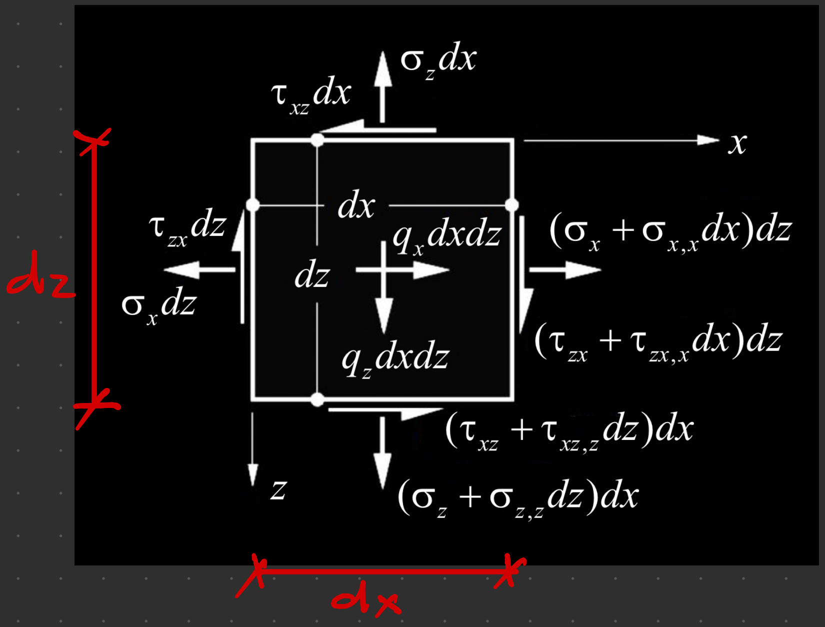

$$\sum n_x = 0:$$

$$-\cancel{\sigma_x dz} + q_x\,dx\,dz + \left(\cancel{\sigma_x} + \overbrace{\frac{d\sigma_x}{dx}}^{\text{Variation of } \sigma_x \text{ along } x}\cdot dx\right)dz$$

$$-\cancel{\tau_{xz}\,dx} + \left(\cancel{\tau_{xz}} + \frac{d\tau_{xz}}{dz}\cdot dz\right)dx = 0$$

$$\Rightarrow q_x\,\cancel{dz}\,\cancel{dx} + \frac{d\sigma_x}{dx}\cdot\cancel{dx}\,\cancel{dz} + \frac{d\tau_{xz}}{dz}\cdot\cancel{dz}\,\cancel{dx} = 0$$

$=$

$$\boxed{\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xz}}{\partial z} + q_x = 0}$$

