2 In-plane loading – walls and beams

2.1 Stress fields

Learning objectives

Within this chapter, the students are able to:

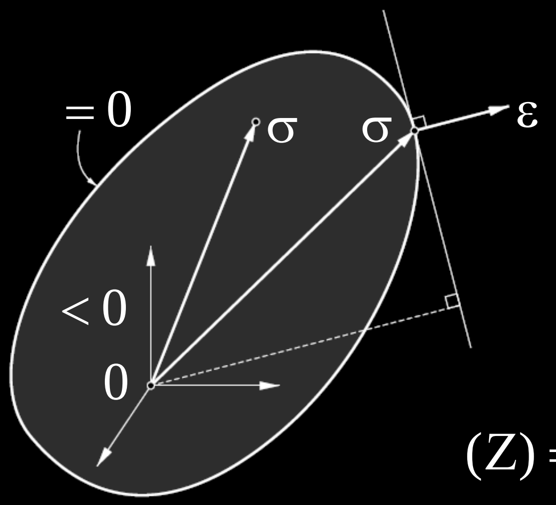
- create simplified stress fields and strut-and-tie models for walls, beams and frames, including discontinuity regions, as a combination of basic stress fields and strut-and-tie models.
  - discuss the differences and similitudes between stress fields and strut-and-tie models.
  - identify the critical regions of a simplified stress field or strut-and-tie model and formulate detailed stress fields allowing for the verification of those regions.
  - verify most frequent nodal zones.
- assess the applicability of stress fields and strut-and-tie models, particularly concerning (i) the presence of transversal reinforcement, (ii) the selection of suitable effective compressive strength, (iii) the proper detailing of nodal zones and (iv) the existence of relevant 3D effects in 3D structures made of 2D elements. Whenever 3D effects are present, the students are able to create 2D stress fields and strut-and-tie models capturing those effects.

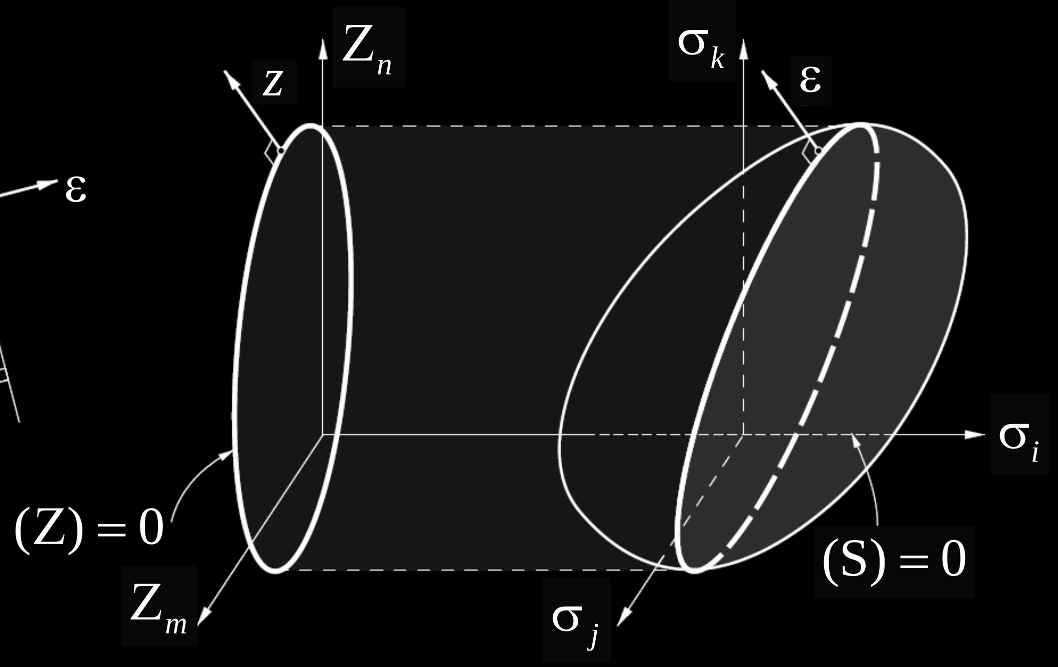
Stress fields

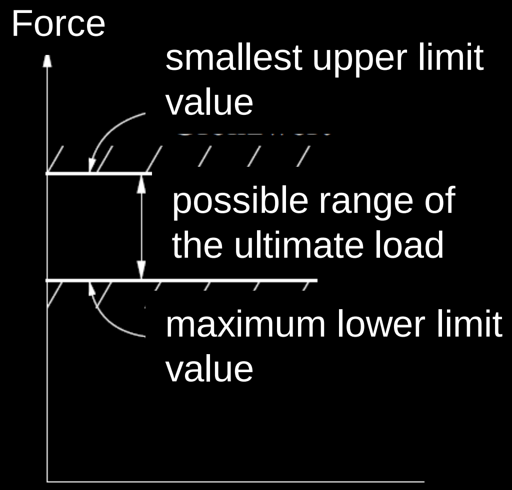
Strut-and-tie models and stress fields: Historical development

- Originally, solutions followed primarily the main load path, the dimensions of the struts being of second importance. Such models have persisted until today («Strut-and-tie models», e.g. Schlaich et al., 1984 and 1987).
- Since about 1975, strut-and-tie models (truss models) have been used in combination with the assumption of a limited concrete compressive strength f_c. The dimensions of the struts and nodal zones result from the assumption of f_c.
- The resulting strut-and-tie models (truss models) are statically admissible (discontinuous) stress fields according to the lower bound (static) theorem of the theory of plasticity and, therefore, are based on a consistent theoretical basis.
- Computer-aided methods for the development of stress fields have been developed at various universities (e.g. the Compatibility Stress Field Method, CSFM, developed at ETH Zürich in collaboration with the company IDEA StatiCa). The use of these methods is starting to become more common in practice. These methods will be discussed in the chapter about numerical modelling.

Stress fields





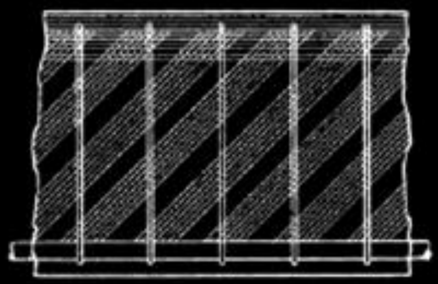


- The application of stress fields is based on the theory of plasticity.
- ETH Zurich played a central role in their development - namely Professors Bruno Thürlimann and Peter Marti.
- Internationally this approach is known as the "Zurich School". It is based on consistent mechanical models, which are verified with large-scale tests.

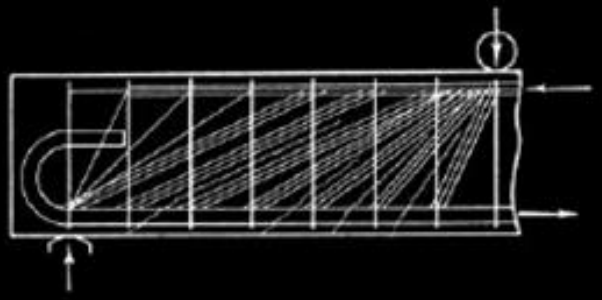
Stress fields

Early truss models (descriptive)

K. W. Ritter, «Die Bauweise Hennebique» (1899)

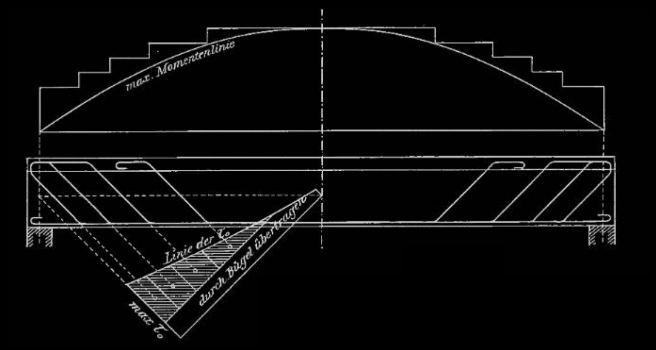


E. Mörsch, «Der Eisenbetonbau» (1908)

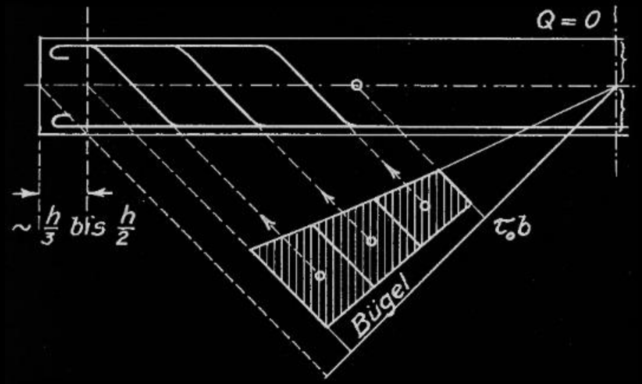


E. Mörsch, «Der Eisenbetonbau» (1922)

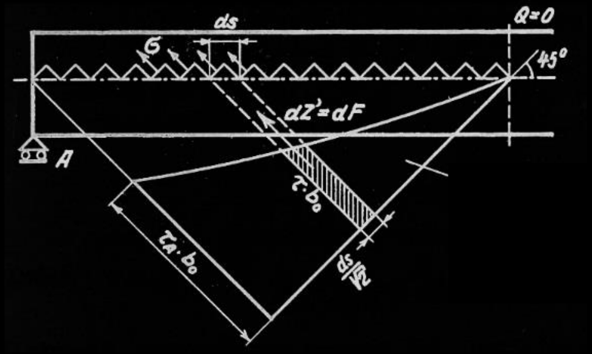
Elastic stress fields with principal tensile stresses (semi-empirical)



E. Mörsch, «Der Eisenbetonbau» (1908)



M. Ritter, «Vorlesung Massivbau» (ca. 1940)

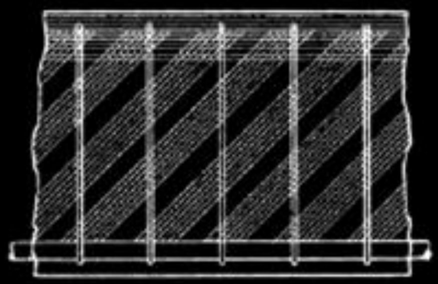


P. Lardy, «Vorlesung Massivbau» (1951)

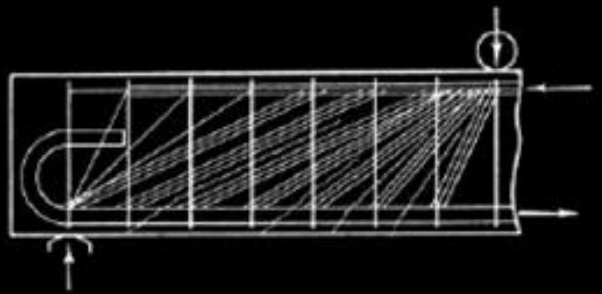
Stress fields

Early truss models (descriptive)

K. W. Ritter, «Die Bauweise Hennebique» (1899)

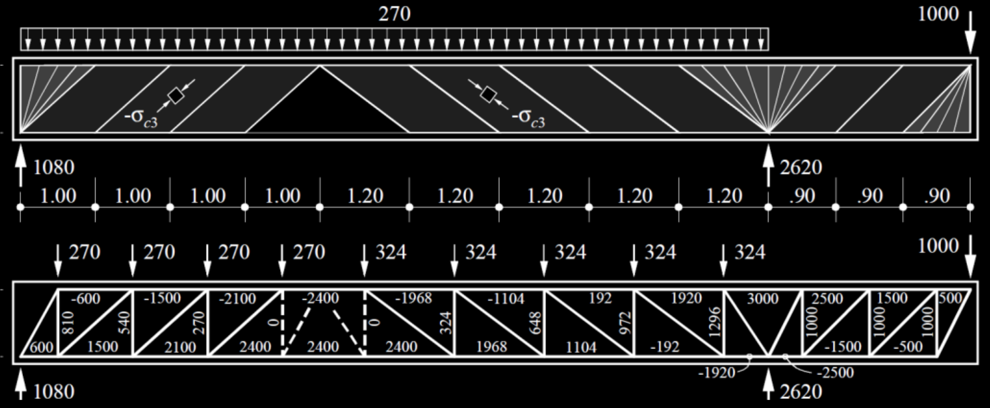


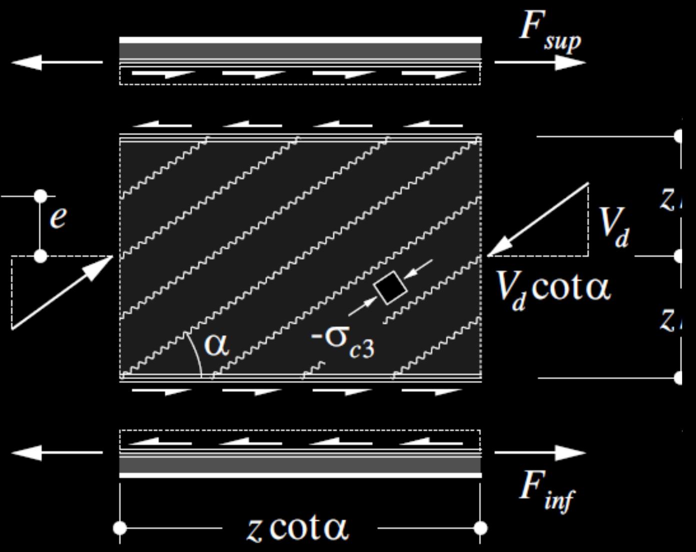
E. Mörsch, «Der Eisenbetonbau» (1908)

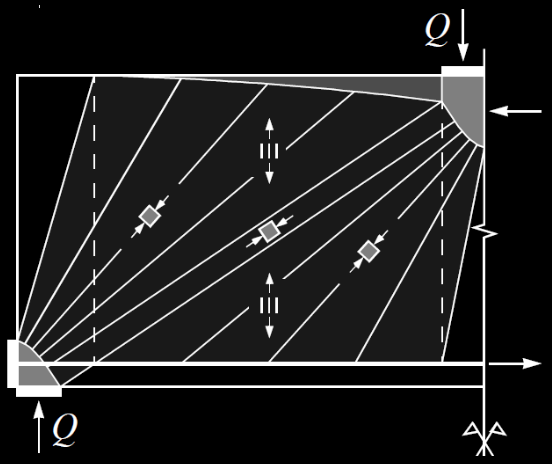


E. Mörsch, «Der Eisenbetonbau» (1922)

Current strut-and-tie models / Stress fields: theory of plasticity = consistent foundation







Structrural concrete at the ETH - former professors

Karl Culmann
1821-1881

Prof. 1855-1881
(→ Ritter)

Karl Wilhelm Ritter
1847-1906

Emil Mörsch
1872-1950

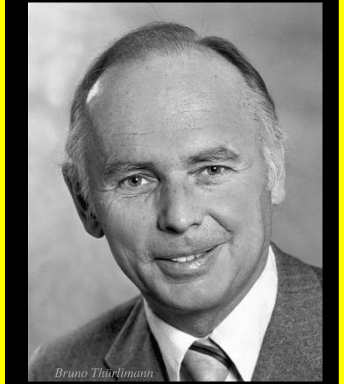
Arthur Rohn
1878-1956

Max Ritter
1884-1946

Pioneers in the application of the theory of plasticity to structural concrete members

Pierre Lardy
1903-1958

Prof. 1946-1958
(→ Thürlimann)

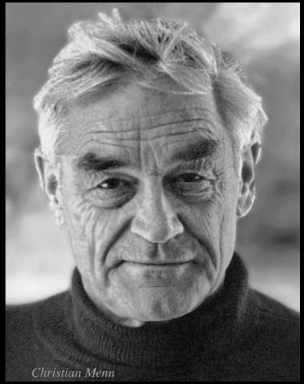


Bruno Thürlimann
1923-2008

Prof. 1960-1990
(→ Marti)

Hugo Bachmann
1935

Prof. 1969-2000
(→ Stojadinovic)



Christian Menn
1927-2018

Prof. 1971-1992
(→ Vogel)

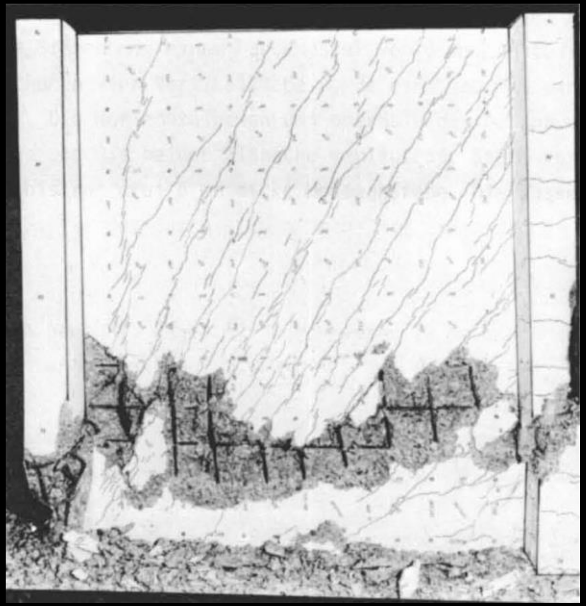
Peter Marti
1949

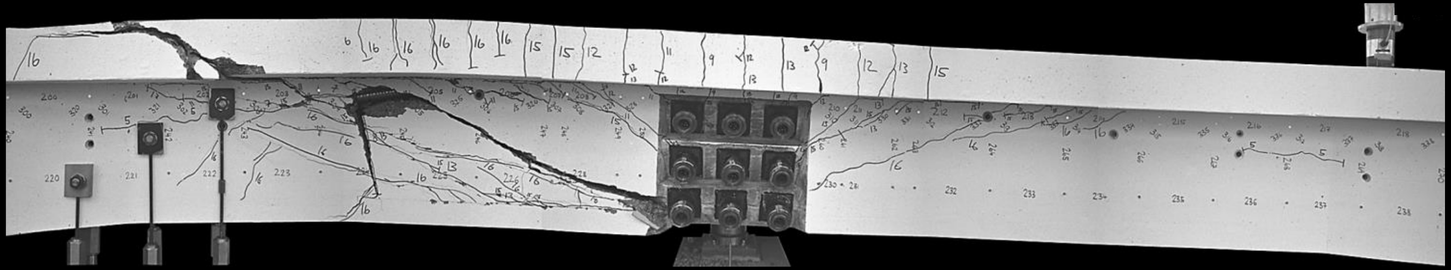
Prof. 1990-2014
(→ Kaufmann)

Plastic design methods

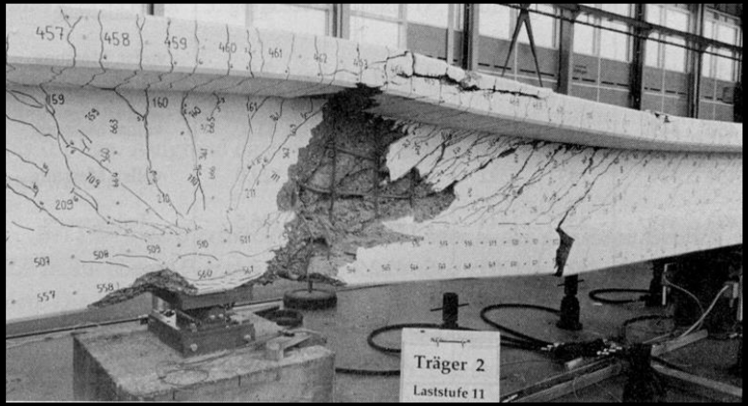
Bachmann / Thürlimann
(1965)

Maier / Thürlimann
(1985)



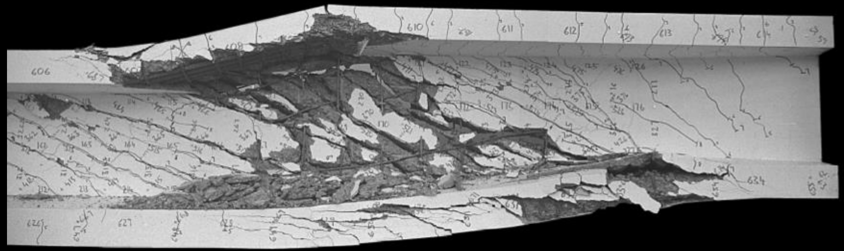


Stoffel / Marti
(1995)



Sigrist / Marti
(1992)

Kaufmann / Marti
(1995)



Stress fields

Principles for the design of stress fields

There are usually several suitable stress fields to solve the same problem. Designers select the most suitable stress field and dimension the reinforcement accordingly.

The consideration of the following principles usually ensures an economic design (the requirement for stiffness also follows from the principle of the minimum complementary energy):

- Simplicity (usually only orthogonal reinforcement is used)
- Stiffness (e.g. short ties)
- Efficiency (consider minimum reinforcement in the calculation)

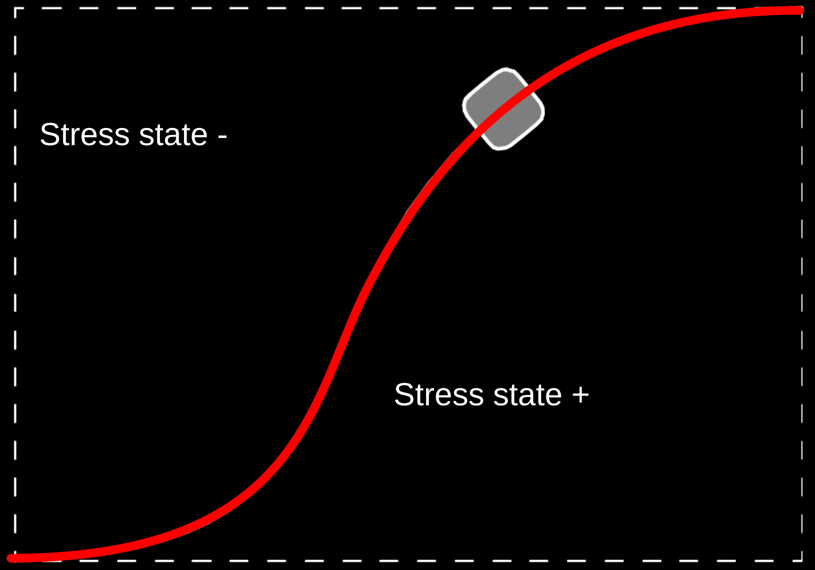
A scaled drawing of the model is highly recommended.

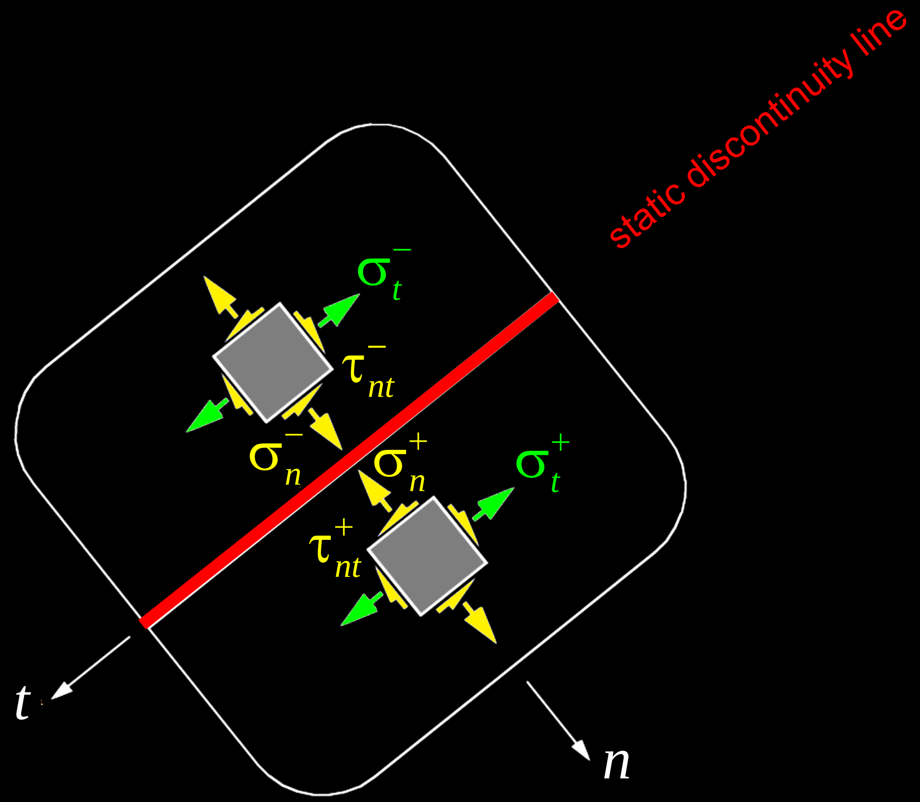
In any case, sufficient minimum reinforcement should be used
(ρ = 0.1...0.3 %, depending on the region).

Particular attention should be paid to the choice of the effective concrete compressive strength, that should account for the non ideally plastic behaviour of concrete (see separate chapter) and has a decisive influence on the geometry of the model.

Stress fields

Stress discontinuities





Lower bound theorem of the theory of plasticity: equilibrium must be fulfilled

→ Normal stresses parallel to the discontinuity line may have a discontinuity ($\sigma_t^- \neq \sigma_t^+$ is admissible)

→ Normal stresses perpendicular to the discontinuity line and shear stresses must be continuous ($\sigma_n^- = \sigma_n^+$ and $\tau_{nt}^- = \tau_{nt}^+$ must be fulfilled)

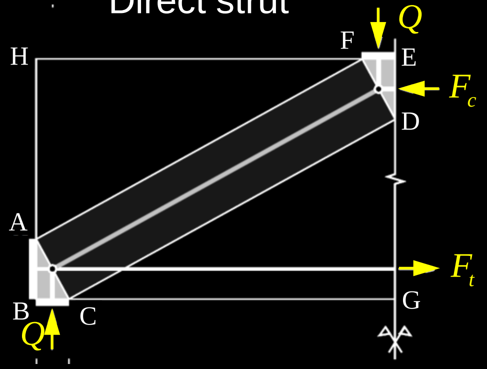
Stress fields

Basic models for beams

(a) without activation of transverse reinforcement

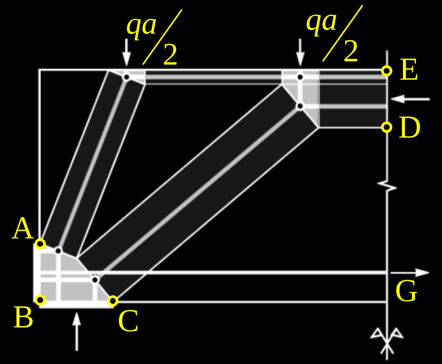
Point load (*Q*)

Direct strut

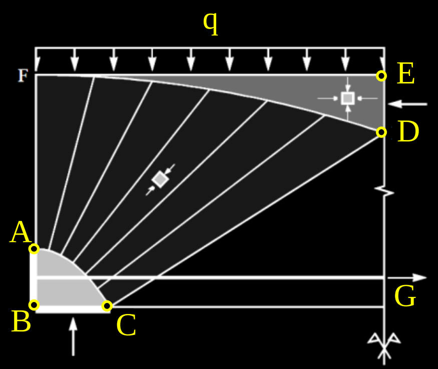


Distributed load (*q*)

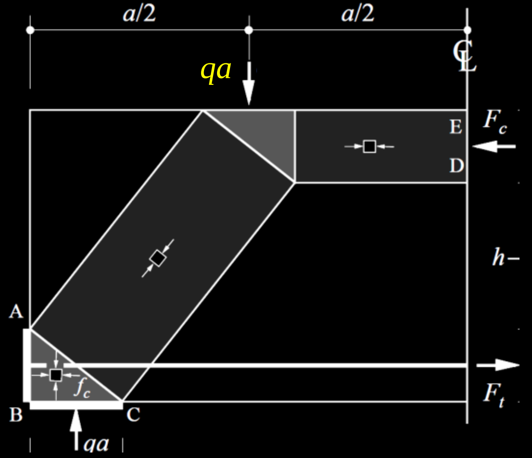
Polygonal fan



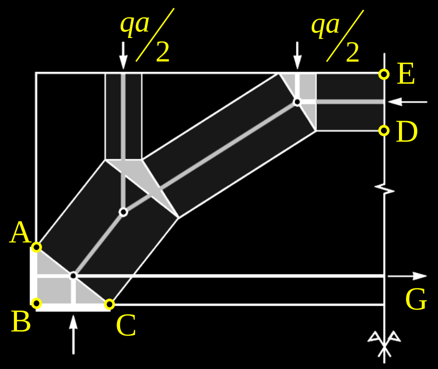
Fan



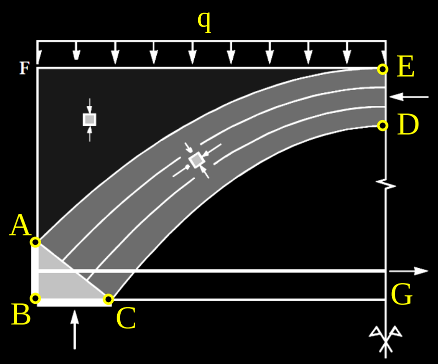
Direct strut



Polygonal arch



Arch

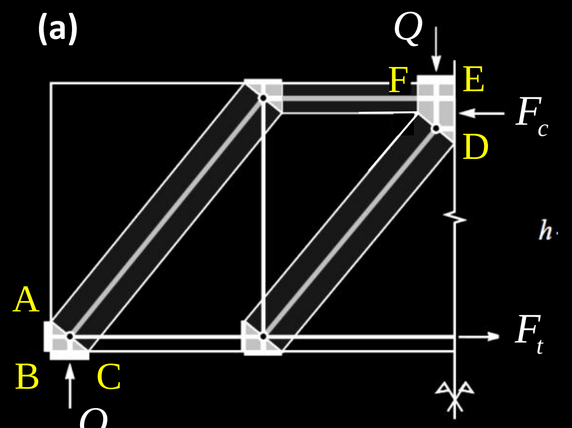


Stress fields

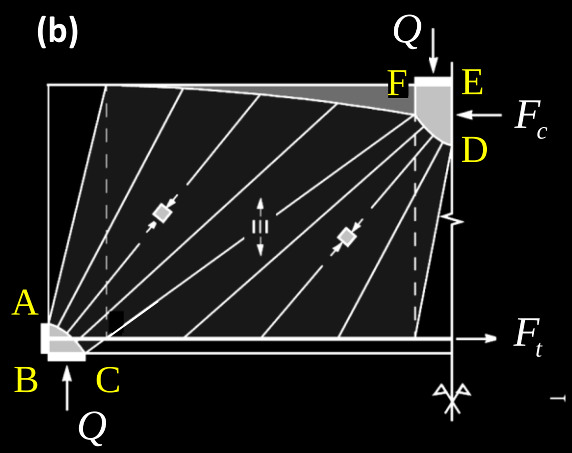
Basic models for beams

(b) with activation of transverse reinforcement and point load (*Q*)

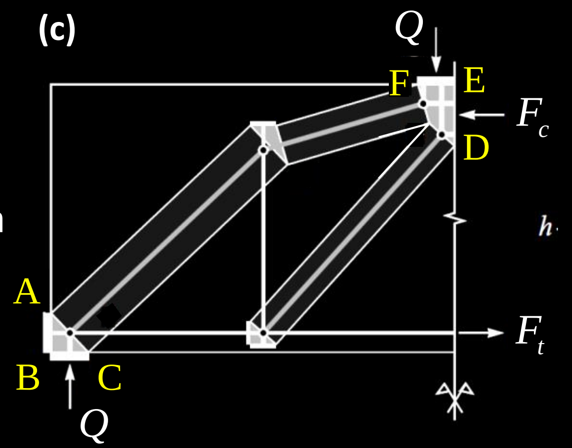
Truss



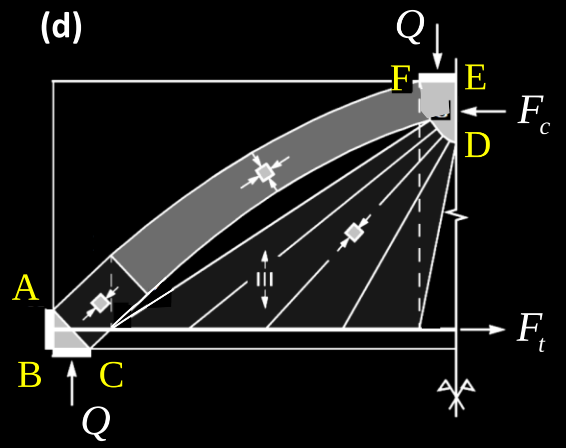
Double fan



Truss + polygonal arch

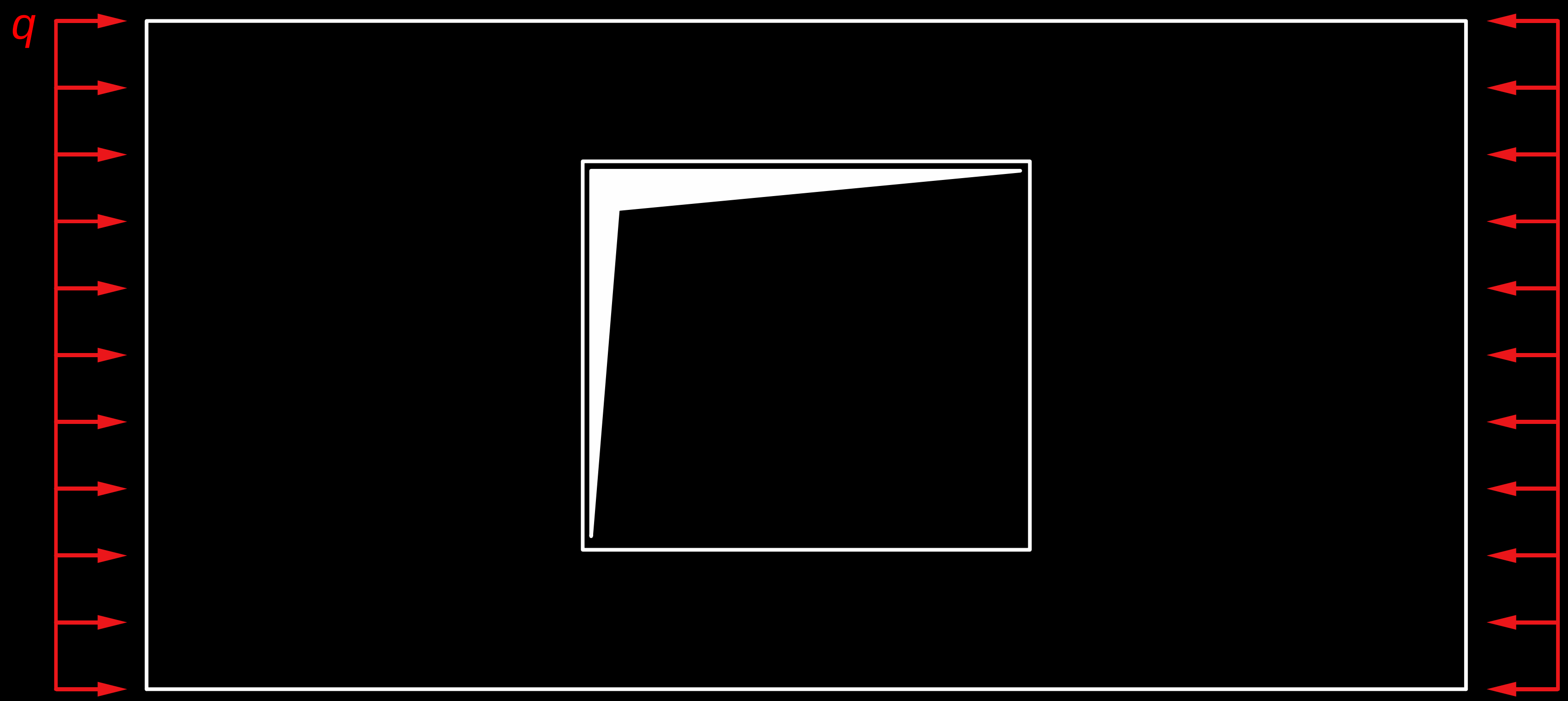


Arch + fan



Stress fields

In-class exercise



Stress fields

Derivation of direct strut mechanism (point load + no activation of transverse reinforcement)

