\tau_{bs1} s_r}{E_{sh} \varnothing} \quad \text{(bare steel} - \Delta\varepsilon_1, \ \Delta\varepsilon_1 = \frac{\tau_{bs1} s_r}{E_{sh}\varnothing}\text{)} \qquad \sigma_{sr} = f_{sy} + \left( \varepsilon_{sm} - \frac{f_{sy}}{E_s} \right) \cdot E_{sh} + \frac{\tau_{bs1} \cdot s_r}{\varnothing_s}$$



# Hybrid reinforcement (SFRC and conventional reinforcing bars)

**Critical fibre residual tensile stress**

- Addition of steel fibres has favourable effects:
  - Increase in ultimate load
  - Reduced crack spacing and crack widths
  - Stiffer behaviour while reinforcing bars are elastic

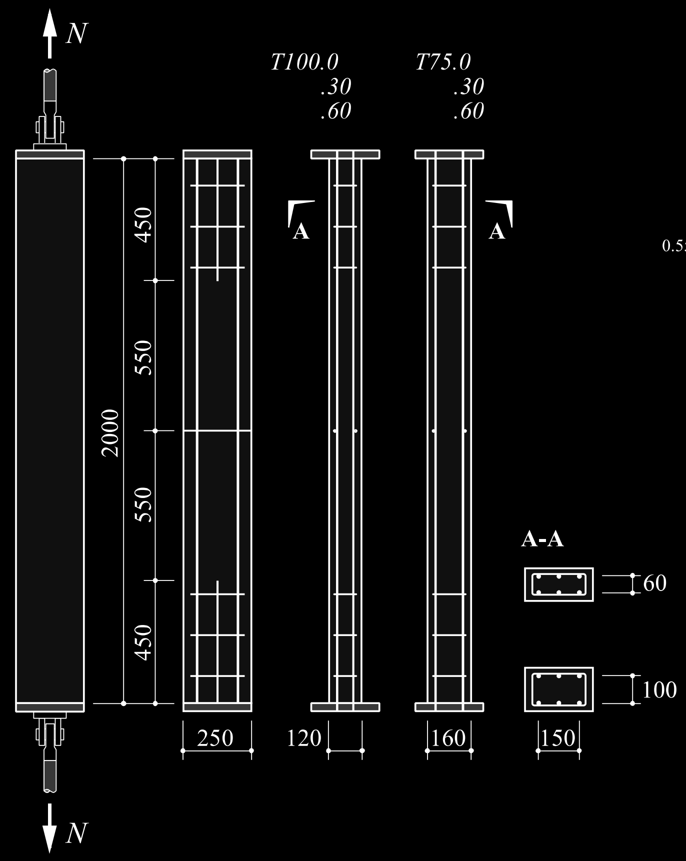
**However**:

- SFRC is softening
- Moderate-high fibre dosages combined with low-moderate conventional reinforcement ratios may result in a softening response of a tension chord (that would be hardening without fibres)
- A softening response occurs if at any point the differential loss in force due to the softening behaviour of SFRC is greater than the differential force increase due to hardening of the reinforcing bars
- Differentiating the tensile force and setting it to zero leads to: $N' = A_c \frac{d}{dw}\left(\sigma_{s,r}\rho_s + \sigma_{cf}\right) = 0$

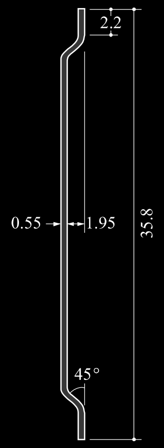
# Hybrid reinforcement (SFRC and conventional reinforcing bars)