$\sigma_x = -2$ MPa
$\sigma_z = 5$ MPa
$\tau_{xz} = 5$ MPa

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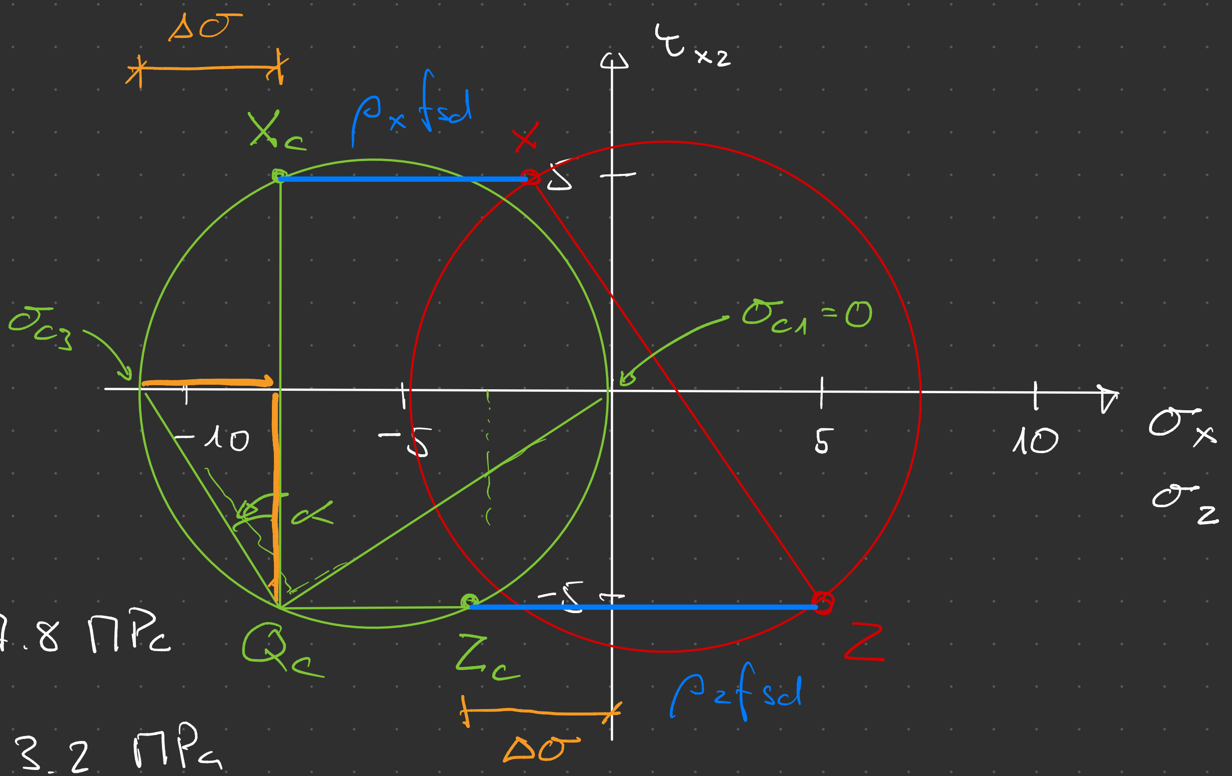
$\rho_x = 1.3\%$
$\rho_z = 1.9\%$
$f_{sd} = 435$ MPa

Reinforcement



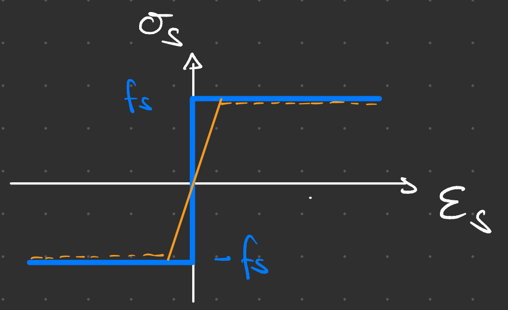
$$\boxed{\begin{matrix}\sigma_{xc}\\ \sigma_{zc}\end{matrix}} \begin{matrix}=\\=\end{matrix} \boxed{\begin{matrix}\sigma_{x}\\ \sigma_{z}\end{matrix}} \begin{matrix}-\\-\end{matrix} \boxed{\begin{matrix}\rho_x \cdot f_{sd}\\ \rho_z \cdot f_{sd}\end{matrix}} \begin{matrix}= \ldots = -7.8 \text{ MPa}\\ = \ldots = -3.2 \text{ MPa}\end{matrix}$$

$f_{c,min} = \sigma_{c3} = \sigma_{xc} + \sigma_{zc} = \underline{-11 \text{ MPa}} \longrightarrow f_c = k_c \cdot f_{cd} \Longleftrightarrow \underline{\underline{f_{cd} = 20 \text{ MPa}}}$