Equilibrium:

$$F_c = b_w c f_c = A_s f_{sy} = F_t$$

$$Qa = b_w c f_c (h-c)$$

$$c = \frac{h}{2} - \sqrt{\frac{h^2}{4} - \frac{Qa}{b_w f_c}}$$

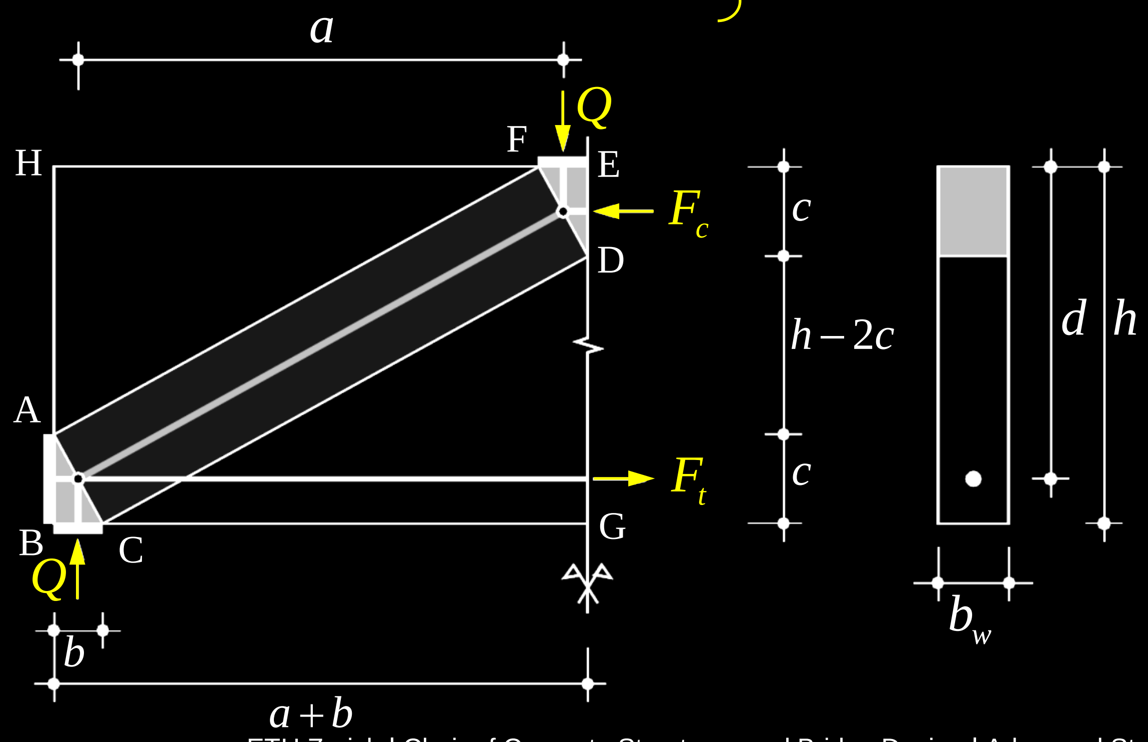
$$A_s = b_w c \frac{f_c}{f_{sy}}$$

$$\rho = A_s / (b_w d)$$

$$\omega = \rho (f_{sy} / f_c)$$

$$Q = \frac{b_w f_c h^2}{a} \cdot \frac{\omega(1-\omega/2)}{(1+\omega/2)^2} \quad \left(\omega \le \frac{2}{3}\right)$$

$$Q = \frac{b_w f_c h^2}{4a} \quad \left(\omega \ge \frac{2}{3}\right)$$



$$b = \frac{Q}{b_w f_c}$$

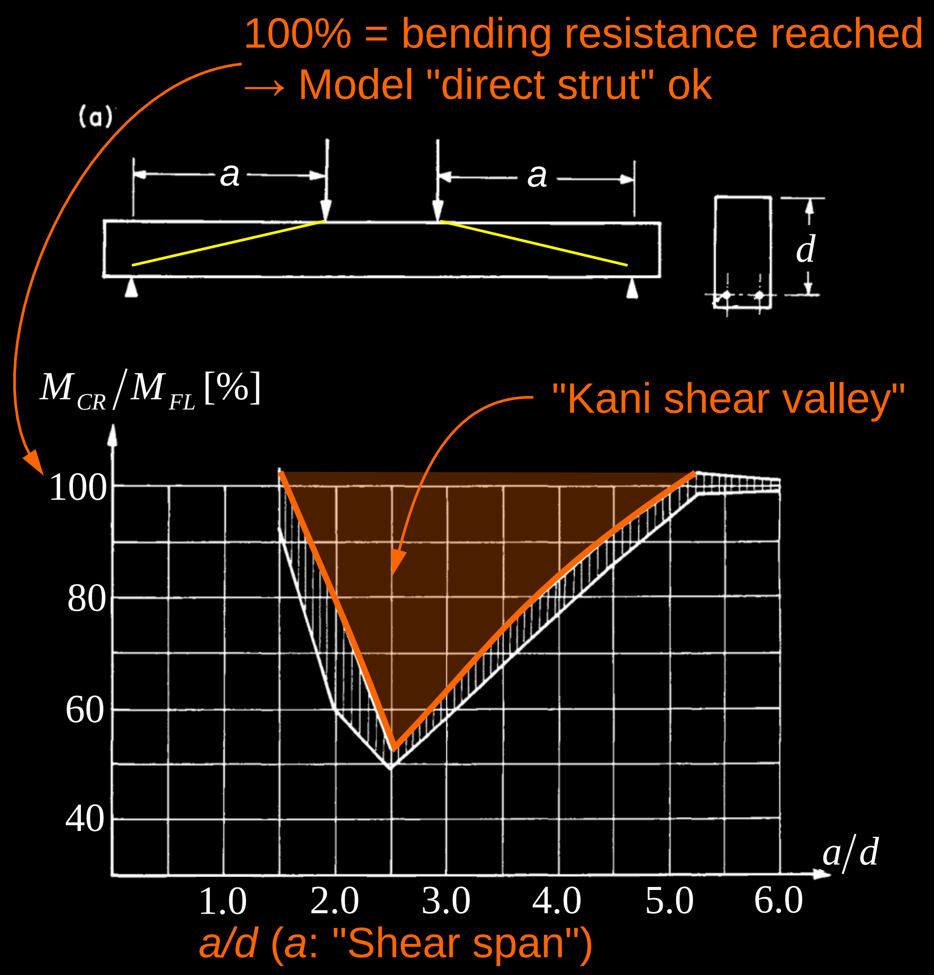
$$c = \omega d, \quad \omega = \frac{A_s f_{sy}}{b_w d f_c}$$

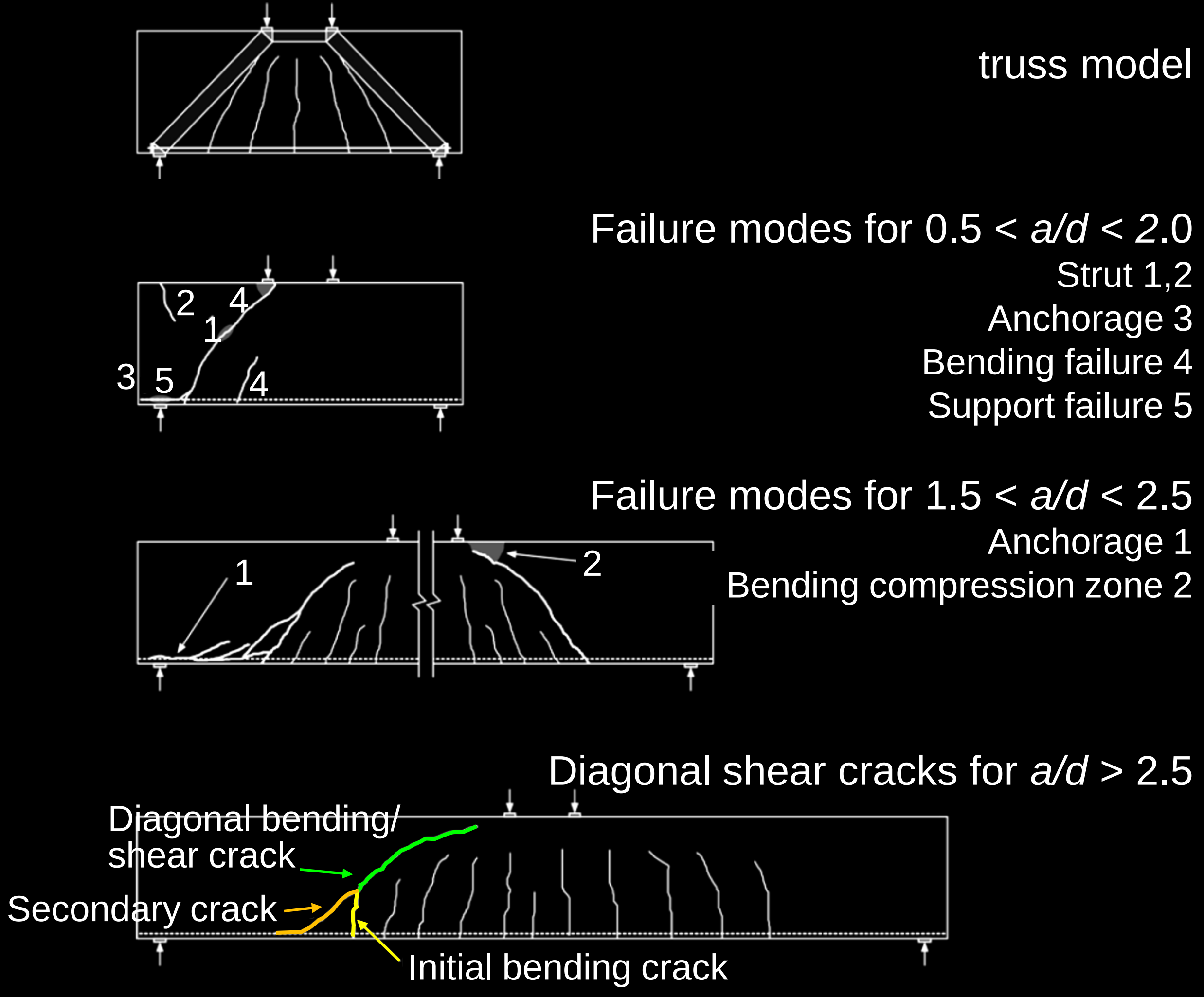
$$F_c = F_t = \omega d b_w f_c$$

Stress fields

Limits of applicability of direct strut model

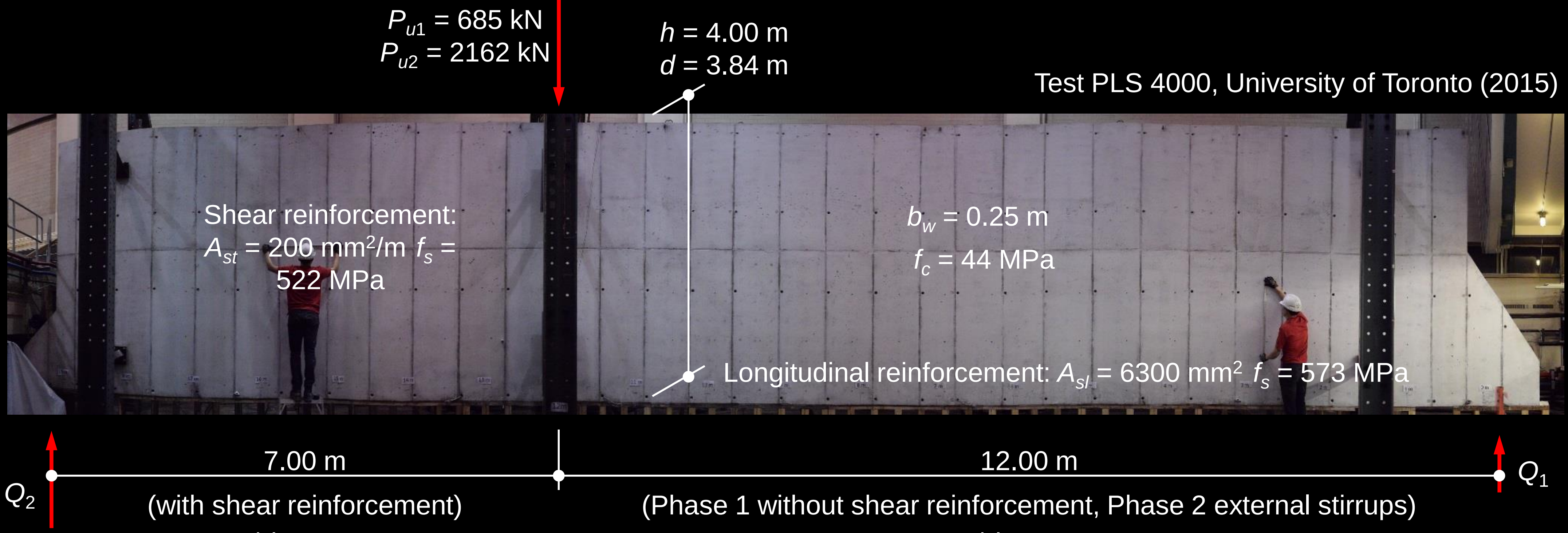
G.N.J. Kani (“The Riddle of Shear Failure and its Solution”, 1964): Results of experiments without stirrups

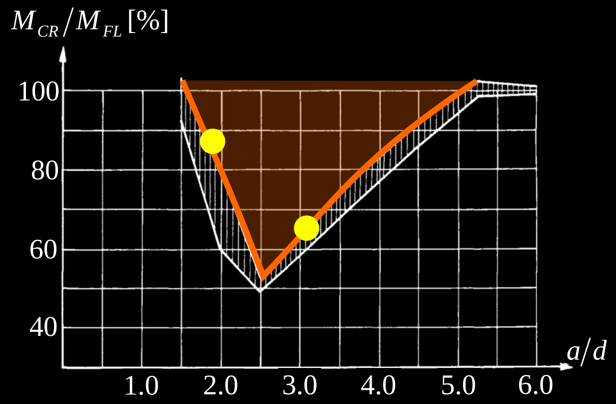




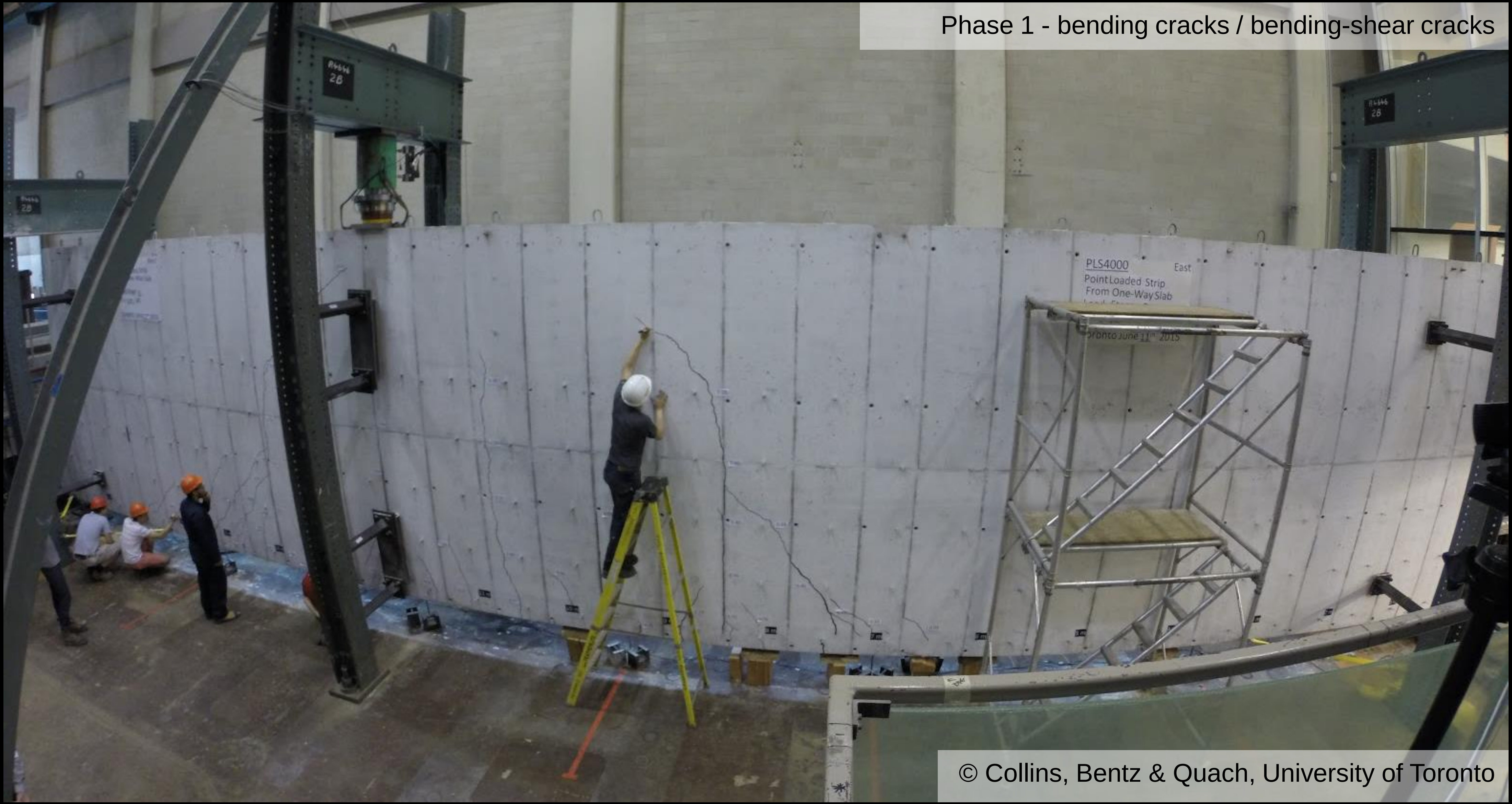
[based on Sherwood 2008]

Stress fields





Stress fields



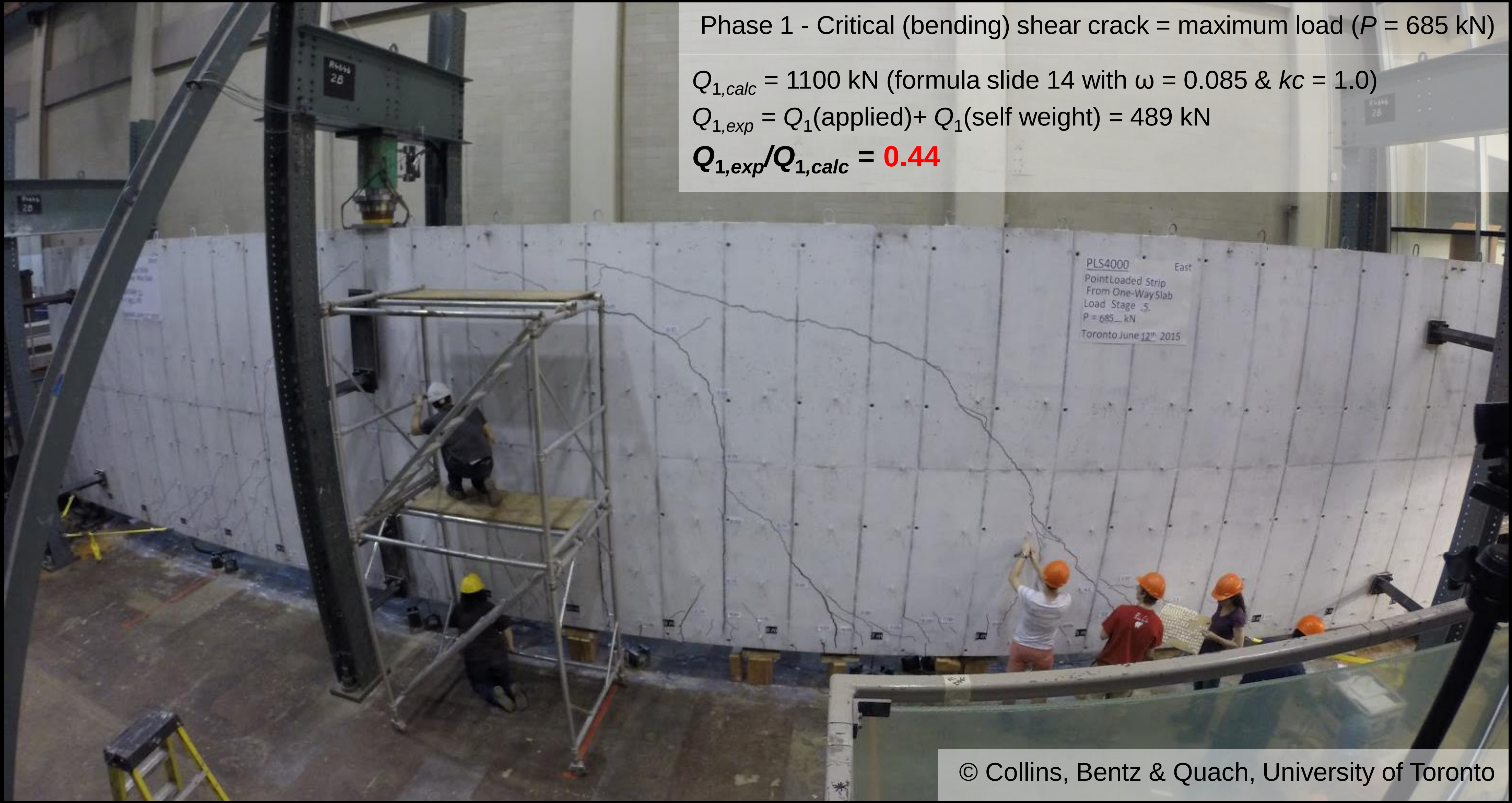
Stress fields

Phase 1 - Critical (bending) shear crack = maximum load (P = 685 kN)

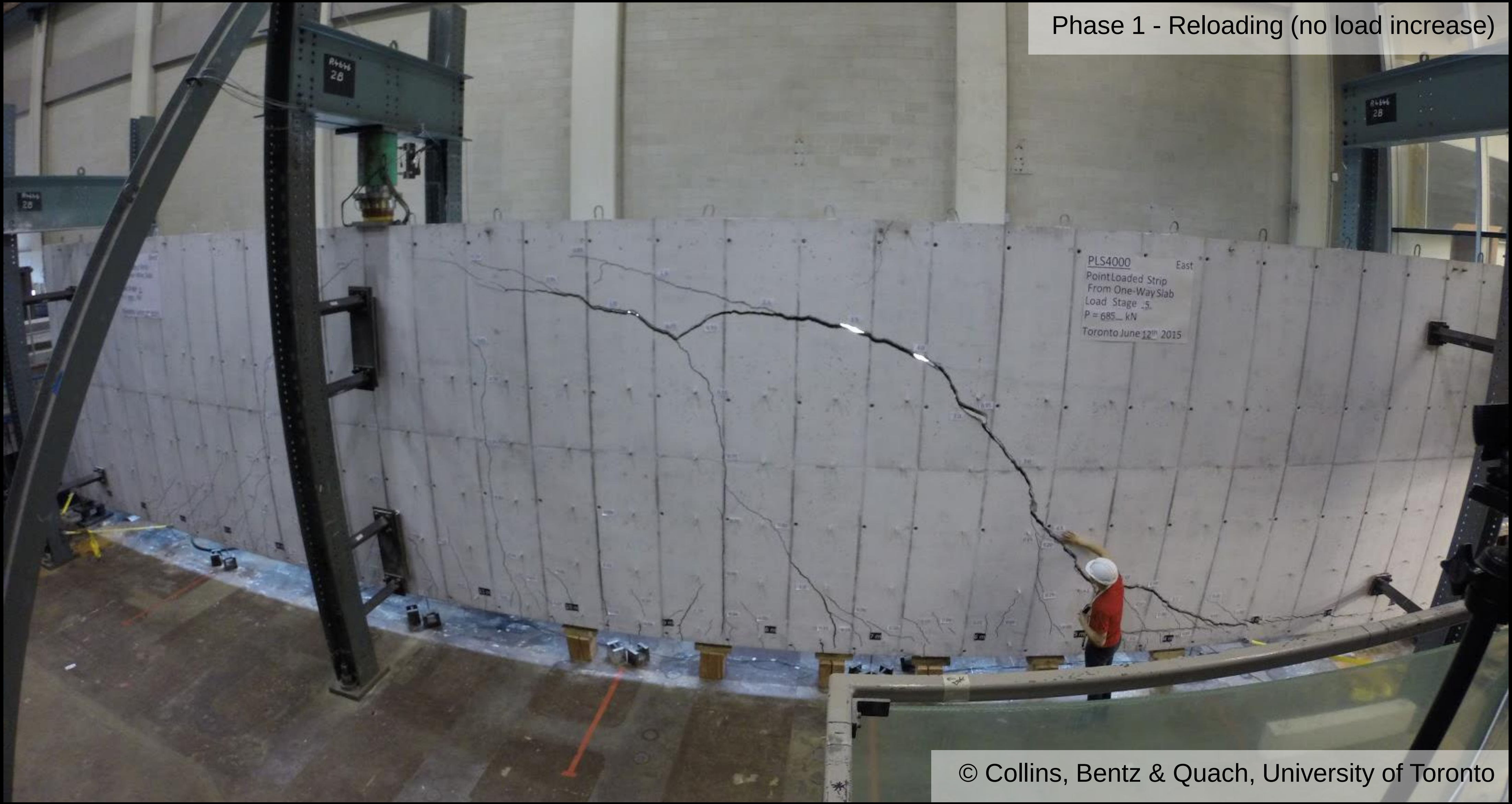
$Q_{1,calc}$ = 1100 kN (formula slide 14 with ω = 0.085 & kc = 1.0)

$Q_{1,exp}$ = Q_1(applied)+ Q_1(self weight) = 489 kN

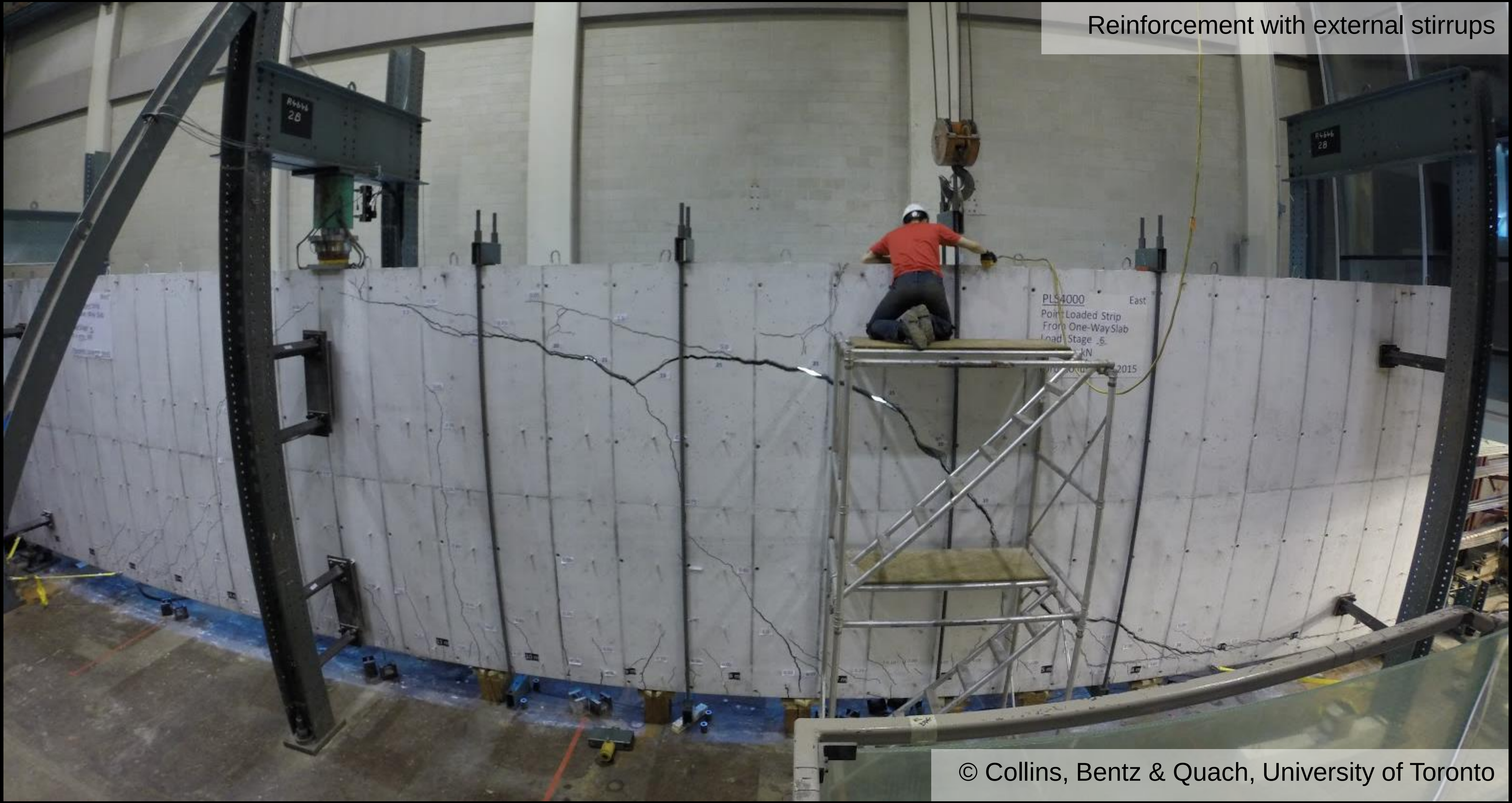
$\mathbf{Q_{1,exp}/Q_{1,calc}}$ **= 0.44**



Stress fields



Stress fields



Reinforcement with external stirrups

Stress fields

Stress fields

Limits of applicability of direct strut model

The presented stress fields for beams without transverse reinforcement are strong simplifications of reality:

- The tension chord is modelled like reinforcement without bond, but with an anchor plate that anchors the entire load. In reality, bond stresses lead to successive crack formation, and only for loads significantly higher than the cracking load a direct strut mechanism occurs.
- If no minimum reinforcement is placed in the member, there is the possibility that a diagonal crack penetrates into the compression field and the structure fails before the desired load-bearing mechanism is achieved. This is associated with a brittle failure (scale effect!).

The behaviour can be improved by prestressing the tension chord, which forces the direct strut mechanism.

In any case, a load transfer by a direct strut mechanism (without prestressing) is only meaningful in squat elements. In slender elements, the nodal zones' dimensions become very large and the anchorage of the reinforcement becomes problematic because the entire tensile force must be anchored behind the bearing!

These problems can be solved by providing transverse reinforcement (or by the activation of the vertical minimum reinforcement, which must **always** be placed).

Stress fields

Fan and arch mechanisms: beams without activation of transverse reinforcement & distributed load (see also [4])

The figure shows 4 possible models for the same problem. The formation of a fan or an arch mechanism depends, among other aspects, on:

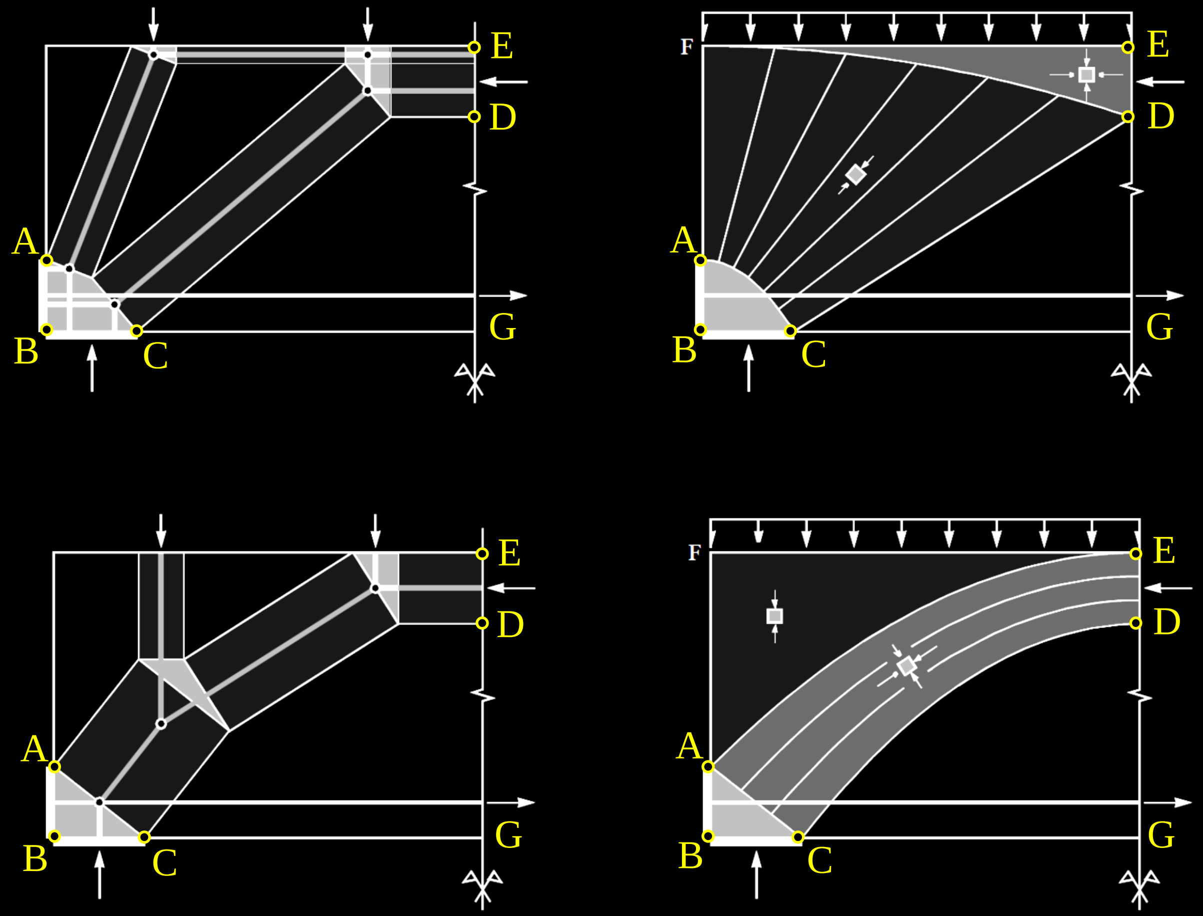
- Slenderness of the element
- Amount of reinforcement
- Loading history

The strut geometry and the dimensions of the bearings are selected such that a biaxial compressive stress state is obtained in the nodal zone ABC in all examples: $\sigma_{c1} = \sigma_{c3} = - f_c$

$\rightarrow$ The location of points A to E, as well as the lower bound of the ultimate load, is identical in all models.

