5 Slabs

In-depth study and additions to Stahlbeton II

5.5 Influence of shear forces

Learning objectives

Within this chapter, the students are able to:

- compare the shear behaviour of a slab with and without transverse reinforcement and explain the arising differences of forces in the sandwich covers when using a sandwich model.
- determine the “actual” (according to SIA 262) punching resistance of a slab without punching reinforcement and understand the verification procedure.

Slabs - Influence of shear forces

Shear resistance of slabs - General remarks (→ Stahlbeton II)

- Slabs, especially those with shear reinforcement (three-dimensionally reinforced), are generally very ductile structures.
- On the other hand, a shear failure of slabs without shear reinforcement is very brittle → practically impossible to redistribute the internal forces (therefore, no stress relief of the affected areas by internal force redistribution)!
- Often slabs are designed according to the lower bound theorem of the theory of plasticity. In doing so the maximum shear forces occurring in the course of the load history can deviate significantly from the shear load in the calculated (bending) failure state (*).
  For a safe design, the shear force at each point of the slab should, therefore, strictly speaking, be checked during the entire load history (internal force redistribution under the same external loads).
- In practice, shear structural safety is usually only checked in the state of maximum internal force redistribution, which is also the basis for the bending design. This is associated with considerable uncertainties, especially since the shear forces resulting from FE calculations scatter strongly (they are determined numerically as derivatives of the bending moments, one order of magnitude less accurate).
  In case of doubt, a ductile behaviour must be ensured by arranging a shear reinforcement!

(*) also applies to a design based on linear elastic FE calculations (= equilibrium state), since crack formation, residual stress states due to settlements, construction process, etc. can never be completely recorded or correctly modelled!

The subject of this chapter is the influence of shear forces on the behaviour of slabs.

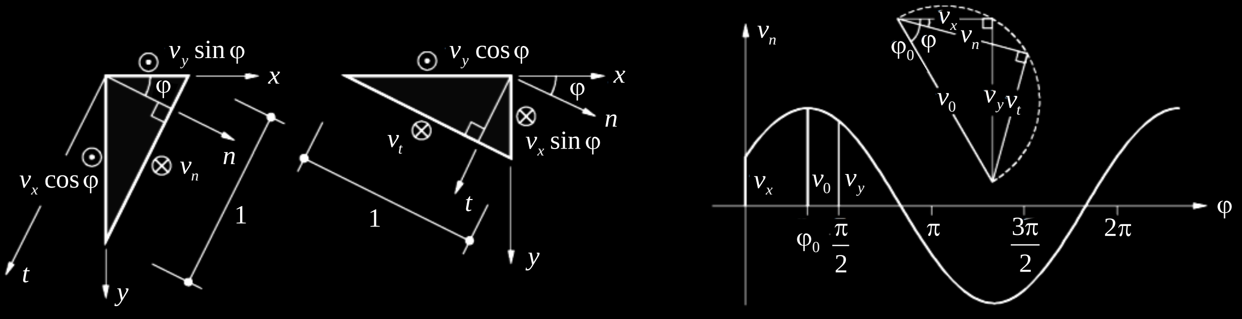
This is essentially a repetition from the lecture Stahlbeton II with selective additions.

Slabs - Influence of shear forces

Shear resistance of slabs - General remarks

- In a slab, the principal shear force $v(\varphi_0) = v_0$ is carried in the direction φ_0 at every point. Perpendicular to it the shear force is zero: $v = v(\varphi_0 \pm \pi/2) = 0$.

→ Measure for shear stress: nominal shear stress $\tau_{nom} = v_0 / z$
(with z = lever arm of the internal forces).



$$v_n = v_x \cos\varphi + v_y \sin\varphi \qquad v_t = -v_x \sin\varphi + v_y \cos\varphi \qquad v_0 = \sqrt{v_x^2 + v_y^2} \qquad \tan\varphi_0 = \frac{v_y}{v_x}$$

Repetition from Stahlbeton II:

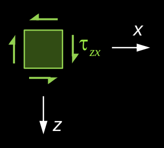
Principal shear force and associated direction.