## Experimental results: Uniaxial tension

Test setup

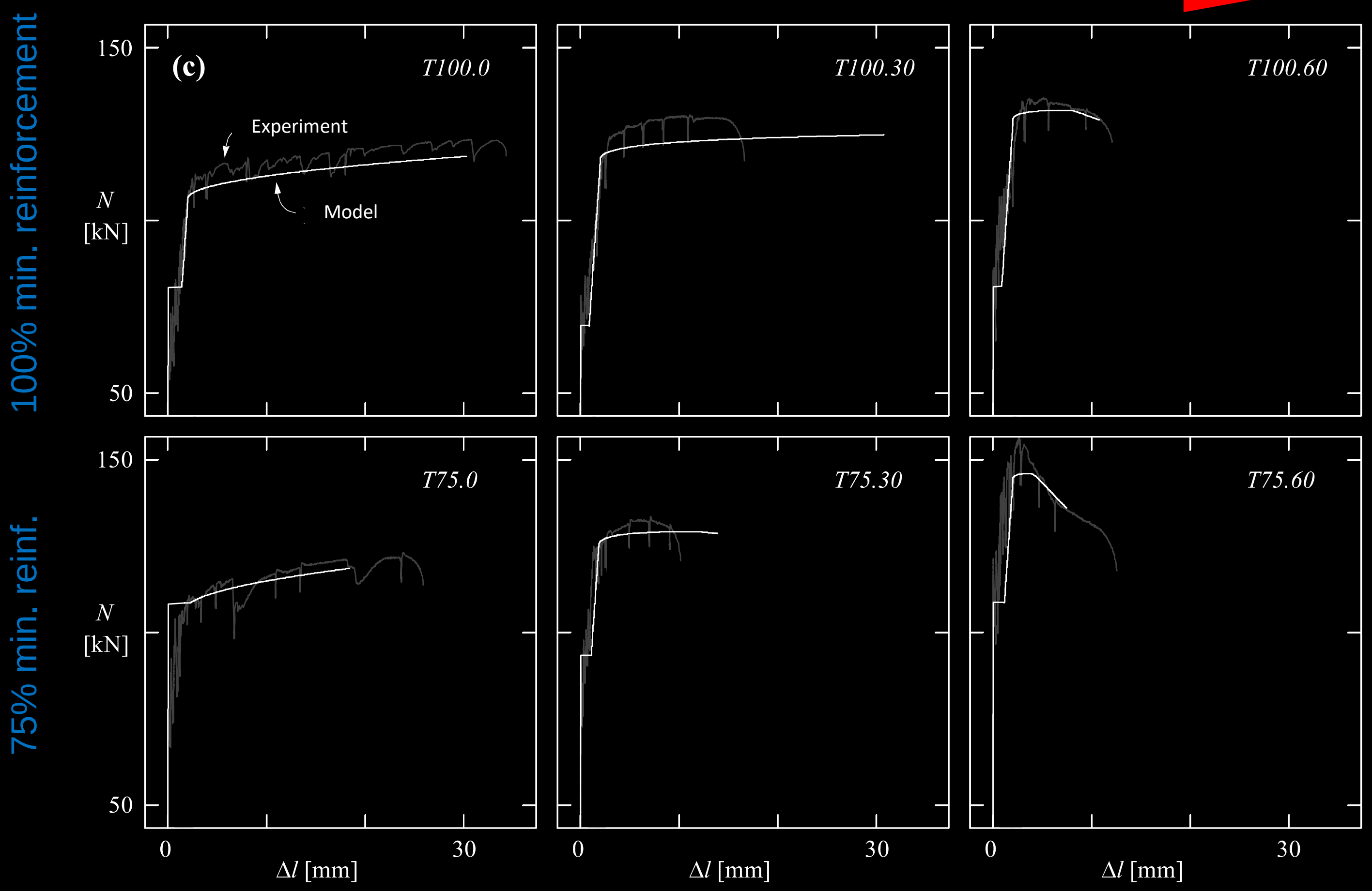



End-hooked fibres
(Type 3D 65/35)




Test results vs. model

increasing fibre content (0, 30, 60 kg/m³)

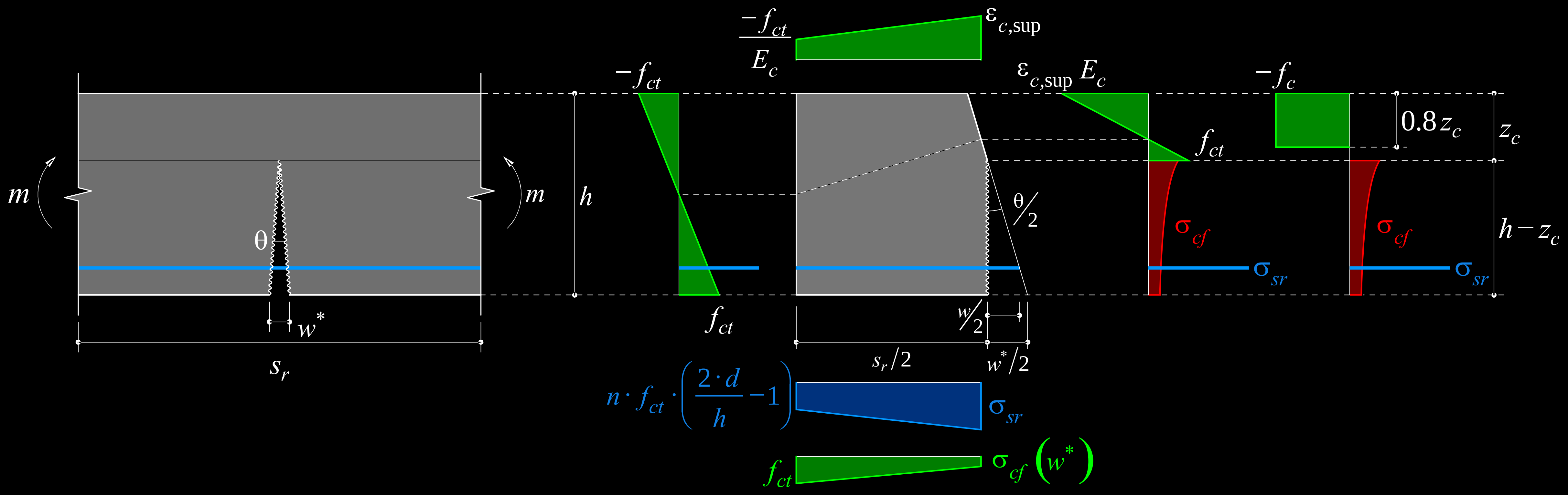



[ Source: Pfyl, 2003 ]