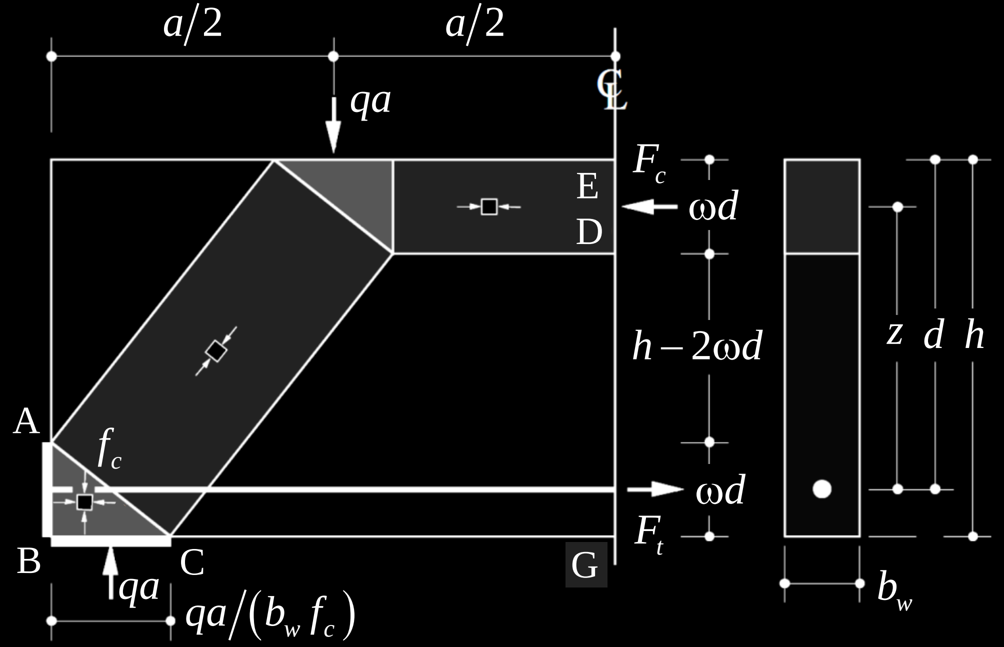
Note: Elastically, the stiffer model tends to form minimum complementary energy, i.e.

$$U^* = \int \varepsilon(\sigma)\, d\sigma \rightarrow \min$$



Stress fields

Since the location of points A...E is identical in all the models, the geometry can be defined from any of them, or even from a simpler model:



Geometric/mechanical reinforcement ratio:

$$\rho = \frac{A_s}{b_w d} \qquad \omega = \rho \frac{f_y}{f_c} \qquad h = d\left(1 + \omega/2\right)$$

Equilibrium:

$$\omega d b_w f_c d\left(1 - \omega/2\right) = \frac{qa^2}{2}\left(1 - \frac{q}{b_w f_c}\right)$$

Solutions of the quadratic equation resulting from the equilibrium condition:

$$q = \frac{b_w f_c}{2}\left(1 - \sqrt{1 - \frac{8h^2}{a^2} \cdot \frac{\omega\left(1-\omega/2\right)}{\left(1+\omega/2\right)^2}}\right) \qquad \text{for } \omega \le 2/3$$

$$q = \frac{b_w f_c}{2}\left(1 - \sqrt{1 - \frac{2h^2}{a^2}}\right) \qquad \text{for } \omega \ge 2/3$$

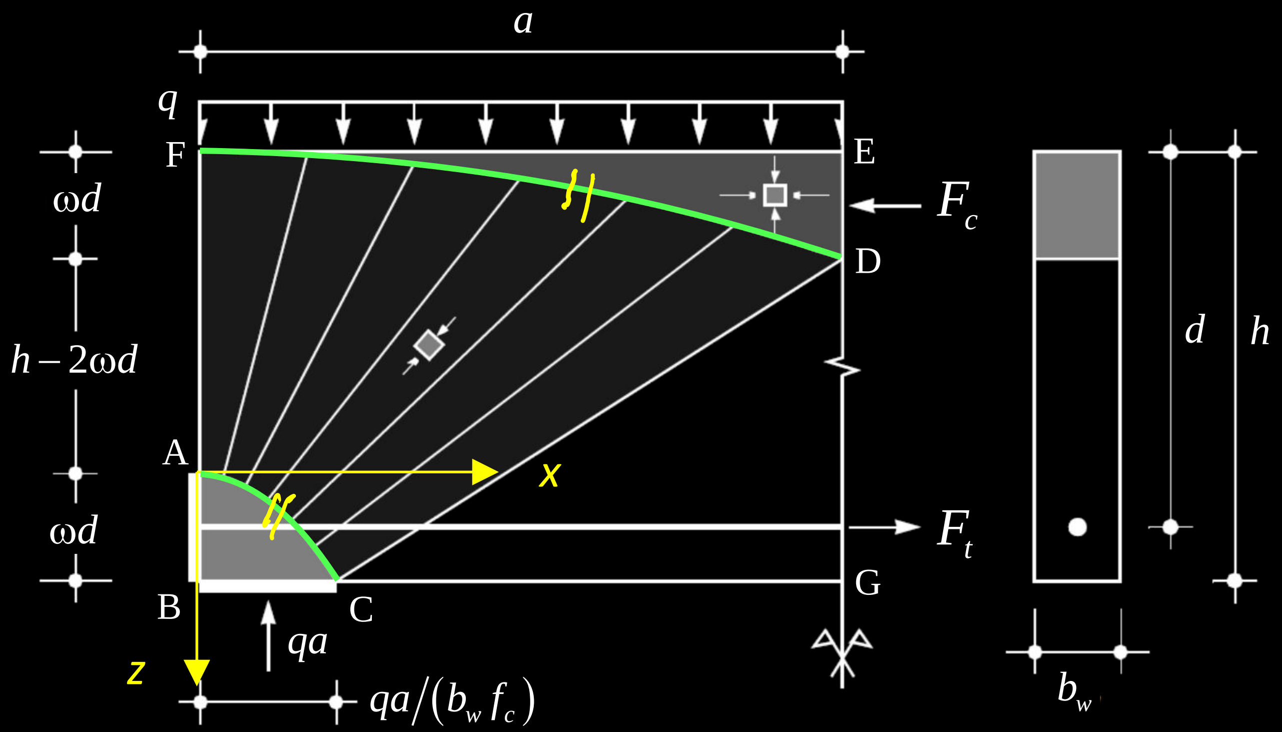
It is also possible to formulate a relationship for the mechanical reinforcement ratio required to resist a certain load *q* (see [4]).

Stress fields

The exact geometry of the fan borders is rarely required in practice. If necessary, a differential equation for these curves can be formulated with an equilibrium condition on a differential fan element (see Annex).

The trajectory of the lower fan boundary AC is given by: $z = \dfrac{f_c - q}{2qd\left(1-\omega/2\right)}x^2$

The upper edge of the fan DF is also a second degree parabola.



Stress fields

Load suspension mechanism

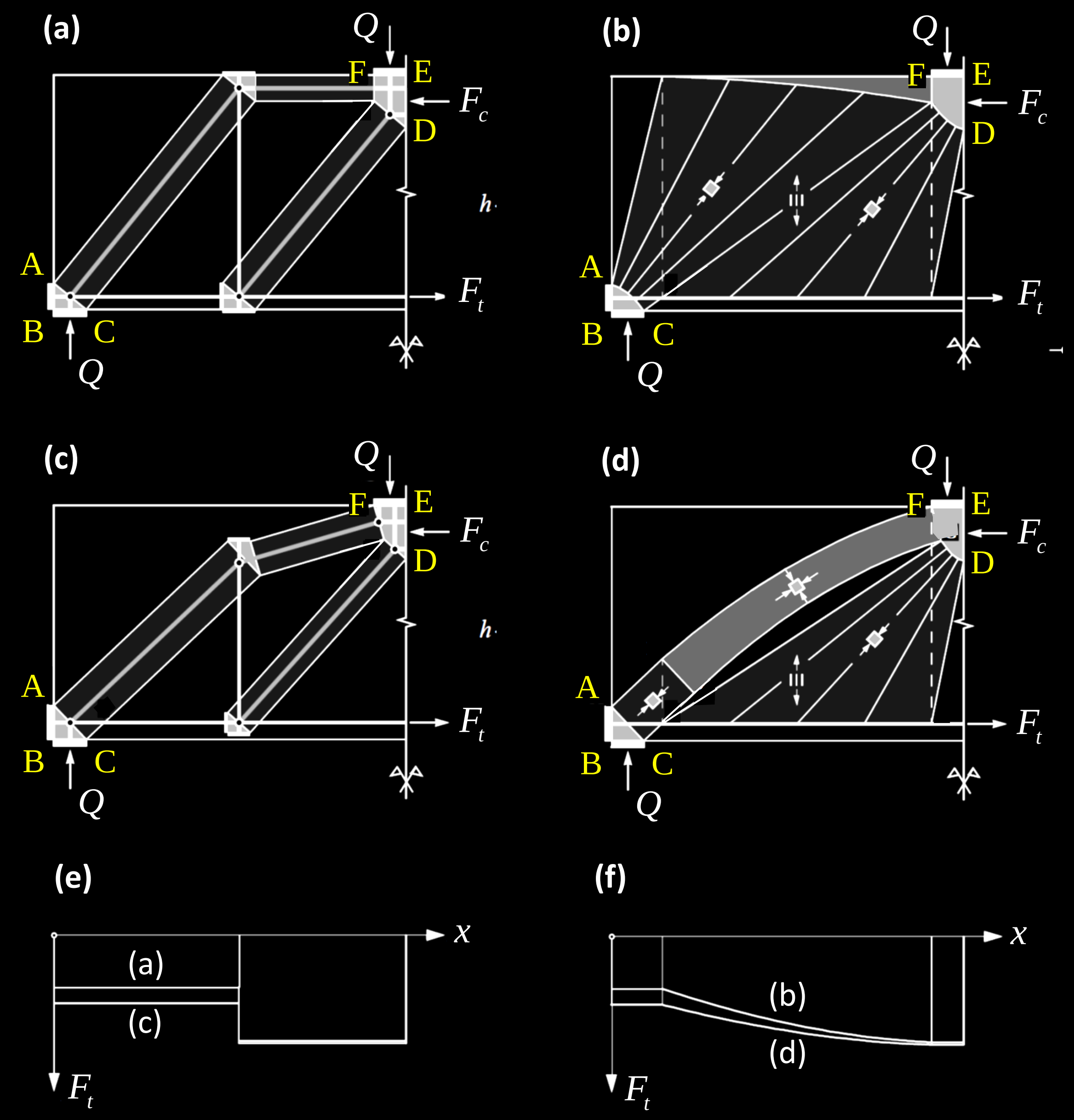
The minimum amount of vertical reinforcement (stirrups) can be activated by a load suspension mechanism. This reduces the tensile force to be anchored behind the support.

In all four models the bearing and the load introduction plates (B-C, E-F) are identical. Therefore, the lower bounds of the ultimate load are identical in all models as well.

The stress fields (right) can be derived from the simple strut-and-tie models (left).

The entire load can be suspended (upper models) or only a part of it (lower models).

The distribution of the force in the tension chord (lowest row) and the force to be anchored behind the support can be derived from the respective stress fields.



Stress fields

Load suspension mechanism

(a,b): Load *Q* totally suspended

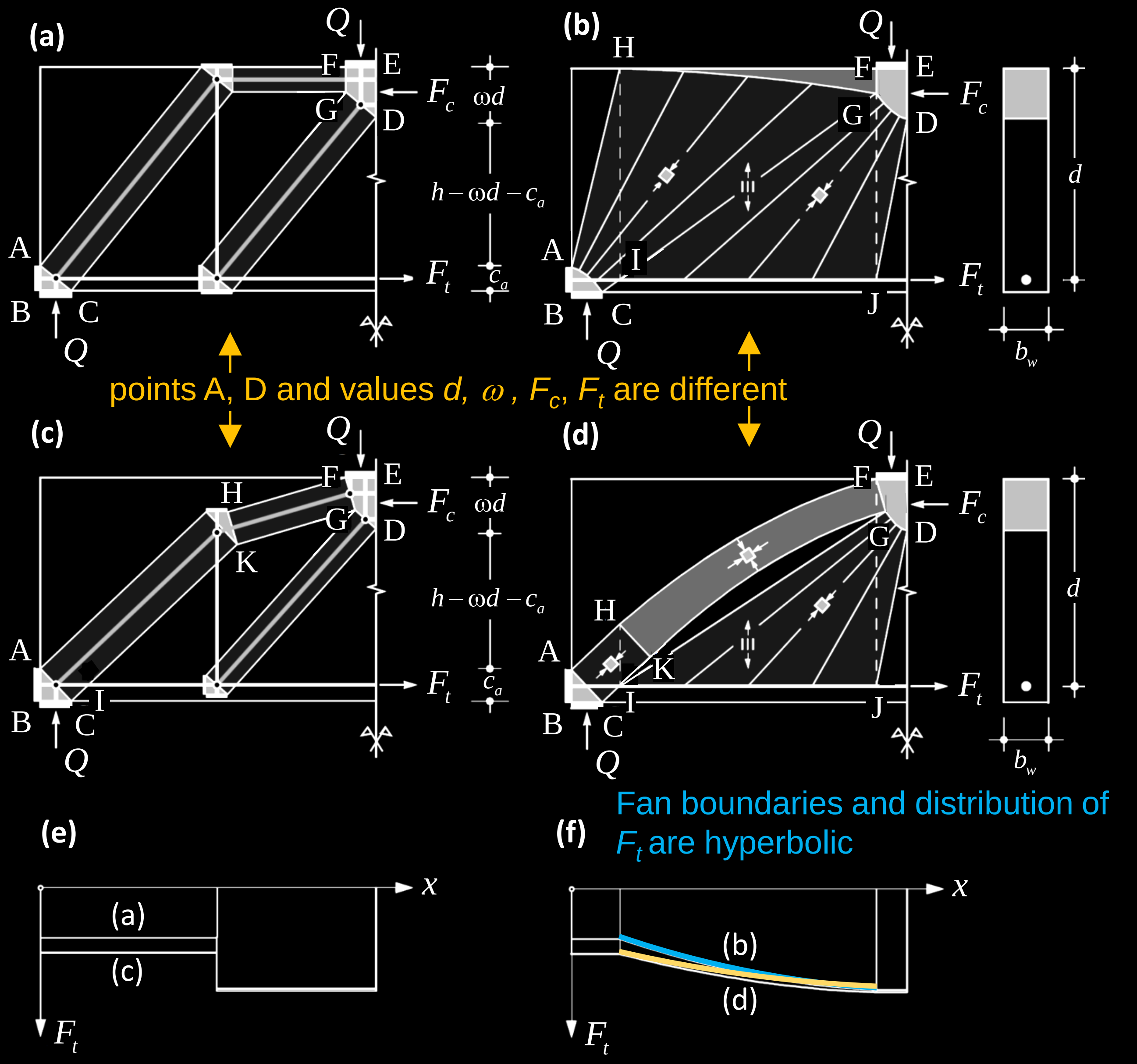
- Larger amount of stirrups required, but smaller tensile force to be anchored

(c,d): Load *Q* partially suspended

- Lower amount of stirrups required, but larger tensile force to be anchored

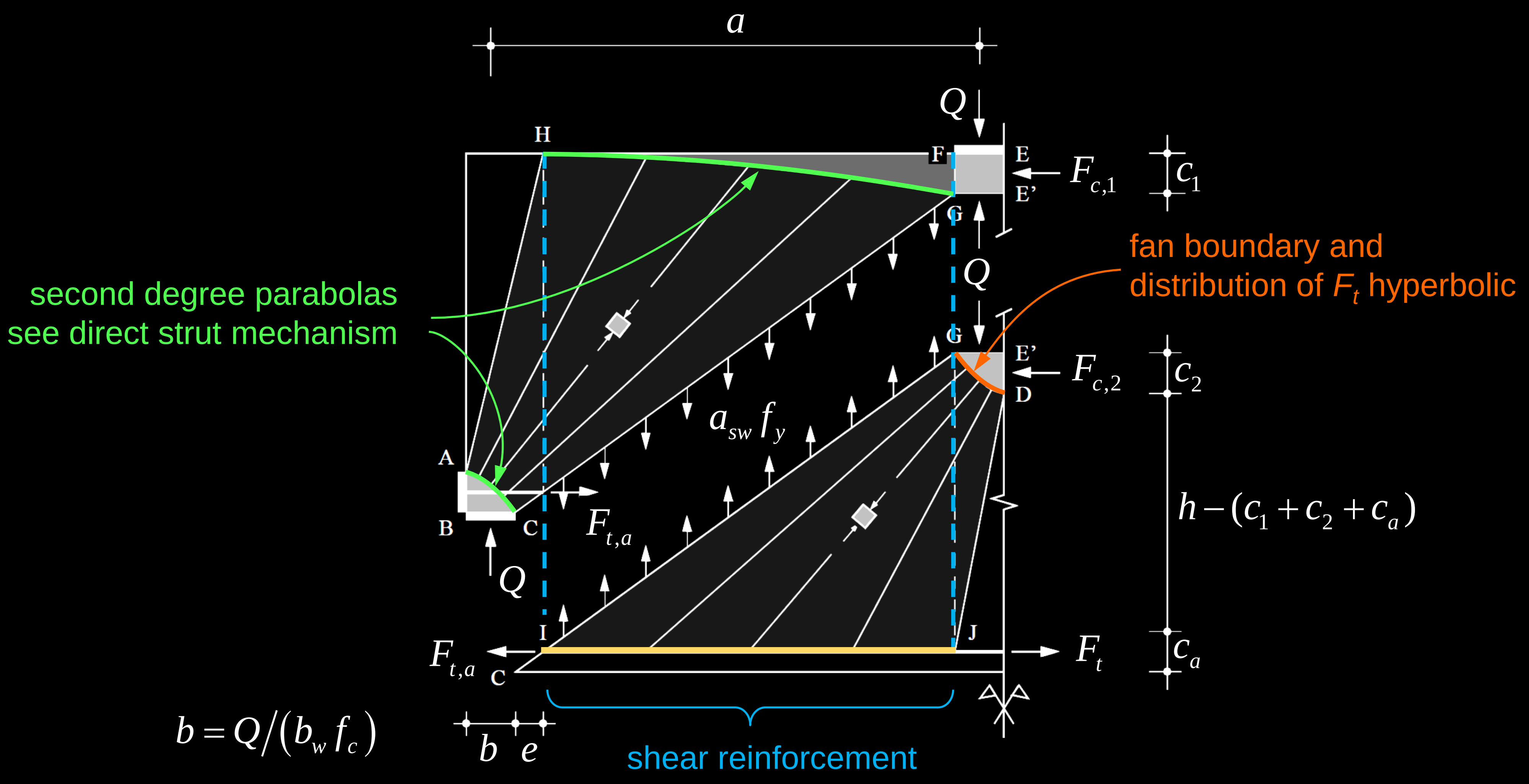
Tension chord force distribution F_t

- (a,c) steeped, (b,d) continuous
- F_t in all cases lower than with direct strut mechanism



Stress fields

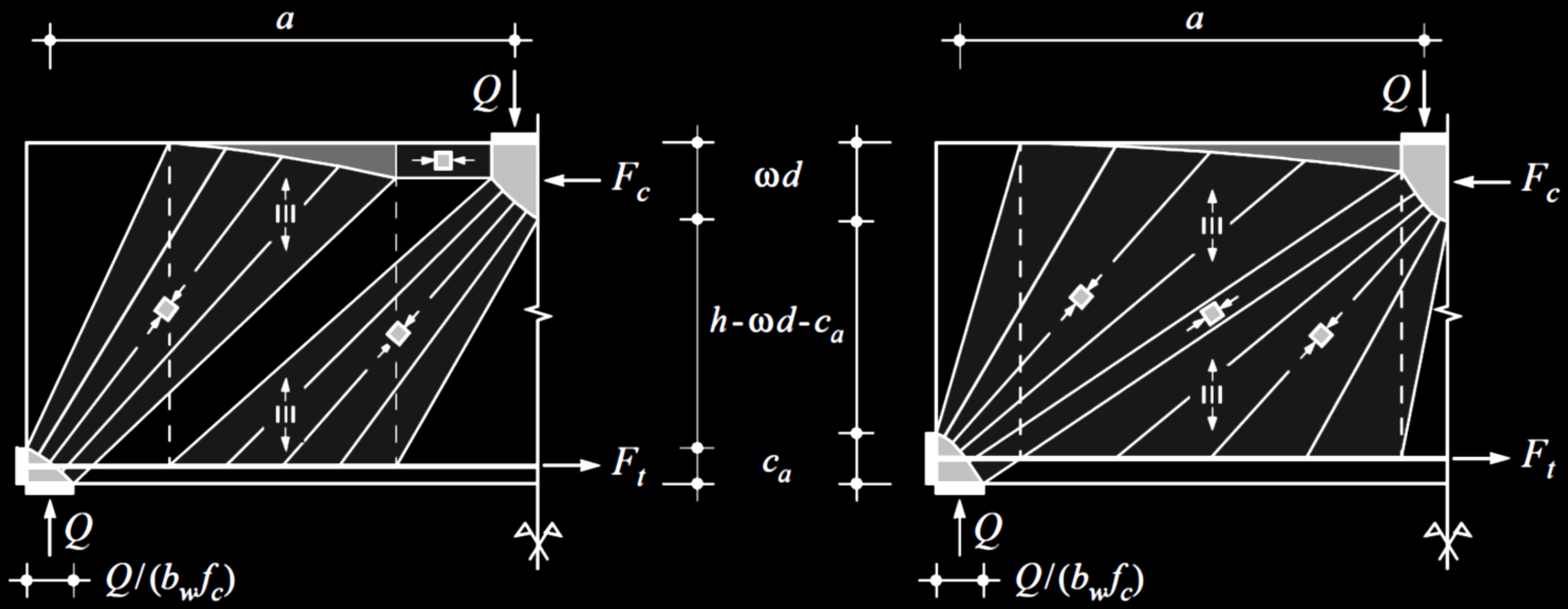
Load suspension mechanism (detail model b)



Stress fields

Load suspension mechanism: combination of basic models

Other possible stress fields (concentrated shear reinforcement, combined direct strut and suspension mechanisms suspension)



Information Sheet: Strut-and-Tie Model

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Advanced Structural Concrete

Information Sheet: Strut-and-Tie Model

(101-0127-00L)

Stress fields and strut-and-tie models (STMs) are common tools for practicing engineers to design and assess membrane elements such as beams and walls. For design, the engineer is free to choose a suitable solution and dimension the reinforcement accordingly, whereas for assessment the engineer needs to verify the load-bearing capacity of the chosen solution. The design principles of simplicity, stiffness, and efficiency should be followed.

Here, a short step-by-step approach for the choice of a suitable STM is given, shown with an example of a simply supported beam subjected to partially distributed load q (see 0. in the figure below). The enumeration corresponds to the number in Figure 1.

1. Solve the static system.
2. Define locations and forces of chords ($F = \frac{M}{z}$).
3. If there are distributed loads, select a suitable stress field inclination α (usually 25..45°)
4. Propose a basic STM. Ensure the force flow and that equilibrium in all nodes is fulfilled.
5. Calculate the forces in the STM.
6. Calculate the required amounts of reinforcement A_s.
7. Detailing (verify concrete forces and define the location of reinforcement)
   a. Nodal zones
   b. Critical details with stress fields

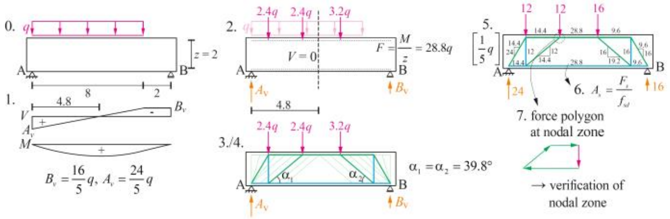


Figure 1: Example of a suitable strut-and-tie model for a simply supported beam with partially distributed load q.

see website

Stress fields

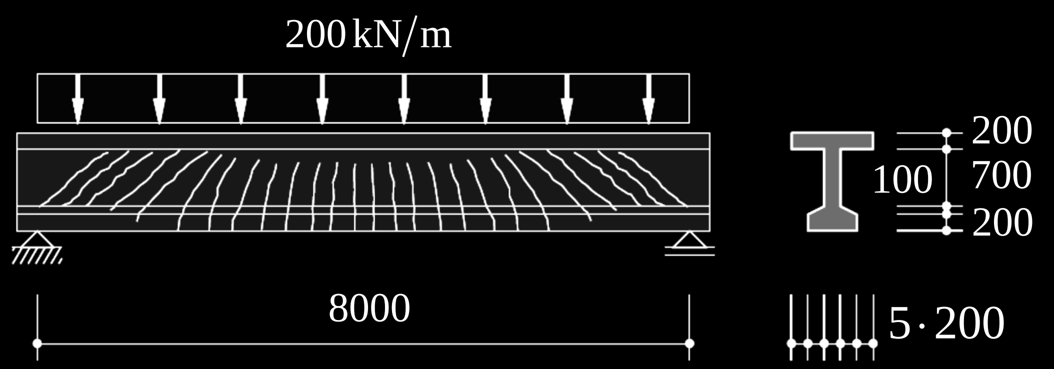
Beam - Example 1 (see [4] p. 66 ff)

Beam with distributed load, expected crack pattern

→ Idealization as a plane element

→ Upper/lower chord reduced to the centre of gravity of the axes: "Stringer"

→ Web modelled as plane membrane element



Possible strut-and-tie model

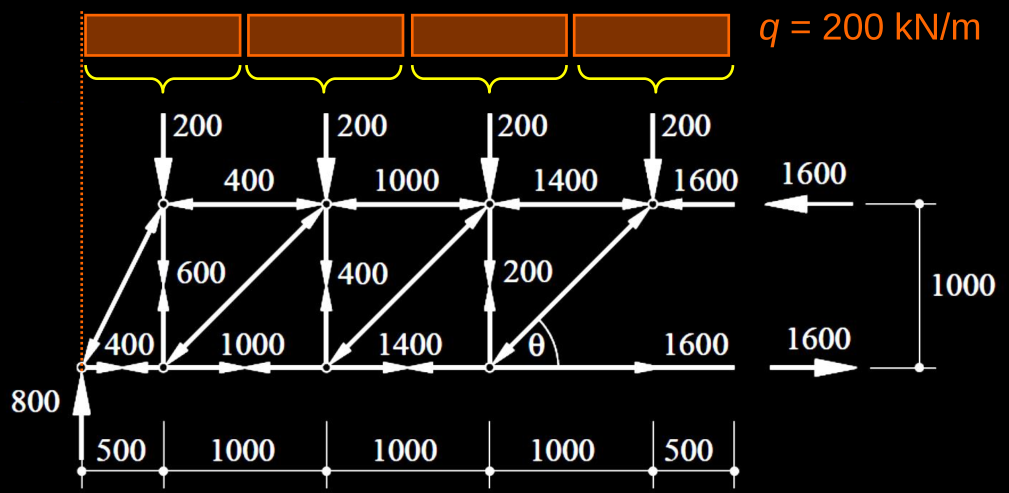
→ Upper chord and struts (concrete) = compression forces

→ Lower chord (longitudinal reinforcement) and vertical ties (stirrups) = tensile forces

→ Distributed load reduced to statically equivalent individual loads in the nodes of the upper chord

→ Correct geometry: nodes chosen in such a way that equivalent static nodal forces can be applied (thus the first compression diagonal is steeper!)

→ If required, strut-and-tie models can be refined into stress fields



(forces in kN, dimensions in mm)

Stress fields

Beam - Example 1 (see [4] p. 66 ff)

Various possible strut-and-tie models

→ Different inclinations of the diagonal concrete struts

→ Flatter struts:
  - more longitudinal reinforcement
  - less shear reinforcement

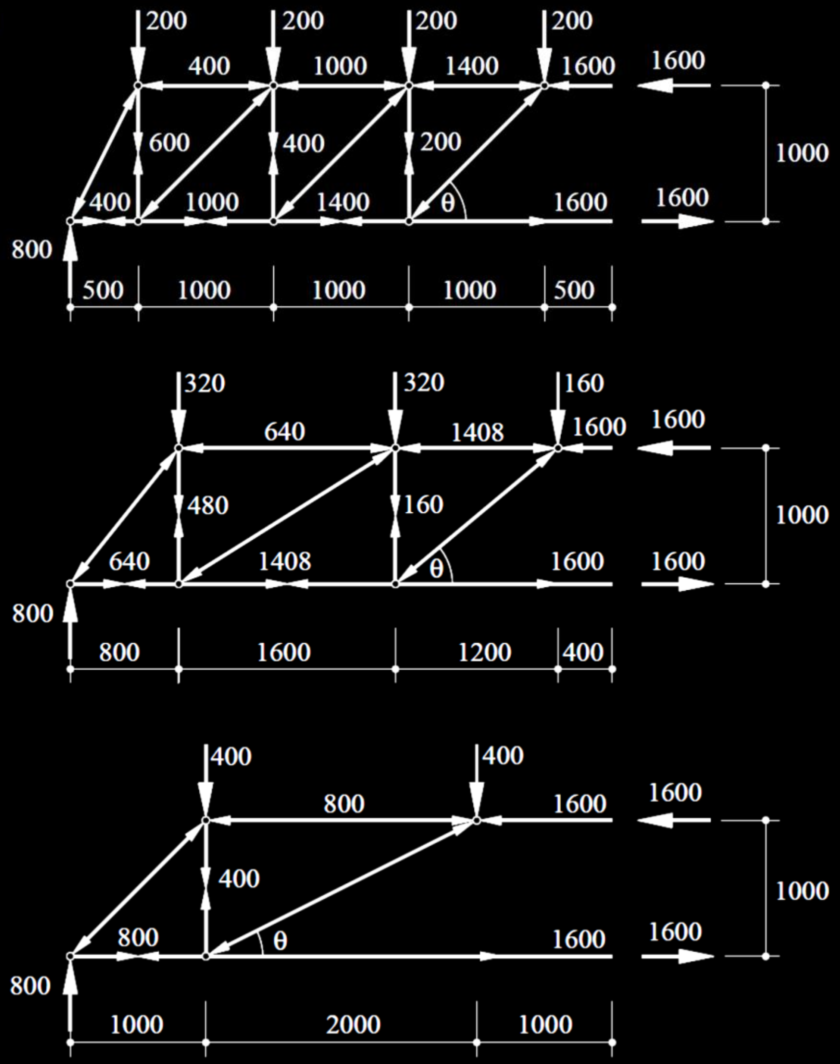
→ The influence of the inclination of the diagonal concrete compression field on the total reinforcement volume is low

Note:

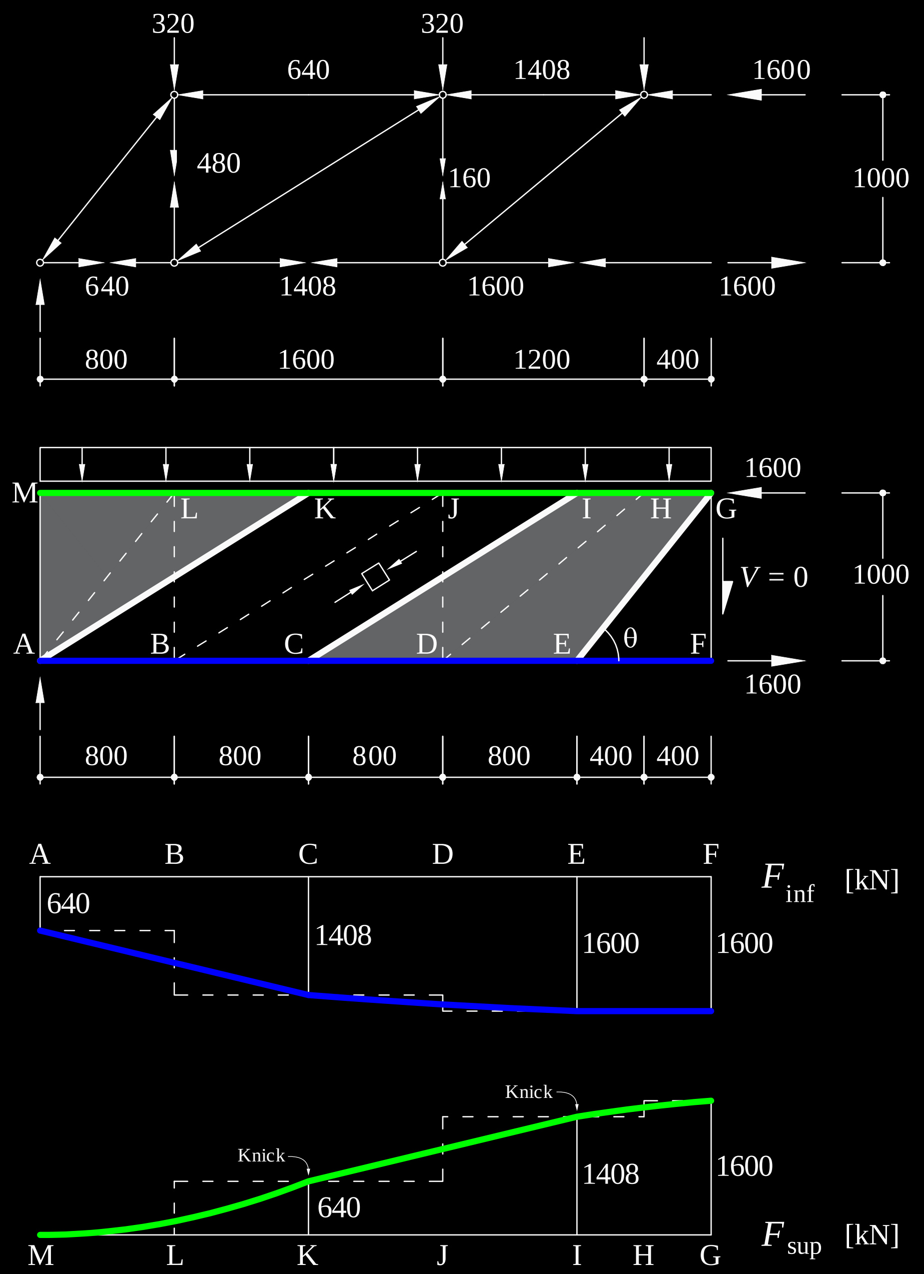
→ Assessment of existing bridges designed according to earlier standards (inclined principal tensile stresses): The ultimate limit state verification is often only possible with very flat inclinations

→ Very flat inclinations lead to large vertical strains in the web → Concrete compressive strength is affected, brittle failure of stirrups can occur

→ SIA 262: $k_c = \dfrac{1}{1,2 + 55\varepsilon_1} \leq 0.65$



Stress fields



Beam - Example 1 (see [4] p. 66 ff)

Strut-and-tie model and corresponding stress field

→ Dashed lines = lines of action of the stress resultant of the individual elements of the stress field = struts and ties

→ Resultants of the stress fields = values of the truss forces

→ Tension and compression stringers AF and GM, fan CEGI, fan AKM centred at support point A, parallel compression tie ACIK, vertical ties CEIK and ACKM

→ Determination of the chord forces = Stringer forces: Equilibrium of the load acting along the chord axes and the forces acting in the individual elements.

