2 In-plane loading – membrane elements

2.4 Equilibrium and yield conditions

Learning objectives

Within this chapter, the students are able to:

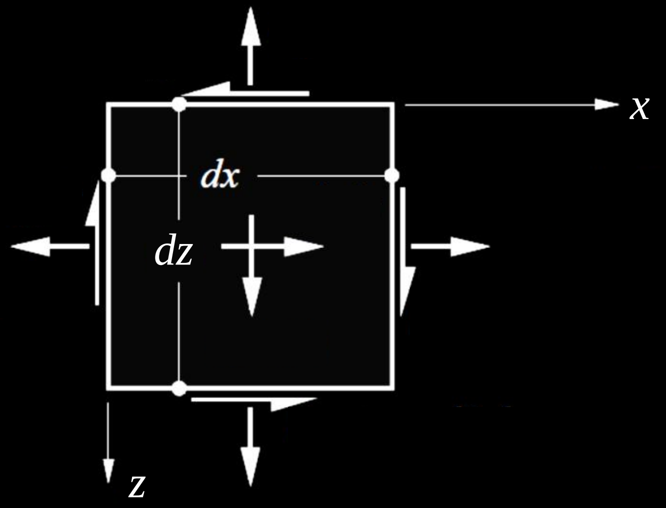
- identify the relevance of membrane elements in structural concrete, and how they can be used to design a more general shell structure.
- assess the equilibrium of reinforced concrete membrane elements as a combination of concrete and reinforcement.
  - combine the yield conditions of concrete and reinforcement to determine the yield conditions of membrane elements with orthogonal reinforcement.
  - distinguish and explain the different yield regimens.
  - design membrane elements with orthogonal reinforcement either with yielding of both reinforcements (regime 1) or with concrete crushing and yielding of the longitudinal reinforcement (regime 2).
  - illustrate the behaviour of a membrane element with skew reinforcement and yielding of both reinforcements (regime 1) by means of Mohr’s circles.

Membrane elements - Introduction

Definition

The analysis of membrane elements presented in this chapter is valid for:

- In-plane loaded elements
- Homogeneously loaded (i.e. no variations of stresses)
- Homogeneously distributed reinforcing bars → steel and bond stresses can be modeled by equivalent stresses uniformly distributed over the thickness and in the transverse direction between the reinforcing bars

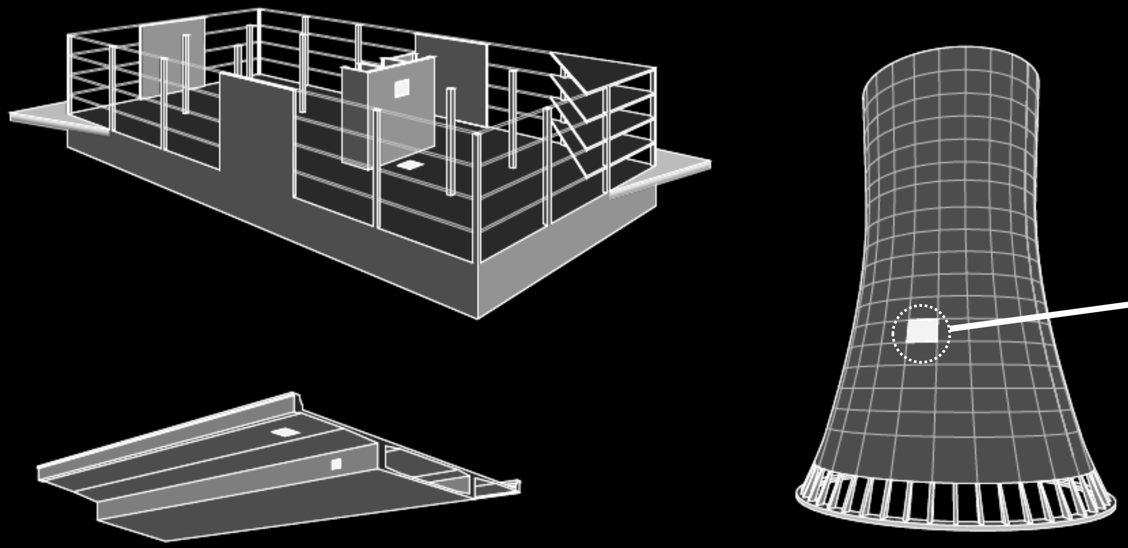


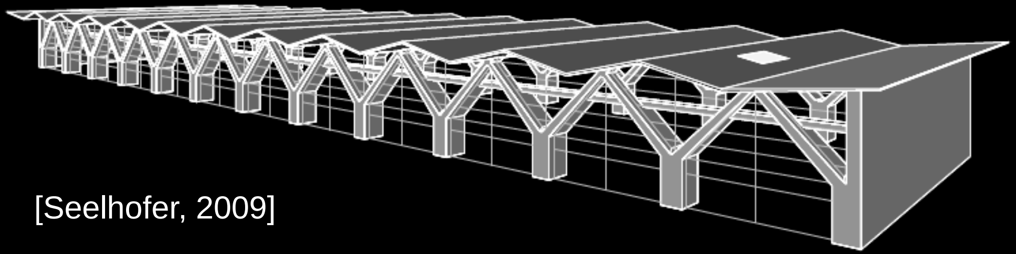
Only very few structural elements fulfil these criteria and can be directly designed as a single membrane element. Why study this theoretical case?

The local behaviour of a plane structure subjected to a general loading (i.e. in-plane forces, bending moments, twisting moments, and transverse shear) can be modelled by a combination of membrane elements (sandwich or layered approaches). With numerical approaches, the behaviour of most structures can be modelled by the superposition of membrane elements (see the following slide).

Membrane elements - Introduction

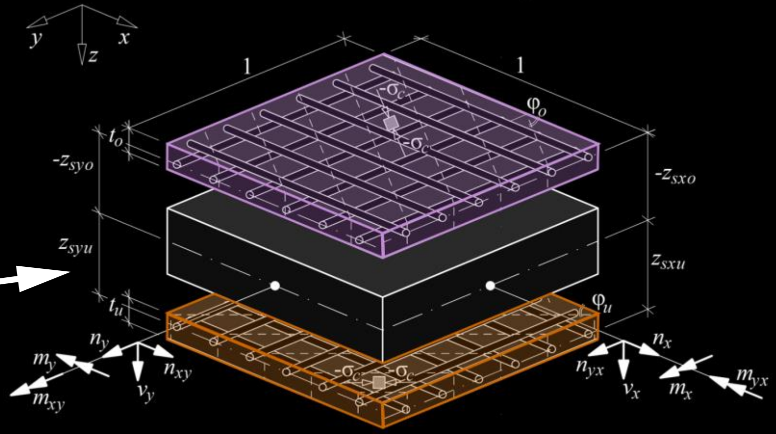
Modelling of structures composed by plane elements





[Seelhofer, 2009]

Generally loaded shell element
(8 stress resultants)



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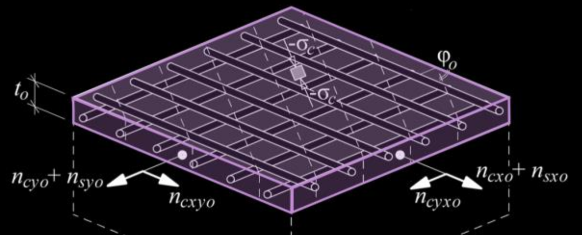
Membrane element



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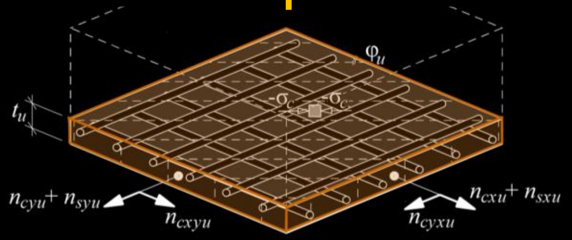
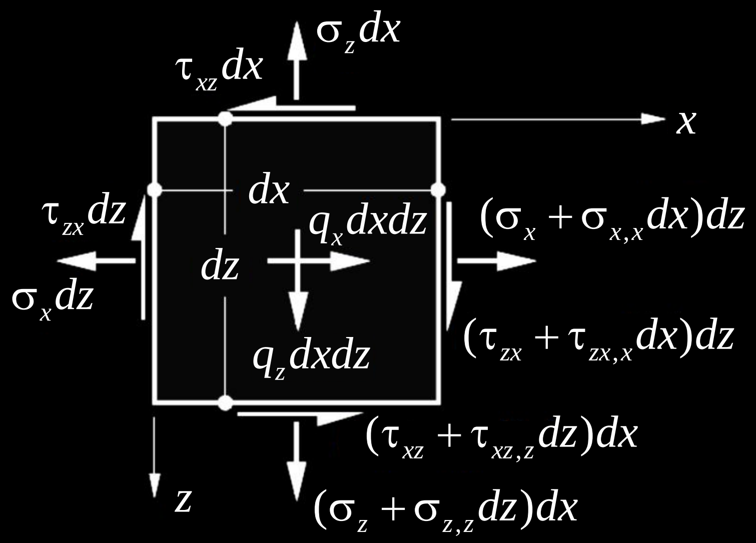
Membrane element



Membrane elements - Equilibrium

Equilibrium conditions



A stress component is taken as positive if it acts in a positive (negative) direction on an element face where a vector normal to the face is in a positive (negative) direction relative to the axis considered.

Positive membrane forces correspond to positive stresses

Indices: 1-direction of the stress, 2-direction of the normal vector

Equilibrium in directions *x*, *z*:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + q_x = 0$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \sigma_z}{\partial z} + q_z = 0$$

Or in membrane forces
(σ, τ constant over membrane element thickness *h*):

$$\frac{\partial n_x}{\partial x} + \frac{\partial n_{xz}}{\partial z} + h \cdot q_x = 0$$

$$\frac{\partial n_{zx}}{\partial x} + \frac{\partial n_z}{\partial z} + h \cdot q_z = 0$$

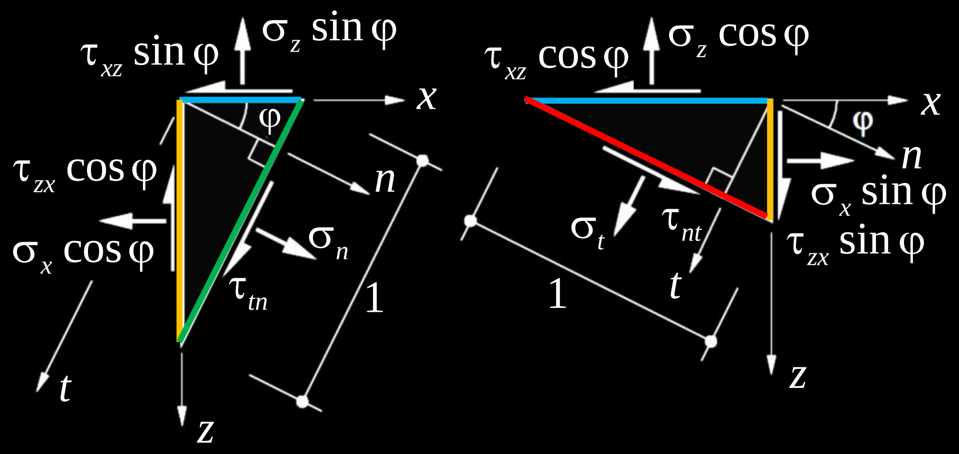
$$\left(n_x = h\sigma_x \quad n_z = h\sigma_z \quad n_{xz} = h\tau_{xz} \right)$$