# Hybrid reinforcement (SFRC and conventional reinforcing bars)

**Bending**

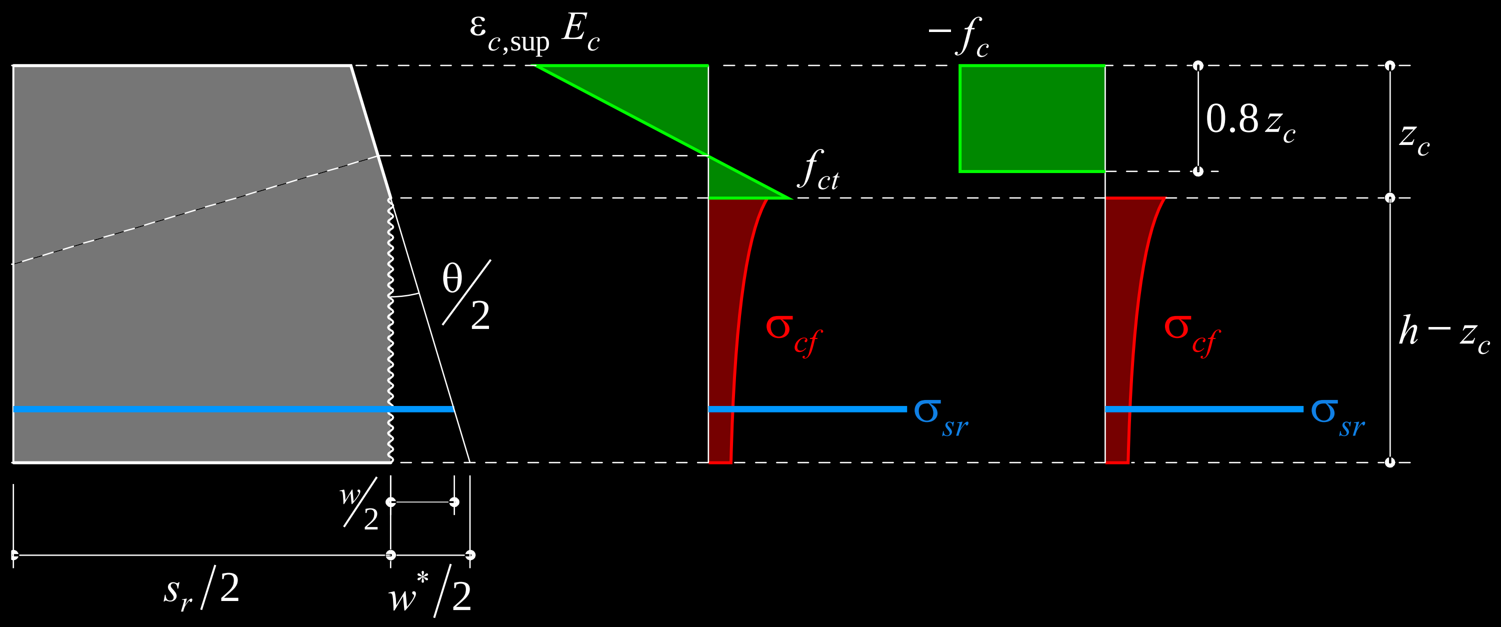
- Same assumptions on strains and crack kinematics as for SFRC elements in *bending* (slide 21)
- Crack width is determined from the average steel strains neglecting the elongation of the concrete between the cracks
- Steel stresses are determined from the tension chord model (slide 28)



# Hybrid reinforcement (SFRC and conventional reinforcing bars)

## Bending – simplified stress distribution in concrete

- Assuming a simplified stress distribution for the concrete in compression, the *m*-χ-relationship can be essentially determined from equilibrium alone, leading to a much simpler expression
- The crack spacing is determined from to the modified tension chord model using a reinforcement ratio of ρ* (see Stahlbeton I)




$$\sum F_H = 0: \quad a_s \cdot \sigma_{sr} + \sigma_{crm} \cdot (h - z_c) = 0.8 \cdot z_c \cdot f_c$$

$$\sum M = 0: \quad m = a_s \cdot \sigma_{sr} \cdot (d - 0.4 \cdot z_c) + \sigma_{crm} \cdot (h - z_c) \cdot (\zeta_{ct} + 0.6 \cdot z_c)$$

$$\xi = \frac{2 \cdot \theta \cdot (h - z_c)}{l}$$

Average tensile fibre stress

$$\sigma_{crm} = \frac{\sigma_{cf0}}{3} \cdot \left(\xi^2 - 3 \cdot \xi + 3\right)$$

Centroid of tensile fibre stress

$$\zeta_{ct} = (h - z_c) \cdot \frac{3 \cdot \xi^2 - 8 \cdot \xi + 6}{4 \cdot \xi^2 - 12 \cdot \xi + 12}$$