→ Force distribution parabolic along fan edges CE, GI and KM, and linear along the edges of the parallel compression field (AC and IK)

→ Vertical ties CEIK and ACKM: uniformly distributed forces (100 kN m^{-1} and 300 kN m^{-1} respectively)

→ Chord forces from stress field and strut-and-tie model are identical in sections CK and EI (stirrup forces = discontinuity lines of the vertical ties)

Stress fields

Beam - Example 1 (see [4] p. 66 ff)

Propagation of the compressive force into the upper flange

→ Simple 45º truss model (can be refined into a stress field)

→ Applied longitudinal force = Gradient of the longitudinal force in the compression stringer = horizontal component of the compression forces in fans and parallel compression tie along GM

→ The longitudinal force is supported by inclined struts on compression stringers (arranged in the centre of gravity axes of the upper flange).

→ This results in transverse tensile forces → requires transversal reinforcement

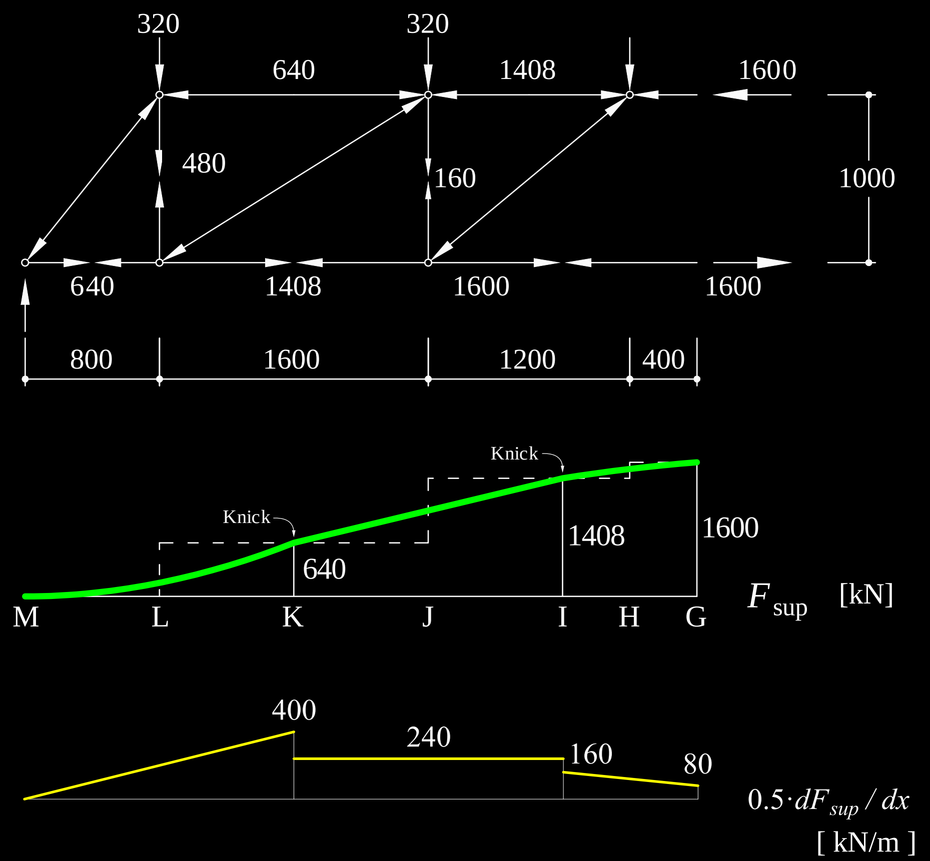
→ Consideration of web width = 200 mm in the strut-and-tie model = reduction of the transversal reinforcement of the upper flange

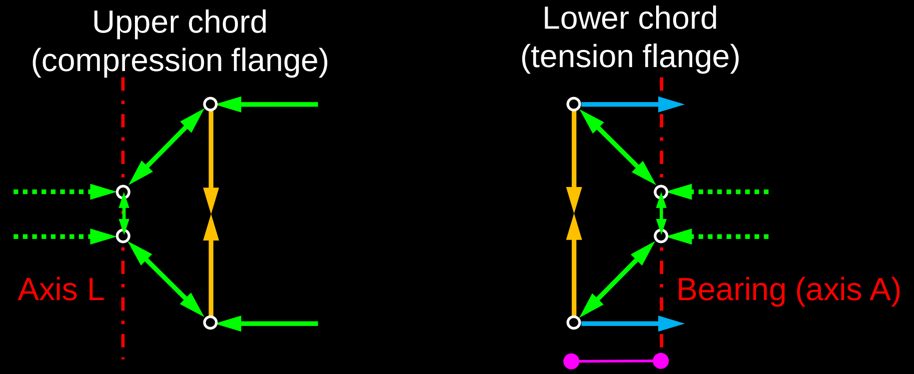
Propagation of the tension chord force into the lower flange

→ Analogous considerations (load is spread by means of transversal reinforcement to the longitudinal reinforcement bars distributed in the flange)

→ Load spreading at the support A requires extending the longitudinal reinforcement above the support (in the order of half of the flange width to fully activate all tensile reinforcement)

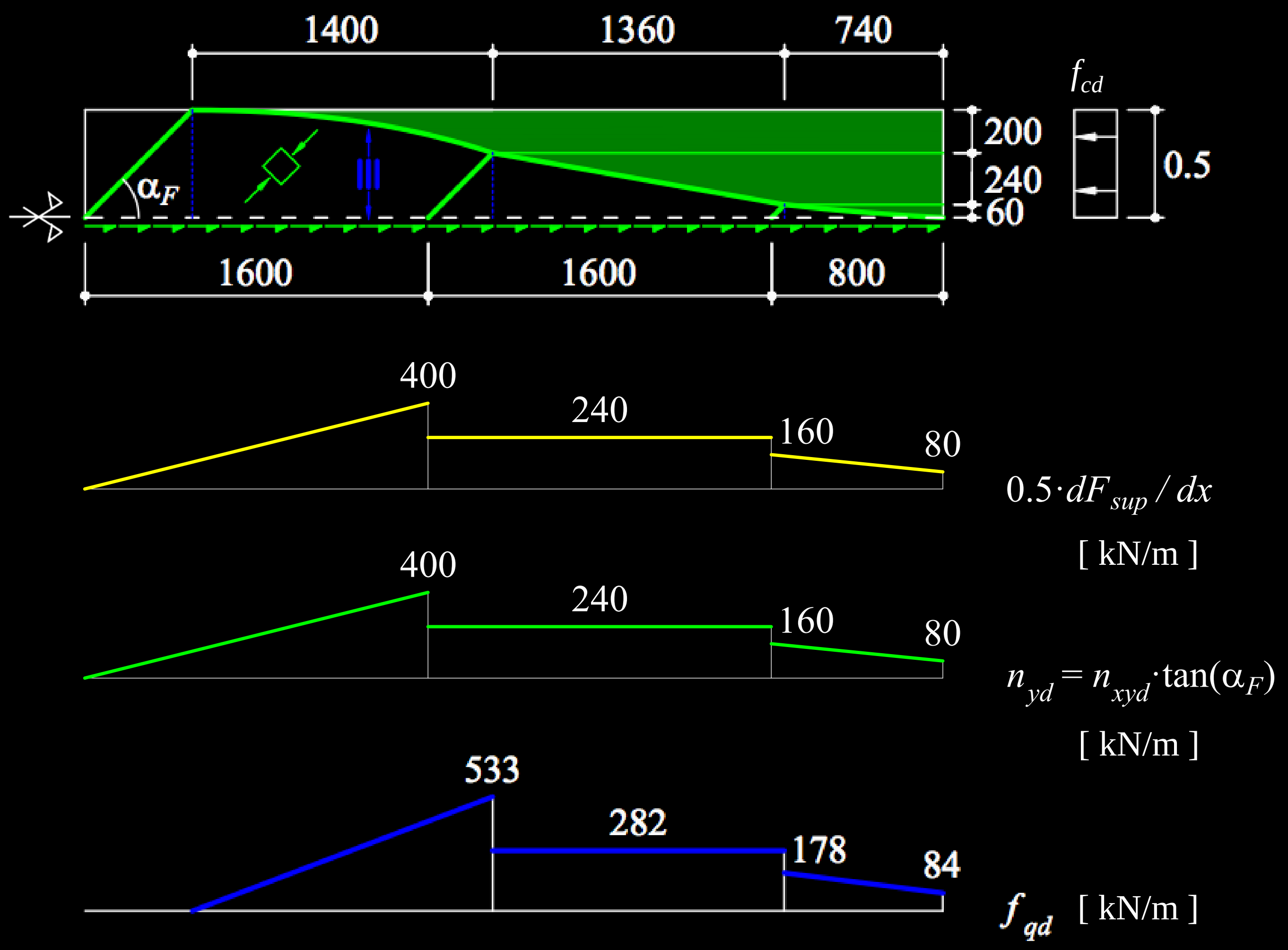
→ Without extending the longitudinal reinforcement above the support all the required tensile capacity (640 kN) at the support should be provided exclusively by the reinforcement below the web.



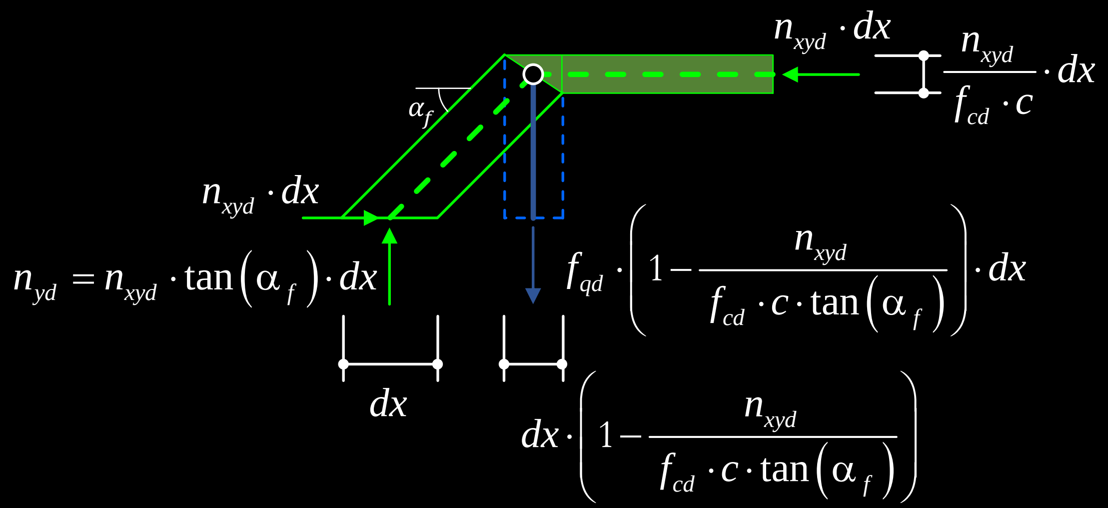


Stress fields - Transversal shear

Example – Top view of a T-beam



General:

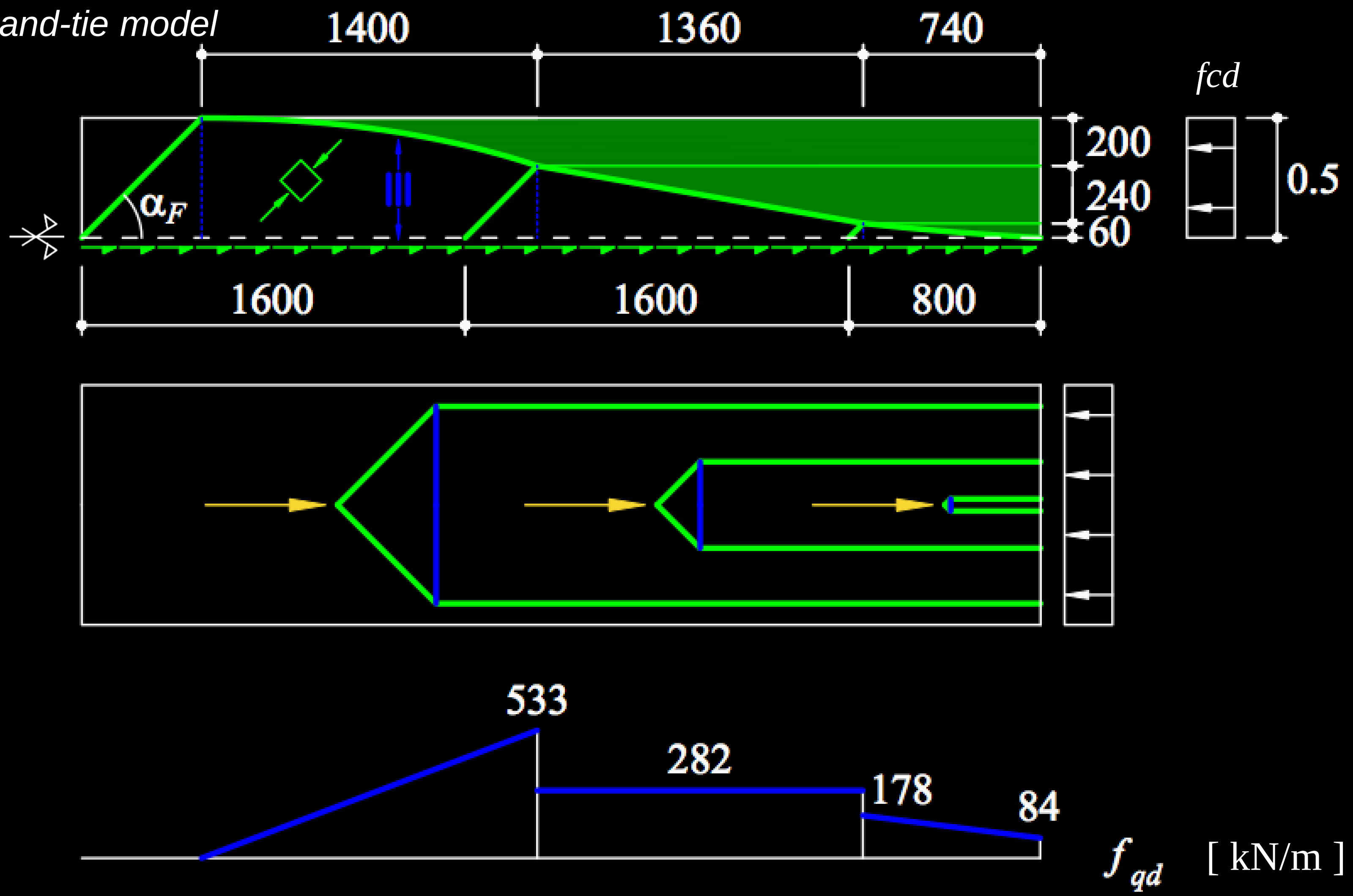


$$f_{qd} = n_{xyd} \cdot \tan(\alpha_f) \cdot \left(1 - \frac{n_{xyd}}{f_{cd} \cdot c \cdot \tan(\alpha_f)}\right)^{-1}$$

Stress fields - Transversal shear

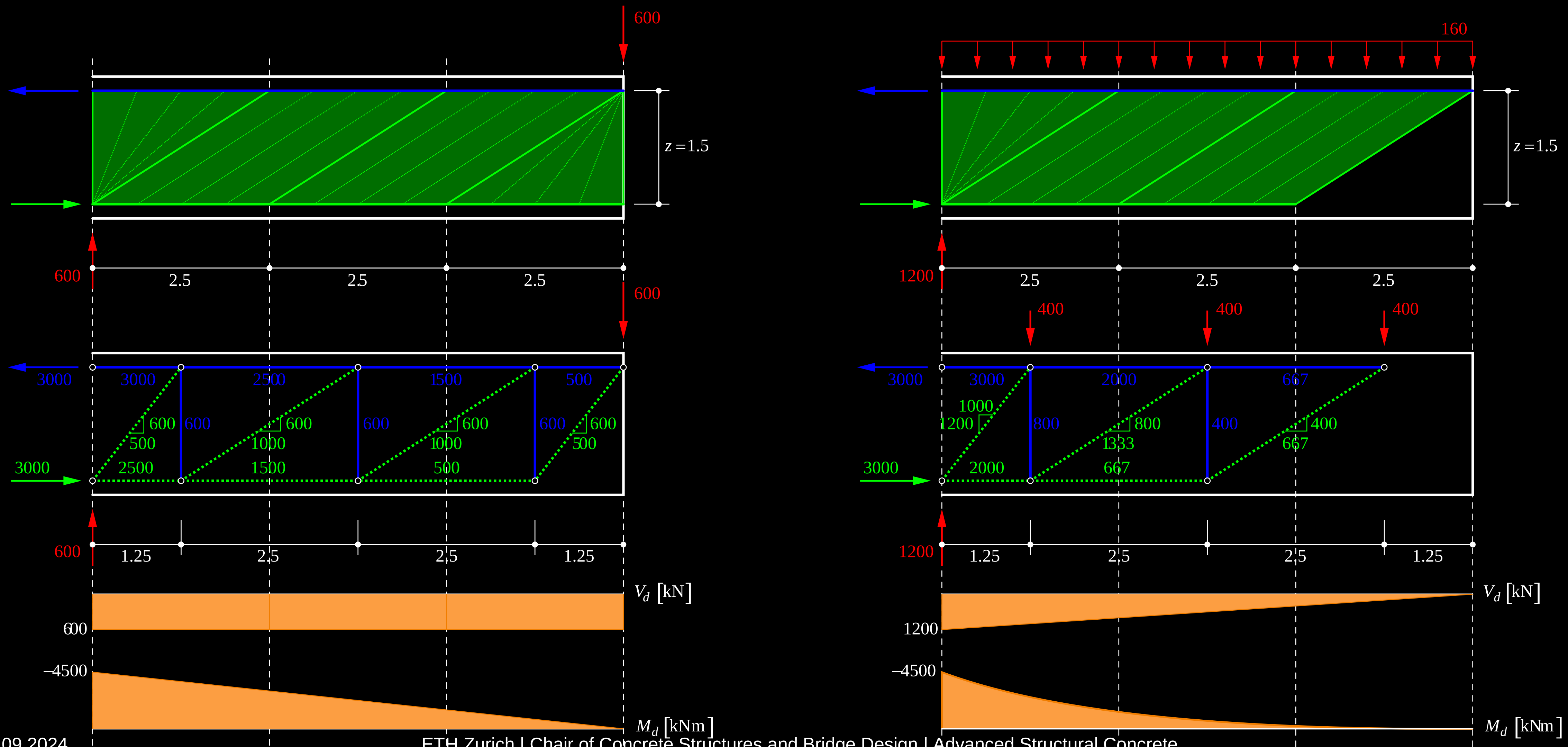
Example – Top view of a T-beam

... and the associated strut-and-tie model



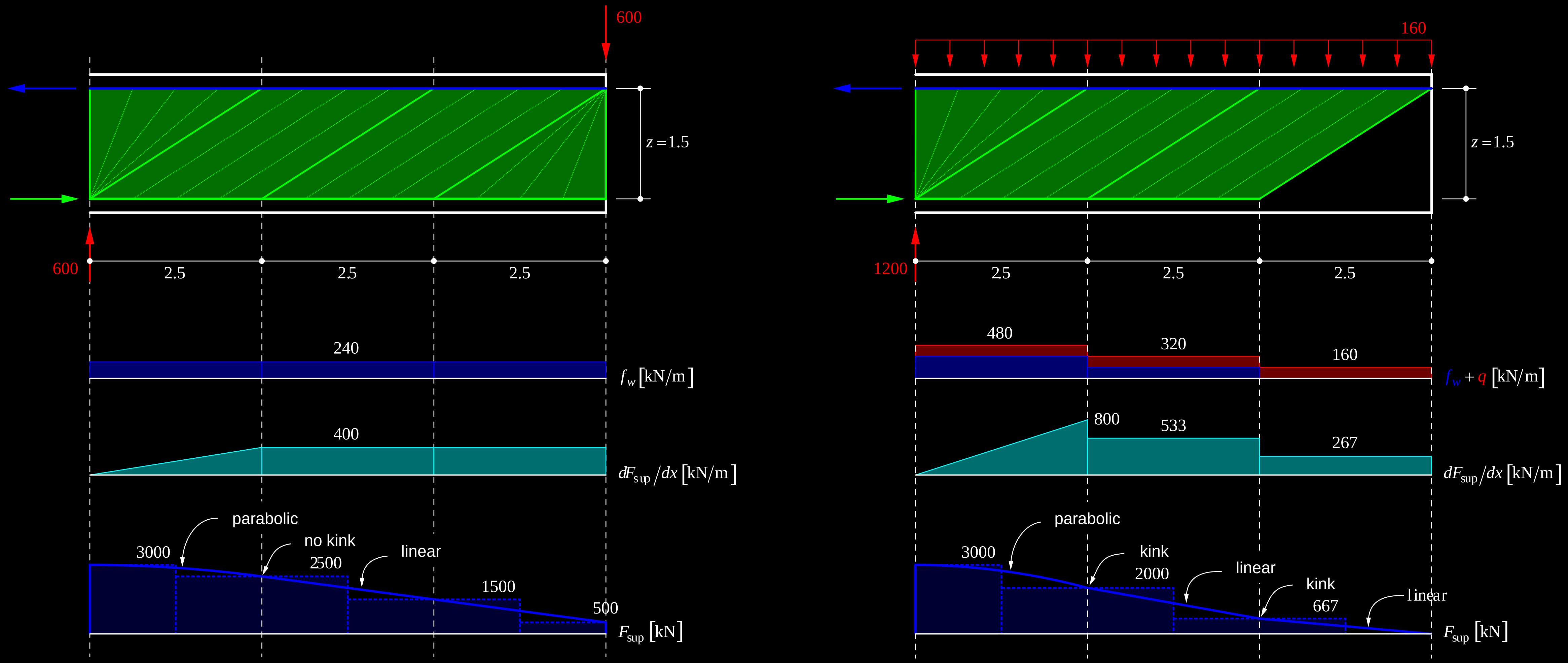
Stress fields

Example 2: Cantilever beam with point and distributed load



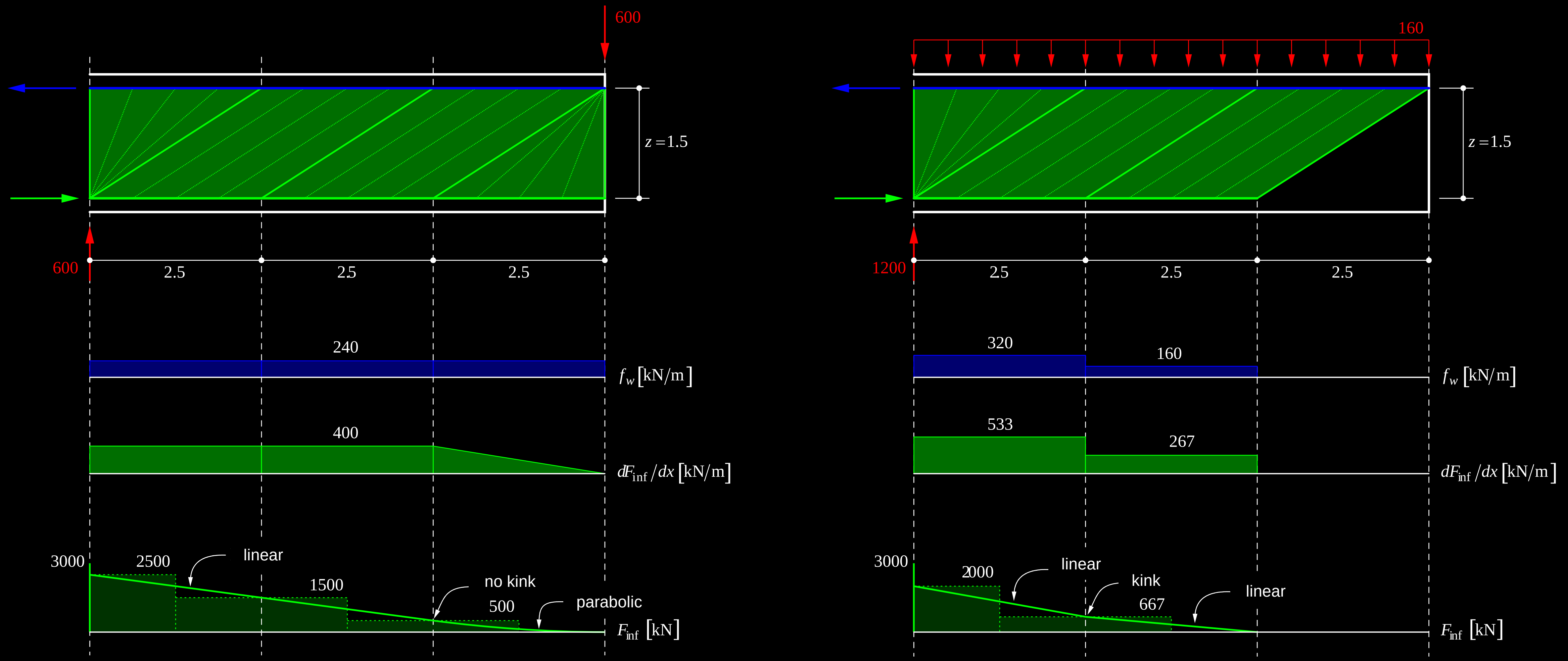
Stress fields

Example 2: Cantilever beam with point and distributed load



Stress fields

Example 2: Cantilever beam with point and distributed load



Stress fields

Nodal zones

(a) General nodal zones: Struts with $\sigma_A \neq \sigma_B \neq \sigma_C$
(Forces in equilibrium!)

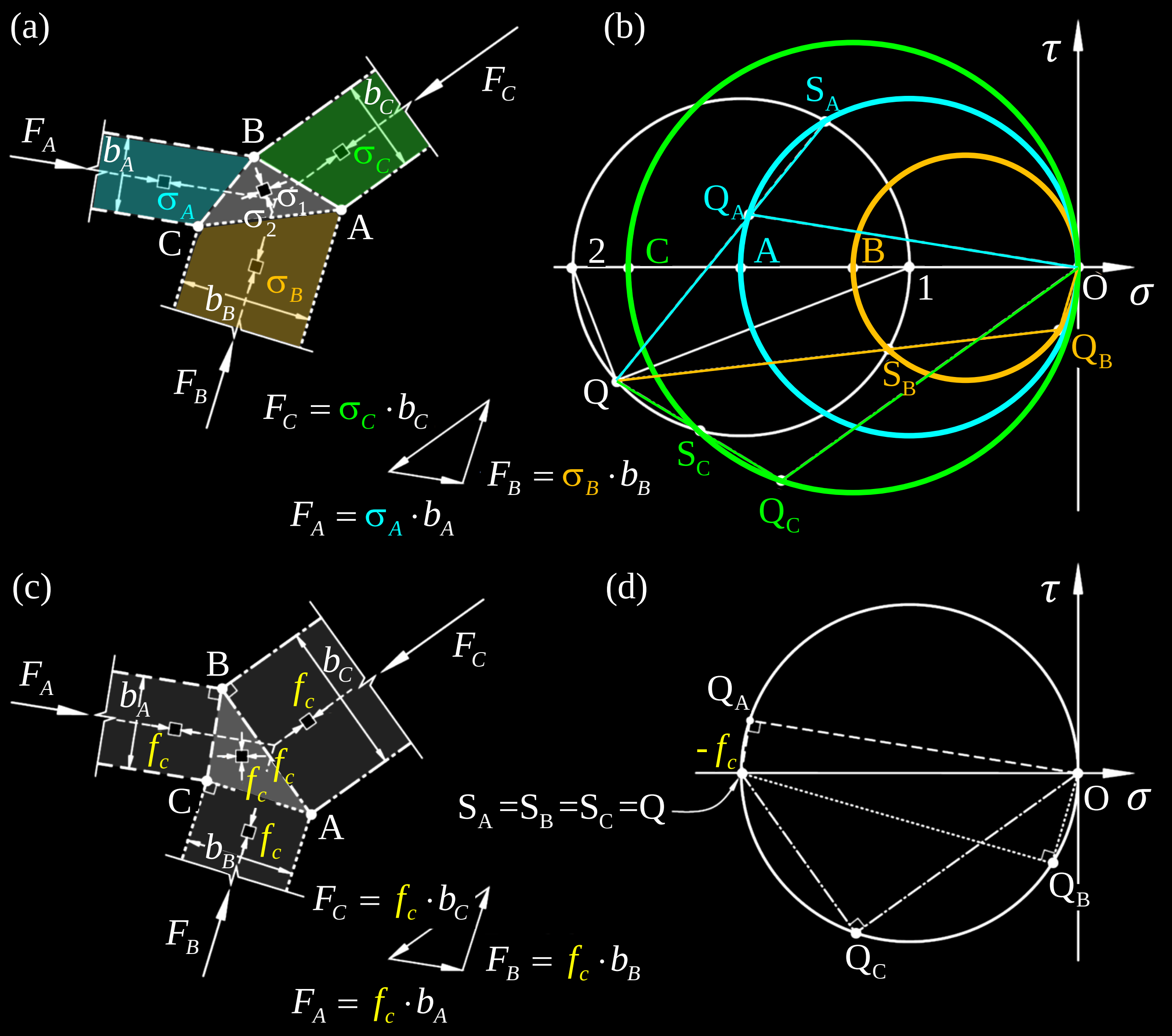
→ Compressive stress in the nodal zones $\sigma_2 < \min(\sigma_A, \sigma_B, \sigma_C)$, except if node boundary ⊥ corresponding strut

→ Connecting line of the poles of Mohr's circles of stress states on both sides of a discontinuity line // stress discontinuity line

(c) nodal zone with $\sigma_A = \sigma_B = \sigma_C$ (relevant in practice)

→ Node boundaries ⊥ struts, node geometry affine to polygon of strut forces (equilibrium)

→ «Hydrostatic» stress condition $\sigma_1 = \sigma_2 = f_c$
(strictly speaking not hydrostatic, as $\sigma_3 = 0$)



Stress fields

Nodal zones

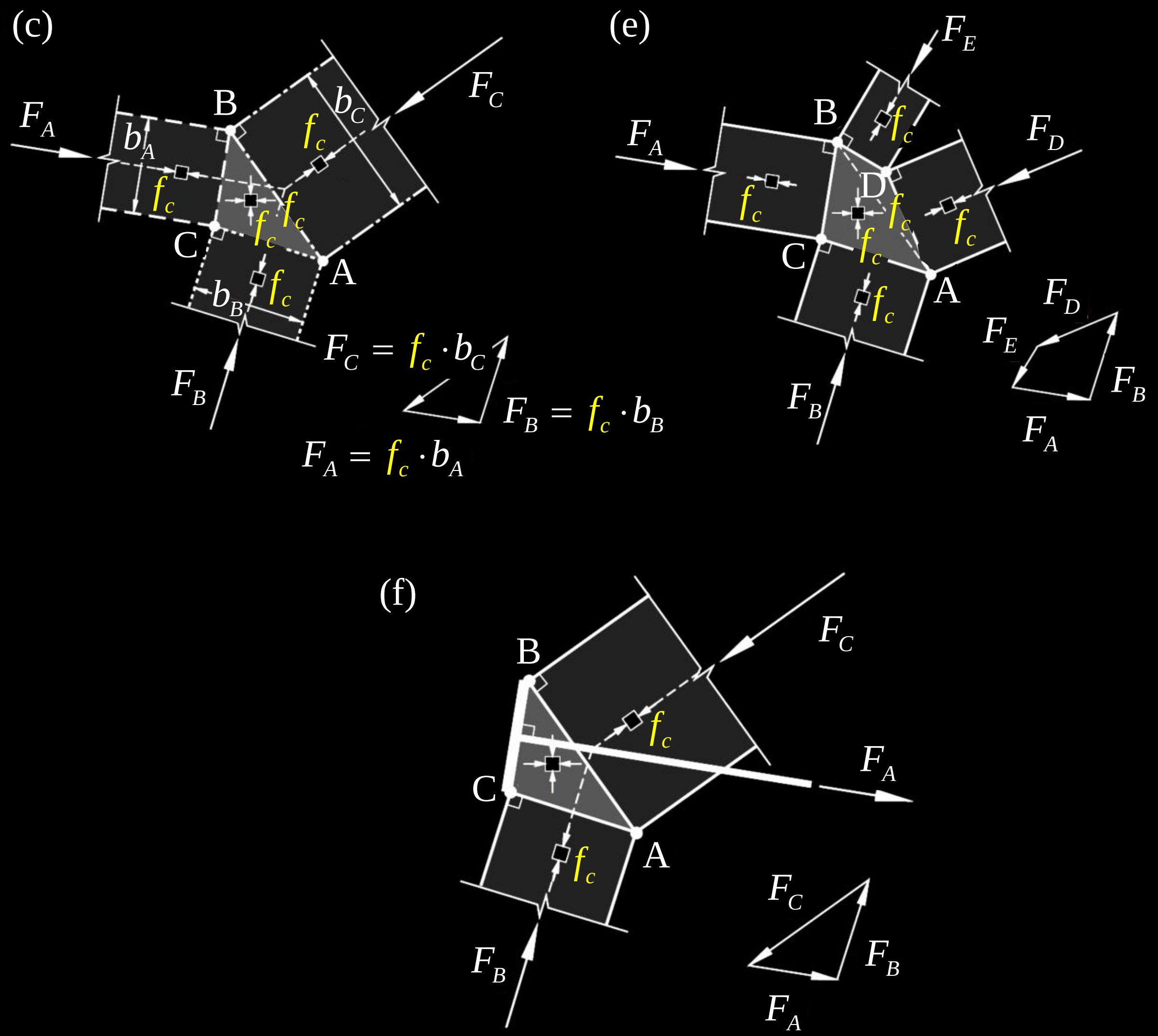
(e) Replacement of a strut (C) by two statically equivalent struts (D, E)

→ Only the shape of the node boundary around the replaced strut changes, the remaining boundaries stay the same.

