2 In-plane loading

2.6 Numerical modelling

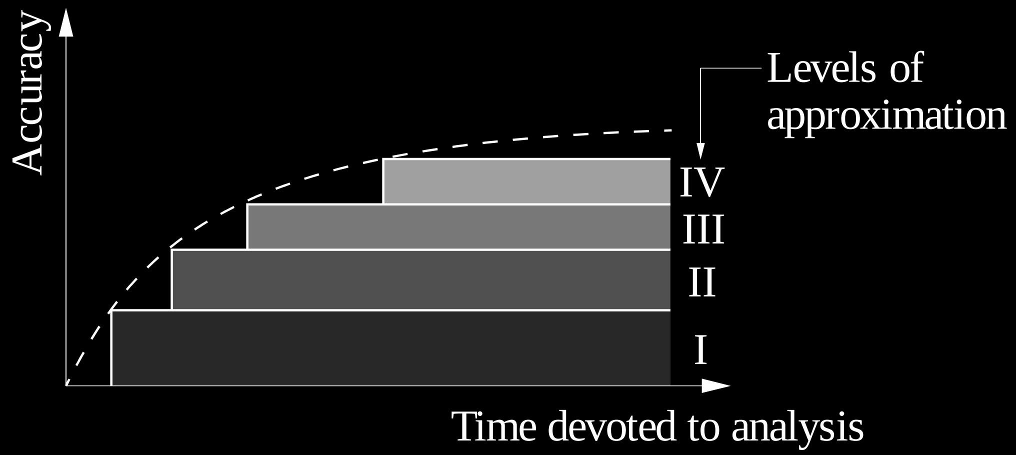
Learning objectives

Within this chapter, the students are able to:

- select the most suitable numerical model for each structural concrete problem, clearly differentiating design and assessment-oriented approaches.
    - recognise the higher probability of making mistakes when increasing modelling complexity and the necessity to cross-check numerical models' results with simple handmade analysis.
    - identify how to discretise a structural member with a combination of spine, planar, multilayer, and three-dimensional elements.
    - discuss the workflow of selected numerical models.
- recall the main assumptions of the Compatible Stress Field Method, its range of applicability and the similitudes and differences to already studied equilibrium and compression field approaches.

Introduction

Levels of Approximation (LoA)

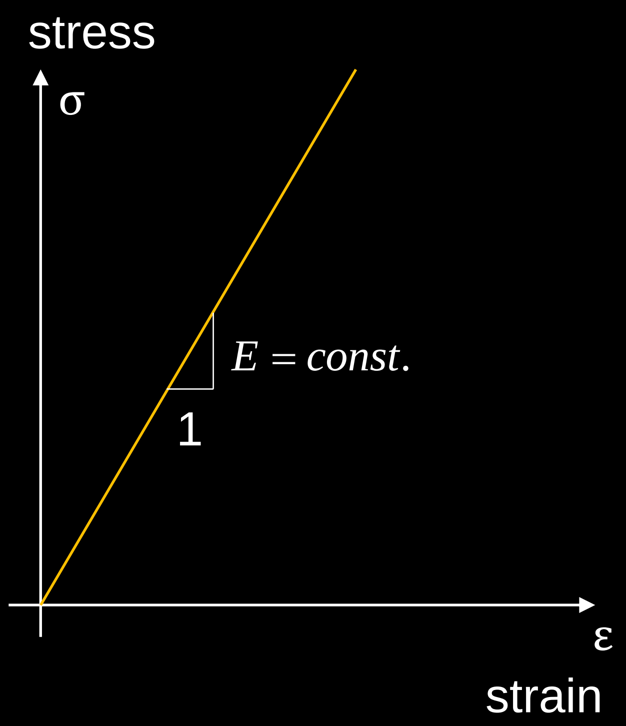


[Muttoni, 2018]

- From simple analyses (handmade) to nonlinear calculations (specific software)
- With every new LoA the knowledge on the behavior of the structure increases
- While a low LoA tends to be conservative, a higher LoA does not always predict a higher load (hidden brittle mechanisms can be captured with high LoA)
- More complex models also increase the probability of making a modelling mistake → engineer should always cross check with simple hand calculations!

Linear vs. non-linear finite element analysis

Linear



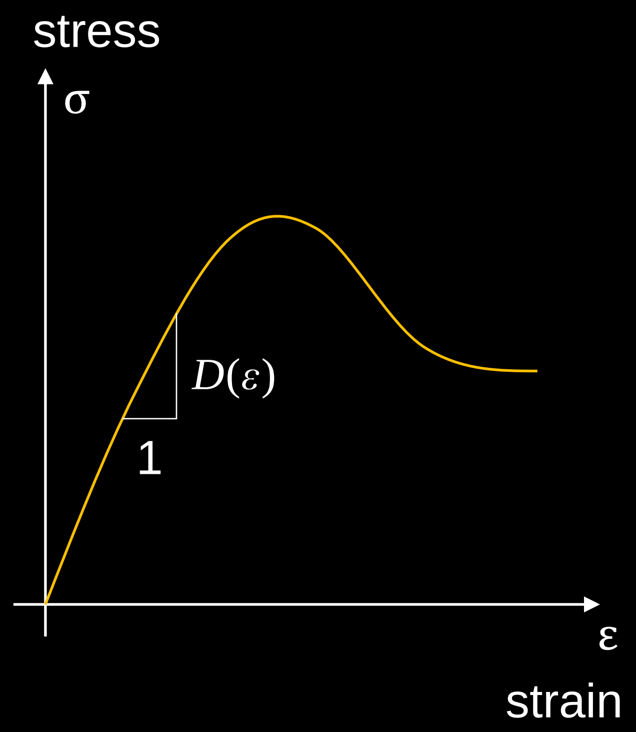
Stress-strain relationship: $\sigma = E \cdot \varepsilon$

FE solves $K \cdot u = f \rightarrow u = K^{-1} \cdot f$

Stresses depend linearly on strains.

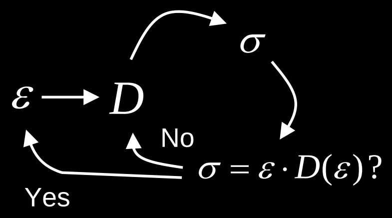
→ directly obtain stresses with *E* and strains

Non-linear



Stress-strain relationship: $\sigma = \varepsilon \cdot D(\varepsilon)$

FE solves iteratively $K(u) \cdot u = f$

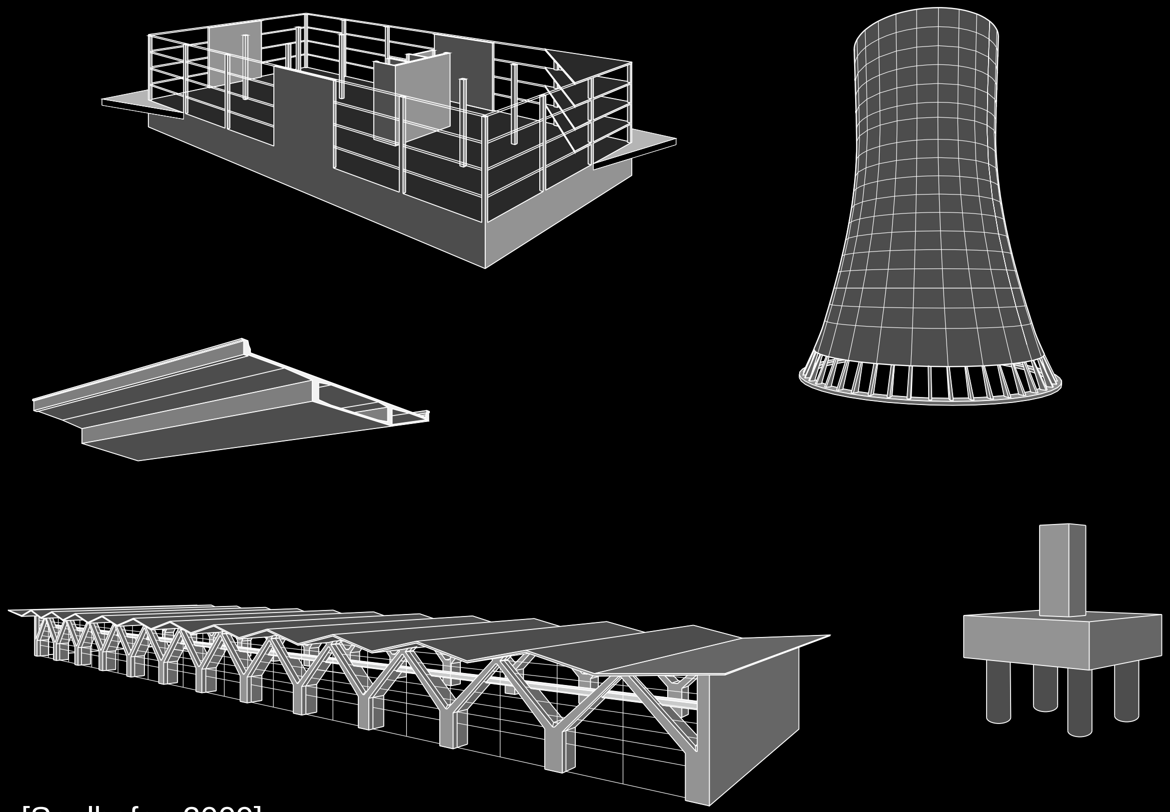


Stresses depend non-linearly on strains.

→ stiffness matrix is obtained iteratively depending on strains / stresses and whether equilibrium is fulfilled

Introduction

Modelling of structures



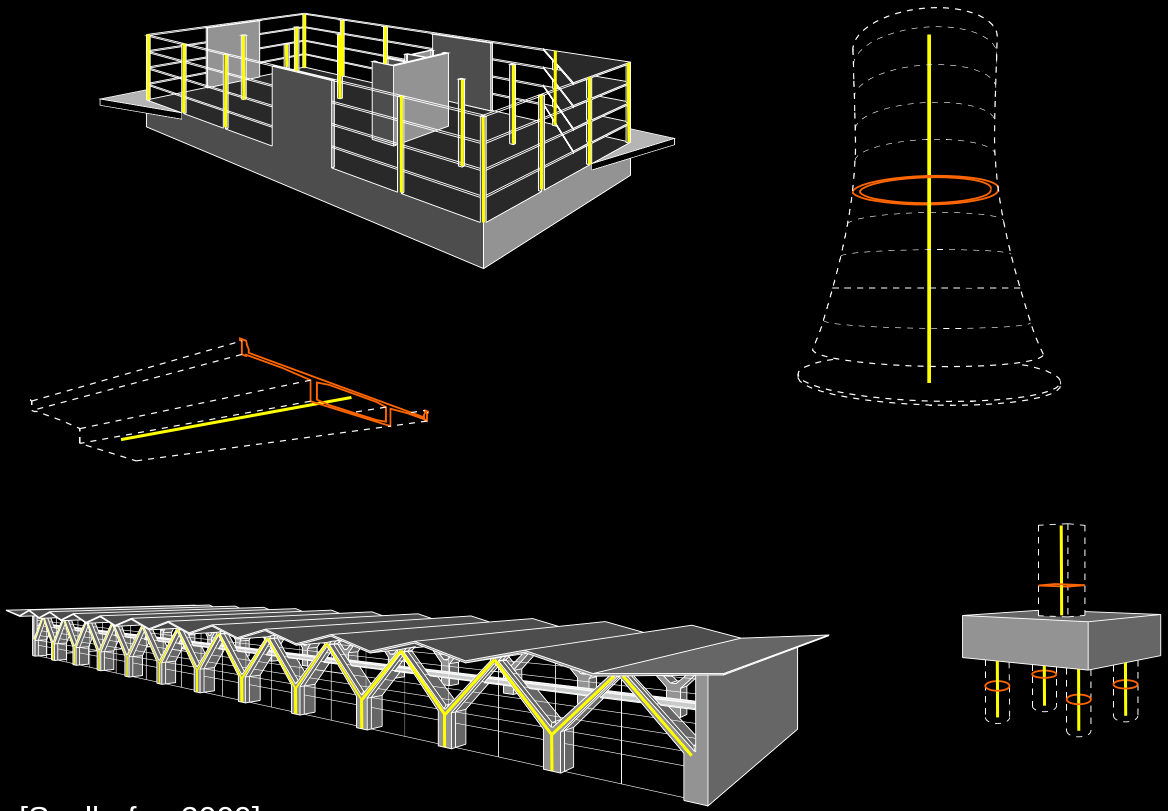
[Seelhofer, 2009]

Structures can be modelled with linear or non-linear approaches and with

- 1D elements (spine)
- 2D elements
- 2D multilayer elements
- 3D elements

Introduction

Modelling of structures



[Seelhofer, 2009]

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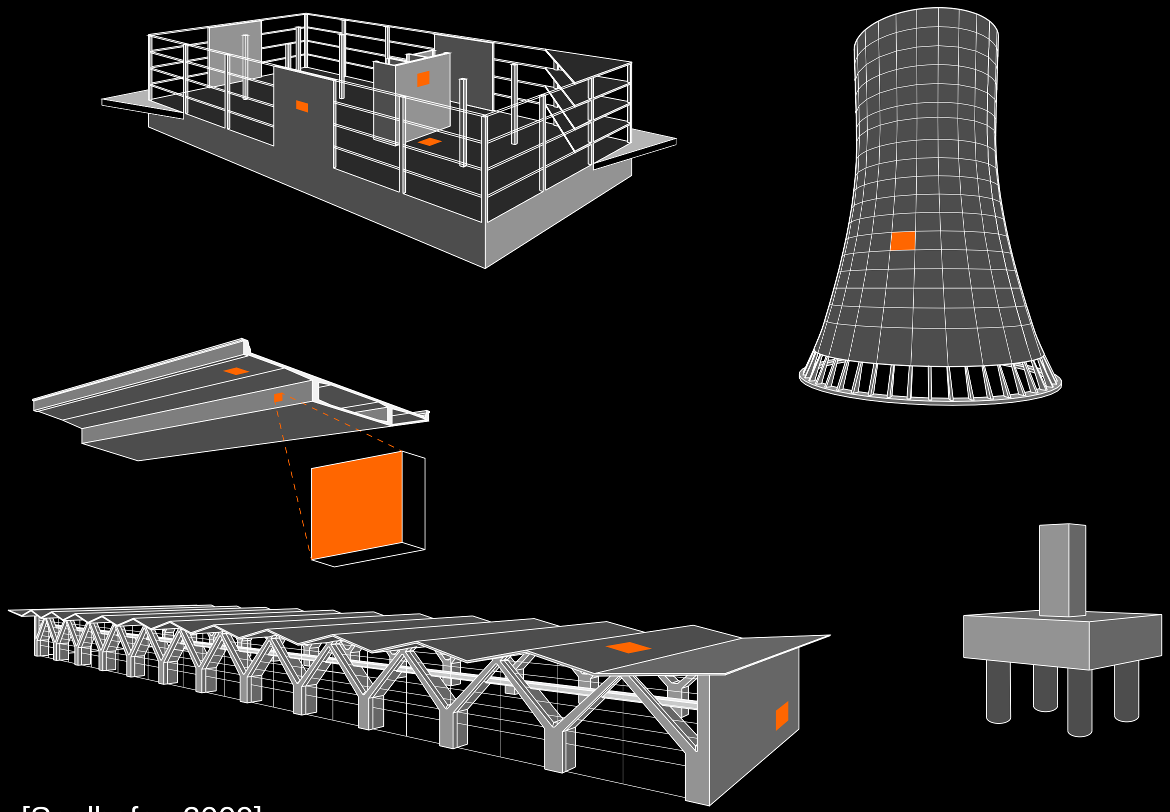
Introduction

Modelling of structures



[Seelhofer, 2009]

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- 2D elements
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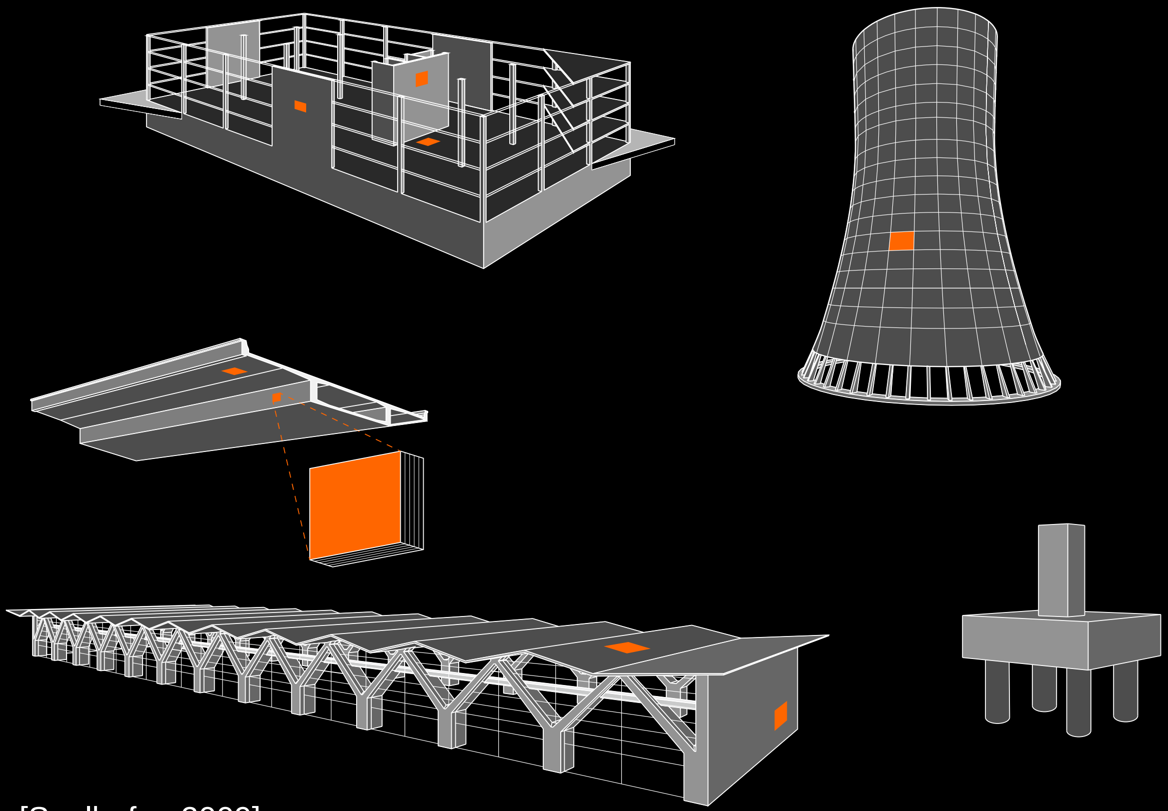
- 3D elements

Introduction

Modelling of structures



[Seelhofer, 2009]

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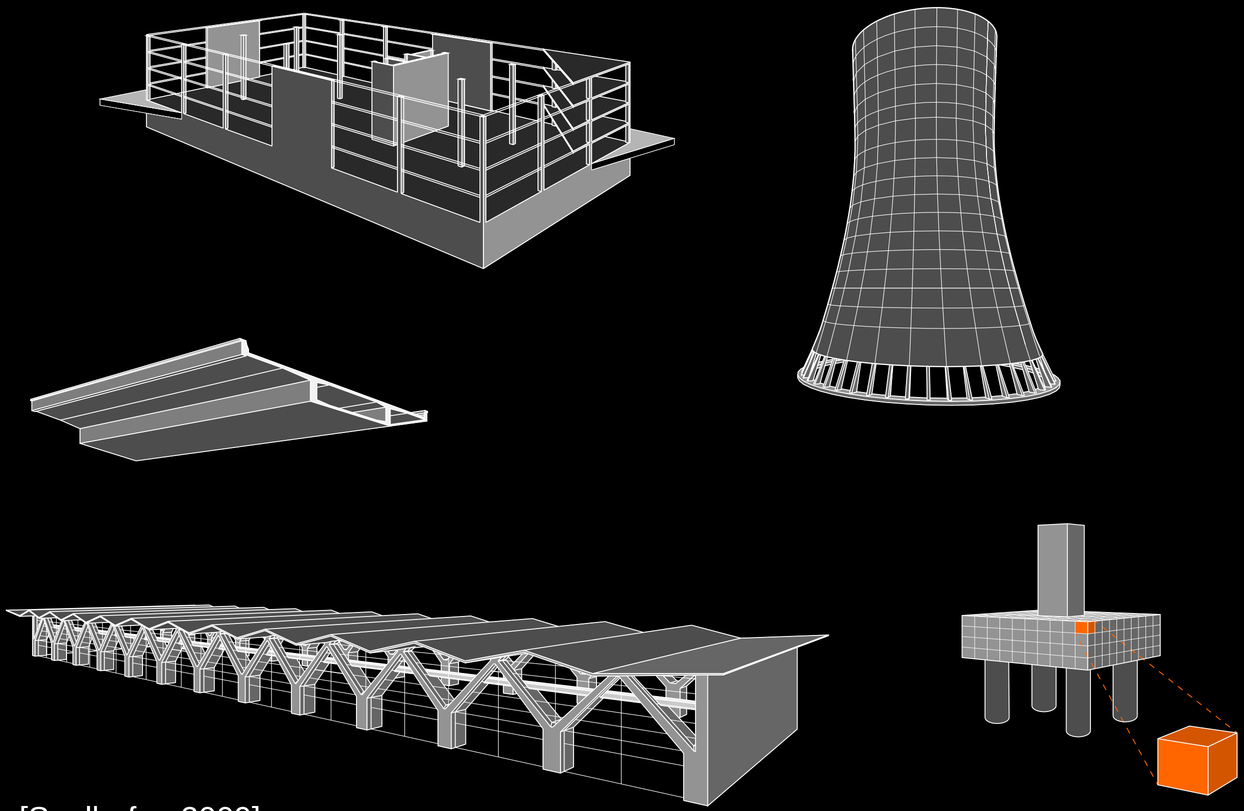
- 3D elements

Introduction

Modelling of structures



[Seelhofer, 2009]

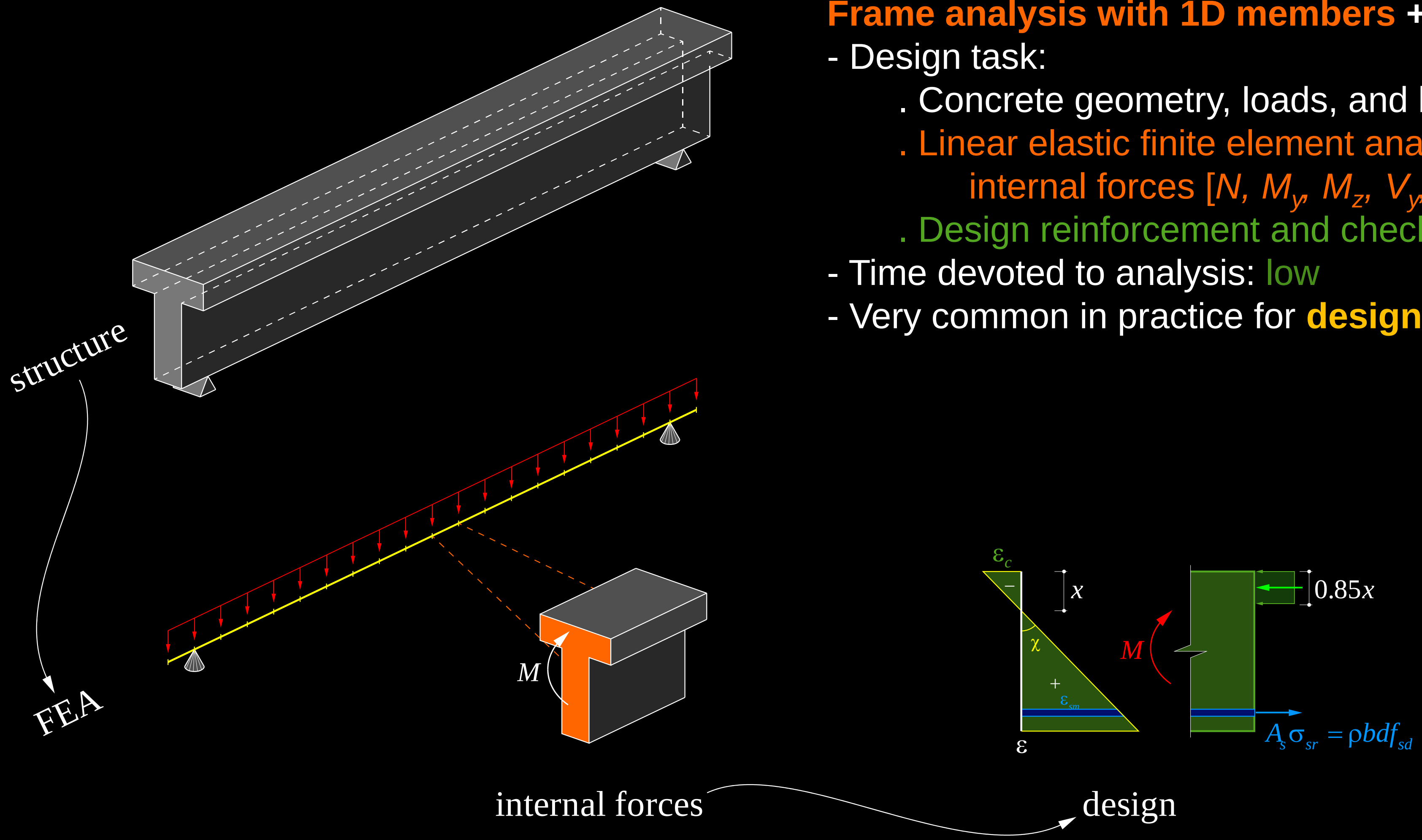
Structures can be modelled with linear or non-linear approaches and with

