5 Slabs

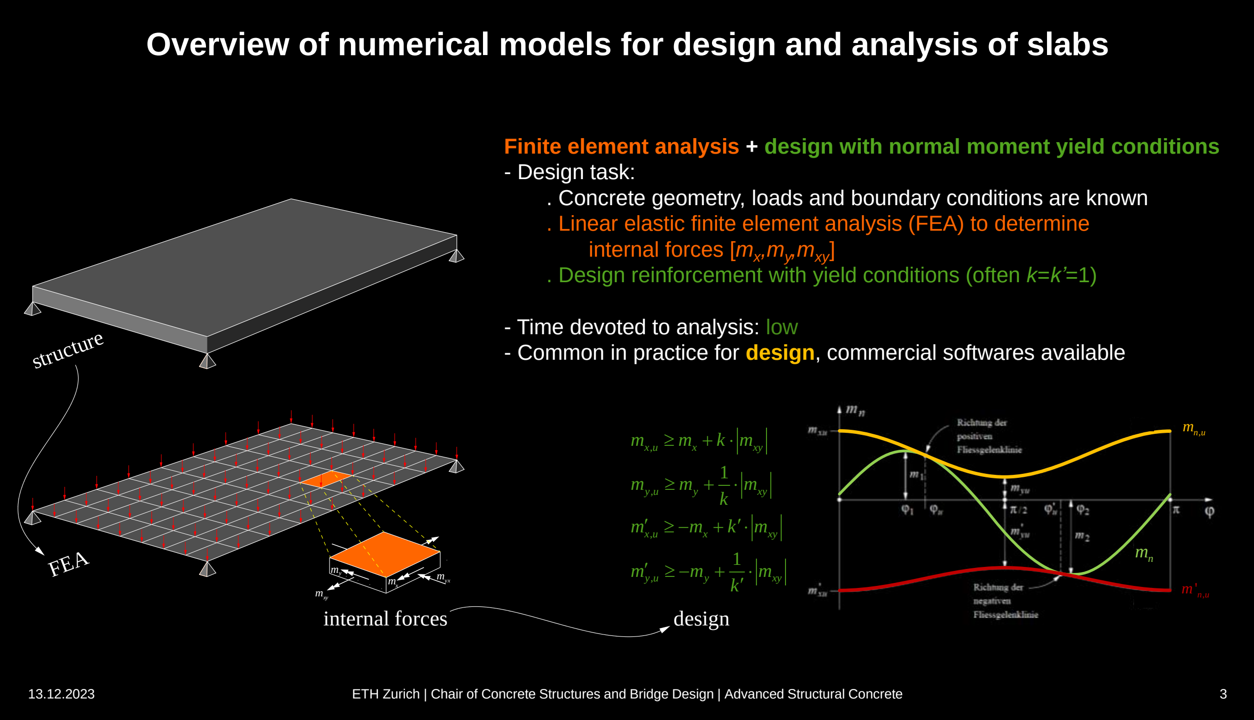
In-depth study and additions to Stahlbeton II

5.8 Numerical modelling of slabs

Learning objectives

Within this chapter, the students are able to:

- select the most suitable numerical model for slabs, clearly differentiating design and assessment-oriented approaches.
    - recognise the underlying mechanical models and understand their limitations and assumptions.
    - discuss the workflow of selected numerical models.