$\quad \hookrightarrow 0.55$

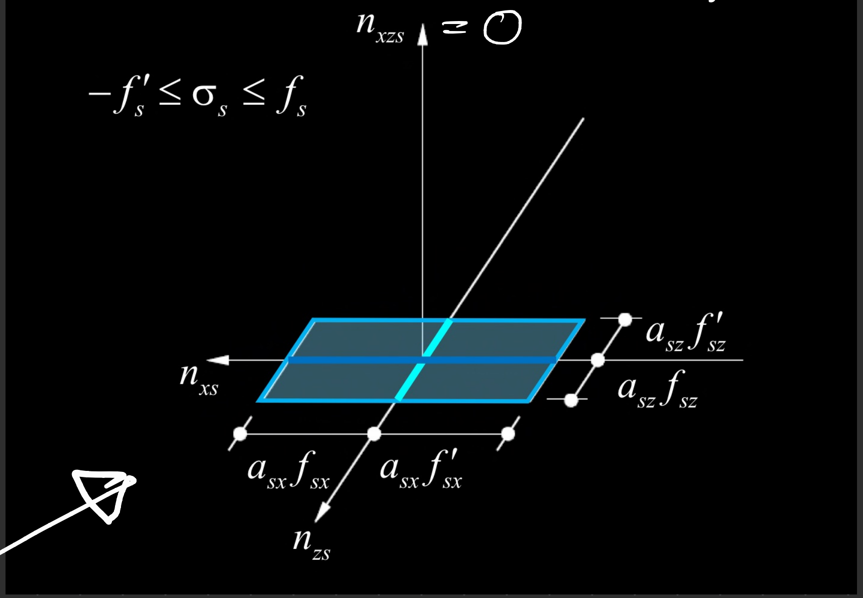
$\tan\alpha = \dfrac{\sigma_{zc}}{\tau_{xz}} \Rightarrow \alpha = 32.7^\circ \geq \alpha_{min} = 30^\circ$