Steel stresses at crack

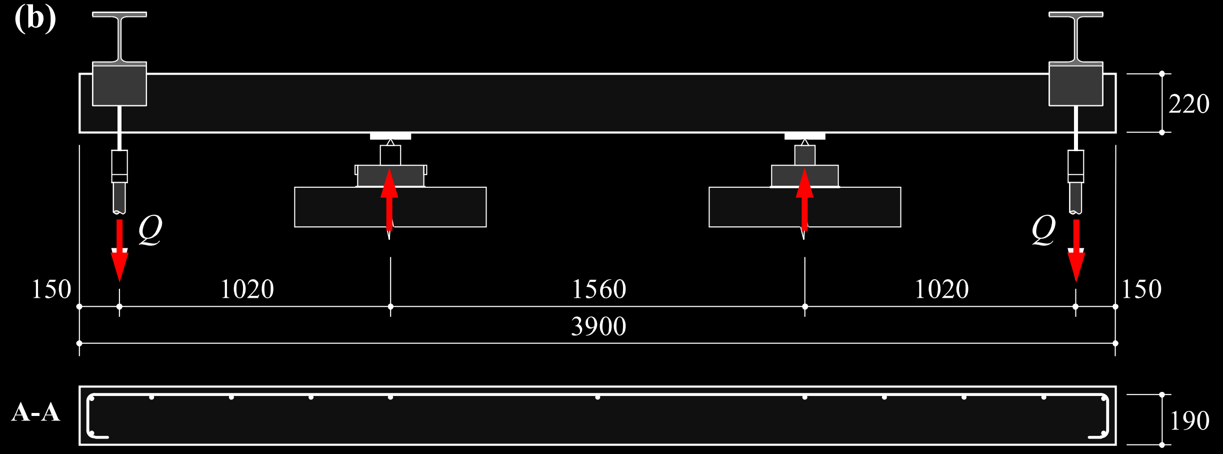
$\sigma_{sr}$ = acc. to TCM (*see slide 28*)

$$\varepsilon_{sm} \approx \frac{w}{s_r} \qquad \chi \approx \frac{w}{s_r \cdot (d - z_c)}$$

# Hybrid reinforcement (SFRC and conventional reinforcing bars)

**Critical fibre stress for SFRC members in bending**

- Similar to structural members in tension, SFRC members with conventional reinforcement can exhibit a softening or hardening behaviour in bending, depending on the fibre content.
- The total response results from the superposition of the softening behaviour of SFRC and the hardening behaviour of conventionally reinforced concrete members.
- Softening occurs if $m' = \frac{dm}{dw} < 0$
- Experimental study with 4-point-bending tests