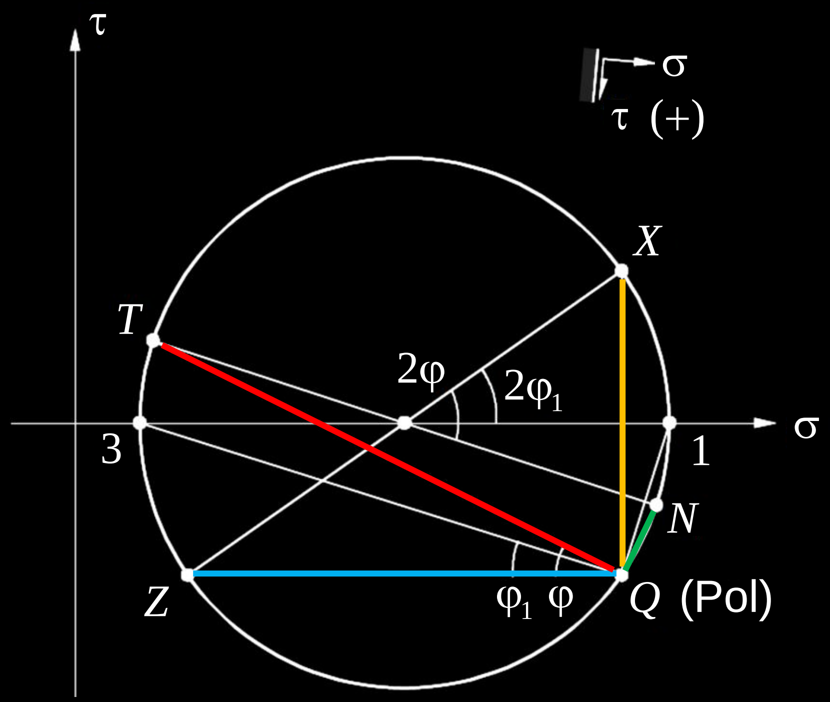
With (moment condition $M_y = 0$):

$\tau_{zx} = \tau_{xz}$ resp. $n_{zx} = n_{xz}$

Membrane elements - Stress transformation

Stress transformation: Mohr's circle





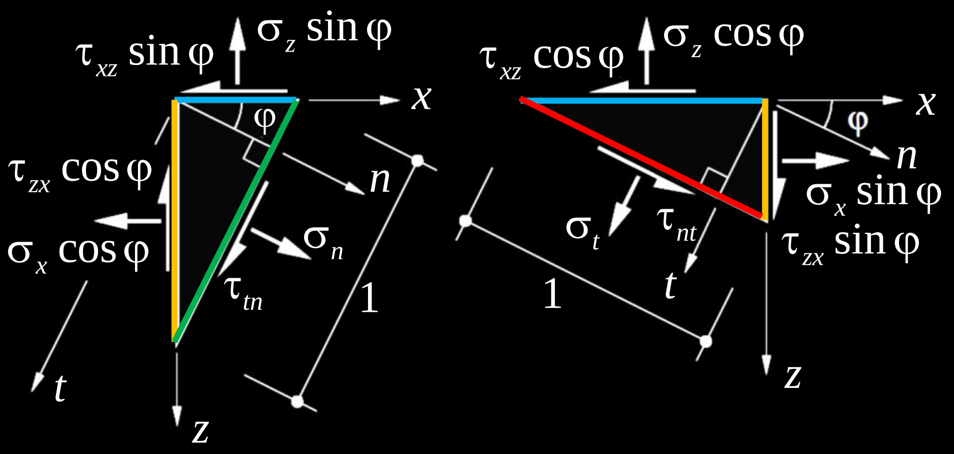
$$\sigma_n = \sigma_x \cos^2\varphi + \sigma_z \sin^2\varphi + 2\tau_{xz} \sin\varphi \cos\varphi$$

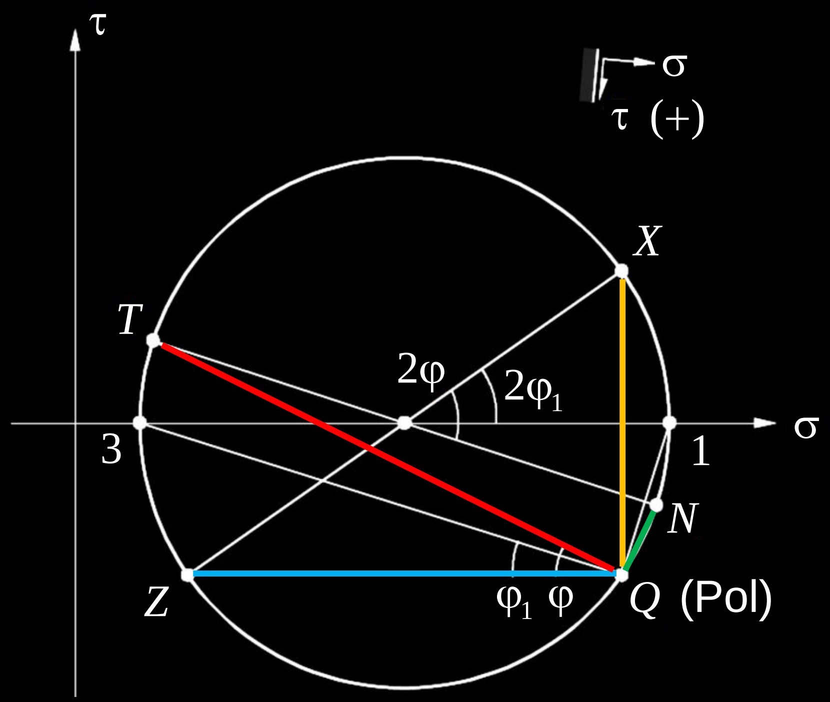
$$\sigma_t = \sigma_x \sin^2\varphi + \sigma_z \cos^2\varphi - 2\tau_{xz} \sin\varphi \cos\varphi$$

$$\tau_{nt} = (\sigma_z - \sigma_x)\sin\varphi \cos\varphi + \tau_{xz}(\cos^2\varphi - \sin^2\varphi)$$

Membrane elements - Stress transformation

Stress transformation: Mohr's circle





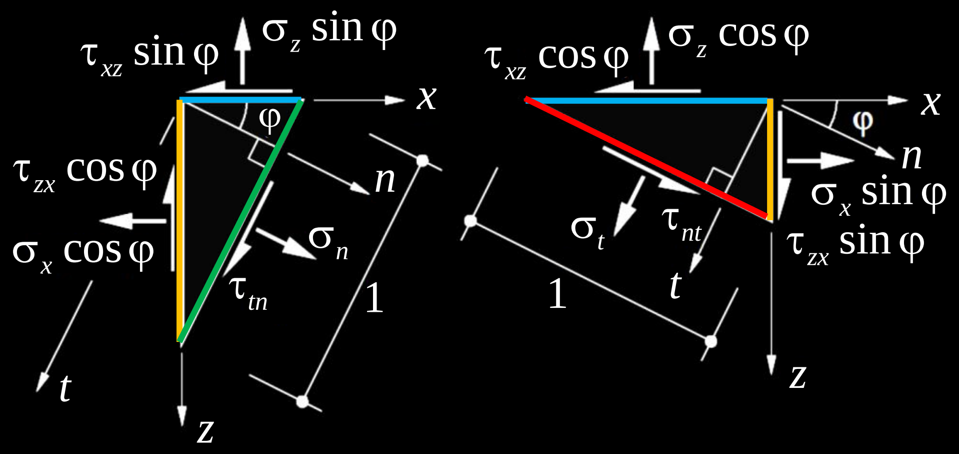
$$\sigma_n = \frac{\sigma_x+\sigma_z}{2} + \frac{\sigma_x-\sigma_z}{2}\cos 2\varphi + \tau_{xz}\sin 2\varphi$$

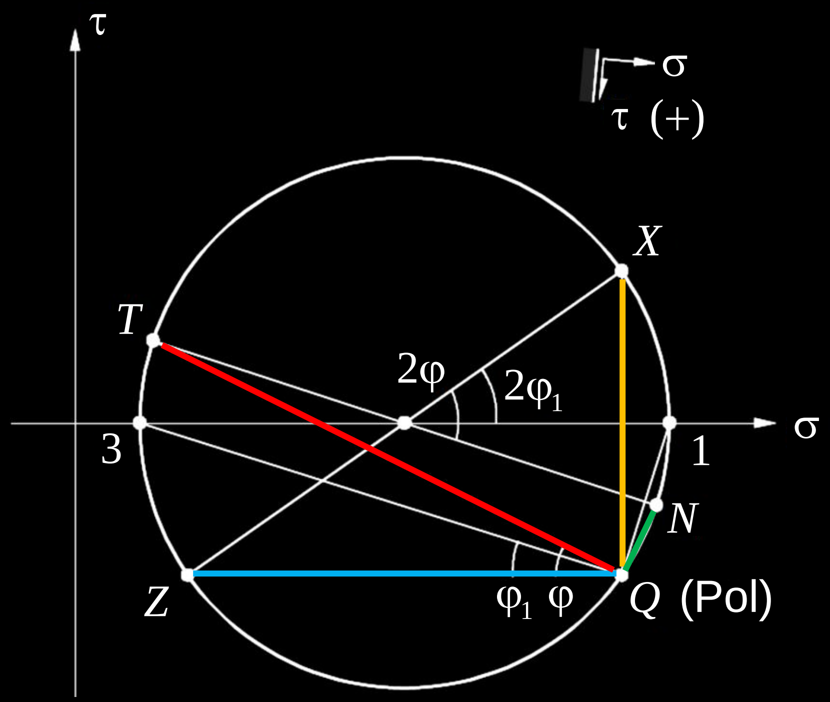
$$\sigma_t = \frac{\sigma_x+\sigma_z}{2} + \frac{\sigma_x-\sigma_z}{2}\cos 2\varphi - \tau_{xz}\sin 2\varphi$$

$$\tau_{tn} = -\frac{\sigma_x-\sigma_z}{2}\sin 2\varphi + \tau_{xz}\cos 2\varphi$$

$$\boxed{\begin{aligned}\cos 2\varphi &= \cos^2\varphi - \sin^2\varphi\\ 1 &= \sin^2\varphi + \cos^2\varphi\\ \sin 2\varphi &= 2\sin\varphi\cos\varphi\end{aligned}}$$

Stress transformation: Mohr's circle





$$\sigma_n = \frac{\sigma_x+\sigma_z}{2} + \frac{\sigma_x-\sigma_z}{2}\cos 2\varphi + \tau_{xz}\sin 2\varphi$$

$$\sigma_t = \frac{\sigma_x+\sigma_z}{2} + \frac{\sigma_x-\sigma_z}{2}\cos 2\varphi - \tau_{xz}\sin 2\varphi$$