Overview of numerical models for design and analysis of slabs

Finite element analysis + **design with normal moment yield conditions**

- Design task:
  - . Concrete geometry, loads and boundary conditions are known
  - . Linear elastic finite element analysis (FEA) to determine internal forces [m_x,m_y,m_{xy}]
  - . Design reinforcement with yield conditions (often k=k'=1)

- Time devoted to analysis: low
- Common in practice for **design**, commercial softwares available

$$m_{x,u} \geq m_x + k \cdot |m_{xy}|$$

$$m_{y,u} \geq m_y + \frac{1}{k} \cdot |m_{xy}|$$

$$m'_{x,u} \geq -m_x + k' \cdot |m_{xy}|$$

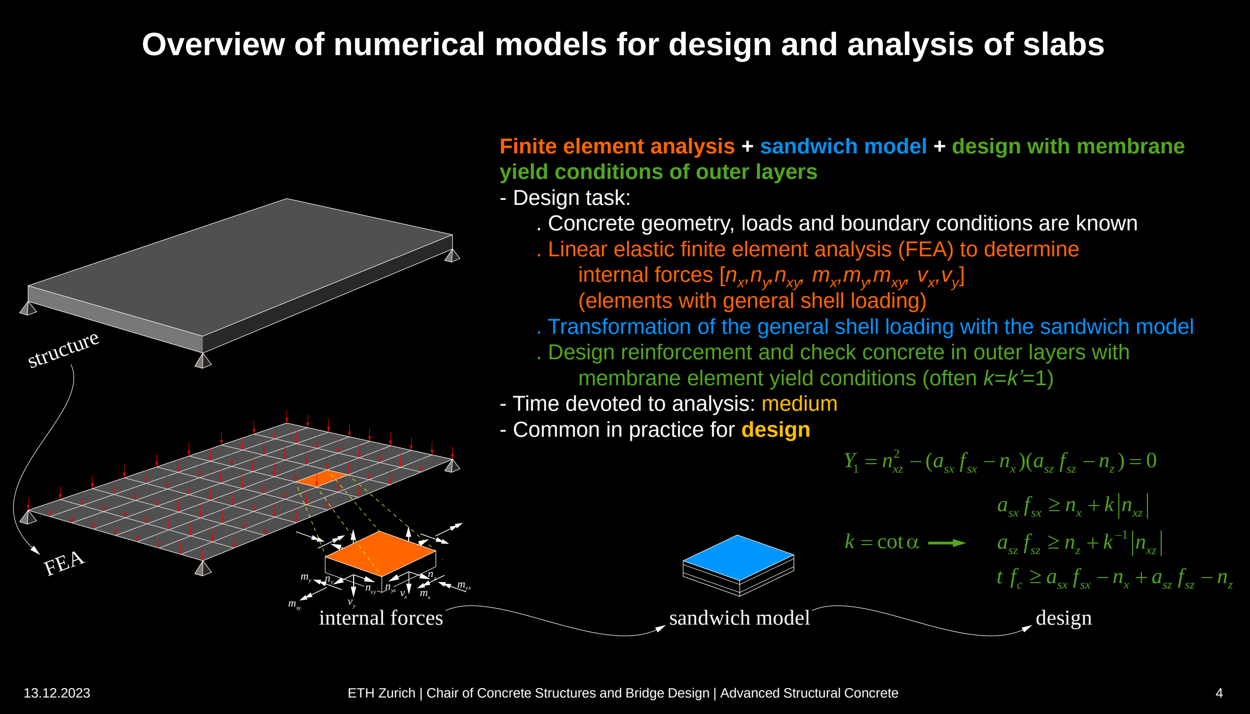
$$m'_{y,u} \geq -m_y + \frac{1}{k'} \cdot |m_{xy}|$$

Finite element analysis + design with normal moment yield conditions:

The reinforcement of slabs modelled with 2D elements can be easily designed by using the normal moment yield conditions already presented in the course. The procedure is analogous to the design of structures subjected exclusively to membrane loading (in-plane loading). First, the moments are calculated, typically with linear elastic Finite Element Analysis, and then the limit analysis normal moment yield conditions (rigid-ideal plastic material idealization, i.e. non-linear behaviour) are used for structural design. The linearised normal moment yield conditions (*k*=*k'*=1) are frequently implemented in commercial softwares.