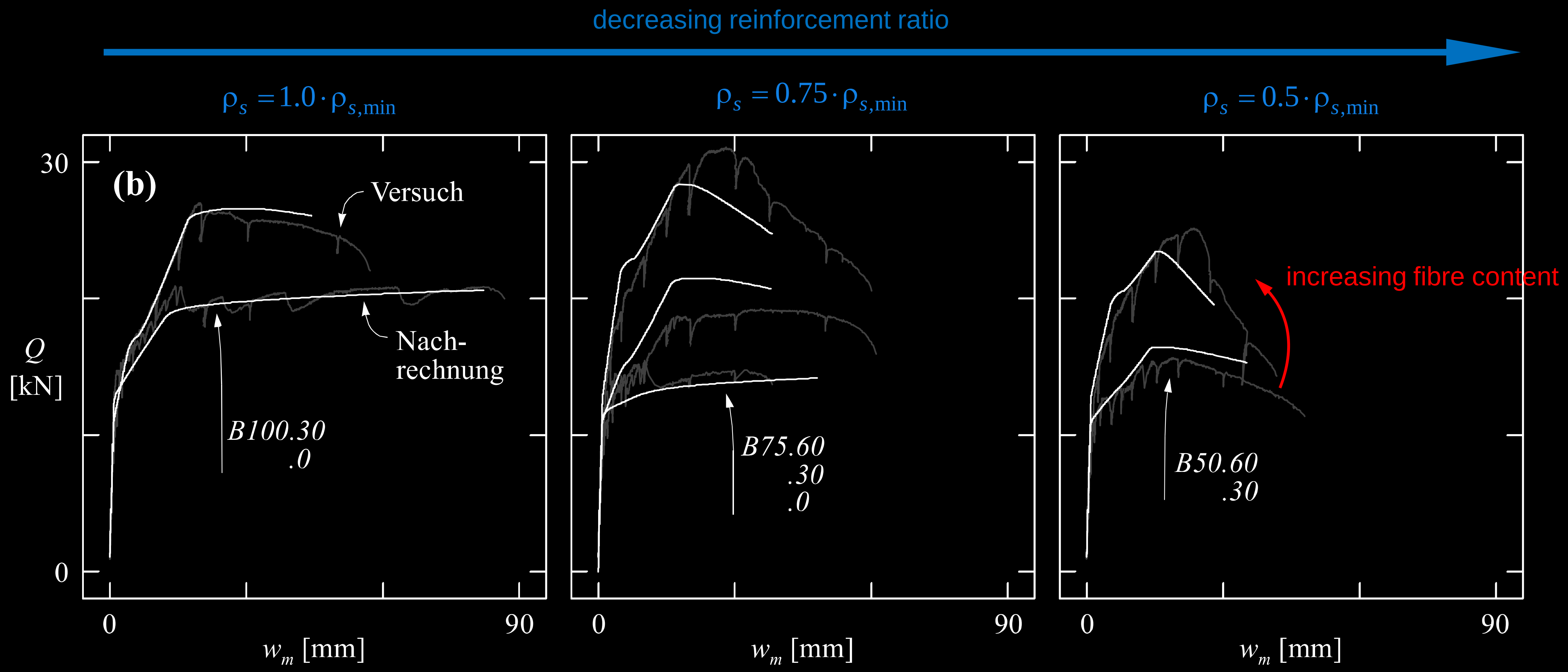



[ Source: Pfyl, 2003 ]

# Hybrid reinforcement (SFRC and conventional reinforcing bars)

**Experimental results: 4-point bending**

Test results vs. model



# Ultra High Performance Fibre Reinforced Concrete (UHPFRC)

**What is Ultra High Performance Fibre Reinforced Concrete?**

Ultra high strength fibre reinforced concrete with a compressive strength up to 200 MPa thanks to special mix composition:

- very high cement content (ca. 3 times more than ordinary concrete) $\rightarrow$ high cost and $CO_2$ emissions
- very high fibre content (>2% of steel and/or other fibres, often «fibre cocktail» of different types) $\rightarrow$ high cost
- very  low w/c-ratios (< 0.25), high density and low porosity $\rightarrow$ high durability
- small aggregate (grain) size, usually not larger than 2 mm (rather a fine mortar than "concrete")

Advantages:

- high compressive strength
- high durability, very low permeability (watertightness)
- tensile strength (strain hardening mixes only)

Drawbacks:

- high cost (cement and fibre content, additives and admixtures, often patented technology (Ductal ®, Ceracem ®, …)
- high $CO_2$-emissions (cement content, fibre content, fine aggregates)
- high shrinkage (typically around 1‰, can be reduced with heat treatment)
- many mixes strain softening in spite of high fibre dosage

# Ultra High Performance Fibre Reinforced Concrete (UHPFRC)

**What is Ultra High Performance Fibre Reinforced Concrete?**

Design aspects:

- SIA MB 2052 *Ultra-Hochleistungs-Faserbeton (UHFB) - Baustoffe, Bemessung und Ausführung* provides a basis for the dimensioning of UHPFRC
- even if strain hardening mixes are used (mandatory for structures according to MB 2052), no redistribution of action effects is allowed due to limited ductility (rupture strain in tension: few microstrains only)
- the limited ductility, high cost and $CO_2$ emissions are limiting factors for a widespread application of UHPFRC
- applications should focus on elements and parts where the high strength is really needed (lightweight prefabricated elements, connections, …)

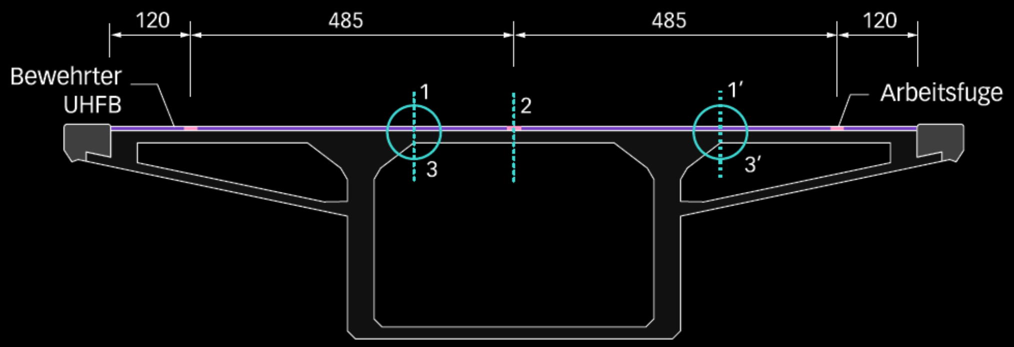
Some alternatives to UHPFRC (both even more expensive than UHPFRC and hardly used in large scale elements):

- SIFCON = Slurry infiltrated concrete (extremely high fibre contents are packed in the formwork, then the cement mix is poured in the spaces between the fibres)
- ECC = Engineered cementitious composites (microfibre cocktail, relatively low tensile strength, but strain hardening with high ductility, rupture strain in tension of several %)

# Ultra High Performance Fibre Reinforced Concrete (UHPFRC)

**Examples (selection)**

Bridge decks (overlay): Strengthening of Viaduc de Chillon




**Die Verstärkung mit einer Schicht bewehrtem UHFB** erhöht den Biegewiderstand um 73 % (Biegezug im UHFB, Schnitte 1 und 1') beziehungsweise 33 % (Biegedruck im UHFB, Schnitt 2).
Der Querkraftwiderstand (Schnitte 3 und 3') der verstärkten Fahrbahn im Endzustand (C30/40+UHFB) ist 20 % höher als ohne Verstärkung im heutigen Zustand (C60/70).[2]