→ Useful when considering fan stress fields (node dimensions based on the stress resultant = define node dimensions on a simple strut-and-tie model; exact shape of the boundary is usually not relevant)

(f) Treatment of tensile forces

→ Anchored behind the nodal zone, treated like a compressive force (see constructive solutions in the next slide)



Information Sheet: Nodal zones

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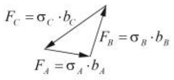
Advanced Structural Concrete

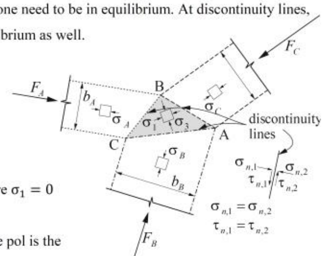
Information Sheet: Nodal Zone Verification[1]

(101-0127-00L)

For practice, the stresses in the struts are usually assumed to be equal and the width is adapted accordingly. This leads to a biaxial uniform stress state[2]: $\sigma_1 = \sigma_2 = f_c$, which simplifies the verification.

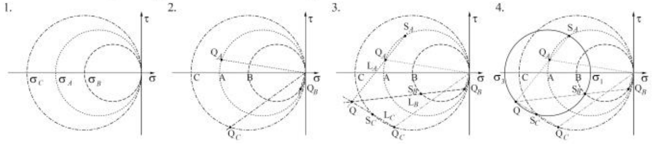
For a general nodal zone ($\sigma_A \neq \sigma_B \neq \sigma_C$, often called CCC node), the approach of the verification is explained as follows. First, the acting forces on the nodal zone need to be in equilibrium. At discontinuity lines, normal and shear stresses, σ_n and τ_n, need to be in equilibrium as well.





Approach:

1. Draw the Mohr's circles of each acting strut. Here $\sigma_1 = 0$ is assumed, but this does not need to be the case.
2. For each strut, find the corresponding pol Q_i. The pol is the intersection of the Mohr's circle and the principal direction 3 of each strut starting at σ_1 (if starting at σ_3 it would be principal direction 1). The pol Q_i is the point on the Mohr's circle, around which stresses rotate.
3. With the help of the pol, find the point S_i (σ_{ni}, τ_{ni}) which is the intersection of the Mohr's circle and the line L_i, parallel to the discontinuity line of the node boundaries, passing through the corresponding pol Q_i. The intersection of all L_i is the pol Q of the final Mohr's circle. All points S_i lie on the Mohr's circle of the nodal zone.
4. Finally, the Mohr's circle of the nodal zone can be drawn and the corresponding compressive stresses σ_1 and σ_3 can be read from the diagram.



[1] Presented in Lecture 2.1, Slide 39
[2] Often referred to as "hydrostatic" for simplicity although the stress state is not hydrostatic, because the stress perpendicular to the membrane plane is $\sigma_3 = 0$.

lg/hs

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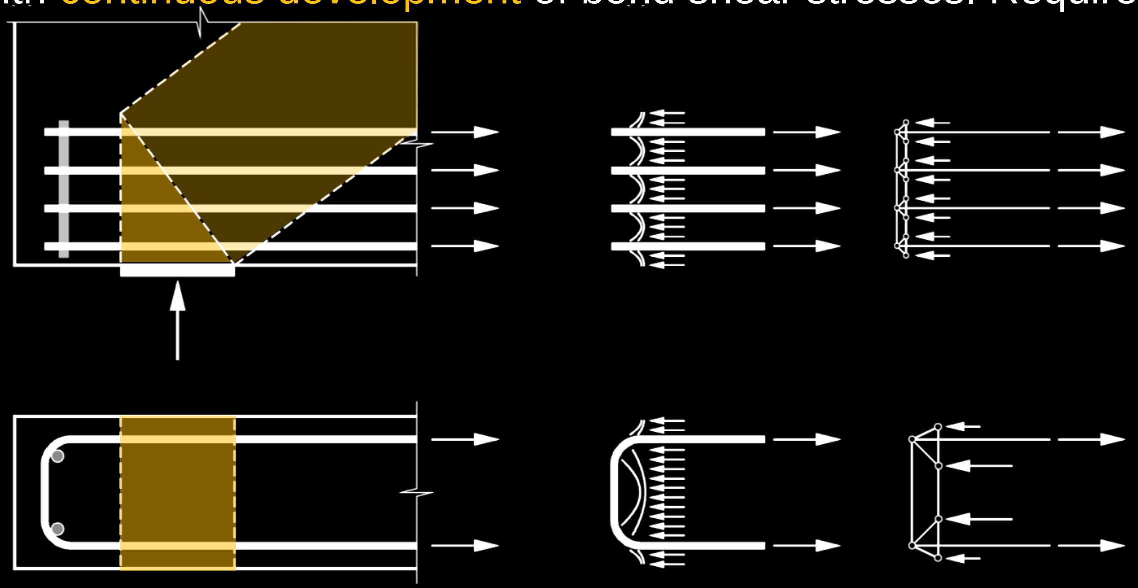
see website

Stress fields

Nodal zones (see [4] p. 64)

- Proper constructive detailing is critical!
- Anchor plates are not frequently used, but sometimes indispensable to anchor high tensile forces.
- Alternative i: U-shaped links see figures below. Local stress field → concrete cover can only be activated by the tensile strength of concrete
- Alternative ii: Use of headed bars (d ≈ 3Ø). Experimentally verified that the anchor length is very short (< 10Ø)

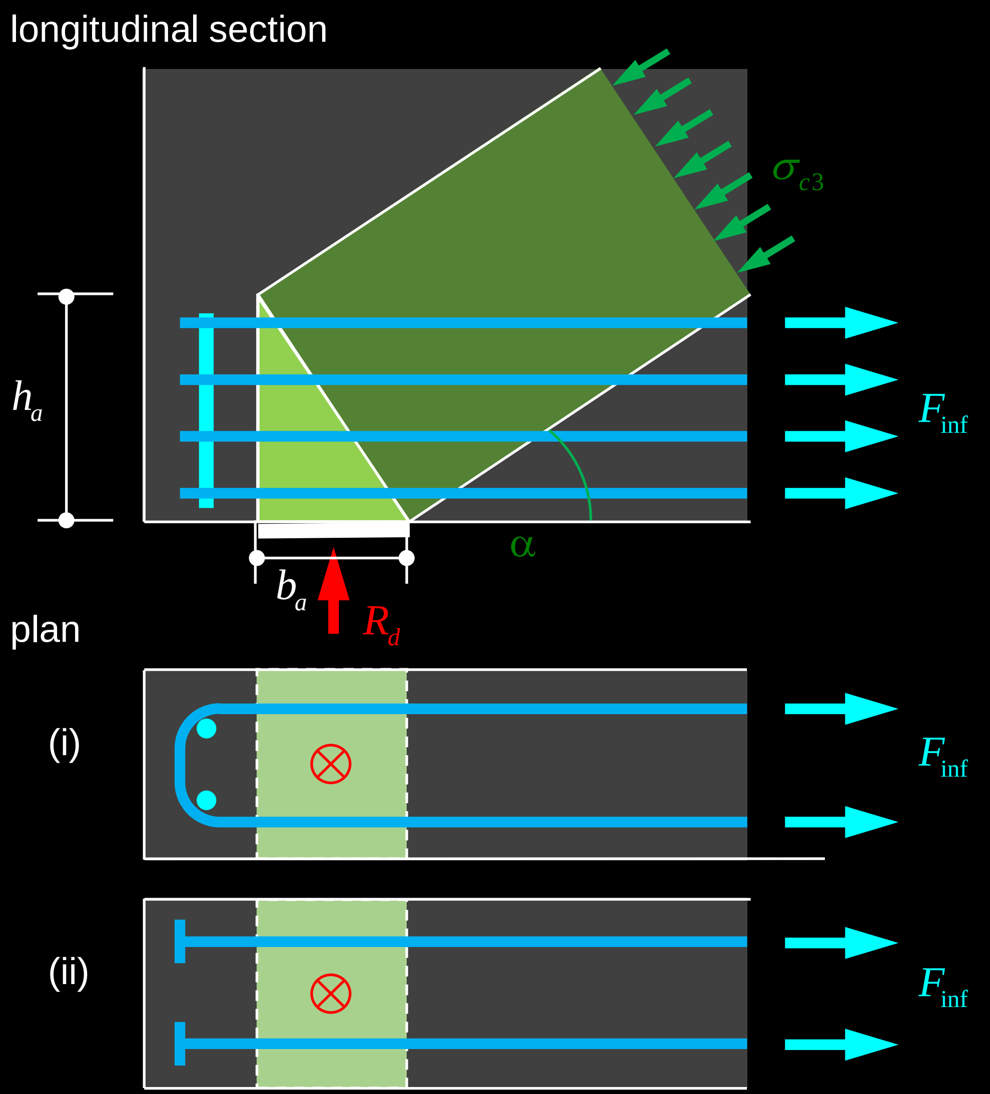
  → Verifcation of the lateral spreading forces!
- Alternative iii: Bent-up flexural reinforcing bars (if enough space to develop a “compression banana" with deviation forces)
- Alternative iv: Stress fields with continuous development of bond shear stresses. Requires larger node dimensions.



Stress fields

Nodal zones (see [4] p. 64)

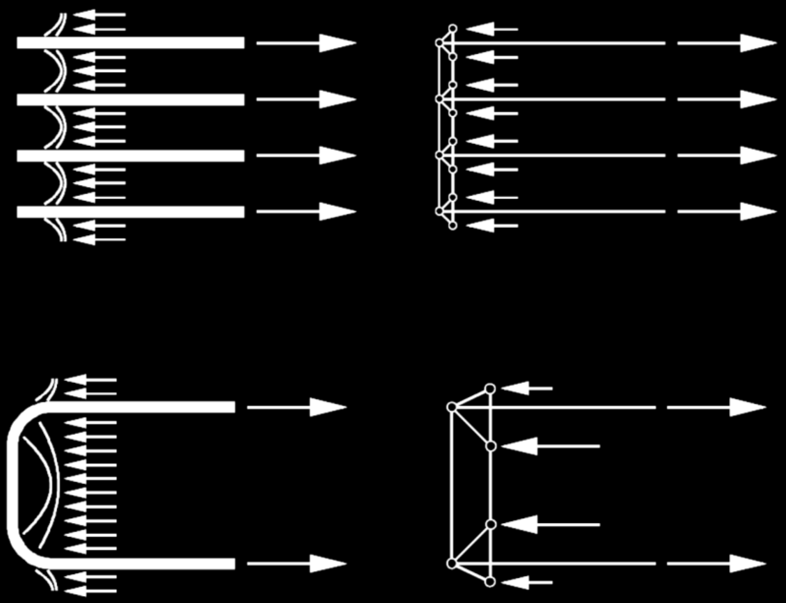
- Proper constructive detailing is critical!
- Simplest solution: nodal zones with $\sigma_h = \sigma_v$ (often referred to as "hydrostatic", but $\sigma_1 = 0$)
- Anchor plates are not frequently used, but sometimes indispensable to anchor high tensile forces.
- Alternative (i): Place U-shaped links, see pictures below. Local stress field → concrete cover can only be activated by the tensile strength of concrete
- Alternative (ii): Headed bars (anchor plate diameter ≈ 3Ø), experimentally verified that the anchor length is very short (< 10Ø). Verify the lateral spreading forces!

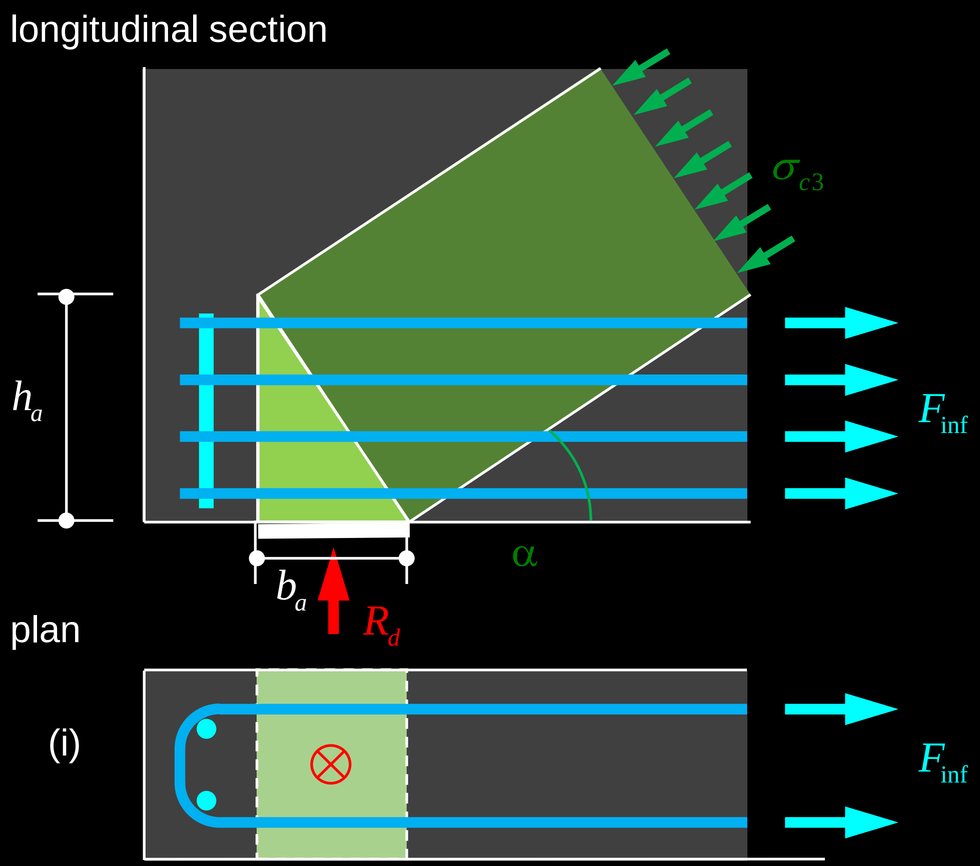


Stress fields

Nodal zones (see [4] p. 64)

- Strictly speaking concrete tensile stresses are required, especially to activate the concrete cover.

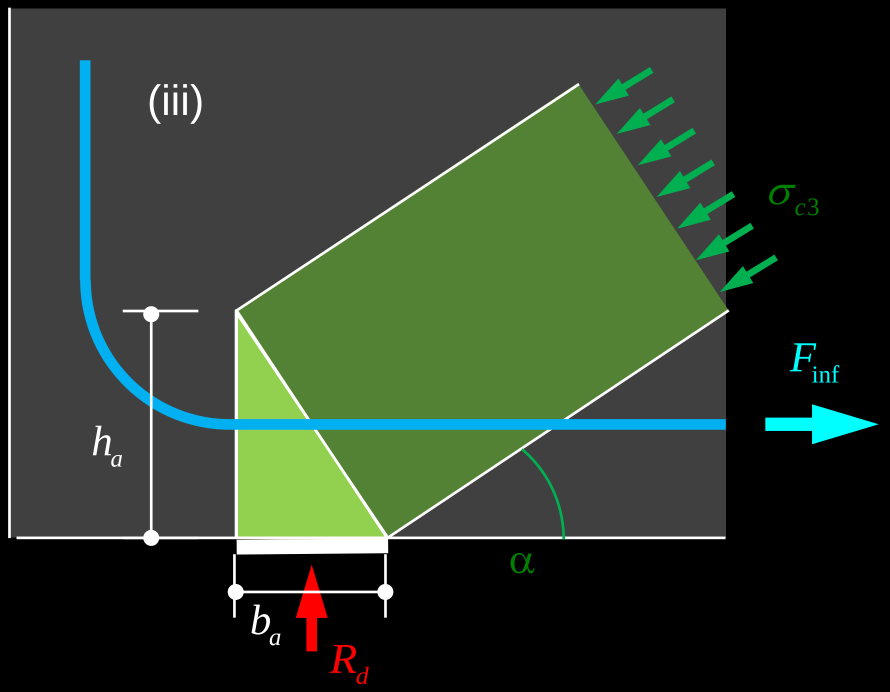




Stress fields

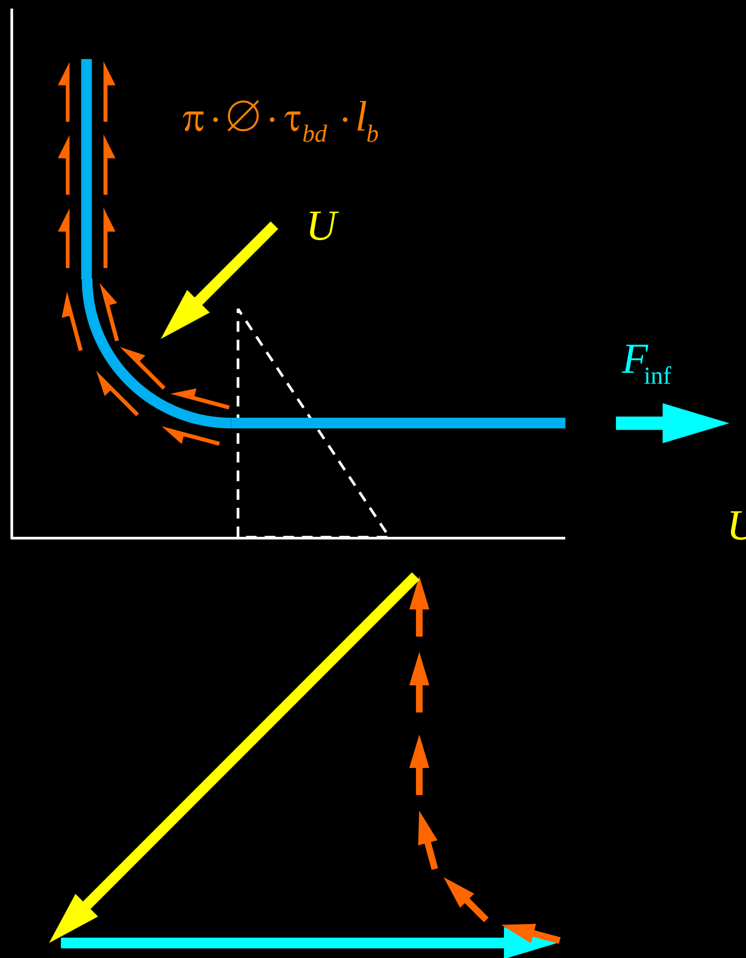
Nodal zones (see [4] p. 64)

longitudinal section

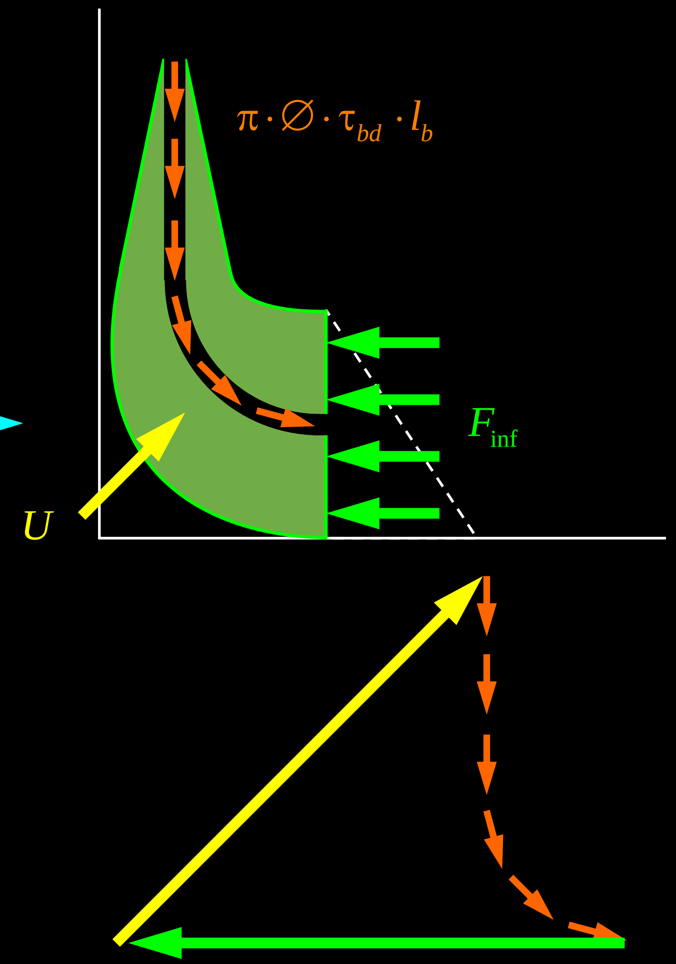


- Solution (iii): Bent-up reinforcing bars can be activated if there is sufficient anchorage length behind the support to anchor it (“compression banana" in the concrete with a deviation force *U*).

reinforcement forces



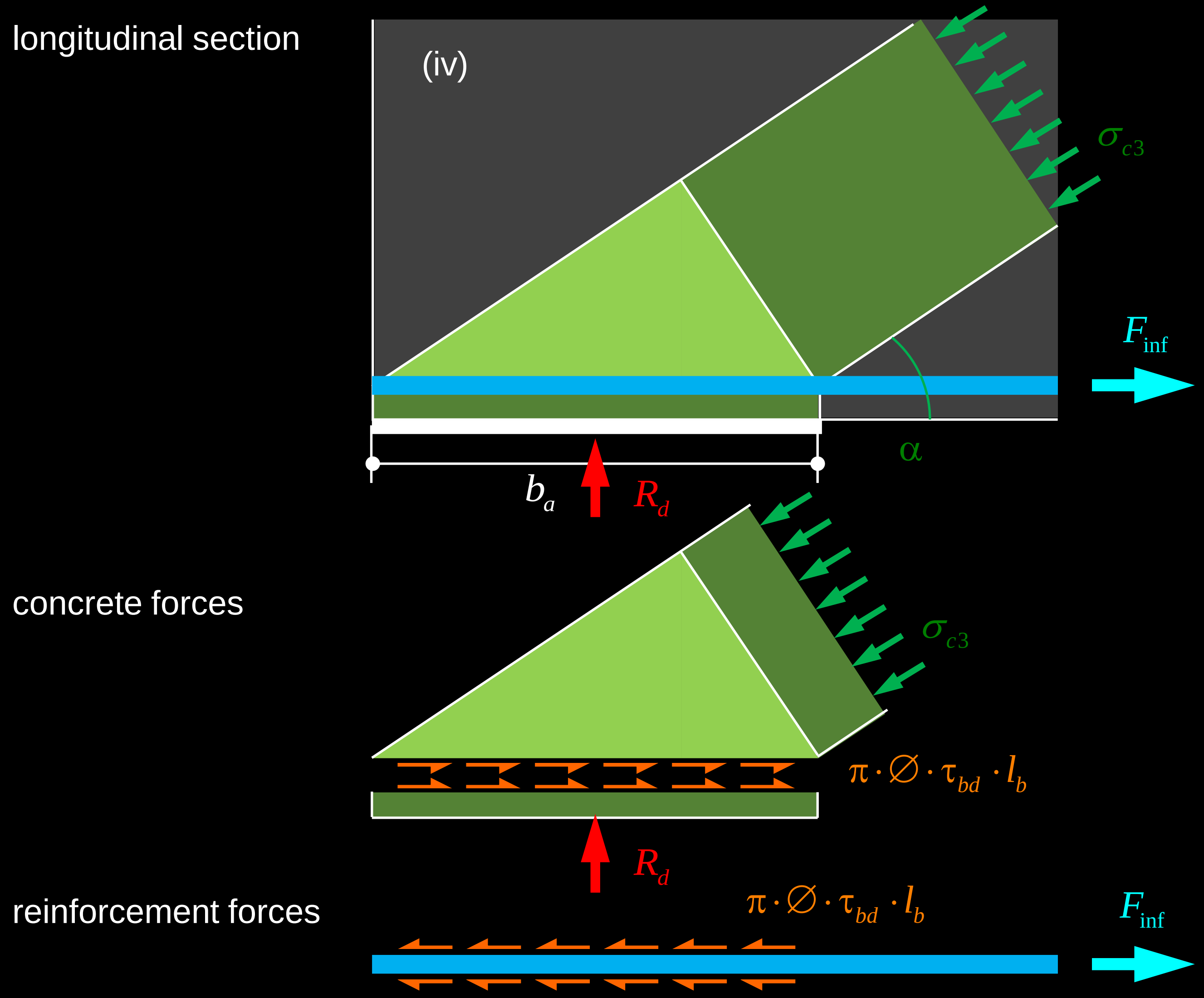
concrete forces



Stress fields

Nodal zones (see [4] p. 64)

- Alternative (iv): Stress field with continuous development of tension chord force through bond-shear stresses.
- Requires larger node dimensions (large anchorage length = node width, despite favorable effect of transverse compression on bond)



Stress fields

Nodal zones (see [4] p. 64)

Disadvantages of solutions (i)-(iii) = "hydrostatic" nodal zone

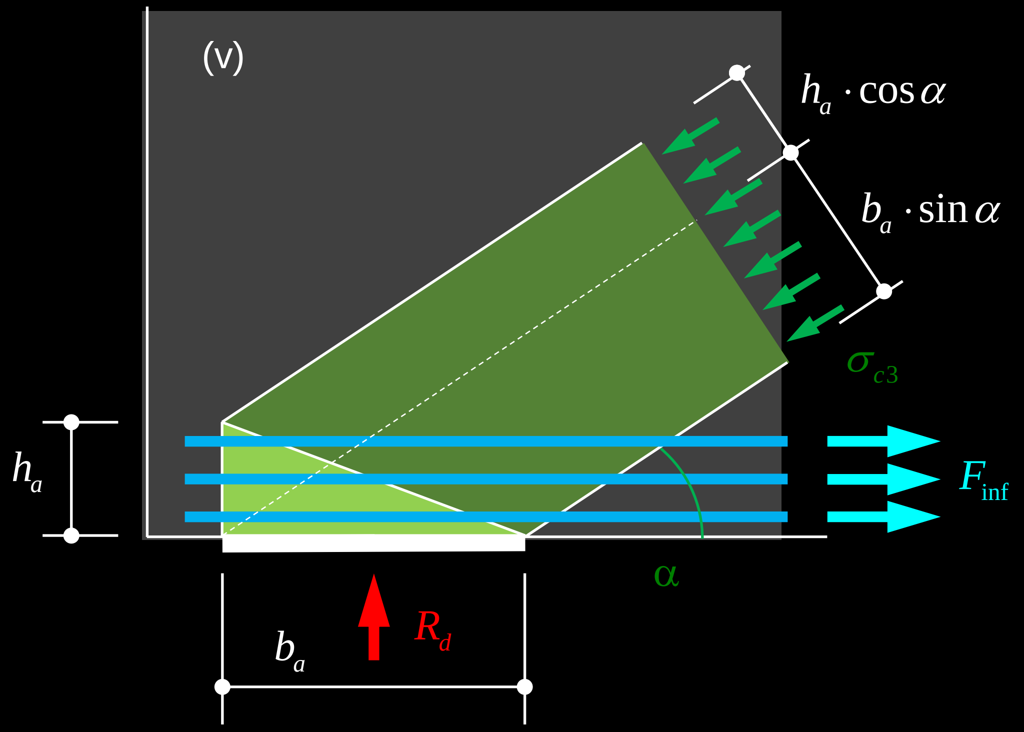
- Require a relatively large height of the nodal zone, which reduces the effective depth of the beam.
- Do not consider that a higher compressive strength may be applied in the nodal zone than in the strut (different values of k_c).

Disadvantages of the solution (iv) = anchoring via bond-shear stresses

- Requires a large, often impracticable width of the nodal zone (= bearing plate)

Solution (v) (see, e.g. Canadian standard CSA)

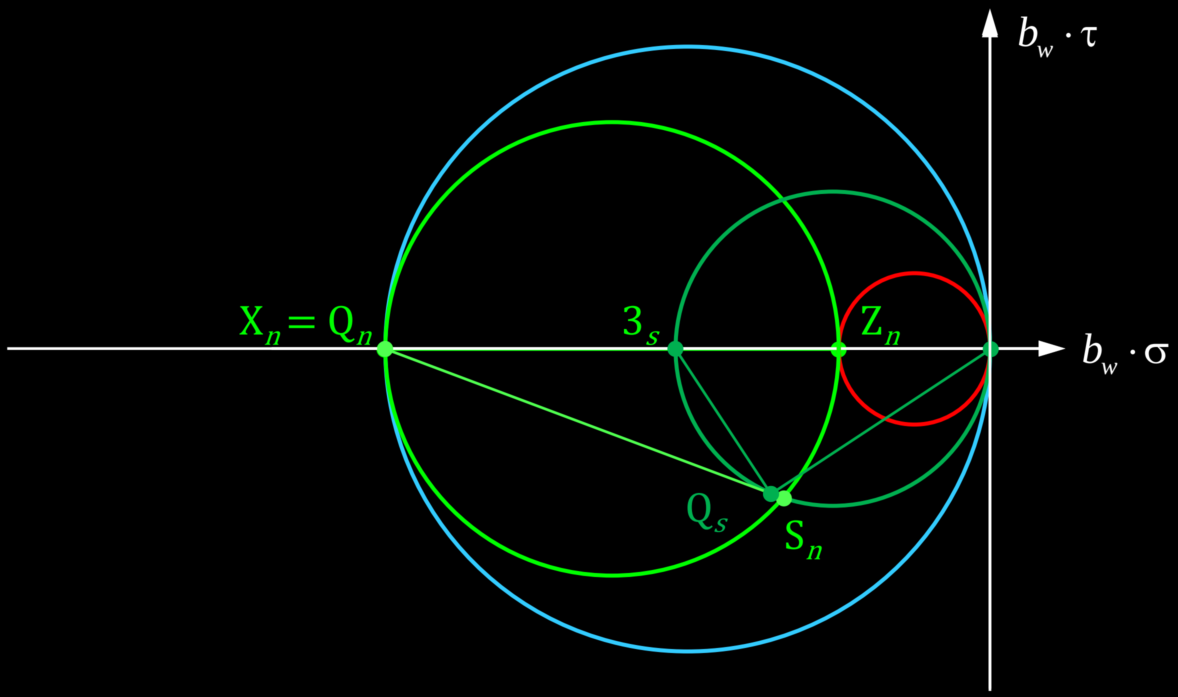
- "free" choice of node height and width, leading to nodal zones with $\boldsymbol{\sigma_h \neq \sigma_v}$
- Compressive stress in strut < Compressive stress in nodal zone

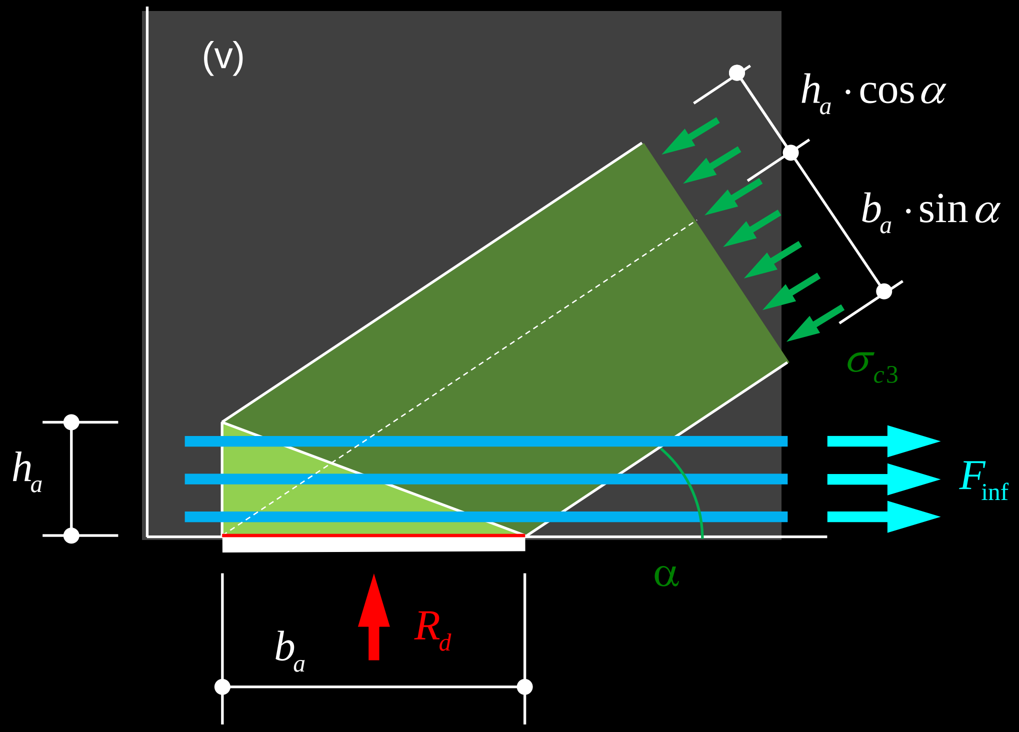


Stress fields

Nodal zones (see [4] p. 64)