Slabs - Influence of shear forces

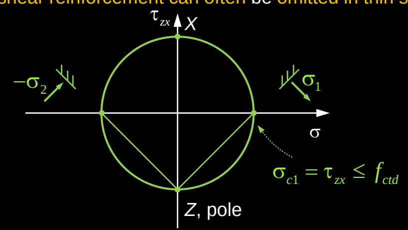
Shear resistance of slabs without shear reinforcement

- Shear stresses in the uncracked (isotropic) state correspond to a principal tensile stress of the same amount, $\sigma_{c1} = |\tau_{zx}|$ (elastic shear flow: $\tau_{max} = 1.5 \cdot \tau_{nom} = 1.5 \cdot v_0 / z$)
- In the case of thin slabs, which according to SIA 262 may be designed without shear reinforcement, the tensile strength of the concrete is implicitly taken into account (which is usually even slightly higher than the permissible value for insignificant components). This can be justified on the following reasons:
  - Higher redundancy than beam structures (biaxial load-bearing, beneficial compressive membrane forces neglected in the design)
  - Shear stress generally lower (except in the vicinity of concentrated loads and supports)
  - No failure at first shear crack formation under moderate shear stress (if crack roughness is sufficient and longitudinal reinforcement has reserves)

→ In contrast to beam structures (minimum shear reinforcement mandatory), shear reinforcement can often be omitted in thin slabs.



$$\sigma_1 = \tau_{xz} = \frac{v_d}{b_w \cdot z}$$



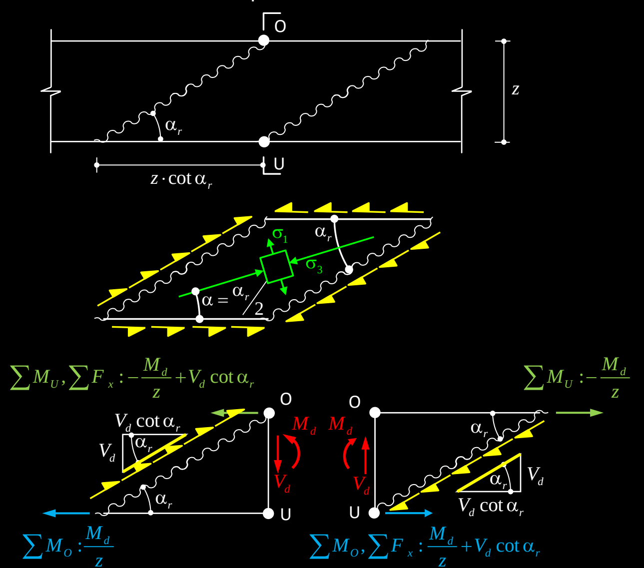
- NB: Longitudinal compressive stresses reduce the principal tensile stress. In earlier editions of SIA 262 (then SIA 162), the shear resistance of prestressed beams was verified on this basis.

Repetition from Stahlbeton II:

"Nominal shear stresses" in the uncracked state

Slabs - Influence of shear forces

Web tension failure - Component without shear reinforcement



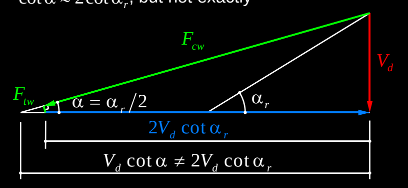
In thin slabs, no failure occurs at first shear crack formation under moderate shear stress, provided that the crack roughness (aggregate interlock) is sufficient and the longitudinal reinforcement has reserves.

(The additional tensile forces in the longitudinal reinforcement due to shear are twice as large as with shear reinforcement!)

Forces acting on a vertical cut:

$\alpha = \alpha_r / 2$

$\cot\alpha \approx 2\cot\alpha_r$, but not exactly



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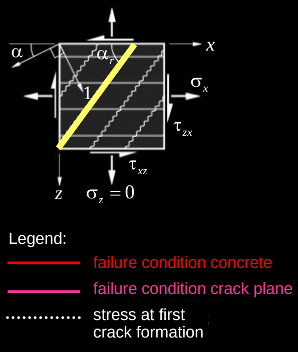
Repetition from Stahlbeton II:

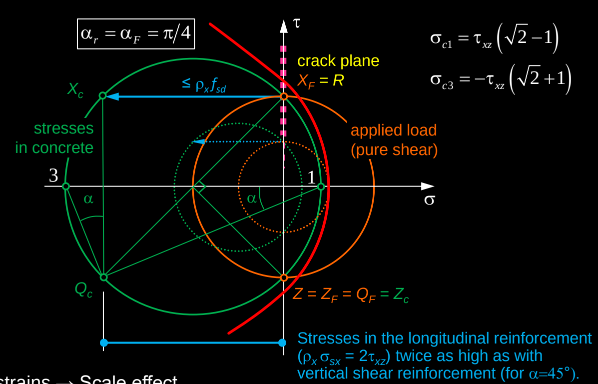
Model for beams without shear reinforcement with a set of parallel, rough cracks transferring pure shear stresses. The longitudinal reinforcement must be able to absorb additional tensile forces. These are twice as high as those in a beam with shear reinforcement and a parallel compression field of inclination $\alpha = \alpha_r$ in the web.

