2 In-plane loading – walls and beams

2.2 Stress fields with prestressing

Learning objectives

Within this chapter, the students are able to:

- recognise the suitability of treating prestressing as equivalent forces for the analysis of 2D and 3D structures.
- create simplified stress fields and strut-and-tie models including prestressing as anchorage and deviation forces.

2 In-plane loading – walls and beams

2.2 Stress fields with prestressing

Basis

Repetition from Stahlbeton II (Vorspannung)

Prestressing of framed structures (SB II)

- Prestressing = controlled application of forces to the structure or building component
- Anchorage, deviation and friction forces act between the prestressing steel and the structural member without prestressing tendon.
- Prestressing generates a residual stress state and causes deformations of the structure.
- In statically indeterminate systems, restraint forces result from restrained deformations.
- The load-bearing behaviour of prestressed beams can be investigated analogously to passively reinforced structures by means of cross-sectional analyses. Note that the strain difference $\Delta\varepsilon$ between prestressing steel and concrete is "frozen" during the injection of the prestressing duct.
- There are two alternative possibilities for treating prestressing:

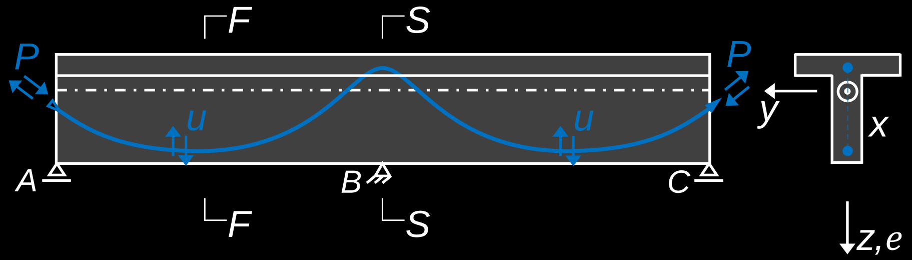
Residual stress state acting on the entire structure or building component including the prestressing tendon	**Anchorage, deviation and friction forces** acting on the structural member without prestressing tendon
Also referred to as **prestressing treated as resistance**	Also referred to as **prestressing treated as load**

- Both possibilities lead to the same result (with consistent application). The only difference are the boundaries of the system.
- Depending on the specific problem, one or the other option is more convenient.

Prestressing of framed structures (SB II)

Treatment of prestressing / definition of system under consideration (2)

Entire structure / element

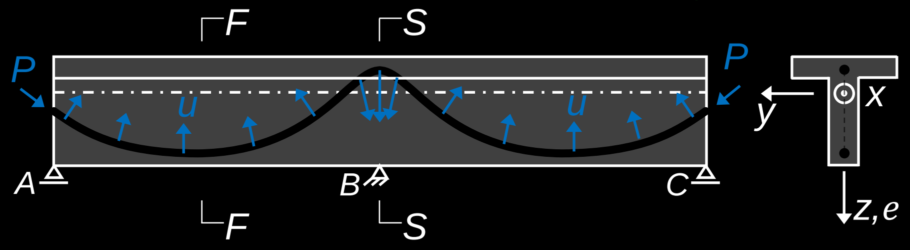


Prestressing causes a residual stress state in the cross-sections: The tensile force in the tendon is in equilibrium with the (compression) forces in the reinforced concrete section. The residual stress state corresponds to strains and curvatures → deformations of the structure.

The internal actions contain only the restraint actions $M_{ps}(P)$, $V_{ps}(P)$, $N_{ps}(P)$. Actions on the total cross-section :

$$
\begin{aligned}
M &= M_{g,q} + M_{ps} \\
V &= V_{g,q} + V_{ps} \\
N &= N_{g,q} + N_{ps}
\end{aligned}
$$

Structure / element without prestressing tendon



The prestressing corresponds to anchorage, deviation and friction forces acting on the structure without the tendon. These loads result in the so-called internal actions due to prestressing $M_c(P)$, $V_c(P)$, $N_c(P)$ and deformations (compatible with the arrangement of supports).

The internal actions contain the total internal forces due to prestressing $M_c(P)$, $V_c(P)$, $N_c(P)$. Actions on the cross-section without the prestressing tendon:

$$
\begin{aligned}
M_c &= M_{g,q} + M_c(P) &&= M_{g,q} + M_{ps} - P\cos\beta_p \cdot e \\
V_c &= V_{g,q} + V_c(P) &&= V_{g,q} + V_{ps} - P\sin\beta_p \\
N_c &= N_{g,q} + N_c(P) &&= N_{g,q} + N_{ps} - P\cos\beta_p
\end{aligned}
$$

$$
\begin{Bmatrix}
-P\cos\beta_p \cdot e \approx -P \cdot e \\
-P\sin\beta_p \\
-P\cos\beta_p \approx -P
\end{Bmatrix}
$$

2 In-plane loading – walls and beams

2.2 Stress fields with prestressing

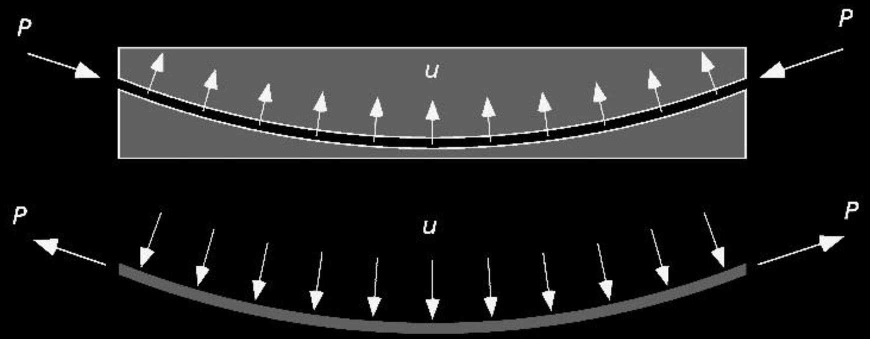
Particularities in membrane, slab and shell structures

Additions to Stahlbeton II (Vorspannung)

Prestressing of membrane, slab and shell structures

Treatment of prestressing in membrane, slab and shell structures
The treatment of prestressing as a residual stress state in the total system is deemed to fail in two-dimensional or three-dimensional structures because the residual stress state due to prestressing cannot be uniquely determined (internal static indeterminacy, unknown spreading of compressive force, reference cross-section unclear, etc.).