Solution (v) (see Canadian standard CSA, among others): Stresses



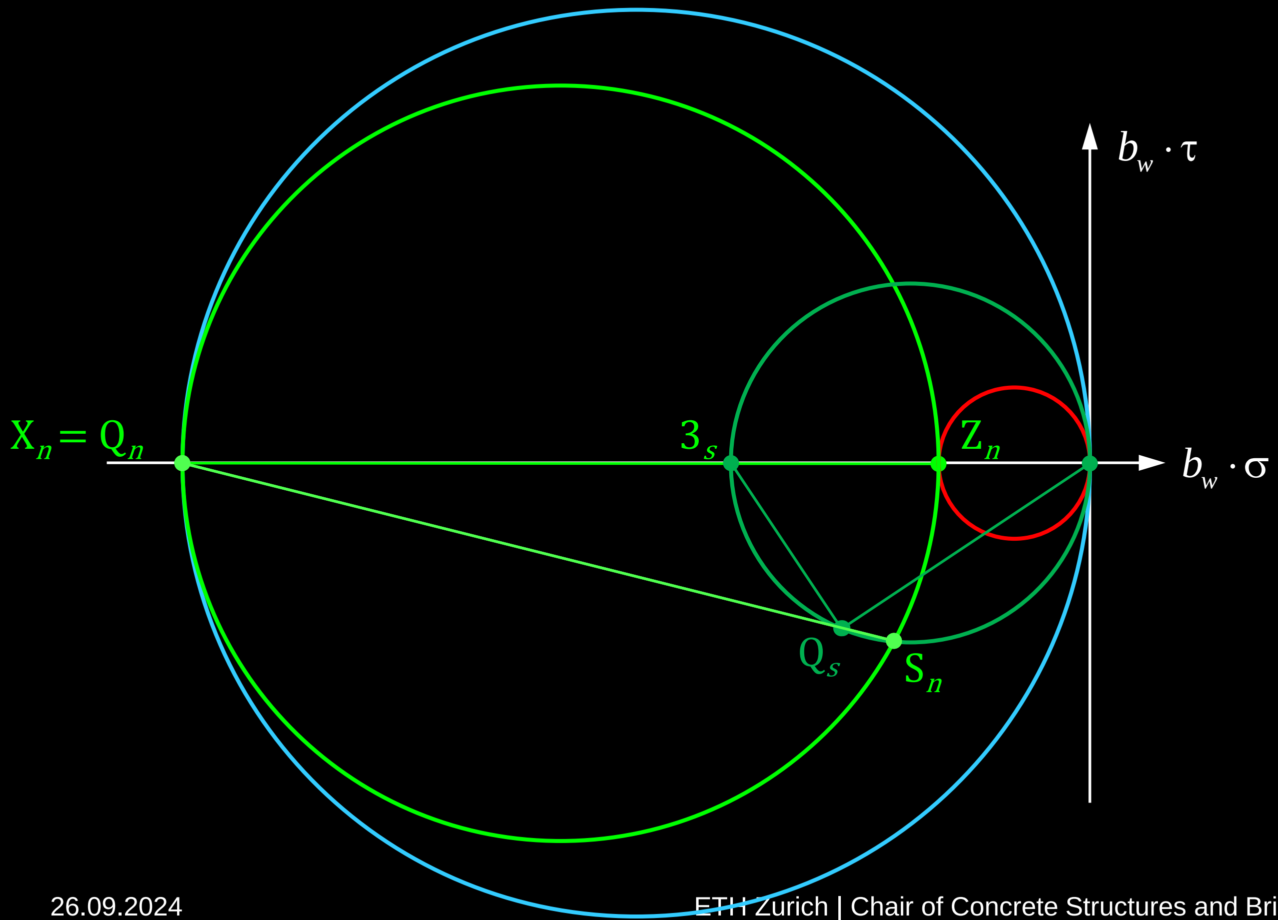


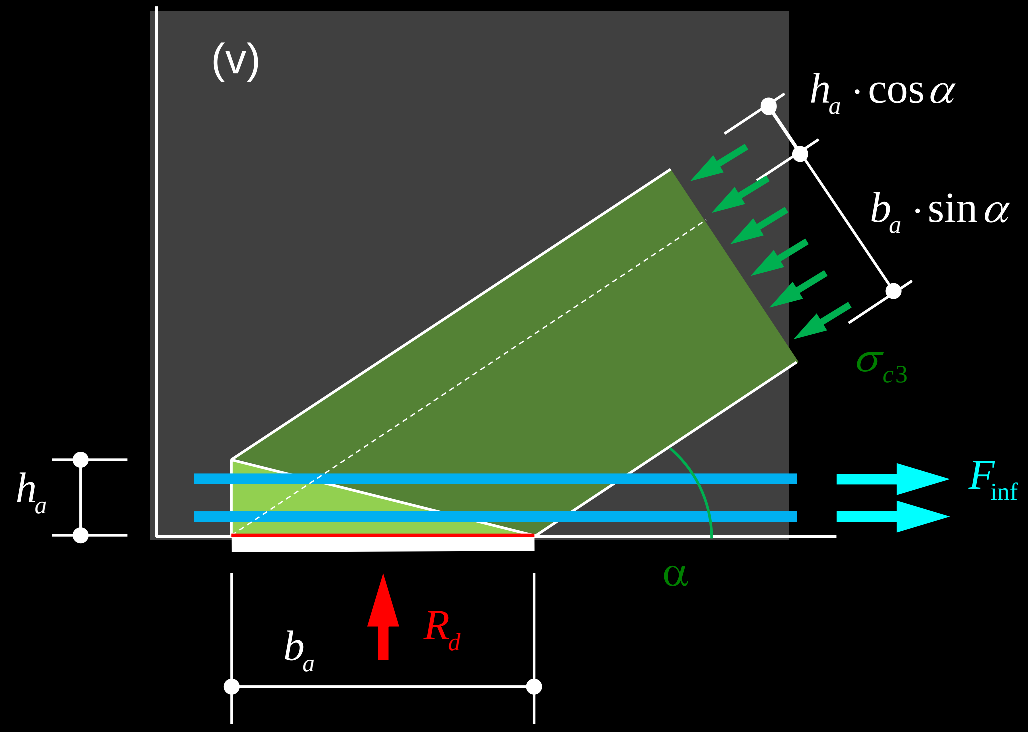
- horizontal node boundary $\sigma_v = -R_d / (b_a \cdot b_w)$
- vertical node boundary $\sigma_h = -F_{\text{inf}} / (h_a \cdot b_w)$
- **N:** nodal zone (σ_v, σ_h = like node boundaries)
- **S:** strut $-\sigma_{c3} = \dfrac{-R_d}{(b_a \cdot \sin\alpha + h_a \cdot \cos\alpha) \cdot b_w \cdot \sin\alpha}$

Stress fields

Nodal zones (see [4] p. 64)

Solution (v) (see Canadian standard CSA, among others): Stresses
(Alternative with even smaller node height)





horizontal node boundary $\sigma_v = -R_d \,/\, (b_a \cdot b_w)$

vertical node boundary $\sigma_h = -F_{\text{inf}} \,/\, (h_a \cdot b_w)$

N: nodal zone (σ_v, σ_h = like node boundaries)

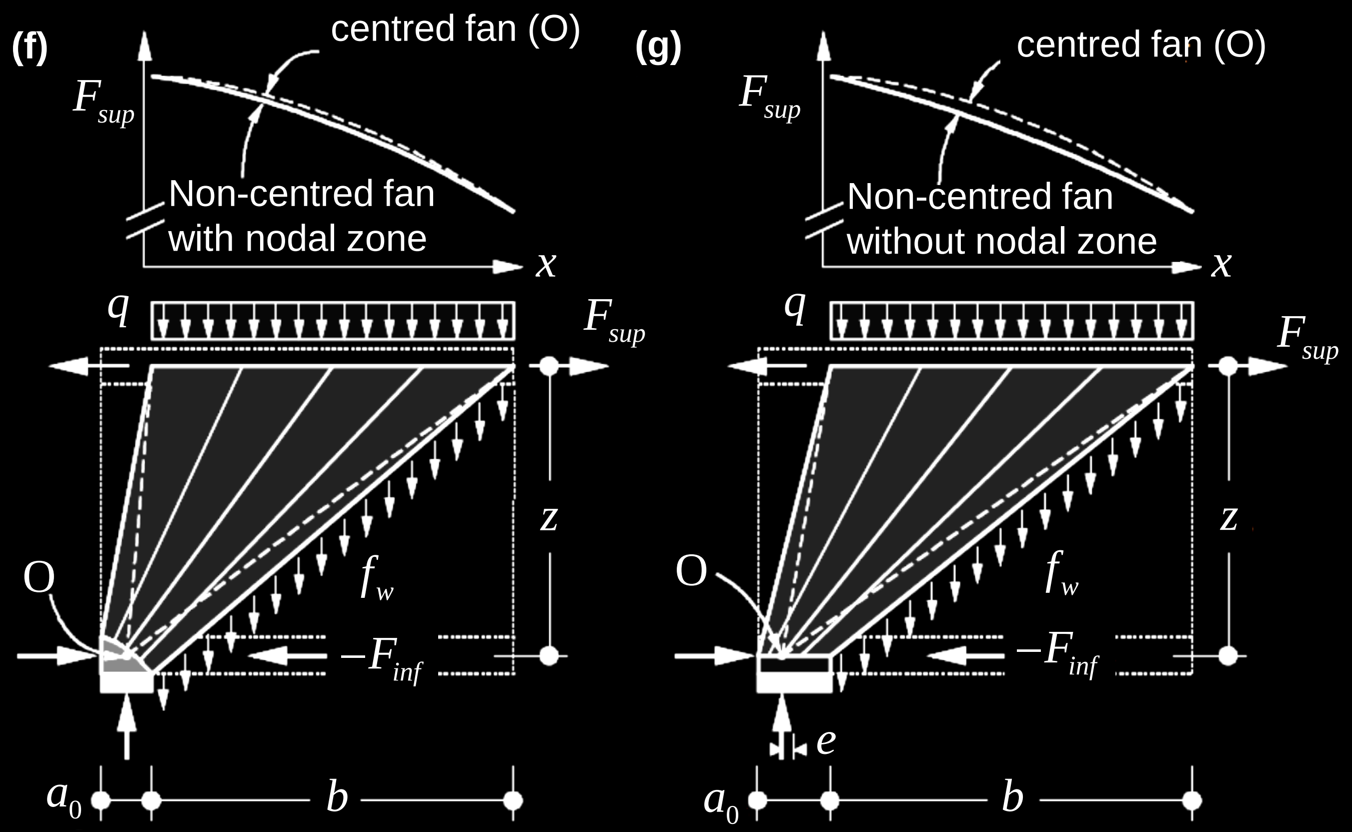
S: strut $-\sigma_{c3} = \dfrac{-R_d}{(b_a \cdot \sin\alpha + h_a \cdot \cos\alpha) \cdot b_w \cdot \sin\alpha}$

Stress fields

Concrete compressive stresses in fans: supports (see [4] p. 70 ff)

→ (f) Usual solution:
Non-centred fans with nodal zone, see stress fields for membrane elements with rectangular cross section (in the nodal zone: $-\sigma_{c3} < f_c$ → define the dimensions of the bearing plate accordingly)

→ (g) Less suitable: Non-centred fans without nodal zone (requires longer length for the same f_c; bond must be checked)

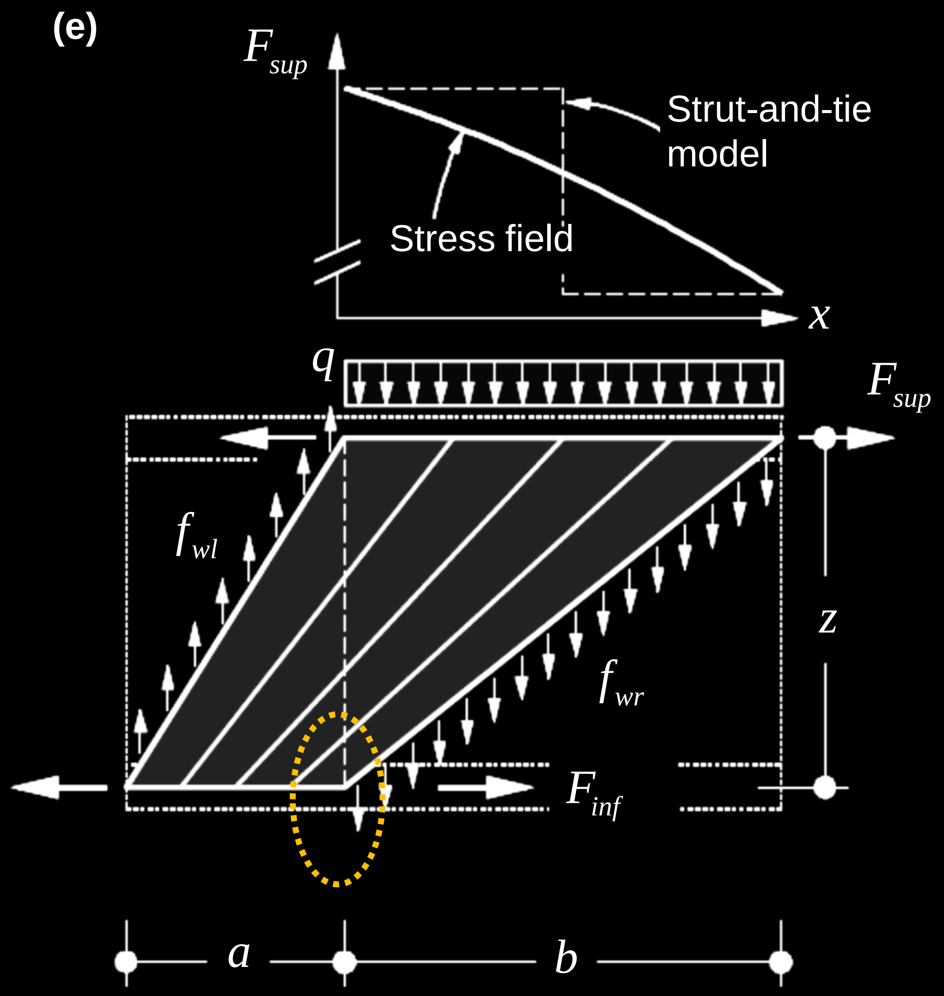


→ Chord force distribution F_{sup} in the fan area can be checked conservatively supposing a centred fan, provided that the height of the nodal zone according to (f) is in the flange (check with F_{inf} calculated assuming a centred fan). Otherwise the effective depth must be reduced (iteratively).

Stress fields

Concrete compressive stresses in fans: fan instead of paralell field (see [4] p. 70 ff)

→ (e) Not convenient: In centred fans with large changes of the inclination, the compressive stresses at the bottom end of the flattest trajectory are much bigger than in adjacent parallel compression fields, because the point with the flattest inclination, i.e. maximum (1+ cot$^2\alpha$), and the largest stirrup force f_w coincide.



Note: In the adjacent parallel fields (for q = 0):

$$-\sigma_{c3} = \frac{f_w(1+\cot^2\alpha)}{b_w}$$

$$f_w = \frac{V}{z\cot\alpha}$$

$$\rightarrow -\sigma_{c3} = \frac{V}{b_w z}(\tan\alpha + \cot\alpha) = \frac{V}{b_w z \sin\alpha\cos\alpha}$$

i.e. the flatter the diagonal compression field, the higher the stresses

Stress fields

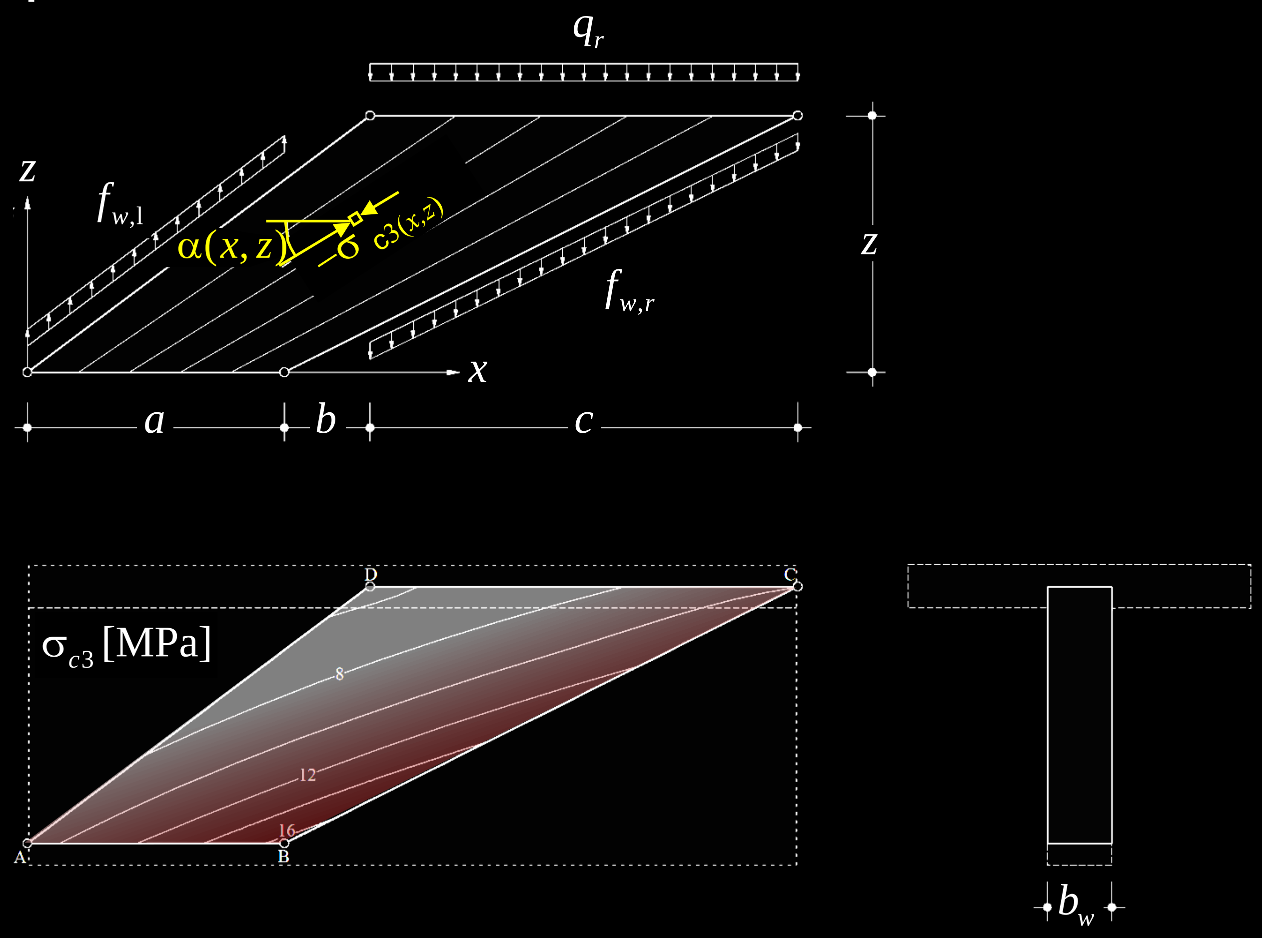
Concrete compressive stresses in fans: fan instead of paralell field

Numerical example

→ Concrete compressive stresses vary significantly with small changes of α (in adjacent parallel compression fields approx. 5 MPa to 10 MPa, but in point B 16 MPa!)

→ Difference to nodal zones in supports: no transversal compression (vertical) due to the reaction and no transversal restraint (horizontal) due to the bearing plate or adjacent fans
⇨ situation much worse

→ Strong changes of the inclinations are very unfavourable and should be avoided!

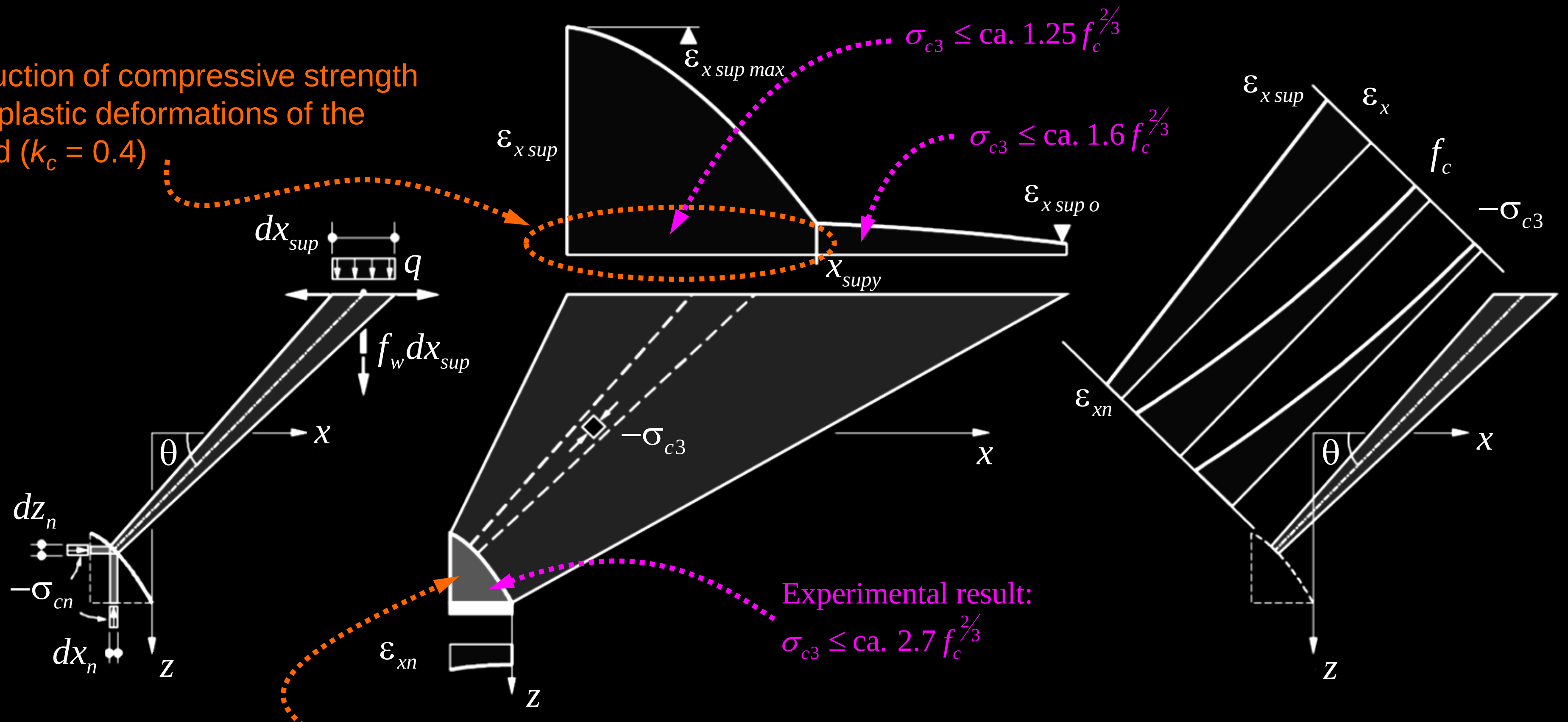


[from Marti and Stoffel 1999]

Stress fields

Concrete compressive stresses in fans: variable compressive strength

Reduction of compressive strength with plastic deformations of the chord (k_c = 0.4)



Verification in the support nodal zone with increased compressive strength due to transverse compression, k_c = 1.0 (with confinement reinforcement even higher)

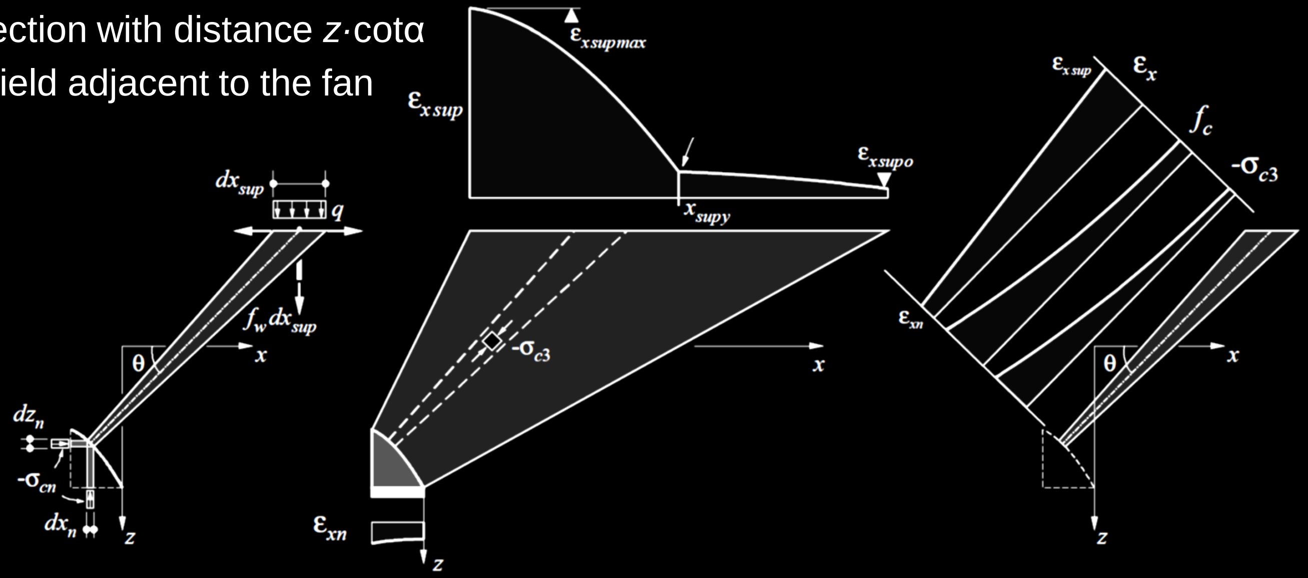
Stress fields

Concrete compressive stresses in fans

→ Concrete compressive stresses vary hyperbolically along the trajectories
→ Strain state also varies along the trajectories, which modifies the effective concrete compressive strength as well

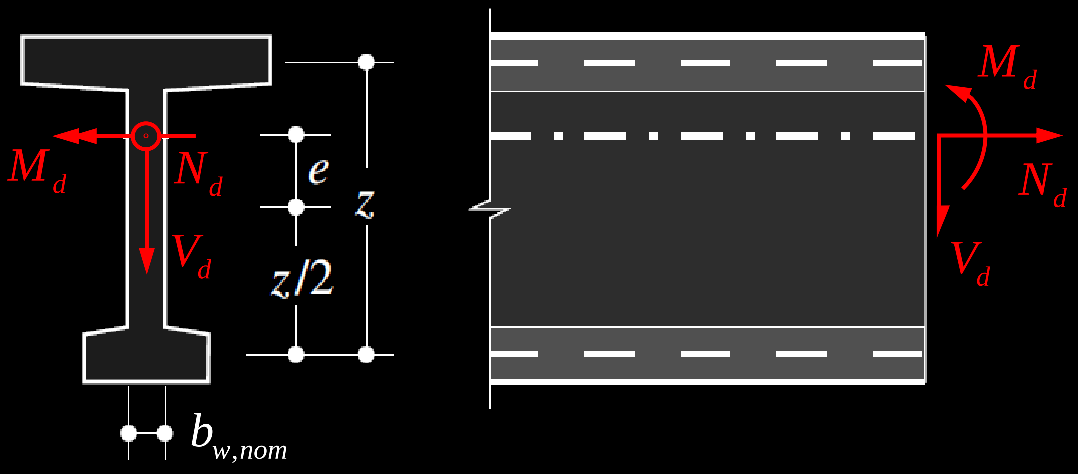
SIA 262:, $k_c = \dfrac{1}{1{,}2 + 55\varepsilon_1} \leq 0.65 \qquad \varepsilon_1 = \varepsilon_x + \left(\varepsilon_x + 0{,}002\right)\cot^2 a$

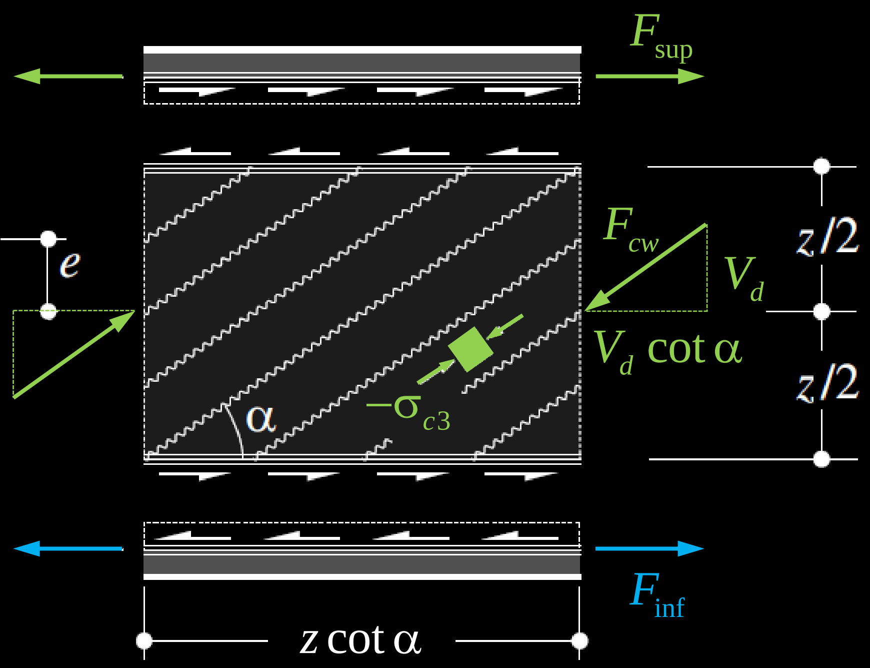
→ Verification of concrete compressive strength in a fan is complex
→ Under normal conditions, no failure occurs in the fan as long as the tension chord reinforcement does not yield.
→ Verify by checking the compressive stresses in the nodal zone (with increased strength due to transverse restraint or transverse compression) as well as in the parallel compression field adjacent to the fan (with yielding of the chord reinforcement in the area under consideration = incl. fans with reduced strength).
→ SIA 262: Cross-sectional analysis = nominal verification in the section with distance $z \cdot \cot\alpha$ to the support, corresponds to a verification in the compression field adjacent to the fan
→ Approximation without analysis of the strain state k_c = 0.55, when the tension chord reinforcement yields in the nodal zone: reduction of k_c = 0.4)



Stress fields

Beam - "Cross-sectional analysis" (parallel chords)





$$F_{sup} = \frac{-M_d + N_d \cdot e}{z} + \frac{N_d}{2} + \frac{|V_d| \cdot \cot\alpha}{2}$$

$$F_{inf} = \frac{M_d - N_d \cdot e}{z} + \frac{N_d}{2} + \frac{|V_d| \cdot \cot\alpha}{2}$$

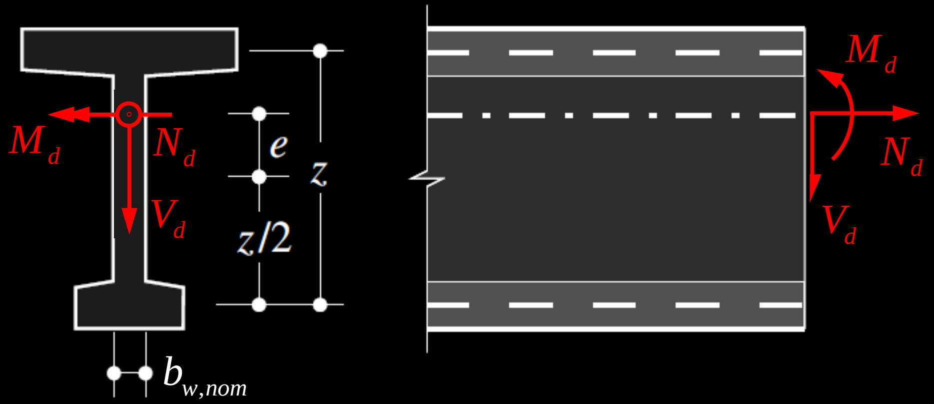
→ Dimensioning by "cross-sectional analysis" is possible, provided that all static and geometric values along the beam axis vary only gradually (not abruptly!).

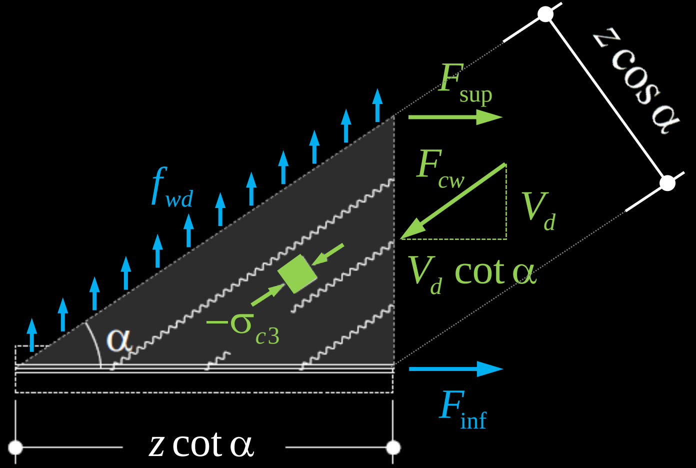
→ Internal forces (*M*,*N*) should be related to the centroidal axis; for consideration of prestressing see haunched beams

→ Inclination of the concrete compressive field theoretically freely chosen; restrictions to avoid premature ruptures of the stirrups or aggregate interlock (SIA 262: Normal case 30...45°)

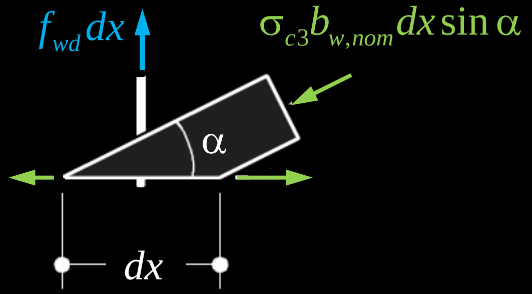
Stress fields

Beam - "Cross-sectional view" (parallel chords)





- Equilibrium at the sectional member (upper right figure)
  Force in the reinforcement f_{wd} :
  $$f_{wd} = |V_d| / (z\cot\alpha) \leq a_{sw} f_{sd}$$
- Equilibrium at the differential element (bottom right figure)
  Concrete compressive stress σ_{c3} in the web:
  $$-\sigma_{c3d} = |V_d| / (b_{w,nom} z)(\tan\alpha + \cot\alpha) \leq k_c f_{cd} \quad \text{mit} \quad b_{w,nom} = b_w - k_H \sum \varnothing_H$$



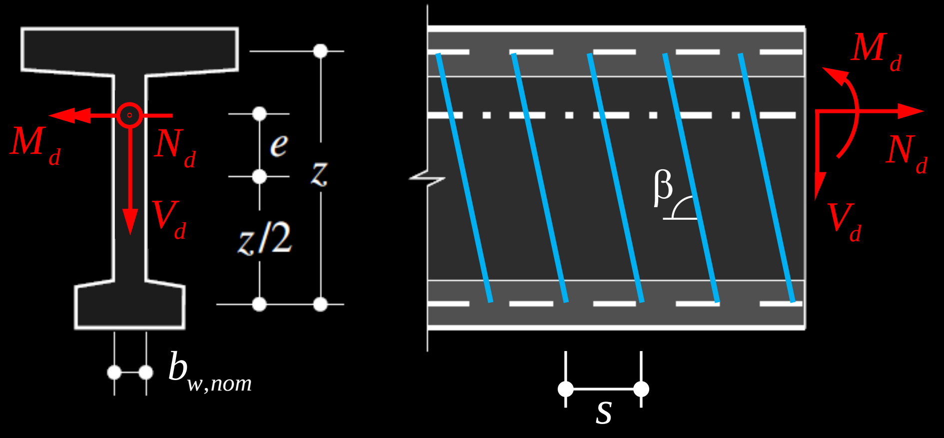
- Ducts in the web disturb the compressive stress field → Reduce web width (see above), where k_H = 0.5 (steel) or k_H = 0.8 (plastic) applies for injected ducts, k_H = 1.2 for non-injected ducts.
- Compressive stresses are minimal for truss inclinations of 45°; for flatter inclinations the stresses progressively increase and the concrete compressive strength decreases (k_c).

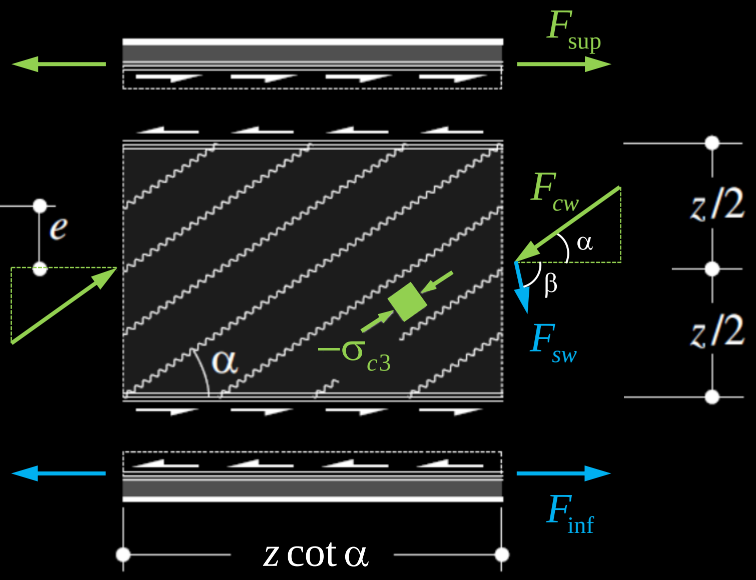
Note: Actually this is not a cross-sectional design, since stirrups are determined for a certain length ("staggering effect"); a cross-sectional design for shear force is strictly speaking not possible.