The normal moment yield condition overestimates the resistance for large twisting moments with respect to the reinforcement directions as well as for high reinforcement ratios. In many cases, this overestimation is compensated by the favourable effect of the membrane forces, which are usually neglected in the design. However, caution is required close to corner supports in which an approximate state of pure twisting can be generated.

Linear elastic FEA are also frequently used for the shear design of slabs. Here, it should be noted that the accuracy of the transverse shear forces is an order of magnitude less than that of the bending moments (as they are obtained as derivative of the bending moments).



Overview of numerical models for design and analysis of slabs

Finite element analysis + sandwich model + design with membrane yield conditions of outer layers

- Design task:
  - . Concrete geometry, loads and boundary conditions are known
  - . Linear elastic finite element analysis (FEA) to determine internal forces [n_x, n_y, n_{xy}, m_x, m_y, m_{xy}, v_x, v_y] (elements with general shell loading)
  - . Transformation of the general shell loading with the sandwich model
  - . Design reinforcement and check concrete in outer layers with membrane element yield conditions (often *k*=*k'*=1)
- Time devoted to analysis: medium
- Common in practice for **design**

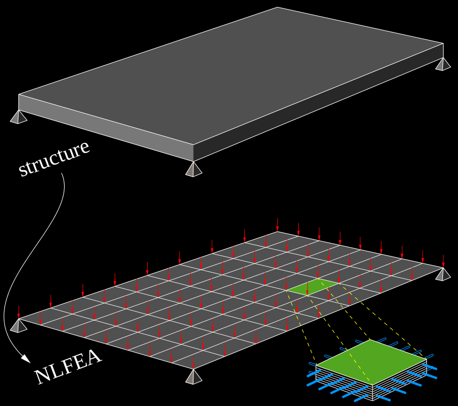
$$Y_1 = n_{xz}^2 - (a_{sx} f_{sx} - n_x)(a_{sz} f_{sz} - n_z) = 0$$

$$k = \cot\alpha \longrightarrow \quad \begin{aligned} a_{sx} f_{sx} &\geq n_x + k|n_{xz}| \\ a_{sz} f_{sz} &\geq n_z + k^{-1}|n_{xz}| \\ t\, f_c &\geq a_{sx} f_{sx} - n_x + a_{sz} f_{sz} - n_z \end{aligned}$$

Finite element analysis + sandwich model + design with membrane yield conditions of outer layers:

In the case of slabs subjected to both in-plane loading as well as bending and twisting moments (e.g., if membrane action is not negligible in a slab), the design for the general shell loading can still be done based on the yield conditions of membrane elements. This approach was already presented in the section about numerical modelling of in-plane loaded structures. Hence, all the comments presented there apply also to this case. The internal forces are calculated in a first step typically with linear elastic FEA, which only requires the concrete geometry, loads, and boundary conditions to be known. In a second step, the sandwich model can be applied in order to transform the general shell loading in two states of membrane loading in the outer layers. The outer layers can be designed using the limit analysis membrane yield conditions (rigid-ideal plastic material idealisation, i.e. non-linear behaviour).

Overview of numerical models for design and analysis of slabs



[Thoma, 2018]

Cracked Membrane Model Usermat (CMM-Usermat)

- Assessment task
    - . Concrete geometry, loads and reinforcement are known
    - . Non-linear finite element analysis (NLFEA) → Compatible stress fields

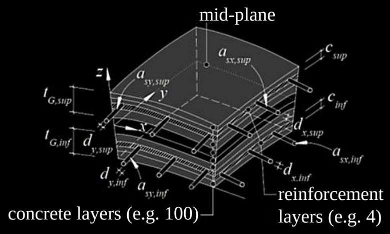
      Multilayer shell element

      Reinforcement and Concrete are modelled as a composite

      Tension stiffening according to TCM (2D)
- Time devoted to analysis: high
- Used at ETHZ for research and expert consulting

constitutive relationship

$\sigma \xleftrightarrow{\sigma(\varepsilon)} \varepsilon$



Cracked Membrane Model Usermat (CMM-Usermat):