- 1D elements (spine)
- 2D elements
- 2D multilayer elements
- 3D elements

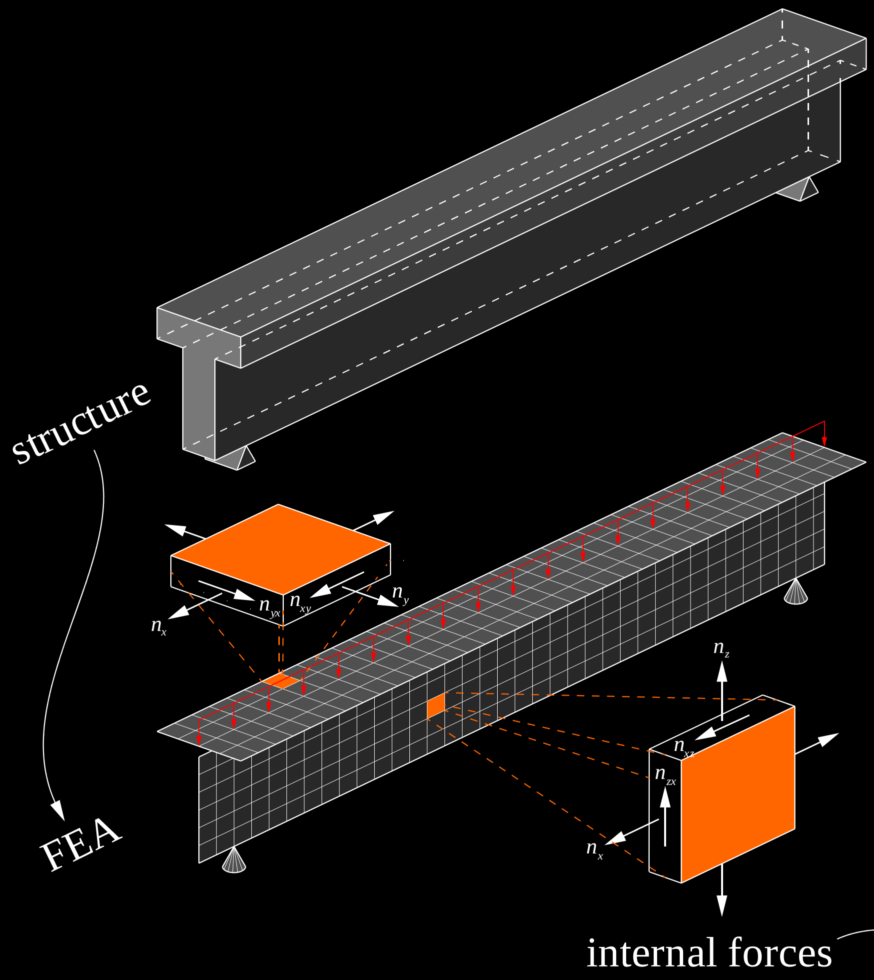
Overview of numerical models for structural design and analysis

Frame analysis with 1D members + cross section design

- Design task:
  - . Concrete geometry, loads, and boundary conditions are known
  - . Linear elastic finite element analysis (FEA) to determine internal forces [$N, M_y, M_z, V_y, V_z, T_x$]
  - . Design reinforcement and check concrete
- Time devoted to analysis: low
- Very common in practice for **design**, commercial software available



Overview of numerical models for structural design and analysis

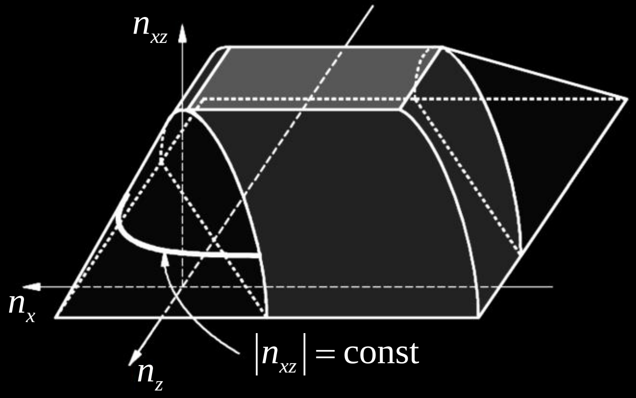


2D analysis + **design with membrane yield conditions**

- Design task:
    - . Concrete geometry, loads and boundary conditions are known
    - . Linear elastic finite element analysis (FEA) to determine internal forces [n_x, n_z, n_{xz}] (elements with only membrane loading)
    - . Design reinforcement with yield conditions (k=1) and check concrete
- Time devoted to analysis: low
- Common in practice for **design**, commercial software available

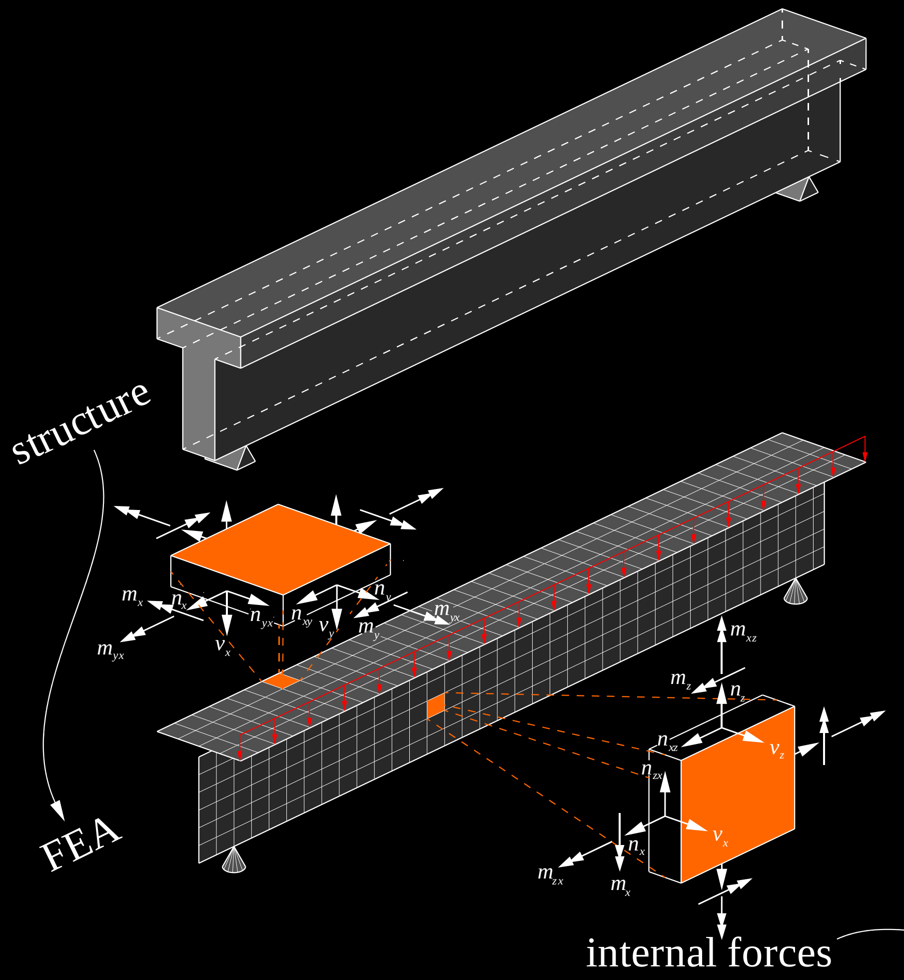
$$Y_1 = n_{xz}^2 - (a_{sx} f_{sx} - n_x)(a_{sz} f_{sz} - n_z) = 0$$

$$k = \cot\alpha \longrightarrow \begin{array}{l} a_{sx} f_{sx} \geq n_x + k\left|n_{xz}\right| \\ a_{sz} f_{sz} \geq n_z + k^{-1}\left|n_{xz}\right| \end{array}$$



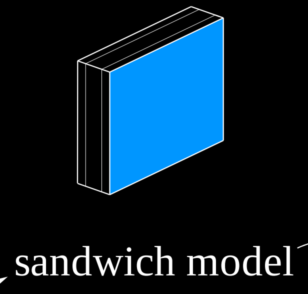
design

Overview of numerical models for structural design and analysis



2D analysis + sandwich model + design with membrane yield conditions of outer layers

- Design task:
  - . Concrete geometry, loads, and boundary conditions are known
  - . Linear elastic finite element analysis (FEA) to determine internal forces [n_x, n_z, n_{xz}, m_x, m_z, m_{xz}, v_x, v_z] (elements with general shell loading)
  - . Transformation of the general shell loading to the sandwich model
  - . Design reinforcement in the outer layers with yield conditions (k=1) and check concrete
- Time devoted to analysis: medium
- Common in practice for **design**

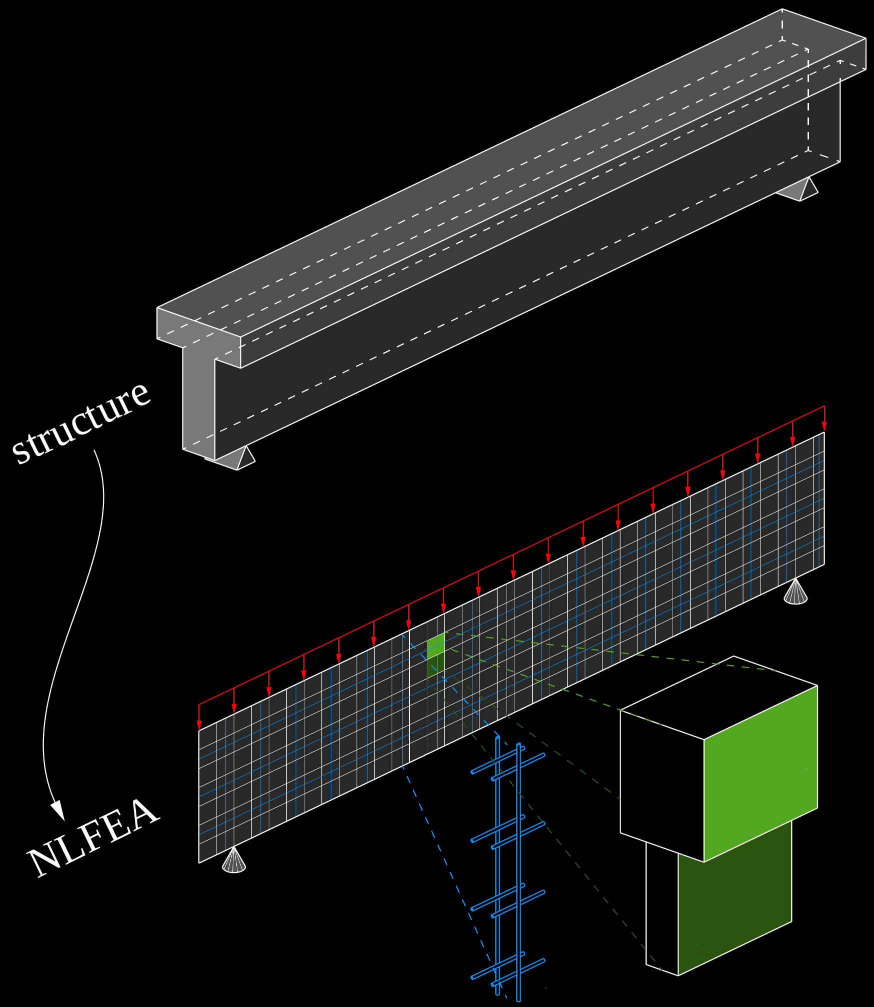


$$Y_1 = n_{xz}^2 - (a_{sx} f_{sx} - n_x)(a_{sz} f_{sz} - n_z) = 0$$

$$k = \cot\alpha \longrightarrow \begin{array}{l} a_{sx} f_{sx} \ge n_x + k|n_{xz}| \\ a_{sz} f_{sz} \ge n_z + k^{-1}|n_{xz}| \end{array}$$

design

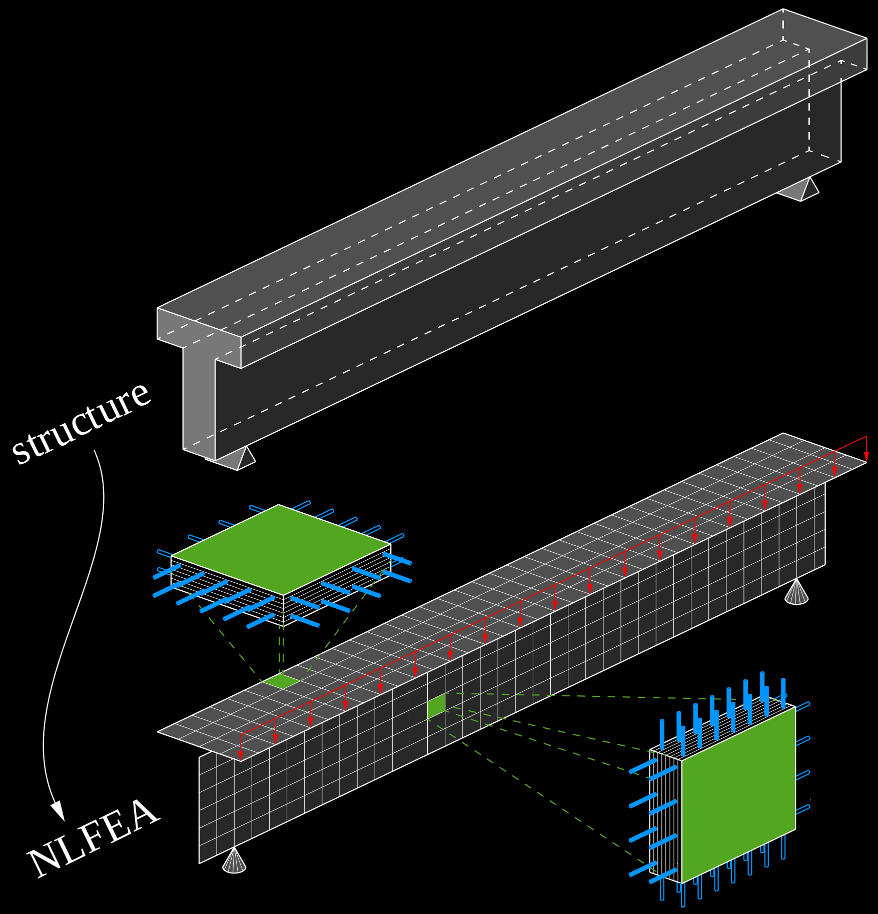
Overview of numerical models for structural design and analysis



Compatible Stress Field Method (CSFM)

- Assessment task
  - . Concrete geometry, loads, and reinforcement are known
  - . Non-linear finite element analysis (NLFEA) → $\sigma \xleftrightarrow[\text{}]{\sigma(\varepsilon)} \varepsilon$ (constitutive relationship)
    - Compatible stress fields
    - Structures with only in-plane loading
    - Reinforcement and concrete are modelled separately
    - Suitable for Discontinuity Regions
    - Tension stiffening according to TCM & POM (1D)
- Time devoted to analysis: medium
- Commercial software available → Idea StatiCa Detail
- Increasingly used in practice for **assessment** and **design**

Overview of numerical models for structural design and analysis



[Thoma, 2018]

Cracked Membrane Model Usermat (CMM-Usermat)

- Assessment task
  - . Concrete geometry, loads, and reinforcement are known
  - . Non-linear finite element analysis (NLFEA) → constitutive relationship $\sigma \xleftrightarrow{\sigma(\varepsilon)} \varepsilon$
    - Compatible stress fields
    - Multilayer shell element
    - Reinforcement and concrete are modelled as a composite
    - Tension stiffening according to TCM (2D)
- Time devoted to analysis: high
- Used at ETHZ for research and expertise

Overview of numerical models for structural design and analysis

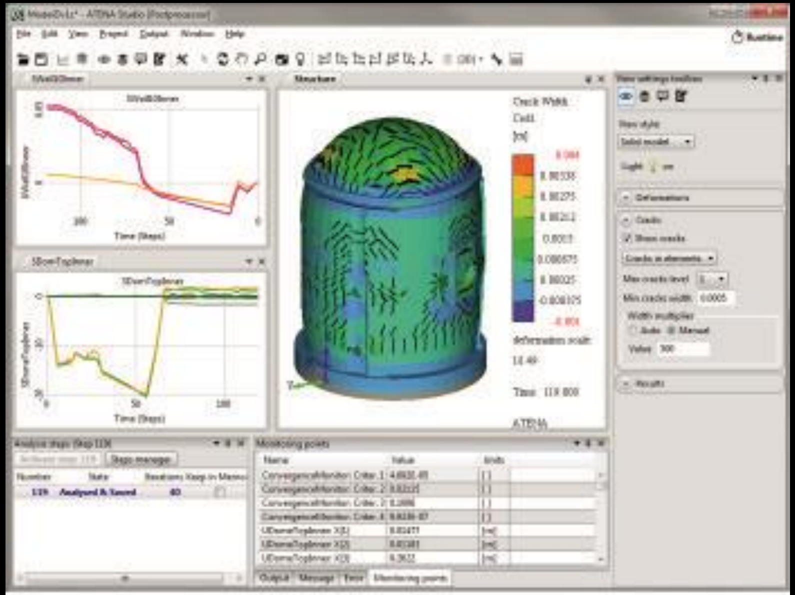


Fig 2: Over-pressure simulation of nuclear containment

[Cervenka, 2020]

Full non-linear finite element analyses

- Assessment task
    - . Concrete geometry, loads, and reinforcement are known
    - . Non-linear finite element analysis (NLFEA) → $\sigma \xleftrightarrow{\sigma(\varepsilon)} \varepsilon$ constitutive relationship
        - Many available models (usually very complex)
        - Tensile strength usually considered for equilibrium
        - Not compliant with structural design codes
- Time devoted to analysis: very high
- Many commercial software available (Ansys, Abaqus, Atena, Diana…)
- Not a design tool. Rarely used in practice for assessment (skilled users)

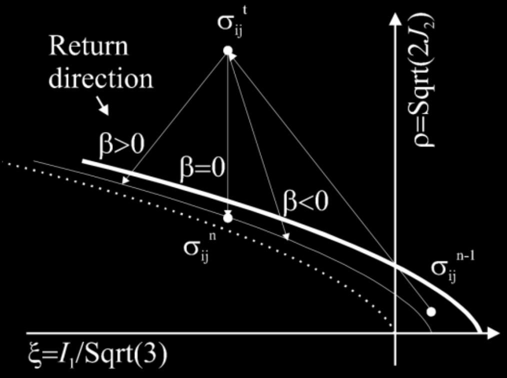


Fig. 2-22 Plastic predictor-corrector algorithm.

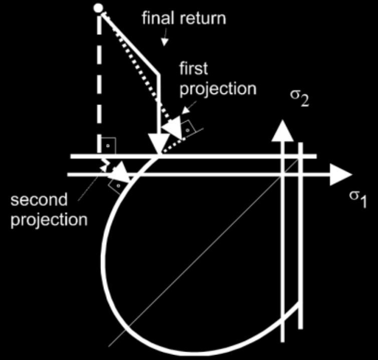
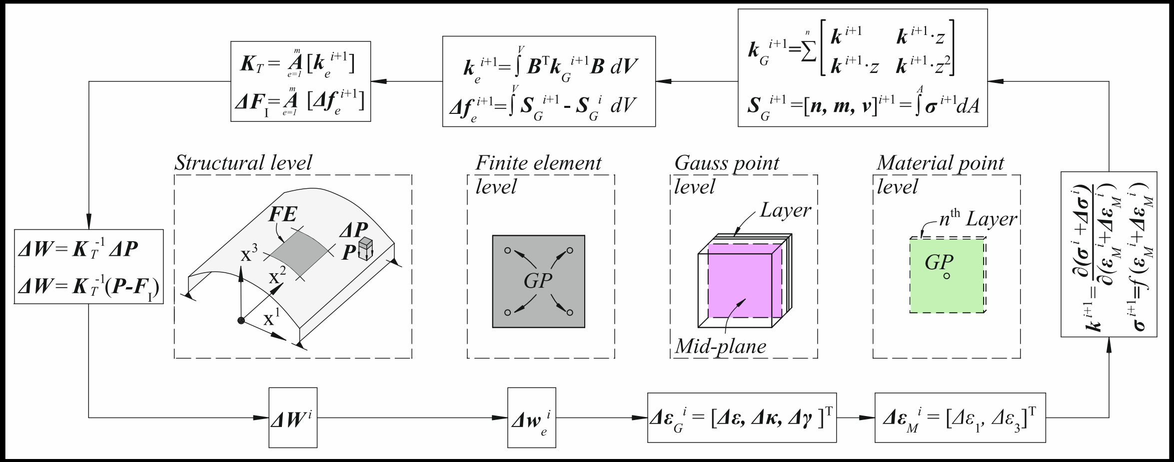


Fig. 2-23. Schematic description of the iterative process (2.73). For clarity shown in two dimensions.