[ Source: espazium – Tec 21 ]

# Ultra High Performance Fibre Reinforced Concrete (UHPFRC)

**Examples (selection)**

Bridge decks (overlay): Strengthening of Viaduc de Chillon

[ Source: espazium – Tec 21 ]

# Ultra High Performance Fibre Reinforced Concrete (UHPFRC)

**Examples (selection)**

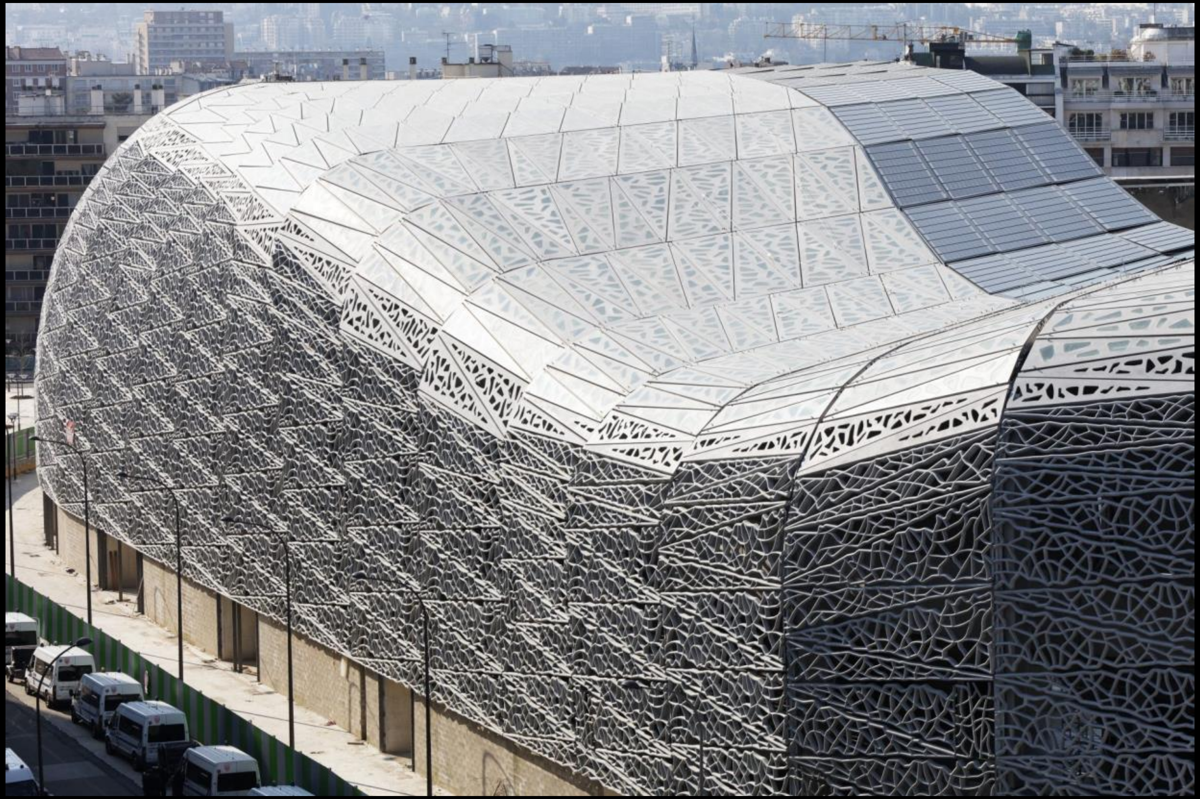
Precast bridge girders: DURA Technology Sdn. Bhd.