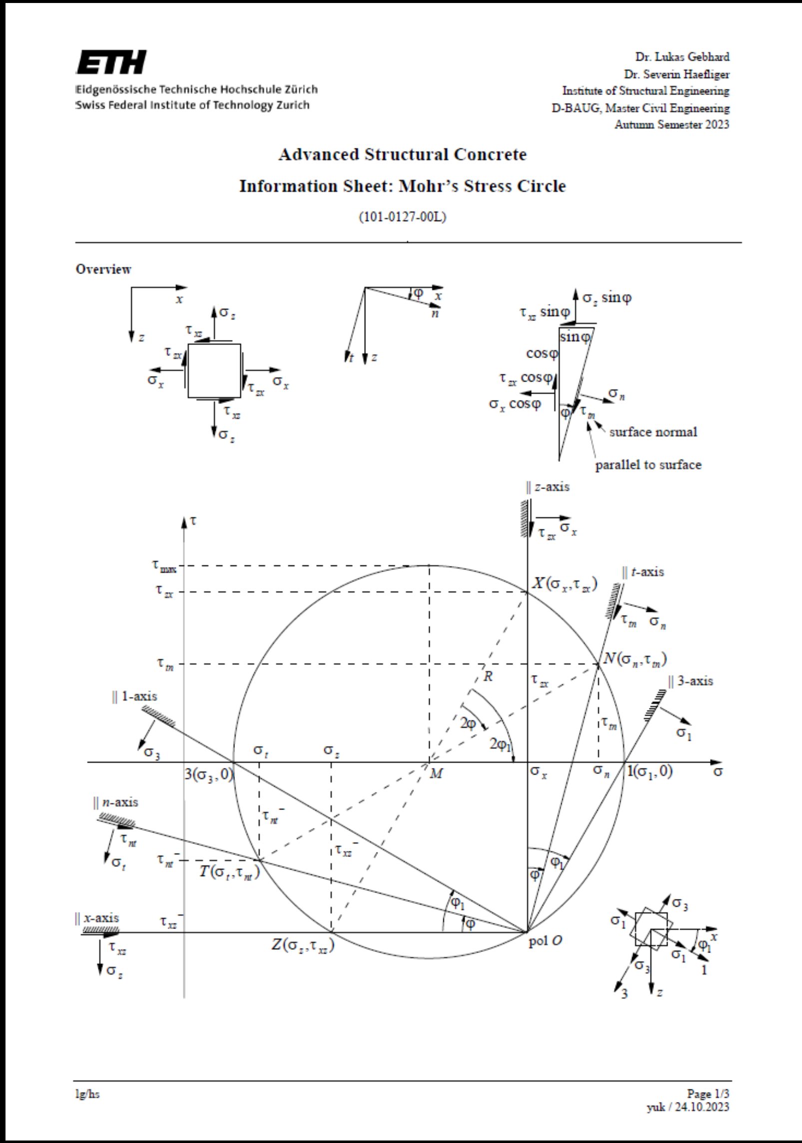
$$\tau_{tn} = -\frac{\sigma_x-\sigma_z}{2}\sin 2\varphi + \tau_{xz}\cos 2\varphi$$

$$\tau_{nt} = \tau_{tn} = 0 \quad \rightarrow \quad \varphi_1 = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xz}}{\sigma_x-\sigma_z}\right)$$

$$\sigma_{1,3} = \underbrace{\frac{\sigma_x+\sigma_z}{2}}_{\text{Centre /}} \pm \underbrace{\frac{\sqrt{(\sigma_x-\sigma_z)^2+4\tau_{xz}^2}}{2}}_{\text{Radius Mohr's circle}}$$

Membrane elements - Stress transformation

Stress transformation: Mohr's circle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Autumn Semester 2023

Advanced Structural Concrete

Information Sheet: Mohr's Stress Circle

(101-0127-00L)

Overview

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yuk / 24.10.2023

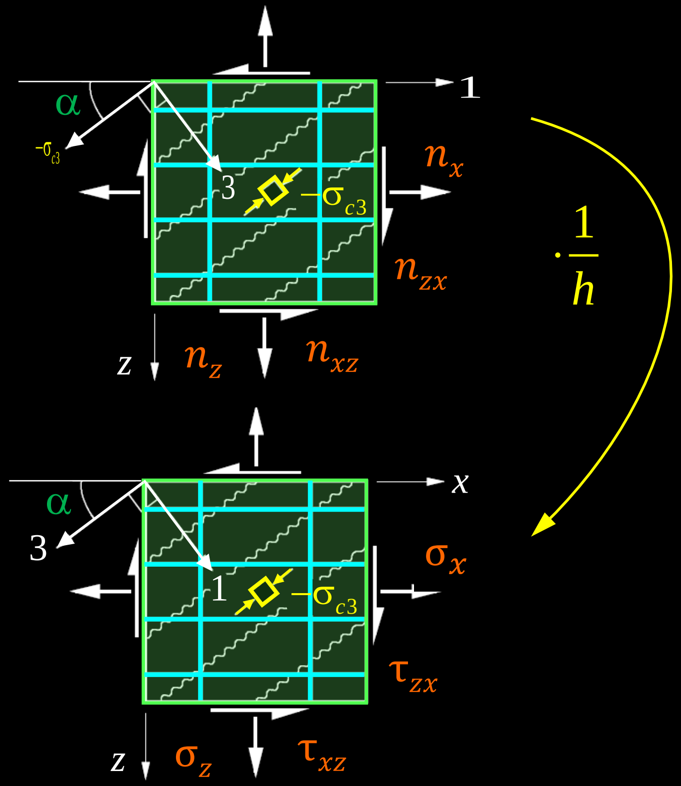
Membrane elements - Equilibrium

Equilibrium («reinforced concrete = concrete + reinforcement»)
Orthogonally reinforced element (reinforcement directions *x, z*):

- Concrete is homogeneous and isotropic, absorbs compressive stresses ≤ f_c in any direction but no tensile stresses
- Reinforcement only carries forces in the direction of the bar, up to a maximum value f_s and is distributed and anchored in such a way that equivalent distributed stresses can be expected
- Perfect bond between concrete and reinforcement

In membrane forces:

$$n_x = n_{xc} + n_{xs} = n_{xc} + a_{sx}\sigma_{sx}$$
$$n_z = n_{zc} + n_{zs} = n_{zc} + a_{sz}\sigma_{sz}$$
$$n_{xz} = n_{xzc} + n_{xzs} = n_{xzc}$$
$$(n_x = h\sigma_x \quad n_z = h\sigma_z \quad n_{xz} = h\tau_{xz})$$



In equivalent stresses:

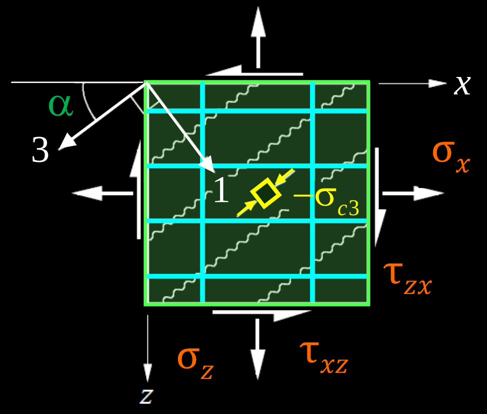
$$\sigma_x = \sigma_{xc} + \rho_x\sigma_{sx}$$
$$\sigma_z = \sigma_{zc} + \rho_z\sigma_{sz}$$
$$\tau_{xz} = \tau_{xzc}$$

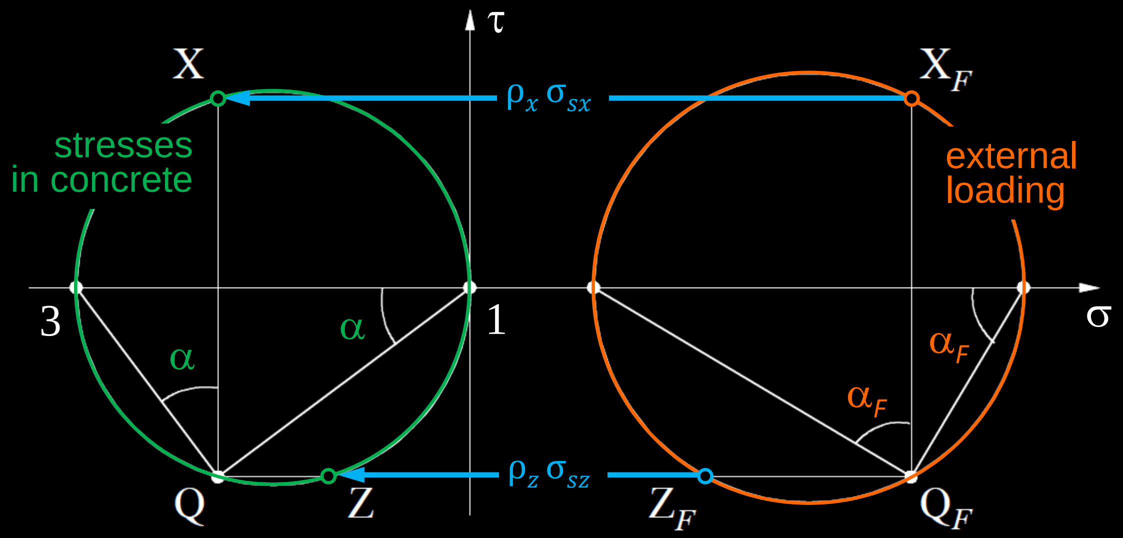
(reinforcement ratios $\rho_x = a_{sx}/h$, $\rho_z = a_{sz}/h$)

Membrane elements - Equilibrium

Equilibrium («reinforced concrete = concrete + reinforcement»)
Orthogonally reinforced element (reinforcement directions *x, z*):

Representation with Mohr's circles (straightforward for orthogonal reinforcement, since $\tau_{xzs} = 0$):





$$\sigma_x = \sigma_{xc} + \rho_x \sigma_{sx} = \sigma_{c3} \cos^2 \alpha + \rho_x \sigma_{sx}$$
$$\sigma_z = \sigma_{zc} + \rho_z \sigma_{sz} = \sigma_{c3} \sin^2 \alpha + \rho_z \sigma_{sz}$$
$$\tau_{xz} = \tau_{xzc} = -\sigma_{c3} \sin \alpha \cos \alpha$$

x, *z*: directions of reinforcement, behaviour not isotropic (also not for $a_{sx} = a_{sz}$)!

α: Principal direction of concrete compression

Membrane elements - Equilibrium

In-class exercise

Given a membrane element under the following conditions:

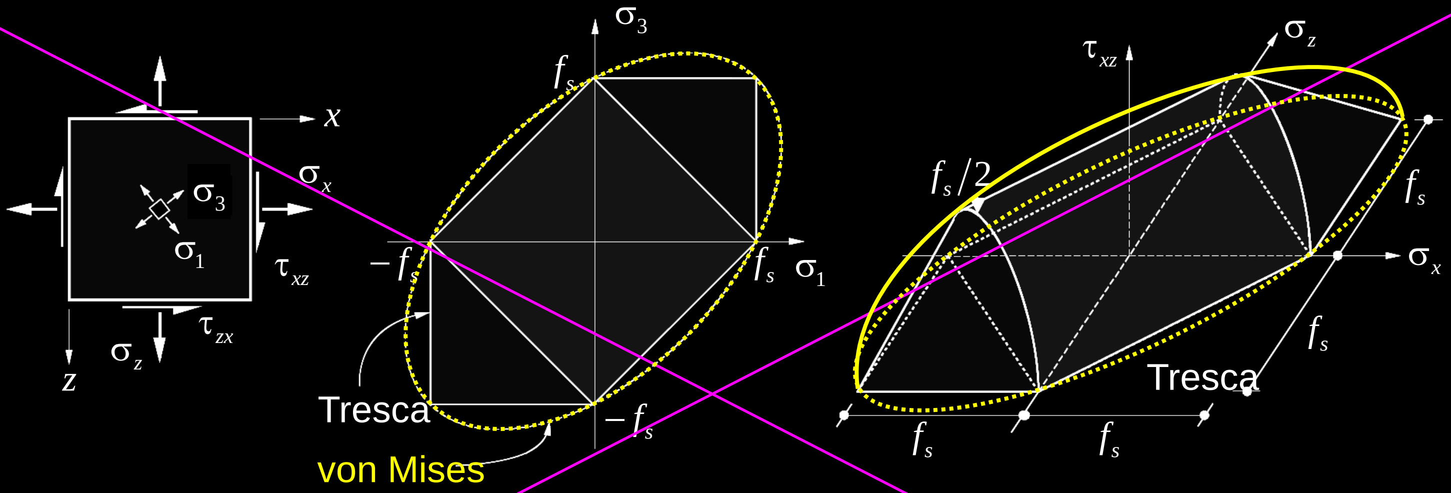
- Loading: σ_x = -2 MPa, σ_z = 5 MPa, τ_{xz} = 5 MPa
- Reinforcement: f_{sd} = 435 MPa, ρ_x = 1.3%, ρ_z = 1.9%

What is the minimum concrete strength f_c, you need to choose for the element to carry the loads?

What is the inclination of the occurring cracks?

Membrane elements - Yield conditions

Yield conditions of Tresca and v. Mises for plane stress conditions
(not suitable for reinforced concrete, not even for "isotropic reinforcement"!)