[Cervenka, 2020]

Annex

Cracked Membrane Model Usermat (CMM-Usermat)



[Thoma, 2018]

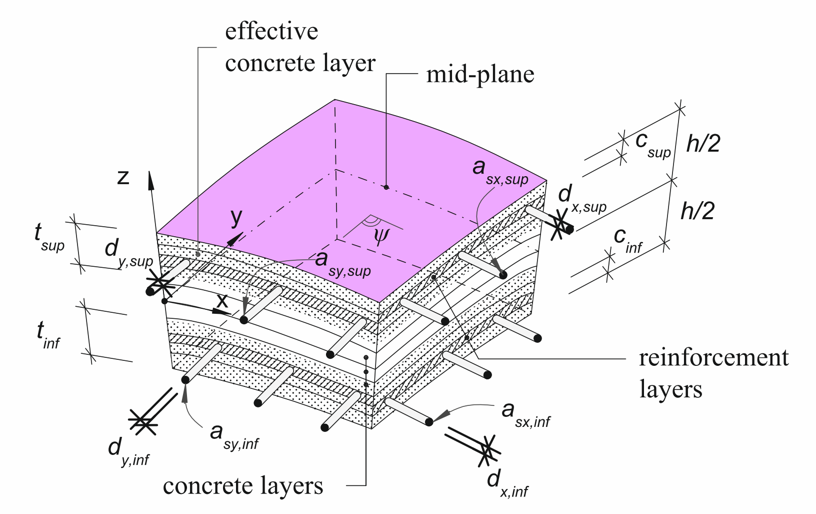
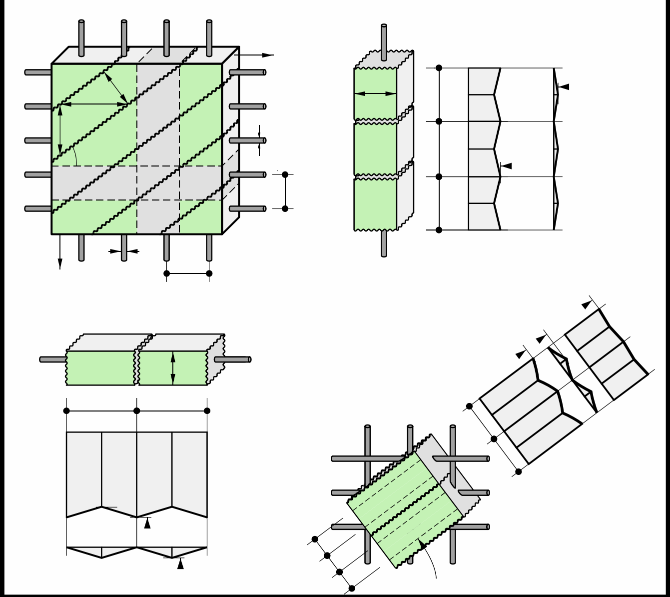


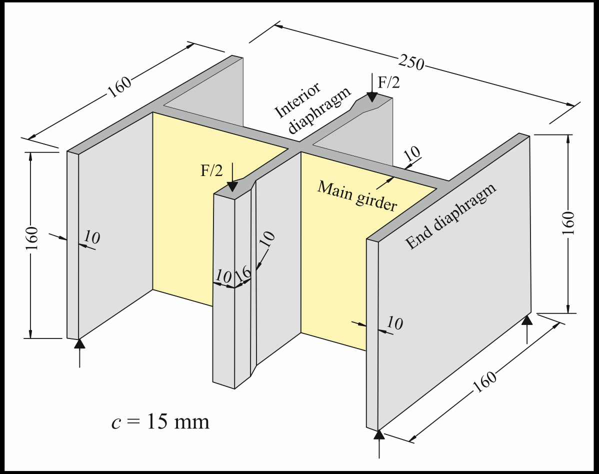
Fig. 3. Multilayer shell element – Cracked Membrane Model Usermat input parameters.

[Thoma, 2018]



[Kaufmann, 1998]

Cracked Membrane Model Usermat (CMM-Usermat)



Comparison between experiment and CMM-Usermat calculation

- Reinforced concrete shear wall: IWT2 (from Leonhardt and Walther)
  . Indirectly supported plate with indirect load introduction

- Results
  . Measured and calculated load-deformation curves agree well
  . Same failure mechanism at exactly the same location
  . Crack pattern at failure are also sufficiently similar

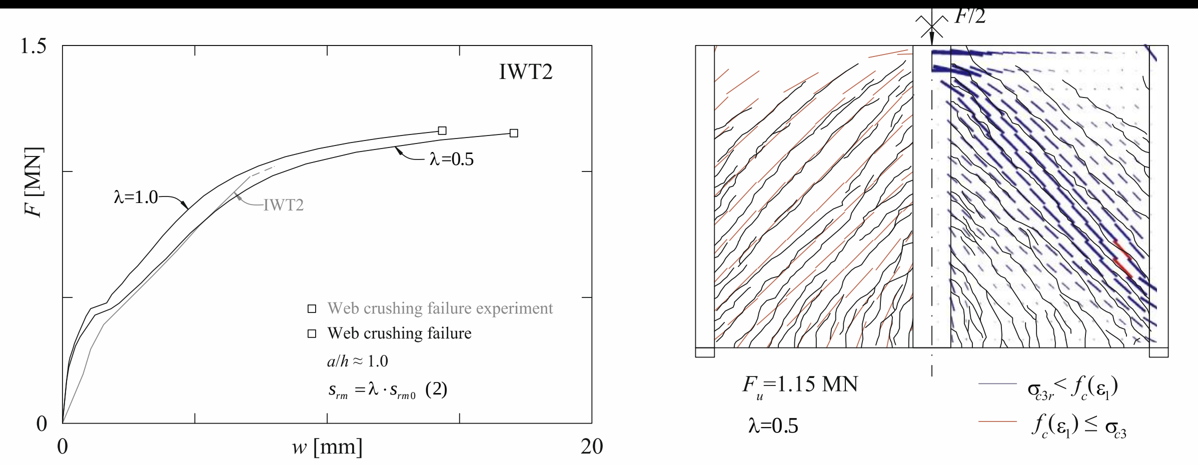


Fig. 6. Plate test IWT2 carried out by Leonhardt and Walther [15]: (a) load–deformation diagram; (b) crack pattern and principal compressive stress σ_{c3} in the main girder.

[Thoma, 2018]

Compatible Stress Field Method

Compatible Stress Field Method (CSFM) - Implemented in commercial software Idea StatiCa Detail

Continuous stress fields = Computer-aided stress fields

Scope

- Simple method for efficient, code-compliant design and assessment of discontinuity concrete regions
- Including serviceability and deformation capacity verifications
- Direct link to conventional RC design: standard material properties, concrete tensile strength totally neglected for equilibrium (only its influence to the stiffness is accounted for)

Inspirations

- EPSF FE-implementation (strain compatibility, automatic determination of concrete reduction factor from strain state)
- Tension Chord Model TCM and Cracked Membrane Model CMM (tension stiffening, ductility and serviceability checks)

Development / Credits

ETH zürich

IDEA StatiCa®

Calculate yesterday's estimates

This project has received partial funding from Eurostars-2
joint programme, with co-funding from the European Union
Horizon 2020 research and innovation programme

Compatible Stress Field Method

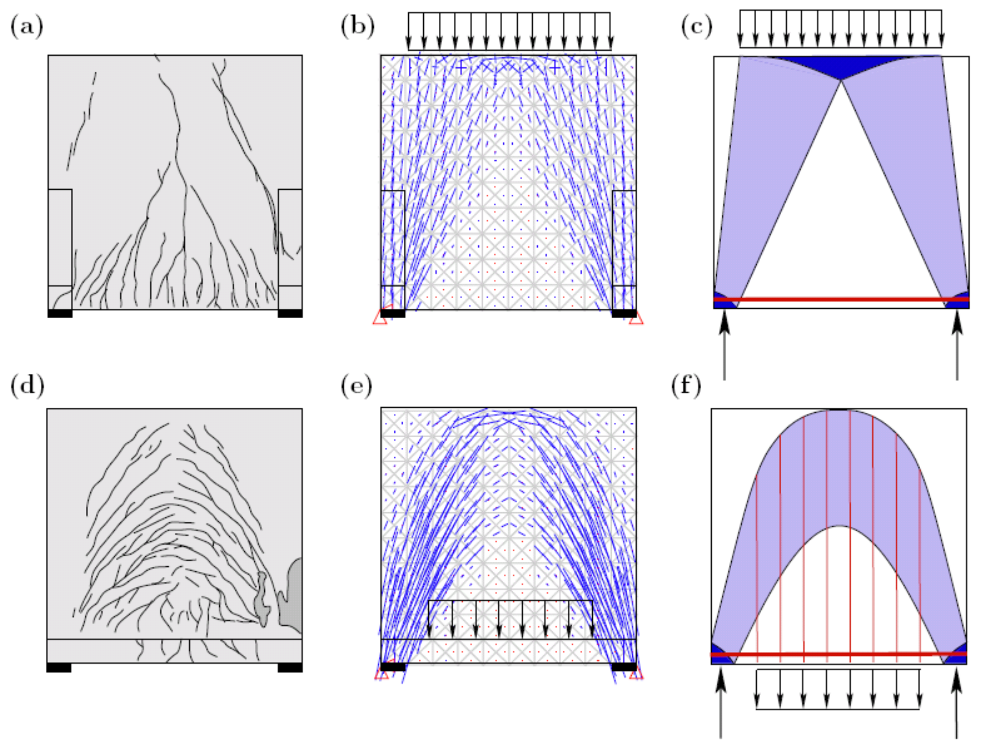
Dimensioning/assesment of Discontinuity Regions: Previously existing computer-aided tools

EPSF elastic plastic stress fields (Fernández Ruiz & Muttoni, 2007)

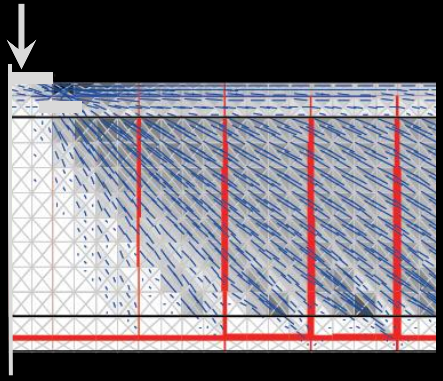
Experimental crack pattern

Numerical results EPSF

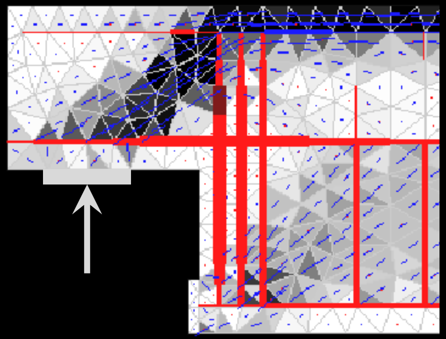
Hand-calculated stress fields



[Muttoni & Fernandez Ruiz, 2007]



- ☺ Maintains advantages of hand calculations (transparent, safe design with $f_{ct} = 0$, consistent detailing)
- ☺ Compressive strength f_c determined automatically from strain state



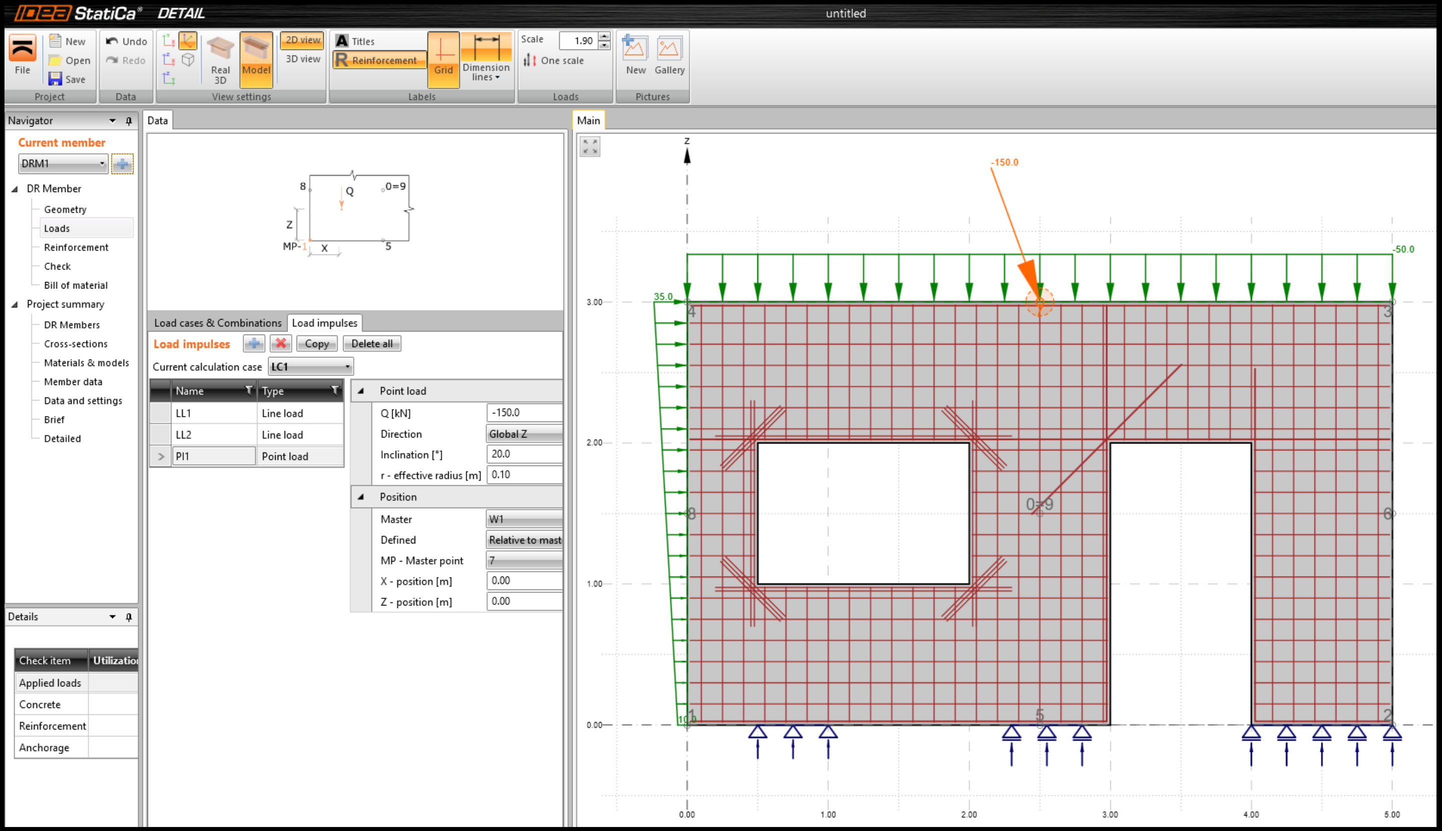
- ☹ Limited user-friendliness
- ☹ Limited use for serviceability
  … no tension stiffening
  … no crack width calculation
- ☹ No check of deformation capacity (perfectly plastic material)

Compatible Stress Field Method

CSFM: design process

1) **Definition of geometry, loads and load combinations**
   a) BIM connections: export data from a global model for the analysis of a detail
   b) Standalone application:

Full definition in standalone user-friendly application



Current member: DRM1

- DR Member
  - Geometry
  - Loads
  - Reinforcement
  - Check
  - Bill of material
- Project summary
  - DR Members
  - Cross-sections
  - Materials & models
  - Member data
  - Data and settings
  - Brief
  - Detailed

Load cases & Combinations | Load impulses

Load impulses: Copy, Delete all

Current calculation case: LC1

Name	Type
LL1	Line load
LL2	Line load
PI1	Point load

Point load

Q [kN]: -150.0
Direction: Global Z
Inclination [°]: 20.0
r - effective radius [m]: 0.10

Position

Master: W1
Defined: Relative to mast
MP - Master point: 7
X - position [m]: 0.00
Z - position [m]: 0.00

Details

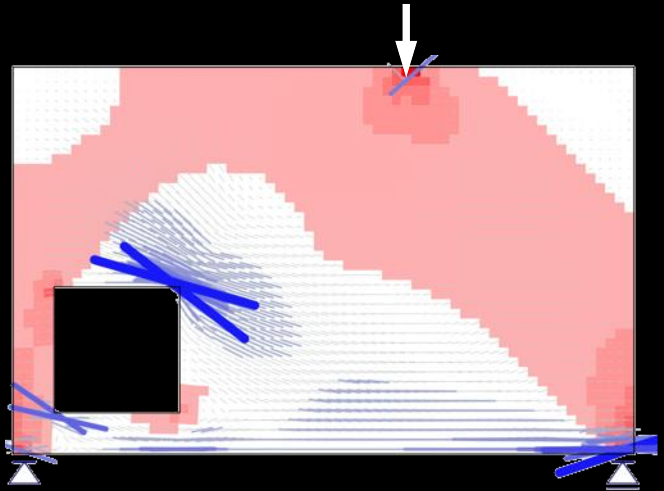
Check item	Utilizatio
Applied loads	
Concrete	
Reinforcement	
Anchorage	

Compatible Stress Field Method

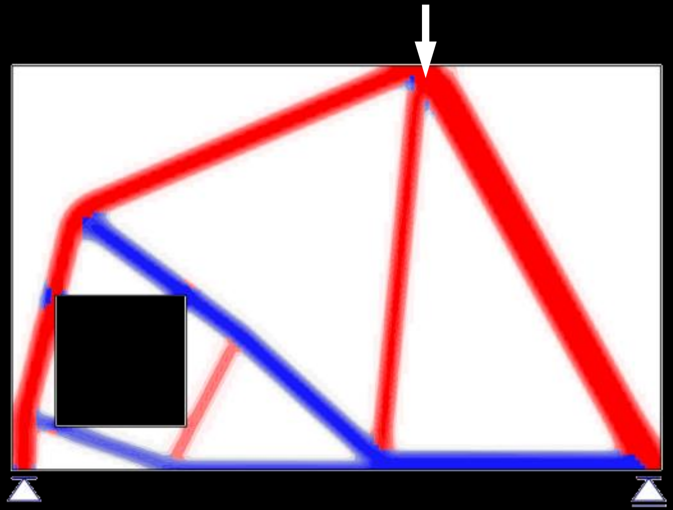
CSFM: design process

2) **Reinforcement design**
   a) Location of reinforcement: definition by user. Several design tools are provided to identify where the reinforcement is required (for complex regions):

Linear elastic stress flow



Topological optimization

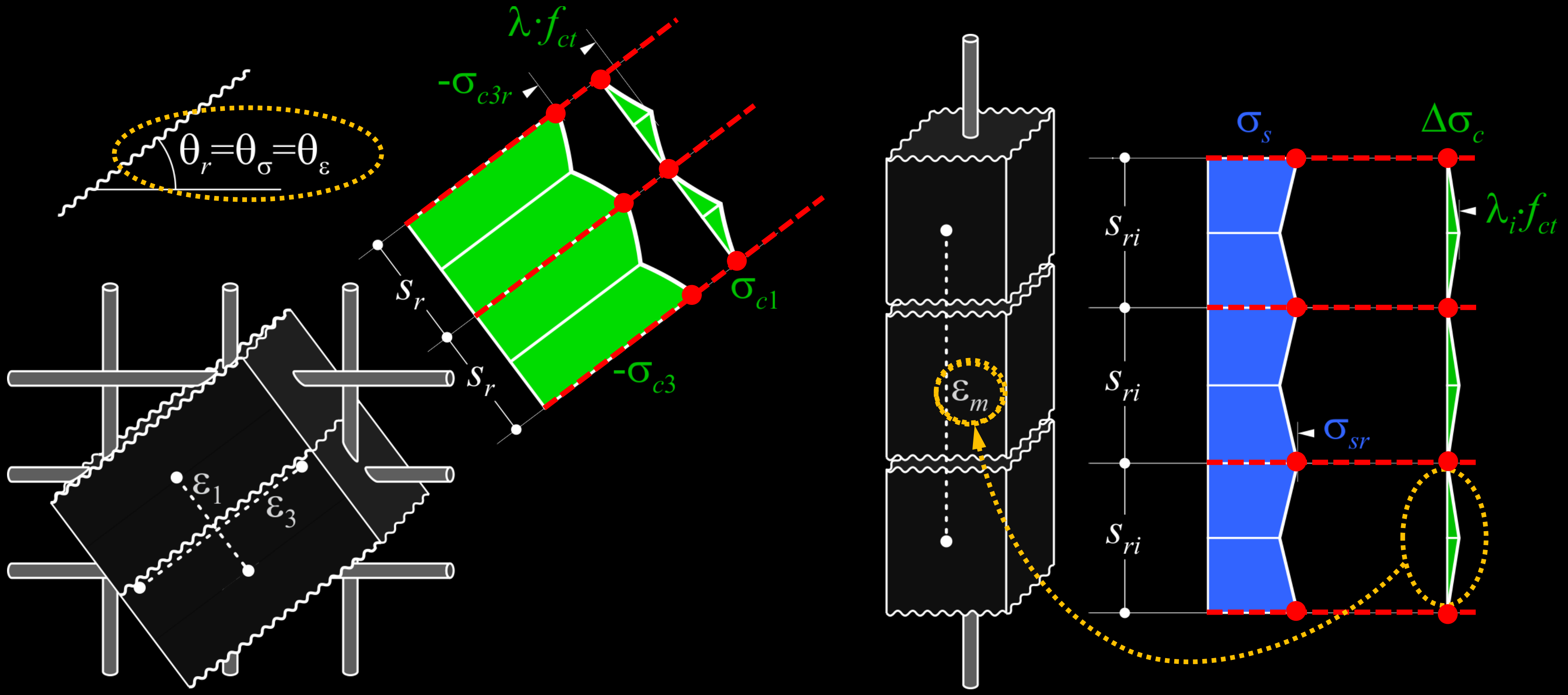


   b) Amount of reinforcement: can be automatically designed for all or part of the reinforcement. Not yet released in current version

3) **Verification models** to check all code requirements
   a) Load-bearing capacity
   b) Serviceability verifications (deformations, crack width…)

Compatible Stress Field Method

CSFM verification model: main assumptions



based on [Kaufmann and Marti, 1998]

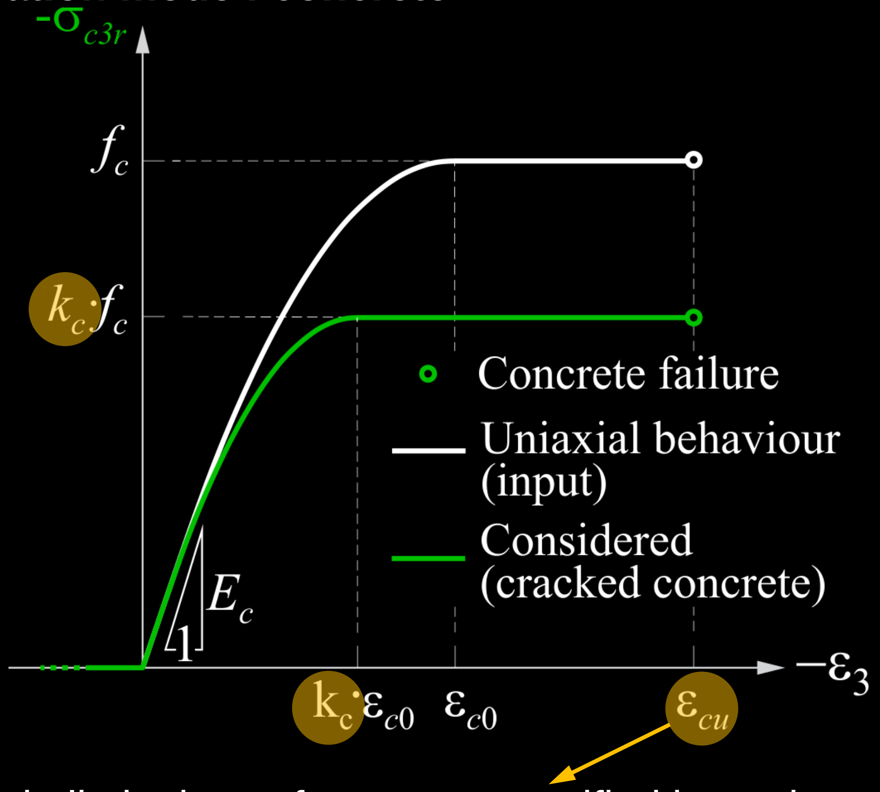
Main assumptions:

- Fictitious, rotating, stress-free cracks ($\sigma_{c1,r}=0$) without slip
- Average strains
- Equilibrium at cracks:
  i. Maximum stresses: $-\sigma_{c3,r}$ / $\sigma_{s,r}$
  ii. Concrete tensile strength neglected except for tension-stiffening: ε_m

Suitable for elements with minimum transversal reinforcement. Slender elements without shear reinforcement might lead to unconservative results.