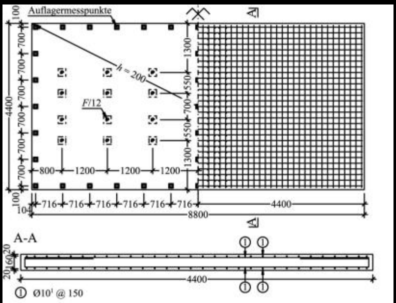
The Cracked Membrane Model with fictitious rotating stress-free cracks (CMM-R) was already presented in the chapter on in-plane loaded structures. This model has been implemented by Prof. Karel Thoma at HSLU as an ANSYS user-defined material, which can be applied to analyse structures with 2D elements. The use of a multilayer approach makes it suitable to analyse shell structures with any kind of loading. The material properties can be automatically generated from the concrete and reinforcement grades, based on the prescriptions of structural design codes.

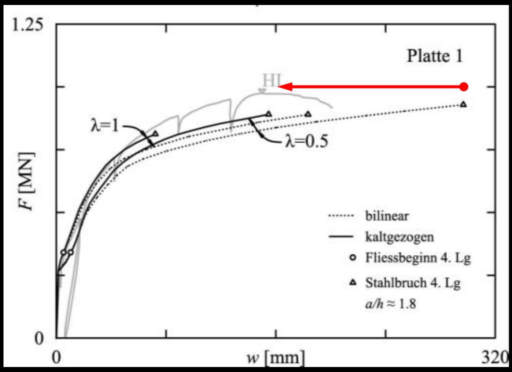
In this approach, the structure is analysed by means of several membrane elements in which the concrete and the reinforcement are modelled together as a composite. The Cracked Membrane Model is very accurate for capturing the global behaviour of the structure, but does not yield accurate results in parts of the structure with static and/or geometric discontinuities.

It should be noted that non-linear analysis is way less frequent in slabs than in-plane loaded structures, as they are very time-consuming, and the amount of redistributions is smaller in slabs.

Furthermore, it should be noted that many commercial software packages (ANSYS, ABAQUS, DIANA, ATENA, SOFISTIK, …) include nonlinear finite elements suitable for slabs and shells. However, the underlying models often lack mechanical consistency and deviate from modern design philosophy, e.g. by accounting for the tensile strength of concrete in ULS design.

Overview of numerical models for design and analysis of slabs



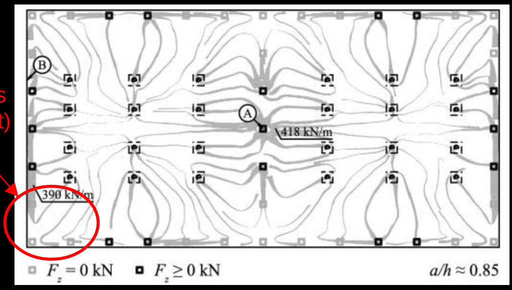


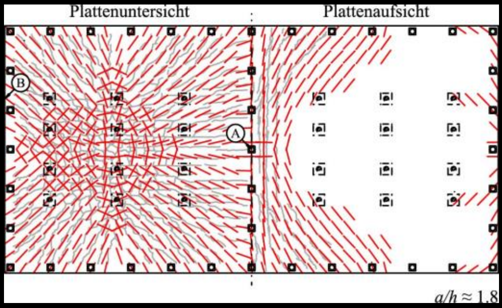
Deformations are overestimated with bilinear idealization of steel

[Niederberger/Thoma, 2010]

Overview of numerical models for design and analysis of slabs

Lifting of the corner areas
(similar to the experiment)





[Niederberger/Thoma, 2010]

$$v_0 = \sqrt{v_x^2 + v_y^2}$$

$$\tan \varphi_0 = {}^{v_y}/_{v_x}$$