Tresca

$$Max(|\sigma_1|,|\sigma_3|,|\sigma_1-\sigma_3|)-f_s=0$$

- Principal stress plane: hexagon
- Space: two elliptical cones and connecting elliptical cylinder

v. Mises

$$\sigma_x^2-\sigma_x\sigma_z+\sigma_z^2+3\tau_{xz}^2-f_s^2=0$$

- Principal stress plane: ellipse circumscribed by the Tresca hexagon
- Space: Ellipsoid circumscribed by the yield condition of Tresca

Membrane elements - Yield conditions

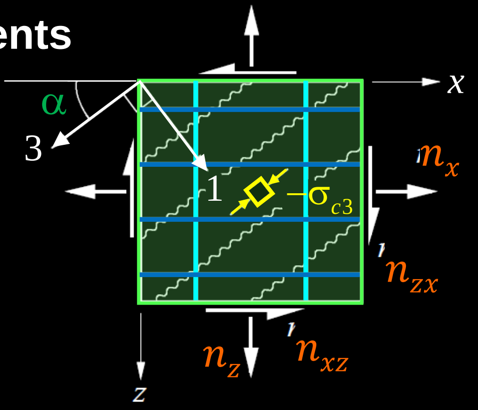
Yield condition for orthogonally reinforced membrane elements

(«Reinforced concrete = concrete + reinforcement»):

Membrane thickness *h*

Concrete and steel perfectly plastic, perfect (rigid) bond

Replace designations f_c and f_y (tension) or f_y' (compression) at design with $f_c = k_c f_{cd}$ and $f_y = - f_y' = f_{sd}$

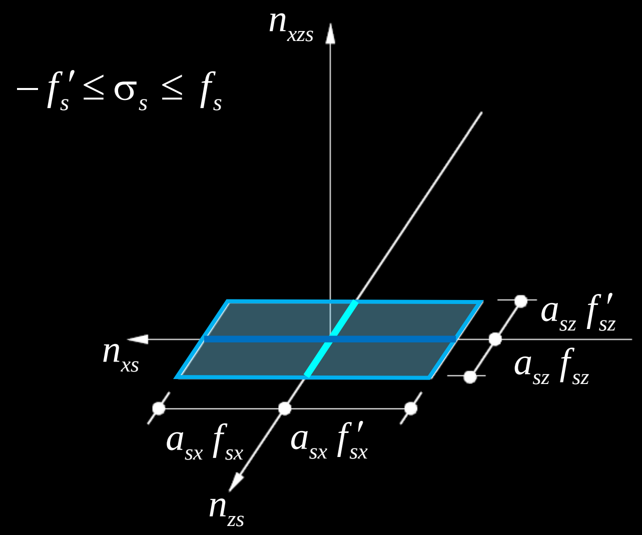


$$n_x = n_{xc} + n_{xs} = n_{xc} + a_{sx}\sigma_{sx}$$

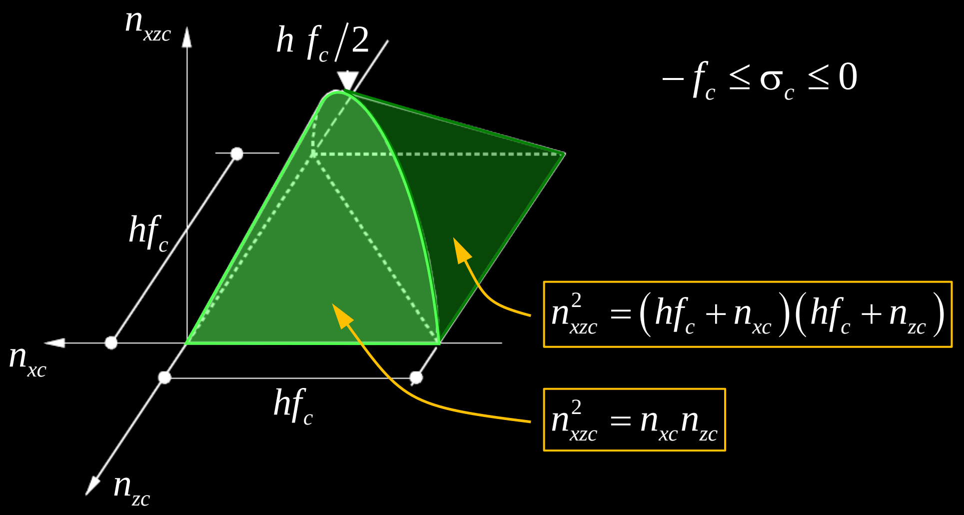
$$n_z = n_{zc} + n_{zs} = n_{zc} + a_{sz}\sigma_{sz}$$

$$n_{xz} = n_{xzc}$$

Yield condition reinforcement:
(absorbs only forces in its direction)



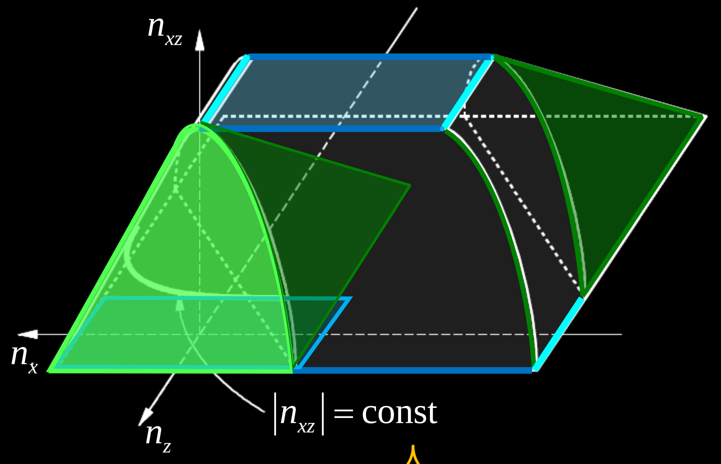
Yield condition concrete:
(homogeneous, isotropic, with f_{ct} = 0)

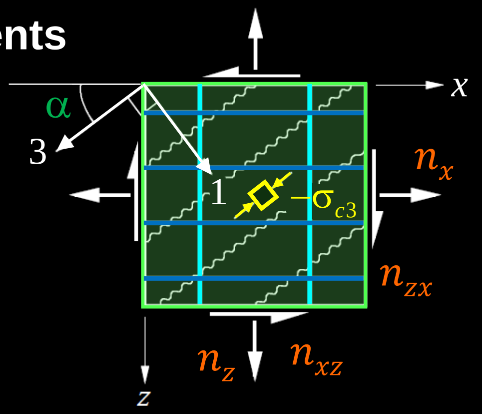


Membrane elements - Yield conditions

Yield condition for orthogonally reinforced membrane elements

Geometric linear combination concrete + reinforcement



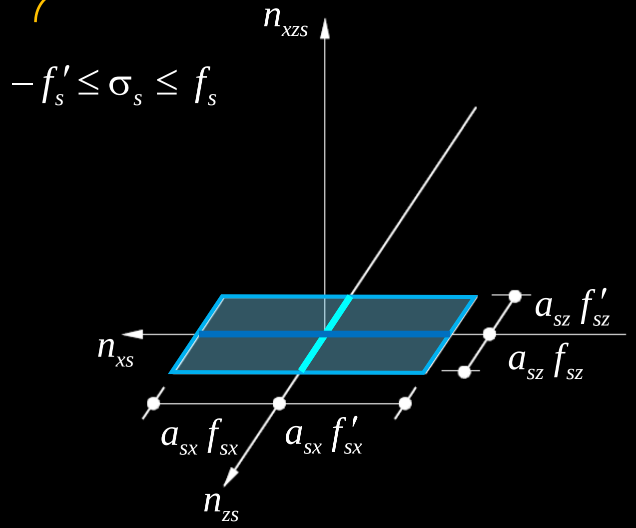


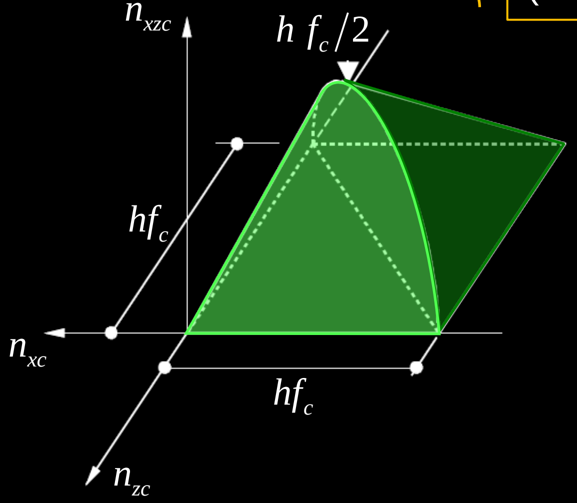
$$n_x = n_{xc} + n_{xs} = n_{xc} + a_{sx}\sigma_{sx}$$

$$n_z = n_{zc} + n_{zs} = n_{zc} + a_{sz}\sigma_{sz}$$

$$n_{xz} = n_{xzc}$$

Procedure:
Move the yield surface $Y_c = 0$ with its origin along yield surface $Y_s = 0$
(or vice versa $Y_s = 0$ along $Y_c = 0$)





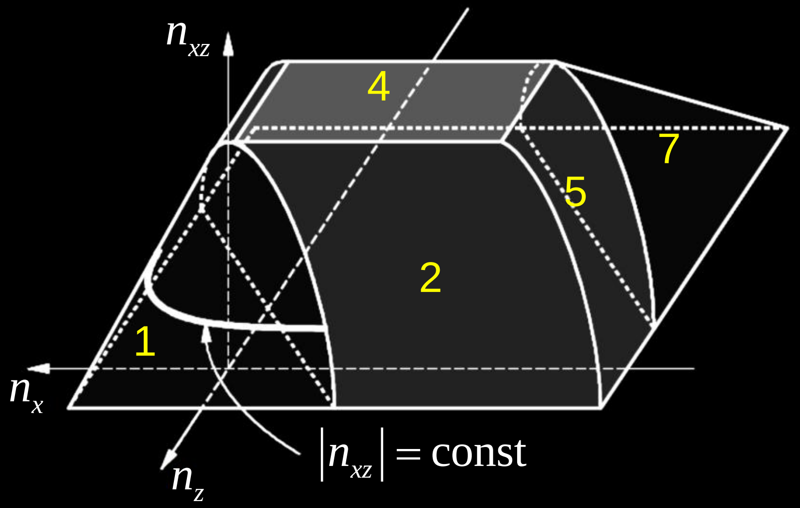
Membrane elements - Yield conditions

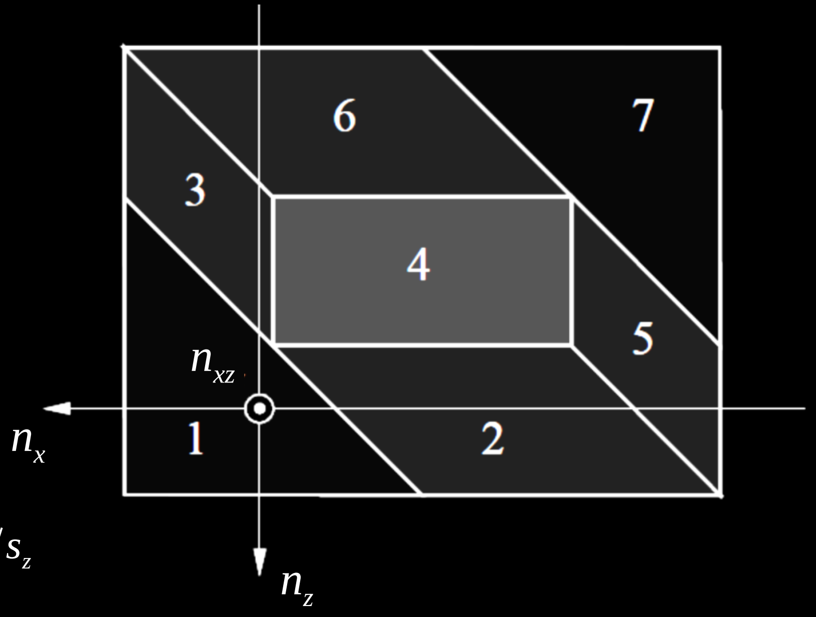
Yield conditions / yield regimes reinforced concrete

Linear combination of the yield conditions, i.e. shifting the yield condition of the concrete (origin) along the yield condition of the reinforcement.