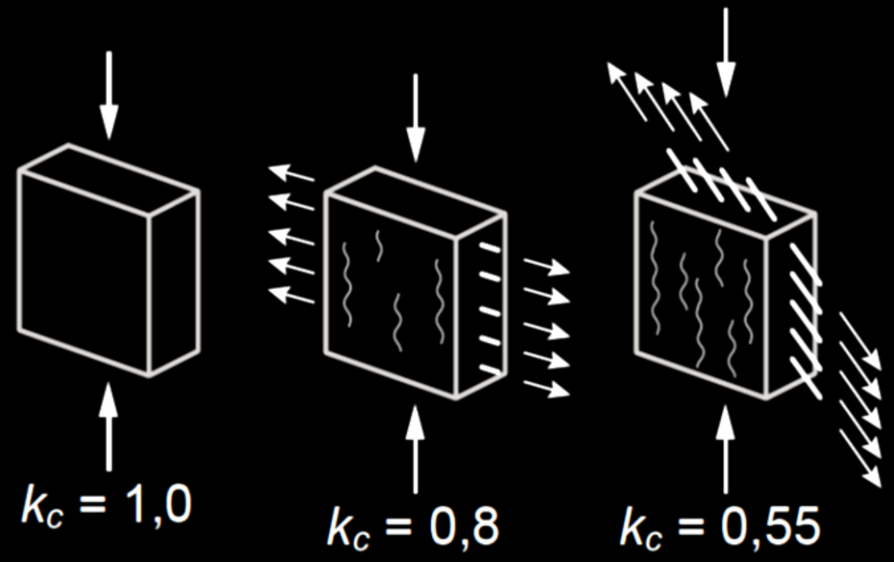
Compatible Stress Field Method

CSFM verification model: concrete



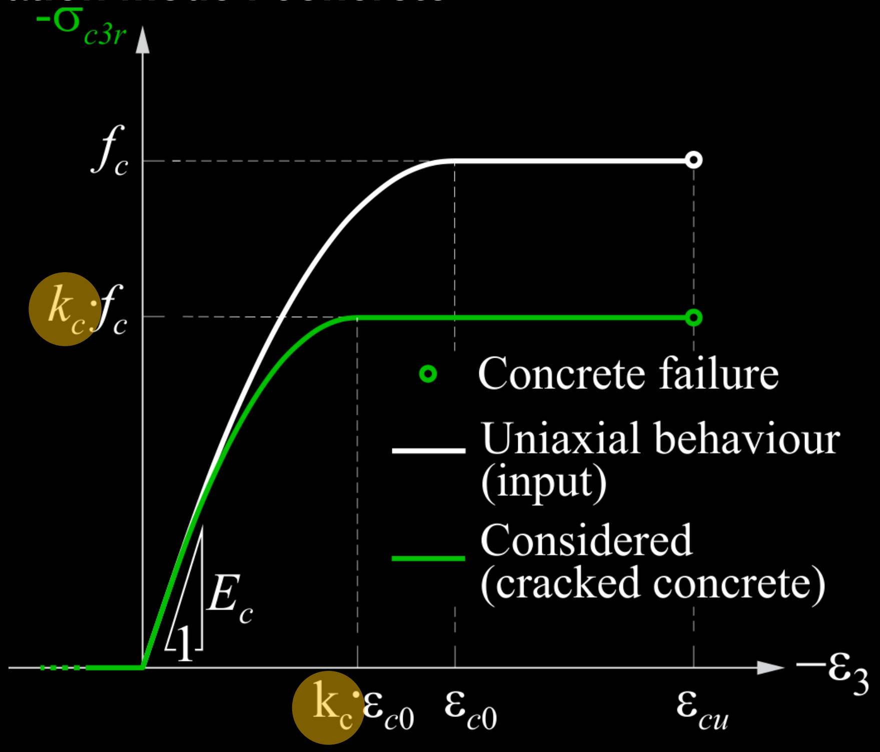
- Strain limitations of concrete specified by codes (explicitly considers the increasing brittleness of concrete with strength).
- Imposed to the average strain over a characteristic crushing band length.

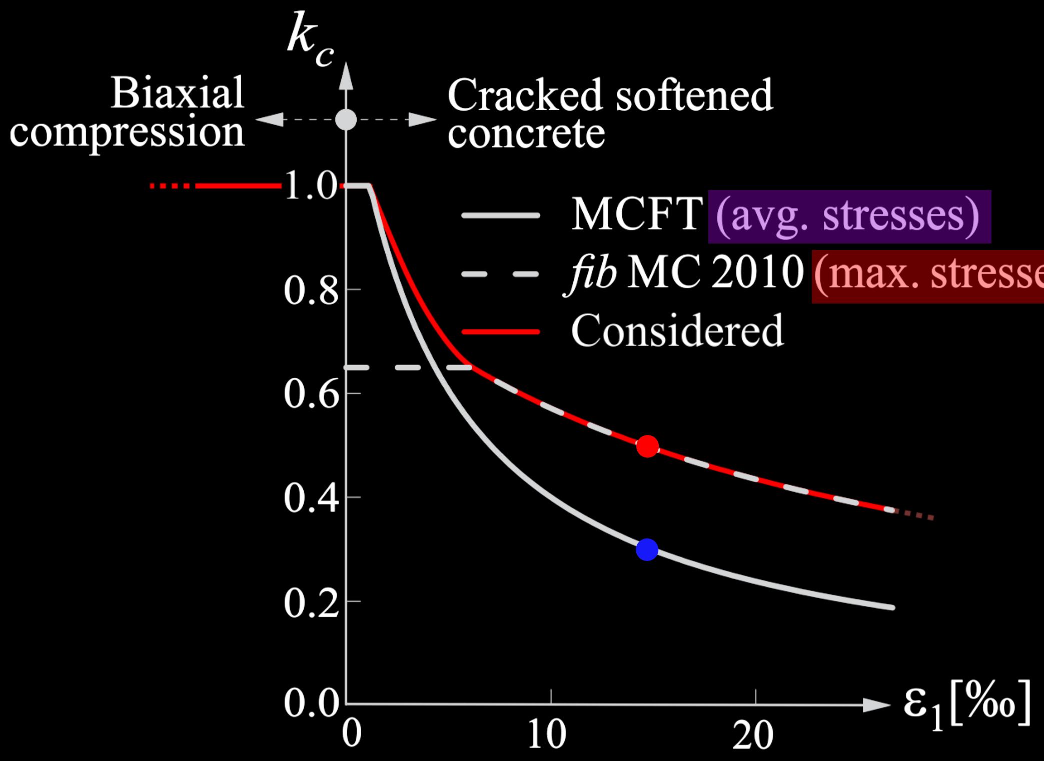
- k_c discrete values for hand calculations



Compatible Stress Field Method

CSFM verification model: concrete





- Strain limitations of concrete specified by codes (explicitly considers the increasing brittleness of concrete with strength).
- Imposed to the average strain over a characteristic crushing band length.

- k_c (compression softening) automatically computed based on the transversal strain state.
- Use of *fib* MC 2010 / SIA 262:213 proposal for shear verifications (consistent with considered max. stresses) extended for general cases.

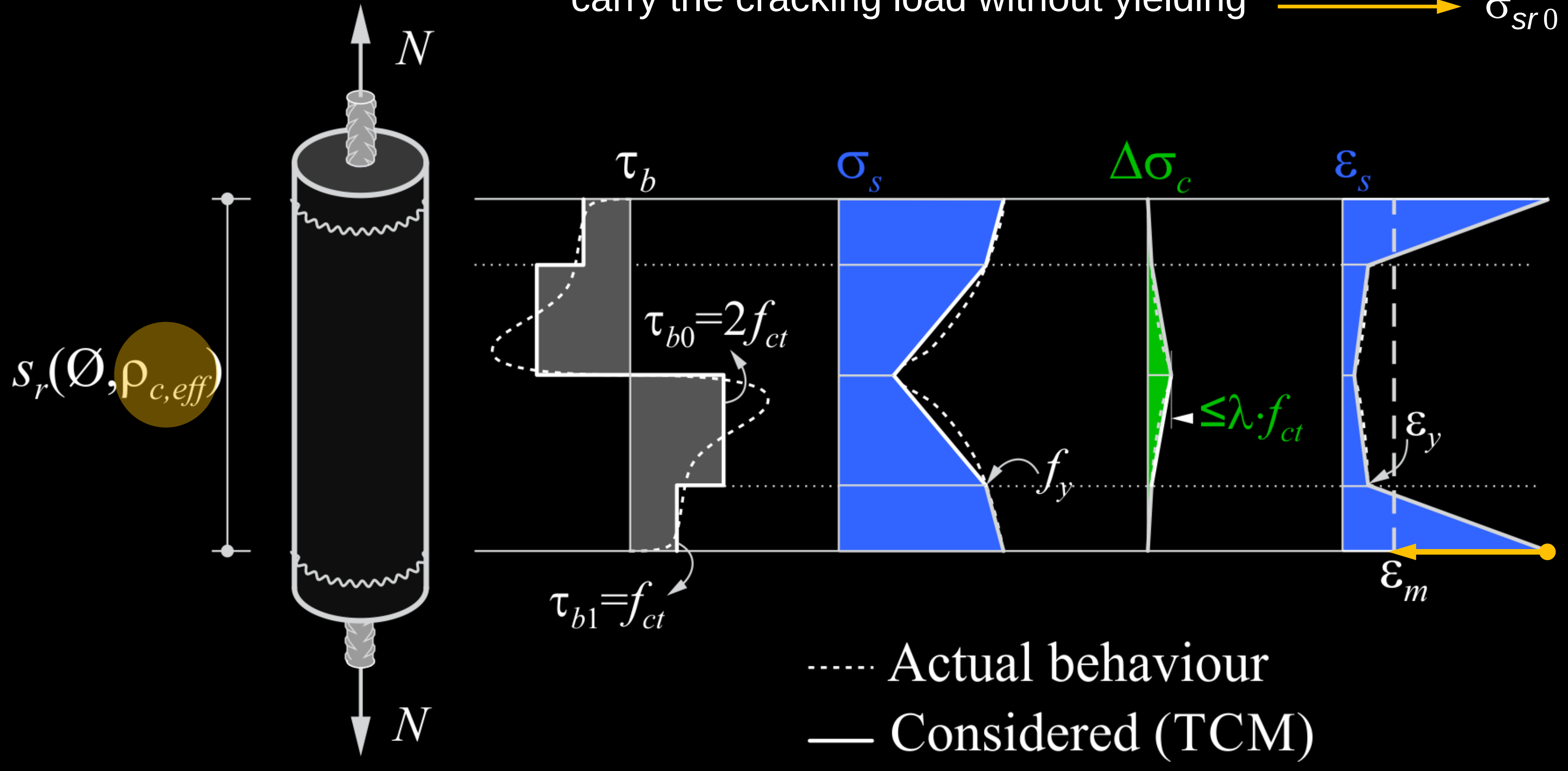
Compatible Stress Field Method

CSFM verification model: tension stiffening

Stabilized crack pattern

for $\rho > \rho_{cr} \approx 0.6\%$ → Reinforcement is able to carry the cracking load without yielding

$$\sigma_{sr0} = f_y = f_{ctm}\left(\frac{1}{\rho_{cr}} + n - 1\right)$$



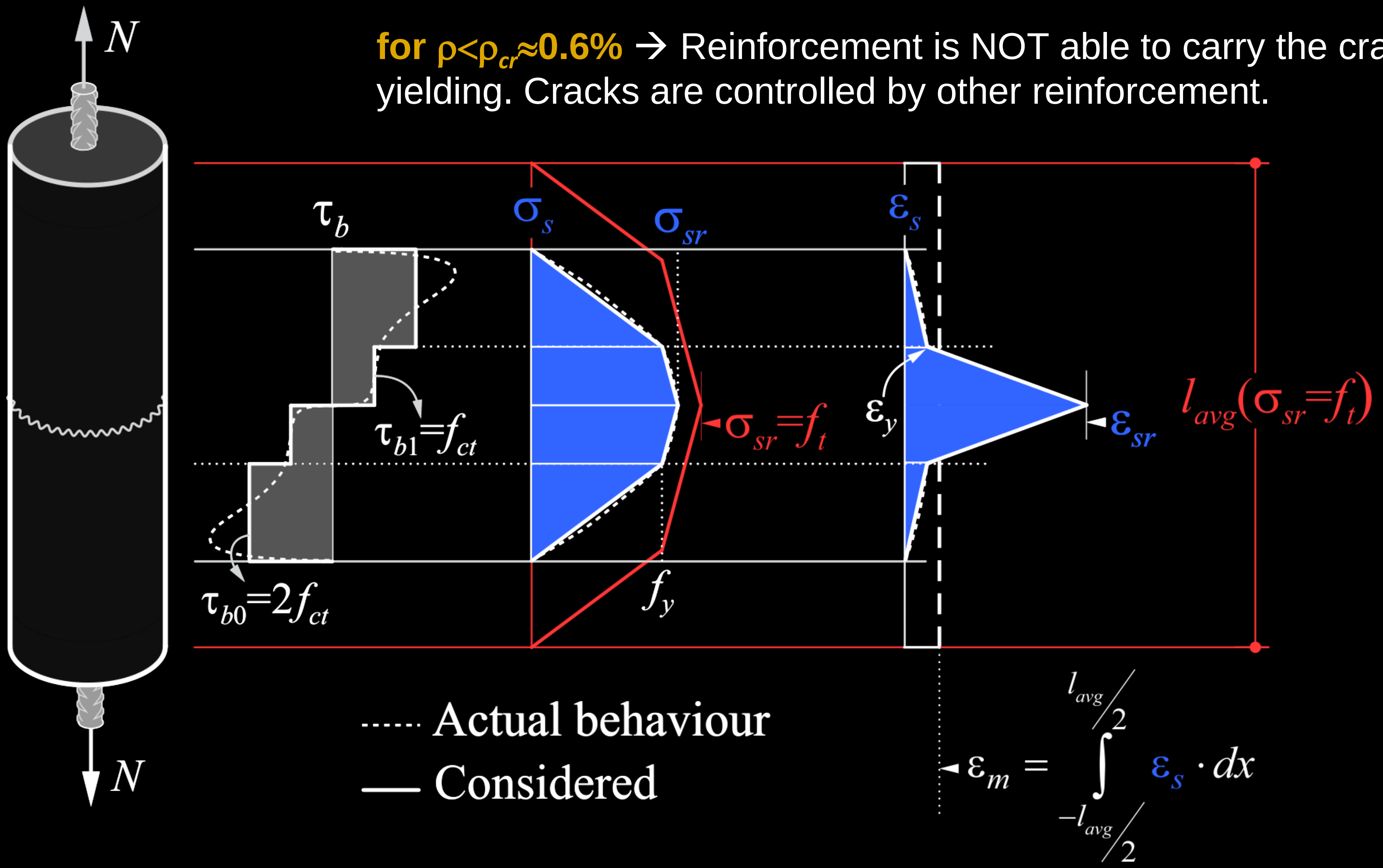
- Implementation of **Tension Chord Model** (TCM) [Alvarez, 1998; Marti et al., 1998]
- Average crack spacing: assumed λ=0.67

Compatible Stress Field Method

CSFM verification model: tension stiffening

Non-stabilized crack pattern

for $\rho<\rho_{cr}\approx0.6\%$ → Reinforcement is NOT able to carry the cracking load without yielding. Cracks are controlled by other reinforcement.



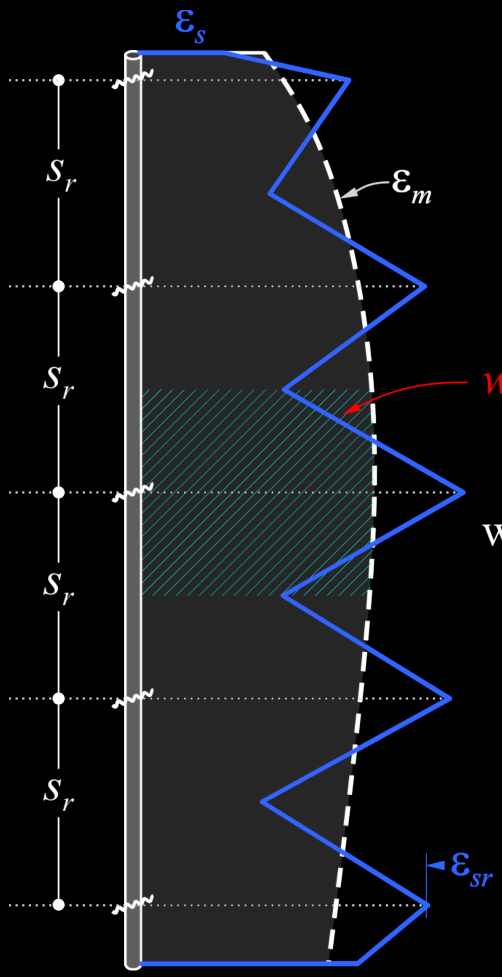
$$\varepsilon_m = \int_{-l_{avg}/2}^{l_{avg}/2} \varepsilon_s \cdot dx$$

- Independent cracks are assumed + bond model of Tension Chord Model.
- Crack localization (size effect): stiffness of the whole rebar embedded in concrete > local stiffness near the crack

  (considered average strain over l_{avg}).

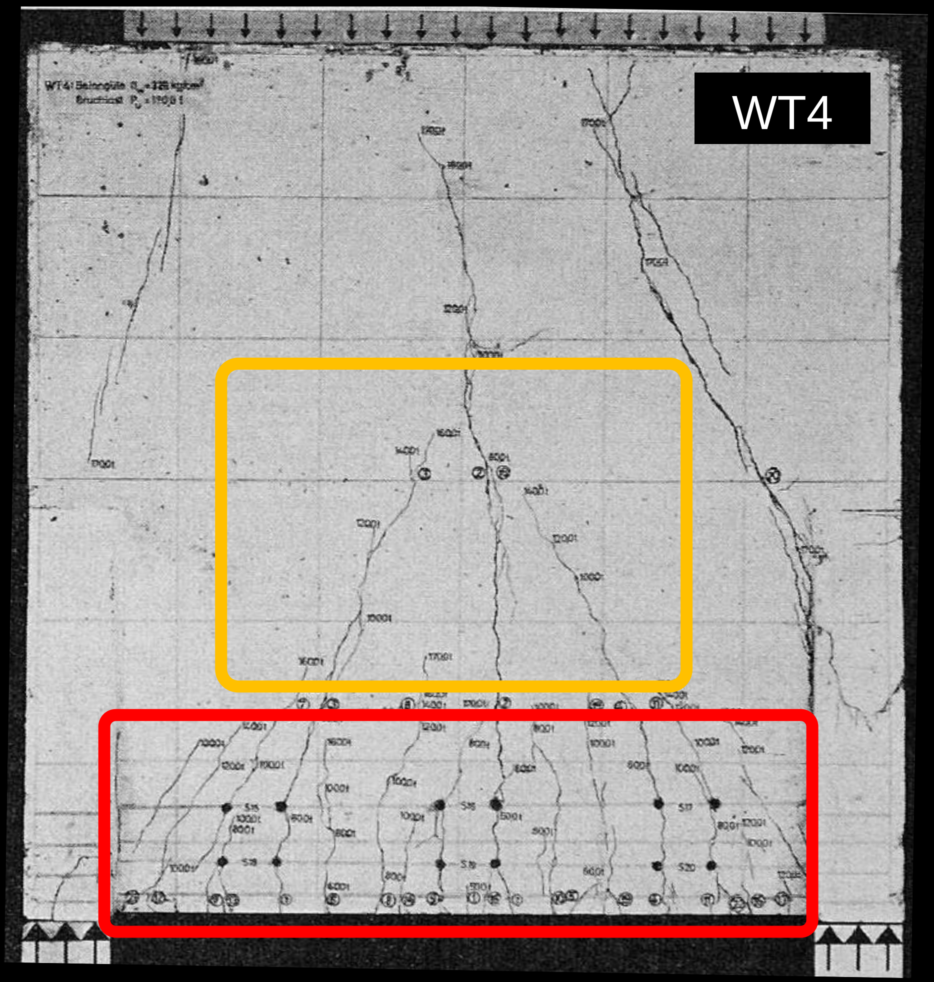
Compatible Stress Field Method

CSFM verification model: crack width – stabilized crack pattern



$$w_b = \int_{-s_r/2}^{s_r/2} (\varepsilon_m - \varepsilon_{cm}) \cdot dx$$

with $\varepsilon_{cm} = \dfrac{\lambda \cdot f_{ctm}}{2 \cdot E_s}$

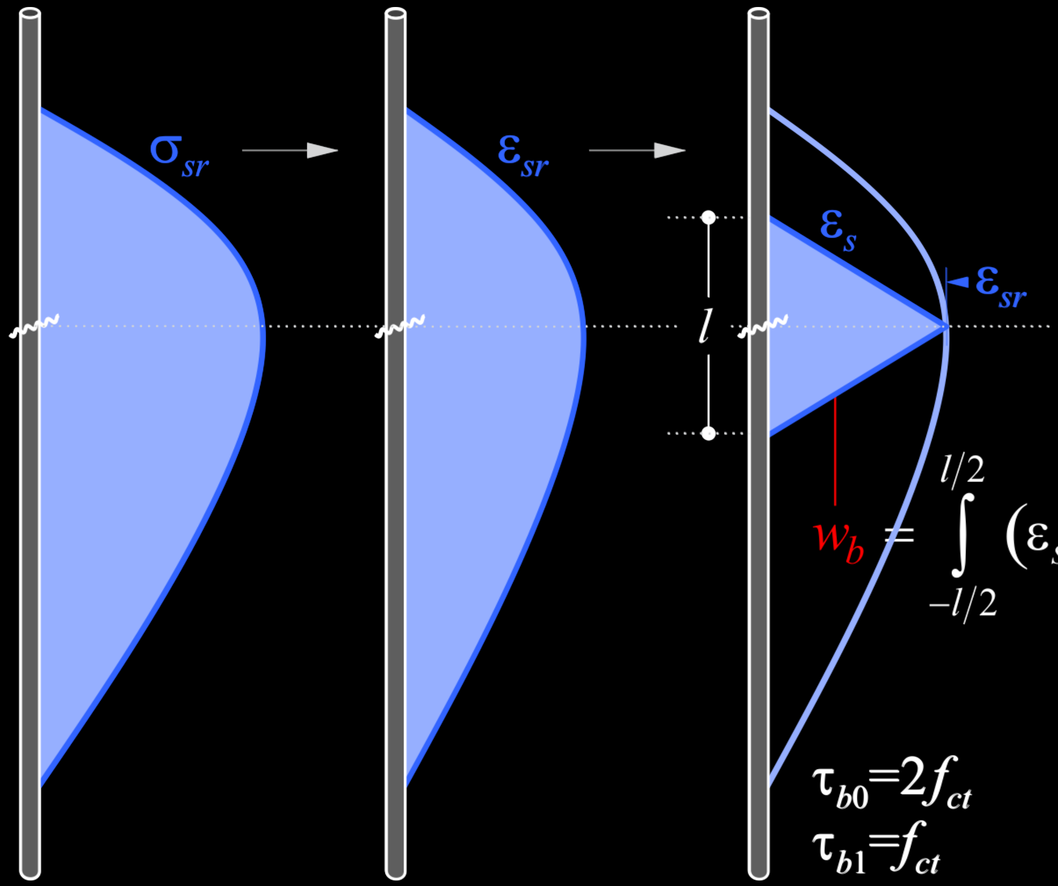


[Walther, 1967]

Compatible Stress Field Method

CSFM verification model: crack width – non-stabilized crack pattern

Assumed independent cracks at SLS



$$w_b = \int_{-l/2}^{l/2} (\varepsilon_s - \varepsilon_c)\cdot dx \simeq \int_{-l/2}^{l/2} \varepsilon_s \cdot dx$$

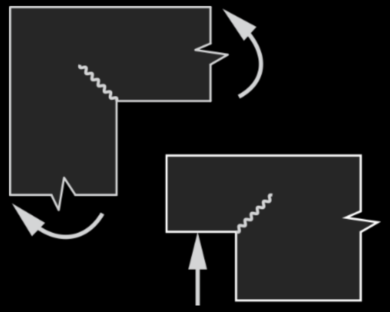
$\tau_{b0}=2f_{ct}$

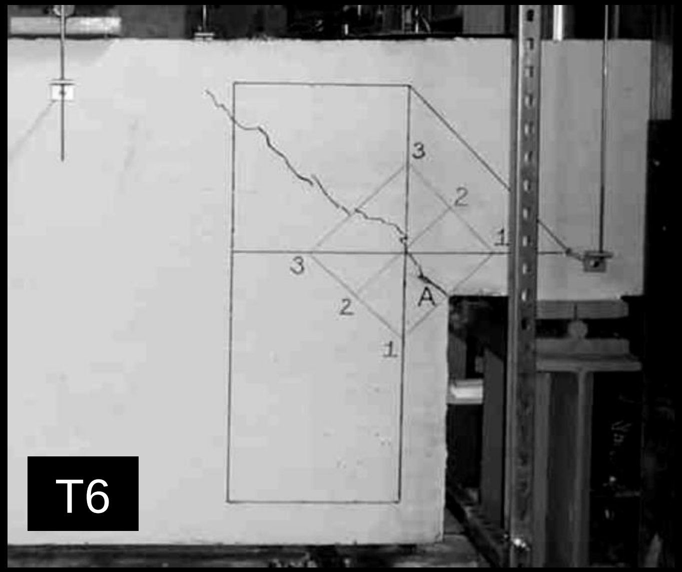
$\tau_{b1}=f_{ct}$

Considered for:

a) Regions with ρ<0.6%

b) Cracks triggered by geometric discontinuities at low loads





[Zhu et al., 2003]

