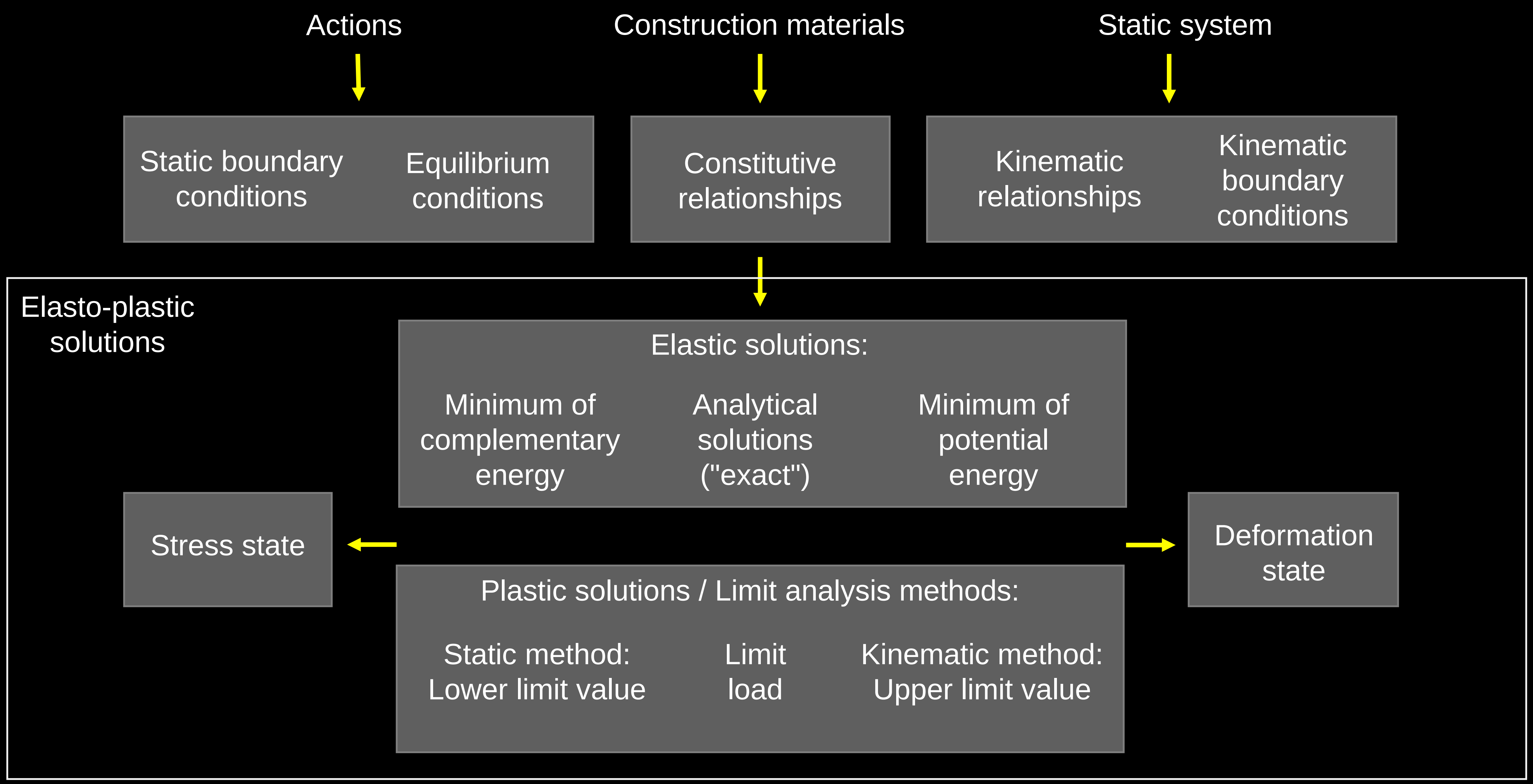
1. Introduction

Learning objectives

Within this chapter, the students are able to:

- identify and distinguish the different methods to design/analyse concrete structures:
  - describe the differences between elastic solutions and plastic solutions (limit analysis methods).
  - differentiate between the approaches required to design a new structure and to assess an existing one.
  - explain the main assumptions and theorems of limit analysis and their consequences on structural design practice.

Methods for structural analysis and design



Theory of plasticity – Limit analysis

Lower bound (static) theorem

Every loading for which it is possible to specify a statically admissible stress state that does not infringe the yield condition is not greater than the ultimate load.

(statically admissible: a stress state satisfying equilibrium and static boundary conditions)

Upper bound (kinematic) theorem

Every loading that results from equating the work of external forces for a kinematically admissible deformation state with the associated dissipation work is not less than the limit load.

(kinematically admissible: kinematic relationships and kinematic boundary conditions are fulfilled)

Compatibility theorem

A load for which a complete solution can be specified is equal to the ultimate load.

(complete solution: statically admissible stress state that does not infringe the yield condition and a compatible kinematically admissible state of deformation can be specified for that load.)

Theory of plasticity – Limit analysis

Main consequences of the theorems of limit analysis

- Residual stresses and restraints have no influence on the ultimate load (as long as the resulting deformations remain infinitesimally small).
  (NB: This applies only to limit analysis methods; in elastic solutions and particularly in stability problems, the failure load depends on residual stresses and restrains)
- Adding (subtracting) weightless material cannot decrease (increase) the ultimate load.
- Raising (lowering) the yield limit of the material in any region of a system cannot decrease (increase) its ultimate load.
- The ultimate load that can be calculated with a yield surface circumscribing (inscribing) the effective yield surface forms an upper (lower) bound to the effective ultimate load.

