5 Slabs

In-depth study and additions to Stahlbeton II

5.5 Influence of shear forces

Learning objectives

Within this chapter, the students are able to:

- compare the shear behaviour of a slab with and without transverse reinforcement and explain the arising differences of forces in the sandwich covers when using a sandwich model.
- determine the “actual” (according to SIA 262) punching resistance of a slab without and with punching reinforcement and understand the verification procedure.

Slabs - Influence of shear forces

Shear resistance of slabs - General remarks (→ Stahlbeton II)

- Slabs, especially those with shear reinforcement (three-dimensionally reinforced), are generally very ductile structures.
- On the other hand, a shear failure of slabs without shear reinforcement is very brittle → practically impossible to redistribute the internal forces (therefore, no stress relief of the affected areas by internal force redistribution)!
- Often slabs are designed according to the lower bound theorem of the theory of plasticity. In doing so the maximum shear forces occurring in the course of the load history can deviate significantly from the shear load in the calculated (bending) failure state (*).
  For a safe design, the shear force at each point of the slab should, therefore, strictly speaking, be checked during the entire load history (internal force redistribution under the same external loads).
- In practice, shear structural safety is usually only checked in the state of maximum internal force redistribution, which is also the basis for the bending design. This is associated with considerable uncertainties, especially since the shear forces resulting from FE calculations scatter strongly (they are determined numerically as derivatives of the bending moments, one order of magnitude less accurate).
  In case of doubt, a ductile behaviour must be ensured by arranging a shear reinforcement!

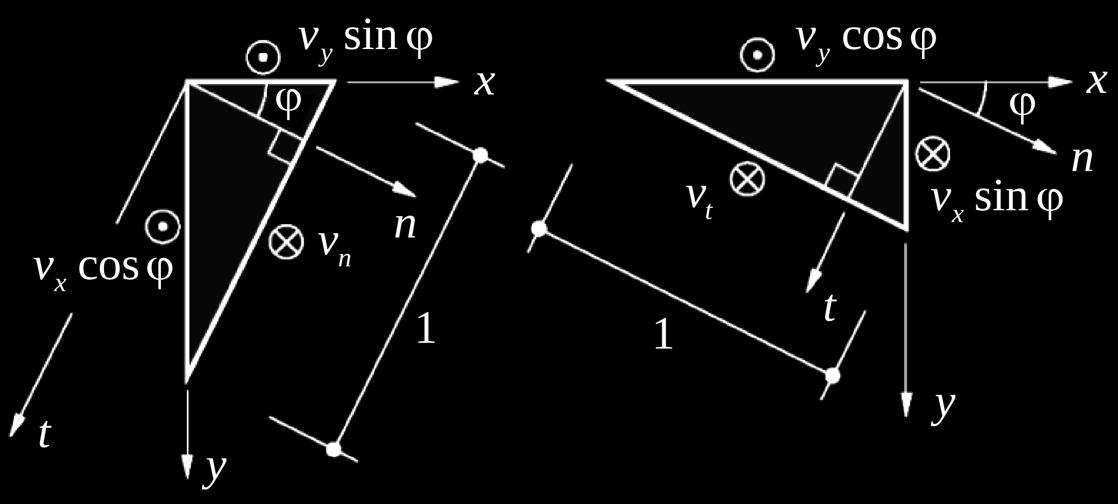
(*) also applies to a design based on linear elastic FE calculations (= equilibrium state), since crack formation, residual stress states due to settlements, construction process, etc. can never be completely recorded or correctly modelled!

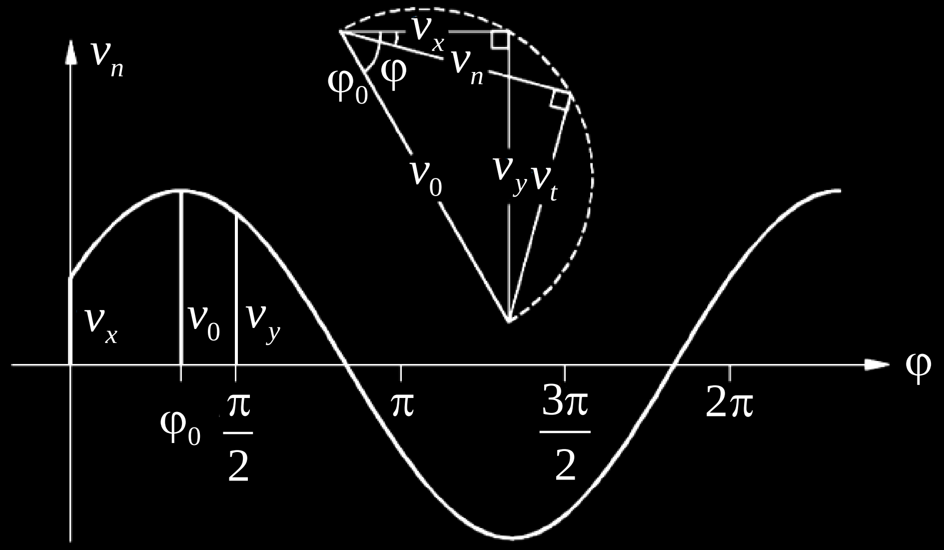
Slabs - Influence of shear forces

Shear resistance of slabs - General remarks

- In a slab, the principal shear force $v(\varphi_0) = v_0$ is carried in the direction φ_0 at every point. Perpendicular to it the shear force is zero: $v = v(\varphi_0 \pm \pi/2) = 0$.

→ Measure for shear stress: nominal shear stress $\tau_{nom} = v_0 / z$
(with z = lever arm of the internal forces).





$$v_n = v_x \cos\varphi + v_y \sin\varphi \qquad v_t = -v_x \sin\varphi + v_y \cos\varphi$$

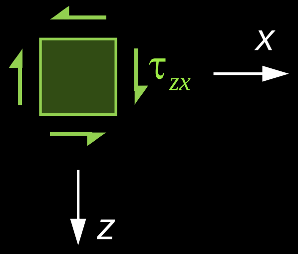
$$v_0 = \sqrt{v_x^{\,2} + v_y^{\,2}} \qquad \tan\varphi_0 = \frac{v_y}{v_x}$$

Slabs - Influence of shear forces

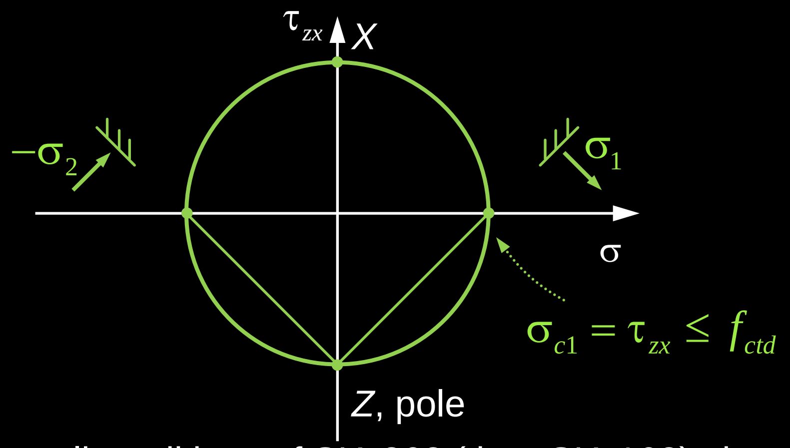
Shear resistance of slabs without shear reinforcement

- Shear stresses in the uncracked (isotropic) state correspond to a principal tensile stress of the same amount, $\sigma_{c1} = |\tau_{zx}|$ (elastic shear flow: $\tau_{max} = 1.5 \cdot \tau_{nom} = 1.5 \cdot v_0 / z$)
- In the case of thin slabs, which according to SIA 262 may be designed without shear reinforcement, the tensile strength of the concrete is implicitly taken into account (which is usually even slightly higher than the permissible value for insignificant components). This can be justified on the following reasons:
  - Higher redundancy than beam structures (biaxial load-bearing, beneficial compressive membrane forces neglected in the design)
  - Shear stress generally lower (except in the vicinity of concentrated loads and supports)
  - No failure at first shear crack formation under moderate shear stress (if crack roughness is sufficient and longitudinal reinforcement has reserves)

→ In contrast to beam structures (minimum shear reinforcement mandatory), shear reinforcement can often be omitted in thin slabs.



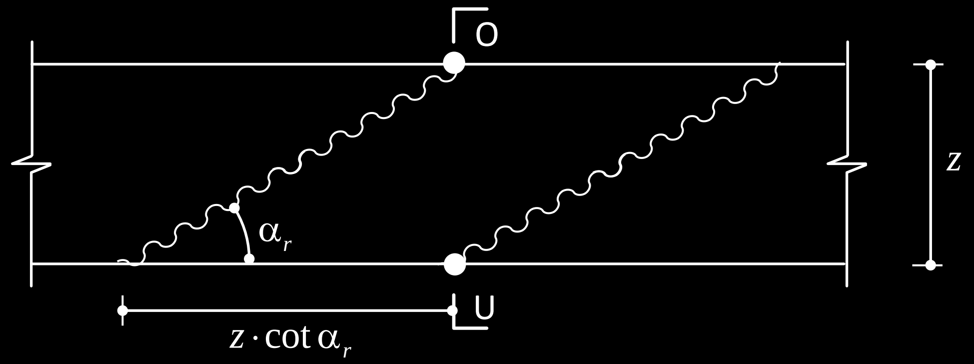
$$\sigma_1 = \tau_{xz} = \frac{v_d}{b_w \cdot z}$$

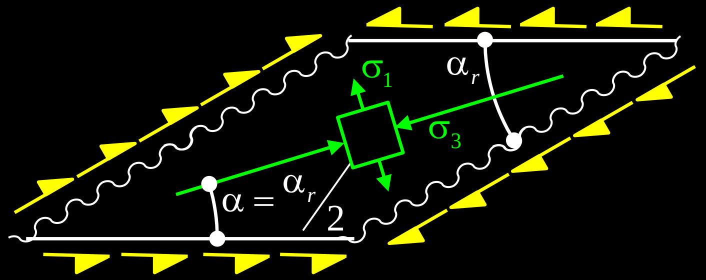


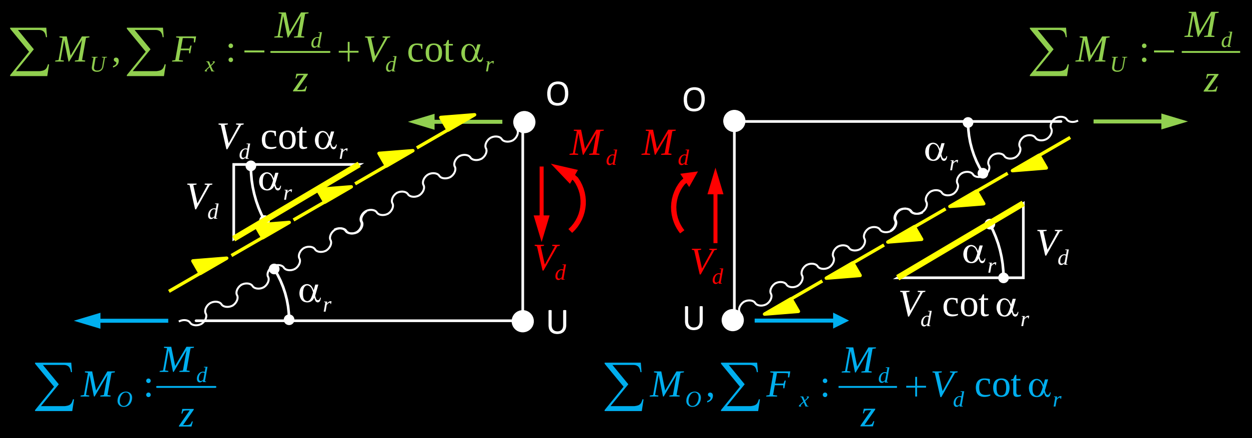
- NB: Longitudinal compressive stresses reduce the principal tensile stress. In earlier editions of SIA 262 (then SIA 162), the shear resistance of prestressed beams was verified on this basis.

Slabs - Influence of shear forces

Web tension failure - Component without shear reinforcement







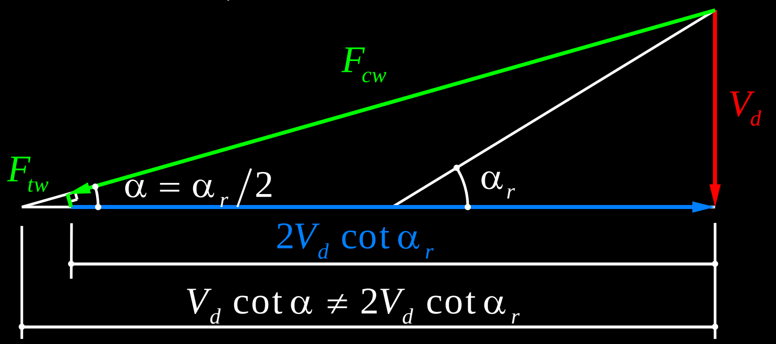
In thin slabs, no failure occurs at first shear crack formation under moderate shear stress, provided that the crack roughness (aggregate interlock) is sufficient and the longitudinal reinforcement has reserves.

(The additional tensile forces in the longitudinal reinforcement due to shear are twice as large as with shear reinforcement!)

Forces acting on a vertical cut:

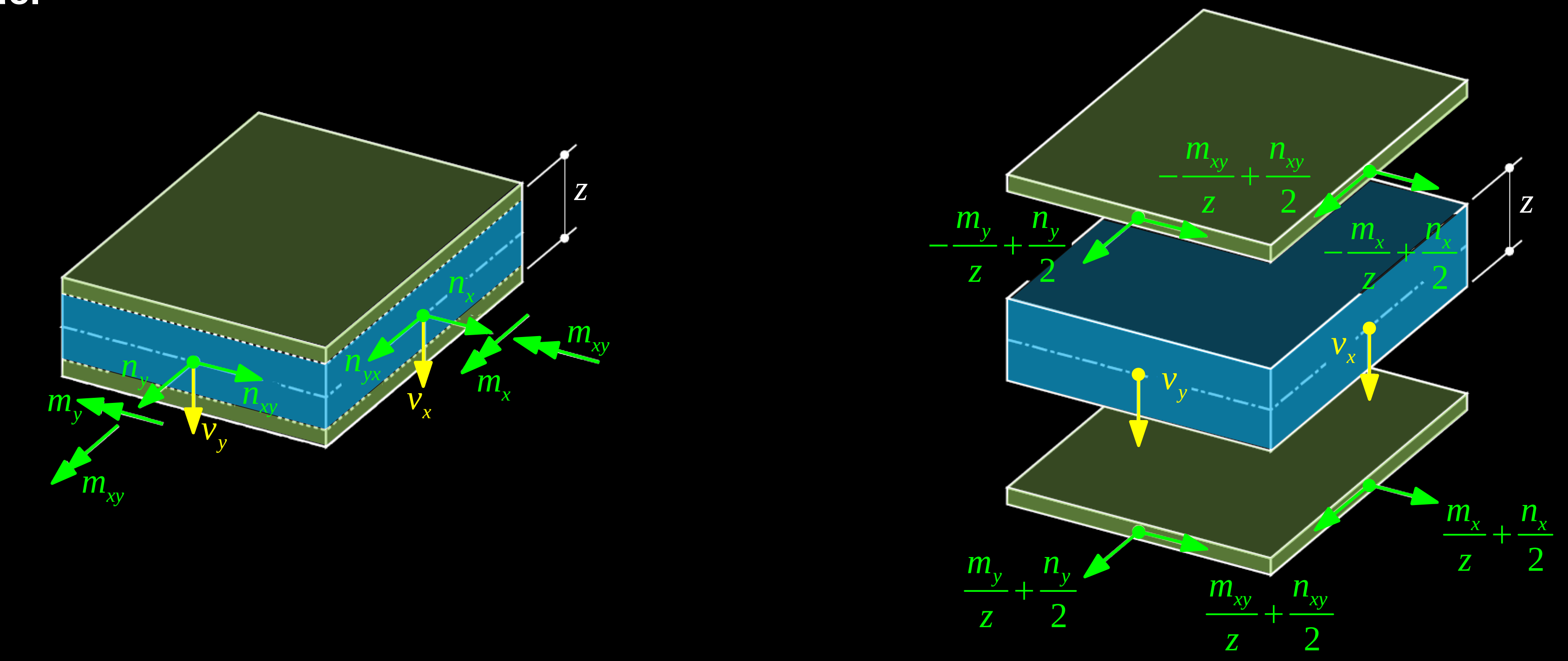
$\alpha = \alpha_r / 2$

$\cot\alpha \approx 2\cot\alpha_r$, but not exactly



Slabs - Influence of shear forces

Sandwich model



Equilibrium solution (general shell loading):

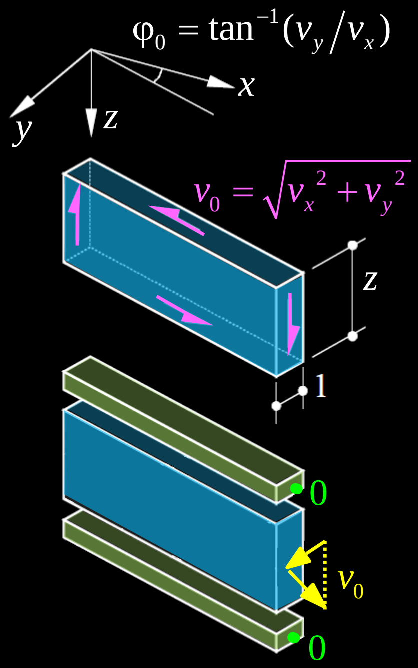
- Sandwich covers carry bending and twisting moments as well as possible membrane forces
  → plane loading, treatment as membrane elements with corresponding reinforcement
  (→ see yield conditions for membrane elements)
- Sandwich core absorbs shear forces
  → Sandwich core absorbs principal shear force v_0 in direction φ_0 and can be treated like the web of a beam in this direction

NB: High membrane (compression) forces: core can also be used for this (take into account interaction with *v*)