Compatible Stress Field Method

CSFM & IdeaStatiCa Detail implementation: additional information

Theoretical description of CSFM method & experimental validation

- “Computer-aided stress field analysis of discontinuity concrete regions”, J. Mata-Falcón, D. T. Tran, W. Kaufmann, J. Navrátil; Proceedings of the Conference on Computational Modelling of Concrete and Concrete Structures (EURO-C 2018), 641-650, London: CRC Press, 2018.

  https://www.researchgate.net/profile/Jaime_Mata-Falcon/publication/328419485_Computer-aided_stress_field_analysis_of_discontinuity_concrete_regions/links/5bcd7f4da6fdcc03c79ad556/Computer-aided-stress-field-analysis-of-discontinuity-concrete-regions.pdf

- “Compatible Stress Field Design of Structural Concrete: Principles and Validation”, W. Kaufmann, J. Mata-Falcón, M. Weber, D. T. Tran, J. Kabelac, M. Konecny; ISBN 978-3-906916-95-8, ETH Zurich & IDEA StatiCa, 2020. (see additional literature)

Use and installation of Idea StatiCa Detail software:

- Installation of the software: https://www.ideastatica.com/downloads/

  Free educational license might be ordered in https://www.ideastatica.com/educational-license/

- Idea StatiCa Resource Center (tutorials, sample projects…): https://www.ideastatica.com/support-center

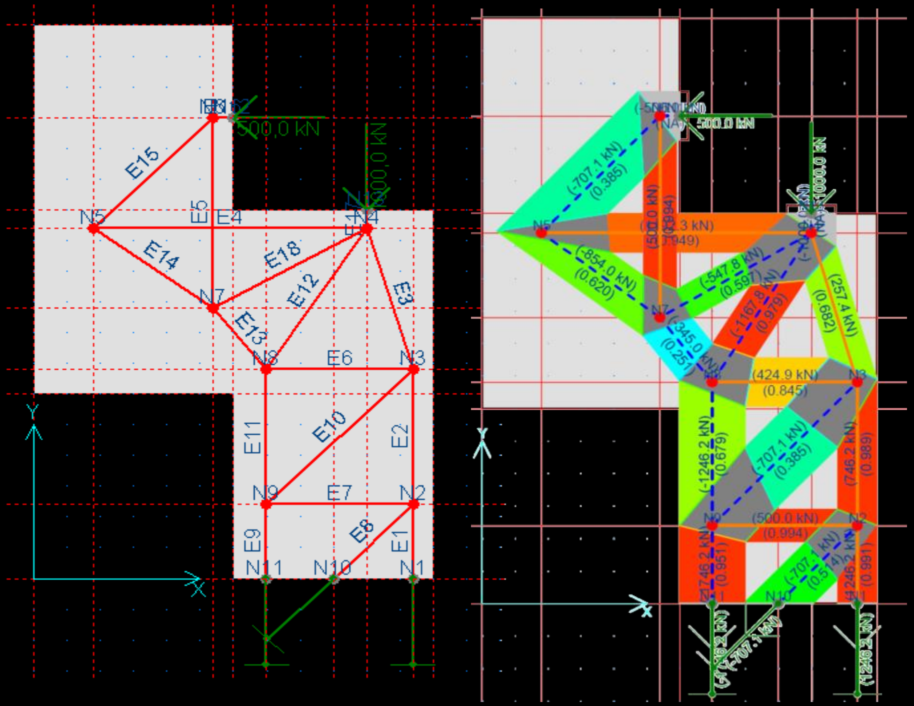
- Practical workshop will be organised for those students interested

Annex

Compatible Stress Field Method

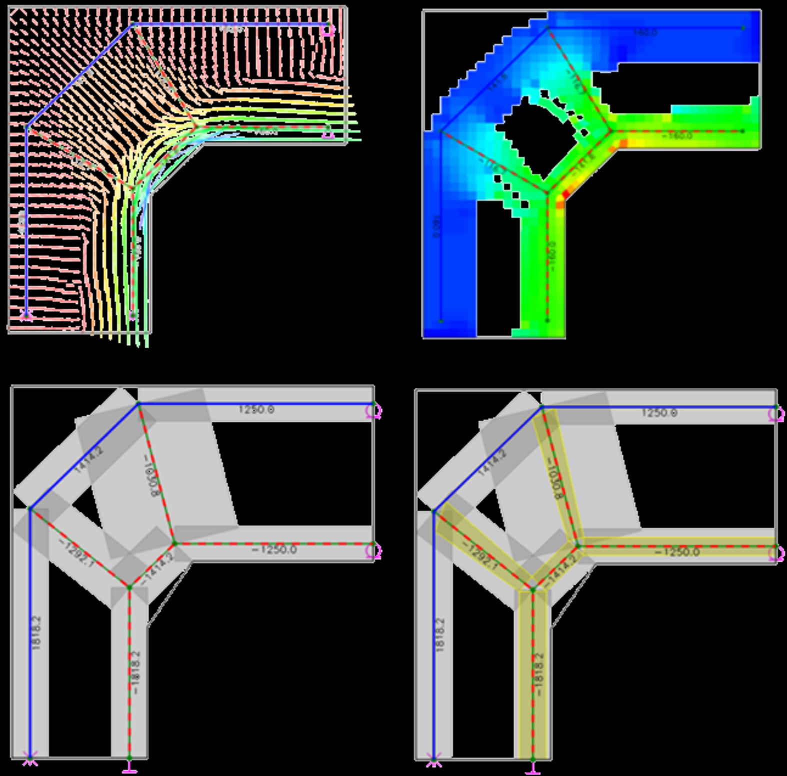
Dimensioning/assesment of Discontinuity Regions: Previously existing computer-aided tools

CAST (Tjhin & Kutchma, 2002)
(strut-and-tie → f_c=? Realistic results?)



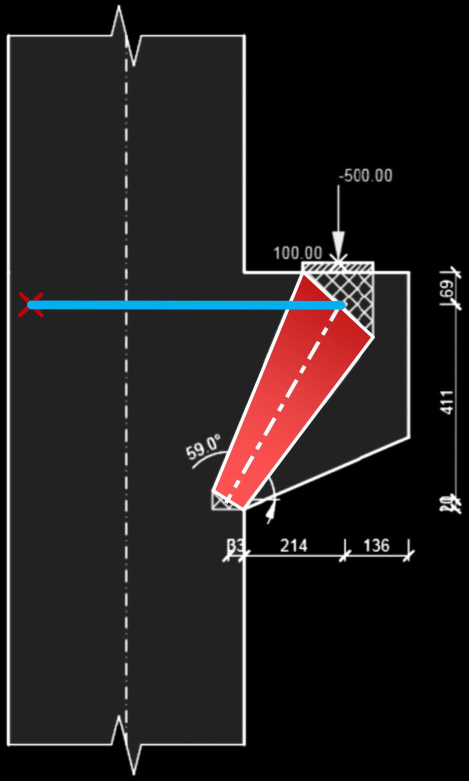
[Mata-Falcón & Sánchez-Sevilla, 2006]

AStrutTie (HanGil)
(strut-and-tie → f_c=? Realistic results?)



[HanGil, 2017]

Idea StatiCa for specific details
(corbels, piles caps…)

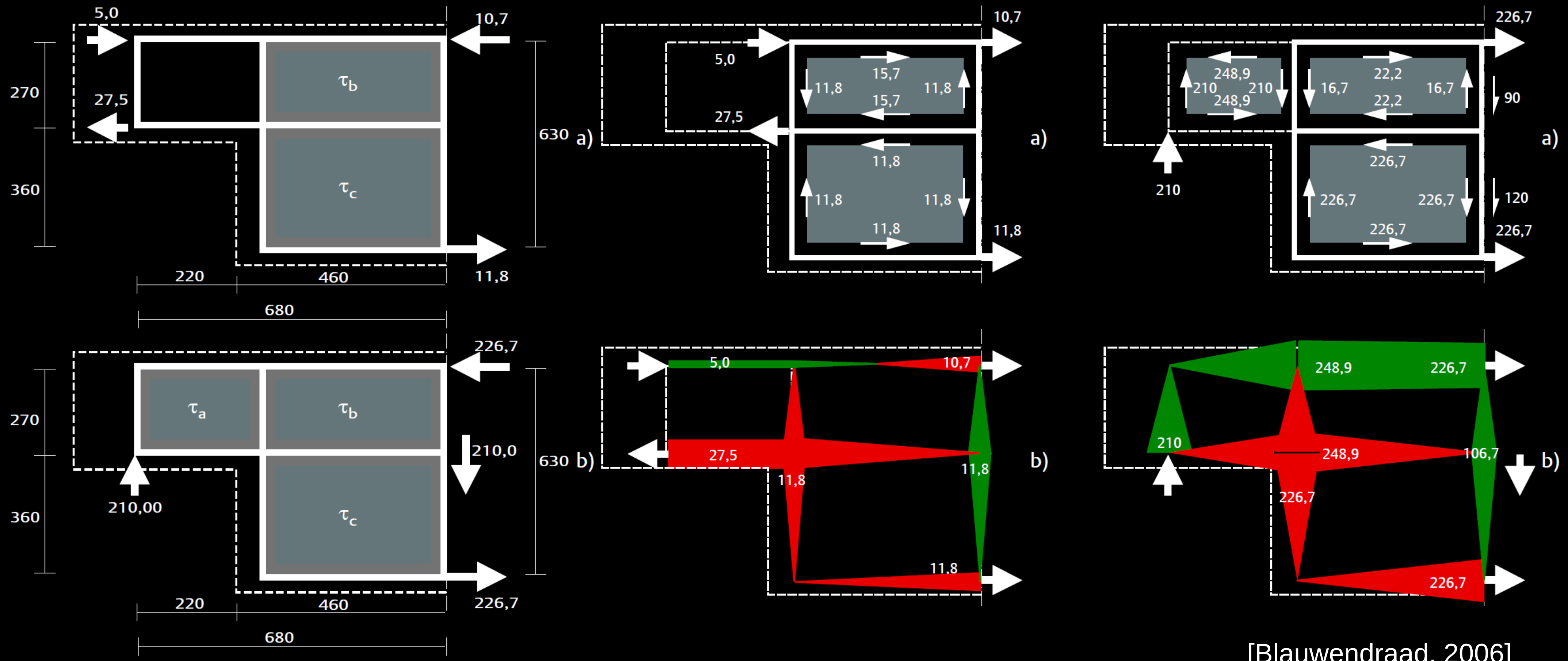


[IDEA, 2017]

Compatible Stress Field Method

Dimensioning/assessment of Discontinuity Regions: Previously existing computer-aided tools

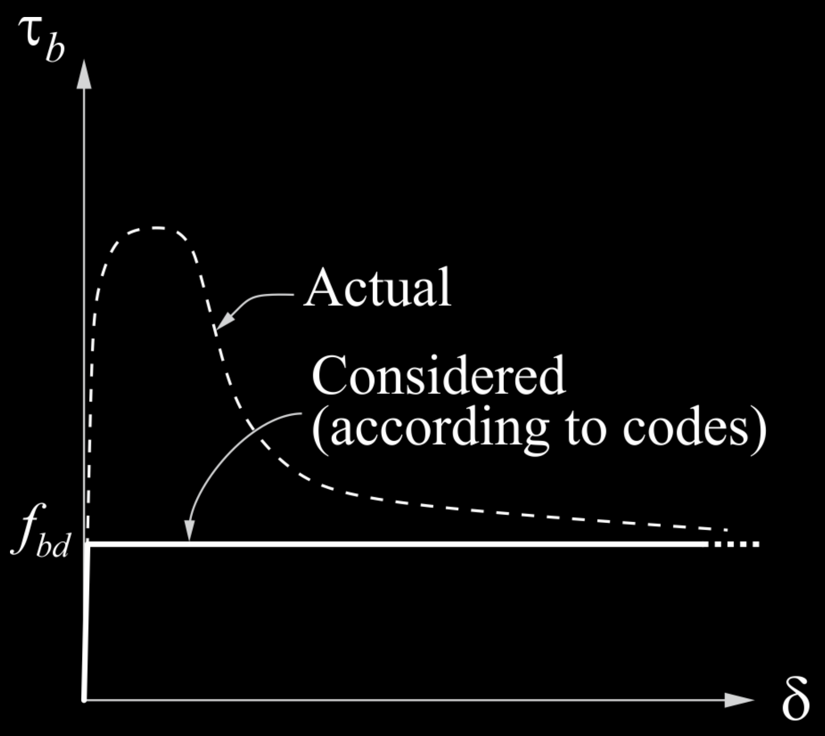
Stringer-Panel Models (Nielsen, 1971; Blaauwendraad & Hoogenboom, 1996; Marti & Heinzmann, 2012)



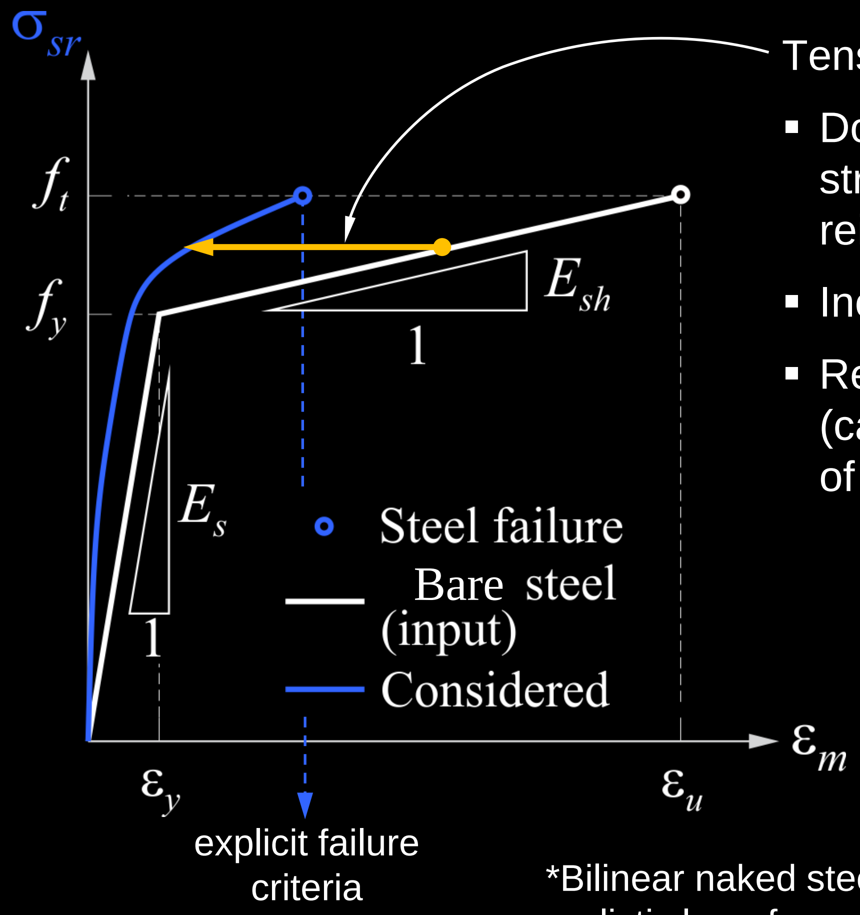
[Blauwendraad, 2006]

Compatible Stress Field Method

CSFM verification model: verification of anchorage length and reinforcement



Bond model used exclusively for anchorage length verifications



Tension-stiffening:

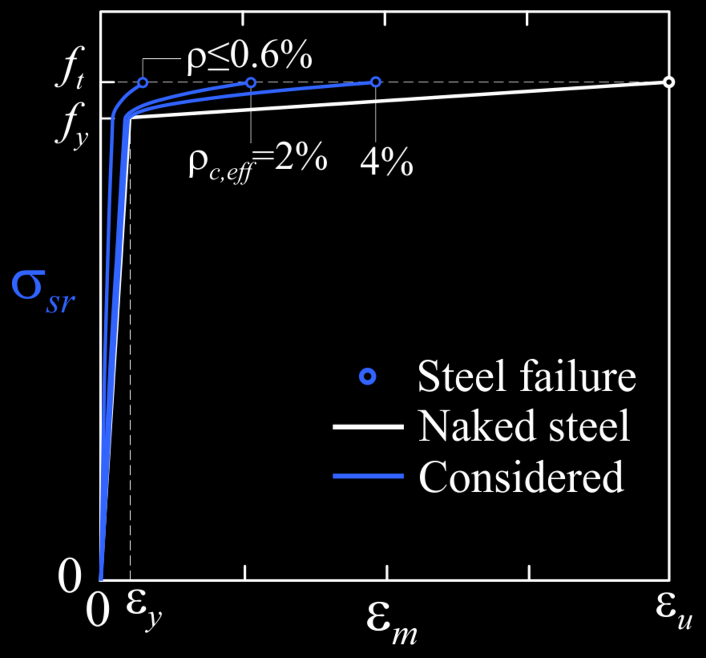
- Does not affect the strength of the reinforcement
- Increases the stiffness
- Reduces the ductility (can reduce the strength of the member)

*Bilinear naked steel input for design. More realistic laws for assessment and experimental validation.

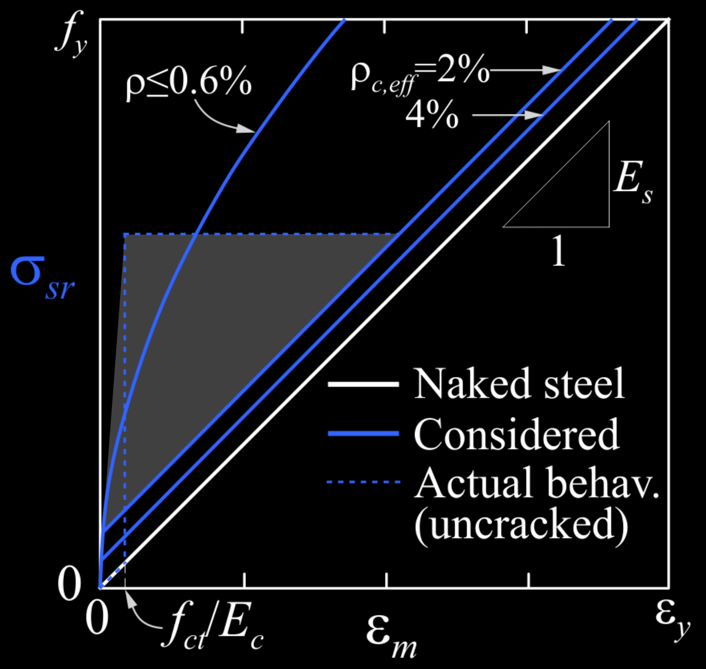
Compatible Stress Field Method

CSFM verification model: tension stiffening

Resultant tension chord behaviour



Steel EU - B500B
$f_t/f_u \approx 1.08$ $\varepsilon_{su} \approx 5\%$



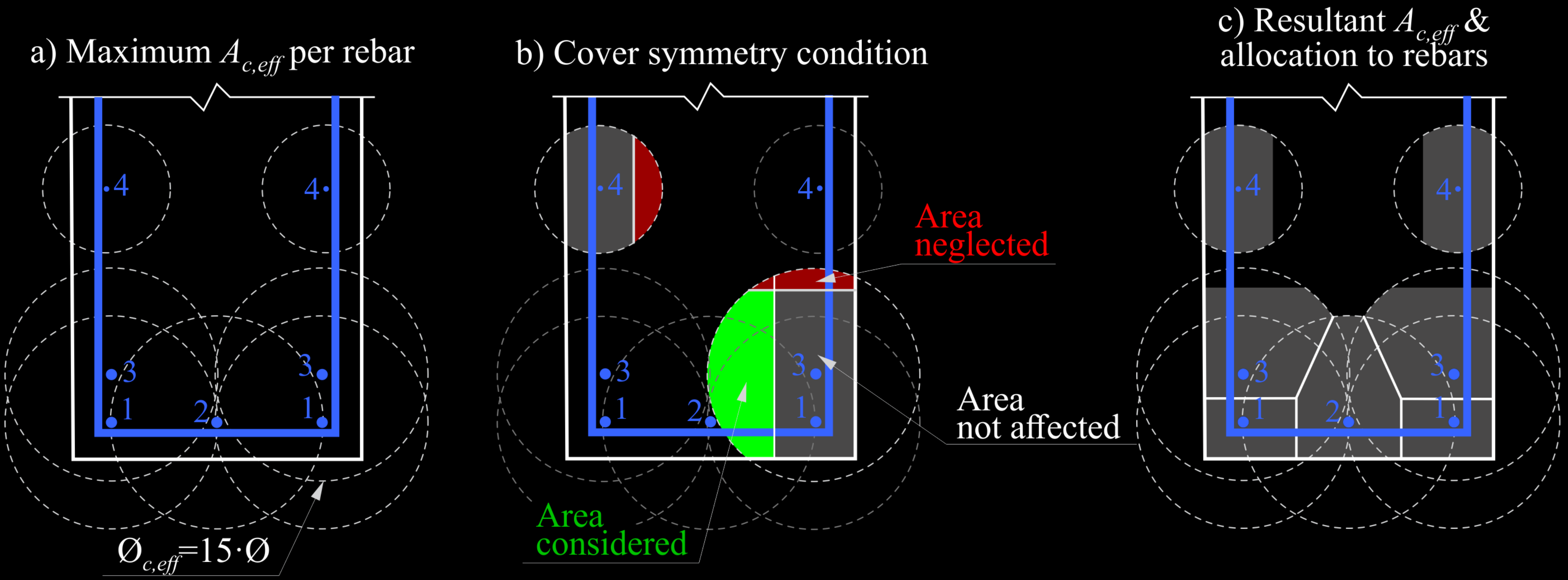
Steel EU - B500B
$f_t/f_u \approx 1.08$ $\varepsilon_{su} \approx 5\%$

- Fully cracked behaviour considered for design.
- Uncracked initial stiffness can be considered for refined verification models.