Influence of shear forces

Shear resistance of slabs without shear reinforcement

- Simple model for shear transmission through aggregate interlock in the first cracks under 45° (pure shear stress in the first cracks) → longitudinal reinforcement needs to resist double of the additional tensile force due to *V*:





NB1: Aggregate interlock depends on crack opening, not strains → Scale effect

NB2: The load-bearing capacity due to aggregate interlock is not necessarily sufficient in regions subjected to high shear stress (slabs in the support area) to avoid brittle failure in the event of initial shear cracking!

The simple model shown on the previous slide can be applied to membrane elements under uniform loading and extended for general crack failure conditions (shear and normal stresses).

On this slide, the stress states in the longitudinal reinforcement and in the concrete for the case of cracks with an inclination of 45° are shown by means of a Mohr's circle. The cracks transmit pure shear stresses (without compressive stress). In the concrete between the cracks, there is a biaxial state of stress with principal stresses $-\tau_{xz}\cdot(\sqrt{2}+1)$ (compression) and $\tau_{xz}\cdot(\sqrt{2}-1)$ (tension).

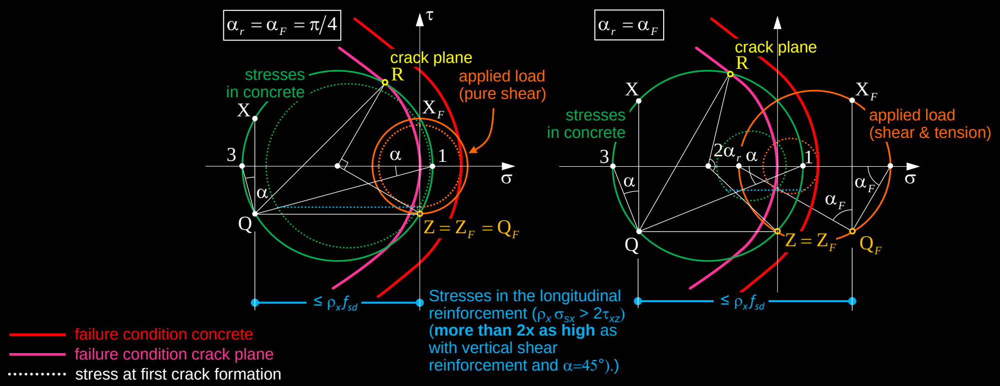
It can be seen that just as in the structural model for beams without shear reinforcement (previous slide), the resulting equivalent reinforcement stresses are twice as high as in a compression field with an inclination of 45° in orthogonally reinforced elements.

Note: The figure on the left shows a more general case with initial crack inclination > 45°, the Mohr's circles on the right are valid for an initial crack direction of 45°.

Influence of shear forces

Shear resistance of slabs without shear reinforcement

- Consideration of more realistic failure criteria for shear transmission by aggregate interlock, i.e. Mohr's envelope. Shear can only be transmitted with compressive stress → even more longitudinal reinforcement required!



NB: There is a scale effect and the validity is limited to moderate shear stresses!

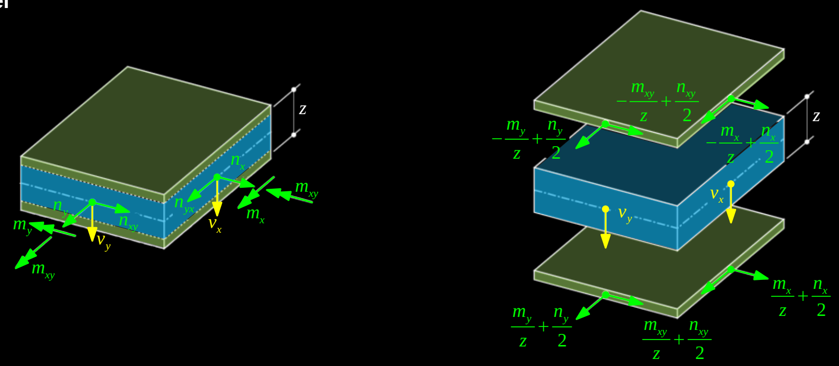
The structural model can be extended by considering realistic relationships for the possible shear and normal stresses at the cracks (aggregate interlock).