«reinforced concrete = steel + concrete»

$$
\begin{aligned}
Y_1 &= n_{xz}^2 - (a_{sx} f_{sx} - n_x)(a_{sz} f_{sz} - n_z) &&= 0 \\
Y_2 &= n_{xz}^2 - (hf_c - a_{sz} f_{sz} + n_z)(a_{sz} f_{sz} - n_z) &&= 0 \\
Y_3 &= n_{xz}^2 - (a_{sx} f_{sx} - n_x)(hf_c - a_{sx} f_{sx} + n_x) &&= 0 \\
Y_4 &= n_{xz}^2 - \left(h\, f_c/2\right)^2 &&= 0 \\
Y_5 &= n_{xz}^2 + (a_{sx} f'_{sx} + n_x)(hf_c + a_{sx} f'_{sx} + n_x) &&= 0 \\
Y_6 &= n_{xz}^2 + (hf_c + a_{sz} f'_{sz} + n_z)(a_{sz} f'_{sz} + n_z) &&= 0 \\
Y_7 &= n_{xz}^2 - (hf_c + a_{sx} f'_{sx} + n_x)(hf_c + a_{sz} f'_{sz} + n_z) &&= 0
\end{aligned}
$$





SN: Reinforcement areas per unit length in *x*- and *z*-direction $a_{sx} = A_{sx}/s_x$ $a_{sz} = A_{sz}/s_z$

Membrane elements - Yield conditions

Yield conditions / yield regimes reinforced concrete

Y_1: Both reinforcements yield in tension
($\sigma_{sx} = f_{sx}$, $\sigma_{sz} = f_{sz}$, $0 \geq \sigma_{c3} \geq -f_c$)

Y_2: z-reinforcement yields in tension, concrete crushes
($\sigma_{sz} = f_{sz}$, $\sigma_{c3} = -f_c$, $-f'_{sx} \leq \sigma_{sx} \leq f_{sx}$)

Y_3: x-reinforcement yields in tension, concrete crushes
($\sigma_{sx} = f_{sx}$, $\sigma_{c3} = -f_c$, $-f'_{sz} \leq \sigma_{sz} \leq f_{sz}$)

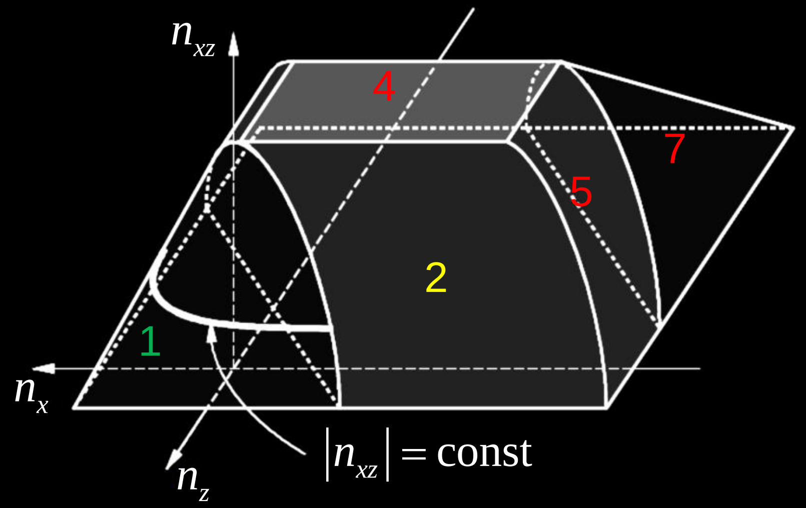
Y_4: Concrete crushes
($\sigma_{c3} = -f_c$, $-f'_{sx} \leq \sigma_{sx} \leq f_{sx}$, $-f'_{sz} \leq \sigma_{sz} \leq f_{sz}$)

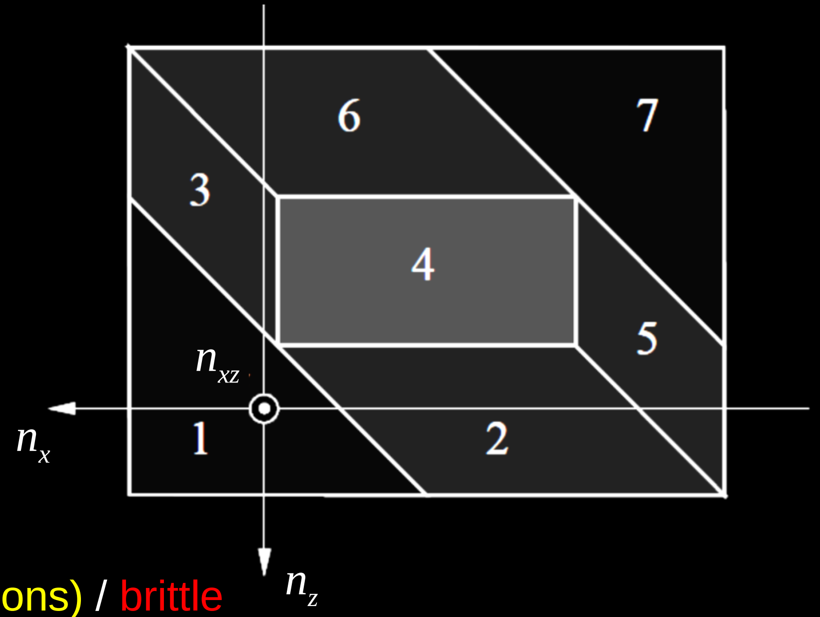
Y_5: x-reinforcement yields in compression, concrete crushes
($\sigma_{sx} = -f'_{sx}$, $\sigma_{c3} = -f_c$, $-f'_{sz} \leq \sigma_{sz} \leq f_{sz}$)

Y_6: z-reinforcement yields in compression, concrete crushes
($\sigma_{sz} = -f'_{sz}$, $\sigma_{c3} = -f_c$, $-f'_{sx} \leq \sigma_{sx} \leq f_{sx}$)

Y_7: Both reinforcements yield in compression, concrete crushes
($\sigma_{sx} = -f'_{sx}$, $\sigma_{sz} = -f'_{sz}$, $\sigma_{c3} = -f_c$)

(mean concrete principal stress also negative)





SN: failure type: very ductile / ductile (except for very flat stress field inclinations) / brittle

Membrane elements - Yield conditions

Strain increments and principal compressive direction

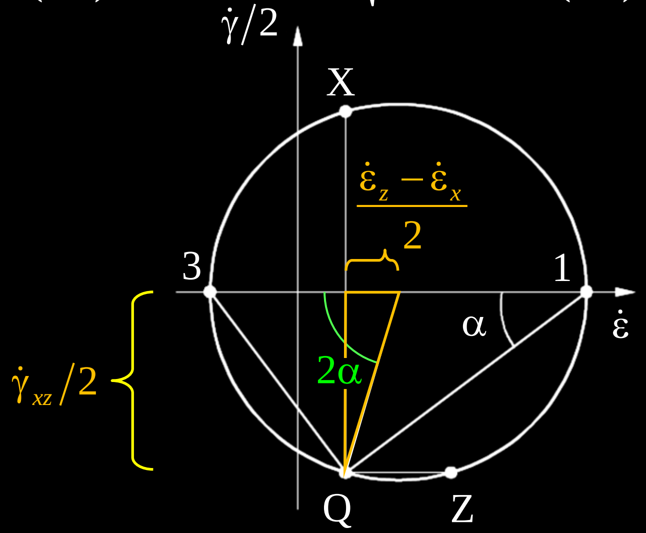
Strain increments are proportional to the components of the outer normal to the yield surface (gradient) in the respective point of the yield surface ($\kappa \geq 0$: any factor):

$$\dot{\varepsilon}_x = \kappa \frac{\partial Y}{\partial n_x}, \qquad \dot{\varepsilon}_z = \kappa \frac{\partial Y}{\partial n_z}, \qquad \dot{\gamma}_{xz} = \kappa \frac{\partial Y}{\partial n_{xz}}$$

Inclination α of the principal compressive direction 3 with respect to the *x*-axis follows from the Mohr's circle of plastic strain increments (principal strain direction = principal compressive direction in concrete):

$$\cot 2\alpha = \frac{\dot{\varepsilon}_z - \dot{\varepsilon}_x}{\dot{\gamma}_{xz}} \quad \text{mit} \quad \cot\alpha = \frac{\cos(2\alpha)+1}{\sin(2\alpha)} = \cot(2\alpha) + \sqrt{\frac{\cos^2(2\alpha)+\sin^2(2\alpha)}{\sin^2(2\alpha)}}$$

$$\cot\alpha = \frac{\dot{\varepsilon}_z - \dot{\varepsilon}_x}{\dot{\gamma}_{xz}} + \sqrt{\left(\frac{\dot{\varepsilon}_z - \dot{\varepsilon}_x}{\dot{\gamma}_{xz}}\right)^2 + 1}$$



$$\begin{aligned}
Y_1 &: \cot^2\alpha = (a_{sx} f_{sx} - n_x)/(a_{sz} f_{sz} - n_z) \\
Y_2 &: \cot^2\alpha = (hf_c - a_{sz} f_{sz} + n_z)/(a_{sz} f_{sz} - n_z) \\
Y_3 &: \cot^2\alpha = (a_{sx} f_{sx} - n_x)/(hf_c - a_{sx} f_{sx} + n_x) \\
Y_4 &: \cot^2\alpha = 1 \\
Y_5 &: \cot^2\alpha = -(a_{sx} f'_{sx} + n_x)/(hf_c + a_{sx} f'_{sx} + n_x) \\
Y_6 &: \cot^2\alpha = -(hf_c + a_{sz} f'_{sz} + n_z)/(a_{sz} f'_{sz} + n_z) \\
Y_7 &: \cot^2\alpha = -(hf_c + a_{sx} f'_{sx} + n_x)/(hf_c + a_{sz} f'_{sz} + n_z)
\end{aligned}$$

Membrane elements - Yield conditions

Dimensioning of reinforcement

Design practice: Usually Regime 1 (ductile failure type; both reinforcements yield before concrete crushing, concrete remains "elastic" = undamaged).