Compatible Stress Field Method

CSFM verification model: effective area of concrete in tension

→ suitable for numerical implementation and valid for automatic definition of $\rho_{c,eff}$ in any region



Maximum concrete area each rebar can activate (concrete at f_{ct})

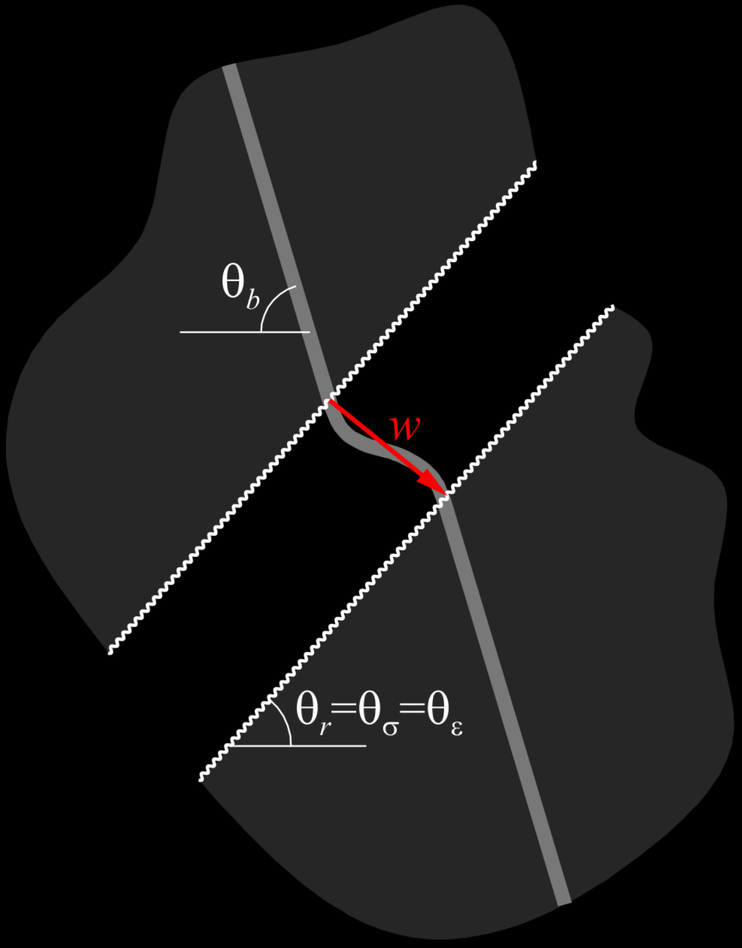
(illustrated for rebars 3 and 4)

Areas used in calculation

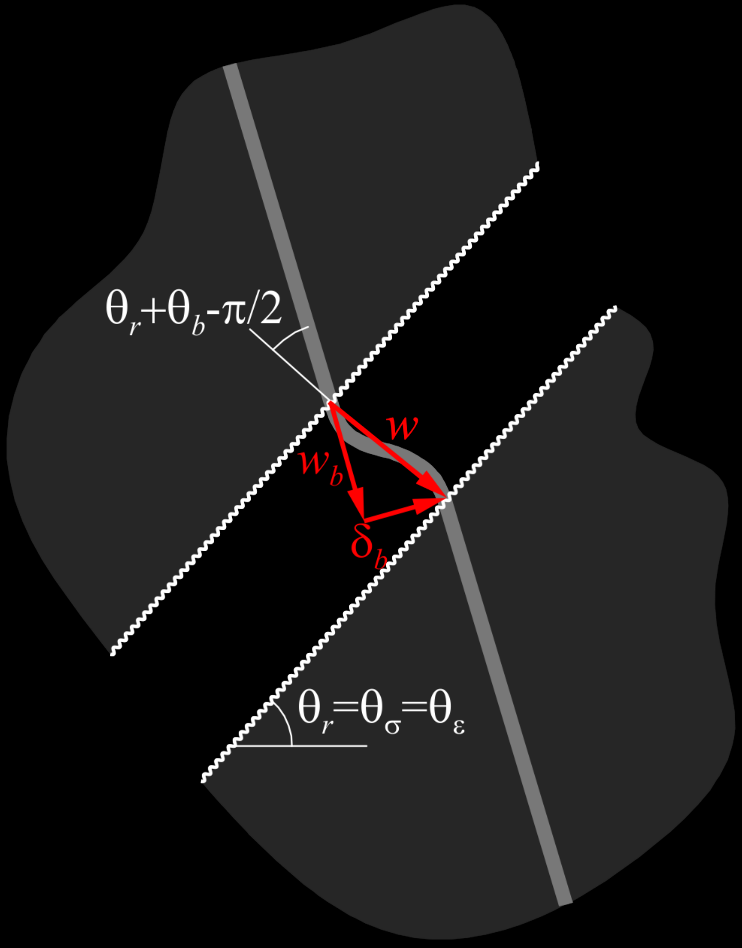
Compatible Stress Field Method

CSFM verification model: crack width – crack kinematic

Considered crack kinematic
(assuming δ=0)



Projection of kinematics
into principal directions of the rebar



$$w=\frac{w_b}{\cos\left(\theta_r+\theta_b-{}^{\pi}\!/_{2}\right)}$$

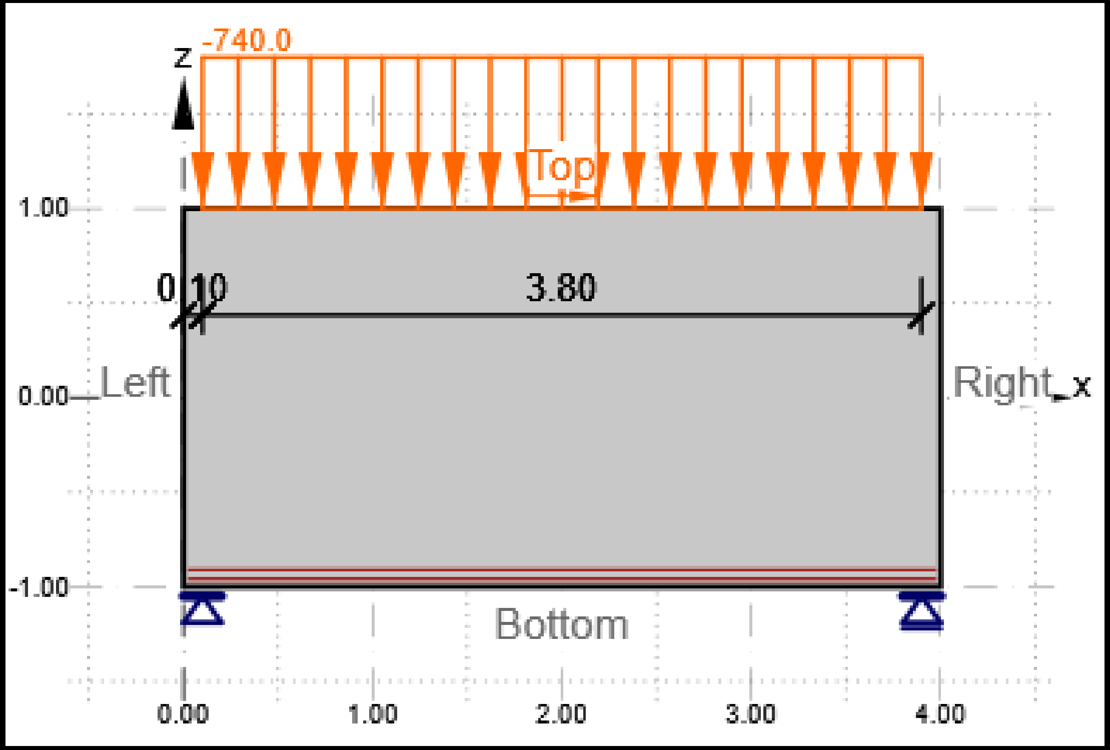
w_b → integration of strains in the rebars

Compatible Stress Field Method

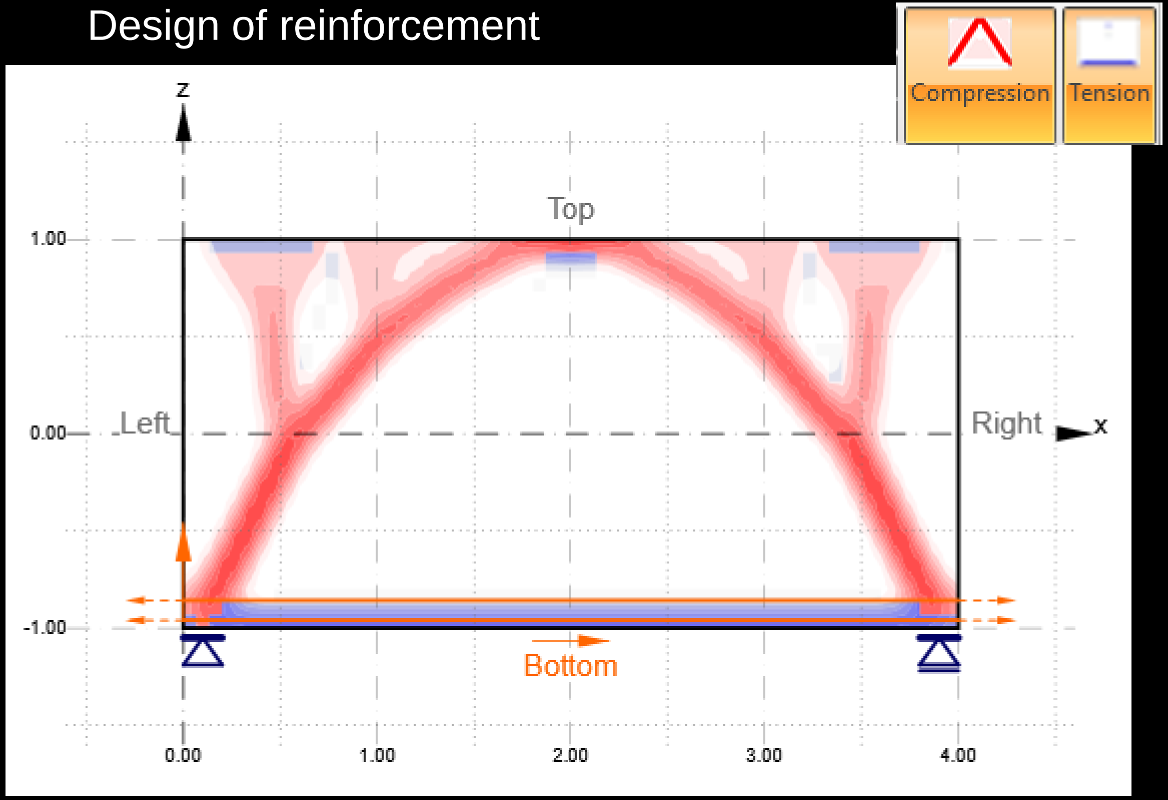
CSFM: practical examples in Idea StatiCa Detail

Deep beam with distributed top load

Problem definition



Design of reinforcement

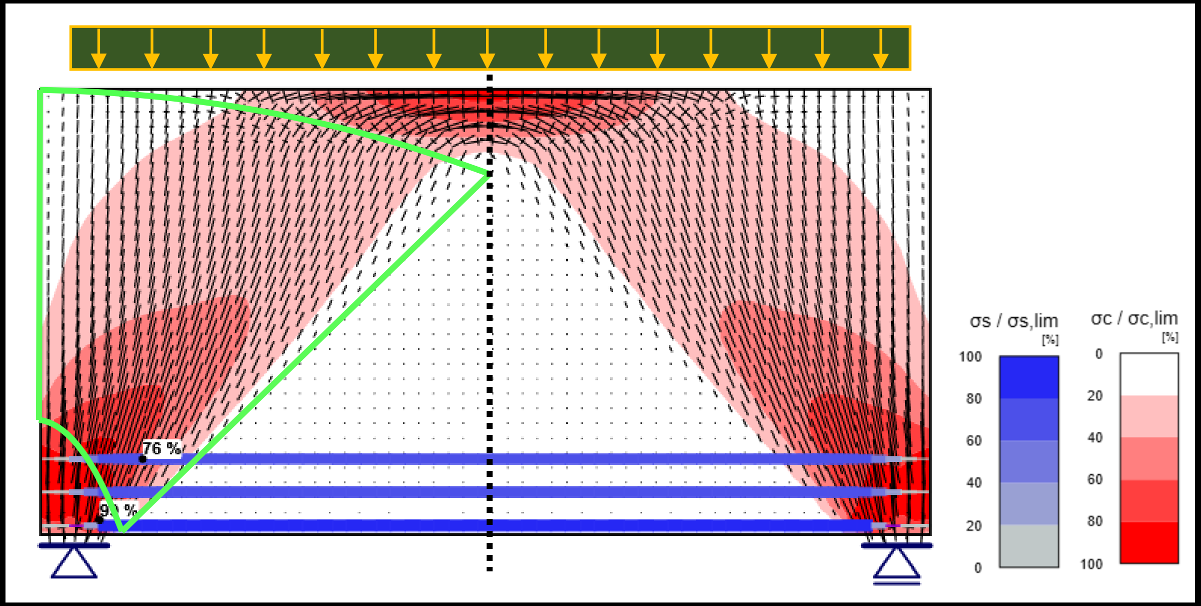


Compatible Stress Field Method

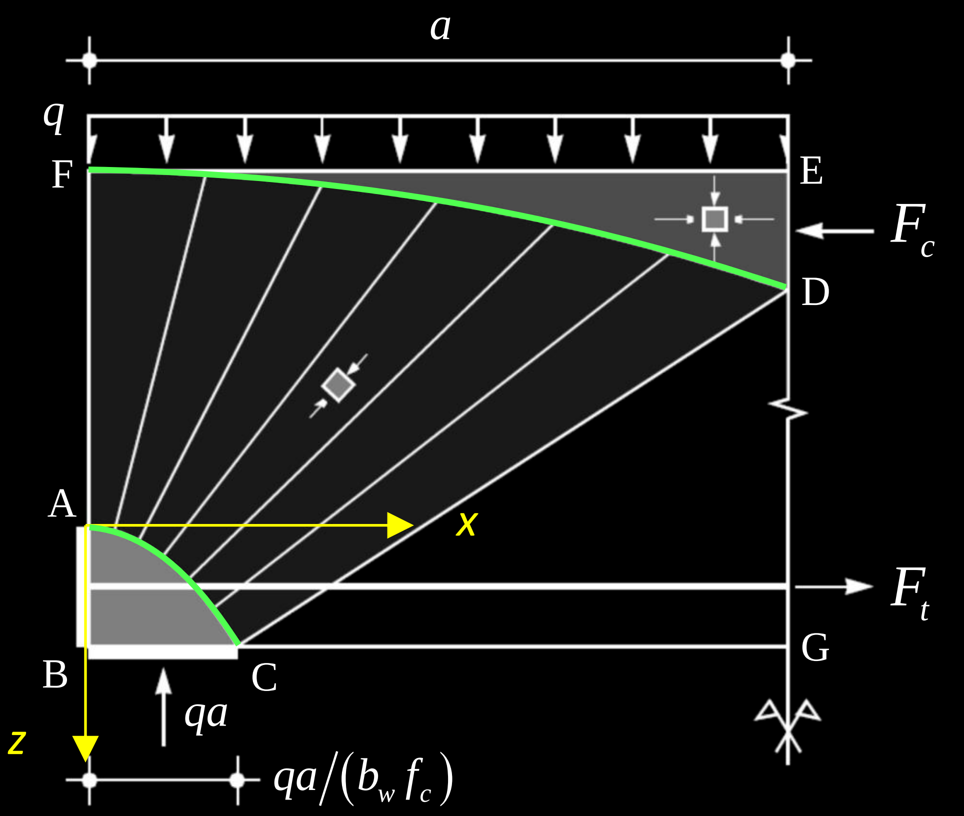
CSFM: practical examples in Idea StatiCa Detail

Deep beam with distributed top load

Compatible stress fields



Discontinuous stress fields

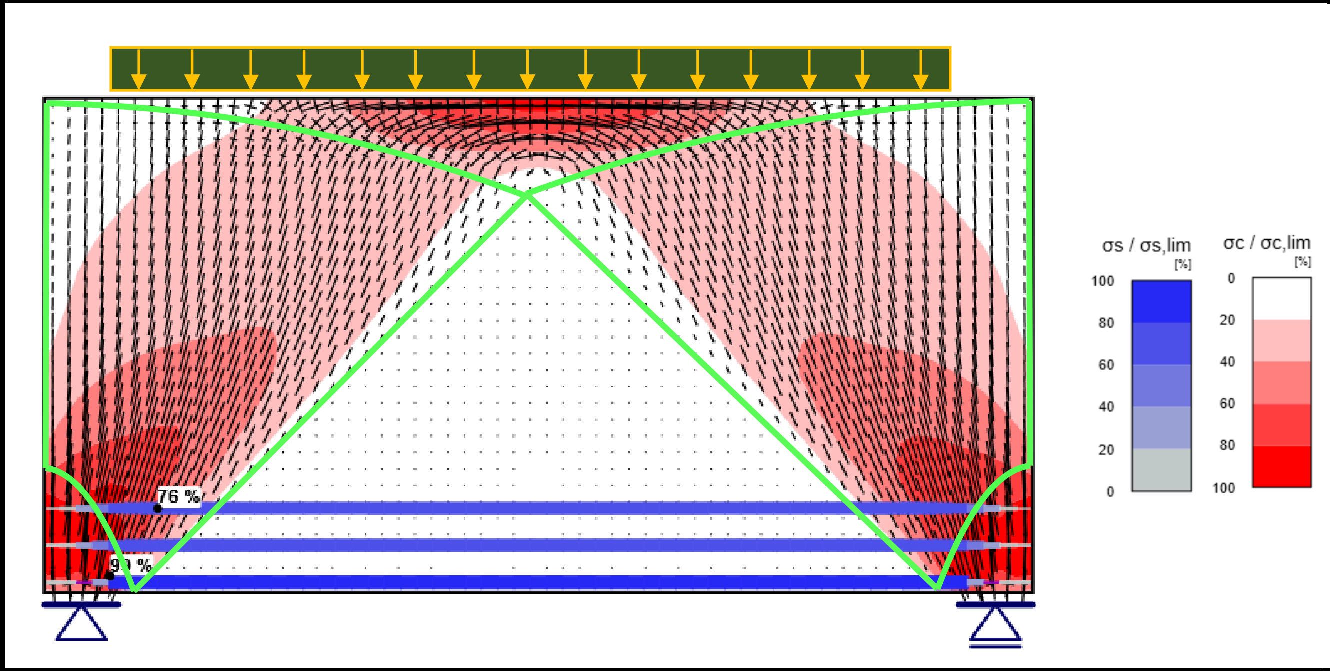


Compatible Stress Field Method

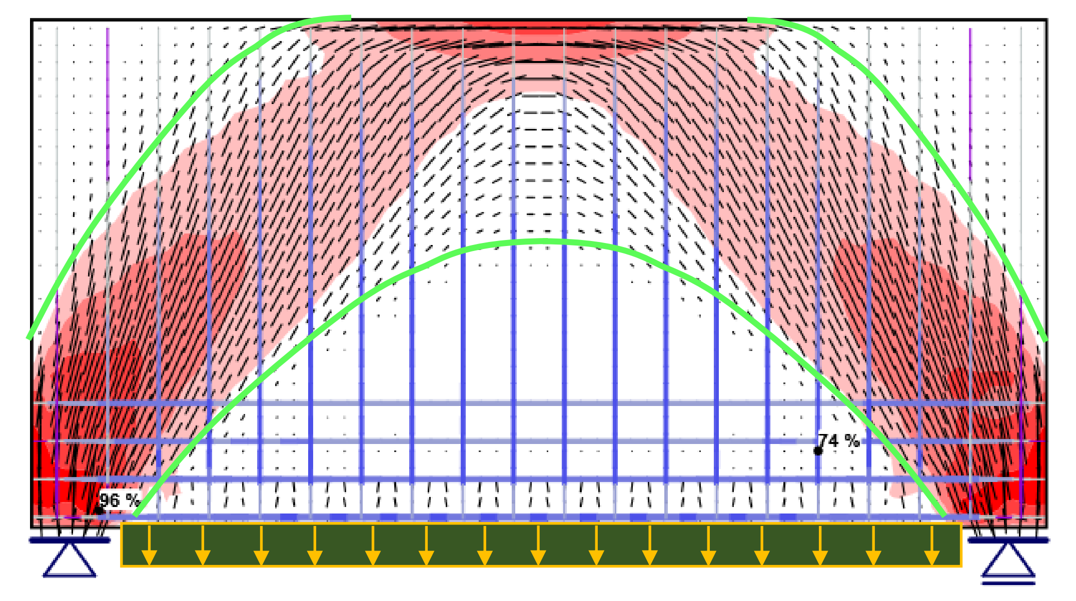
CSFM: practical examples in Idea StatiCa Detail

Deep beam with distributed load

Top load: fan mechanism



Suspended load: arch mechanism



Arch mechanism requires enough capacity of flexural reinforcement; otherwise, the load is suspended until top & fan action is generated

Compatible Stress Field Method

CSFM experimental validation

- Direct tension experiments – Alvarez and Marti (1996)
  - Ultimate limit state
  - Load deformation behaviour
  - Crack width

- Pure bending experiments – Frantz and Breen (1978)
  - Crack width distribution

- Cantilever shear walls – Bimschas, Hannewald and Dazio (2010, 2013)
  - Load deformation behaviour under combined loading
  - Bearing capacity under combined loading

- Beams with low amount of transversal reinforcement – Huber, Huber and Kolleger (2016)
  - Bearing capacity in shear (failures due to insufficient ductility of the transversal reinforcement)

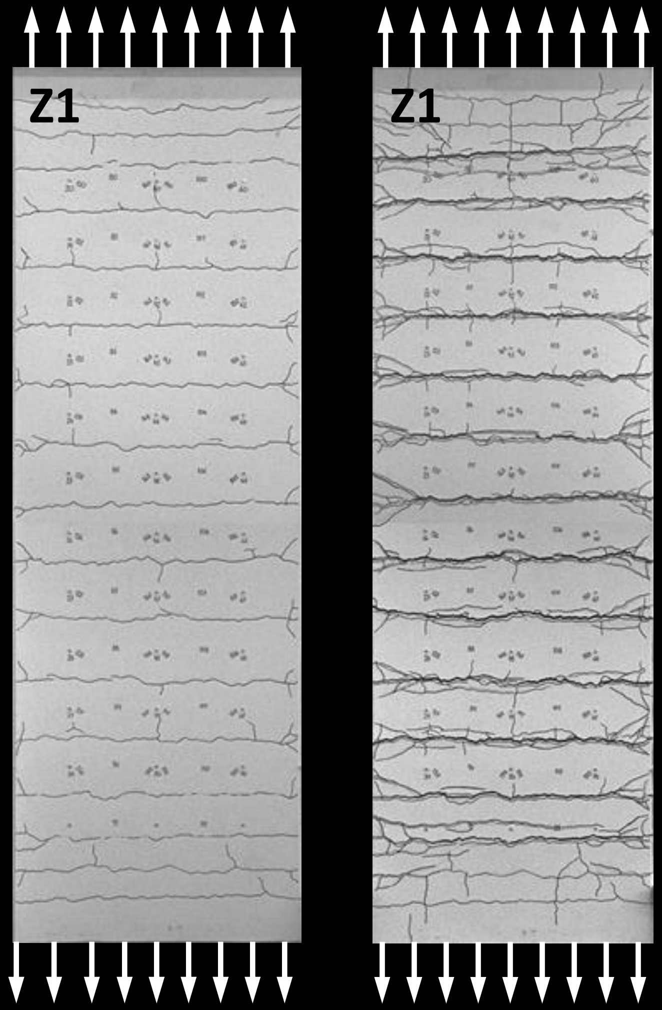
Compatible Stress Field Method

CSFM experimental validation

Alvarez and Marti (1996) - experimental setup/specimens

Specimen	**Z1**	**Z2**	**Z4**	**Z8**
Long. reinforcement	14xØ14 (ρ = 1%)	14xØ14 (ρ = 1%)	14xØ14 (ρ = 1%)	10xØ14 (ρ = 0.7%)
Steel quality (ductility class)	High	High	Normal	High
f_{ck_cube} (MPa)	50	90	50	50

Loading: pure tension



[Avarez and Marti, 1996]