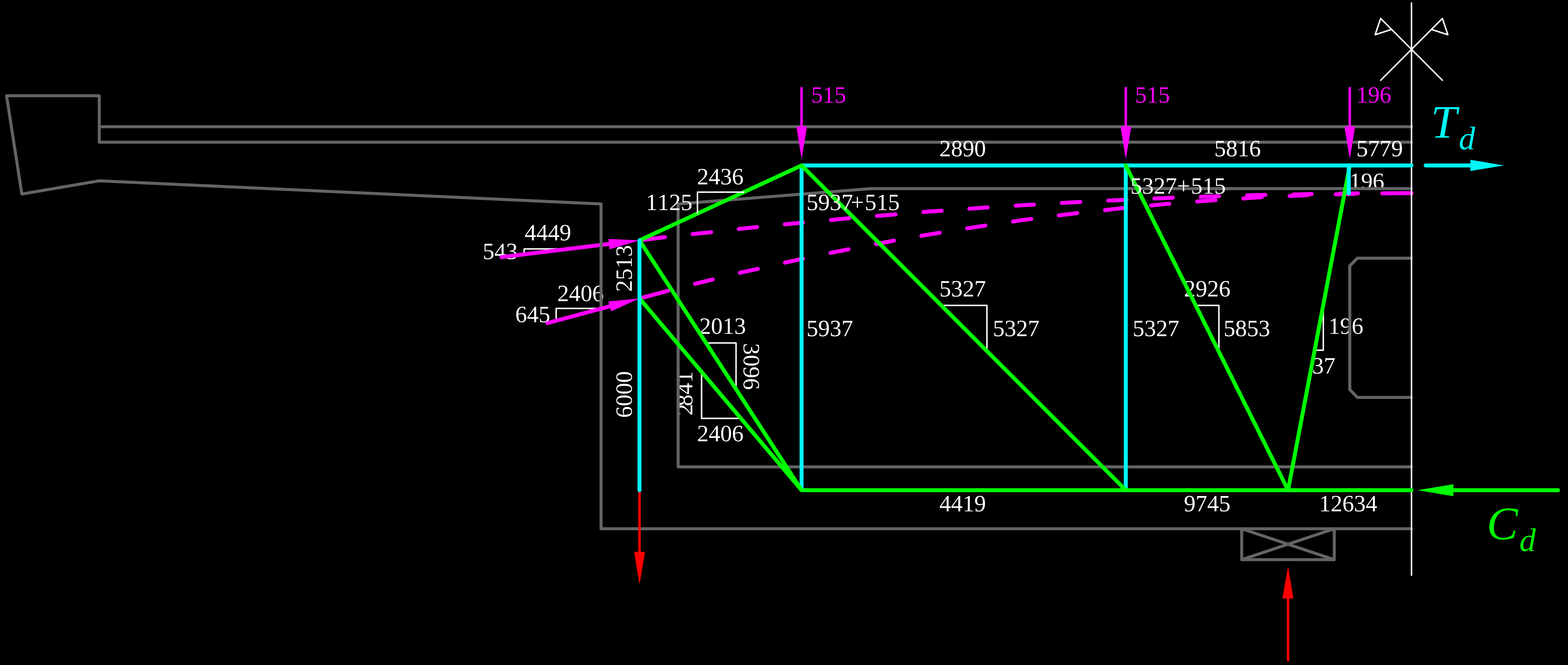
The treatment of the prestressing as anchorage, deviation and friction forces on the subsystem "reinforced concrete structure without prestressing", on the other hand, is possible without any problems. This also allows to visualise the force flow (using stress fields, strut-and-tie models).



In design practice, the anchorage, deviation and friction forces are usually determined considering the prestressing force without any increase. The increase in the prestressing force at ULS could theoretically be investigated with suitable considerations (e.g. stress fields), but the effort is not worthwhile usually (small influence, since the initial preload $0.7f_{pk}$ is only slightly (approx. 3-7%) lower than the design value of the yield stress $f_{p0.1k}$ /1.15). It is more relevant to estimate the influence of long-term losses on the prestressing force.