Application of the theorems of limit analysis

The lower bound theorem of limits analysis is the most used in practice. Typical applications: strut-and-tie models and stress fields for membrane elements, the strip method for slabs.

Many national and international codes are based (in most cases only implicitly, and unknown to many people) on the lower bound theorem.

In practice, the upper limit theorem is particularly helpful in assessing the structural safety of existing structures. (Allows limiting the ultimate load. This is often possible with considerably less effort than the development of a statically admissible stress state that does not violate the yield condition anywhere.)

Dimensioning of new buildings

Ductile design

$x/d < 0.35$, conservative assumption for concrete compressive strength, adequate anchorage lengths, etc.

→ Prevent brittle failures

→ Ensure applicability of plastic design methods

Plastic dimensioning

→ Define desired load path and follow it consistently

→ Structural elements have clearly defined functions

Simple models sufficient

Restraint stresses negligible

Redundancy and robustness

Solving problems conceptually

Designing instead of calculating

Structural Concrete I/II
→ Bachelor's degree
→ Specialisation ... (≠ Construction)

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"The engineer tells the structure how to carry the loads"

~~"The structure tells the engineer what to calculate"~~

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Structural assessment of existing structures

Existing structures

→ Avoid strengthening and retrofitting (expensive)

→ Load path given

→ Take ageing processes into account

Ductility is not given a priori

→ Verify applicability of plastic design methods

→ Evaluate the deformability capacity,
load-deformation behaviour is relevant

⇒

Simple models are often insufficient (not applicable or too conservative)

Combined loading of structural elements

Actual material properties instead of those specified in the design phase

Often numerical approaches required to explore all the load-bearing mechanisms

More demanding than the design of new structures!

Advanced Structural Concrete
→ Master's programme
→ Specialisation in Construction

ANNEX

Theory of plasticity – Limit analysis

Principle of maximum dissipation energy

Equivalent to: convexity of the yield condition + orthogonality of the plastic strain increments to the yield surface
→ Maximum dissipation energy (= basis of the limit analysis of the theory of plasticity)

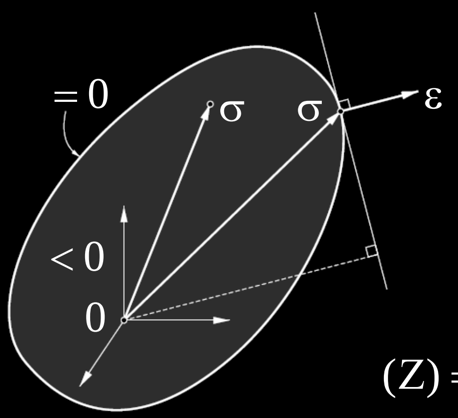
Generalised stresses and deformations

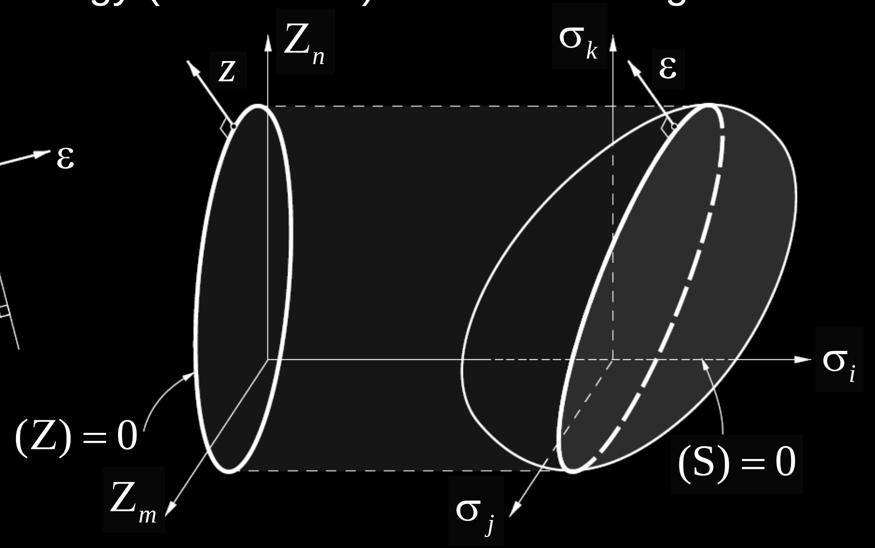
Introduction of kinematic restrictions ↔ Projections of the yield surface (Example: Hypothesis of Bernoulli for bending:
$\boldsymbol{\sigma} \to \{M, N\}$ and $\dot{\boldsymbol{\varepsilon}} \to \{\dot{\chi}, \dot{\varepsilon}_0\}$)

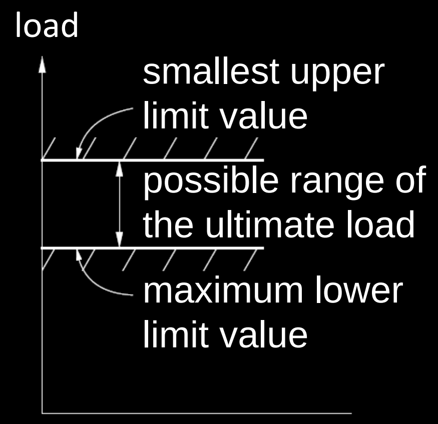
Projected values = *generalised stresses and deformations*

Stress components “lost” in the projection = *generalised reactions*

The principle of maximum dissipation energy (and others) is also valid in generalised quantities







Theory of plasticity – Limit analysis

Concrete - Modified Coulomb yield surface

Normal concrete: $\tan(\varphi) = 0.75 \rightarrow c = f_c/4$, $\varphi =$ approx. 37°