Compatible Stress Field Method

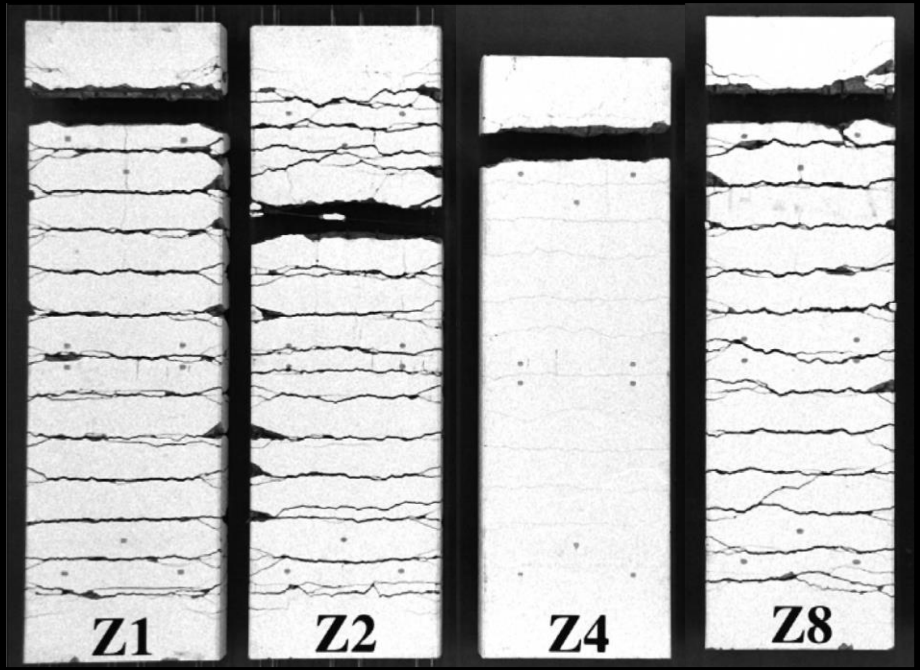
CSFM experimental validation

Alvarez and Marti (1996) - ultimate state

Specimen	**Z1**	**Z2**	**Z4**	**Z8**
Experiment				
V_{exp} (kN)	1294	1295	1275	924
$\varepsilon_{m,exp}$ (%)	6.7	6.8	0.6	6.4
CSFM				
V_{calc} (kN)	1275	1282	1242	918
$\varepsilon_{m,calc}$ (%)	7.0	4.6	0.4	6.5
Safety factor				
Strength: V_{exp}/V_{calc}	**1.01**	**1.01**	**1.03**	**1.01**
Deform. capacity: $\varepsilon_{m,exp}/\varepsilon_{m,calc}$	**0.96**	**1.48**	**1.50**	**0.98**

V: Peak load

ε_m: Average tensile strain



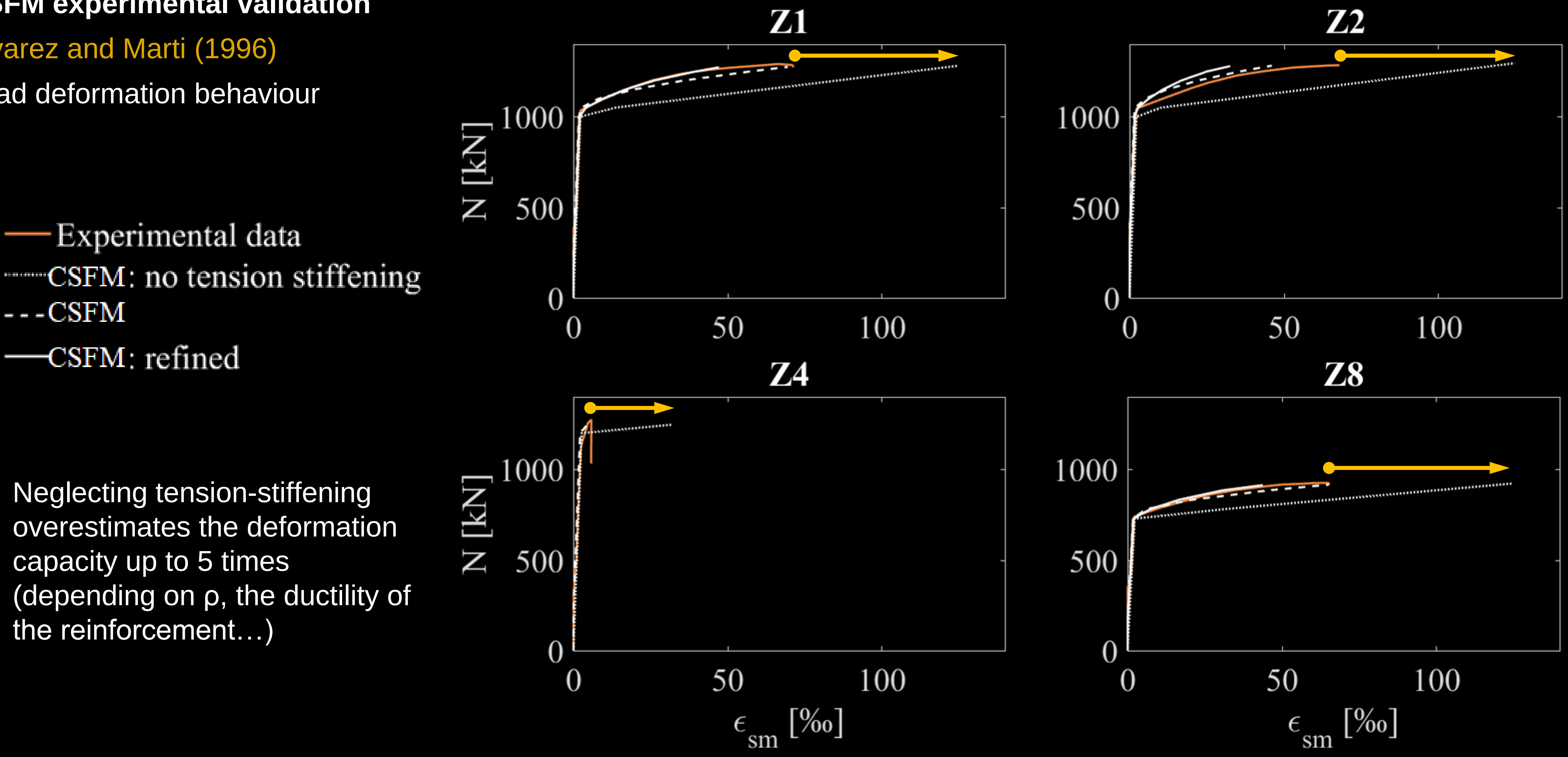
[Avarez and Marti, 1996]

Compatible Stress Field Method

CSFM experimental validation

Alvarez and Marti (1996)

Load deformation behaviour



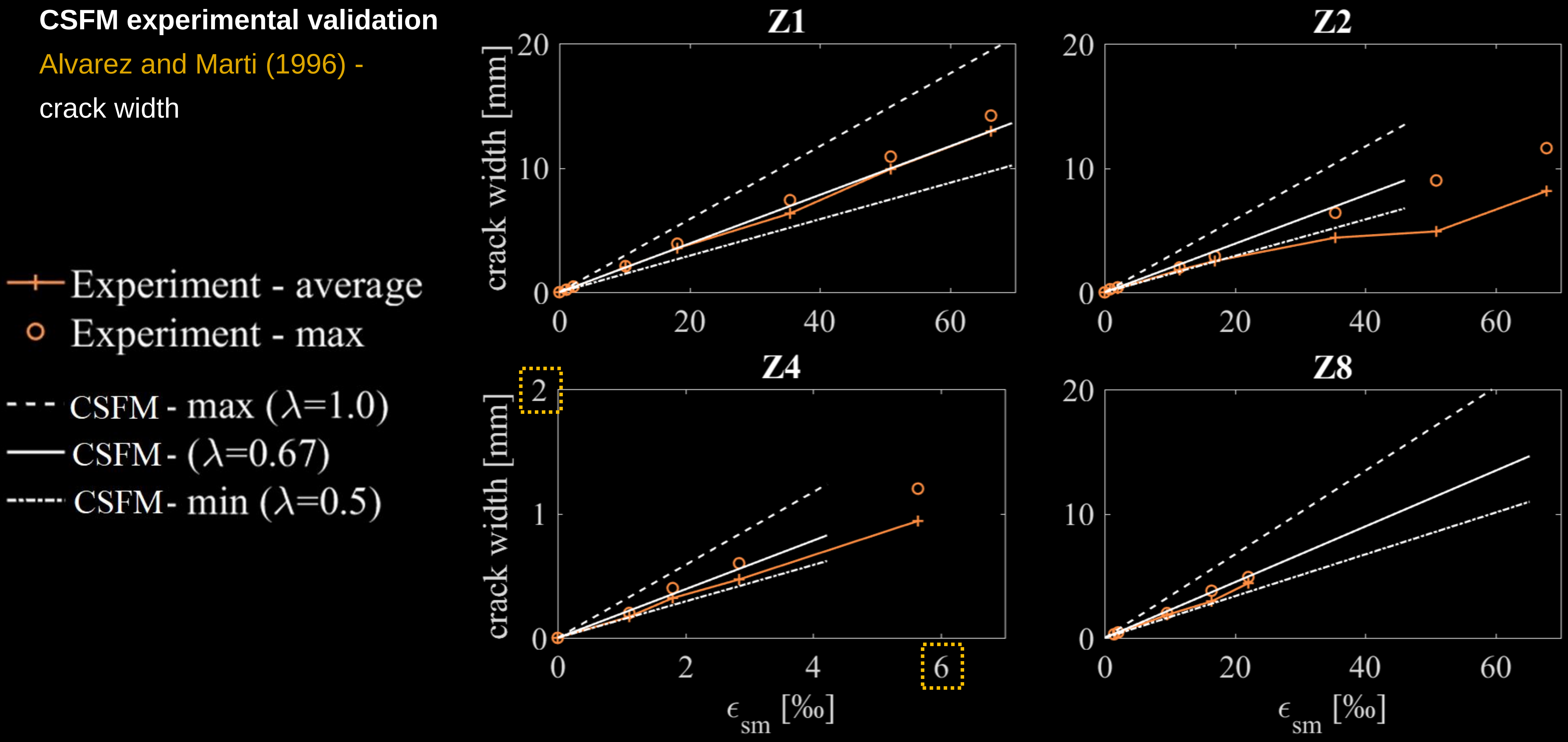
Neglecting tension-stiffening overestimates the deformation capacity up to 5 times (depending on ρ, the ductility of the reinforcement…)

Compatible Stress Field Method

CSFM experimental validation

Alvarez and Marti (1996) -

crack width



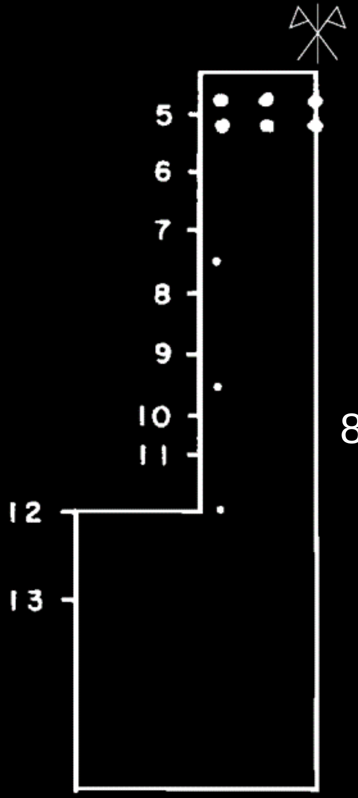
Computergestützte Spannungsfelder

CSFM experimental validation

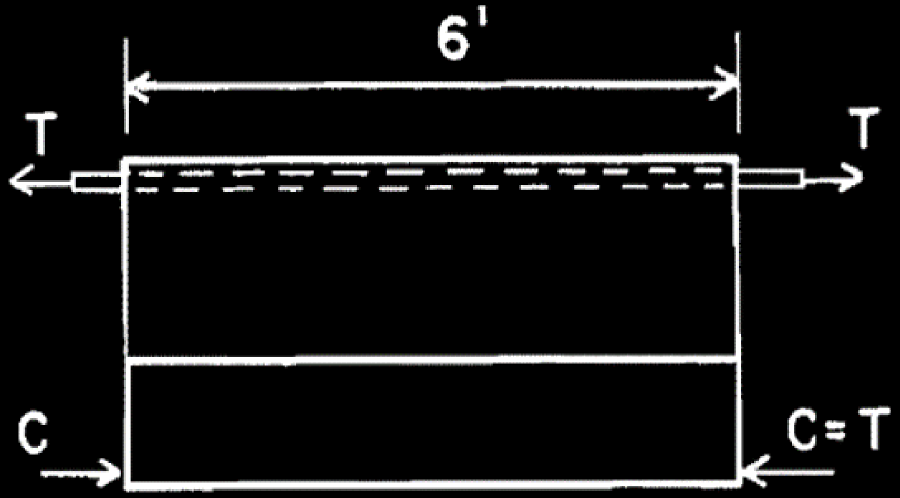
Frantz and Breen (1980) - experimental setup/specimen

Specimen	**RS-3**
Main reinforcement	2xØ15.88 6xØ12.7
Web reinforcement	6xØ6

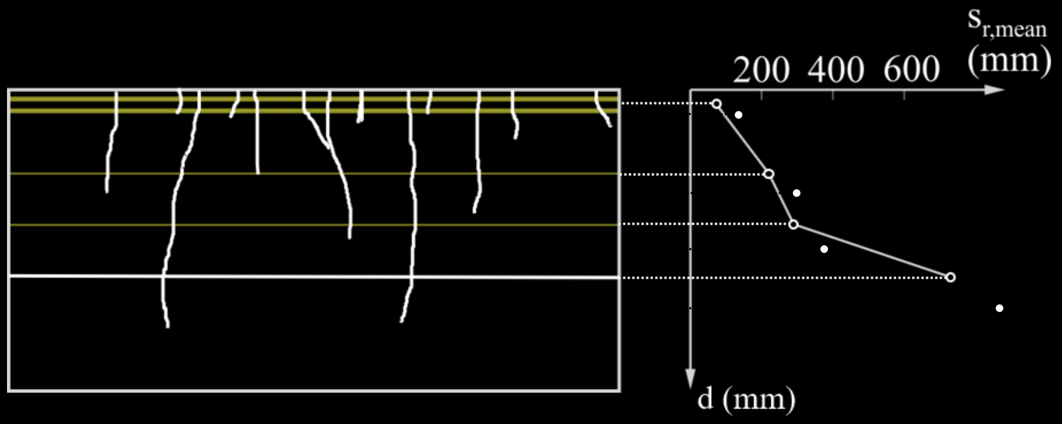
Loading: pure bending



885 mm



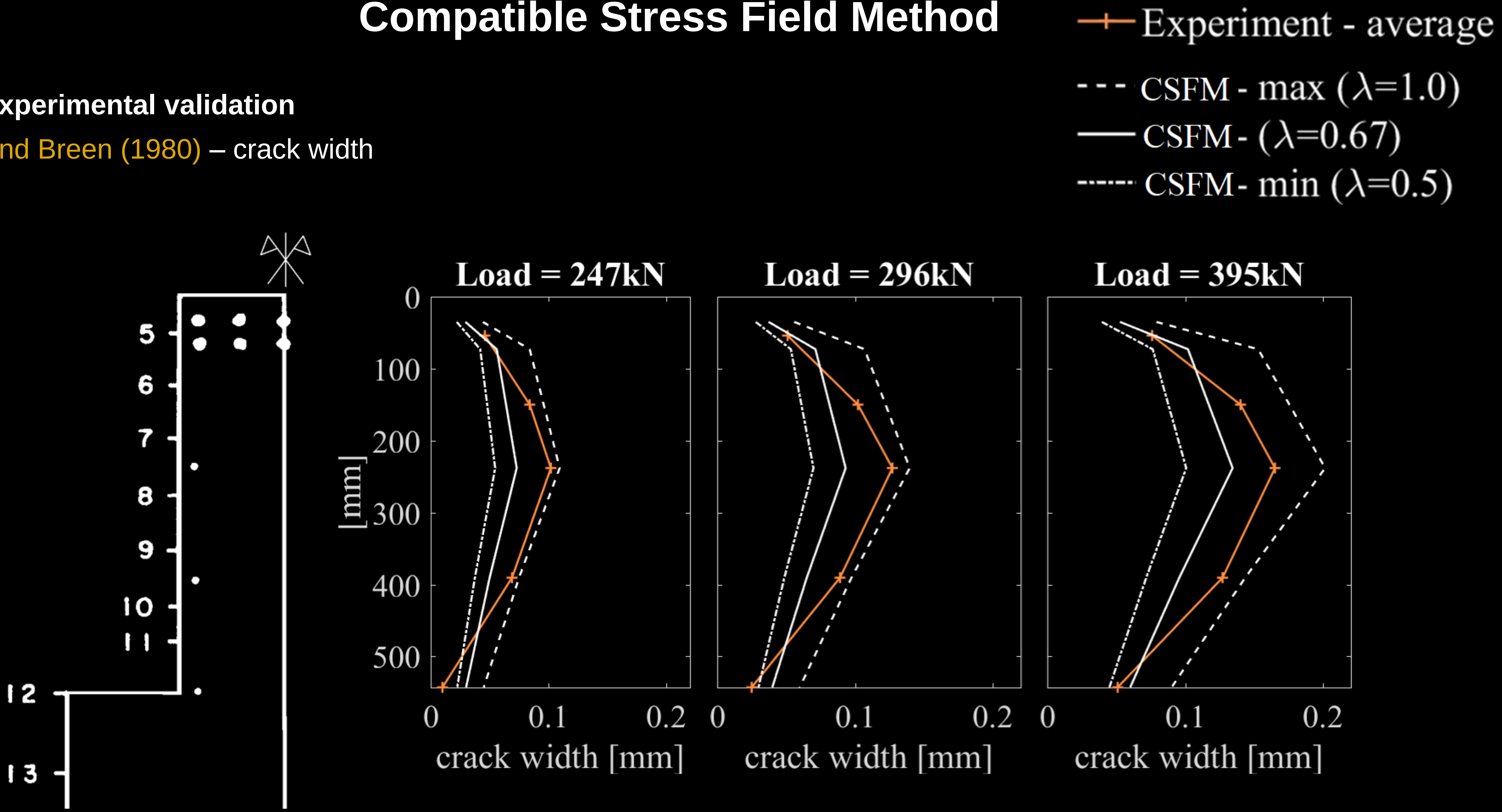
[Frantz and Breen, 1980]



Compatible Stress Field Method

CSFM experimental validation

Frantz and Breen (1980) – crack width



Compatible Stress Field Method

CSFM experimental validation

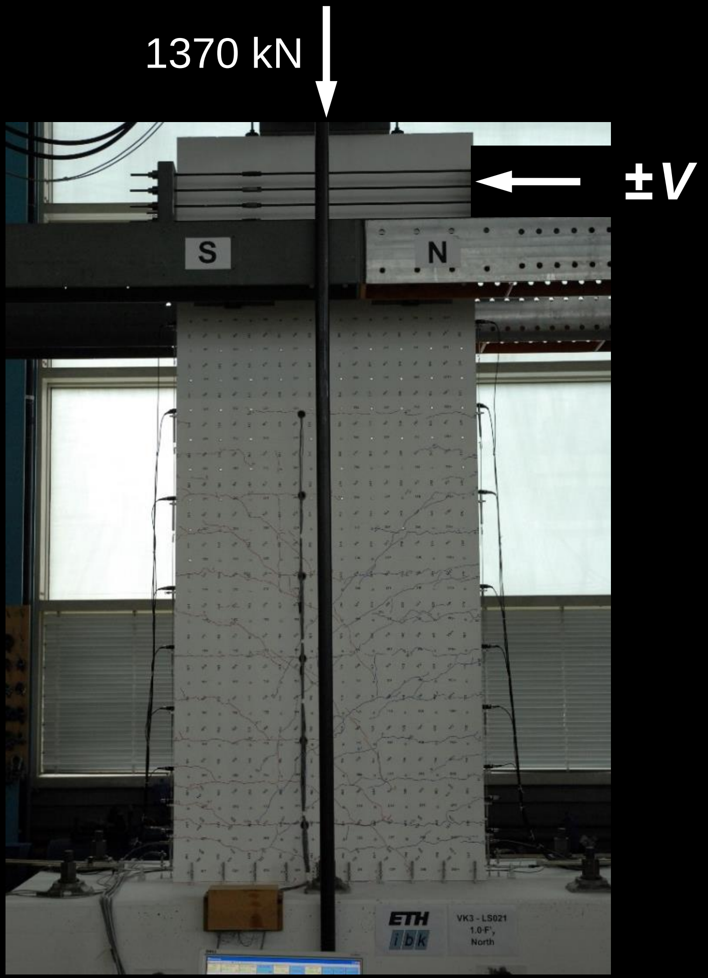
Bimschas et al. (2010, 2013) – experimental setup/specimens

Specimen	**VK1**	**VK3**	**VK6**
Effective height (m)	3.30	3.30	4.50
Section depth (m)	1.50	1.50	1.50
Section width (m)	0.35	0.35	0.35
ρ_{sl} (%)	0.82	1.23	1.23
ρ_{st} (%)	0.08	0.08	0.08

→ ε_u=8.4%

Loading: constant normal force N = -1370kN; quasi-static cyclic loading with increasing amplitudes in horizontal direction.

Note: CSFM aim at describing the backbone of the cyclic response using a monotonic model. Strain penetration into the foundation is not considered.



VK1: first yielding of reinforcement

[Bimschas, 2010]

Compatible Stress Field Method

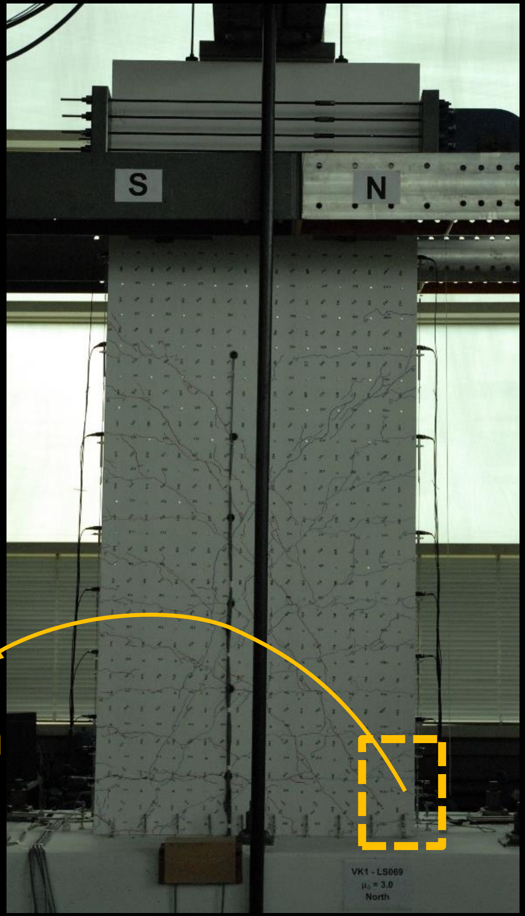
CSFM experimental validation

Bimschas et al. (2010, 2013) – peak load

Specimen	**VK1**	**VK3**	**VK6**
Experiment* V_{exp} (kN)	728	876	647
CSFM V_{calc}(kN)	730	860	650
V_{exp}/V_{calc}	**1.00**	**1.02**	**1.00**

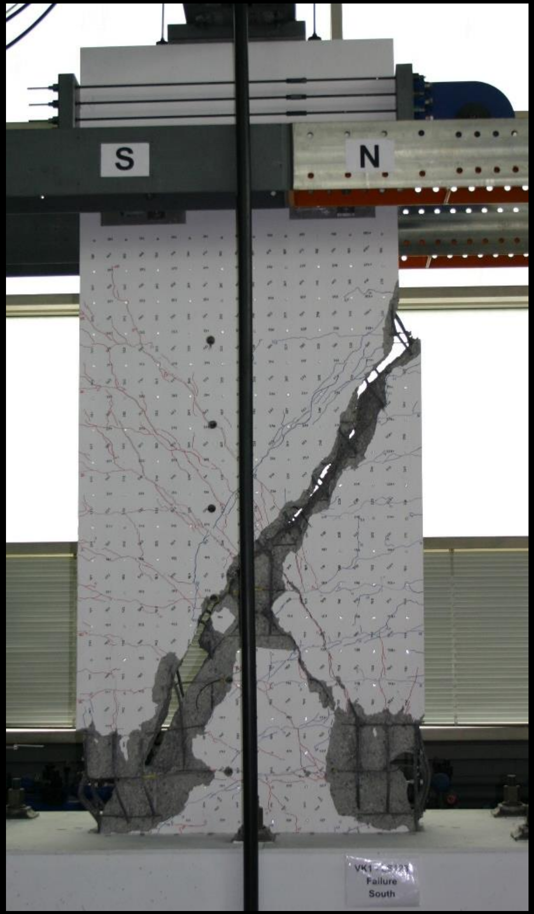
*mean peak horizontal load of North and South directions.

Note: CSFM aims at describing the behaviour of the backbone until concrete peak horizontal strength is reached, (≠ to loss of vertical bearing capacity).