Slabs - Influence of shear forces

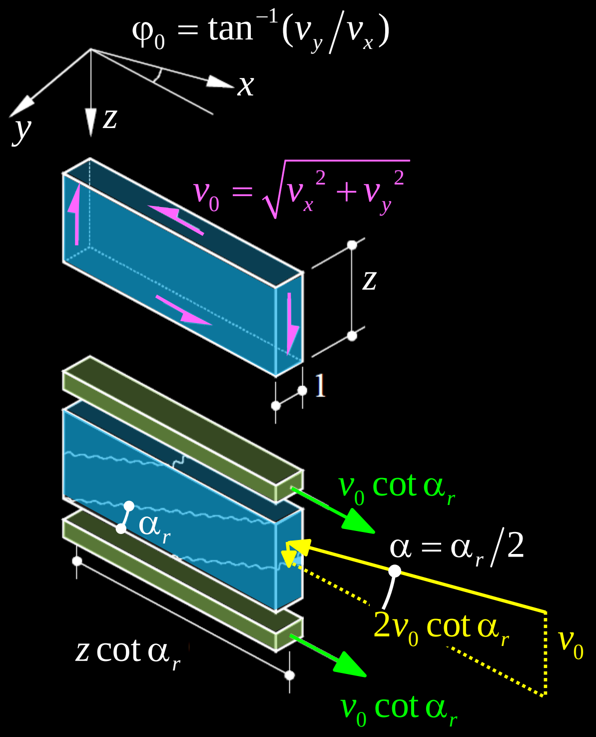
Sandwich model - Core

uncracked (homogeneous)



pure shear

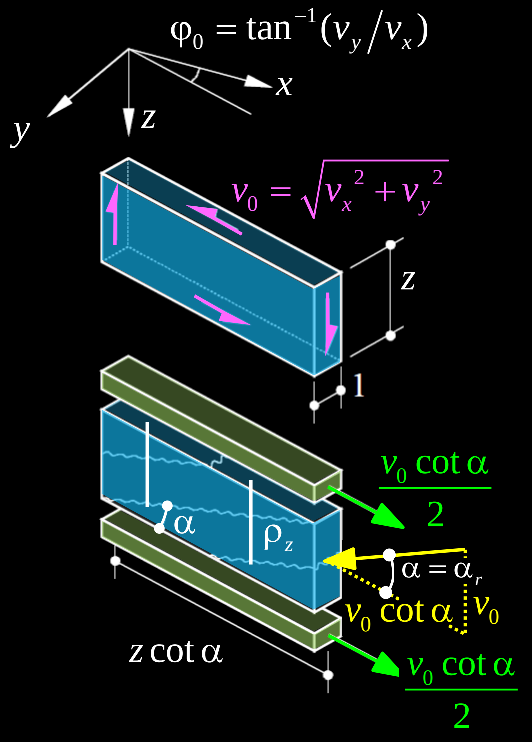
cracked unreinforced



Principal compressive stress in direction $\alpha = \alpha_r / 2$

see previous slides: $2 v_0 \cot \alpha_r \neq v_0 \cot \alpha$

cracked reinforced



required shear reinforcement:

$$\rho_z = \frac{v_0}{f_{sd} \, z \cot \alpha}$$

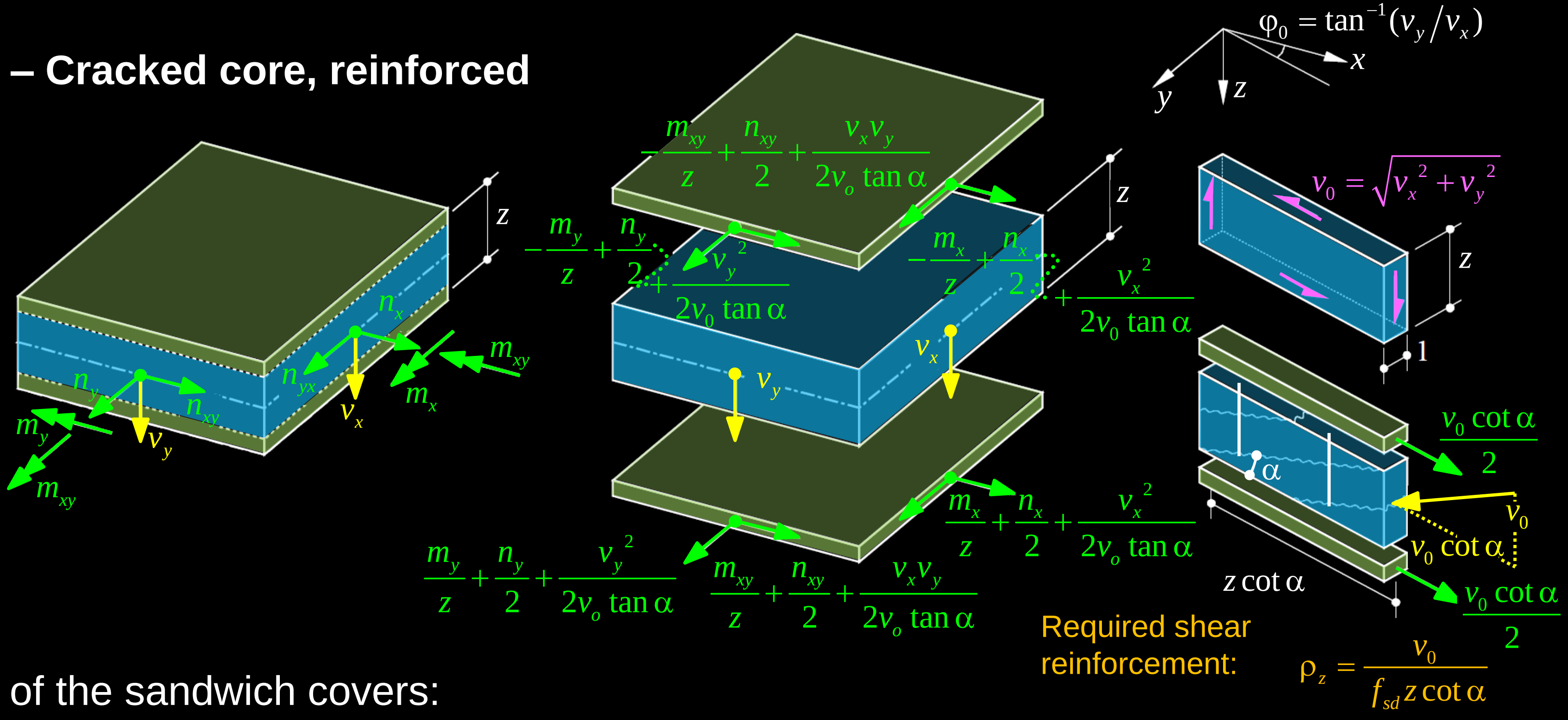
Principal compressive stress in direction $\alpha = \alpha_r$ α_r: crack direction

Principal tensile stress $\sigma_1 = 0$

- Sandwich core carries shear forces
  → Sandwich core carries principal shear force v_0 in the direction φ_0 and can be treated like the web of a beam in this direction.
  Tensile forces in the slab plane are to be carried by the sandwich covers (additional membrane loading).

Slabs - Influence of shear forces

Sandwich model – Cracked core, reinforced



→ Reinforcement of the sandwich covers:

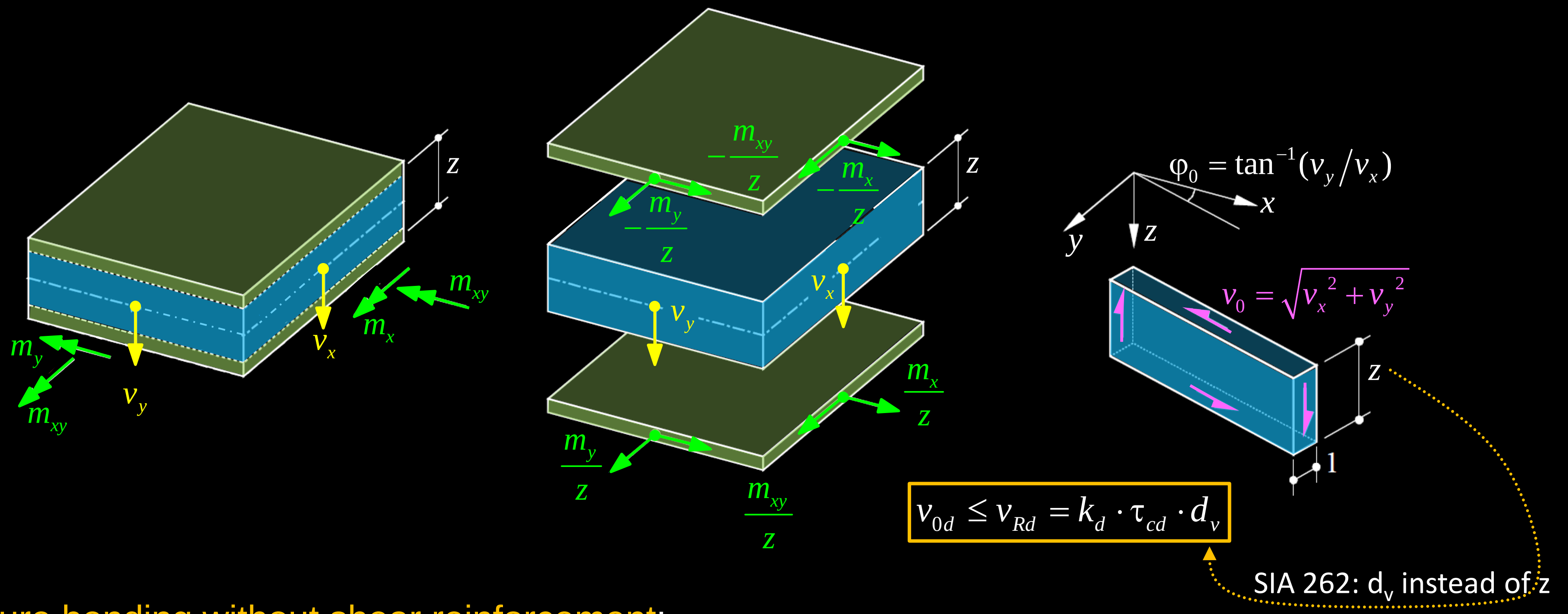
$$a_{sx} f_{sd} \geq \frac{m_x}{z} + \frac{n_x}{2} + \frac{v_x^2}{2v_0 \tan\alpha} + k\left|\frac{m_{xy}}{z} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan\alpha}\right| \qquad a'_{sx} f_{sd} \geq -\frac{m_x}{z} + \frac{n_x}{2} + \frac{v_x^2}{2v_0 \tan\alpha} + k'\left|-\frac{m_{xy}}{z} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan\alpha}\right|$$

$$a_{sy} f_{sd} \geq \frac{m_y}{z} + \frac{n_y}{2} + \frac{v_y^2}{2v_0 \tan\alpha} + \frac{1}{k}\left|\frac{m_{xy}}{z} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan\alpha}\right| \qquad a'_{sy} f_{sd} \geq -\frac{m_y}{z} + \frac{n_y}{2} + \frac{v_y^2}{2v_0 \tan\alpha} \frac{1}{k'}\left|-\frac{m_{xy}}{z} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan\alpha}\right|$$

(The factors k, k' can in principle be selected differently at each point of the slab (avoid abrupt changes or anchor differential reinforcement forces). Selection of the compression field inclination α: Analogous considerations as with beams. (Often $k = k' = \cot\alpha = 1$ is chosen.)

Slabs - Influence of shear forces

Sandwich model - Pure bending, uncracked core



→ Slabs under pure bending without shear reinforcement:
$n_x = n_y = n_{xy} = 0$, $v_{0d} \leq v_{Rd} = k_d \tau_{cd} d_v$

→ Terms with n_x, n_y, n_{xy} disappear

→ Terms with v_x, v_y disappear if an uncracked core is assumed.

→ With aggregate interlock according to slide 4, at least twice the longitudinal reinforcement (2·terms with v_x, v_y) is required as a result of shear force → Reinforcement in slabs without shear reinforcement should not be graded / curtailed too early!

Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement according to SIA 262

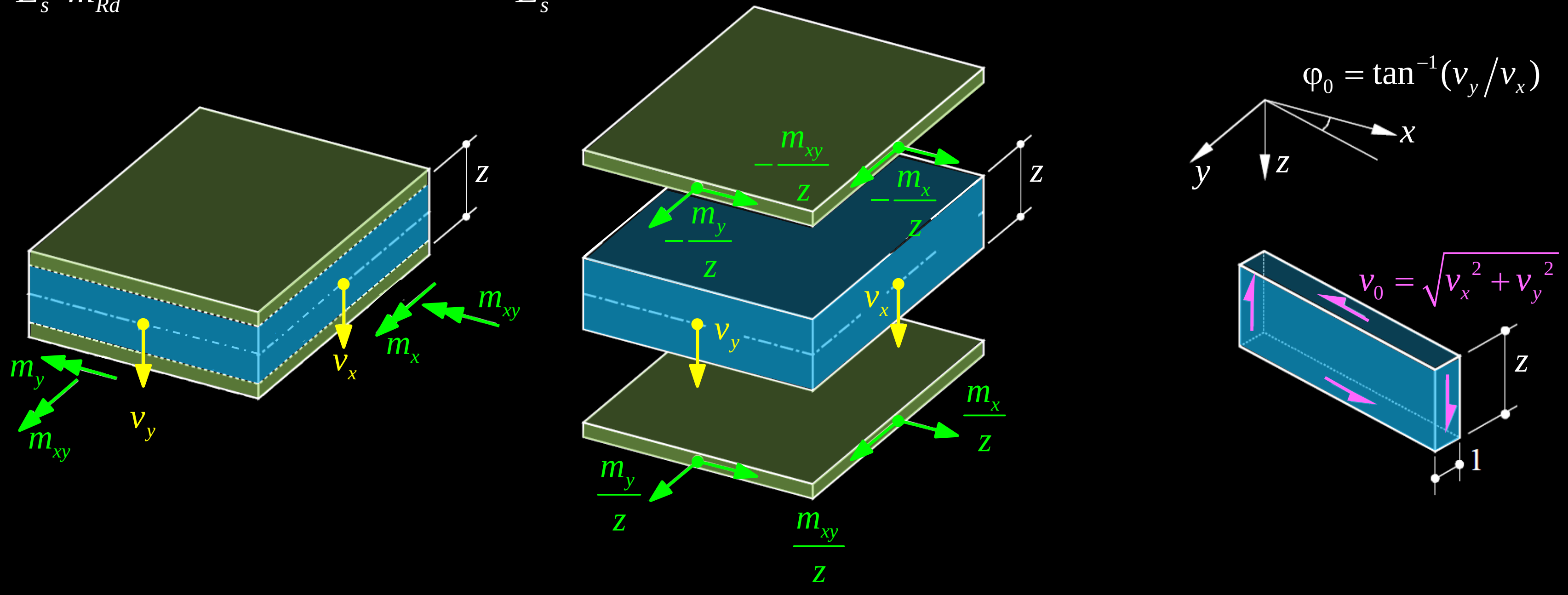
Nominal shear resistance without shear reinforcement

$$v_{Rd} = k_d \cdot \tau_{cd} \cdot d_v \qquad \text{with} \qquad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

$$k_d = \frac{1}{1+\varepsilon_v \cdot d \cdot k_g} \qquad \text{with} \qquad k_g = \frac{48}{16+D_{max}}$$

$$\varepsilon_v = \frac{f_{sd}}{E_s}\frac{m_d}{m_{Rd}} \qquad \text{resp.} \qquad \varepsilon_v = 1.5\frac{f_{sd}}{E_s}$$

- k_d: Reduction factor for static depth of the slab, utilization of longitudinal reinforcement and maximum aggregate size
- d_v: Effective static depth taking into account cross-section discontinuities
- ε_v: Strain of bending reinforcement ($1.5\, f_{sd}/E_s$ applies to plastic deformations, +50% in case of graded longitudinal reinforcement)



Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement according to SIA 262

Nominal shear resistance without shear reinforcement

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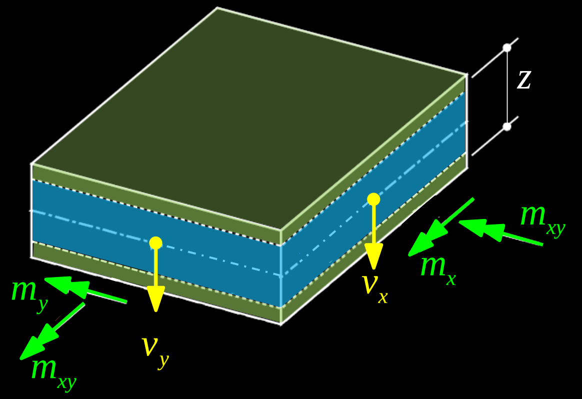
$$\varepsilon_v = \frac{f_{sd}}{E_s}\frac{m_d}{m_{Rd}} \quad \text{resp.} \quad \varepsilon_v = 1.5\frac{f_{sd}}{E_s}$$

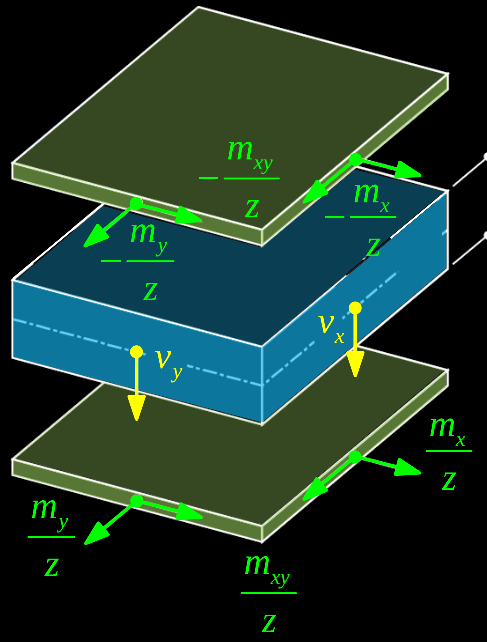
(Pre-)dimensioning, B500B, D_{max} = 32 mm:

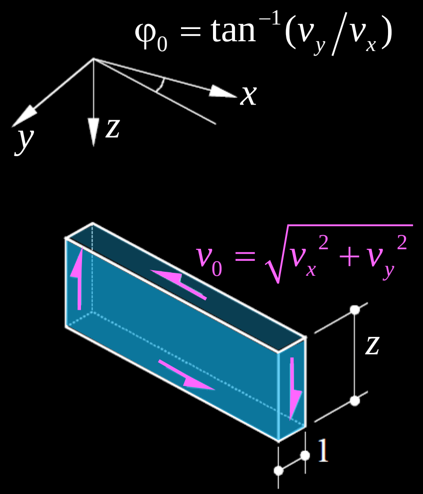
k_g = 1.0; m_d / m_{Rd} = 1.0 (no plastic redistribution)