In the figure on the left, a pure shear load and an inclination of the cracks of 45° is still assumed. However, the cracks cannot transmit pure shear stresses. A compressive stress acting simultaneously is required. It can be seen that with this model even more longitudinal reinforcement is required than in the case of pure shear stress in the crack planes.

In the figure on the right, not a pure shear, but a general load is applied (shear and normal stresses). It is assumed that the cracks run in the direction of the applied load (principal stress direction). The required force in the longitudinal reinforcement can be determined analogously to pure shear.

Slabs - Influence of shear forces

Sandwich model



Equilibrium solution (general shell loading):

- Sandwich covers carry bending and twisting moments as well as possible membrane forces
  → plane loading, treatment as membrane elements with corresponding reinforcement
  (→ see yield conditions for membrane elements)
- Sandwich core absorbs shear forces
  → Sandwich core absorbs principal shear force v_0 in direction φ_0 and can be treated like the web of a beam in this direction

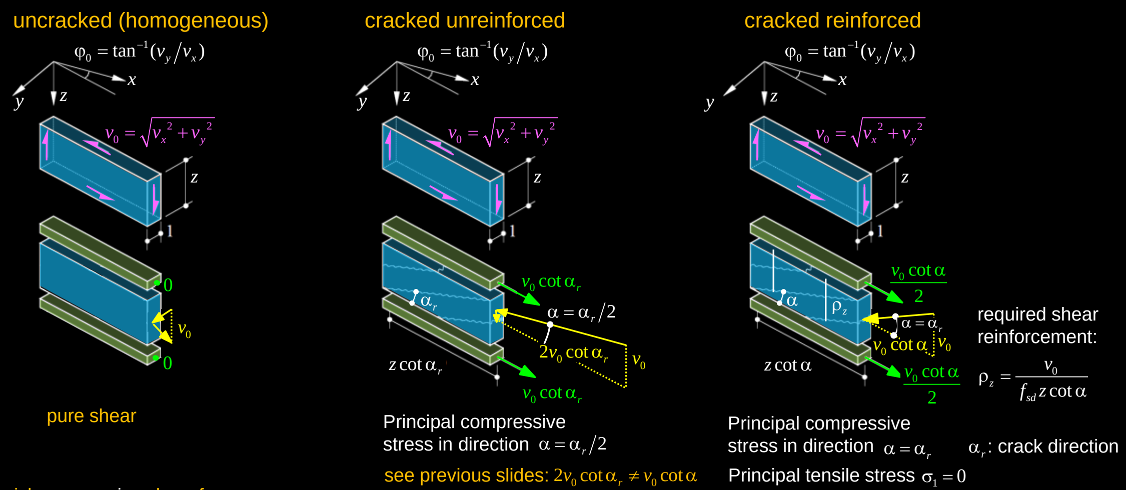
NB: High membrane (compression) forces: core can also be used for this (take into account interaction with v)

Repetition from Stahlbeton II:

The loading of a shell element can be divided between the sandwich covers and the core through statically equivalent forces. The core carries only the transverse (=slab) shear force.

Slabs - Influence of shear forces

Sandwich model - Core



- Sandwich core carries shear forces
  → Sandwich core carries principal shear force v_0 in the direction φ_0 and can be treated like the web of a beam in this direction.
  Tensile forces in the slab plane are to be carried by the sandwich covers (additional membrane loading).

Repetition from Stahlbeton II:

The figure shows three possible model concepts for carrying slab shear forces in the core of the sandwich model. In all three cases it is taken into account that the principal shear force is transferred in the direction φ_0 at every point of the slab (perpendicular shear force = 0).

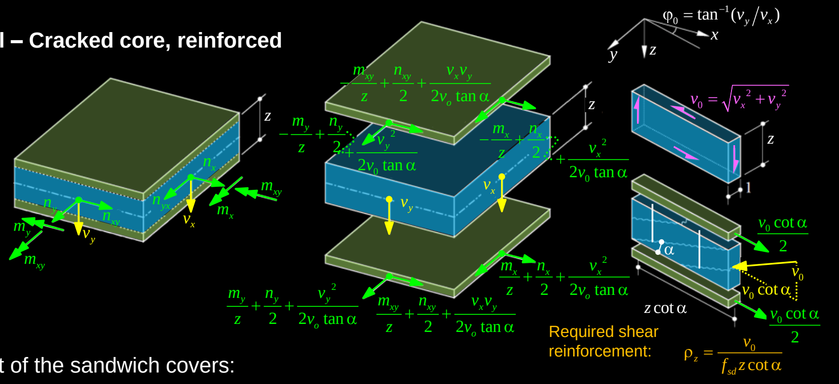
The figure on the left shows the transfer of the shear force in an uncracked core. In this case there is a pure shear stress state (tensile and compressive stresses of the same magnitude under ±45°).