Concrete crushing in compression

VK1: peak strength



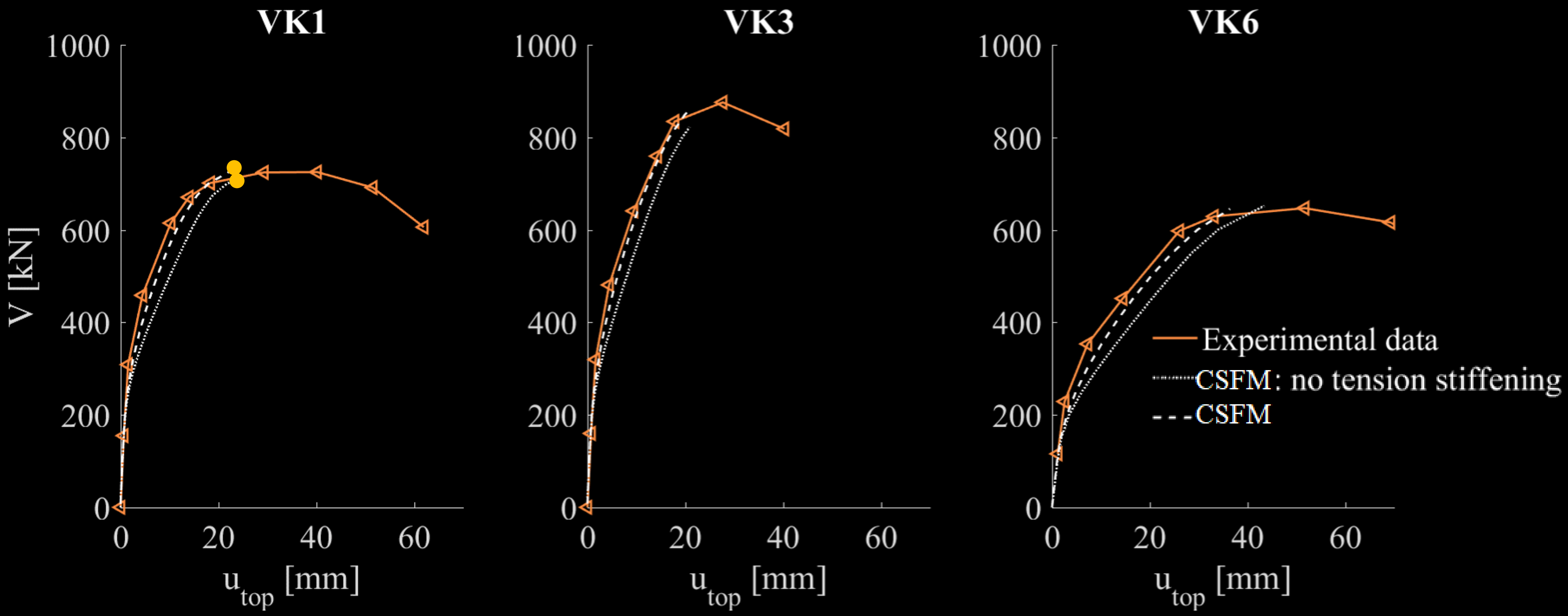
VK1: failure

[Bimschas, 2010]

Compatible Stress Field Method

CSFM experimental validation

Bimschas et al. (2010, 2013) – load deformation behaviour

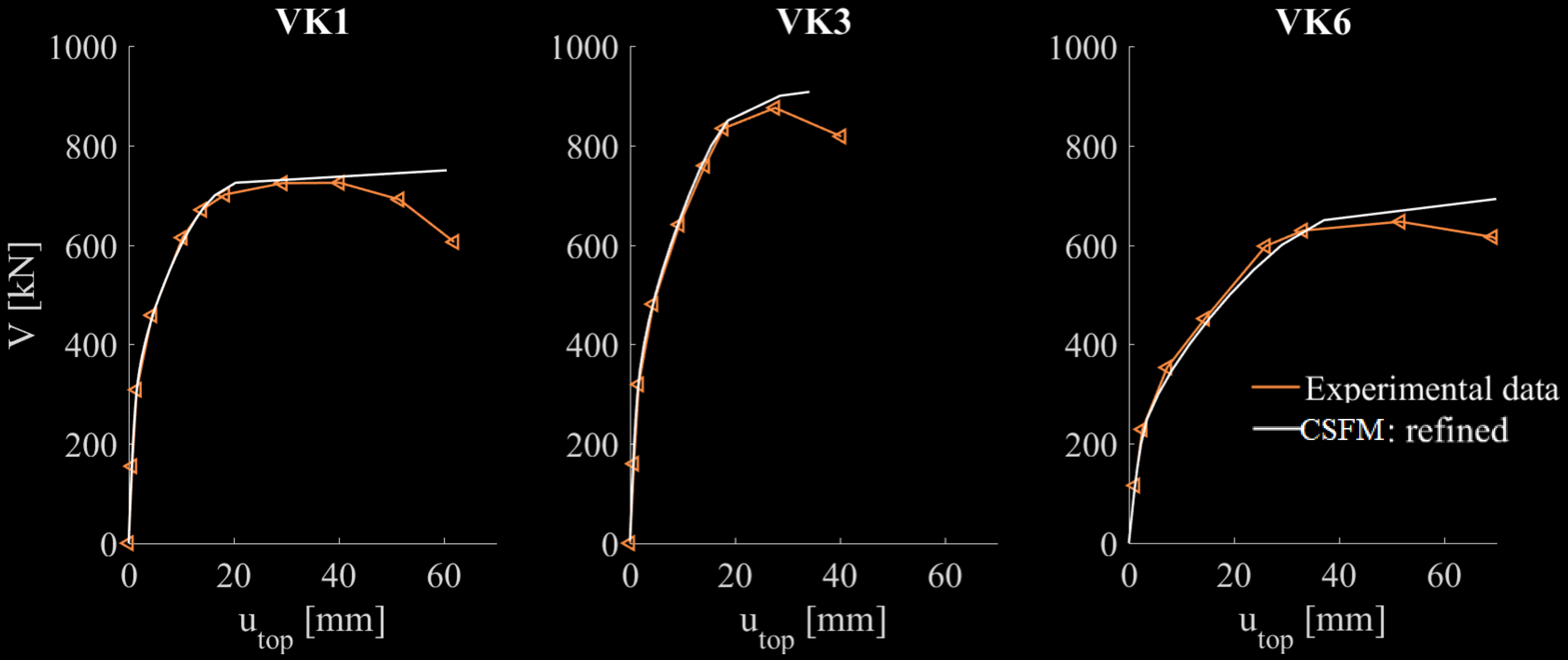


- ● Failure mode: concrete crushing in compression. Failure is considered when the strain limit criteria specified in codes for sectional analysis is reached on average over the crushing band length.

Compatible Stress Field Method

CSFM experimental validation

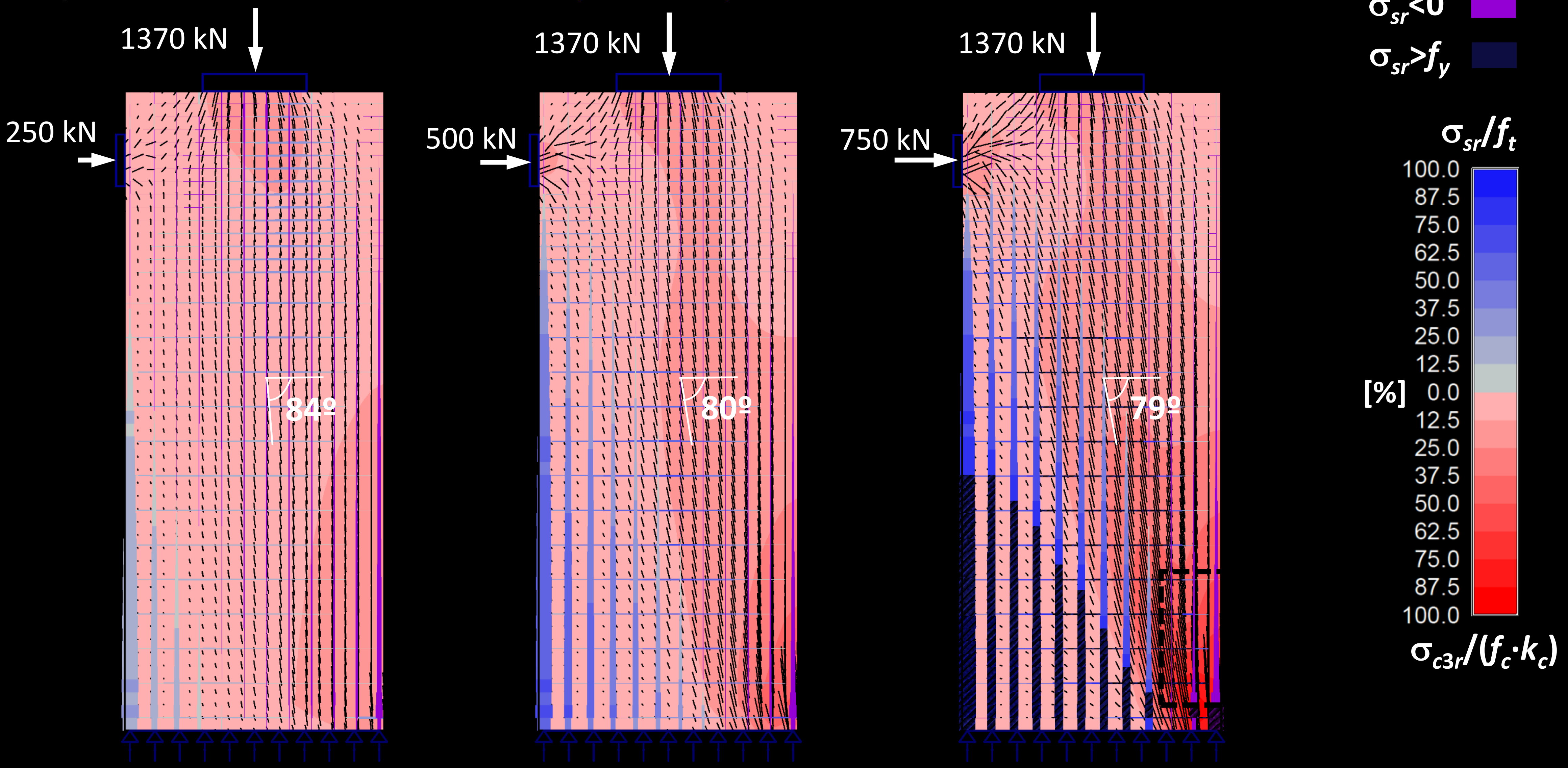
Bimschas et al. (2010, 2013) – stress fields specimen VK1



Note: Refined analysis considers the initial uncracked stiffness, as well as the actual stress-strain relationship of the reinforcement. Moreover, no concrete strain limitation is considered.

Compatible Stress Field Method

CSFM experimental validation: Bimschas et al. (2010, 2013) – load deformation behaviour



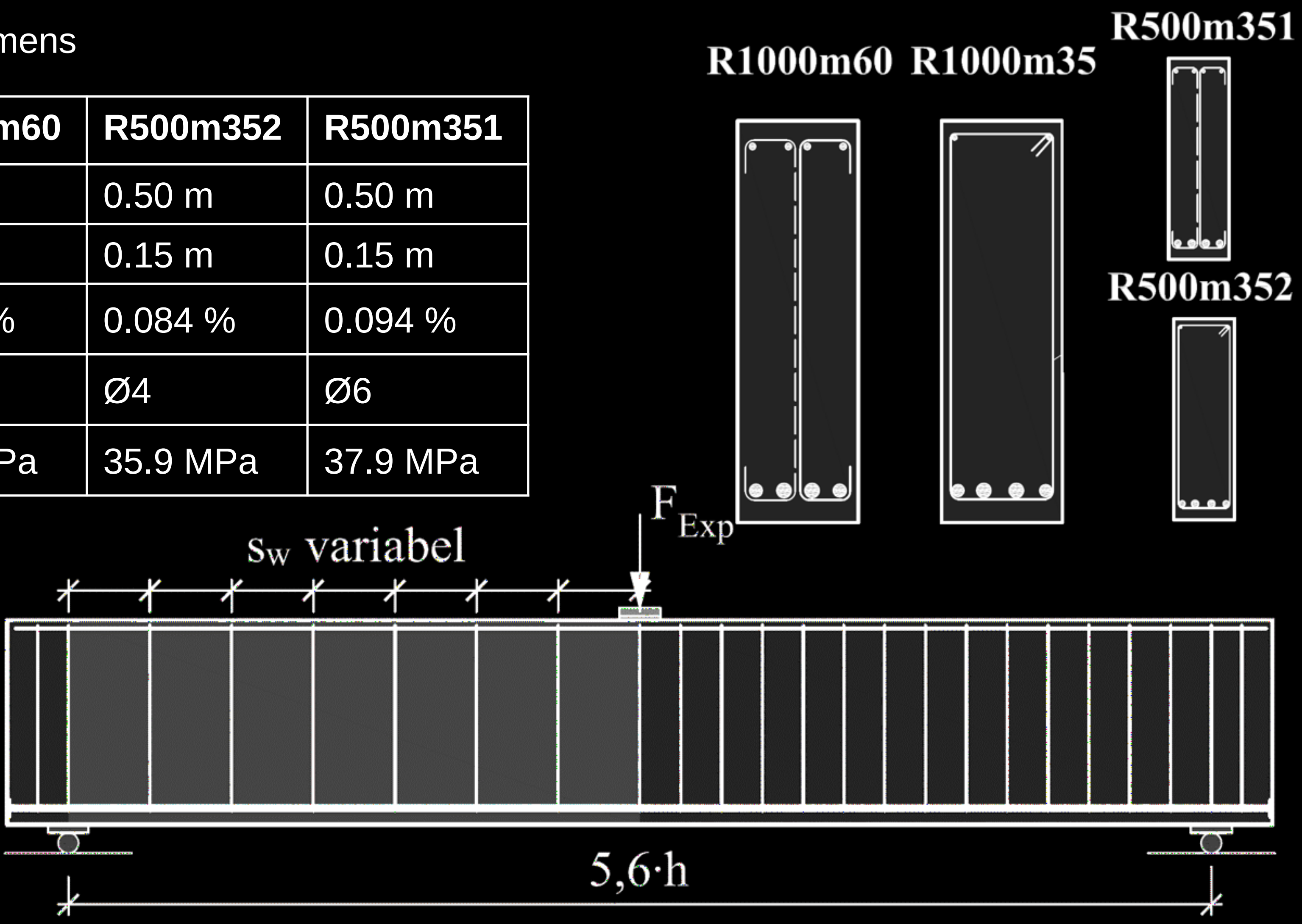
Compatible Stress Field Method

CSFM experimental validation

Huber et al. (2016) – experimental setup/specimens

Specimen	**R1000m35**	**R1000m60**	**R500m352**	**R500m351**
Section depth	1.00 m	1.00 m	0.50 m	0.50 m
Section width	0.30 m	0.30 m	0.15 m	0.15 m
ρ_w	0.094 %	0.094 %	0.084 %	0.094 %
$Ø_w$	Ø6	Ø12	Ø4	Ø6
f_c	29.6 MPa	60.9 MPa	35.9 MPa	37.9 MPa

$Ø_w$ (mm)	**f_y (MPa)**	**f_t (MPa)**	**ε_u (%)**
4	653	710	4.9
6	569	658	3.1
12	552	654	3.4

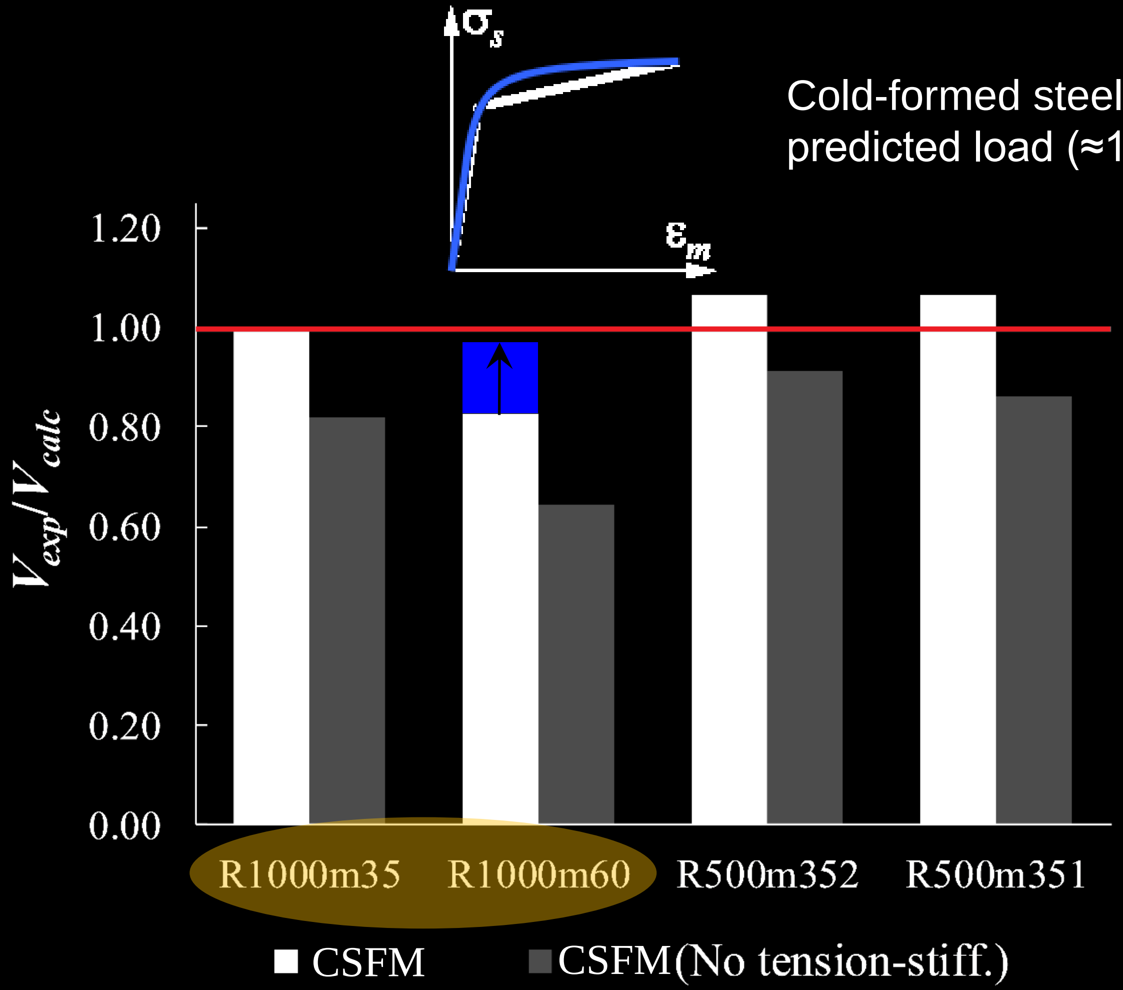


Compatible Stress Field Method

CSFM experimental validation

Huber et al. (2016) – ultimate load

Cold-formed steel with same f_t & f_y → less ductile & lower predicted load (≈10%) than standard bilinear steel law.



CSFM
Mean = 0.99
CoV = 0.11
Stirrup rupture

CSFM (No tension-stiff.)
Mean = 0.81
CoV = 0.14
Concrete crushing

- Neglecting tension stiffening leads to unsafe load predictions and does not capture the real failure mode (stirrup rupture).
- Higher impact of strain localization in real size elements → use of existing experimental databases could underestimate the impact of these failures.

Compatible Stress Field Method

CSFM experimental validation

Huber et al. (2016) – stress fields specimen R1000m35

$\varepsilon_z = 5.4‰$

$\sigma_{srz} = 638\ \text{MPa} = f_t$

$\varepsilon_1 = 6.4‰ \rightarrow k_c = 0.64$

$\sigma_{c3r} = 7.7\ \text{MPa}$

$\sigma_{c3r}/(f_c \cdot k_c) = 0.42$

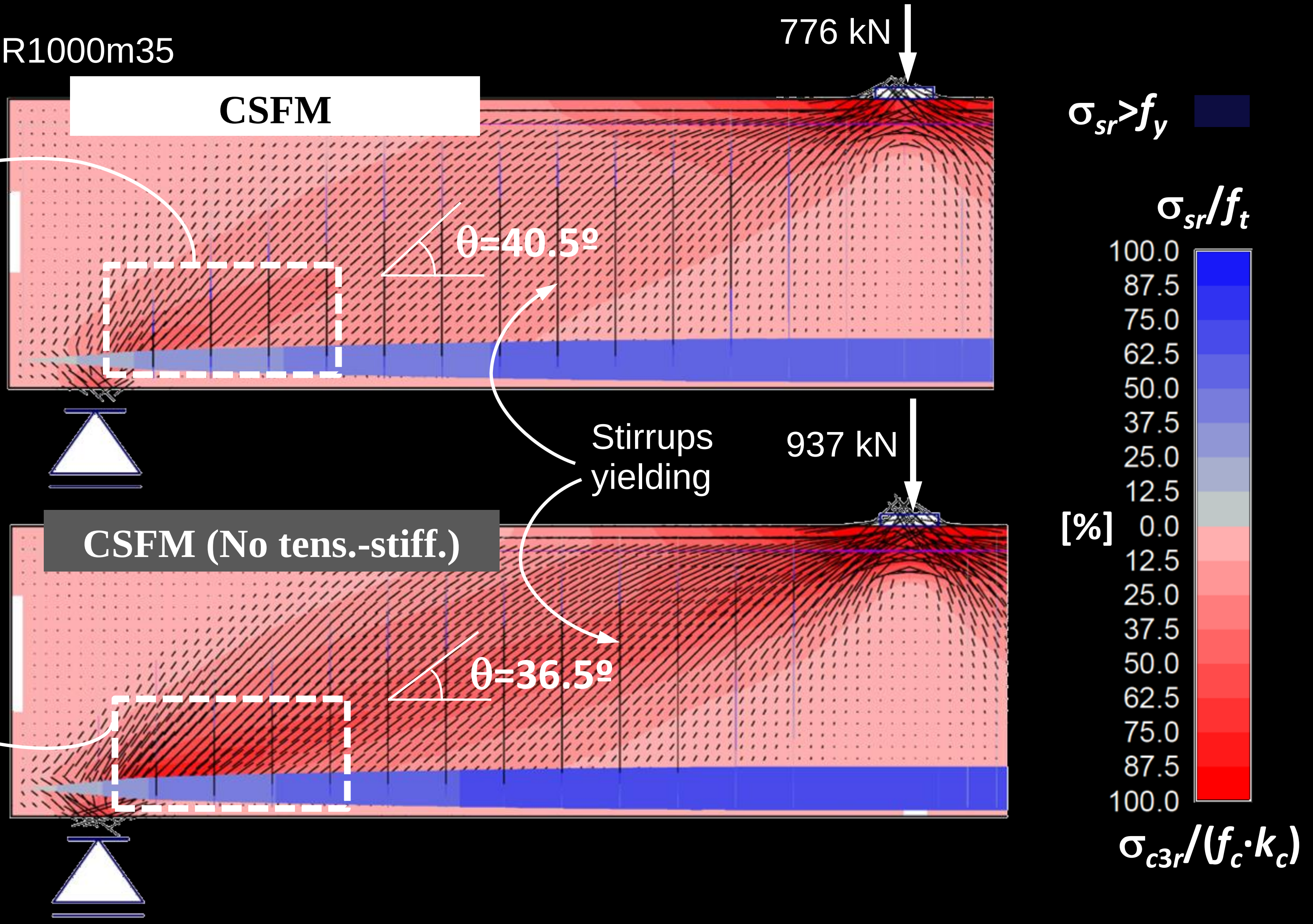
$\varepsilon_z = 20‰$

$\sigma_{srz} = 600\ \text{MPa} < f_t$

$\varepsilon_1 = 23‰ \rightarrow k_c = 0.41$

$\sigma_{c3r} = 12\ \text{MPa}$

$\sigma_{c3r}/(f_c \cdot k_c) = 1.00$



*Results at the most restrictive concrete and steel finite elements (minimum k_c & maximum σ_{srz})