$\rightarrow \varepsilon_v = f_{sd} / E_s$ = 2.12‰

$$\rightarrow v_{Rd} = \frac{\tau_{cd} \cdot d_v}{1+\dfrac{d}{471\text{ mm}}}$$







Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement according to SIA 262

Nominal shear resistance without shear reinforcement

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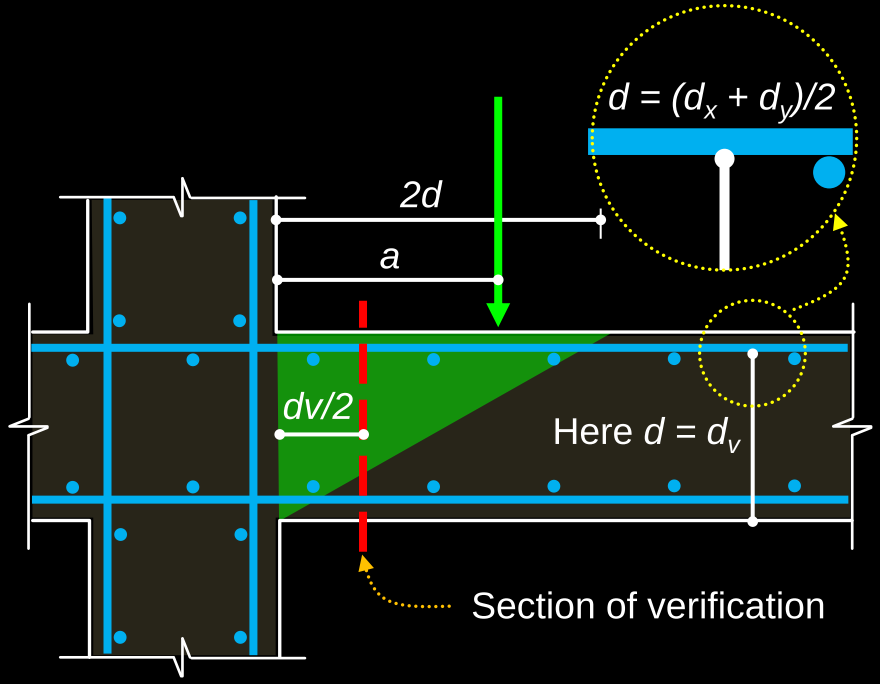
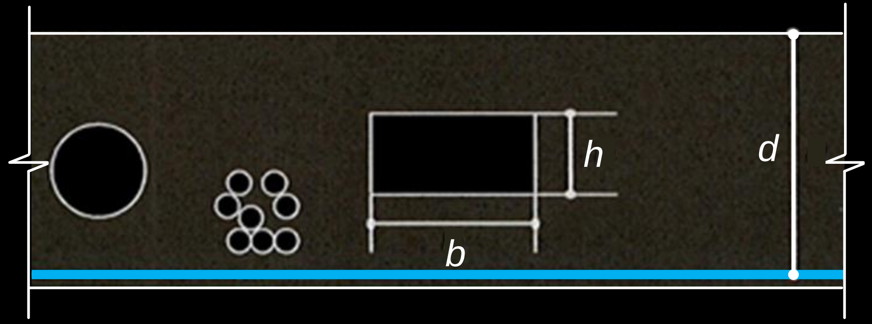
$$k_d = \frac{1}{1+\varepsilon_v \cdot d \cdot k_g} \quad \text{with} \quad k_g = \frac{48}{16+D_{\max}}$$

$$\varepsilon_v = \frac{f_{sd}}{E_s}\frac{m_d}{m_{Rd}} \quad \text{resp.} \quad \varepsilon_v = 1.5\frac{f_{sd}}{E_s}$$

- Control section at a distance d_v /2 from the support edge or edge of the load, if necessary at reinforcement gradations
- Reduction of concentrated loads at distance *a* < 2d from bearing edge with factor a/(2*d*) permissible
- Ducts, pipes:
  Diameter / width / height > d/6
  (for cable bundles: dimension of the entire bundle)
  Reduction of d_v by the largest dimension of the inlay or pipe
  ($d_v = d - \max(b, h)$)





Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement according to SIA 262

Nominal shear resistance without shear reinforcement

$$v_{Rd} = k_d \cdot \tau_{cd} \cdot d_v \qquad \text{with} \qquad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

$$k_d = \frac{1}{1+\varepsilon_v \cdot d \cdot k_g} \qquad \text{with} \qquad k_g = \frac{48}{16+D_{\max}}$$

$$\varepsilon_v = \frac{f_{sd}}{E_s}\frac{m_d}{m_{Rd}} \qquad \text{resp.} \qquad \varepsilon_v = 1.5\frac{f_{sd}}{E_s}$$

- slab with prestress or normal force, with decompression moment m_{Dd} : $\varepsilon_v = \frac{f_{sd}}{E_s}\frac{m_d - m_{Dd}}{m_{Rd} - m_{Dd}}$
  ... m_{Dd} = long-term value of the decompression moment (see chapter punching) accounting for normal forces (e.g. due to restraint by stiff supports)
  ... m_d = incl. moments due to restraint and imposed deformations (e.g. secondary moments from prestressing)
- Concrete compressive strength f_{ck} > 70 MPa: $D_{\max} = 0$, this means $k_g = 3$ ($\rightarrow v_{Rd}(f_{ck})$ is discontinuous at 70 MPa)
- Clear deviation of the principal direction φ_0 of the shear force from the direction of the principal reinforcement by angle ϑ: increase of elongation ε_v with factor $\frac{1}{\sin^4\vartheta + \cos^4\vartheta}$
  (i.e. in the worst case, ϑ = 45°: factor 2)

5 Slabs

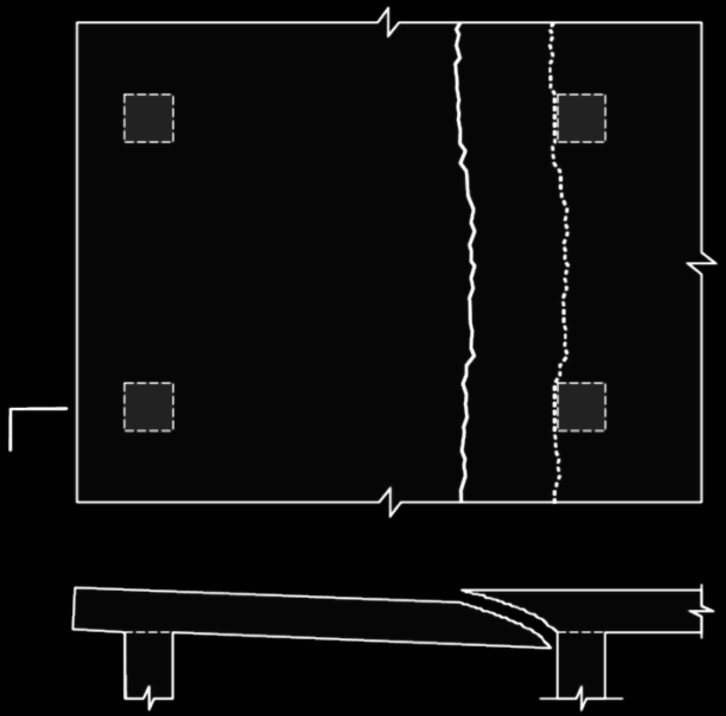
In-depth study and additions to Stahlbeton II

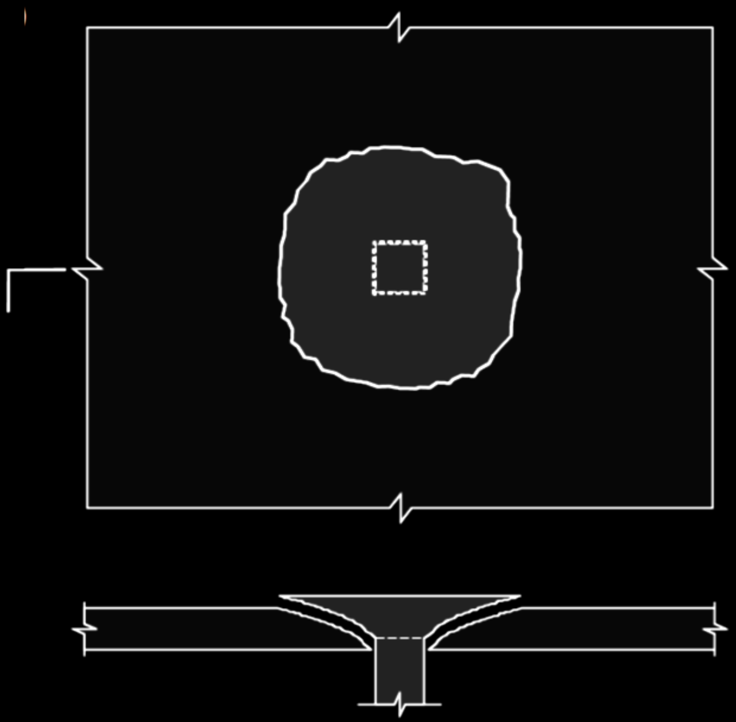
5.6 Punching

Slabs - Influence of shear forces

Slabs without shear reinforcement - Failure mechanisms

- Shear failures as shown in the figure on the left are unlikely in thin slabs. Still, slabs subjected to high loads and primarily carrying in one direction, such as top and bottom slabs of cut-and-cover tunnels may be critical.
- Near concentrated loads (e.g. around columns supporting a flat slab, or supported by a slab on ground), transverse shear forces are often very high. If no shear reinforcement is provided, this can lead to a sudden, very brittle failure (punching).

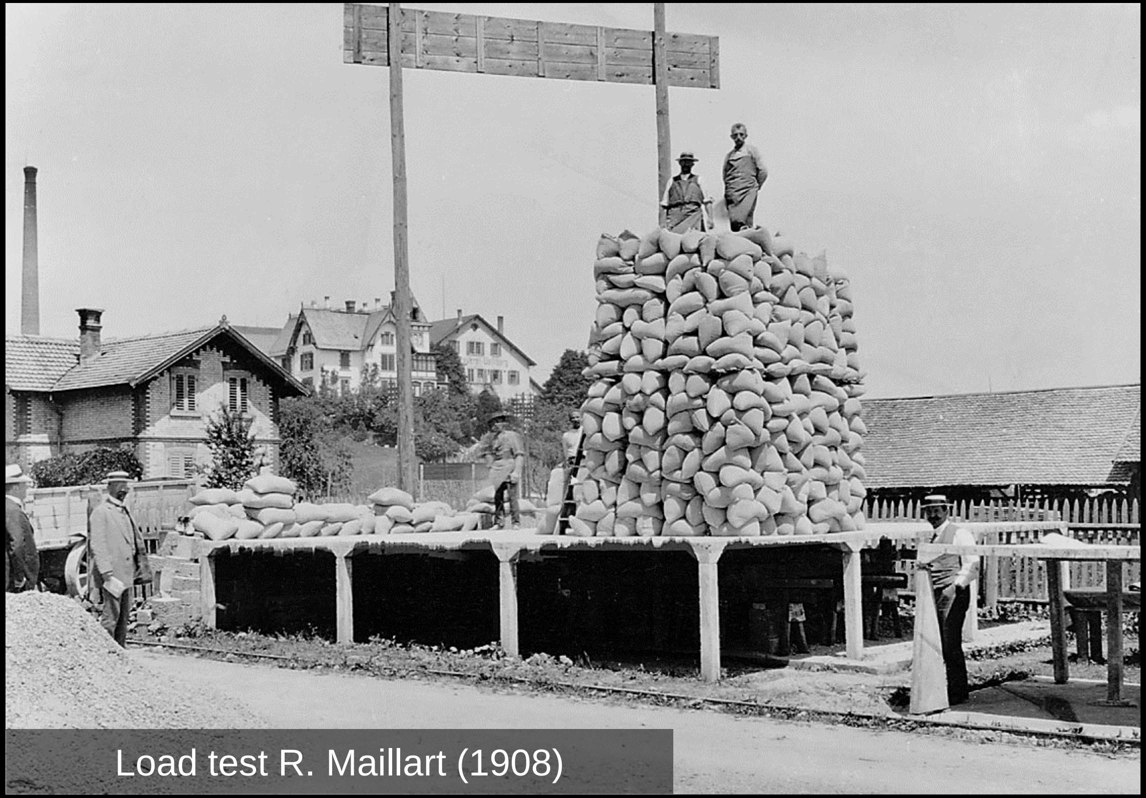




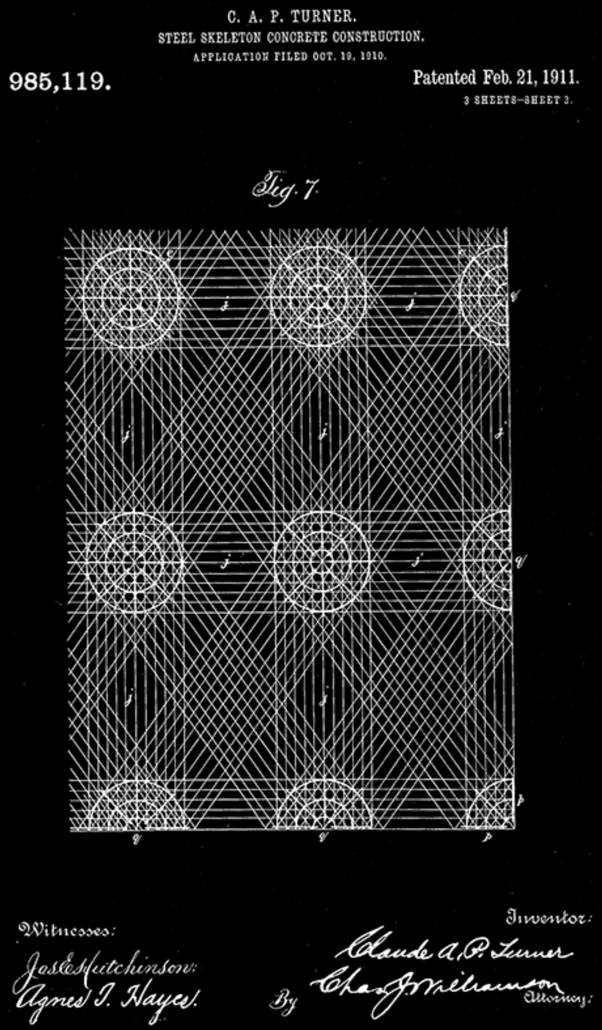
Slabs - Influence of shear forces

Punching

- Flat slabs: load concentration at the supports, maximum v_0 and (m_x, m_y), bending moments with large gradient (elastic solution with point support: m_x *and* $m_y \rightarrow \infty$)
- With respect to the force flow, mushroom slabs are significantly better
- Early days of concrete construction: Flat slabs as a new type of construction
  → Mushroom slab systems Maillart / Turner, fully flat slabs only later



Load test R. Maillart (1908)



Patent specification C.A.P. Turner (1911)

Slabs - Influence of shear forces

Punching

- Flat slabs without shear reinforcement: very brittle failure, progressive collapse possible
- Parking structures are particularly at risk: vehicle fire, corrosion, earth cover exceeding design specification, …
- The punching resistance according to SIA 262 (2003) is significantly reduced with respect to earlier codes (in partial revision 2013 even more strict provisions were introduced)
  → many old buildings are not code-compliant


Gretzenbach (2004)

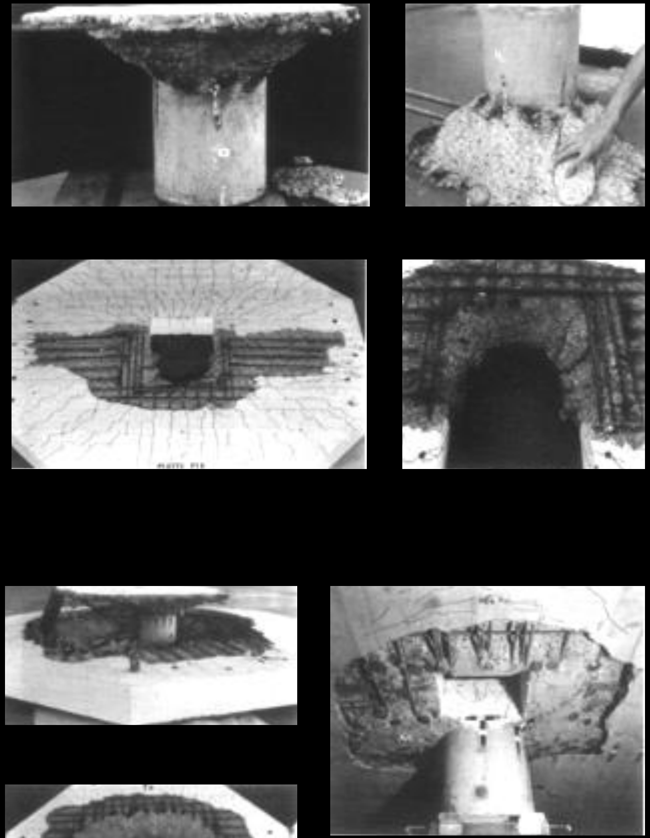

Wolverhampton (1997)

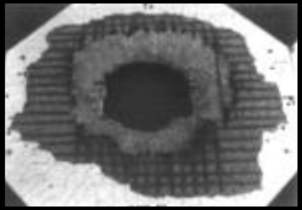

Bluche (1981)

Slabs - Influence of shear forces

Punching

Early on many experimental studies worldwide, including ETH Zurich, EMPA

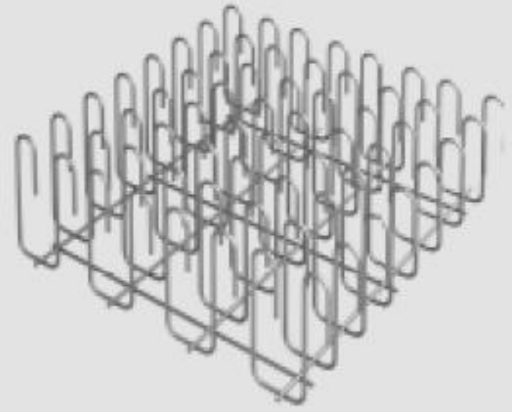




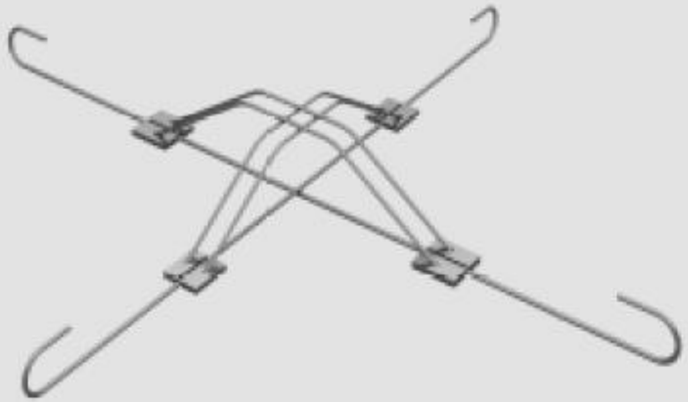
[Thürlimann/Pralong 1979-1984]

Slabs - Influence of shear forces

Conceptual solution to the problem: Punching shear reinforcement (or mushroom slabs!)