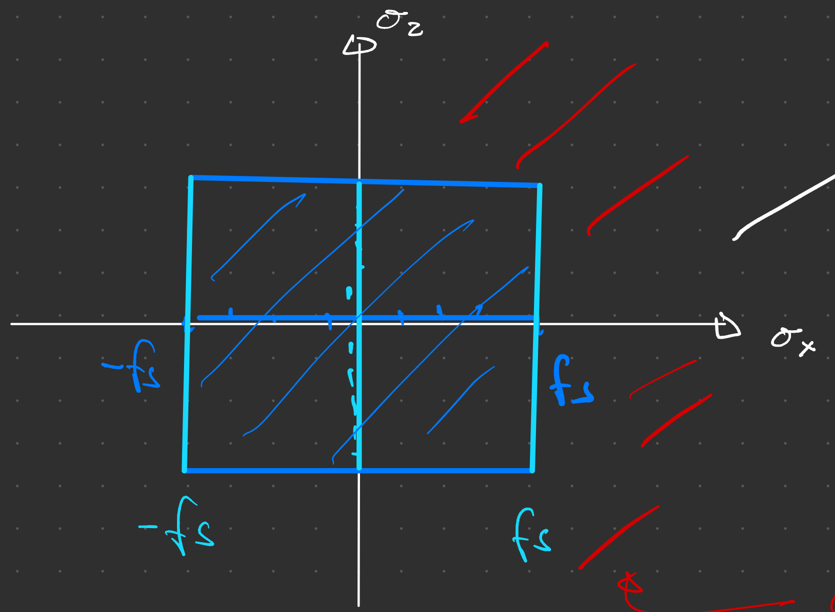
Reinforcement



rigid – ideally plastic

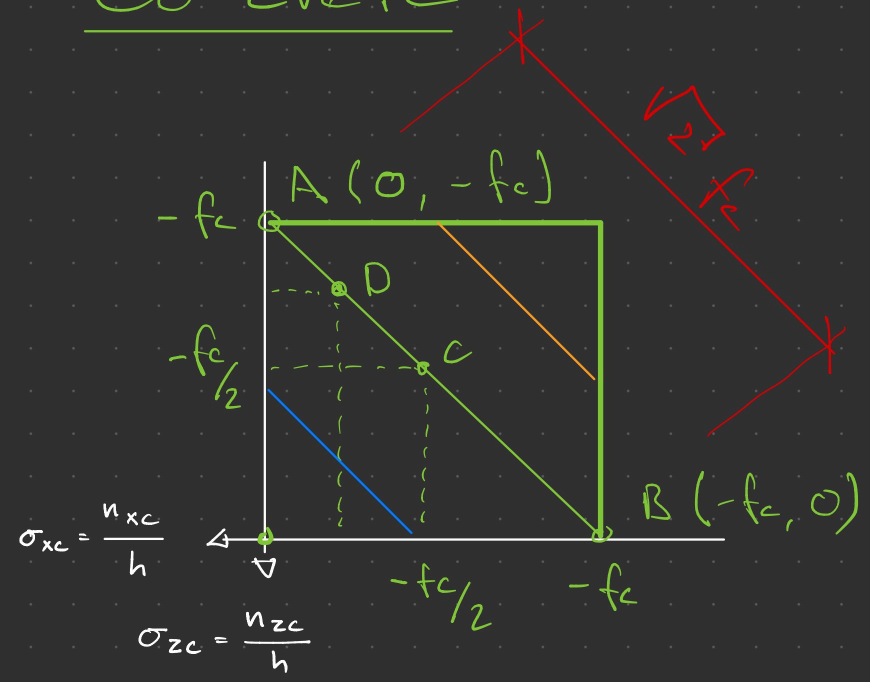
elastic – ideally plastic

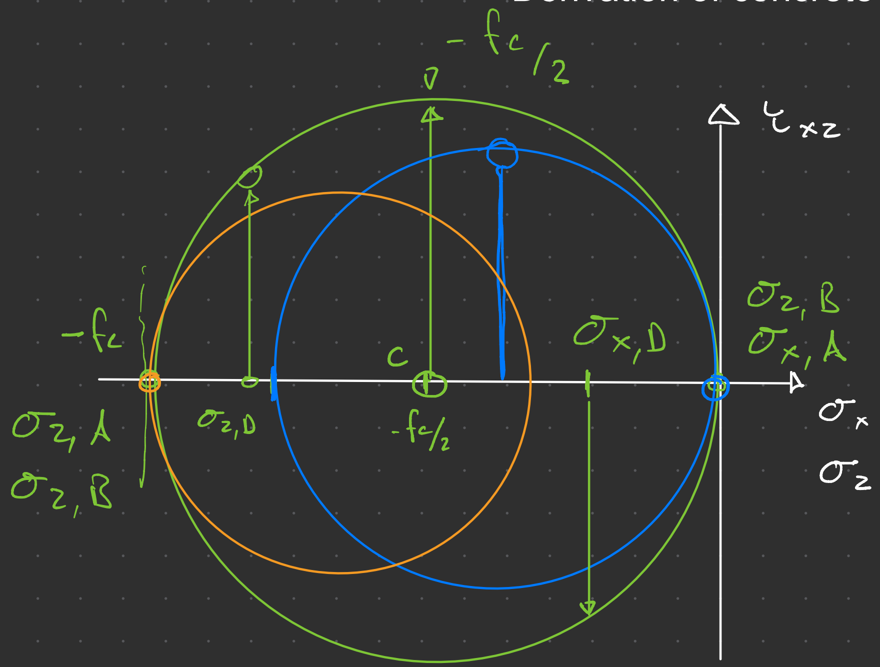


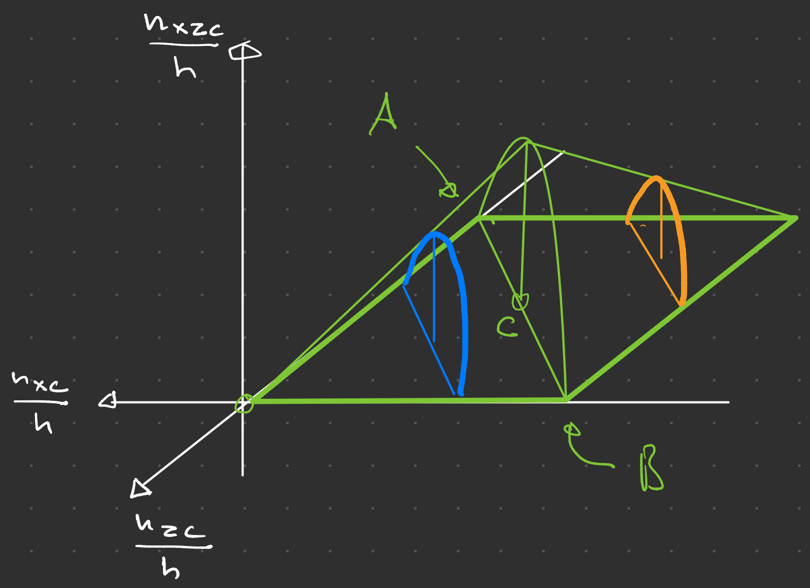


outside yield surface

Concrete


A (0, -fc)
-fc
D
C
-fc/2
B (-fc, 0)
-fc/2
-fc
$\sigma_{xc} = \frac{n_{xc}}{h}$
$\sigma_{zc} = \frac{n_{zc}}{h}$
$\sqrt{2}\, f_c$

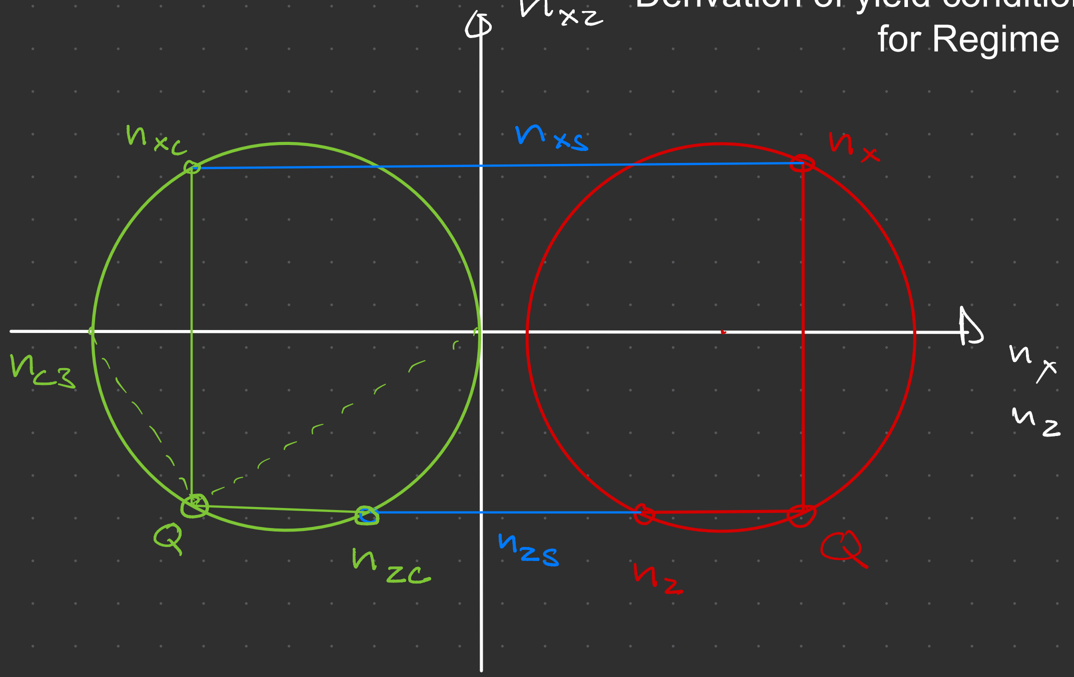

-fc/2
τ_{xz}
-fc
C
$\sigma_{x,D}$
$\sigma_{z,B}$
$\sigma_{x,A}$
$\sigma_{z,A}$
$\sigma_{z,B}$
$\sigma_{z,D}$
-fc/2
σ_x
σ_z


$\frac{n_{xzc}}{h}$
A
C
$\frac{n_{xc}}{h}$
B
$\frac{n_{zc}}{h}$

Derivation of yield condition for Regime 1

(1) $$Y_1 = n_{xz}^2 - (a_{sx} f_{sx} - n_x)(a_{sz} f_{sz} - n_z) = 0$$

(2) $$Y_2 = n_{xz}^2 - (h f_c - a_{sz} f_{sz} + n_z)(a_{sz} f_{sz} - n_z) = 0$$



(1) $Y_c = 0!$ $\quad n_{xz}^2 \overset{!}{=} n_{xz,c}^2$

$$\Rightarrow n_{xz}^2 - n_{xz,c}^2 = 0$$

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$$n_{xz}^2 - n_{xc} \cdot n_{zc} = 0$$