Stress fields

Beam - "Cross-sectional view" (parallel chords)

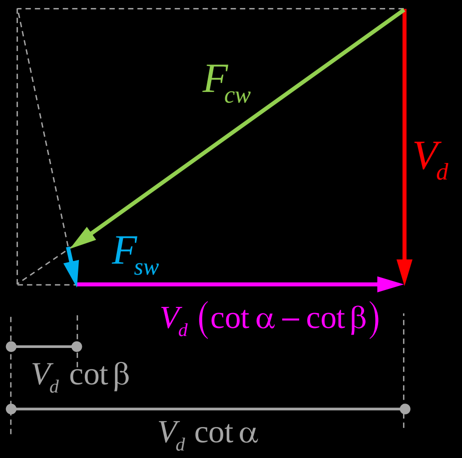
Inclined stirrups





$$F_{sup} = \frac{-M_d + N_d \cdot e}{z} + \frac{N_d}{2} + \frac{V_d \cdot (\cot\alpha - \cot\beta)}{2}$$

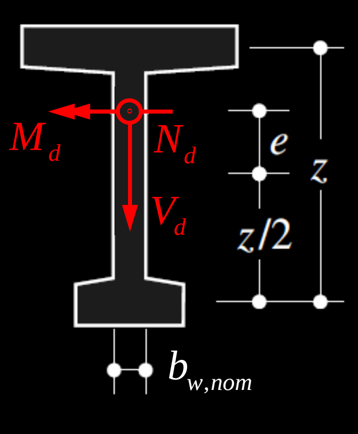
$$F_{inf} = \frac{M_d - N_d \cdot e}{z} + \frac{N_d}{2} + \frac{V_d \cdot (\cot\alpha - \cot\beta)}{2}$$

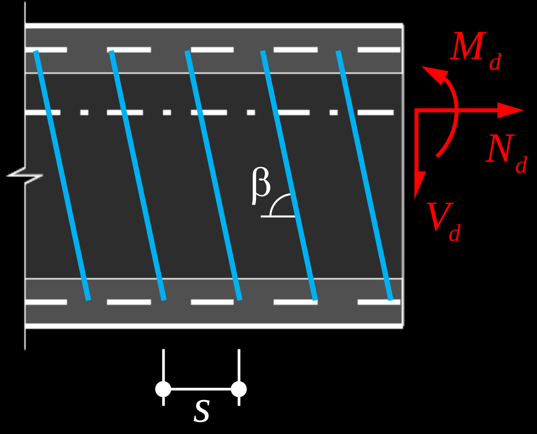


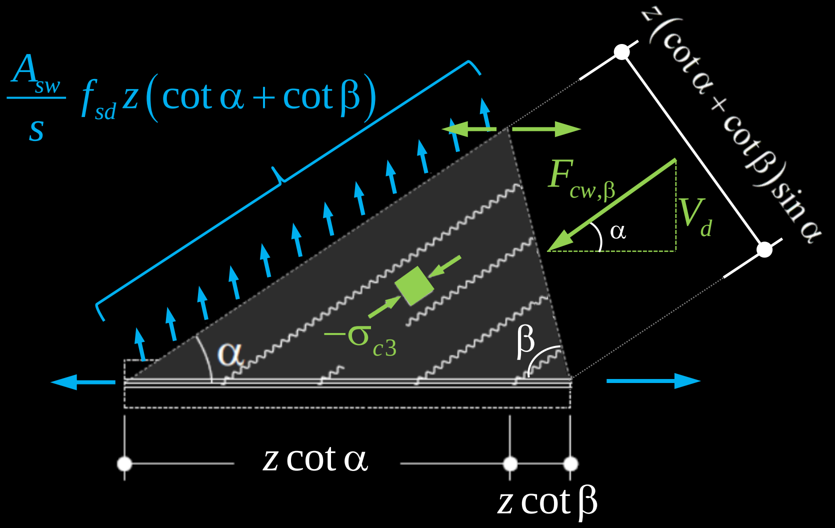
Stress fields

Beam - "Cross-sectional view" (parallel chords)

Inclined stirrups







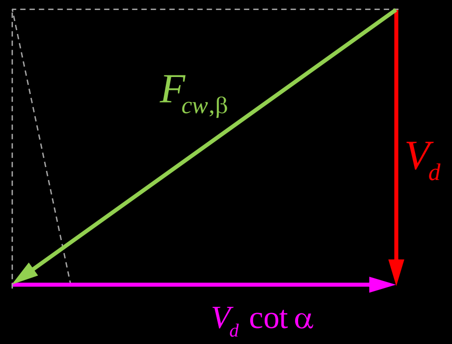
- Resistance of the shear reinforcement:

$$V_{Rd,s} = \frac{A_{sw}}{s} f_{sd} z(\cot\alpha + \cot\beta)\sin\beta \quad \left(= \frac{A_{sw}}{s} f_{sd} z \cot\alpha\right)$$

- Resistance of the concrete compression field:

$$V_{Rd,c} = b_w k_c f_{cd} z(\cot\alpha + \cot\beta)\sin^2\alpha \quad \left(= b_w k_c f_{cd} z \sin\alpha\cos\alpha\right)$$

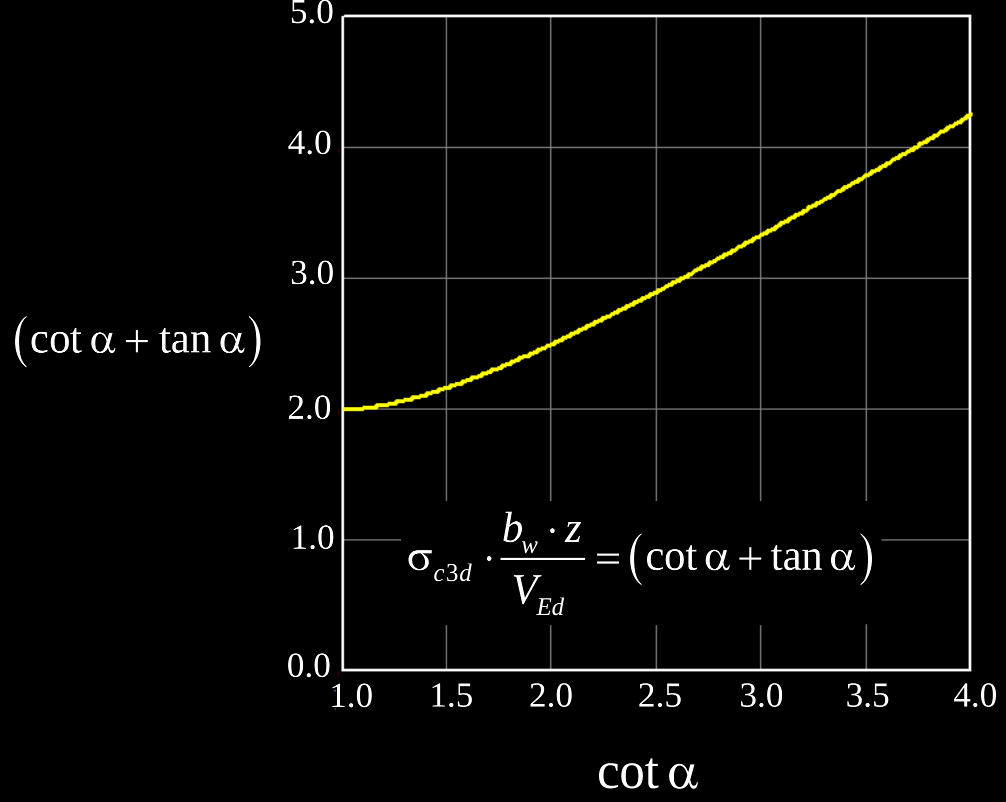
Vertical stirrups: $\beta = \frac{\pi}{2}$

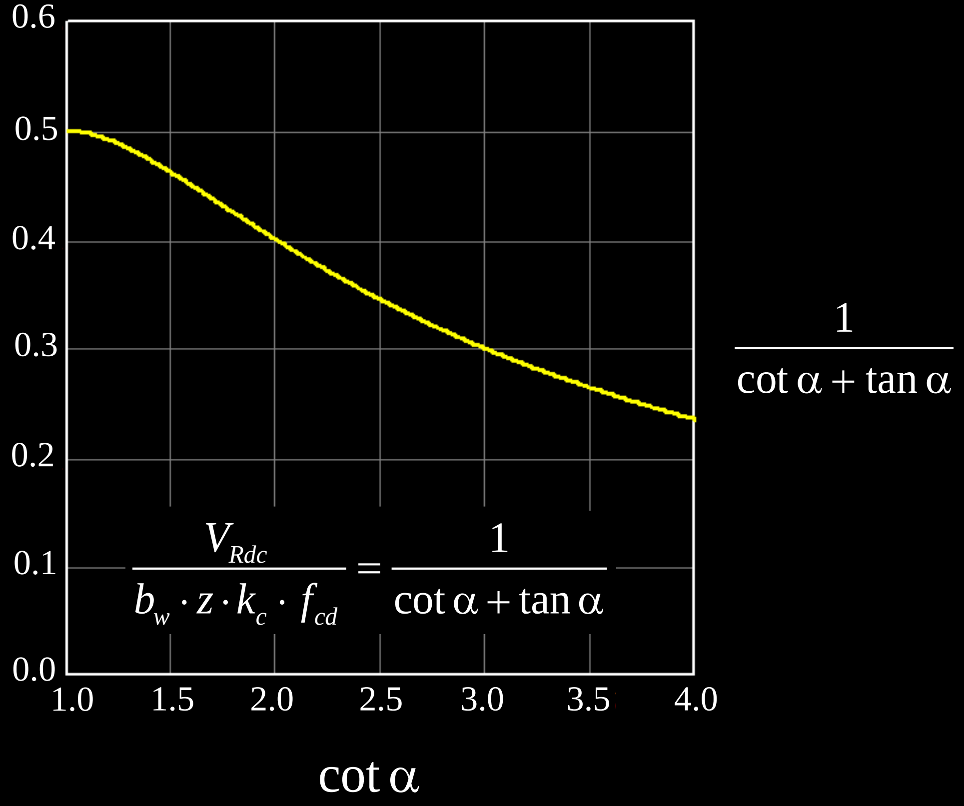


Stress fields

Shear resistance depending on the compression field inclination (web concrete crushing failure)

$$\sigma_{c3d} = \frac{V_{Ed}}{b_w \cdot z} \cdot \left(\cot\alpha + \tan\alpha\right) \le k_c f_{cd} \quad \rightarrow \quad V_{Rdc} = \frac{b_w \cdot z \cdot k_c \cdot f_{cd}}{\cot\alpha + \tan\alpha}$$





→ Concrete compressive stresses increase with flat inclinations

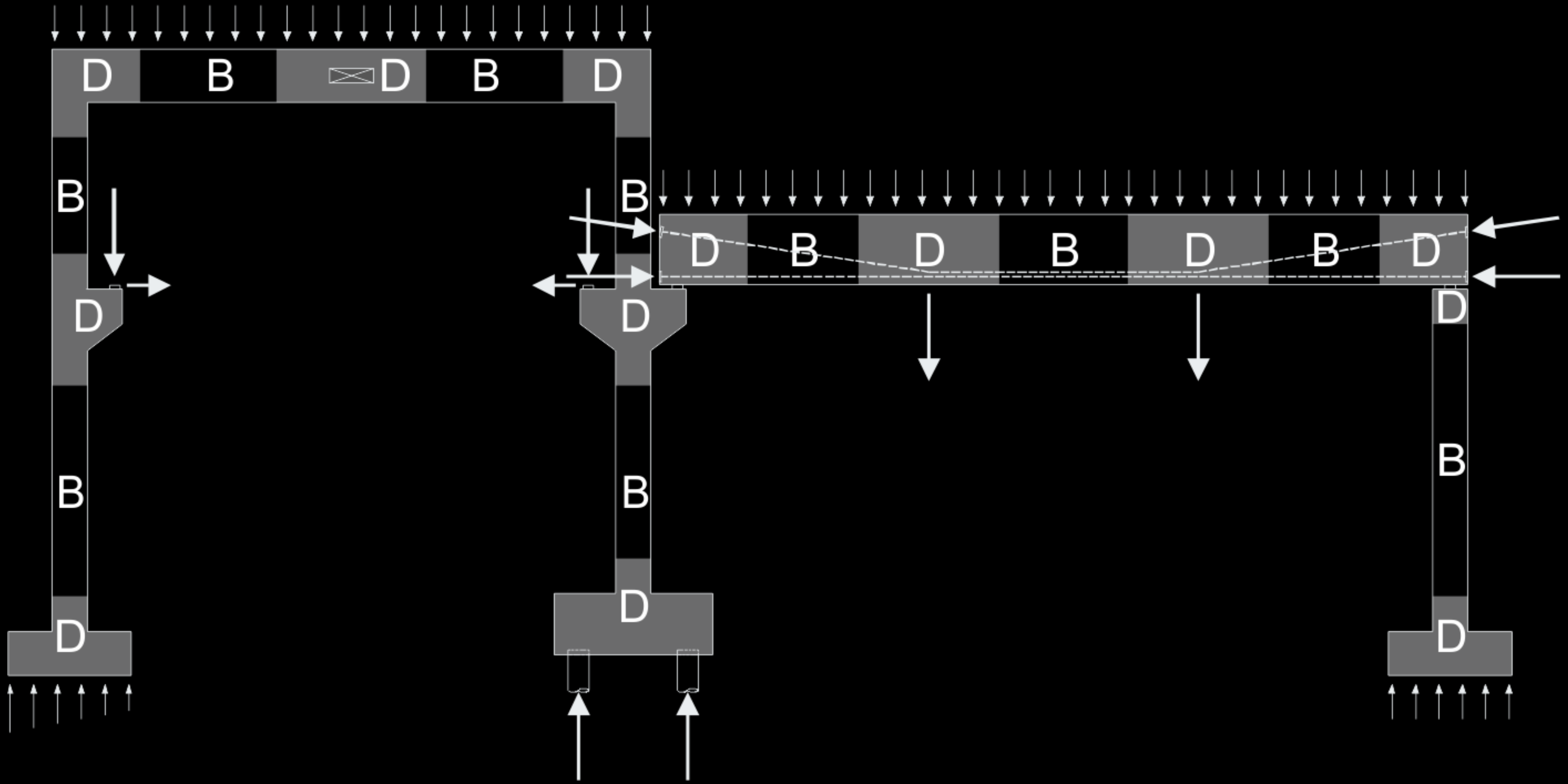
→ Dependence of the effective concrete strength depending on the inclination is not shown in these diagrams

Stress fields

Structural elements with static / geometric discontinuities

B Continuity/Bernoulli regions

D Discontinuity regions: static and/or geometric discontinuities



Stress fields

Structural elements with static / geometric discontinuities

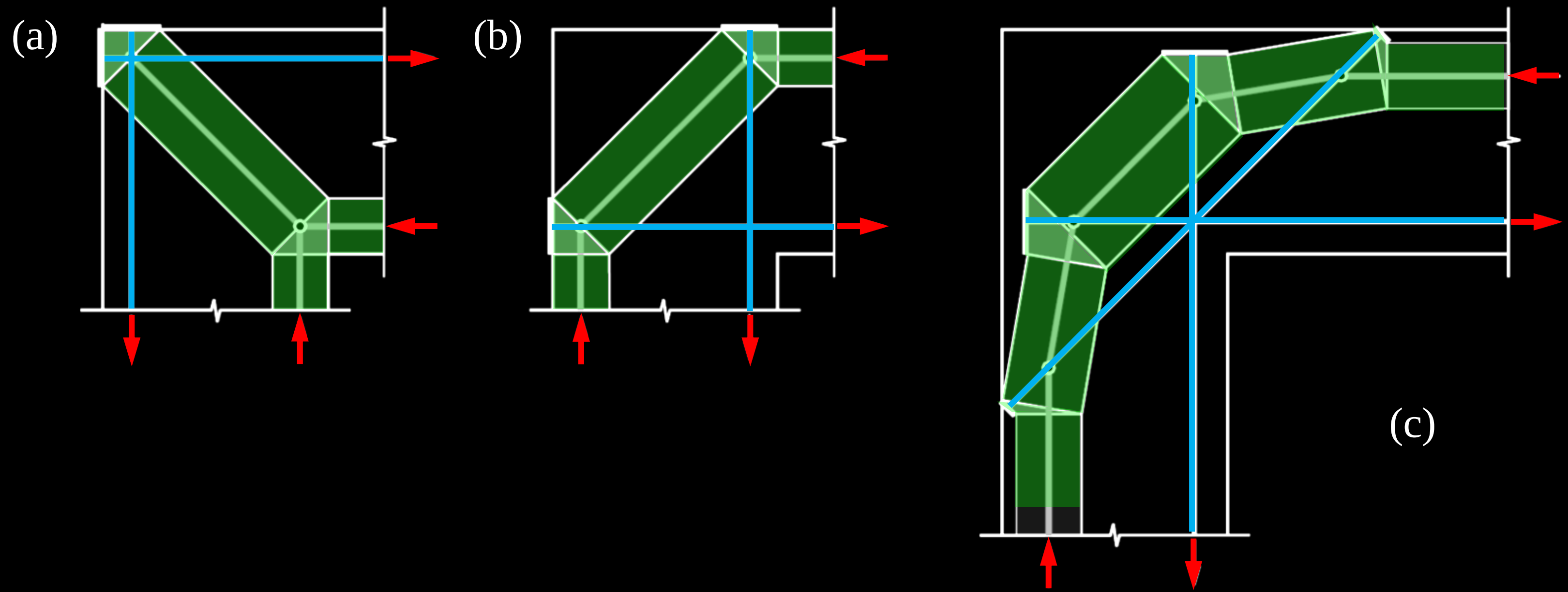
Frame corners under pure bending

→ (a) Closing, (b) Opening moment

→ Especially opening frame corners are tricky and demanding in design

→ Diagonal reinforcement (c) is beneficial for anchoring the reinforcement forces

→ Bending resistance of the adjacent members usually cannot be fully exploited, since the anchoring and the deviation of forces in the corner area cause a reduction of the lever arm in comparison with (a), (b)

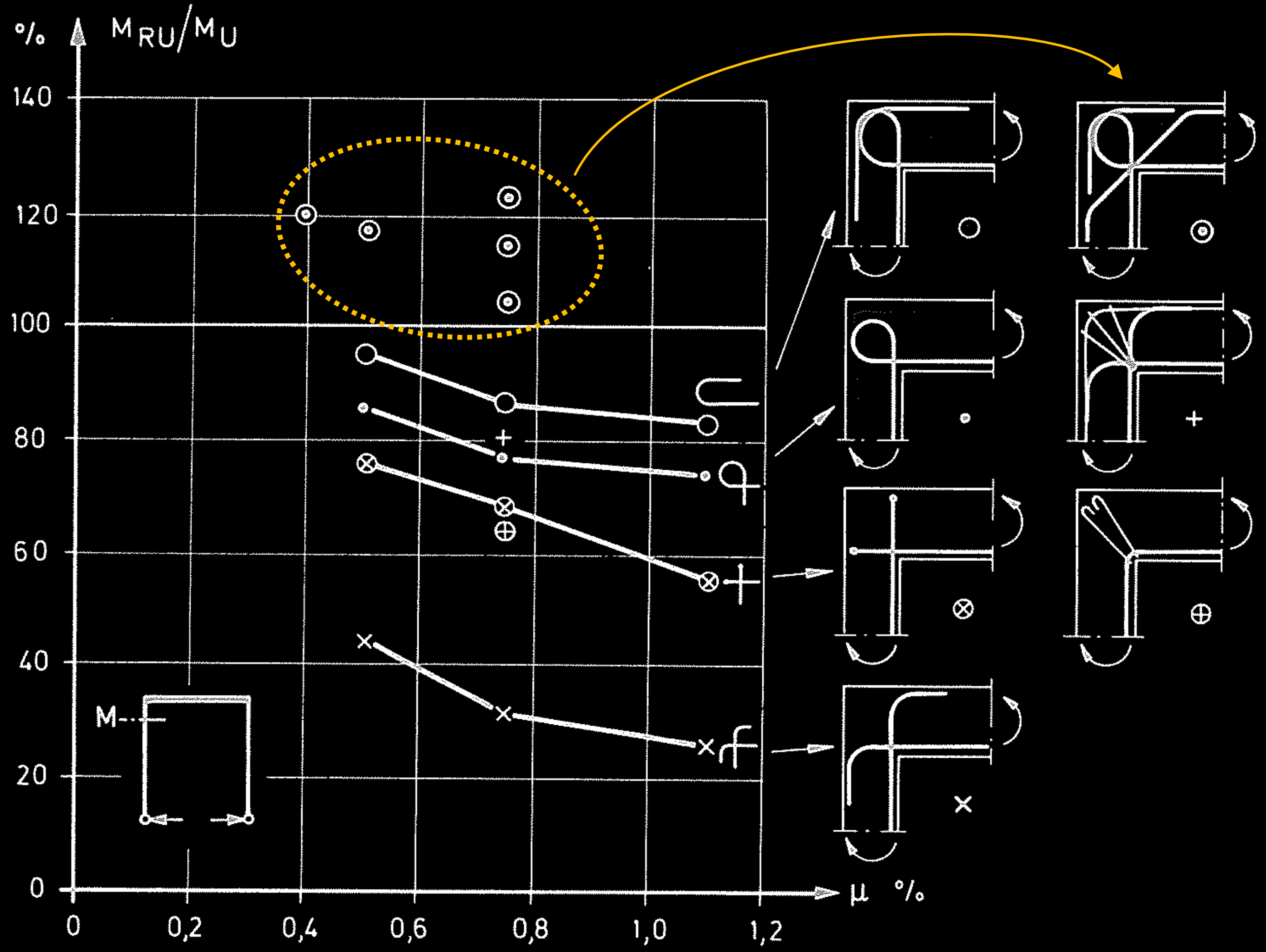


Stress fields

Structural elements with static / geometric discontinuities

Frame corners under pure bending

- → Experiments by Nilsson (1973) confirm the observations of the previous slide
- → Headed reinforcing bars are suitable in frame corners
- → Examples of frame corners with distributed reinforcement, combined loading etc. see e.g. [5].



Stress fields

Structural elements with static / geometric discontinuities

Dapped-end beams (d), (e), (f)

→ (d), (e) possible strut-and-tie models

→ Diagonal reinforcement favourable (f), analogously to the frame corners, superposition of the models (distribution of load can freely be chosen).

→ Serviceability behaviour not covered by stress fields

Corbels (g)

→ (f) Basic case

→ Various other models possible, see e.g. [5]

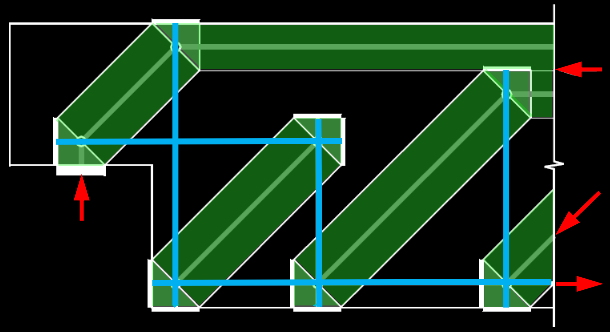
General remarks

→ Stress fields are perfectly suitable for structural elements with static or geometric discontinuities

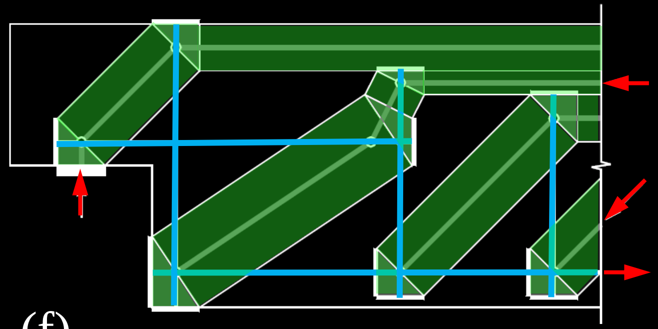
→ Figures only show simple strut-and-tie models

→ Refinement through the introduction of fans, arches, tension and compression chords, etc. enables capturing the load-bearing mechanism of the concrete and the distributed reinforcement over the entire area (as will be shown in the following examples)

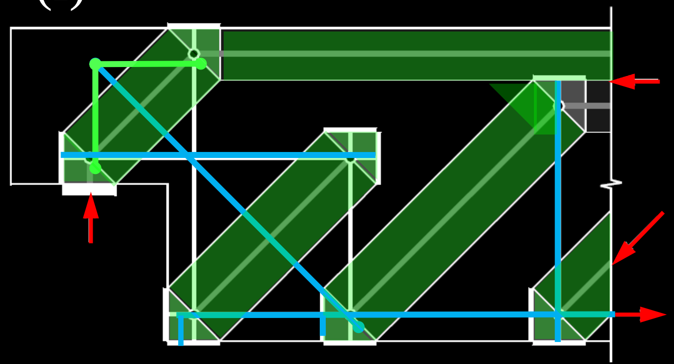
(d)



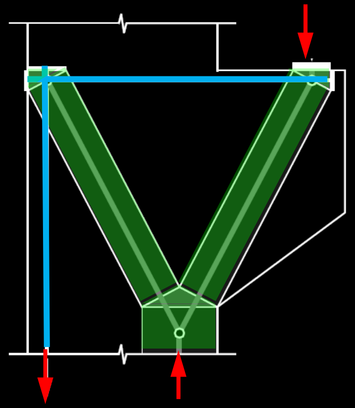
(e)



(f)



(g)

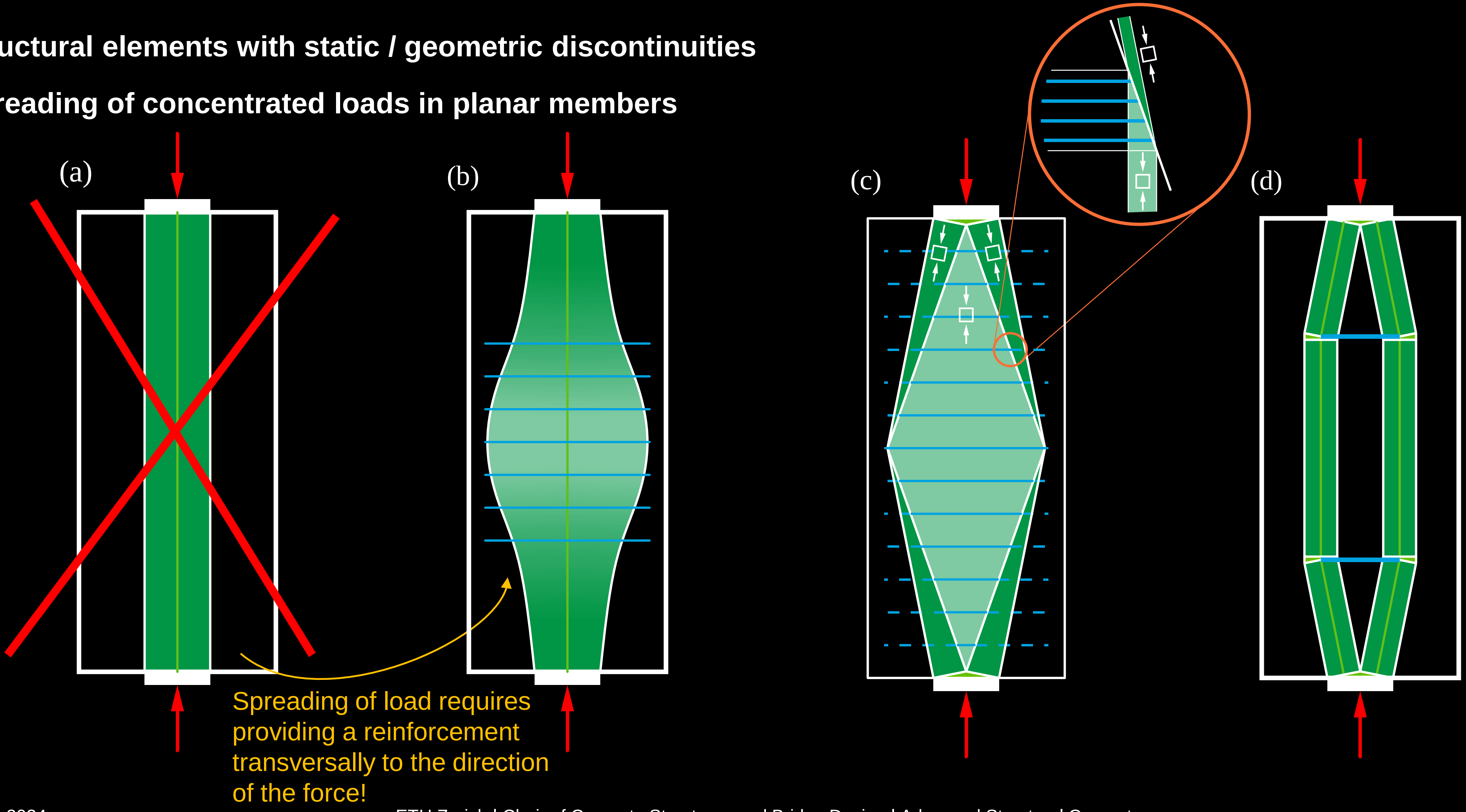


► *Overpass road CV-500 (Valencia, Spain)*

Stress fields

Structural elements with static / geometric discontinuities

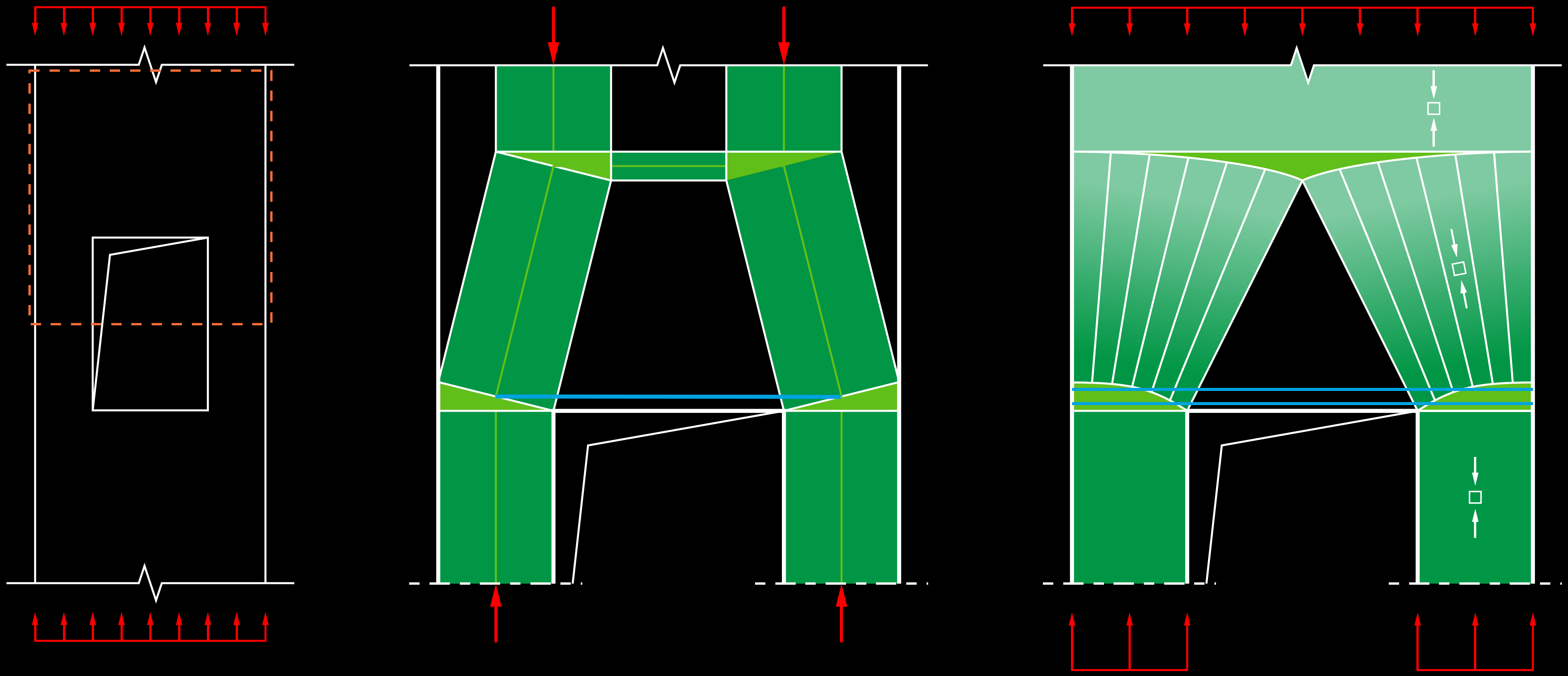
Spreading of concentrated loads in planar members



Stress fields

Structural elements with static / geometric discontinuities

Wall with opening: further models can be built as combination or extrapolation of already known basic models



Stress fields

Comments for practical application («design»)

- In practical applications, the complete determination of the stress state in all components (fan edges, nodal zones, exact tension and chord force curves for fans ...) is not necessary.
- Suitable procedure in practice:
  1. Design the stress field roughly using scale drawings as combination of basic mechanisms
     (if necessary with simplifying assumptions such as centred fans and straight compression chords, see below)
     Most important basis: experience, understanding of the flow of forces, engineering judgement
  2. On this basis, determine sufficiently accurate chord force distributions, shear reinforcement and concrete compressive stresses at critical points.
  3. Determine important constructive details by designing the nodal zones
- Compression zones with variable height of the compression zone, which occur when the forces of the compression stringers cannot be spread into adjacent structural parts such as compression flanges (rectangular cross-sections), make the development of stress fields difficult.
- For the sake of simplicity, the compression zone can be reduced to a straight compression chord even in the absence of a compression flange, whose position (→ static height) should be determined conservatively in order to avoid insufficient concrete dimensions (theoretically correct: resultant of the corresponding part of the fan nodal zone).
- Serviceability behaviour cannot be verified.

Stress fields

Comments for practical application («assessment»)

- Stress fields with combinations of load-bearing mechanisms (arcs and fans, struts and fans or orthogonal and diagonal reinforcement) are very difficult to assess. While they are of secondary importance for design purposes they might be necessary to assess existing structures and avoid unnecessary strengthening measures.

- The combination of load-bearing mechanisms can be easily analysed by means of Compatible Stress Fields (see numerical analysis chapter). This approach allows computing automating the most optimum stress field (i.e. the exact solution according to limit analysis) and accounts for all bearing mechanisms, including minimum reinforcement, whose strength contribution is typically neglected in stress fields.