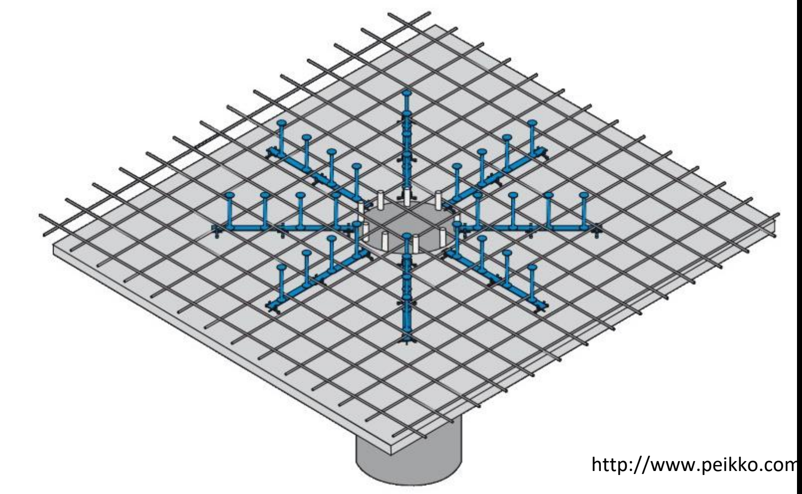


stirrup cage

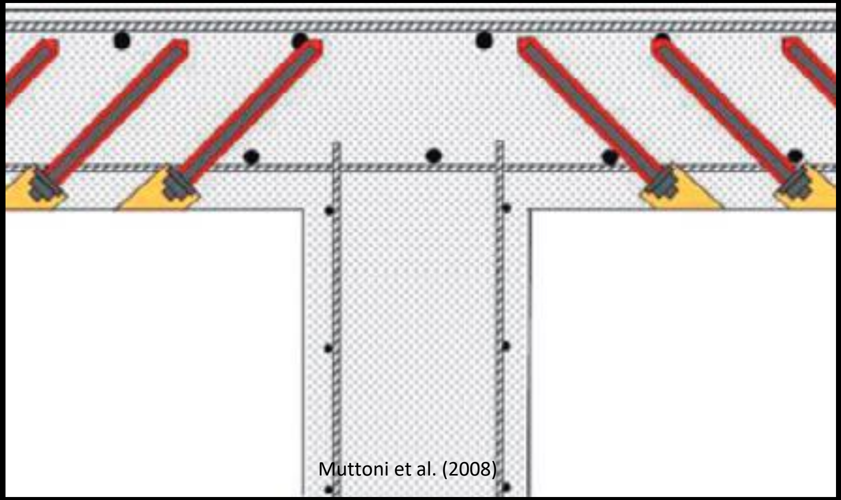


bent reinforcement

Steel Forms



Dowels (Studs)



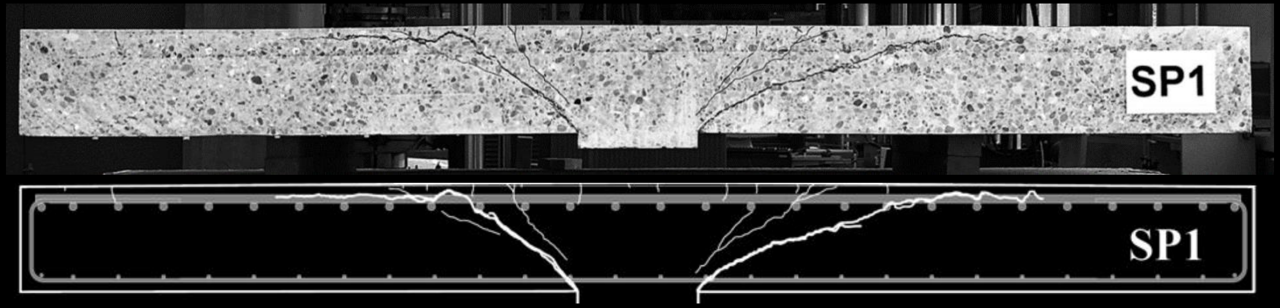
Reinforcing (post-installed) anchors

Slabs - Influence of shear forces

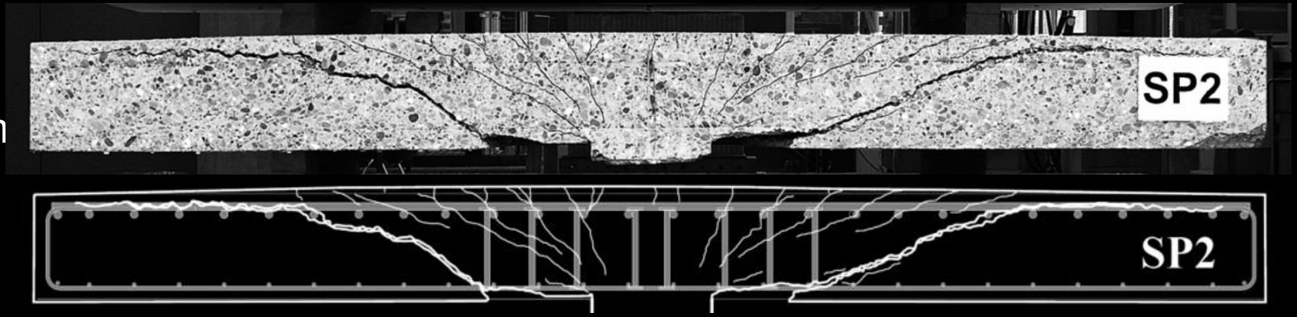
Punching : Types of failure

Example: experiments by Etter, Heinzmann, Jäger, Marti (2009)
IBK report 324

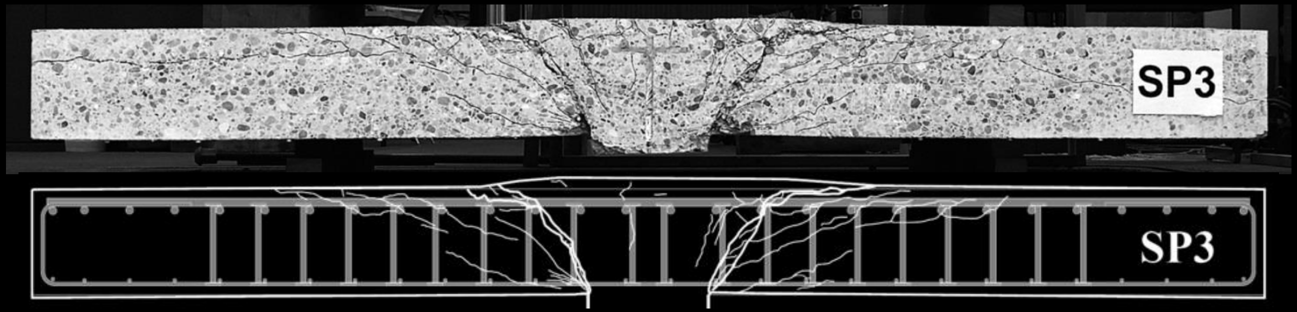
- failure at «inner perimeter»
  (here without punching reinforcement)



- failure at «outer perimeter»
  (section defined by the extent of punching reinforcemen



- «compression strut» failure
  (with high amount and extent of punching reinforcement)

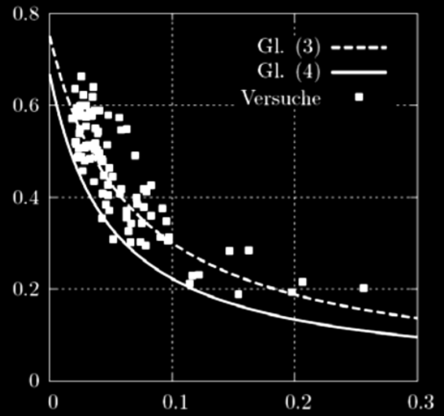


Slabs - Influence of shear forces

Punching: Mechanical model implemented in SIA 262

- Research focus of Prof. Muttoni at ETH Lausanne: Since 2000 various series of experiments (among others with Fernández Ruiz, Guandalini, Guidotti, Lips, Kunz)

[EPFL - ibeton]



→ Governing parameter: State of strain in the support area (→ bending deformations, as already identified e.g. by Kinnunen / Nylander in 1960 and considered in SIA 162/1968 ("Guideline 18"), but not included in standard SIA 162/1989 to avoid complicating the design by using deformation-dependent strength criteria).

→ Model for slabs without shear reinforcement (basis of the design according to SIA 262 and *fib* Model Code 2010): Failure occurs when a critical shear crack is too wide to be able to transfer the shear (hyperbolic failure criterion closely related to relationships for compression softening):

If curvatures due to bending are neglected:
(crack opening) ~ (slab rotation ψ) · (static depth d)

Slabs - Influence of shear forces

Punching resistance of slabs according to SIA 262

Conceptual provisions

- The deformation capacity of slabs subjected to concentrated loads shall be achieved by the following measures:

  → Either ensure a nominal slab rotation (capacity) $\psi > 0.02$ under the design load V_d
  (i.e. do not overdimension bending reinforcement, choose a sufficiently large supporting area and slab thickness)

  → Or provide a punching reinforcement with $V_{Rd,s} \geq V_d /2$ (*)

  Otherwise, imposed deformations must be taken into account in the design (constraint forces due to restrained temperature changes, differential settlements, shrinkage, etc.).
  → May cause strong variation (increase) of the load V_d, very difficult to quantify: avoid!

(*) according to fib Model Code 2010: $V_{Rd,s} \geq V_d /2$ with $\sigma_{sd} = f_{sd}$ (SIA 262: not specified)

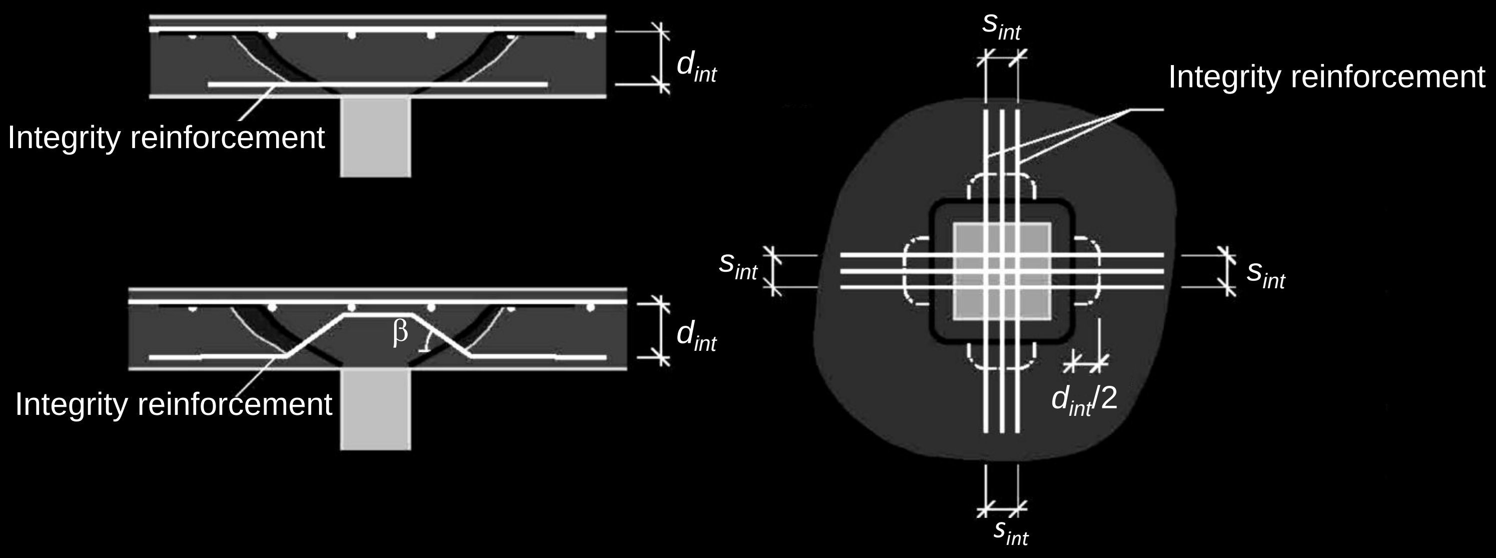
Slabs - Influence of shear forces

Punching resistance of slabs according to SIA 262

Conceptual provisions

- To avoid a progressive collapse (due to punching in spite of a code-compliant design), at least one of the following measures shall be taken:
  → Provide a punching reinforcement with $V_{d,s} \geq V_d/2$ (*)
  → Provide integrity reinforcement preventing a collapse in case of punching (details see SIA 262, 4.3.6.7)

(*) according to fib Model Code 2010: $V_{Rd,s} \geq V_d/2$ with $\sigma_{sd} = f_{sd}$ (SIA 262: not specified)



Slabs - Influence of shear forces

Punching resistance of slabs according to SIA 262

Verification format

The punching resistance is determined on the basis of nominal transverse shear stresses as follows:

$$V_{Rd,c} = k_r \cdot \tau_{cd} \cdot d_v \cdot u \qquad \text{with} \quad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

- k_r Coefficient for static depth of the slab, slab rotation, and maximum aggregate size
- d_v Effective static depth in mm
- u control perimeter

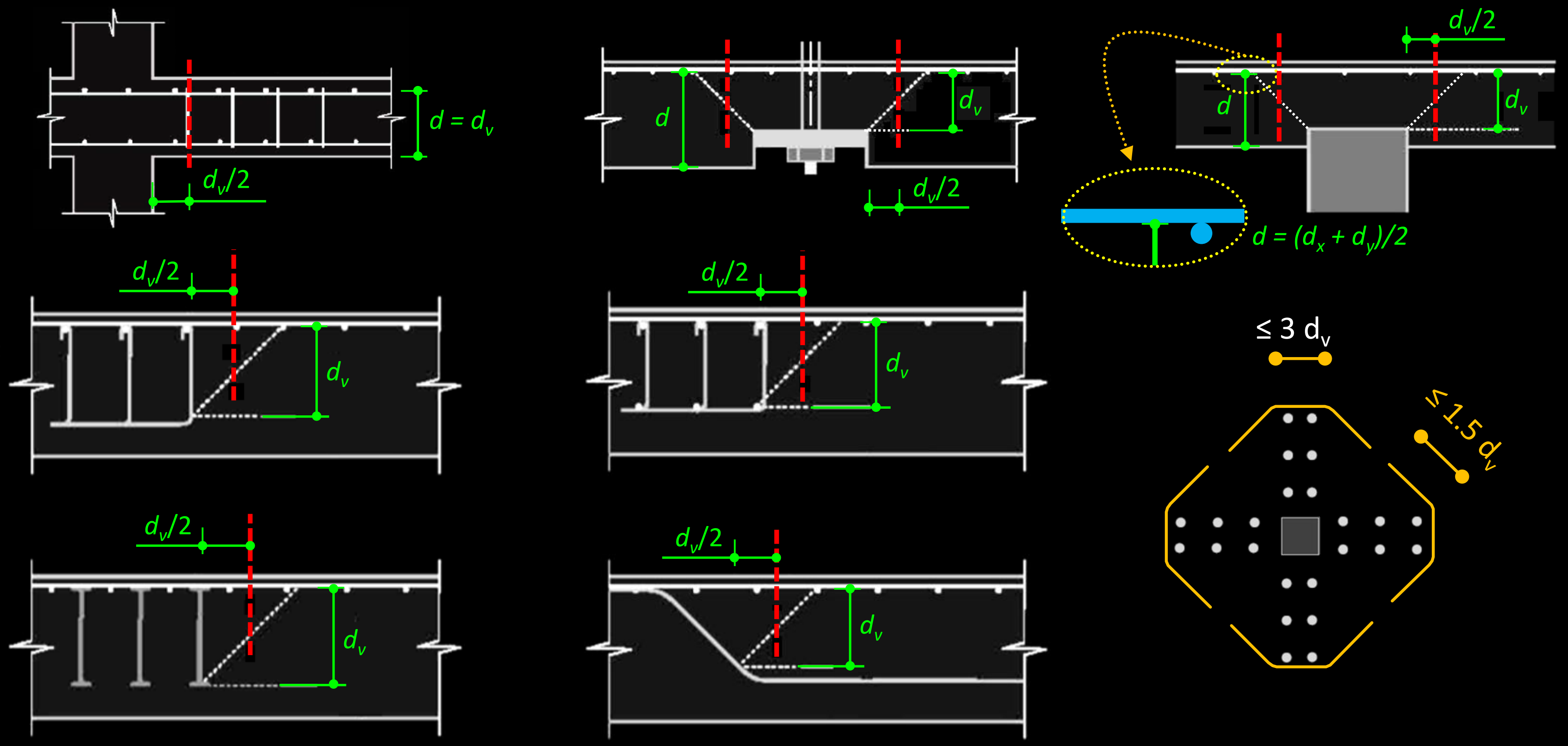
The coefficient k_r depends primarily on the utilisation of the bending reinforcement over the support, which is determined over the width b_s of a nominal "support strip" in each reinforcement direction.

In the following, first the geometrical parameters (effective static depth d_v, control perimeter u, width of the support strip b_s) and then the coefficient k_r are explained.