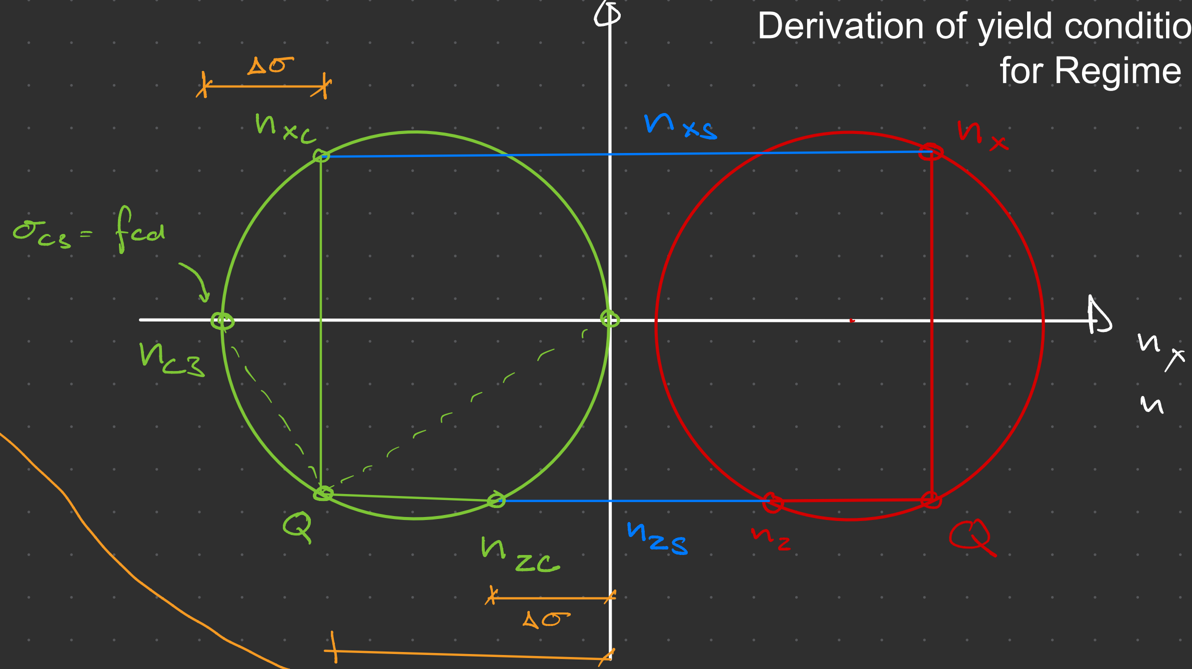
$n_x = n_{xc} + n_{xs}$ $\quad n_{xs} = a_{sx} \cdot f_{sd}$ (yielding)

$n_z = n_{zc} + n_{zs}$ $\quad n_{zs} = a_{sz} \cdot f_{sd}$ (yielding)

$$n_{xz}^2 - (a_{sx} f_{sd} - n_x)(a_{sz} f_{sd} - n_z) = 0$$

$$① \quad Y_1 = n_{xz}^2 - (a_{sx} f_{sx} - n_x)(a_{sz} f_{sz} - n_z) = 0$$

$$② \quad Y_2 = n_{xz}^2 - (h f_c - a_{sz} f_{sz} + n_z)(a_{sz} f_{sz} - n_z) = 0$$