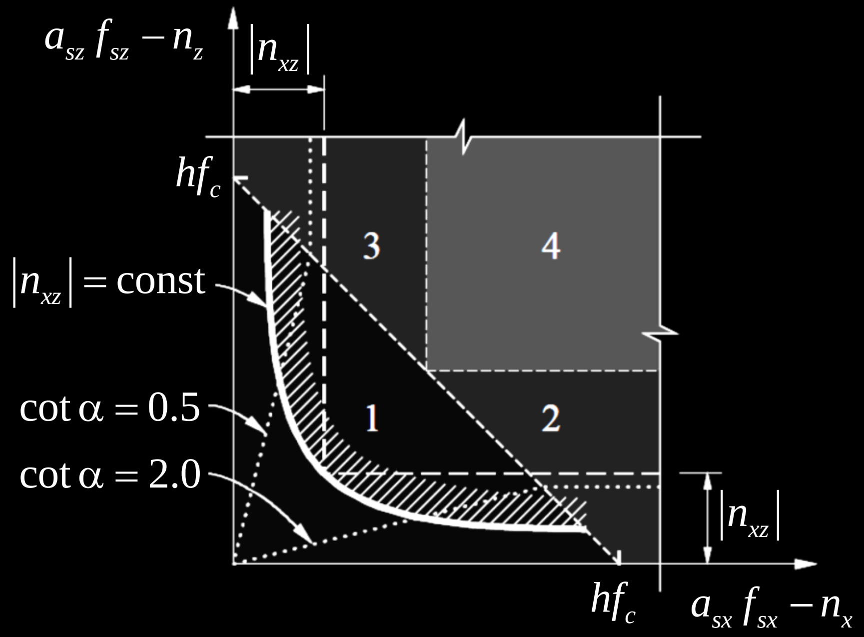
Yield condition for Regime 1 in parametric form (→ direct dimensioning):

$$Y_1 = n_{xz}^2 - (a_{sx} f_{sx} - n_x)(a_{sz} f_{sz} - n_z) = 0$$

$$k = \cot\alpha \longrightarrow \begin{array}{l} a_{sx} f_{sx} \geq n_x + k|n_{xz}| \\ a_{sz} f_{sz} \geq n_z + k^{-1}|n_{xz}| \end{array}$$

Yield condition in Regime 1 is governing (no concrete crushing) if:

$$hf_c \geq a_{sx} f_{sx} + a_{sz} f_{sz} - (n_x + n_z)$$



SN:

→ Value of f_c see next chapter. Approximation according to SIA 262: $f_c = k_c f_{cd}$ (with $k_c = 0.55$)

→ Inclination of the concrete stress field in Regime 1 follows from: $\cot^2\alpha = (a_{sx} f_{sx} - n_x)/(a_{sz} f_{sz} - n_z)$

→ Value $k = \cot\alpha$ can theoretically be freely chosen, in design standards often limited by the condition $0.5 \leq k \leq 2$

→ Use of $k = 1$, i.e. $\alpha = 45°$: "linearised yield conditions", implemented in many FE programs. Safe dimensioning, but this is just one of many possibilities (possibly strongly on the safe side)

Membrane elements - Yield conditions

Web crushing (Regime 2)

If the condition $hf_c \geq a_{sx} f_{sx} + a_{sz} f_{sz} - (n_x + n_z)$ is not satisfied, a failure type where the concrete fails under compression (crushes) is governing.

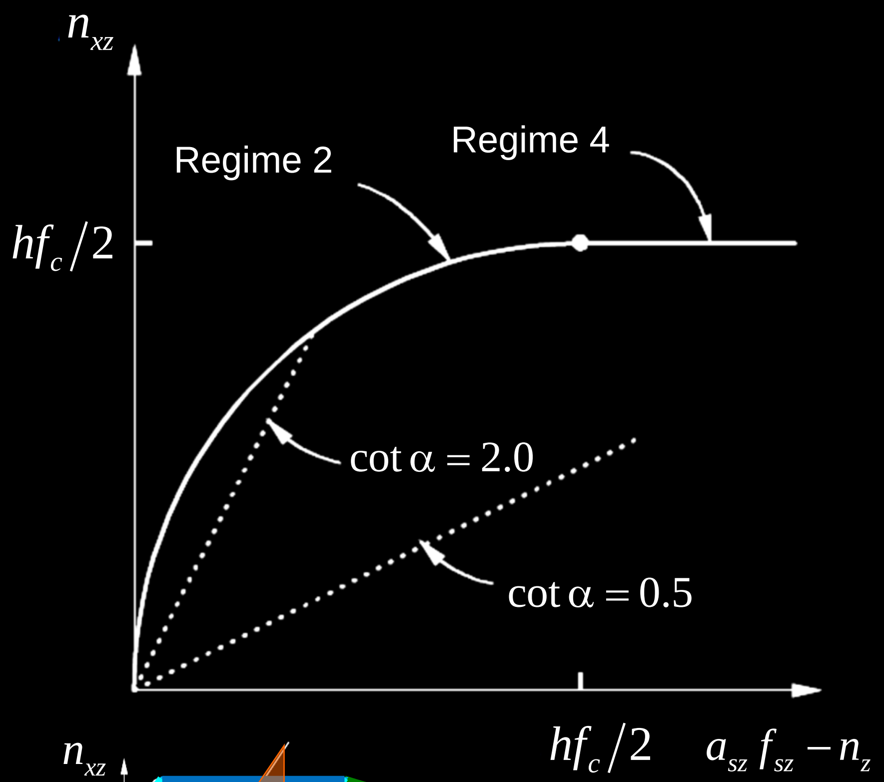
Regime 2 is also of particular practical relevance. It applies if the condition $a_{sx} f_{sx} - n_x > a_{sz} f_{sz} - n_z$ is met.

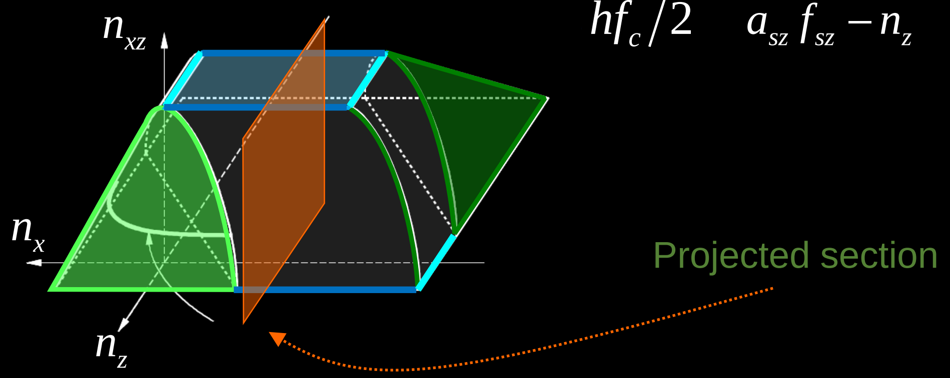
→ Type of failure: Yielding of the *z*-reinforcement with simultaneous concrete compressive failure, called *web crushing*.

→ The corresponding limitation of the shear resistance of the membrane element can be represented as a quarter circle.

→ Limitations for cot α correspond to straight lines in the diagram

SN: Figure on the right = projection of the yield surface to the plane (n_z, n_{xz}), shifted by a_{sz} f_{yz} (n_x = generalised reaction)





Membrane elements – Yield conditions

In-class exercise

Given a membrane element under the following conditions:

- Loading: σ_x = -2 MPa, σ_z = 5 MPa, τ_{xz} = 5 MPa
- Concrete: $k_c \cdot f_{cd}$ = 11 MPa
- Reinforcement: f_{sd} = 435 MPa

What is a possible reinforcement ratio, ρ_x and ρ_z, for the element to carry the loads?

Are other ratios possible?

To which yield regime do the chosen ratios belong?

Membrane elements - Yield conditions

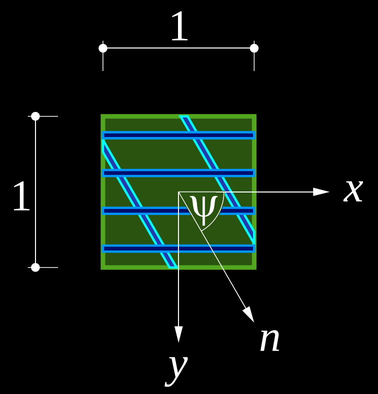
Skew reinforcements

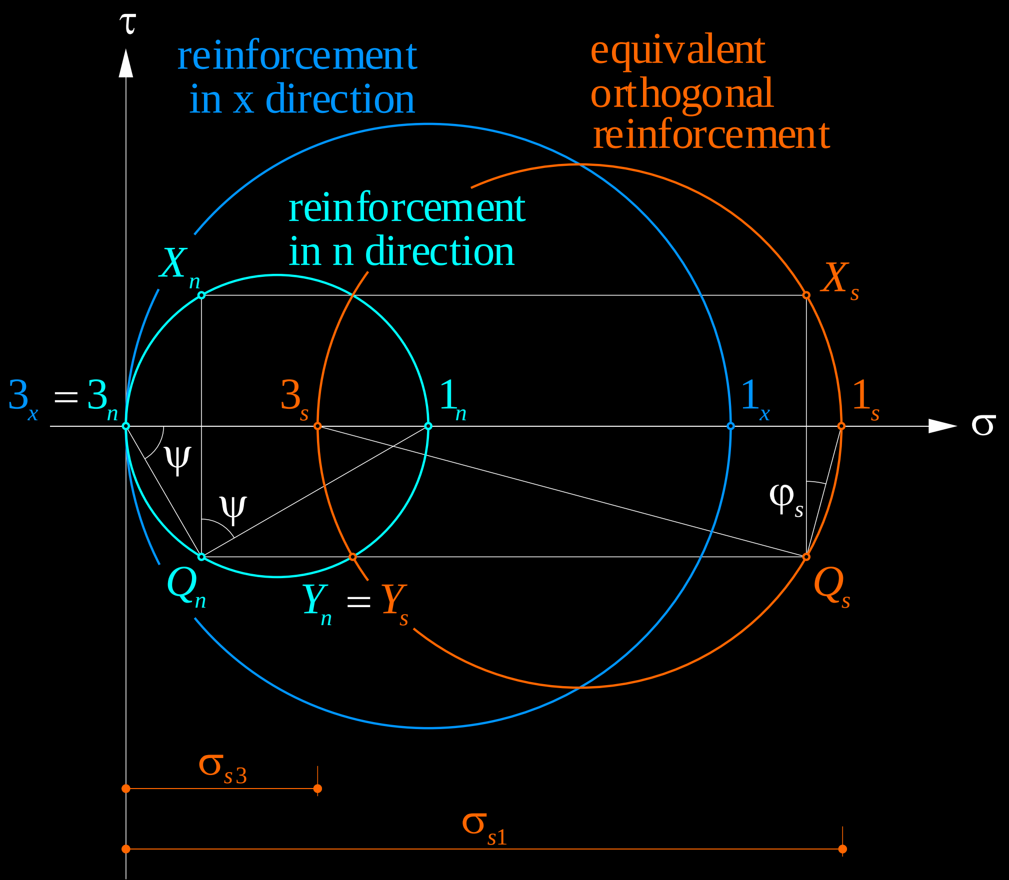
Calculation of the equivalent orthogonal reinforcement (representation with Mohr's circles)

$$\sigma_{1s,3s} = \frac{1}{2}\left[\sum_{k=1}^{m}\rho_k f_{syk} \pm \sqrt{\left(\sum_{k=1}^{m}\rho_k f_{syk}\cos(2\psi_k)\right)^2 + \left(\sum_{k=1}^{m}\rho_k f_{syk}\sin(2\psi_k)\right)^2}\right]$$

$$\tan(2\varphi_s) = \frac{\sum_{k=1}^{m}\rho_k f_{syk}\sin(2\psi_k)}{\sum_{k=1}^{m}\rho_k f_{syk}\cos(2\psi_k)}$$

(see dissertation Seelhofer, 2009)