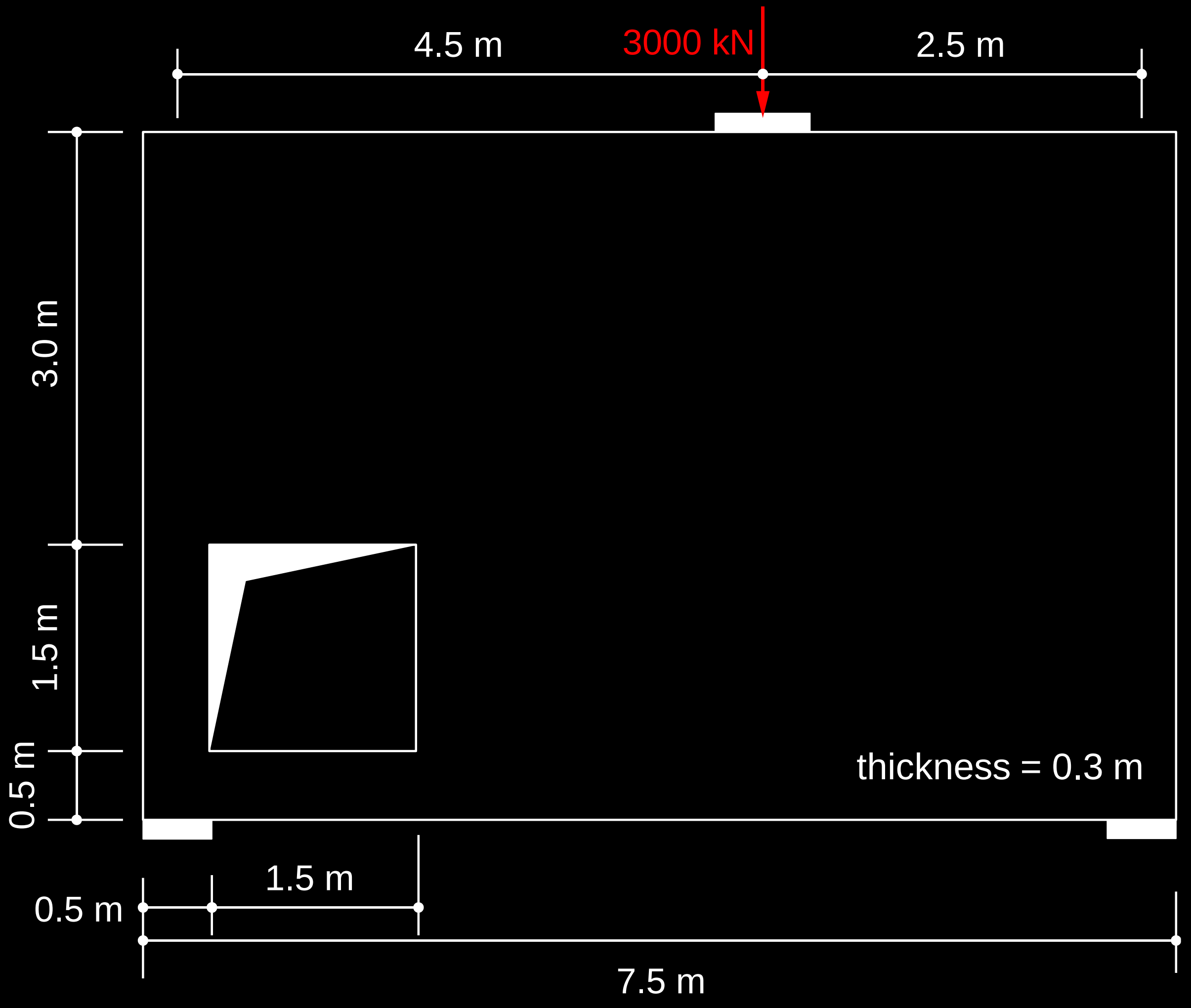
- The Compatible Stress Field Method allows computing the serviceability behaviour (deflections, crack widths…), which is unknown in when using strut-and-tie models and stress field.

- Compatible Stress Fields are very suitable both for assessment and design purposes.

Strut-and-tie model – in-class exercise

In-class exercise:

Discuss possible strut-and-tie models for the following example



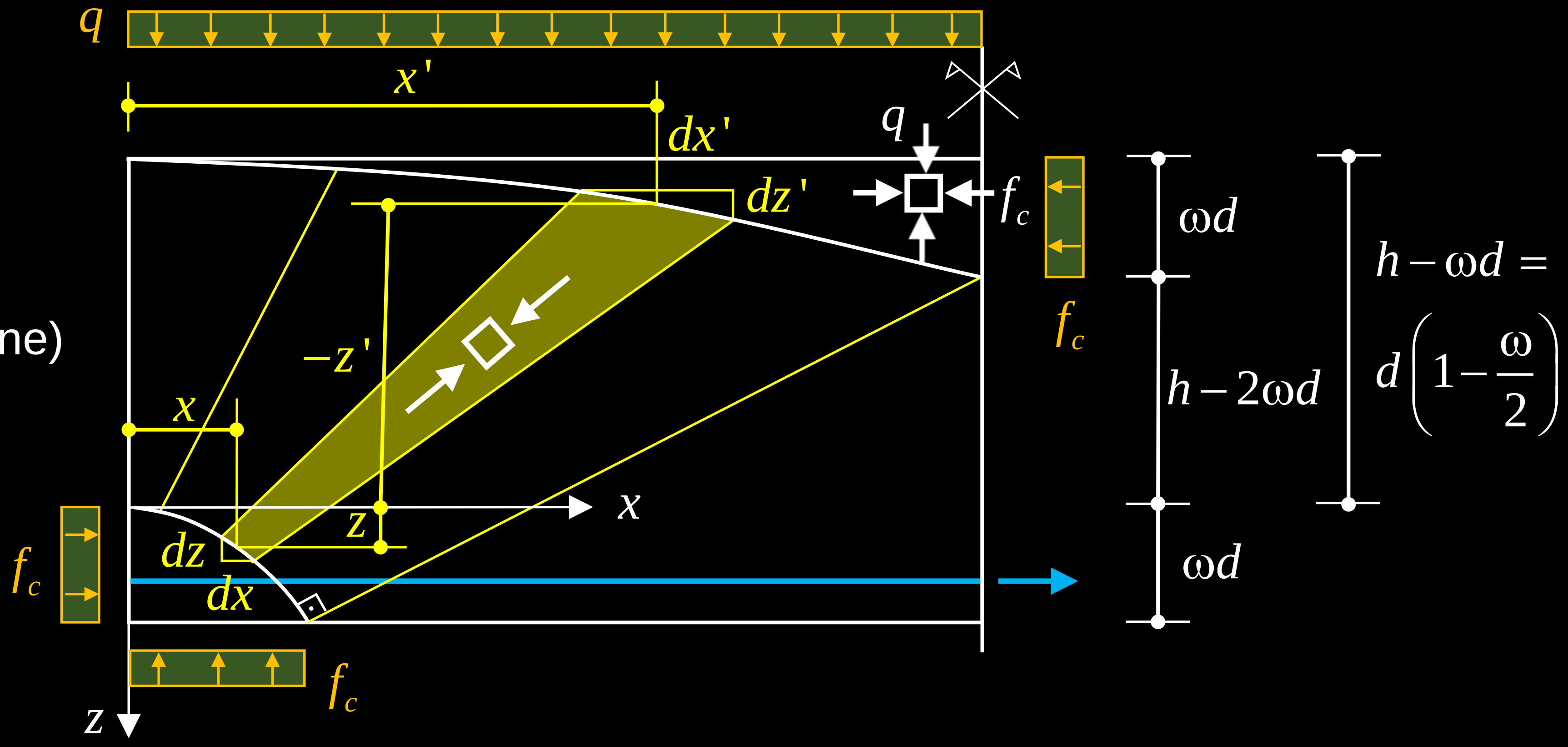
ANNEX

Stress fields

Equilibrium:

$$dx f_c = dx' q; \qquad \frac{dx'}{dx} = \frac{f_c}{q}$$

$$dz f_c = dz' f_c \rightarrow (z - z') = \text{const. (affine)}$$



$$\frac{dz}{dx} = \frac{x' - x}{d\left(1 - \frac{\omega}{2}\right)}$$

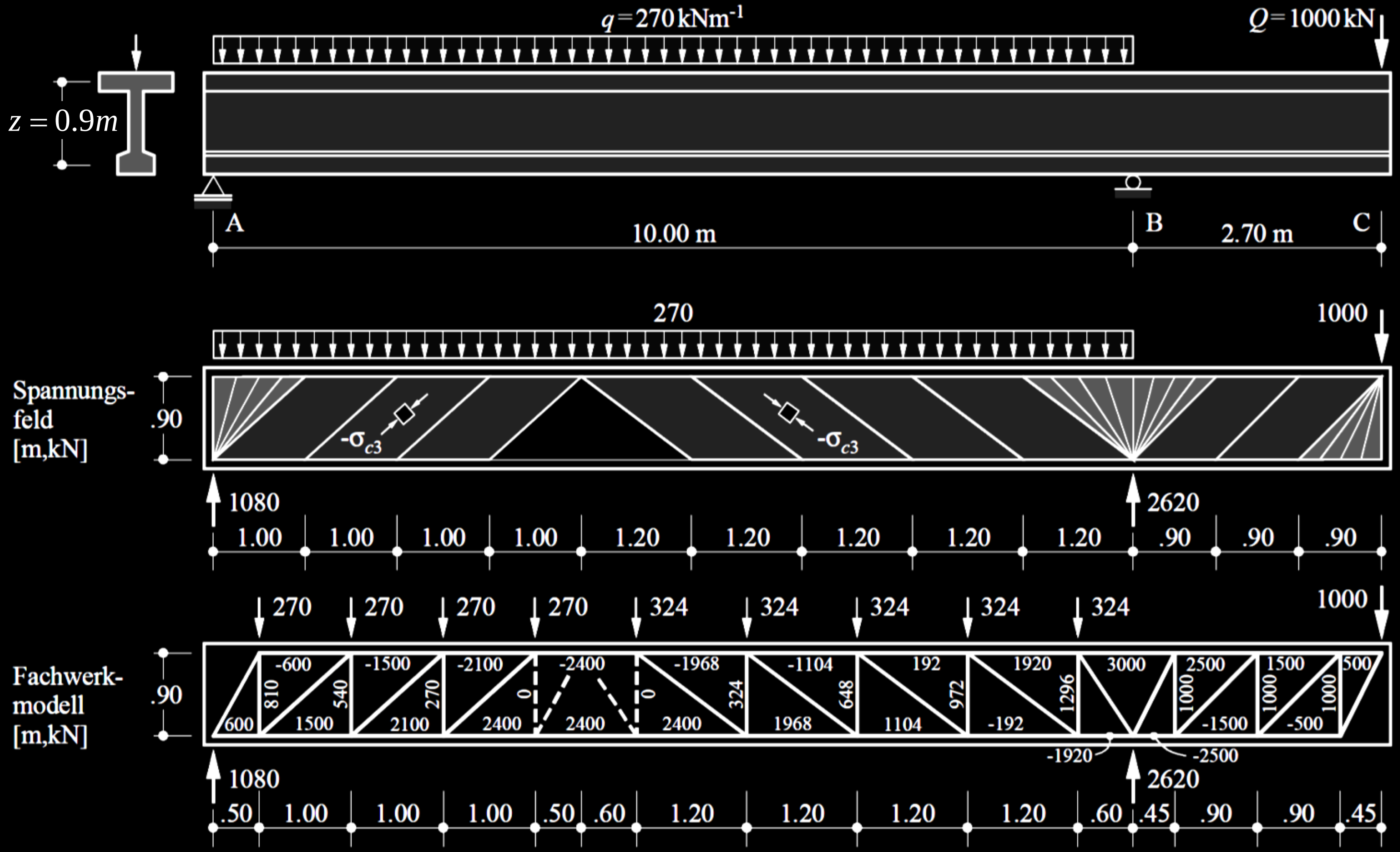
$$\frac{d^2 z}{dx^2} = \frac{\frac{dx'}{dx} - 1}{d\left(1 - \frac{\omega}{2}\right)} = \frac{\frac{f_c}{q} - 1}{d\left(1 - \frac{\omega}{2}\right)}$$

$$\left.\begin{array}{l} \left.\frac{dz}{dx}\right|_{x=0} = 0 \\ z(x = 0) = 0 \end{array}\right\}$$

$$z = \frac{\frac{f_c}{q} - 1}{2d\left(1 - \frac{\omega}{2}\right)} x^2 = \frac{f_c - q}{2qd\left(1 - \frac{\omega}{2}\right)} x^2$$

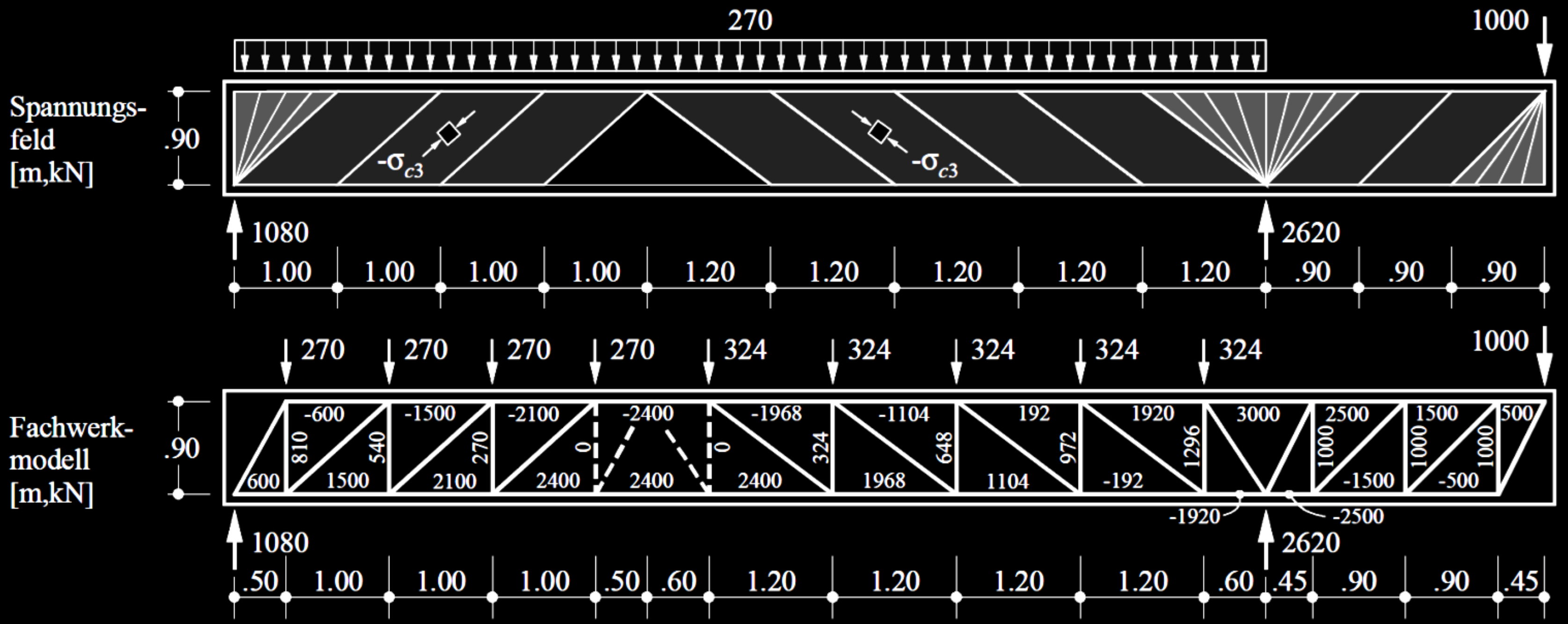
Additional examples

Beam - Example 3 (see [4] p. 66 ff)



Additional examples

Beam - Example 3 (see [4] p. 66 ff)



Construction and elements of the stress field

→ Points of zero shear force 4 m from support A, support B, beam end C

→ Subdivision of the resulting sections into equal sub-sections → Inclinations of the parallel compression fields are $\tan^{-1}(0.9/1.0) = 42.0°$, $\tan^{-1}(0.9/1.2) = 36.9°$ and $\tan^{-1}(0.9/0.9) = 45.0°$.

→ Centred fans (compression trajectories intersect in one point) for concentrated loads

→ Tension chord, compression stringer and vertical ties (shear reinforcement)

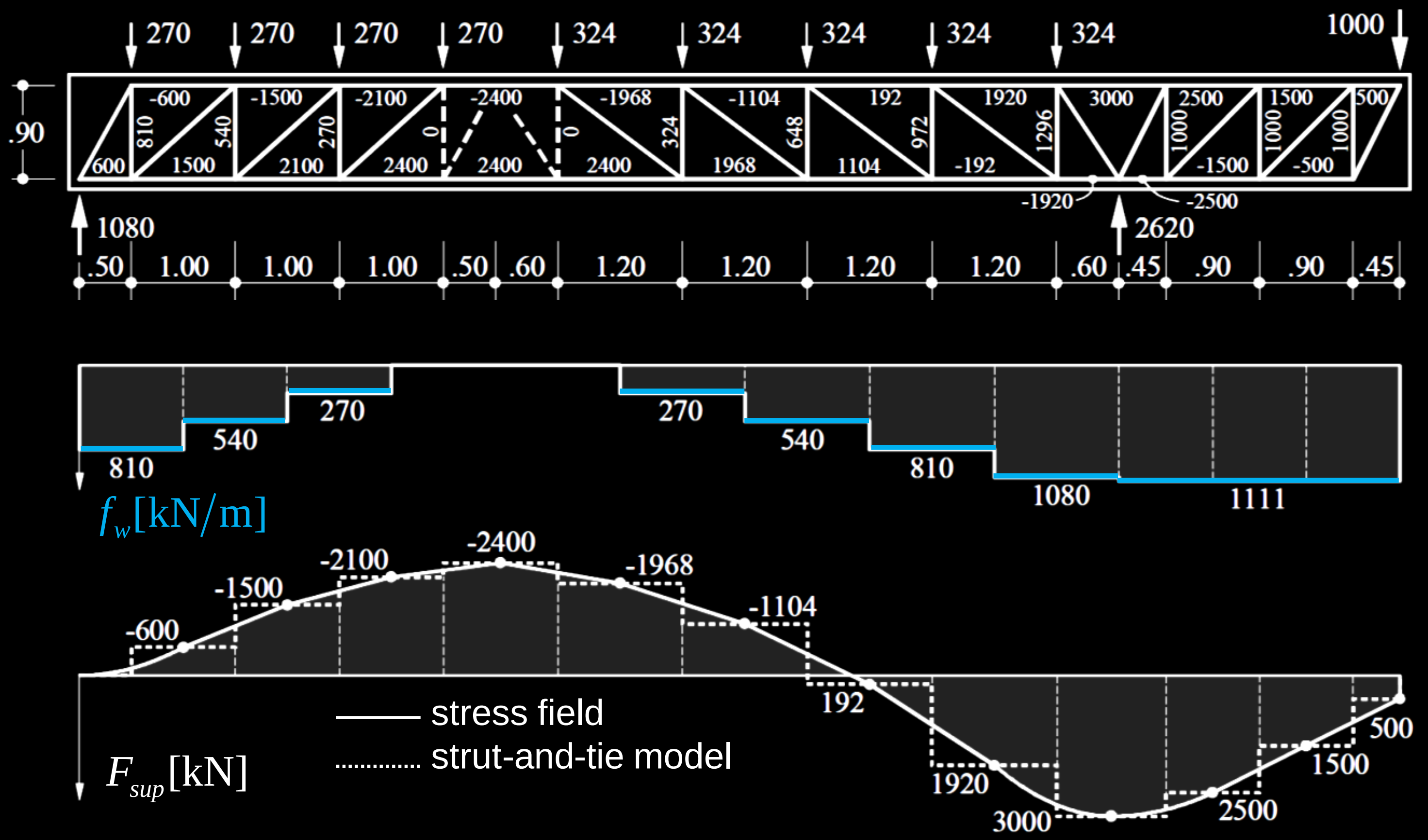
Additional examples

Beam - Example 3 (see [4] p. 66 ff)

Determination of forces in the stress field

→ The stirrup forces f_w (per unit length) can be obtained directly from diagonal cuts at the boundaries of the parallel compression fields or fans; the forces are constant between two boundaries.

→ The forces on the stirrups are constant in certain sections; as the load is applied at the top, the product $f_w \cdot z \cdot \cot\alpha$ is inscribed in the shear force diagram (so-called "staggering effect")



→ Load q_{inf} applied below the upper chord should be suspended by the vertical reinforcement, $\Delta f_w = q_{inf}$

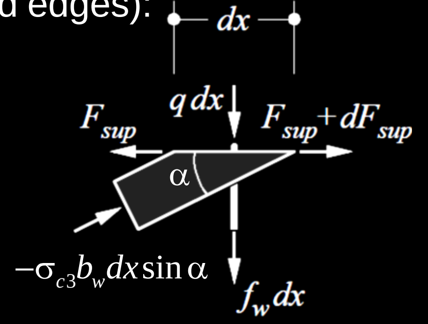
→ Chord forces of the stress field and strut-and-tie model coincide at points (points with numerical values).

Additional examples

Beam - Example 3 (see [4] p. 66 ff)

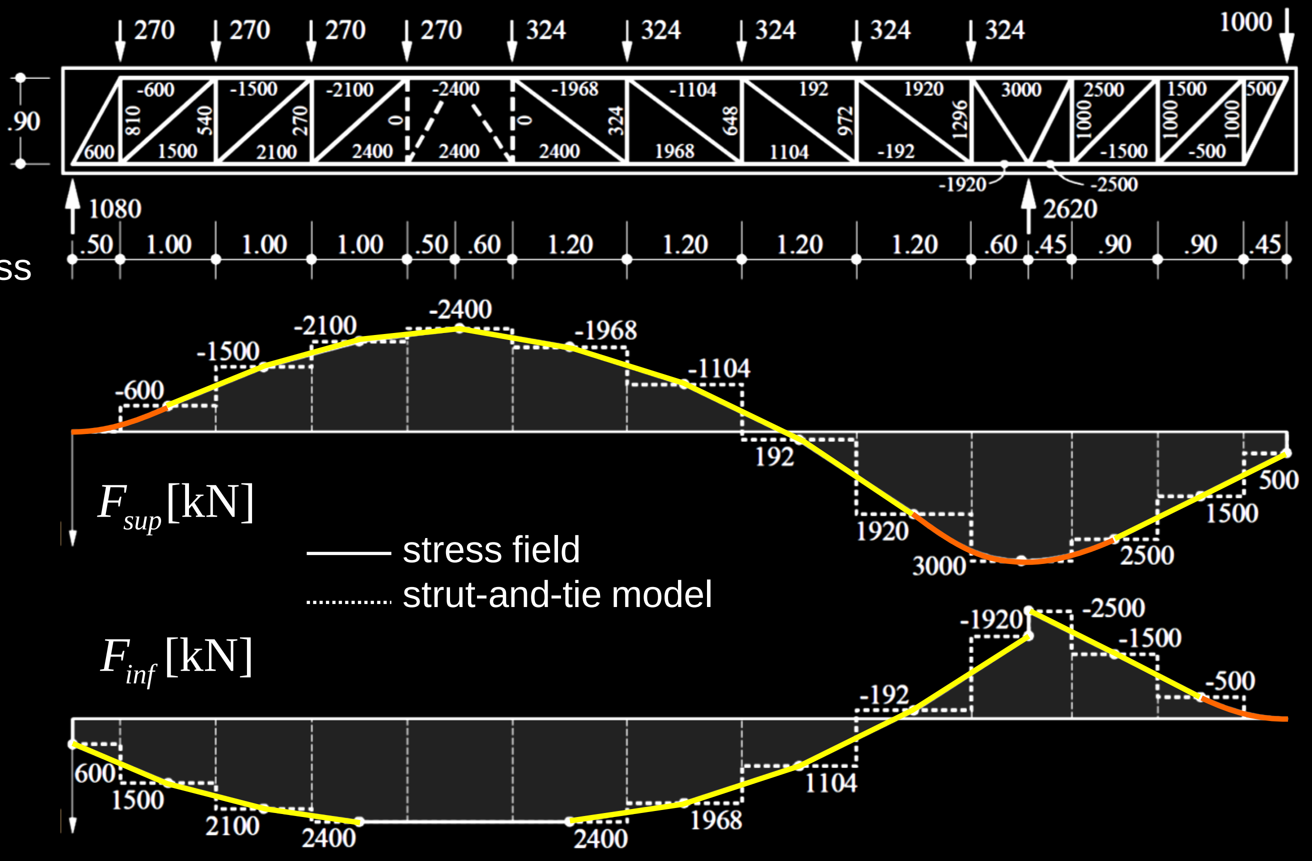
Determination of forces in the stress field

→ Distribution of chord forces F_{sup}, F_{inf} and concrete compr. stress (chord edges):



$$-\sigma_{c3} = \frac{(q + f_w)(1 + \cot^2 \alpha)}{b_w} = \frac{q + f_w}{b_w \sin^2 \alpha}$$

$$\frac{dF_{sup}}{dx} = -(q + f_w)\cot\alpha \qquad \frac{dF_{inf}}{dx} = f_w \cot\alpha$$



→ For a constant applied load q, the chord forces F_{sup}, F_{inf} along parallel compression fields are linear (f_w and cotα constant), along centred fans are parabolic (f_w constant, cotα linear).

→ Concrete compressive stresses are constant in parallel compression fields (along stress trajectories and across the width of the compression field band), they vary hyperbolically along (straight) trajectories of the fans.

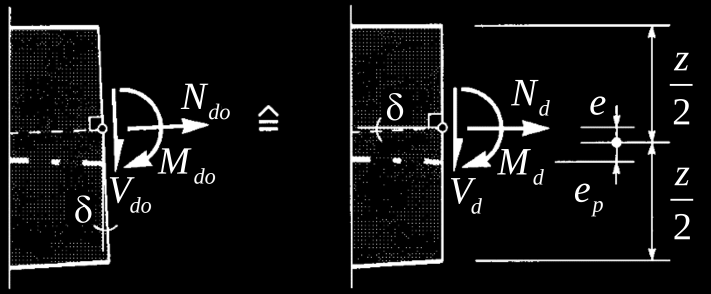
Additional examples

Practical application - Example 4

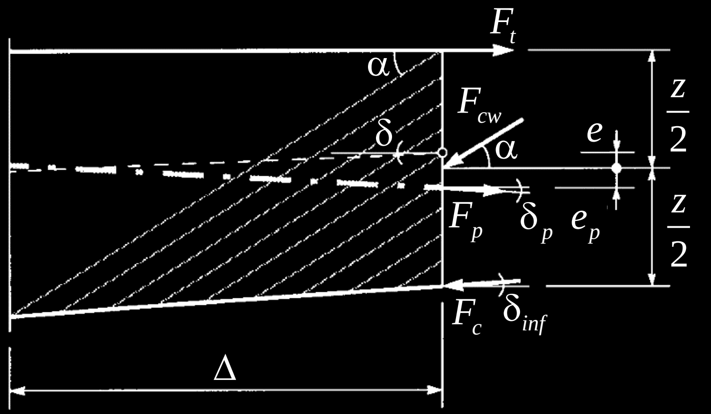
Additional examples

Practical application - Example 4

Beam - "cross-sectional analysis" (haunched beams with inclined prestressing)



$$
\begin{aligned}
N_d &= N_{do}\cos\delta + V_{do}\sin\delta \\
V_d &= -N_{do}\sin\delta + V_{do}\cos\delta \\
M_d &= M_{do}
\end{aligned}
$$



$$
\begin{aligned}
N_d &= F_t - F_{cw}\cos\alpha + F_p\cos\delta_p - F_c\cos\delta_{inf} \\
V_d &= F_{cw}\sin\alpha + F_p\sin\delta_p + F_c\sin\delta_{inf} \\
M_d &= F_t(z/2 - e) + F_{cw}\cos\alpha\cdot e - F_p\cos\delta_p(e_p + e) + \ldots \\
&\quad \ldots F_c\cos\delta_{inf}(z/2 + e)
\end{aligned}
$$

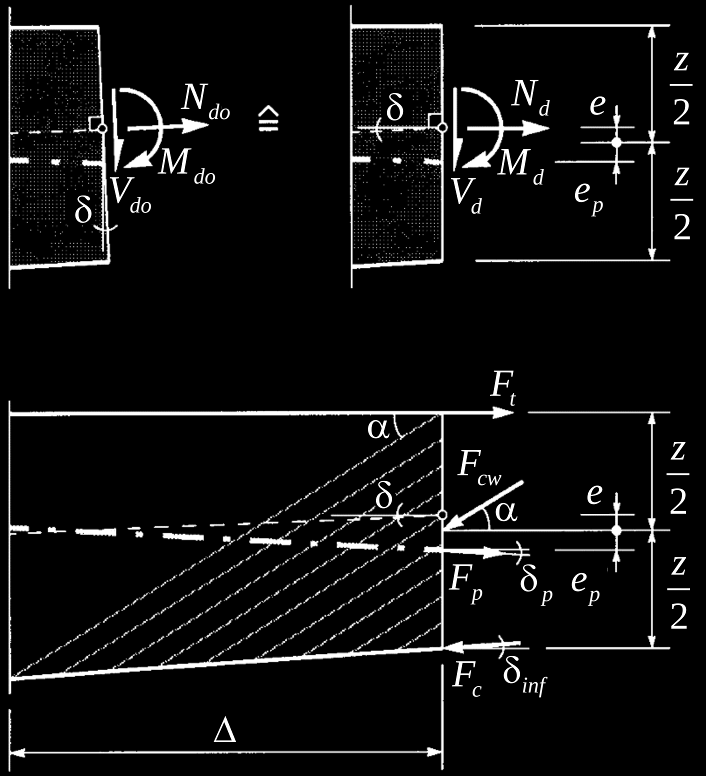
→ Step 1: Translate the resultants from the beam statics calculation (M_{do}, N_{do}, V_{do}) into the reference system of the stress field (M_d, N_d, V_d)

→ Step 2: Formulate equilibrium on a vertical cut

Additional examples

Practical application - Example 4

Beam - "cross-sectional analysis" (haunched beams with inclined prestressing)



$$F_{cw} = \frac{2\left(V_d - F_p \sin\delta_p\right) - \tan\delta_{inf}\left[2\frac{M_d}{z} - N_d\left(1 - \frac{2e}{z}\right) + F_p\cos\delta_p\left(1 + \frac{2e_p}{z}\right)\right]}{\left(1+k\right)\sin\alpha}$$

and

$$F_t = \frac{2k\frac{M_d}{z} + N_d\left(1 + \frac{2ke}{z}\right) + \frac{V_d - F_p\sin\delta_p}{\tan\alpha} - F_p\cos\delta_p\left(1 - \frac{2ke_p}{z}\right)}{1+k}$$

and

$$F_c = \frac{2\frac{M_d}{z} - N_d\left(1 - \frac{2e}{z}\right) - \frac{V_d - F_p\sin\delta_p}{\tan\alpha} + F_p\cos\delta_p\left(1 + \frac{2e_p}{z}\right)}{\left(1+k\right)\cos\delta_{inf}}$$

with

$$k = 1 - \frac{\tan\delta_{inf}}{\tan\alpha}$$

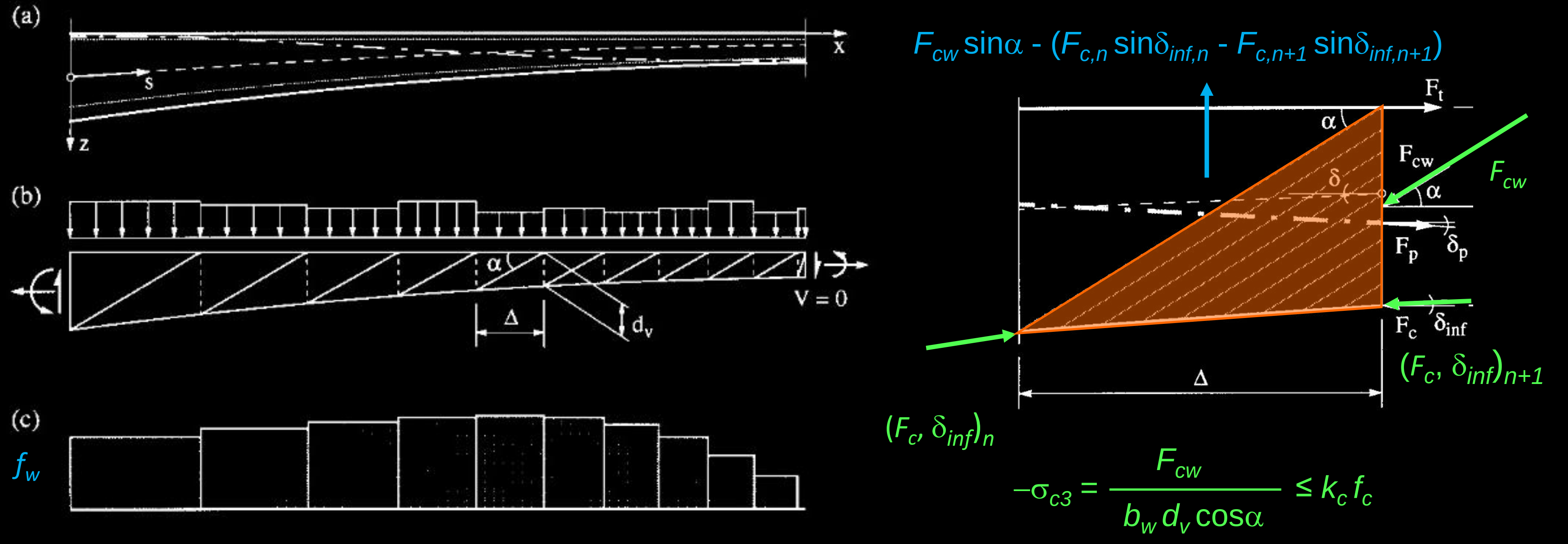
→ Step 3: Determination of the stresses in the stress field and dimensioning of the elements

→ Source: Marti, P. "Shear design of variable-depth girders with inclined prestressing", Pre-stressed concrete in Switzerland, FIP Swiss Group, Zurich 1994, pp. 16-19

Additional examples

Practical application - Example 4

Beam - "cross-sectional analysis" (haunched beams with inclined prestressing)



→ The stirrup forces result from equilibrium at the shown cross sections (a bit smaller than $F_{cw}\sin\alpha$, favourable effect of the deviation forces of the curved lower chord).

→ The deviation forces of curved tendons make the stirrup force variable over the web height, but this can usually be neglected.

→ Verification of the concrete compressive stress or determination of the web width with the specified relation

→ The distribution of the stirrup forces and concrete stress in the web can be controlled by the geometry of the lower chord («correct geometry»: this leads to the most uniform possible stresses over the entire length).

Additional examples

Practical application - Example 4