②

$$n_{xz}^2 - n_{xc} \cdot n_{zc}$$

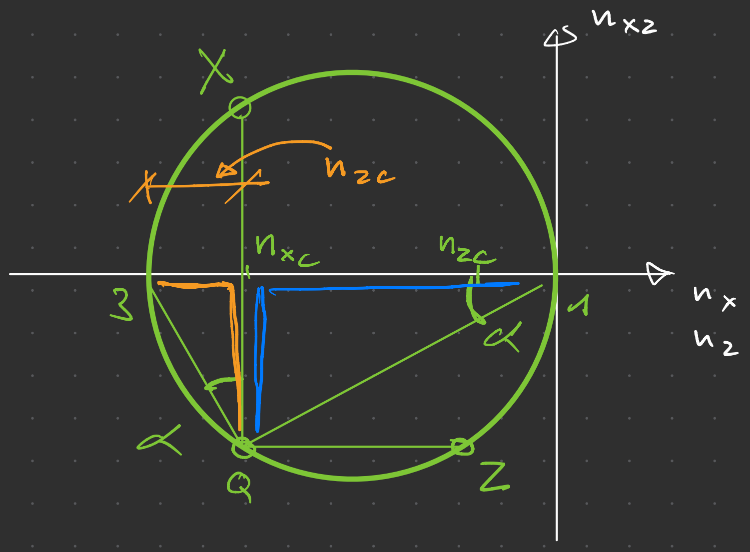
$$\to a_{sz} \cdot f_{sd} - n_z$$

$$a_{sx} \underbrace{\sigma_{sx}}_{=?} - n_x < a_{sx} f_{sd} - n_x$$

$$n_{xc} = h \cdot f_{cd} - n_{zc}$$

$$n_{xz}^2 - \left(h \cdot f_{cd} - (a_{sz} f_{sd} - n_z)\right) \cdot (a_s f_{sd} - n_z)$$

for Y_1



$$\tan\alpha = \frac{n_{zc}}{n_{xzc}}$$

multiply

$$\tan\alpha = \frac{n_{xzc}}{n_{xc}}$$

$$\tan^2\alpha = \frac{n_{zc}}{n_{xc}}$$

$$\cot^2\alpha = \frac{n_{xc}}{n_{zc}} = \frac{a_{sx} f_{sd} - n_x}{a_{sz} f_{sd} - n_z}$$

Decoupling of yield condition for Regime 1

$$Y_1 : \cot^2\alpha = (a_{sx} f_{sx} - n_x)/(a_{sz} f_{sz} - n_z) \longrightarrow (a_{sx} f_{sd} - n_x) = \cot^2\alpha \, (a_{sz} f_{sd} - n_z) \quad (*)$$

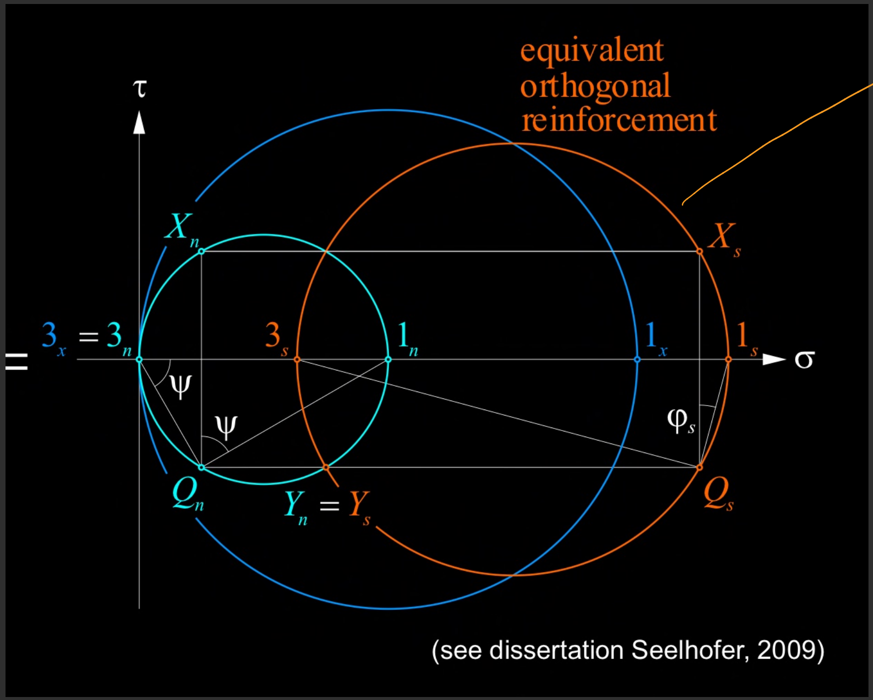
$$Y_1 = n_{xz}^2 - (a_{sx} f_{sx} - n_x)(a_{sz} f_{sz} - n_z) = 0$$