The middle figure shows the transfer of the shear force in a cracked core without shear reinforcement. The load-bearing capacity corresponds to the model shown on the previous slides. The sandwich covers («chords») must absorb twice as much additional force as in the case of shear reinforcement.

The figure on the right shows the transfer of the shear force in a cracked core with vertical shear reinforcement. The load-bearing effect corresponds to a web of a beam with shear reinforcement (see next slide).

Slabs - Influence of shear forces

Sandwich model – Cracked core, reinforced



→ Reinforcement of the sandwich covers:

$$a_{sx} f_{sd} \ge \frac{m_x}{z} + \frac{n_x}{2} + \frac{v_x^2}{2v_0 \tan\alpha} + k\left|\frac{m_{xy}}{z} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan\alpha}\right| \qquad a'_{sx} f_{sd} \ge -\frac{m_x}{z} + \frac{n_x}{2} + \frac{v_x^2}{2v_0 \tan\alpha} + k'\left|-\frac{m_{xy}}{z} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan\alpha}\right|$$

$$a_{sy} f_{sd} \ge \frac{m_y}{z} + \frac{n_y}{2} + \frac{v_y^2}{2v_0 \tan\alpha} + \frac{1}{k}\left|\frac{m_{xy}}{z} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan\alpha}\right| \qquad a'_{sy} f_{sd} \ge -\frac{m_y}{z} + \frac{n_y}{2} + \frac{v_y^2}{2v_0 \tan\alpha} + \frac{1}{k'}\left|-\frac{m_{xy}}{z} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan\alpha}\right|$$

(The factors k, k' can in principle be selected differently at each point of the slab (avoid abrupt changes or anchor differential reinforcement forces). Selection of the compression field inclination α: Analogous considerations as with beams. (Often $k = k' = \cot\alpha = 1$ is chosen.)

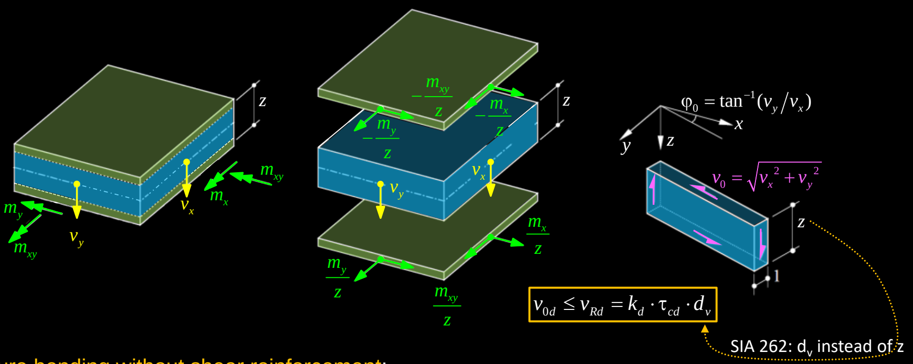
Repetition from Stahlbeton II:

The longitudinal tensile forces due to shear forces, which are to be absorbed by the sandwich covers, result in additional membrane forces in the covers (transformation of the additional «longitudinal» tensile force due to shear in the direction φ_0 in *x*- and *y*-direction). The last terms of the sandwich cover forces and required resistances of the reinforcements shown in the slide correspond to the components of these «chord tensile forces» (see formulas on slide 8 for the components of v_0).

The reinforcement of the sandwich covers can be designed for the resulting forces on the basis of the yield conditions for membrane elements.

Slabs - Influence of shear forces

Sandwich model - Pure bending, uncracked core



→ Slabs under pure bending without shear reinforcement:
$n_x = n_y = n_{xy} = 0$, $v_{0d} \le v_{Rd} = k_d \tau_{cd} d_v$

→ Terms with n_x, n_y, n_{xy} disappear

→ Terms with v_x, v_y disappear if an uncracked core is assumed.

→ With aggregate interlock according to slide 4, at least twice the longitudinal reinforcement (2·terms with v_x, v_y) is required as a result of shear force → Reinforcement in slabs without shear reinforcement should not be graded / curtailed too early!