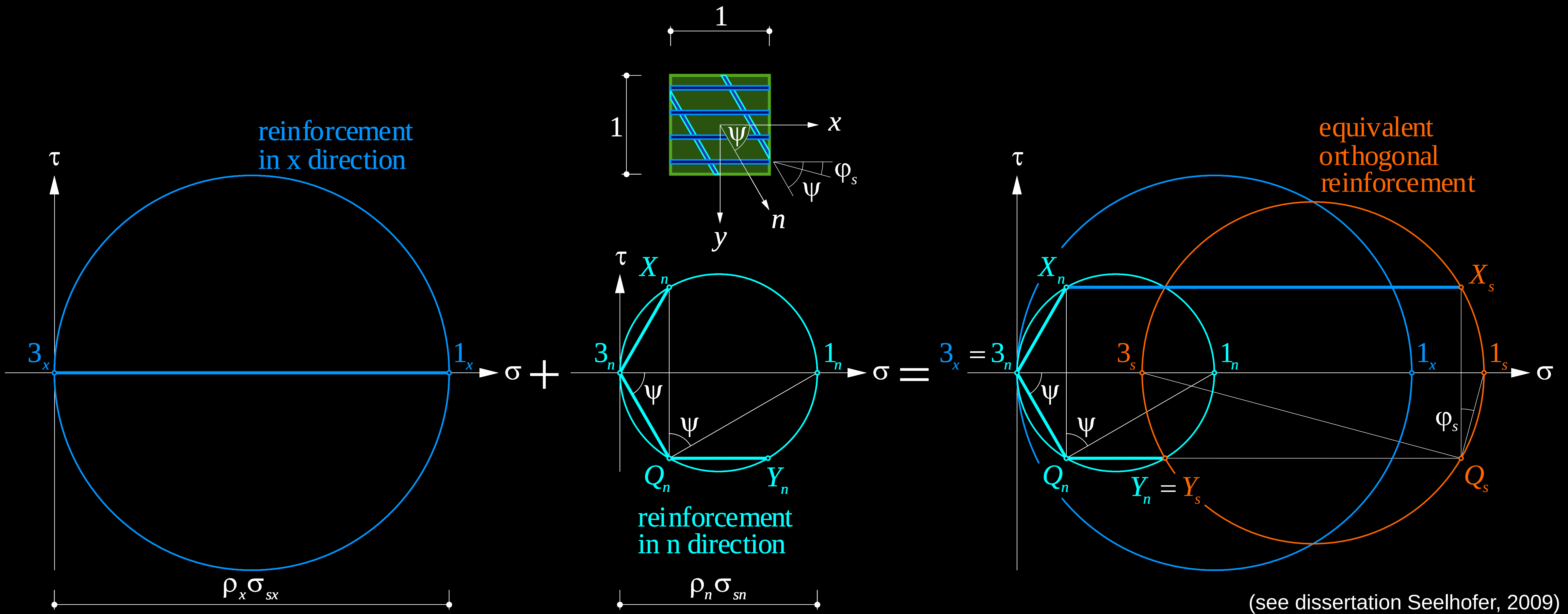




Membrane elements - Yield conditions

Skew reinforcements

Graphical determination of the equivalent orthogonal reinforcement

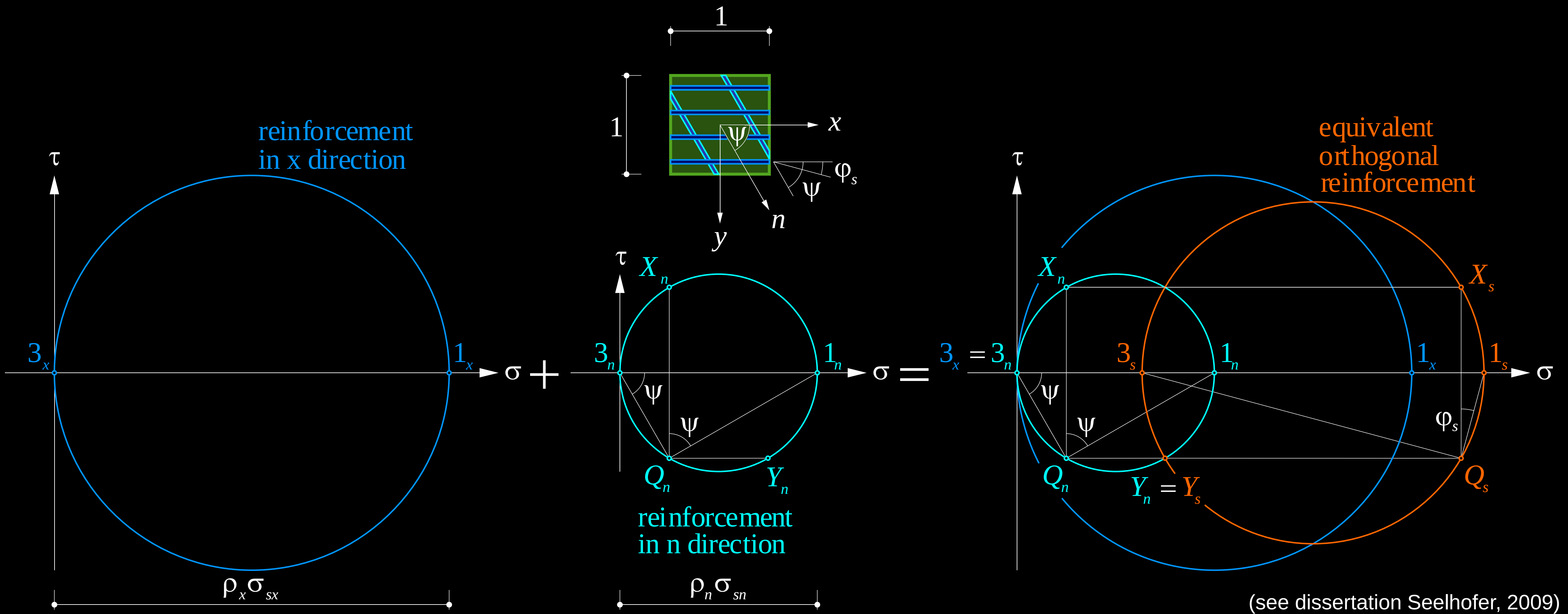


(see dissertation Seelhofer, 2009)

Membrane elements - Yield conditions

Skew reinforcements

Graphical determination of the equivalent orthogonal reinforcement

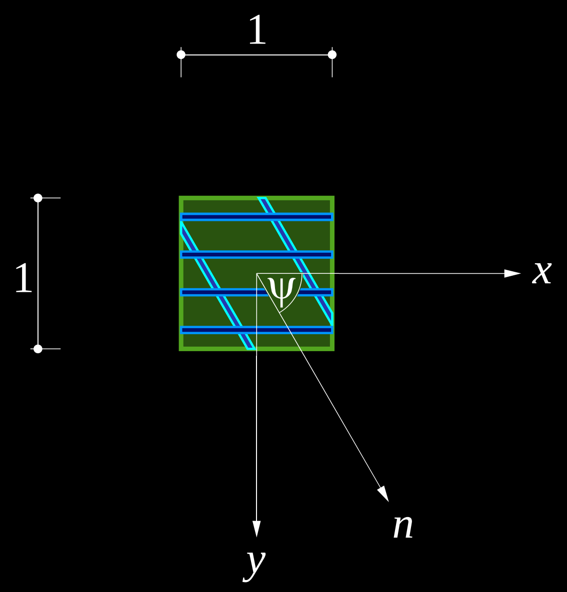


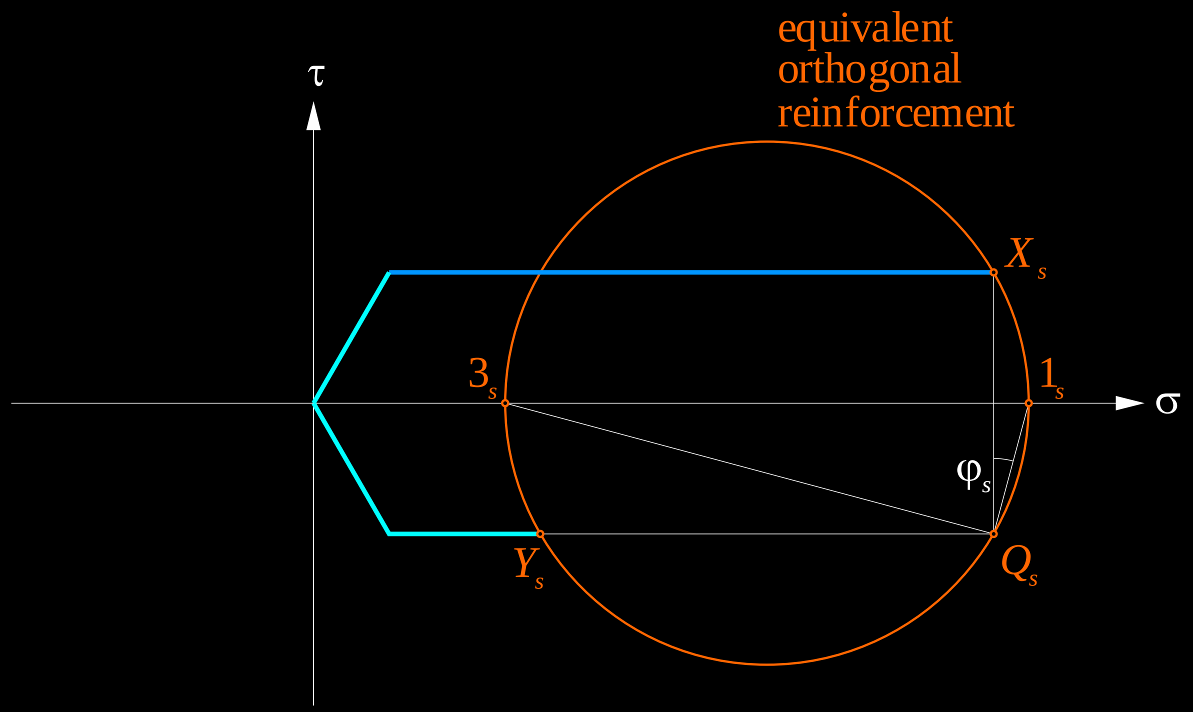
(see dissertation Seelhofer, 2009)

Membrane elements - Yield conditions

Skew reinforcements

Equilibrium («applied stress = concrete + reinforcement»)



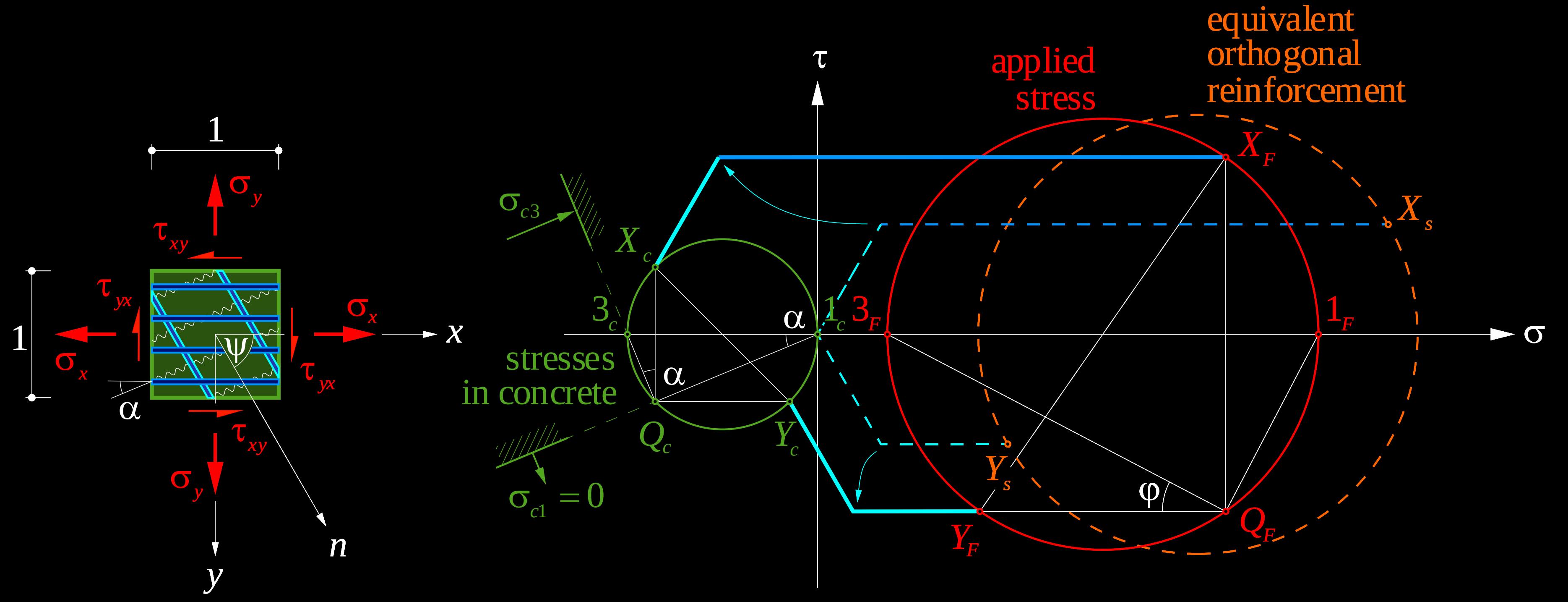


(see dissertation Seelhofer, 2009)