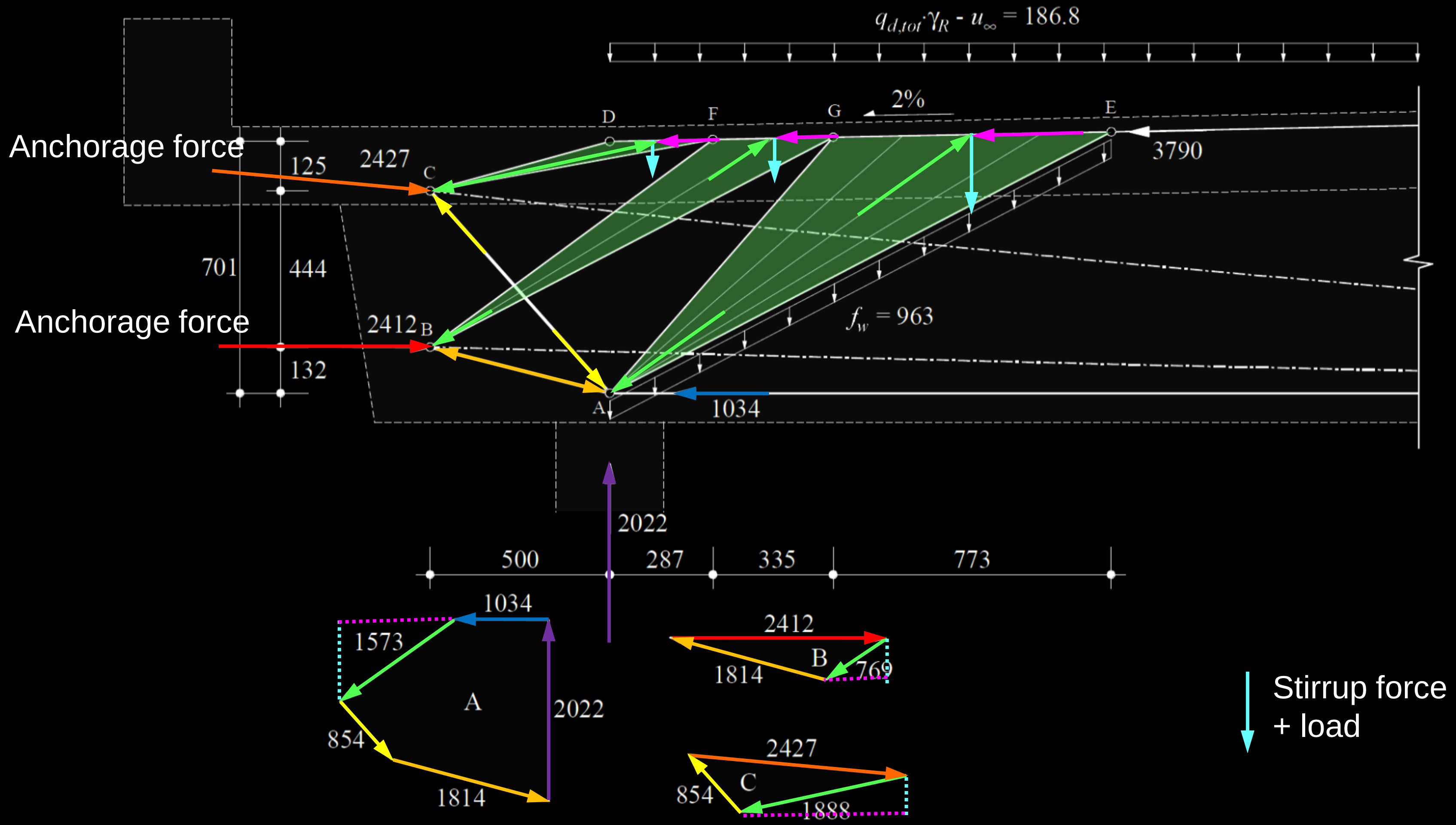
Prestressing of membrane, slab and shell structures

Treatment of prestressing in membrane structures



Prestressing of membrane, slab and shell structures

Treatment of prestressing in membrane structures
Example neglecting the increase in prestressing force [Marti und Stoffel, 1999]



Prestressing of membrane, slab and shell structures

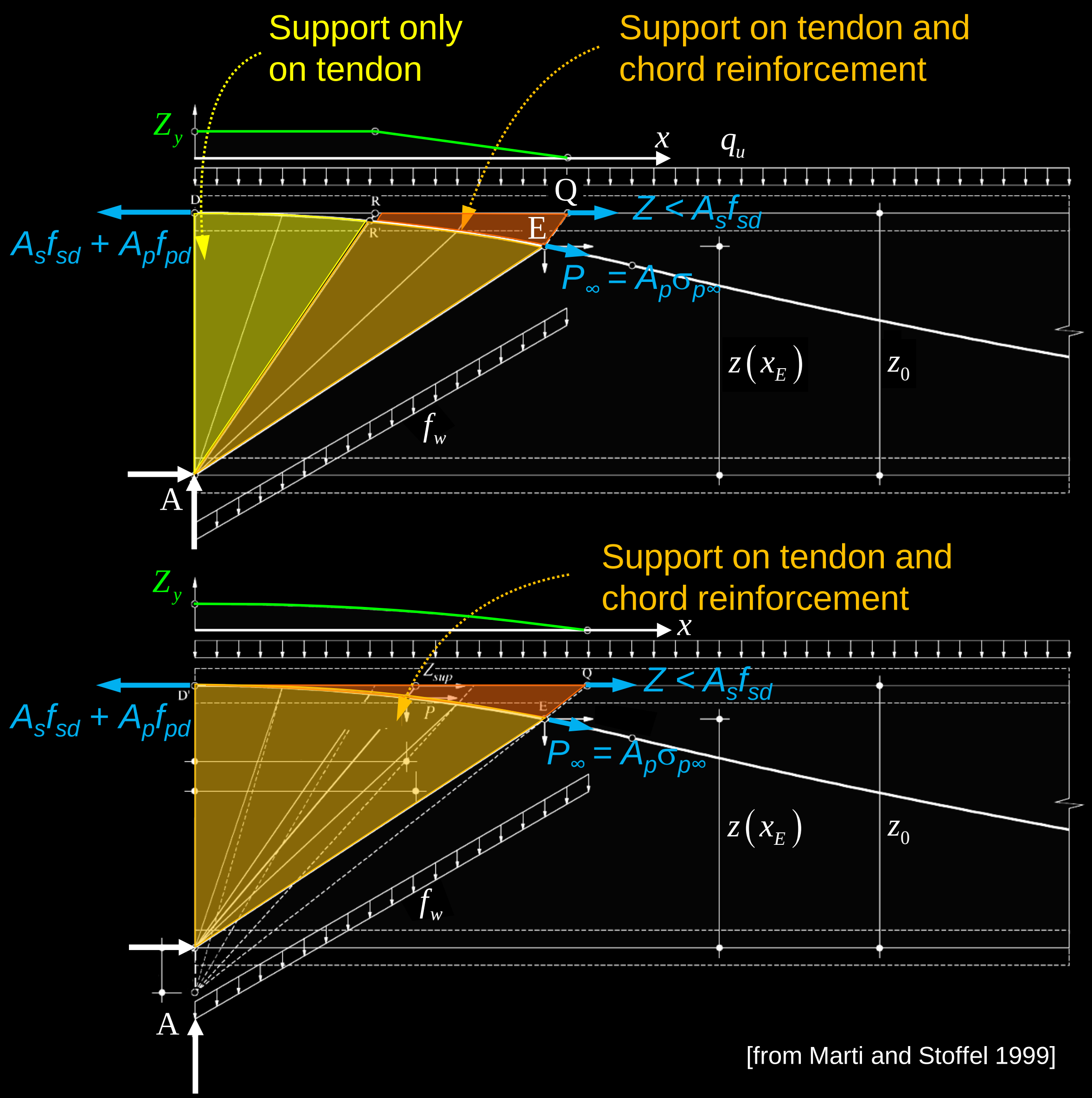
Chord force distribution for centred fan with prestressing