Repetition from Stahlbeton II:

In the case of an uncracked core, there are no longitudinal tensile forces as a result of shear force. However, in the case of a cracked core without shear reinforcement the longitudinal tensile forces would be twice as high as in the case of shear reinforcement. For this reason, the bending reinforcement should not be graded too early for slabs without shear reinforcement.

The reinforcement of the sandwich covers can also be designed for the resulting forces on the basis of the yield conditions for membrane elements.

Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement according to SIA 262

Nominal shear resistance without shear reinforcement

$$v_{Rd} = k_d \cdot \tau_{cd} \cdot d_v \quad \text{with} \quad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

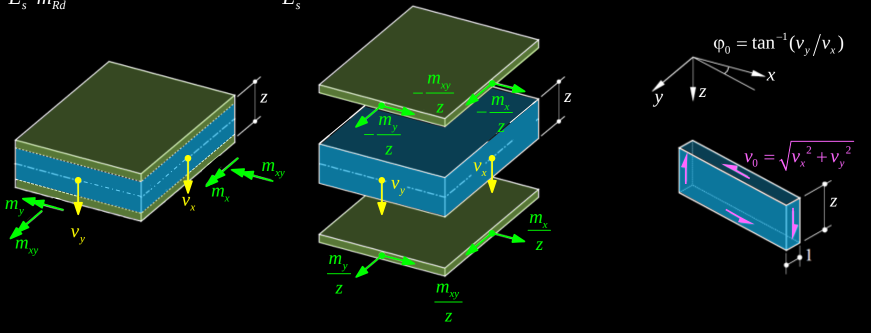
$$k_d = \frac{1}{1+\varepsilon_v \cdot d \cdot k_g} \quad \text{with} \quad k_g = \frac{48}{16+D_{max}}$$

$$\varepsilon_v = \frac{f_{sd}}{E_s}\frac{m_d}{m_{Rd}} \quad \text{resp.} \quad \varepsilon_v = 1.5\frac{f_{sd}}{E_s}$$

k_d: Reduction factor for static depth of the slab, utilization of longitudinal reinforcement and maximum aggregate size

d_v: Effective static depth taking into account cross-section discontinuities

ε_v: Strain of bending reinforcement (1.5 f_{sd}/E_s applies to plastic deformations, +50% in case of graded longitudinal reinforcement)



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Repetition from Stahlbeton II:

The nominal shear resistance without shear reinforcement is determined according to SIA 262 on the basis of the specified relationships. These are based on the concept that a shear failure occurs when a critical shear crack has opened to such an extent that it can no longer transmit the shear stresses required for the transmission of the shear force (see slides 5-6). Therefore, the shear resistance decreases with increasing use of bending reinforcement (which is accompanied by greater chord elongation and thus larger crack openings).

Additional remark:

- In the sandwich model, z was used instead of d_v. Both d and d_v appear in the formulae of SIA 262.

Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement according to SIA 262

Nominal shear resistance without shear reinforcement

$$v_{Rd} = k_d \cdot \tau_{cd} \cdot d_v \quad \text{with} \quad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

$$k_d = \frac{1}{1+\varepsilon_v \cdot d \cdot k_g} \quad \text{with} \quad k_g = \frac{48}{16+D_{\max}}$$

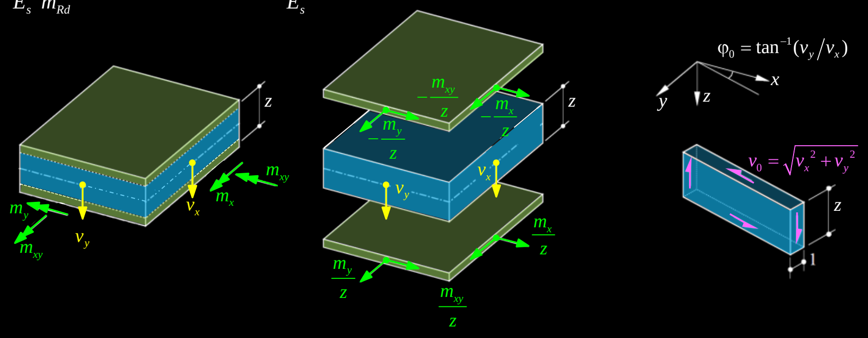
$$\varepsilon_v = \frac{f_{sd}}{E_s}\frac{m_d}{m_{Rd}} \quad \text{resp.} \quad \varepsilon_v = 1.5\frac{f_{sd}}{E_s}$$

(Pre-)dimensioning, B500B, D_{max} = 32 mm:

k_g = 1.0; m_d/m_{Rd} = 1.0 (no plastic redistribution)

$\rightarrow \varepsilon_v = f_{sd}/E_s$ = 2.12‰

$$\rightarrow v_{Rd} = \frac{\tau_{cd} \cdot d_v}{1+\frac{d}{471\text{ mm}}}$$



Repetition from Stahlbeton II:

For preliminary design, the specified simplifications can be used.

Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement according to SIA 262

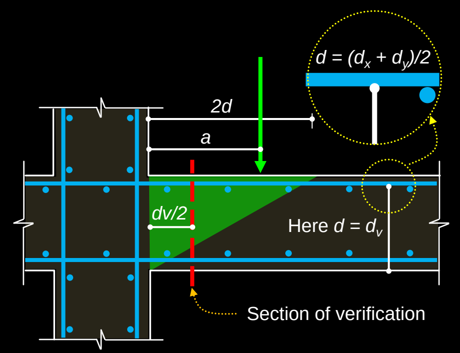
Nominal shear resistance without shear reinforcement

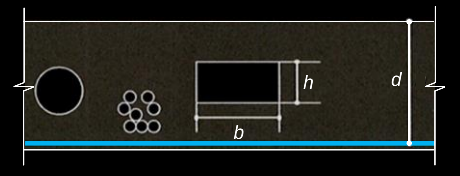
$$v_{Rd} = k_d \cdot \tau_{cd} \cdot d_v \qquad \text{with} \qquad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

$$k_d = \frac{1}{1 + \varepsilon_v \cdot d \cdot k_g} \qquad \text{with} \qquad k_g = \frac{48}{16 + D_{max}}$$

$$\varepsilon_v = \frac{f_{sd}}{E_s} \frac{m_d}{m_{Rd}} \qquad \text{resp.} \qquad \varepsilon_v = 1.5 \frac{f_{sd}}{E_s}$$

- Control section at a distance d_v /2 from the support edge or edge of the load, if necessary at reinforcement gradations
- Reduction of concentrated loads at distance *a* < 2d from bearing edge with factor a/(*2d*) permissible
- Ducts, pipes:
  Diameter / width / height > d/6
  (for cable bundles: dimension of the entire bundle)
  Reduction of d_v by the largest dimension of the inlay or pipe ($d_v = d - \max(b, h)$)





Repetition from Stahlbeton II:

Additional specifications on shear resistance according to SIA 262.

Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement according to SIA 262

Nominal shear resistance without shear reinforcement

$$v_{Rd} = k_d \cdot \tau_{cd} \cdot d_v \quad \text{with} \quad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

$$k_d = \frac{1}{1+\varepsilon_v \cdot d \cdot k_g} \quad \text{with} \quad k_g = \frac{48}{16 + D_{\max}}$$

$$\varepsilon_v = \frac{f_{sd}}{E_s} \frac{m_d}{m_{Rd}} \quad \text{resp.} \quad \varepsilon_v = 1.5 \frac{f_{sd}}{E_s}$$

- slab with prestress or normal force, with decompression moment m_{Dd} : $\varepsilon_v = \frac{f_{sd}}{E_s} \frac{m_d - m_{Dd}}{m_{Rd} - m_{Dd}}$
  ... m_{Dd} = long-term value of the decompression moment (see chapter punching) accounting for normal forces (e.g. due to restraint by stiff supports)
  ... m_d = incl. moments due to restraint and imposed deformations (e.g. secondary moments from prestressing)
- Concrete compressive strength f_{ck} > 70 MPa: $D_{\max} = 0$, this means $k_g = 3$ ($\rightarrow v_{Rd}(f_{ck})$ is discontinuous at 70 MPa)
- Clear deviation of the principal direction φ_0 of the shear force from the direction of the principal reinforcement by angle ϑ: increase of elongation ε_v with factor $\frac{1}{\sin^4\vartheta + \cos^4\vartheta}$
  (i.e. in the worst case, ϑ = 45°: factor 2)

Repetition from Stahlbeton II:

Additional specifications on shear resistance according to SIA 262.

Additional remark:

- The influence of the decompression moment will be explained later (punching)
- The discontinuity of the shear resistance at 70 MPa accounts for the fact that cracks in high strength concrete tend to pass through the aggregates and are therefore smoother than in normal strength concrete, but the chosen value of 70 MPa for the limit cannot be mechanically justified (depending on the strength and shape of the aggregates used and other parameters).