Membrane elements - Yield conditions

Skew reinforcements

Equilibrium («applied stress = concrete + reinforcement»)



(see dissertation Seelhofer, 2009)

Membrane elements - Yield conditions

Skew reinforcements

With skew reinforcements, the determination of the yield conditions becomes mathematically significantly more complicated. For Regime 1, for example, the yield condition becomes:

$$Y_1 = \left(\tau_{xy} - \rho_n f_{sn} \sin\psi \cos\psi\right)^2 - \underbrace{\left(\rho_x f_{sx} + \rho_n f_{sn} \cos^2\psi - \sigma_x\right)}_{\geq 0} \underbrace{\left(\rho_n f_{sn} \sin^2\psi - \sigma_y\right)}_{\geq 0} = 0$$

$$\sigma_\xi = \sigma_x \sin\psi + \sigma_y \cos\psi \cot\psi - 2\tau_{xy} \cos\psi$$

$$\sigma_\eta = \frac{\sigma_y}{\sin\psi}$$

$$\tau_{\xi\eta} = \tau_{\eta\xi} = \tau_{xy} - \sigma_y \cot\psi$$

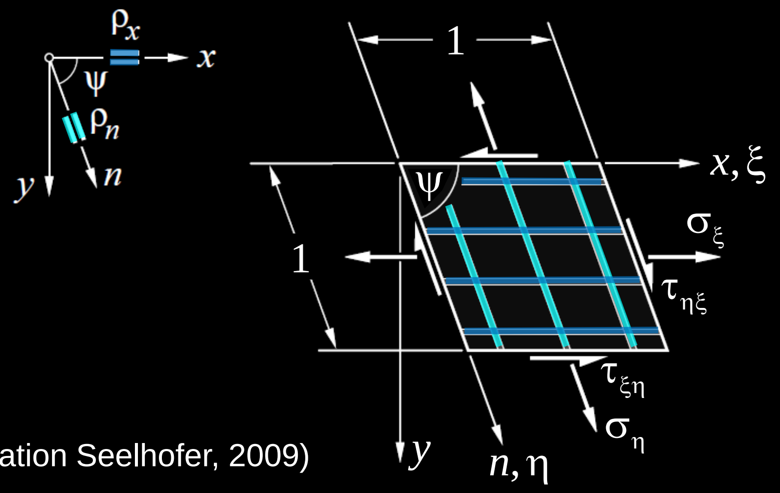
With loads transformed to skew coordinates, the relationships for the direct design of the reinforcement in Regime 1 follow from:

$$k = \left|\cos\psi + \sin\psi \cot\theta\right|$$

$$\rho_x f_{sx} \geq \frac{1}{\sin\psi}\left(\sigma_\xi + k\left|\tau_{\xi\eta}\right|\right) \qquad \rho_n f_{sn} \geq \frac{1}{\sin\psi}\left(\sigma_\eta + k^{-1}\left|\tau_{\xi\eta}\right|\right)$$

and for checking the concrete compressive strength:

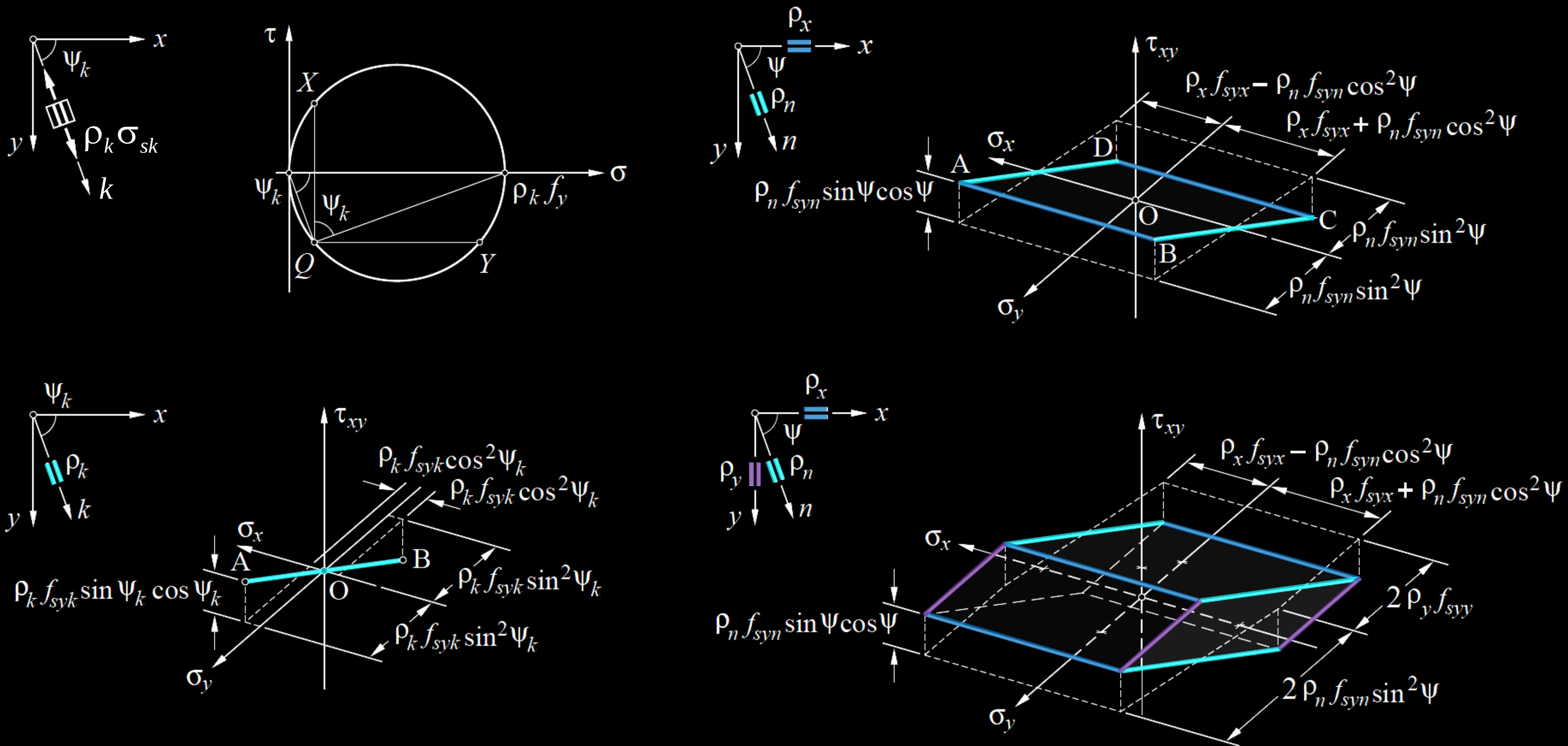
$$\sigma_{c3} = \frac{1}{\sin\psi}\left[2\tau_{\xi\eta} \cos\psi - \left|\tau_{\xi\eta}\right|\left(k + k^{-1}\right)\right] \geq -f_c$$



(see dissertation Seelhofer, 2009)

Membrane elements - Yield conditions

Skew reinforcements



(from dissertation Seelhofer, 2009)

Membrane elements - Yield conditions

Skew reinforcements

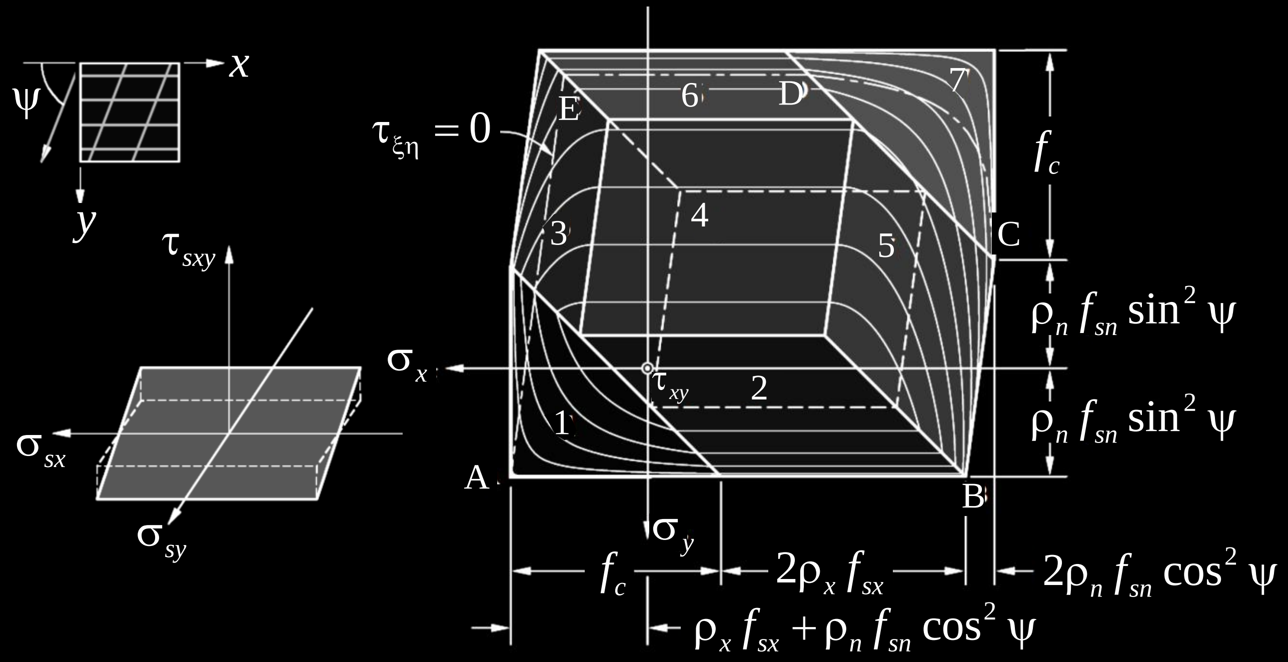
→ Two oblique = skew reinforcement layers: flat, parallelogram-shaped yield figure of the reinforcement, inclined relative to the σ_{sx}-σ_{sy}–plane.

→ Three oblique = skew reinforcement layers: Parallelepiped, can carry load without concrete (imagine a hinged connection of reinforcing bars)

→ Yield surface of a reinforced concrete membrane element results from translation of the concrete yield surface with its origin on the yield surface of the reinforcement (geometric linear combination)

Alternatively, *n* inclined reinforcements can be transformed to the orthogonal *x-z* coordinate system to determine an equivalent orthogonal reinforcement. The yield conditions of the orthogonal reinforcement can then be used, whereby the applied stress is to be transformed into the direction of the equivalent orthogonal reinforcement.

(see dissertation Seelhofer, 2009)



(from dissertation Seelhofer, 2009)

$$\sigma_{sx} = \sum_i \sigma_{si} \cos^2\psi_i$$

$$\sigma_{sy} = \sum_i \sigma_{si} \sin^2\psi_i$$

$$\tau_{sxy} = \sum_i \sigma_{si} \sin\psi_i \cos\psi_i$$

$$\sigma_{s1,2} = \sum_i \sigma_{si} \cos^2\psi_i \frac{\sigma_{sx}+\sigma_z}{2} \pm \frac{\sqrt{\left(\sigma_{sx}-\sigma_{sy}\right)^2 + 4\tau_{sxy}^2}}{2}$$

$$\varphi_{s1} = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{sxy}}{\sigma_{sx}-\sigma_{sz}}\right)$$

Membrane elements - Yield conditions

Summary

→ «Reinforced concrete = steel + concrete»

→ Well suited for design on the basis of FE calculations (limit values)

→ Dimensioning for Regime 1, verification of the prerequisites (no concrete crushing, compressive strength, see compabtibility and deformation capacity)

→ Safe design possible with linearised yield conditions

→ Regime 2 important for beams, "web crushing failure".

→ Skew reinforcements can be treated in the same way (but mathematically more complicated)