→ The distribution of the chord forces between conventional reinforcement (Z) and tendon (P) is not directly determined from equilibrium.

→ Plausible assumption of force distribution:
- Increase in prestressing only in the decompression region (the assumption of P_∞ at the edge of the fan is reasonable, i.e. increase of the prestressing only in the fan area)
- In normal conditions, the fan can never be supported only by the tendon, but it is partially supported by the tendon and the conventional reinforcement.

→ Possible solution see figure on the top: Assumption that in the first area the fan is supported only by the tendon; in the second area it is supported by the tendon and the conventional reinforcement of the tension chord. The position of the points E, Q and the value of f_w can be determined from equilibrium.

→ Alternative solution in the lower figure: fictitious fan for determining the tension chord force (parabolic); the fan is supported over its entire length both on tendon and on the conventional reinforcement. The geometry can also be determined from equilibrium. The stirrup forces are different above and below the tendon.



[from Marti and Stoffel 1999]

2 In-plane loading – walls and beams

2.2 Stress fields with prestressing

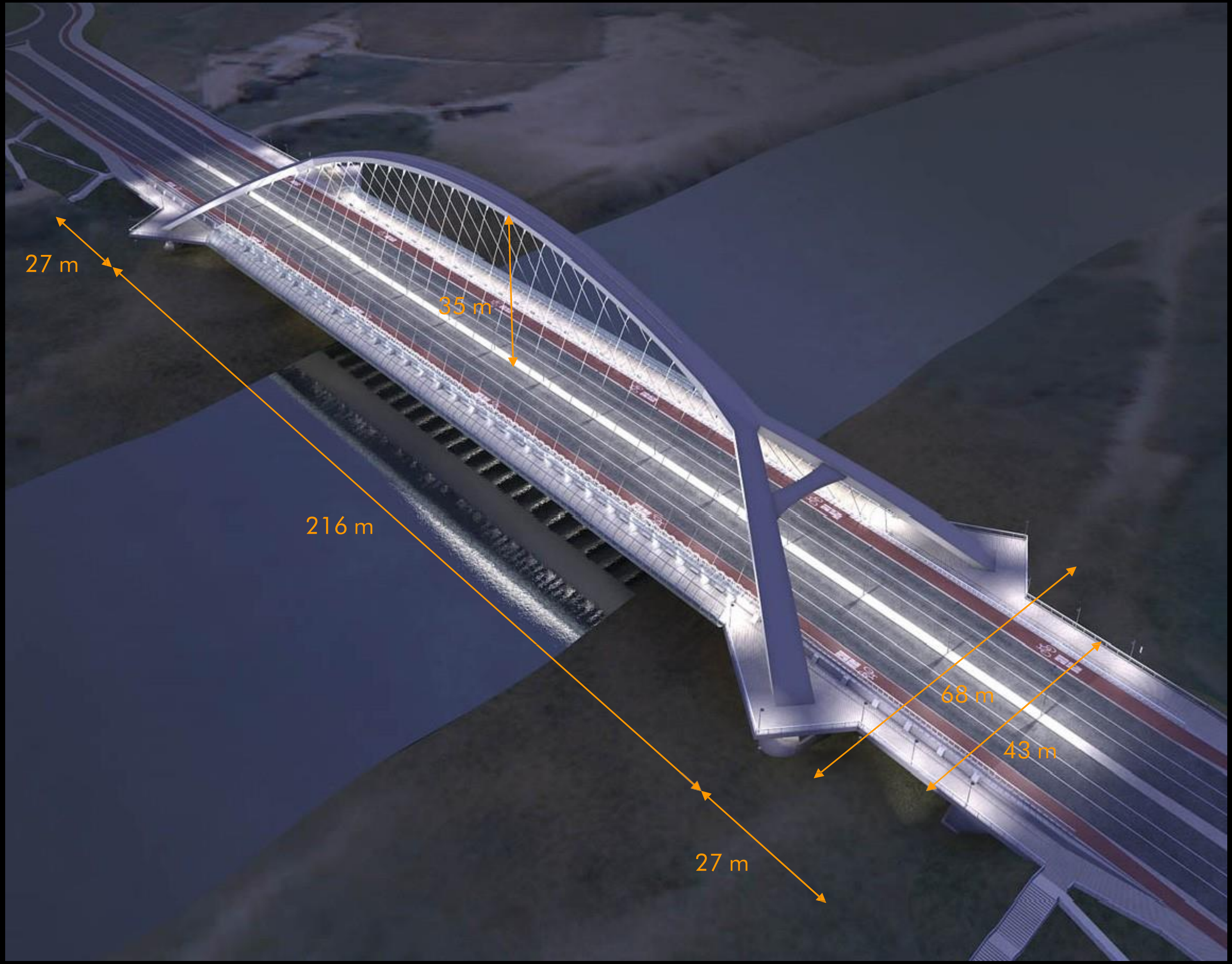
Puente del Tercer Milenio, Zaragoza

Puente del Tercer Milenio, Zaragoza

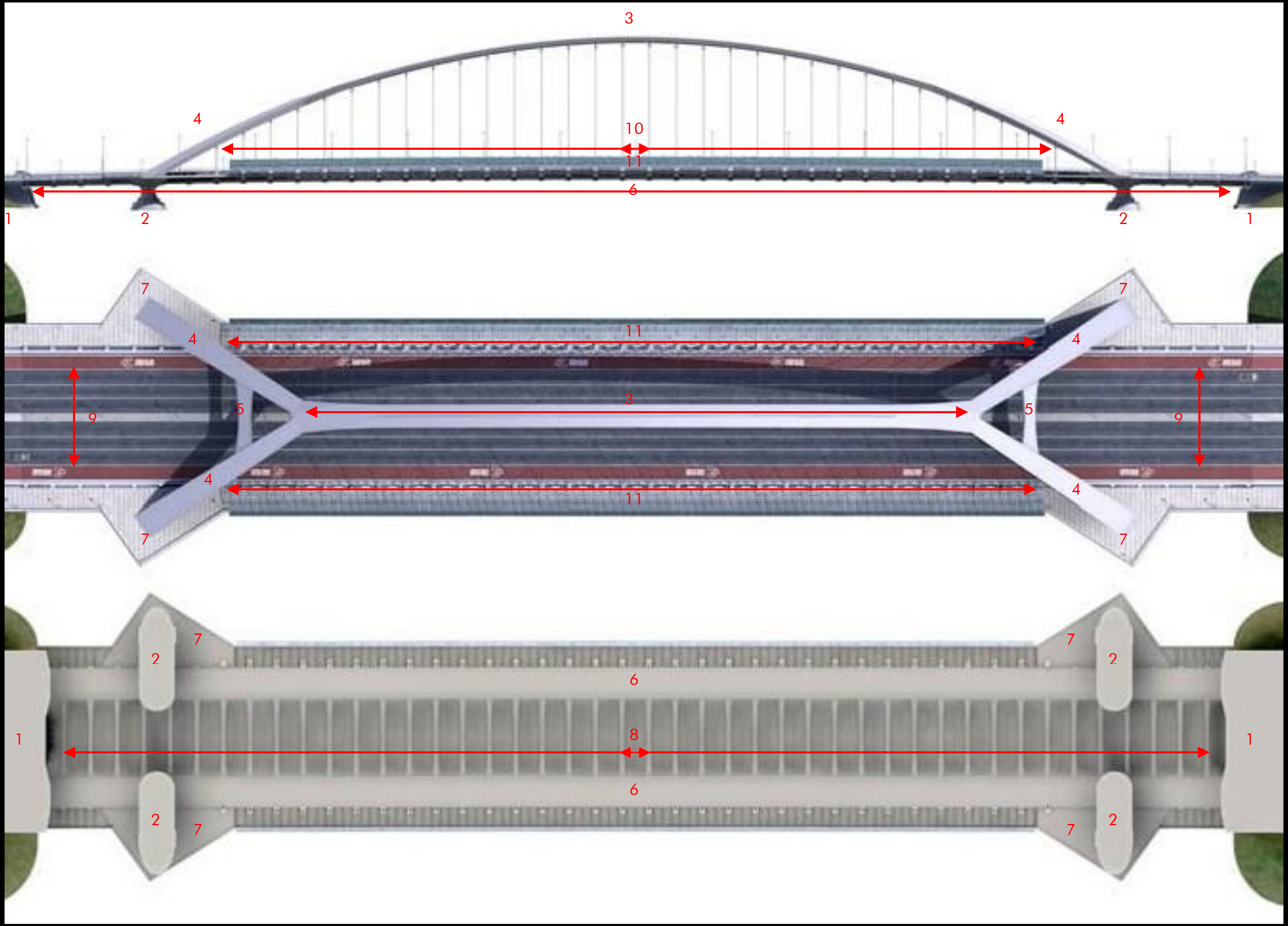
Project / Photos: Arenas & Asociados / Juan José Arenas de Pablo