Influence of shear forces

Derivation of the factor for deviation of the principal direction φ_0 of the shear force from the direction of the principal reinforcement (compression field model for sandwich cover)

Compatibility assuming linear elastic behaviour & stress-free cracks
$\rightarrow \varphi_{1\varepsilon} =$ principal strain direction with $\varphi_{1\varepsilon} = \varphi_{1c}$ resp. $\alpha_r + \varphi_{1\varepsilon} = \pi/2$

$$\varepsilon_x = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)\sin^2\alpha_r \quad = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)\cos^2\varphi_{1\varepsilon}$$

$$\varepsilon_z = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)\cos^2\alpha_r \quad = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)\sin^2\varphi_{1\varepsilon}$$

$$\sigma_{sxr} = E_s\varepsilon_x + \frac{\tau_{b0}s_{rmx}}{\varnothing} \qquad \sigma_{szr} = E_s\varepsilon_z + \frac{\tau_{b0}s_{rmz}}{\varnothing}$$

Neglecting concrete strains and tension stiffening (i.e. $\varepsilon_3 = 0, \tau_{b0} = 0$):

$$\sigma_{sxr} \approx E_s\varepsilon_1\cos^2\varphi_{1\varepsilon} \qquad \sigma_{szr} \approx E_s\varepsilon_1\sin^2\varphi_{1\varepsilon}$$

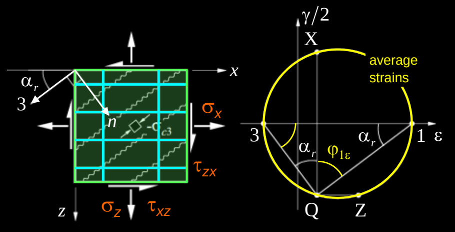
By equilibrium at the cracks, stresses in direction $1\varepsilon=F$ follow as:
(for stress-free cracks only reinforcement stresses act
in this direction; note that generally $\varphi_{1c} \neq \varphi_{1F}$ resp. $\alpha_r \neq \alpha_F$)

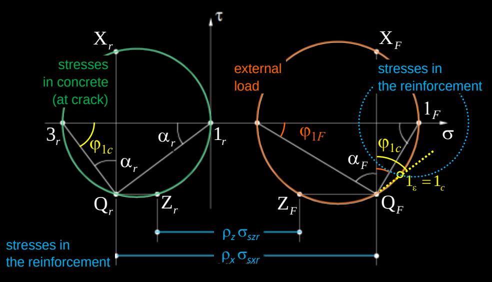
$$\sigma_F(\varphi_{1\varepsilon} = \varphi_{1c}) = \rho_x\sigma_{sxr}\sin^2\alpha_r + \rho_z\sigma_{szr}\cos^2\alpha_r$$

$$= \rho_x\sigma_{sxr}\cos^2\varphi_{1\varepsilon} + \rho_z\sigma_{szr}\sin^2\varphi_{1\varepsilon}$$

$$\rightarrow \sigma_F(\varphi_{1\varepsilon} = \varphi_{1c}) = \rho_x E_s\varepsilon_1\cos^4\varphi_{1\varepsilon} + \rho_z E_s\varepsilon_1\sin^4\varphi_{1\varepsilon}$$

$$\rightarrow \varepsilon_1 = \frac{\sigma_F(\varphi_{1\varepsilon} = \varphi_{1c})}{E_s}\frac{1}{\left(\rho_x\cos^4\varphi_{1\varepsilon} + \rho_z\sin^4\varphi_{1\varepsilon}\right)}$$





The magnification factor $(\sin^4\vartheta+\cos^4\vartheta)^{-1}$ can be derived by considering the deformations of the «sandwich cover» on the flexural tension side using a stress field model.

The tensile force perpendicular to the principal compressive stress direction (= perpendicular to the cracks) can easily be determined from the forces in the reinforcement that cross the crack (assuming that cracks are stress-free). On the other hand, the stresses in the reinforcement can be determined from the principal strain. This results in a relationship between the principal tensile elongation and the tensile force in the corresponding direction. It can be seen that the principal distortion in the isotropic reinforcement is by the factor $(\sin^4\vartheta+\cos^4\vartheta)^{-1}$ greater than it would be the case in reinforcement in the direction of the principal strains.

Additional remark:

- Only in special cases does the principal strain direction correspond to the principal stress direction of the applied load. This means the reinforcement in the crack corresponds to a normal and shear force with respect to the crack direction (as shown above). Thus, the given relation does not link the principal elongation with the applied principal tensile stress (but with the tensile force perpendicular to the principal elongation).