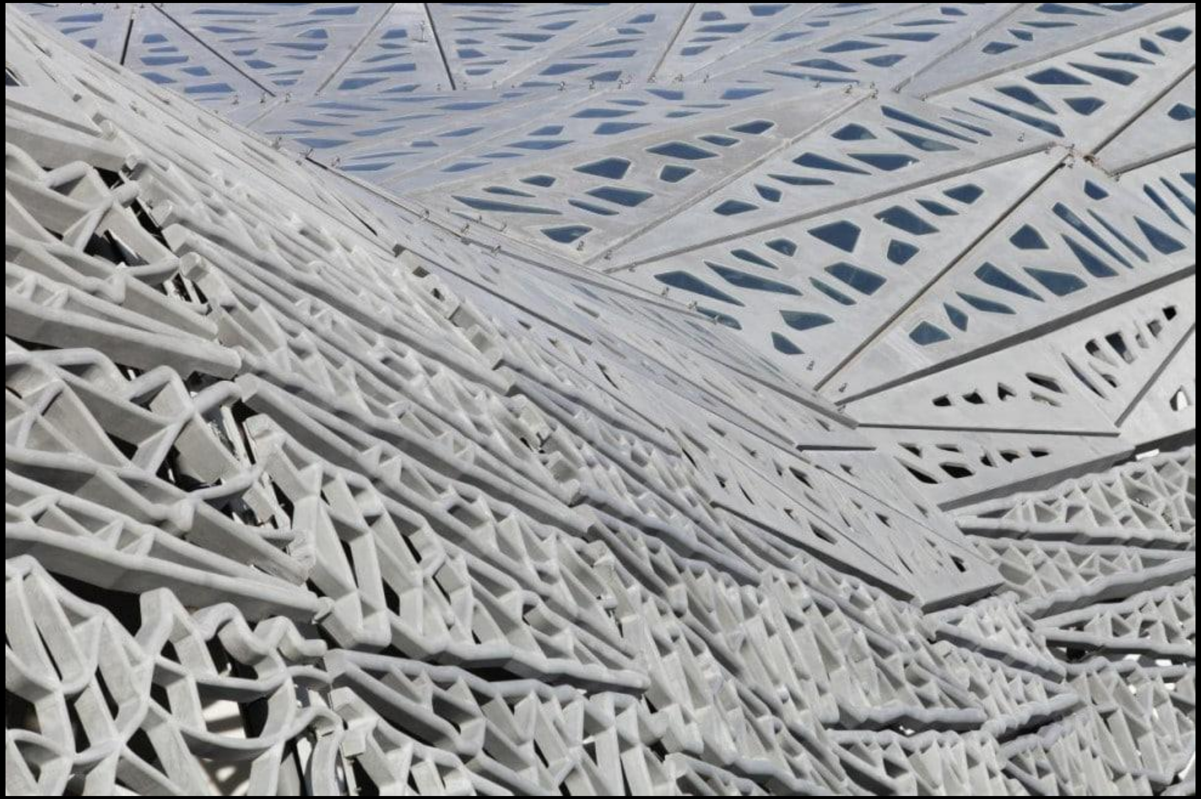

[ Source: DURA Technolog Sdn. Bhd. ]

# Ultra High Performance Fibre Reinforced Concrete (UHPFRC)

**Examples (selection)**

Façade elements: Stade Jean-Bouin, Lamoureux & Ricciotti ingenierie





[ Source: CONSOLIS Group, consolis.com ]

# Summary and conclusions

- Fibre reinforced concrete has been used and investigated in academia over the last five decades.
- The primary objective of adding fibres to concrete is to transmit tension across cracks.
- Practical fibre dosages lead to softening behaviour in tension (initial load drop at cracking, with subsequent gradual pull-out of the fibres in deformation-controlled tests).
- When combined with conventional reinforcing bars (hybrid reinforcement), the tensile stresses carried by the fibres at the cracks result in a more pronounced tension stiffening and by this, reduced crack widths at smaller crack spacings.
- There is a limit to the amount of conventional reinforcing bars that can be replaced by fibres. Beyond this limit, structural concrete members containing fibres will display significantly reduced ductility characteristics.
- SFRC, as well as «new» materials such as UHPFRC, SIFCON and ECC have a high potential for certain applications. However, they also have some drawbacks, which need to be addressed in order to open the way for a more widespread application.

# References

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[3] International Federation for Structural Concrete, *fib Model Code for Concrete Structures*, Ernst & Sohn, 2010

[4] Markić et. al, *Strength and ductility of tension and flexural reinforced concrete members containing steel fibres: critical fibre residual stress*, ASCE, 2018

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[6] Hillerborg, *A Model for Fracture Analysis*, The Lund Institute of Technology, 1978

[7] Sigrist, *Zum Verformungsvermögen von Stahlbetonträgern*, ETH Zürich, 1995

[8] Hansel et. al, *Steel-fibre-reinforced segmental linings: State-of-the-art and completed projects*, Tunnel, 2011

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[10] Brühwiler, *Stahl und Beton effizienter kombiniert*, TEC21, 2014

# ANNEX

# Mechanical behaviour of SFRC

## SFRC members in bending

- The crack width can be determined by integrating the concrete strains over the distance $\pm s_r/2$ (crack element):

$$w = \theta \cdot (h - z_c) = \frac{s_r}{2 \cdot z_c} \cdot \left( \frac{6 \cdot m}{E_c \cdot h^2} - \varepsilon_{c,\sup} \right) \cdot (h - z_c)$$

- Integration of the stresses over the cross section yields the average stresses and the respective centroids for the cracked and uncracked parts of the cross section. Considering only the fibre pull-out phase and Pfyl's model, one gets:

$$\xi = \frac{2 \cdot \theta \cdot (h - z_c)}{l_f}$$

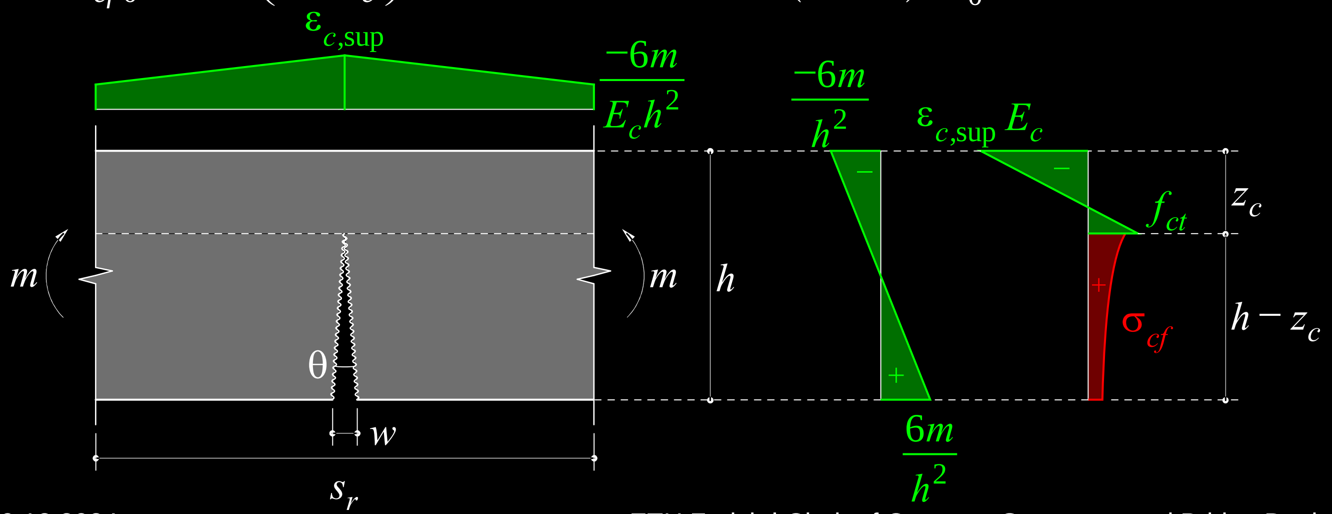
$$\frac{\sigma_{cf}(z)}{\sigma_{cf0}} = 1 - \left( \frac{\xi \cdot z}{h - z_c} \right)^2$$

$$\sigma_{crm} = \frac{1}{h - z_c} \cdot \int_0^{h - z_c} \sigma_{cf}(z) \cdot dz = \sigma_{cf0} \frac{\xi^2 - 3 \cdot \xi + 3}{3}$$

$$\zeta_{ct} = \frac{1}{\sigma_{crm} \cdot (h - z_c)} \cdot \int_0^{h - z_c} \sigma_{cf}(z) \cdot dz = (h - z_c) \cdot \frac{3 \cdot \xi^2 - 8 \cdot \xi + 6}{4 \cdot \xi^2 - 12 \cdot \xi + 12}$$

$$\sigma_{ccm} = \frac{1}{z_c} \cdot \int_{-z_c}^{0} \sigma_{cc}(z) \cdot dz$$

$$\zeta_{cc} = \frac{1}{\sigma_{ccm} \cdot z_c} \cdot \int_{-z_c}^{0} \sigma_{cc}(z) \cdot z \cdot dz$$




$$\sum F_H: \quad \sigma_{crm} \cdot (h - z_c) + \sigma_{ccm} \cdot z_c = 0$$

$$\sum M: \quad m = \sigma_{crm} \cdot (h - z_c) \cdot \zeta_{ct} + \sigma_{ccm} \cdot z_c \cdot \zeta_{cc}$$

→ assume value of $w$

→ solve equations for $z_c(w)$, $\varepsilon_{c,sup}(w)$, $m(w)$

# Mechanical behaviour of SFRC

**SFRC members in bending**

- Further simplifications are possible if the depth of the compression zone is determined as in conventional reinforced concrete (rectangular stress block under $f_{cd}$ acting over 0.8 $z_c$, as shown in slide 32):

$$z_c = \frac{h}{1+\dfrac{2.4 f_{cd}}{\sigma_{cf0}\left(\xi^2 - 3\cdot\xi + 3\right)}} \qquad m = 0.8 f_{cd} z_c \left[0.6 z_c + (h - z_c)\frac{3\cdot\xi^2 - 8\cdot\xi + 6}{4\cdot\xi^2 - 12\cdot\xi + 12}\right] \qquad (0 \le \xi \le 1)$$

$$z_c = \frac{h}{1+\dfrac{2.4 f_{cd}\xi}{\sigma_{cf0}}} \qquad m = 0.8 f_{cd} z_c \left[0.6 z_c + (h - z_c)\frac{h - z_c}{4\xi}\right] \qquad (\xi > 1)$$

NB: The activated strength in the fibres might not reach the required strains for the approximation with a rectangular stress block!

- In many cases, the compression zone depth may be fully neglected without significantly affecting the bending moment, yielding the even simpler expressions:

$$z_c = 0 \qquad m = \frac{\sigma_{cf0} h^2 \left(3\cdot\xi^2 - 8\cdot\xi + 6\right)}{12} \qquad \left(0 \le \xi = \frac{2\theta h}{l_f} \le 1\right)$$

$$z_c = 0 \qquad m = \frac{\sigma_{cf0} h^2}{12\cdot\xi^2} \qquad (\xi > 1)$$

These expressions are useful to determine (estimate) the fibre effectiveness directly from bending tests.