Puente del Tercer Milenio, Zaragoza

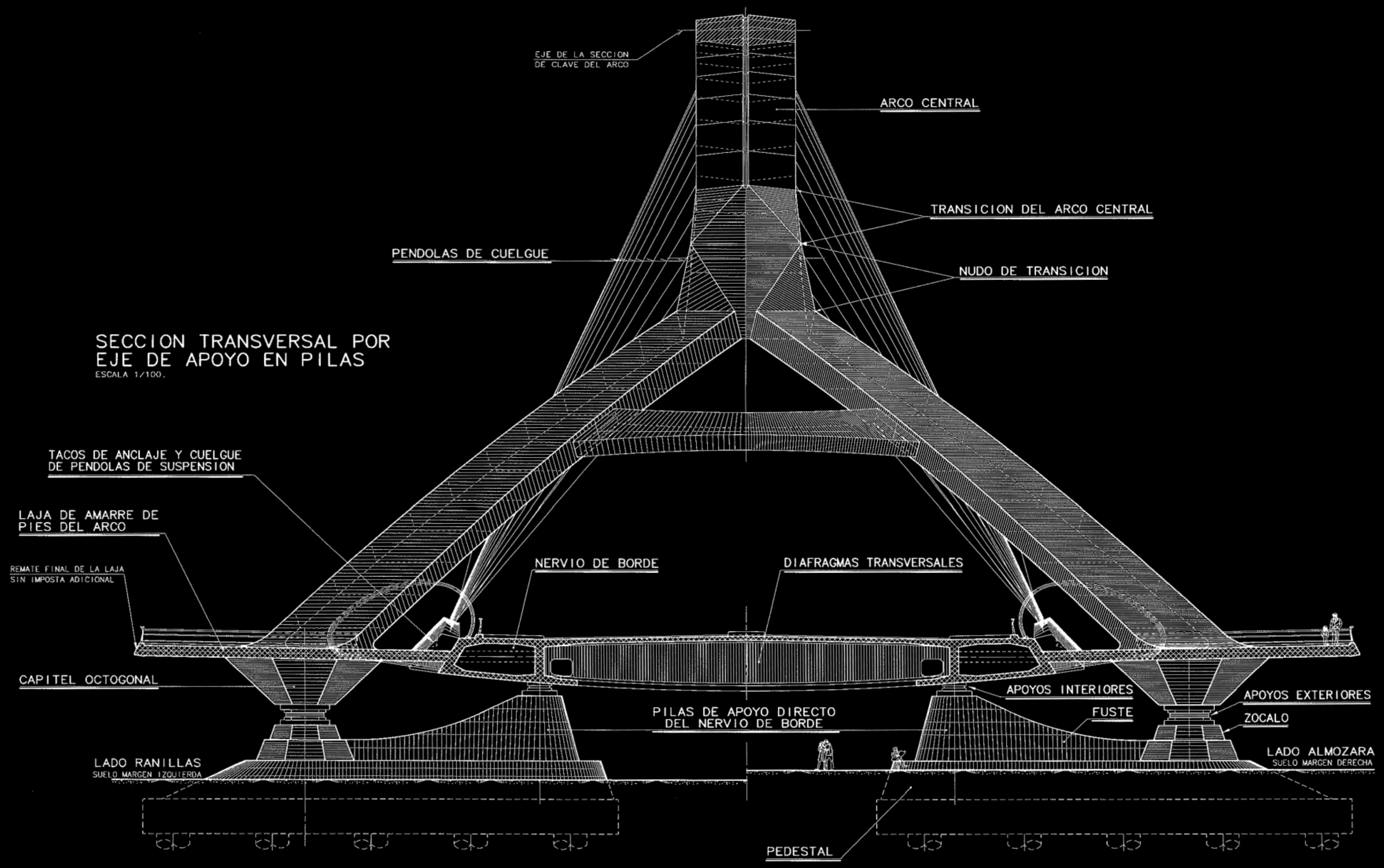
Puente del Tercer Milenio, Zaragoza



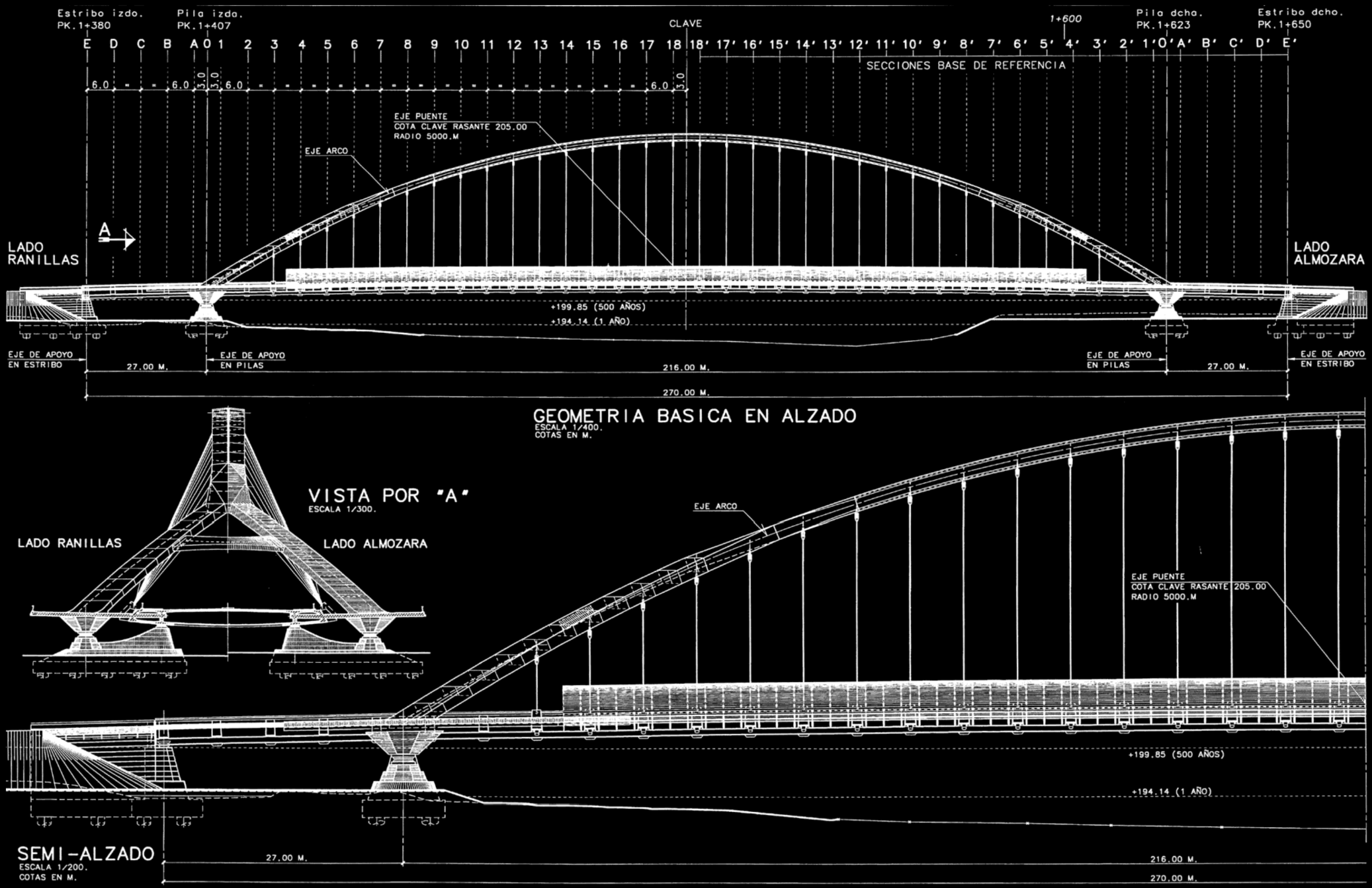
Puente del Tercer Milenio, Zaragoza



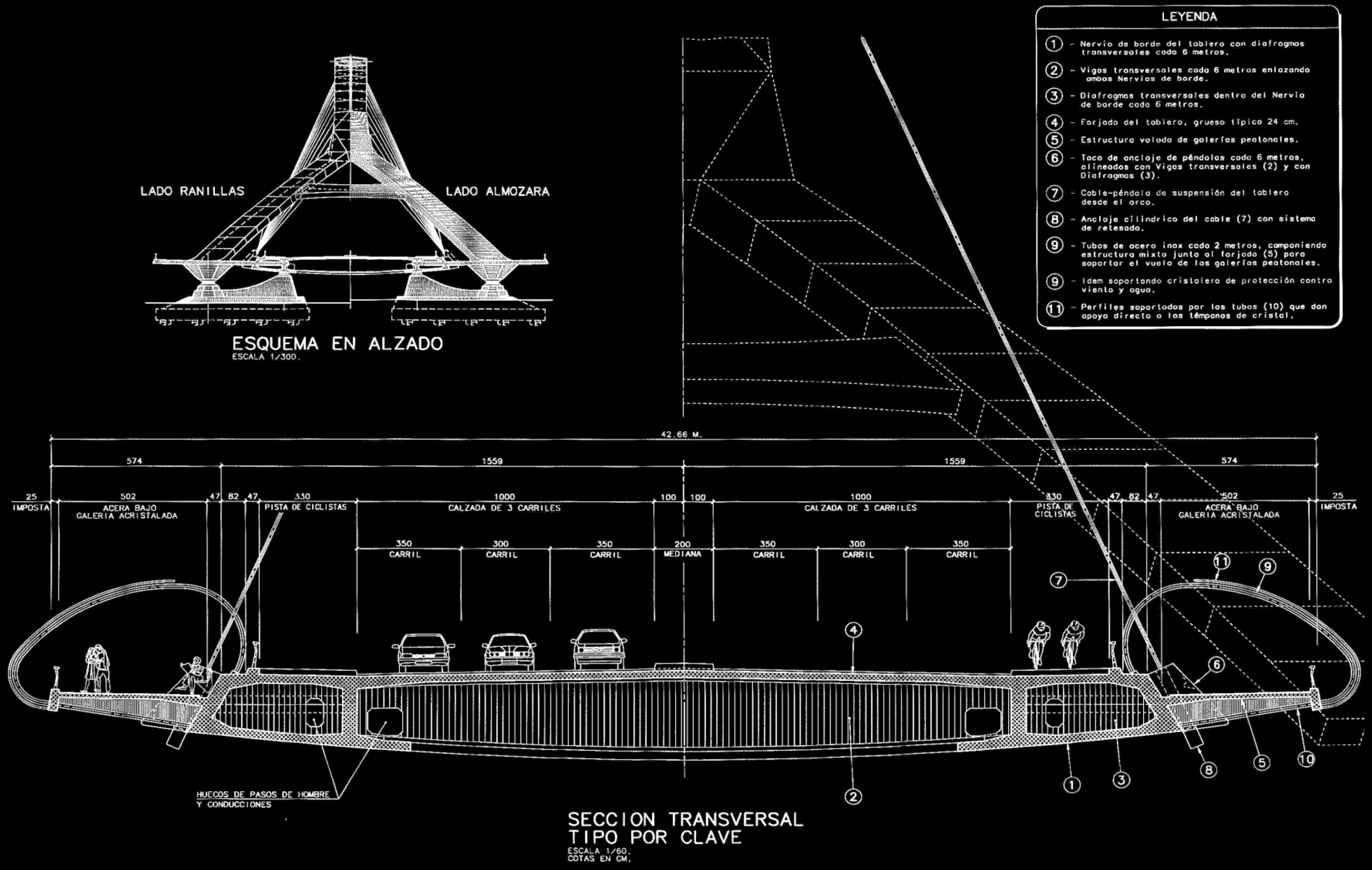
Puente del Tercer Milenio, Zaragoza



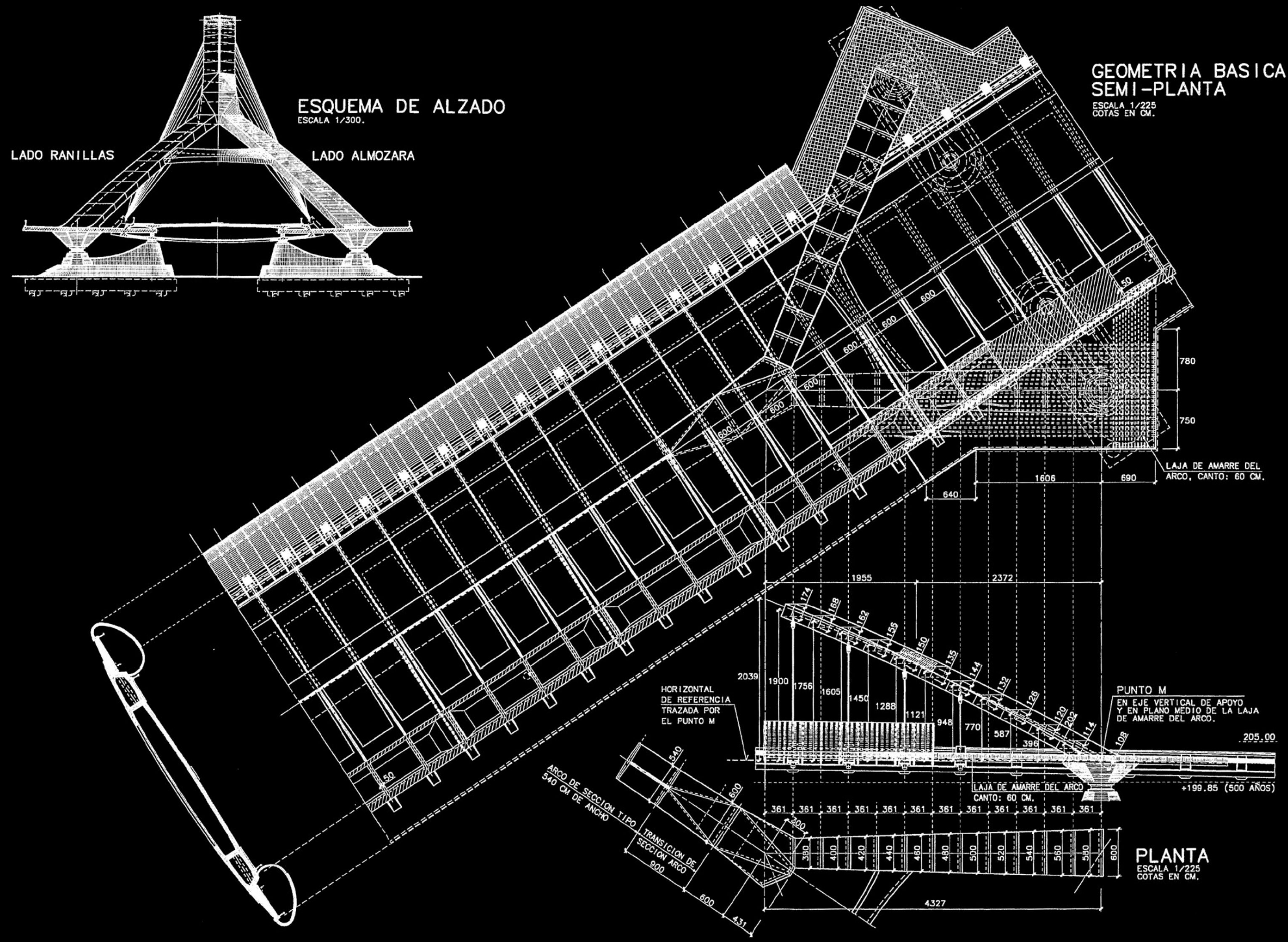
Puente del Tercer Milenio, Zaragoza



Puente del Tercer Milenio, Zaragoza