$$n_{xz}^2 - \cot^2\alpha \, (a_{sz} f_{sd} - n_z)^2 = 0$$

$$(a_{sz} f_{sd} - n_z) = \frac{|n_{zx}|}{\cot\alpha}$$

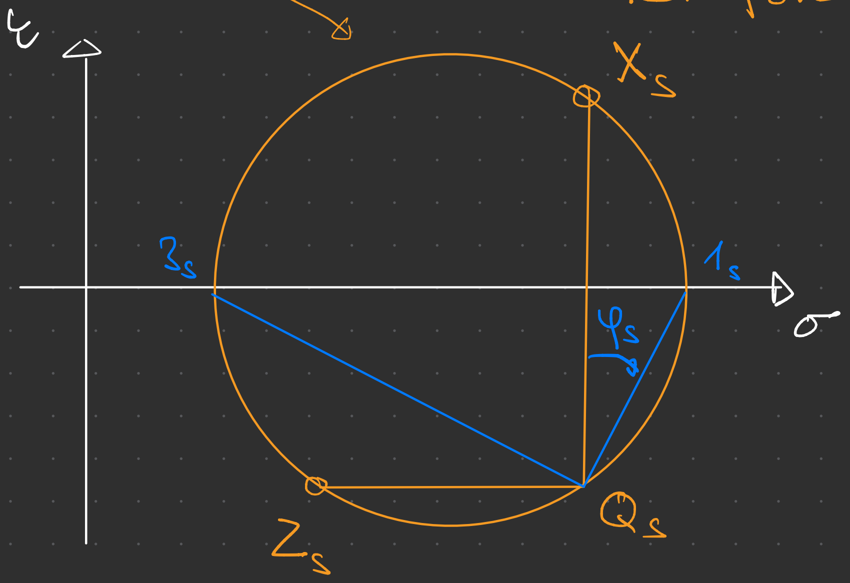
$$(*) \quad (a_{sx} f_{sd} - n_x) = |n_{zx}| \cdot \cot\alpha$$

Approach for using Yield Conditions with skew reinforcement



(see dissertation Seelhofer, 2009)

Reinforcement



$\Rightarrow \sigma_{1s}, \sigma_{3s}$

↳ orthogonal, $\tau_{xz,s} = 0$

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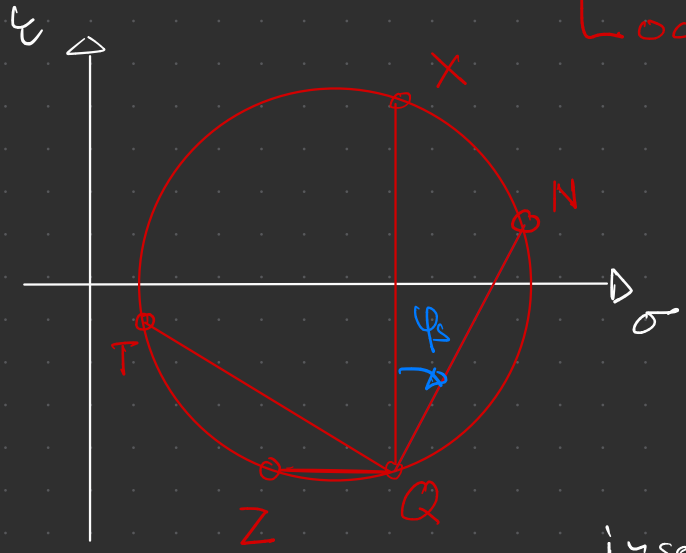

$\Rightarrow \sigma_n, \sigma_t, \tau_{nt}$

n-direction ≙ 1-direction

t-direction ≙ 3-direction

↓

insert $\sigma_n, \sigma_t, \tau_{nt}, \sigma_{s1}, \sigma_{s2}$

in Y_1-condition