Slabs - Influence of shear forces

Punching: Control perimeter and support strip

- Effective static depth d_v according to figures below
- Effective static depth d_v to be taken into account when determining the location of the control perimeter

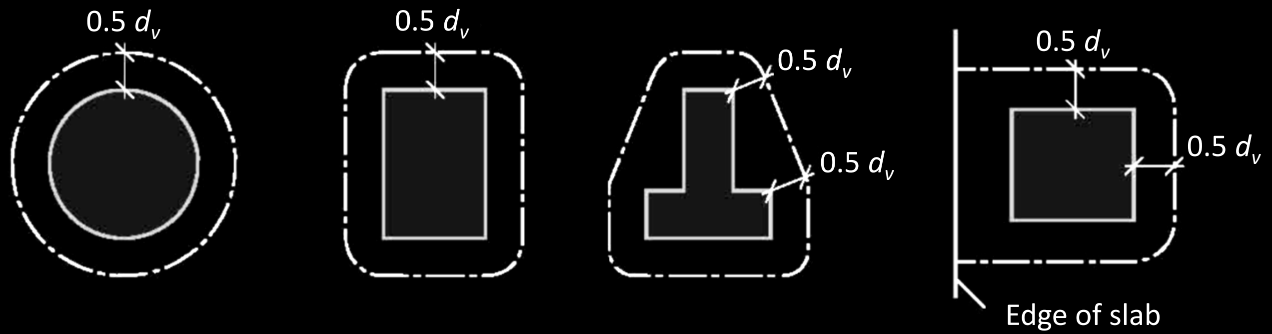


Slabs - Influence of shear forces

Punching: Control perimeter and support strip

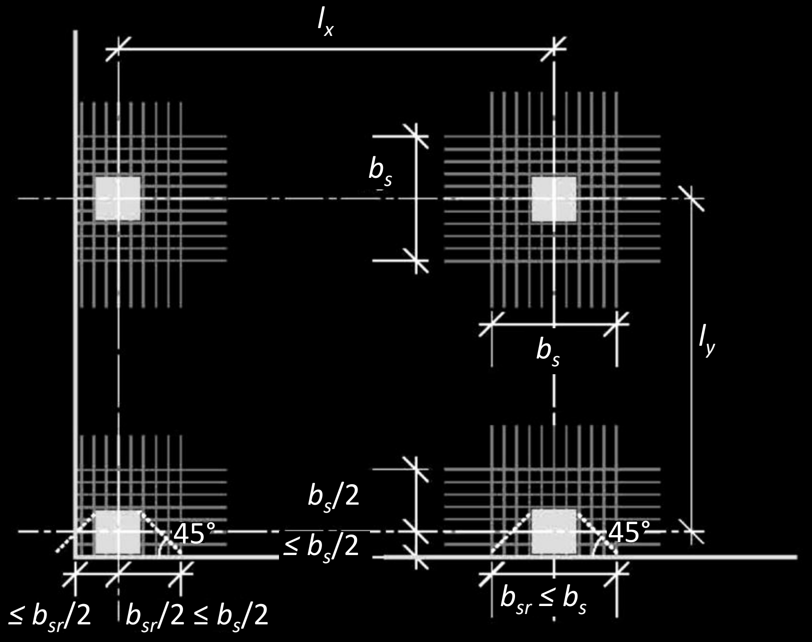
Control perimeter (convex line at distance ≥ $d_v/2$ from support edge → length u)

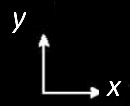
NB: Actions within the control perimeter may be deducted from the design value of the punching load (self weight, foundation stresses, deviation forces from prestressing, etc.)



Support strip (width b_s)

NB: Bending demand m_{sd} and bending resistance m_{Rd} to be used in formulas for k_r (see following slides): mean values over the width of the support strip





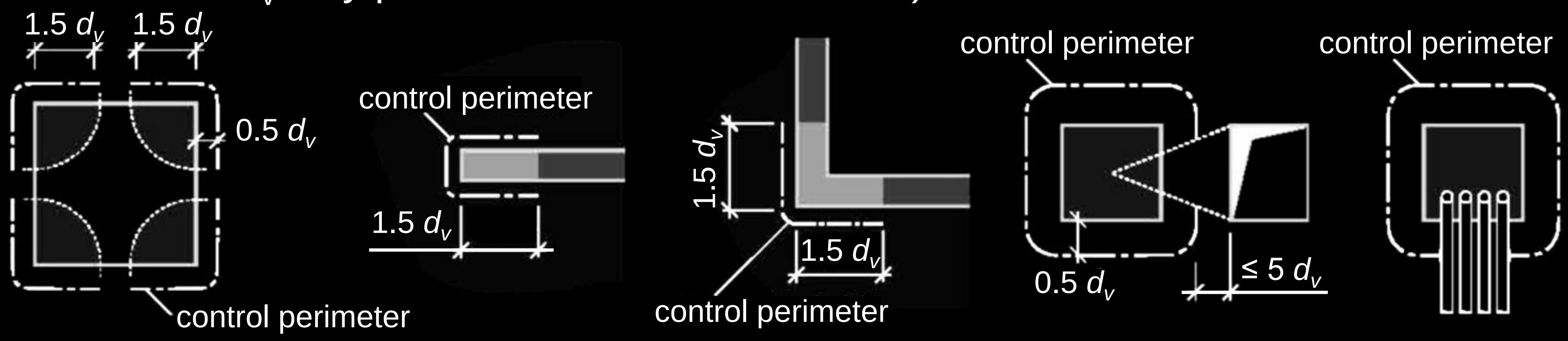
$$b_s = 1{,}5\sqrt{r_{sx} \cdot r_{sy}} \le l_{min}$$

Slabs - Influence of shear forces

Punching: Reference section

Reduction of the length of the control perimeter to account for non-constant distribution of the shear forces along the perimeter

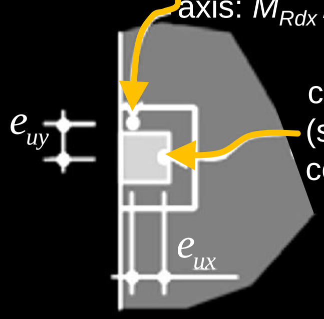
- Consideration of load concentrations in corners, recesses, pipes / ducts, etc.
  (pipes / ducts at a distance < 5d_v only permissible in radial direction)!



- Additional reduction of the control perimeter for moment transmission column-slab by the coefficient k_e (simplifying the curvatures of the control perimeter as corners):

$$k_e = \frac{1}{1 + \frac{e_u}{b}}$$

$$e_u = \sqrt{e_{ux}^2 + e_{uy}^2}$$



Approximation for regularly supported flat slabs, supports rigidly connected, supports do not carry horizontal actions:

- k_e = 0.90 Interior supports
- k_e = 0.75 Wall ends, wall corners
- k_e = 0.70 Edge supports, interior supports with large recesses near the columns
- k_e = 0.65 Corner supports

5 Slabs

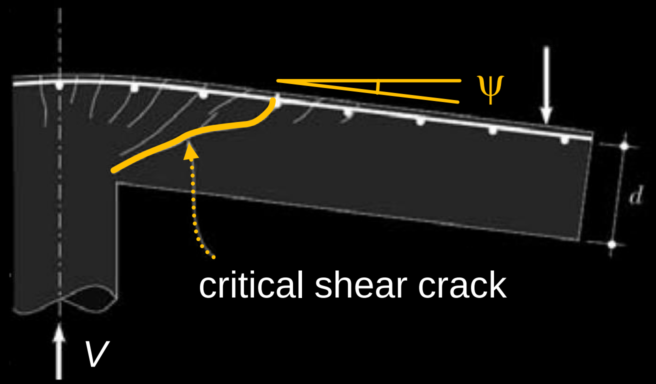
In-depth study and additions to Stahlbeton II

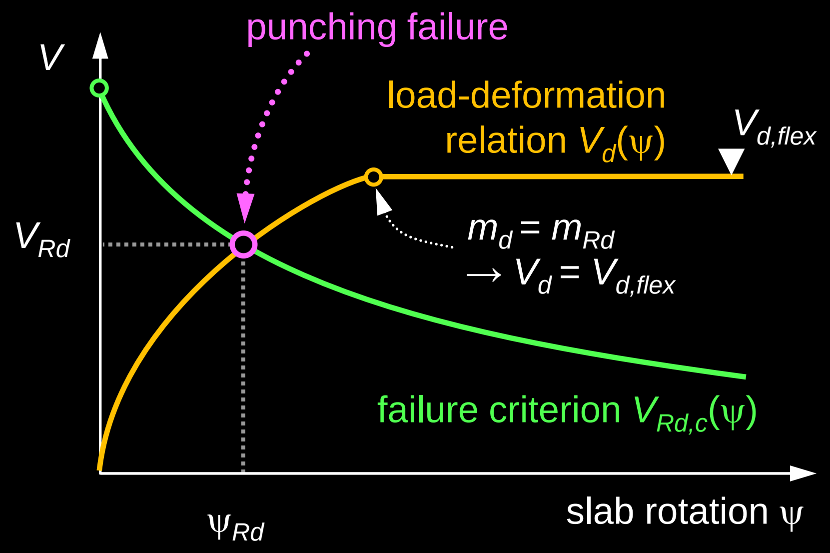
5.6.1 Behaviour without punching reinforcement

Slabs - Influence of shear forces

Punching of slabs without punching reinforcement according to SIA 262

- Basic model: critical shear crack fails if it has opened too much and can no longer transfer the load
- Opening of the critical shear crack (and hence, the punching resistance) is related to the slab rotation ψ through a relationship derived from mechanical considerations and calibrated on experiments → failure criterion $V_{Rd} = V_{Rd}(\psi)$
- An analytical relationship $\psi = \psi(m_{sd}/m_{Rd})$ is established, based on the model, between the slab rotation ψ and the ratio of applied bending moment to bending resistance (m_{sd}/m_{Rd}) in a nominal support strip
- By linking m_{sd} to the support reaction V_d (see following slides) one obtains the load-deformation relationship $\psi = \psi(V_d)$ and hence, $V_d = V_d(\psi)$





Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262

$$V_{Rd,c}(\psi) = k_r \cdot \tau_{cd} \cdot d_v \cdot u \qquad \text{with} \quad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

$$k_r = \frac{1}{0.45 + 0.18 \cdot \psi \cdot d \cdot k_g} \leq 2 \qquad \text{with} \quad k_g = \frac{48}{16 + D_{max}}$$

$$\psi = 1.5 \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd}}{m_{Rd}} \right)^{3/2}$$

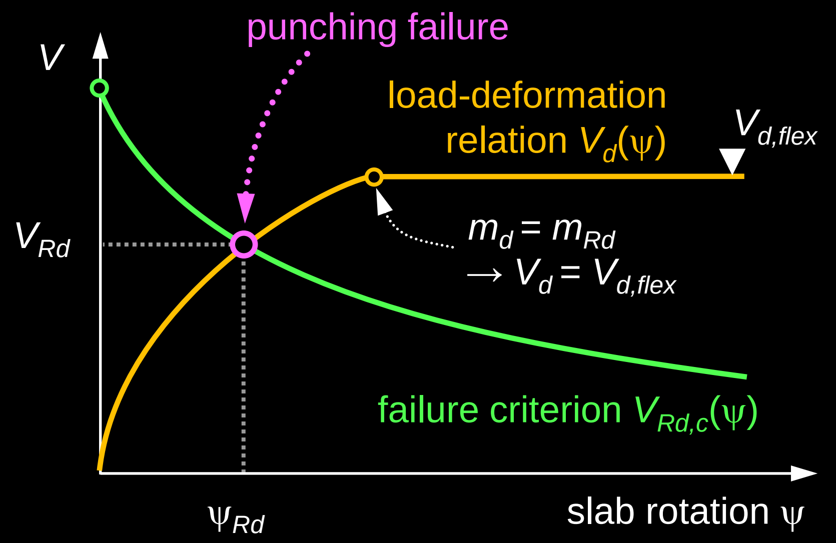
(determine m_{sd}, m_{Rd} und r_s for directions x, y separately, larger value of ψ controls)

- k_r Coefficient for static depth of the slab, slab rotation, and maximum aggregate size
- d_v Effective static depth in mm
- u Control perimeter
- r_s Distance of the point of zero moment (radial moment = 0) from support axis
- m_{sd} Average bending moment in the support strip
- m_{Rd} Average bending resistance in the support strip

Note: The load-deformation relationship does not have to be determined in design (i.e., in the verification whether punching reinforcement is required for a given action V_d).

However, it is needed to calculate the actual punching resistance according to the code.

See the following slides for more details.



Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262

Dimensioning (only governing direction shown (determine ψ_d for m_{sd}, $m_{Rd,}$ and r_s in directions x, y separately, smaller value of V_{Rd} governs)

Given: V_d, support dimensions, static depth (and thus u)

Question: Is punching resistance sufficient without shear reinforcement / are the slab thickness and bending reinforcement sufficient?

Procedure

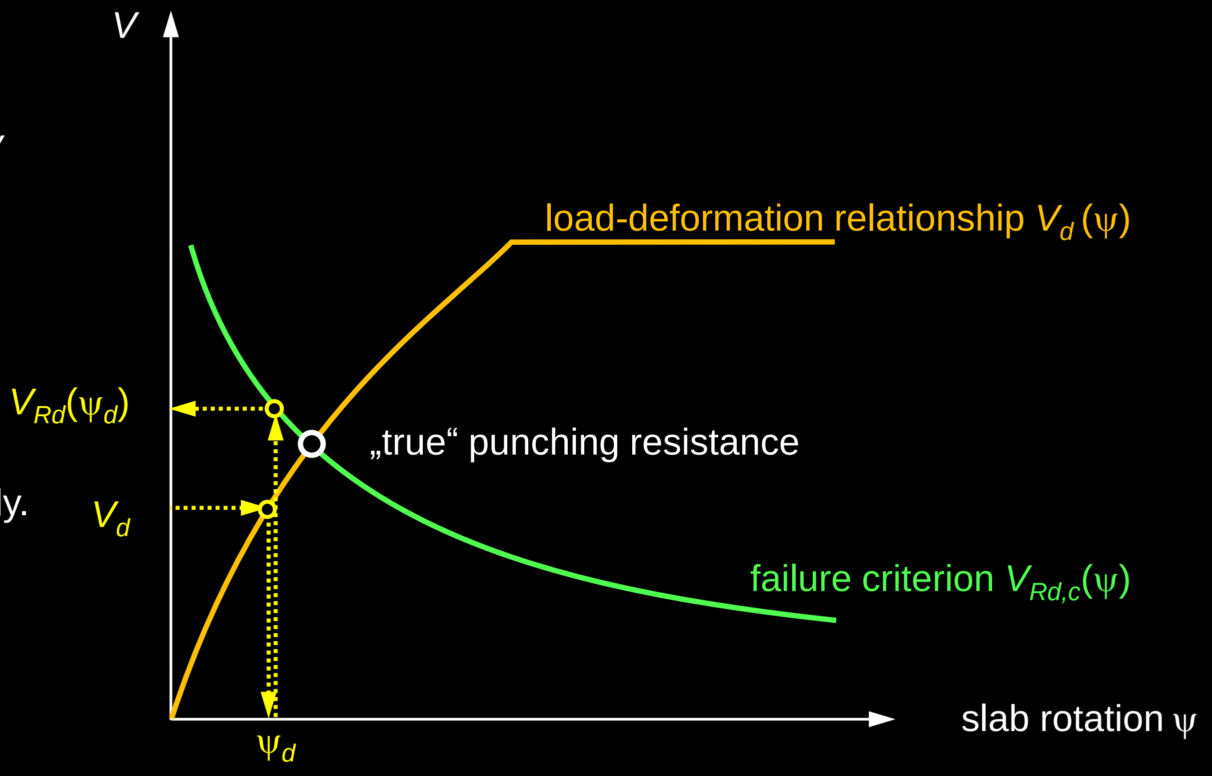
1. Assume d and m_{Rd} (select a reasonable reinforcement)
2. Determine of r_s and m_{sd} $(V_d) \rightarrow \psi_d \rightarrow V_{Rd}(\psi_d)$ per direction x, y (different levels of approximation, see following slides)
3. Increase d and / or m_{Rd}, until $V_{Rd}(\psi_d) > V_d$ (or provide punching reinforcement)

NB: The resulting value $V_{Rd}(\psi_d)$ is greater than the actual punching resistance V_{Rd}.

The «true» value of V_{Rd} would have to be determined iteratively. (intersection of the curves $V_{Rd,c}(\psi)$ and $V_d(\psi)$).

This is unnecessary in design, which can be done without the determination of the load-deformation relationship $V_d(\psi)$.

The determination of the actual punching resistance is explained in more detail in the following slide.



Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262

Check / Verification of existing structures (only the governing direction is shown!)

Given: Support dimension (and thus u), d, m_{Rd}
Question: What is the punching resistance (without shear reinforcement)?