Puente del Tercer Milenio, Zaragoza



2 In-plane loading – walls and beams

2.2 Stress fields with prestressing

In-class exercise

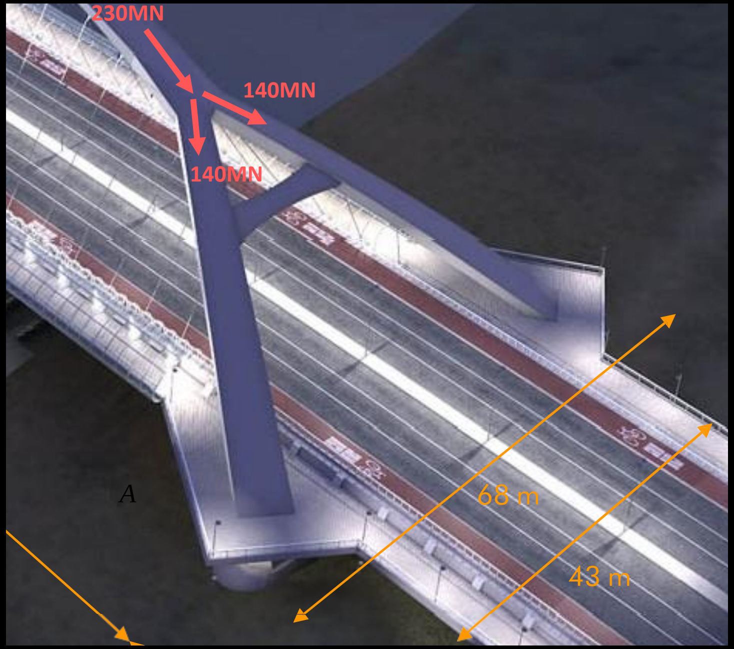
In-class exercise

1) Determine the global force flow and the magnitude of the forces.

   Note that the supports at the arch abutments directly resist the vertical component of the arch force, but are free to move horizontally ($\rightarrow$ full horizontal arch thrust carried by the bridge deck)

2) Determine the in-plane force flow in the tying slabs in more detail, using a strut-and-tie model.

3) Determine the amount of prestressing needed and pre-dimension the tying slabs for membrane elements.



Materials:

Concrete:	C60/70
Steel:	B500B
Prestressing steel:	Y1860