$V_{flex,\,sd}$ Support reaction at which the bending reinforcement yields (in the considered direction)

ψ_{sd} Slab rotation when reaching $V_{flex,\,sd}$

Procedure

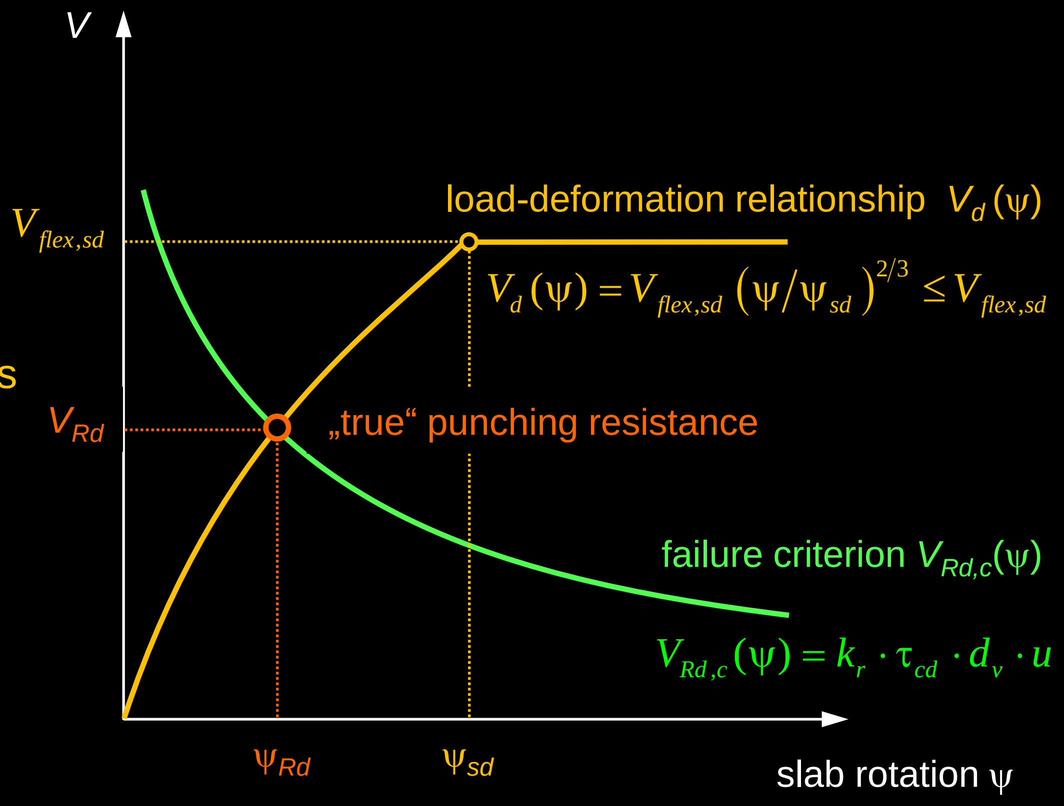
1. Determine the load-deformation relationship $V_d(\psi)$ per direction x, y (for level of approximation 3: factor 1.5 may be reduced to 1.2)

$$\psi = 1.5 \frac{r_s}{d}\frac{f_{sd}}{E_s}\left(\frac{m_{sd}}{m_{Rd}}\right)^{\frac{3}{2}} = \psi_{sd}\left(\frac{m_{sd}(\psi)}{m_{Rd}}\right)^{\frac{3}{2}} \qquad \text{with} \quad \psi_{sd} = 1.5\frac{r_s}{d}\frac{f_{sd}}{E_s}$$

$$V_{flex,sd} = \frac{V_d(\psi)}{m_{sd}(\psi)} m_{Rd} \rightarrow \frac{m_{sd}(\psi)}{m_{Rd}} = \frac{V_d(\psi)}{V_{flex,sd}} \qquad \text{with} \quad \frac{V_d(\psi)}{m_{sd}(\psi)} \text{ from FE slab analysis}$$

$$\rightarrow \psi = \psi_{sd}\left(\frac{V_d(\psi)}{V_{flex,sd}}\right)^{\frac{3}{2}} \qquad \rightarrow V_d(\psi) = V_{flex,sd}\left(\frac{\psi}{\psi_{sd}}\right)^{\frac{2}{3}} \le V_{flex,sd}$$

2. Equating $V_{Rd,c}(\psi) = V_d(\psi) \rightarrow \psi_{Rd}$, $V_{Rd}(\psi_{Rd}) = V_d(\psi_{Rd})$ (direction with smaller value of V_{Rd} controls)



Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262: Levels of approximation (LoA)

(a) Continuously supported flat slabs $0.5 \le l_x / l_y \le 2$, no (small) plastic redistribution ("normal" slab in building construction):

- Level of approximation 1: $r_{sx} = 0.22 \cdot l_x$, $r_{sy} = 0.22 \cdot l_y$ and $m_{sd} / m_{Rd} = 1.0$
- Level of approximation 2: $r_{sx} = 0.22 \cdot l_x$, $r_{sy} = 0.22 \cdot l_y$, estimated bending moments:

$$m_{sd} = V_d \left(\frac{1}{8} + \frac{|e_{u,i}|}{2 b_s} \right)$$ interior colmns

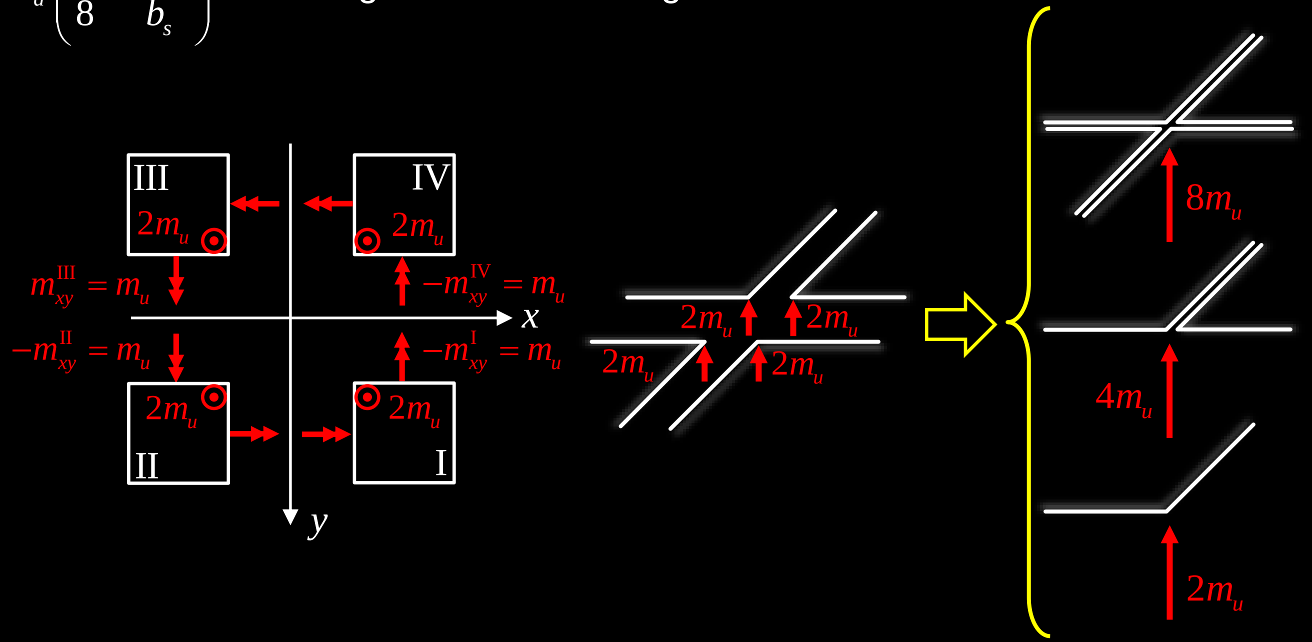
$$m_{sd} = V_d \left(\frac{1}{8} + \frac{|e_{u,i}|}{2 b_s} \right) \ge \frac{V_d}{4}$$ edge columns $\parallel$ edge

$$m_{sd} = V_d \left(\frac{1}{8} + \frac{|e_{u,i}|}{b_s} \right) \ge \frac{V_d}{2}$$ corner columns

$$m_{sd} = V_d \left(\frac{1}{8} + \frac{|e_{u,i}|}{b_s} \right)$$ edge columns $\perp$ edge

The corresponding minimum values result directly from the consideration of the combination of individual slab segments with discontinuous twisting moment fields.

NB: For interior supports an explanation with the moment field (transforming a concentrated load to a uniformly distributed one) is even simpler



Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262:
Levels of approximation (LoA)

(b) Flat slabs with $l_x / l_y < 0.5$ or $l_x / l_y > 2$, slabs with complex geometry or detailed examination required:

- Level of approximation 3: Determination of r_s (distance of the point of zero moment, i.e. radial moment = 0, from support axis) and m_{sd} (mean value of the bending moments including twisting moments in the support strip) from an elastic (usually linear elastic FE) slab calculation. Factor 1.2 instead of 1.5 in formula for ψ:

$$\psi = \cancel{1.5}\ 1.2 \cdot \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd}}{m_{Rd}} \right)^{3/2}$$

Slabs - Influence of shear forces

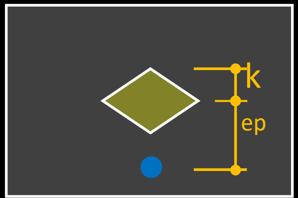
Punching resistance of slabs without punching reinforcement according to SIA 262:
Selected additional provisions (for detailing provisions see SIA 262, 5.5.3)

- Bending resistance m_{Rd} = mean value over support strip, taking prestressing into account.
  (reinforcement must generally be fully anchored at a distance of $2.5 \cdot d_v$ from the control perimeter, but at most at the point of zero bending moment in the respective direction. In the case of edge and corner supports, the reinforcement perpendicular to the edge must be fully anchored → hairpin shaped reinforcement).
- Prestressed slabs with decompression moment m_{Dd}:

$$\psi = (1.5 \text{ or } 1.2) \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd} - m_{Dd}}{m_{Rd} - m_{Dd}} \right)^{3/2}$$

... m_{Dd} = long-term value (shrinkage, creep, relaxation) under consideration of normal forces due to restraints
(for m_{Dd}, only the part of the compressive force that is effective in the support strip may be taken into account)
... m_{sd} = incl. constraints (e.g. secondary moments due to prestressing)
... prestress with unfavourable effect must be taken into account where applicable
... use signs of m_{sd}, m_{Rd}, and m_{Dd} consistently, otherwise might obtain completely wrong results!

NB1: The decompression moment is generally: $m_{Dd} = P \cdot (e_p + k)$. If the prestressing is considered as anchorage and deviation forces ("on the load side"), the contribution $P \cdot e_p$ to m_{Dd} *is* already considered in the correspondingly reduced bending moments m_{sd}. The bending resistance m_{Rd} is also smaller by the amount $P \cdot e_p$ (only the increase in prestressing force as resistance) → only the portion $P \cdot k$ can be subtracted in the numerator and the denominator, taking into account the distribution of P over the slab width and, if necessary, the reduction of P by normal forces due to restraints.



NB2: In addition, the contribution of inclined prestressing forces to the punching resistance may be taken into account (even if prestressing is considered on the load side; the support reaction V_d does not reflect the isostatic effect of prestressing).

Slabs - Influence of shear forces

Punching resistance of prestressed slabs without punching reinforcement according to SIA 262

Check / Verification of existing structures (only the governing direction is shown!)

Given: Support dimension (and thus *u*), *d*, m_{Rd}

Question: What is the punching resistance (without shear reinforcement)?

$V_{flex,sd}$ Support reaction at which the bending reinforcement yields (in the considered direction)

ψ_{sd} Slab rotation when reaching $V_{flex,sd}$

m_{dec} Decompression moment (see slides 37 and 39)

Procedure

1. Determination of the load-deformation relationship $V_d(\psi)$ per direction *x*, *y* (for LoA 3 replace factor 1.5 by 1.2)

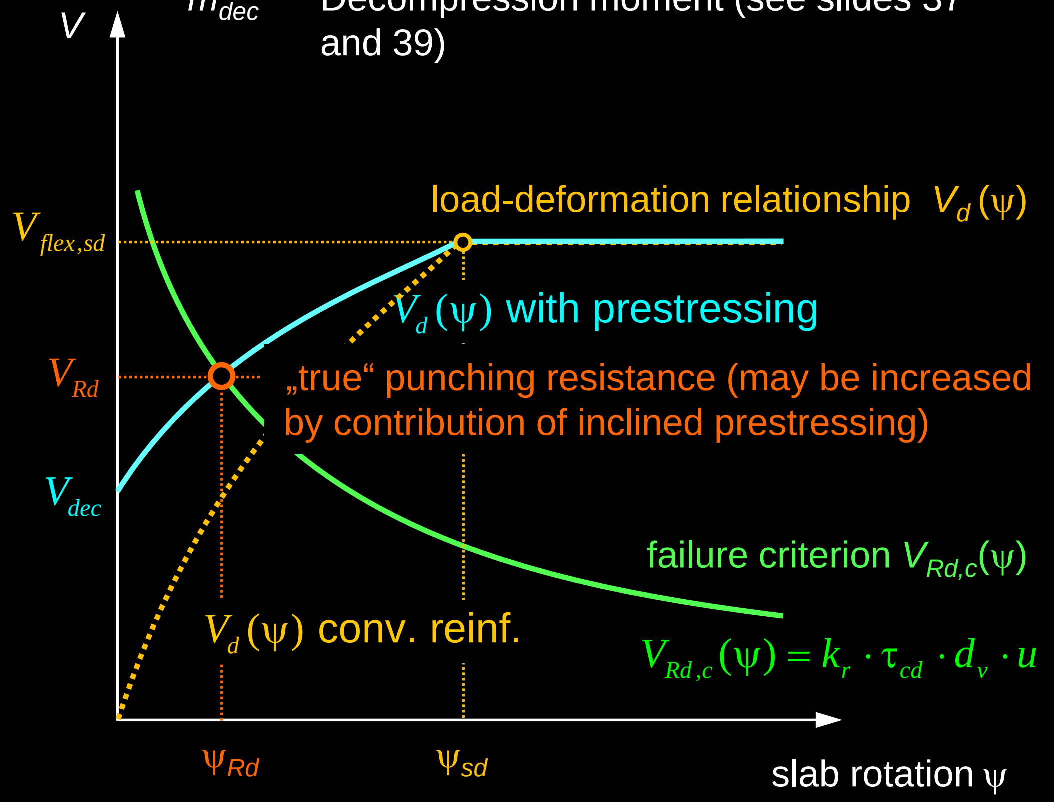
$$\psi = 1.5\frac{r_s}{d}\frac{f_{sd}}{E_s}\left(\frac{m_{sd}-m_{dec}}{m_{Rd}-m_{dec}}\right)^{3/2} = \psi_{sd}\left(\frac{m_{sd}(\psi)-m_{dec}}{m_{Rd}-m_{dec}}\right)^{3/2} \quad \text{mit } \psi_{sd} = 1.5\frac{r_s}{d}\frac{f_{sd}}{E_s}$$

$$V_{flex,sd} = \frac{V_{d,gq}(\psi)}{m_{sd,gq}(\psi)}m_{Rd},\; V_{dec,sd} = \frac{V_{d,gq}(\psi)}{m_{sd,gq}(\psi)}m_{dec} \rightarrow \frac{m_{sd}(\psi)-m_{dec}}{m_{Rd}-m_{dec}} = \frac{V_d(\psi)-V_{dec}}{V_{flex,sd}-V_{dec}}$$

(with $\frac{V_{d,gq}(\psi)}{m_{sd,gq}(\psi)}$ from FE slab analysis)

$$\rightarrow \psi = \psi_{sd}\left(\frac{V_d(\psi)-V_{dec}}{V_{flex,sd}-V_{dec}}\right)^{3/2} \qquad \rightarrow V_d(\psi) = V_{dec} + (V_{flex,sd}-V_{dec})\left(\frac{\psi}{\psi_{sd}}\right)^{2/3} \le V_{flex,sd}$$

2. Equating $V_{Rd,c}(\psi) = V_d(\psi) \rightarrow \psi_{Rd}$, $V_{Rd}(\psi_{Rd}) = V_d(\psi_{Rd})$
   (direction with smaller value of V_{Rd} controls)



Slabs - Influence of shear forces

Punching resistance of prestressed slabs without punching reinforcement according to SIA 262

Prestress taken into account on the resistance side:

$$\psi = 1.5 \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd} - m_{dec}}{m_{Rd} - m_{dec}} \right)^{3/2}$$

mit $m_{sd} = m_{gq,d} + m_{ps}$: Design value of the bending moment in the support strip (negative)

$m_{gq,d}$: Design value of the bending moment due to vertical loads (negative)

m_{ps} : Secondary moment due to prestress (usually positive)

$m_{dec} = -P_\infty (e_p + k)$: Decompression moment (negative)

P_∞ : Prestressing force at t=∞ (positive) (reduce if normal force does not fully act in the support strip!)

e_p : Eccentricity of prestress (in the support strip), here positive upwards (upper side of slab)

k : extent of core (positive, usually = h/6)

m_{Rd} : Design value of the bending resistance $-(A_s f_{sd} \cdot z_s + A_p f_{pd} \cdot z_p)$ (negative)

(All moments used with the usual sign convention, i.e. tension at bottom = positive -> support moments negative)

If the prestressing is taken into account on the resistance side, the proportion of the inclined prestressing force (sum of the vertical components on the decisive circumference) can either be added to the punching resistance or subtracted from the design value of the column reaction = reduced punching load ($V_{d,red} = V_d - \Delta V_d(P)$, $\Delta V_d(P) = \Sigma(P_\infty \sin\alpha_p)$), but **not** both!

Slabs - Influence of shear forces

Punching resistance of prestressed slabs without punching reinforcement according to SIA 262

Prestress taken into account on the load side (as anchor and deviation forces):

$$\psi = 1.5 \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd} - m_{dec}}{m_{Rd} - m_{dec}} \right)^{3/2}$$

mit $m_{sd} = m_{gq,d} + m_p$: Design value of the bending moment in the support strip (negative)

$m_{gq,d}$: Design value of the bending moment due to vertical loads (negative)

m_p : Bending moment due to prestress (Long-term stresses $P_\infty e_p$ and secondary moments, positive)

$m_{dec} = -P_\infty k$: Decompression moment (negative)

P_∞ : Prestressing force at t=∞ (positive) (reduce if normal force does not fully act in the support strip!)

k : extent of core (positive, usually = $h/6$)

(the part $P_\infty e_p$ of prestressing is already included in m_{sd}, do not use here a second time!)

m_{Rd} : Design value of the bending resistance $-(A_s f_{sd} \cdot z_s + A_p f_{pd} \cdot z_p - P_\infty e_p)$ (negative)

(prestressing contribution reduced by $P_\infty e_p = A_p \sigma_{p\infty} e_p$, since $P_\infty e_p$ is already considered in m_p)

(All moments used with the usual sign convention, i.e. tension at bottom = positive -> support moments negative)

m_{sd}, m_{Rd} and m_{dec} differ all by the same value $P_\infty e_p$, compared to considering prestress as resistance side ⇨ same result!

Even if the prestress is introduced on the load side, the proportion of the inclined prestressing force to the punching resistance can be taken into account: The column reaction, which is used as punching load, does not reflect the isostatic effect of prestress (would be different if the integral of the shear forces along the control perimeter was used as load).

5 Slabs

In-depth study and additions to Stahlbeton II

5.6.2 Behaviour with punching reinforcement

Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

The following verifications must be carried out for slabs with punching reinforcement:

- Resistance of the first concrete compression strut next to the supported area
- Resistance of the punching reinforcement (reinforced zone)
- Punching verification (without punching reinforcement) outside the reinforced zone

Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

Minimum required resistance of punching reinforcement:

... resp. in order to neglect imposed deformations in the design and / or avoid the necessity of an integrity reinforcement

$$V_{d,s} \geq V_d - V_{Rd,c}$$

$$V_{d,s} \geq \max \begin{Bmatrix} V_d - V_{Rd,c} \\ V_d / 2 \end{Bmatrix}$$

Resistance of punching reinforcement (normal: inclination $\beta = 90°$):
(A_{sw}: only punching reinforcement within distance 0.35...1.0 $\cdot d_v$ of the supported area is taken into account)

$$V_{Rd,s} = \sum A_{sw} k_e \sigma_{sd} \sin\beta$$

Nominal stress in the punching reinforcement:
(f_{bd}: design value of the bond stress)
(NB: according to fib Model Code 2010: $V_{d,s} \geq V_d / 2$ with $\sigma_{sd} = f_{sd}$)

$$\sigma_{sd} = \frac{E_s \psi}{6}\left(1 + \frac{f_{bd}}{f_{sd}} \frac{d}{\varnothing_{sw}}\right) \leq f_{sd}$$

Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

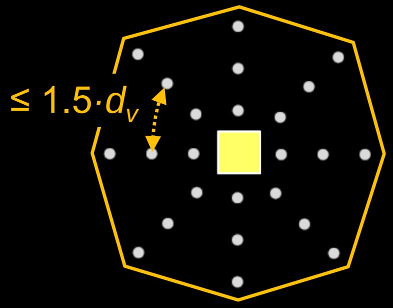
Resistance of the first concrete compression strut:
(Factors > 2 and according to SIA 262 > 3.5 are admissible, provided that the effectiveness of the reinforcement is experimentally proven)

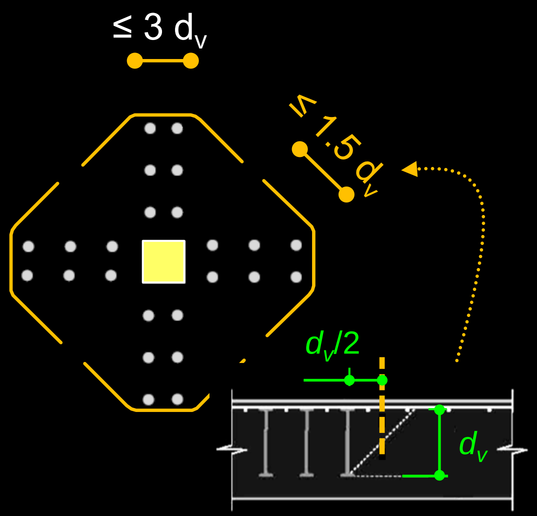
$$V_{Rd,\max} = 2 \cdot k_r \tau_{cd} d_v u \leq 3.5 \cdot \tau_{cd} d_v u$$
$$= 2 \cdot V_{Rd,c} \text{ mit } k_r \leq 1.75$$

Punching verification (without punching reinforcement) outside the reinforced zone

(supported surface defined by out reinforcement, control perimeter according to figure)

$$V_{Rd,c} = k_r \tau_{cd} d_v u$$

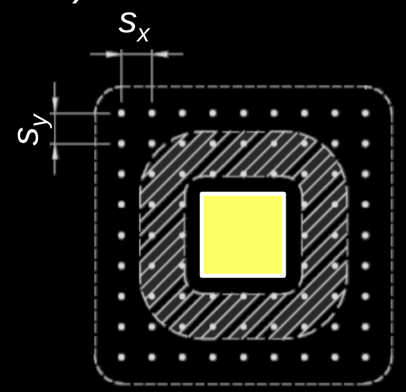


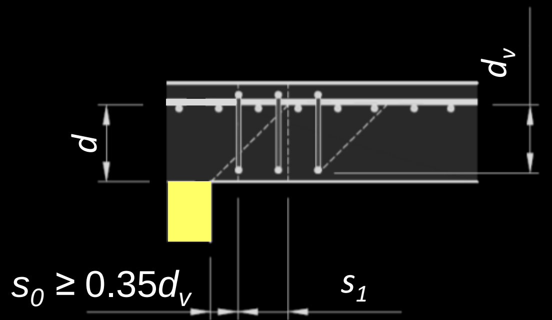


Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262:
Selected additional provisions (for detailing provisions see SIA 262, 5.5.3)

Resistance of the punching reinforcement:
(A_{sw}: punching shear reinforcement only at a distance of 0.35…1.0·d_v from the supported surface)





$$V_{Rd,s} = \sum A_{sw} k_e \sigma_{sd} \sin\beta$$

SIA 262 5.5.3.8: At least two legs in radial direction

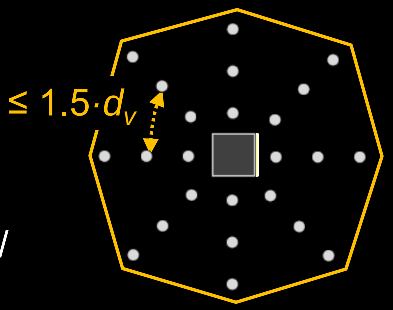
SIA 262 5.5.3.10: Full anchorage in compression and tension zone

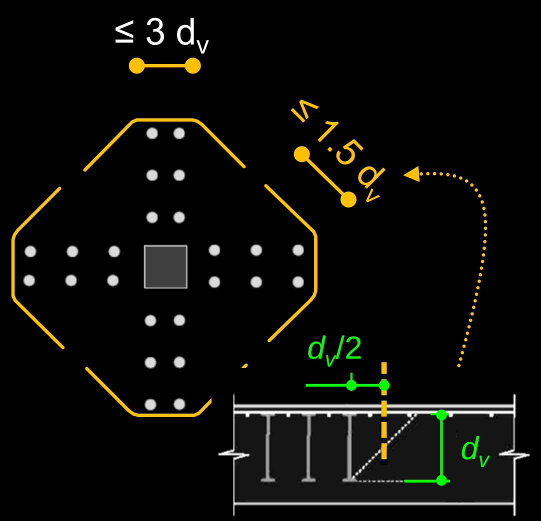
Arrangement of the punching reinforcement within the distance $s_0 < s_1$ from the supported surface:

- radial distance and maximum Ø, see SIA 262, Tab. 20 and Fig. 39
- tangential distance in the second ring ≤ 1.5·d_v

Generally provide the same cross-section A_{sw} per «ring»
(rings geometrically similar to control perimeter)

Punching reinforcement in straight radial rows: same radial distance of dowels / vertical reinforcement satisfies the condition of equal A_{sw} per ring



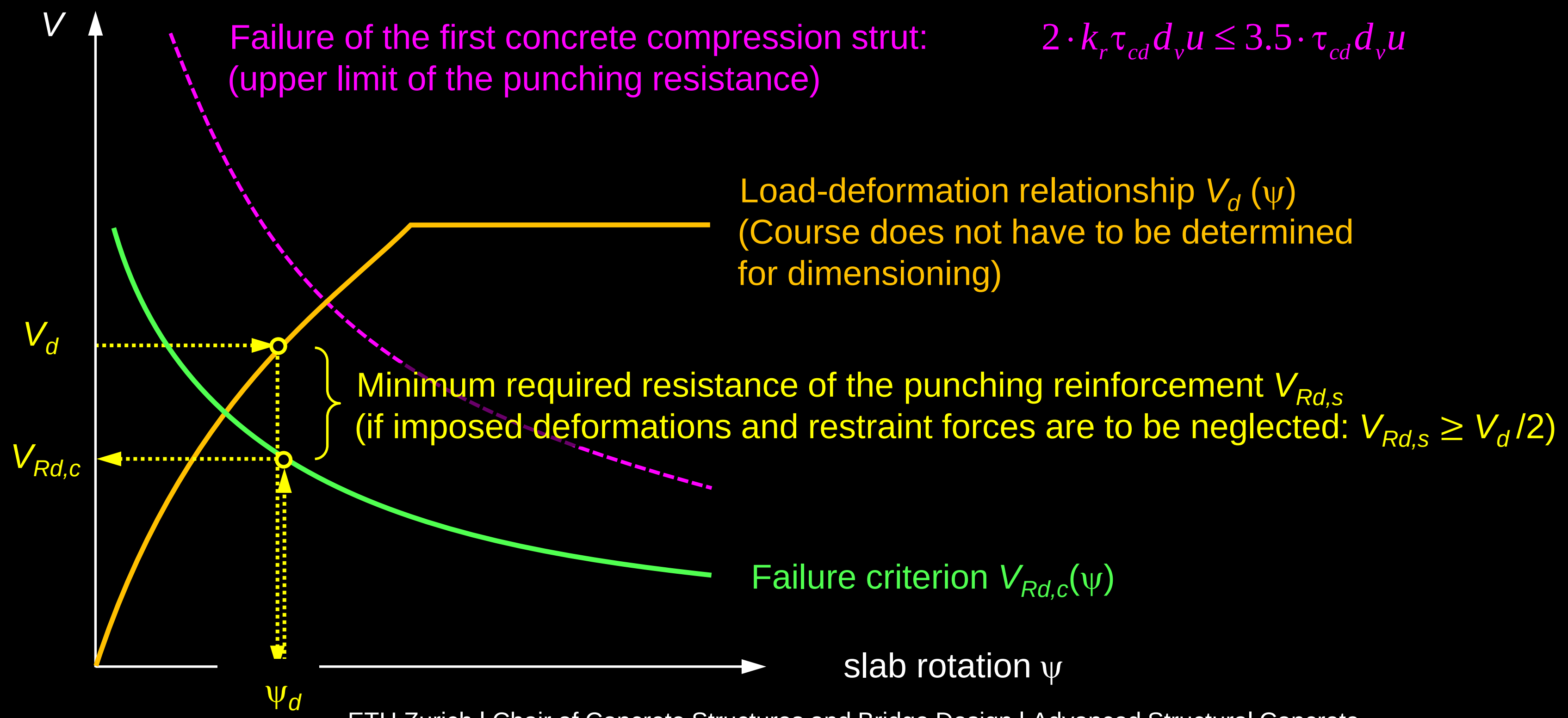


Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

Dimensioning (only governing direction shown (determine ψ_d for m_{sd}, m_{Rd} and r_s per directions x, y, smaller value of V_{Rd} is governing)

1. Determination of $V_{Rd,c}$ (= same as determination V_{Rd} without punching reinforcement, see slides above)
2. Required resistance $V_{Rd,s} \geq V_{d,s} = V_d - V_{Rd,c}$ ($\geq V_d/2$ if constraint forces are to be neglected)
3. Check that failure of the first compression strut is not governing $V_{Rd} \leq 2 \cdot V_{Rd,c}$
4. Definition of the size of the reinforced area (such that outside, $V_{Rd,c}$ alone is sufficient)

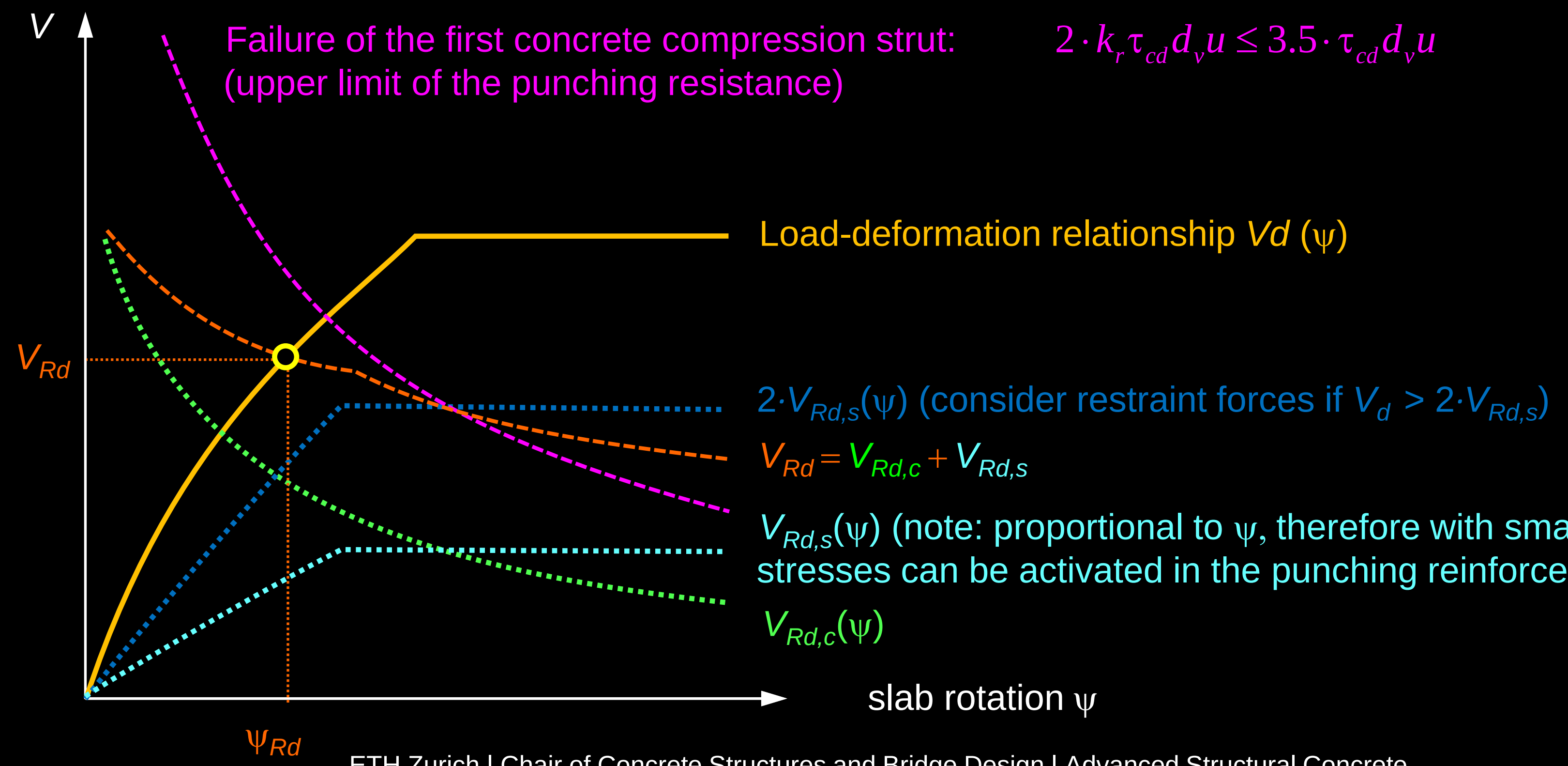


Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

Check / Verification of existing structures

1. Determination of load-deformation relationship $V_d\,(\psi)$ and punching resistance $V_{Rd}\,(\psi) = V_{Rd,c} + V_{Rd,s} \rightarrow$ equate, intersection = V_{Rd}
2. Check $V_{Rd} \geq V_d$ (V_d incl. Imposed deformations and restraint forces, if $V_d > 2 \cdot V_{Rd,s}$)
3. Check that failure of the first compression strut is not governing $V_{Rd} \leq 2 \cdot V_{Rd,c}$
4. Verify the size of the reinforced area with separate verification



NB: Procedure for the determination of $V_d\,(\psi)$ and $V_{Rd,c}$ analogous to that without punching reinforcement; $V_{Rd,s}$ proportional to ψ)

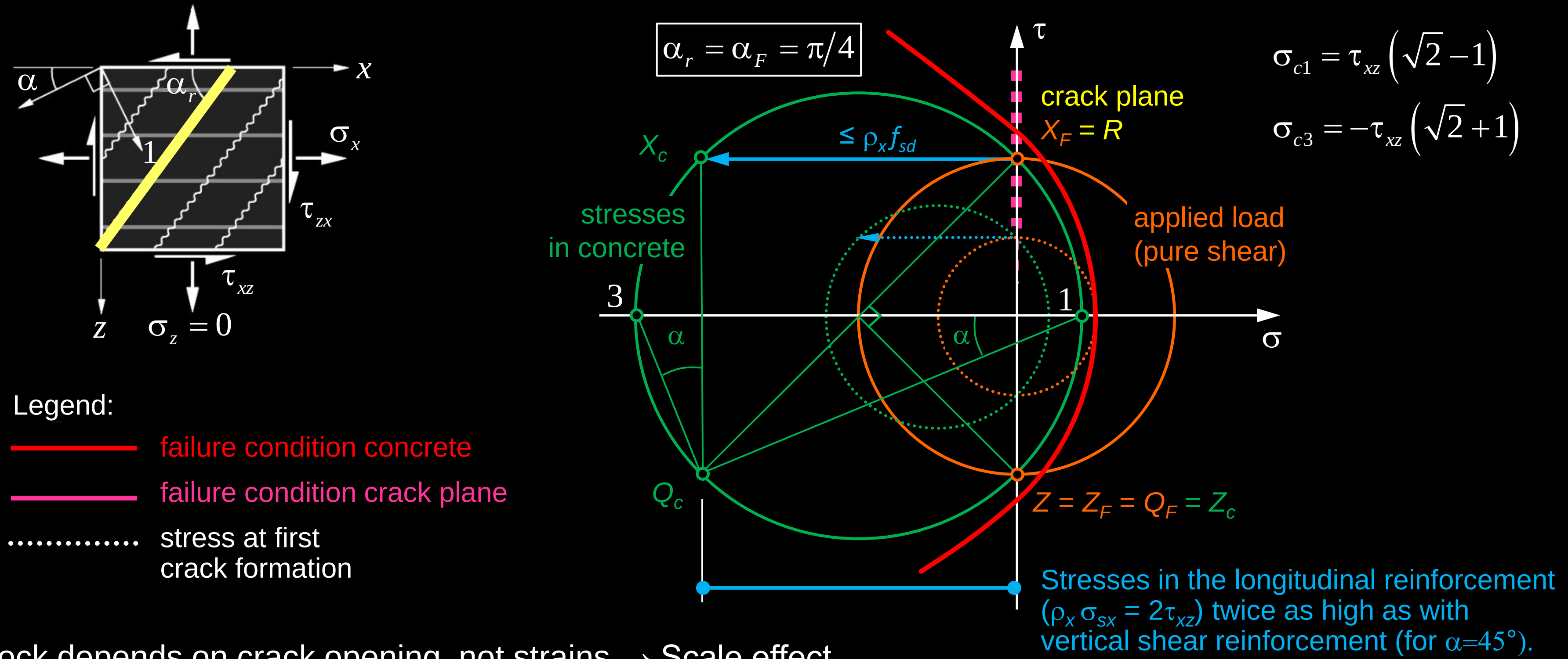
5 Slabs

Appendix

Influence of shear forces

Shear resistance of slabs without shear reinforcement

- Simple model for shear transmission through aggregate interlock in the first cracks under 45° (pure shear stress in the first cracks) → longitudinal reinforcement needs to resist double of the additional tensile force due to *V*:



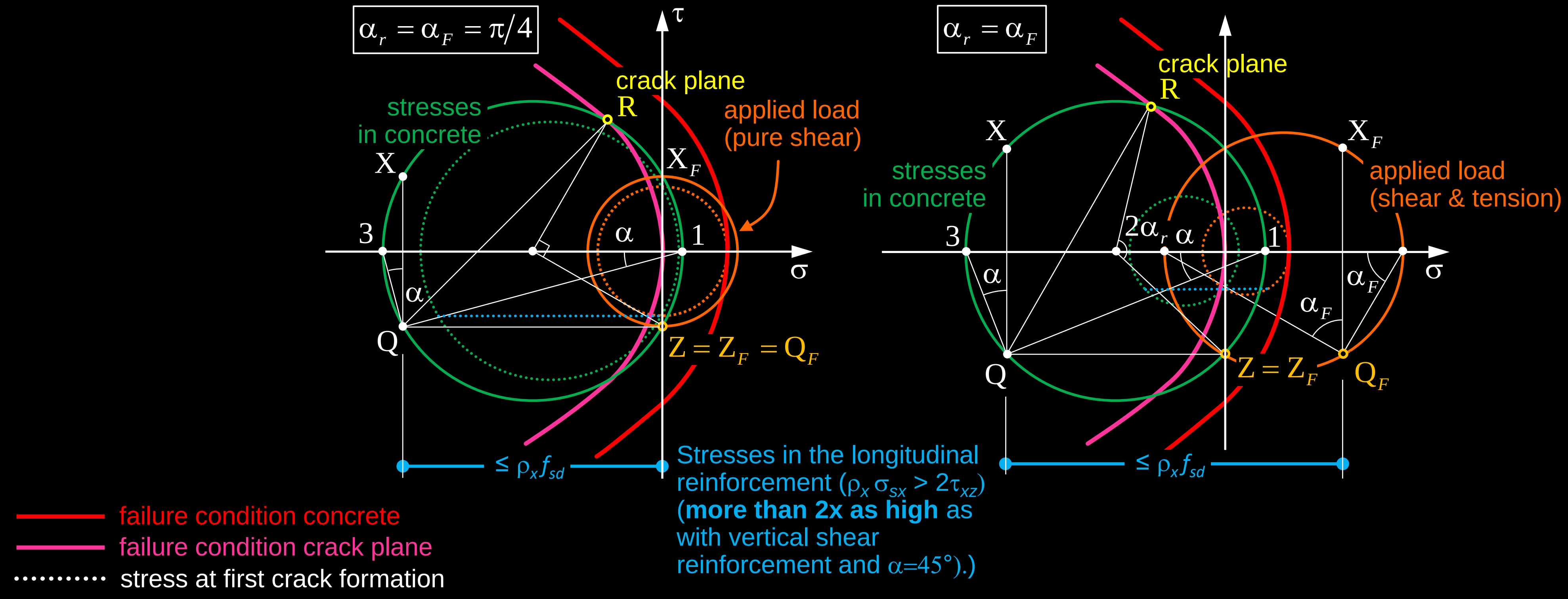
NB1: Aggregate interlock depends on crack opening, not strains → Scale effect

NB2: The load-bearing capacity due to aggregate interlock is not necessarily sufficient in regions subjected to high shear stress (slabs in the support area) to avoid brittle failure in the event of initial shear cracking!

Influence of shear forces

Shear resistance of slabs without shear reinforcement

- Consideration of more realistic failure criteria for shear transmission by aggregate interlock, i.e. Mohr's envelope. Shear can only be transmitted with compressive stress → even more longitudinal reinforcement required!



NB: There is a scale effect and the validity is limited to moderate shear stresses!

Influence of shear forces

Derivation of the factor for deviation of the principal direction φ_0 of the shear force from the direction of the principal reinforcement (compression field model for sandwich cover)

Compatibility assuming linear elastic behaviour & stress-free cracks
$\rightarrow \varphi_{1\varepsilon} =$ principal strain direction with $\varphi_{1\varepsilon} = \varphi_{1c}$ resp. $\alpha_r + \varphi_{1\varepsilon} = \pi/2$

$$\varepsilon_x = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)\sin^2\alpha_r \quad = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)\cos^2\varphi_{1\varepsilon}$$

$$\varepsilon_z = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)\cos^2\alpha_r \quad = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)\sin^2\varphi_{1\varepsilon}$$

$$\sigma_{sxr} = E_s\varepsilon_x + \frac{\tau_{b0}s_{rmx}}{\varnothing} \qquad \sigma_{szr} = E_s\varepsilon_z + \frac{\tau_{b0}s_{rmz}}{\varnothing}$$

Neglecting concrete strains and tension stiffening (i.e. $\varepsilon_3 = 0, \tau_{b0} = 0$):

$$\sigma_{sxr} \approx E_s\varepsilon_1\cos^2\varphi_{1\varepsilon} \qquad \sigma_{szr} \approx E_s\varepsilon_1\sin^2\varphi_{1\varepsilon}$$

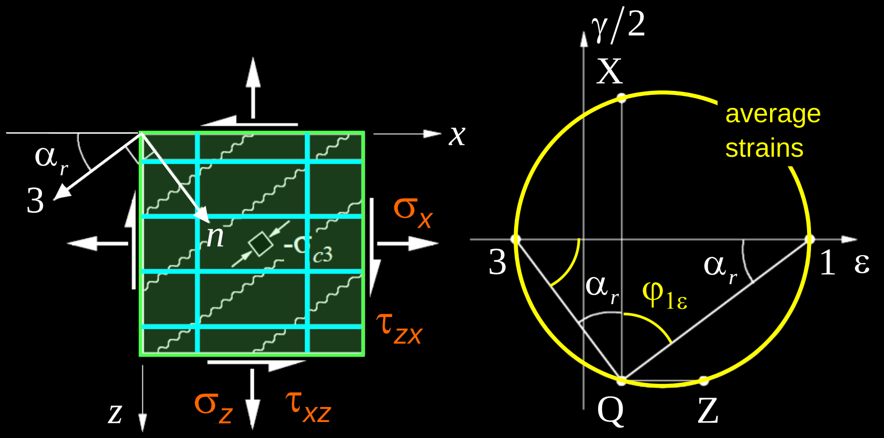
By equilibrium at the cracks, stresses in direction 1ε=F follow as:
(for stress-free cracks only reinforcement stresses act
in this direction; note that generally $\varphi_{1c} \neq \varphi_{1F}$ resp. $\alpha_r \neq \alpha_F$)

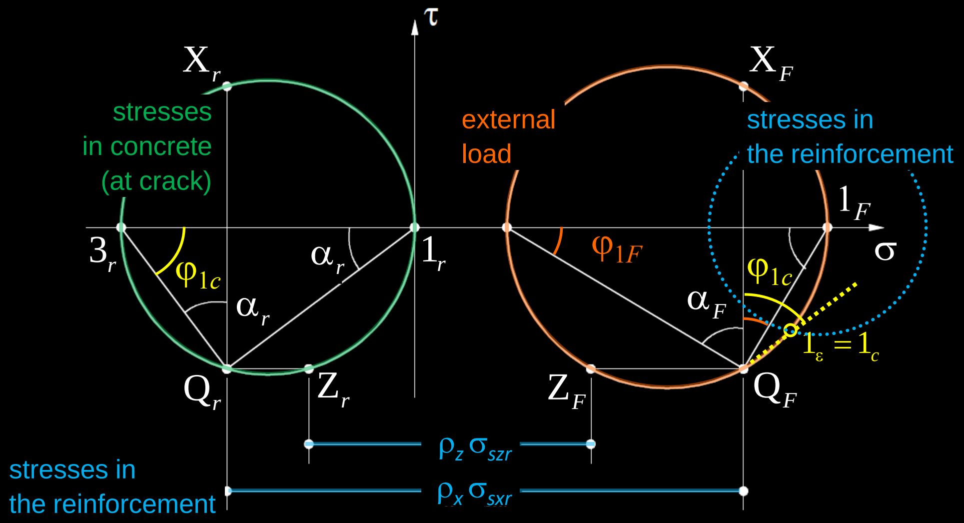
$$\sigma_F(\varphi_{1\varepsilon} = \varphi_{1c}) = \rho_x\sigma_{sxr}\sin^2\alpha_r + \rho_z\sigma_{szr}\cos^2\alpha_r$$

$$= \rho_x\sigma_{sxr}\cos^2\varphi_{1\varepsilon} + \rho_z\sigma_{szr}\sin^2\varphi_{1\varepsilon}$$

$$\rightarrow \sigma_F(\varphi_{1\varepsilon} = \varphi_{1c}) = \rho_x E_s\varepsilon_1\cos^4\varphi_{1\varepsilon} + \rho_z E_s\varepsilon_1\sin^4\varphi_{1\varepsilon}$$

$$\rightarrow \varepsilon_1 = \frac{\sigma_F(\varphi_{1\varepsilon} = \varphi_{1c})}{E_s}\,\frac{1}{\left(\rho_x\cos^4\varphi_{1\varepsilon} + \rho_z\sin^4\varphi_{1\varepsilon}\right)}$$





Additions - Elastic sheets

Kirchhoff's slab theory
(rigid linear elastic slabs with small deflections)

The fourth-order differential equation results from the equilibrium and compatibility conditions for linear elastic behaviour (inhomogeneous bipotential equation) :

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \Delta\Delta w = \frac{q}{D} \quad mit \quad D = \frac{Eh^3}{12(1-\nu^2)}$$

beam in x-direction | additional term | beam in y-direction

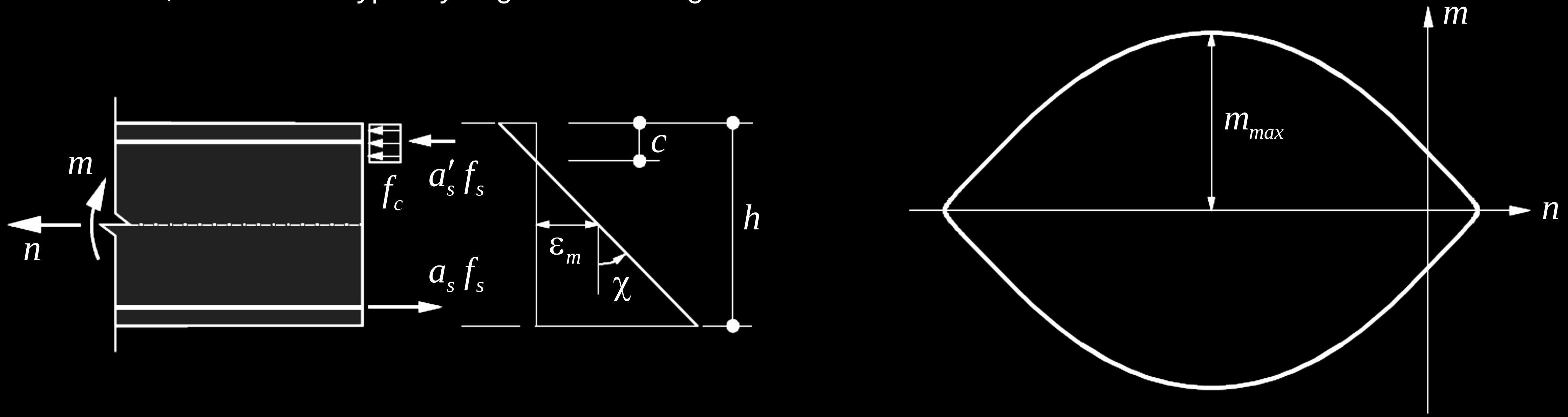
Only two boundary conditions can be adapted to the solution, but there are three variables at the boundary (moments m_n, m_{tn} and shear force v_t) → Support force (see slabs part 1), thus the following boundary conditions:

- clamped slab edge: $w = 0$ $\frac{\partial w}{\partial x} = 0$ thus $\frac{\partial^2 w}{\partial x \partial y} = 0$ and thus $m_{xy} = 0$. m_n and v_n are the support reations.
- simply supported slab edge: $m_x = 0$ $\Delta w = 0$ resulting support force $v_n + m_{tn,t} = m_{n,n} + 2m_{nt,t}$
- free slab edge: $m_n = 0$ disappearing support force $v_n + m_{tn,t} = m_{n,n} + 2m_{nt,t} = 0$

Additions - Membrane action

Development of membrane forces

- Cracking leads to deformations in the middle plane of the slab already in the serviceability limit state (dilatancy)
- The resulting deformations are rarely possible without constraint
  → Compressive membrane forces in cracked areas
  → Usually increase of bending resistance
- Membrane force can usually only be roughly estimated (depending on geometry, deformations of the slab middle plane, stiffness of the membrane support).
- Therefore, this effect is typically neglected in design.

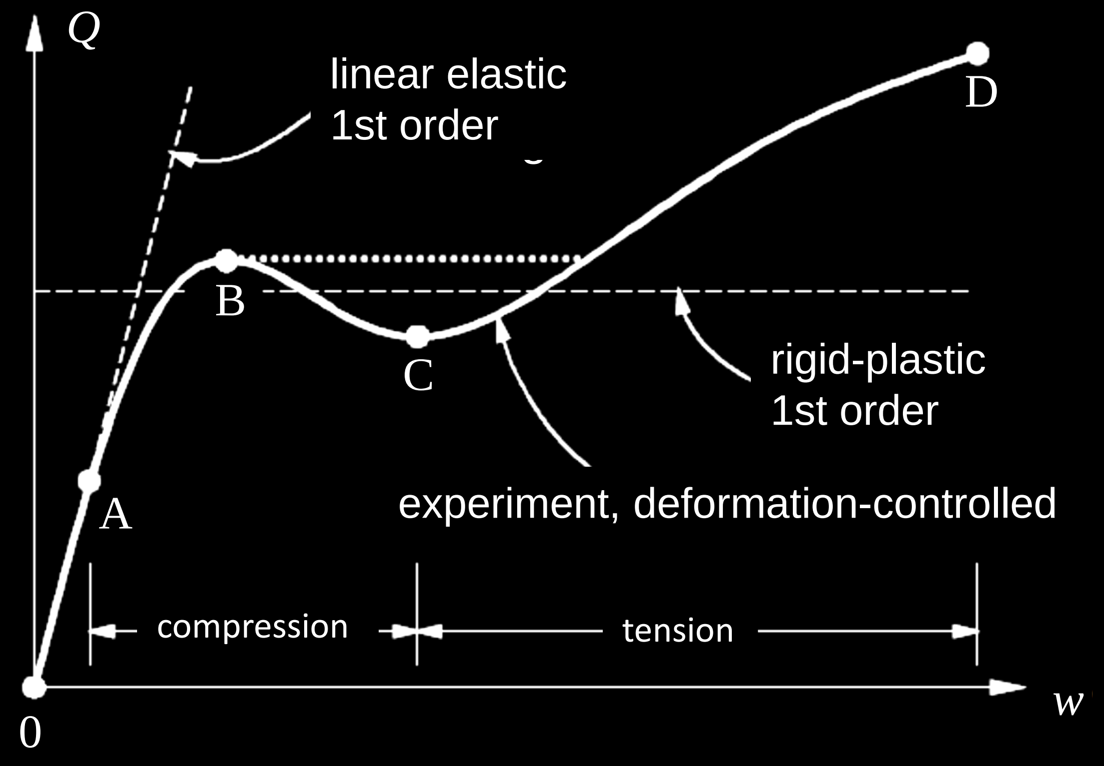


Additions - Membrane action

Development of membrane forces

Behaviour (qualitative)

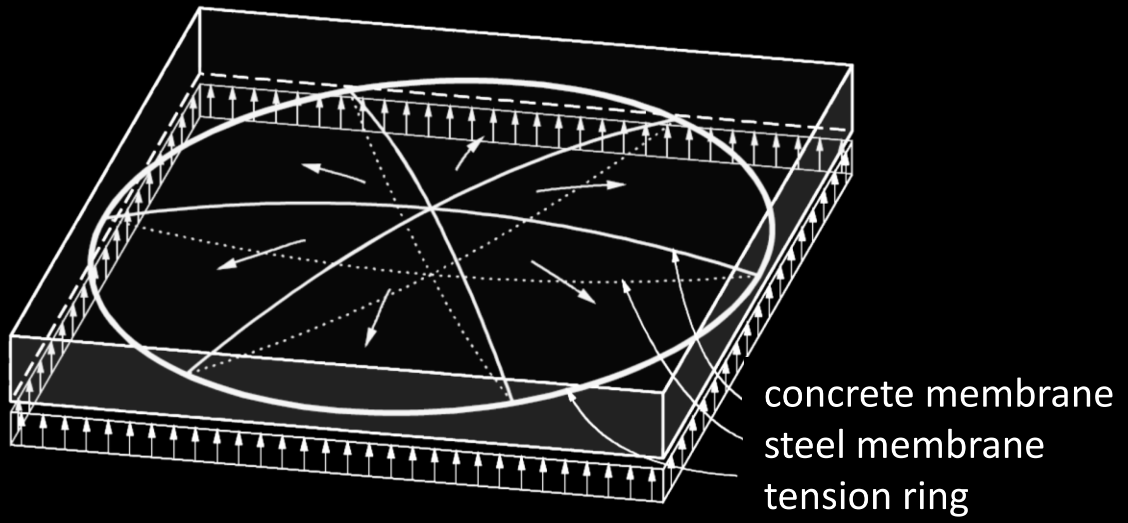
1. Linear elastic (OA)
2. Crack formation, build-up of compressive membrane forces (AB)
3. Maximum load (B) > Load capacity for rigid-ideally plastic behaviour without membrane action (M-N interaction)
4. Load decreases if deformation controlled, compressive membrane forces are reduced (BC); (Load-controlled: «snap-through" effect of the slab)
5. With external membrane support, build-up of tensile membrane forces with increasing deflection. Failure load often >> first maximum (with large deformations, can only be measured with a corresponding calculation)



Additions - Membrane effect

Spatial model for load-bearing capacity

- Membrane support not by bearing, but by tension ring (uncracked area of the slab)
- Load transfer: Compression membrane (concrete) and tension membrane (reinforcement: conventional or e.g. prestressing without bond)
- Without horizontal membrane support (external or by a tension ring), the membrane forces of the concrete and reinforcement membranes are in equilibrium → not an actual membrane effect.
- Membrane action can be used to explain the load-bearing capacity of an unreinforced slab (at the location of the membrane support, horizontal **and** vertical components of the membrane forces need to be resisted)



Additions - Membrane effect

Model for the load-bearing capacity (Ritz, 1978)

- Model for load-bearing behaviour of slab strips prestressed without bond with membrane effect
- Load carried by bending or membrane effect (of concrete and steel membrane), depending on stiffness ratios (if membrane support is missing, no actual membrane effect)