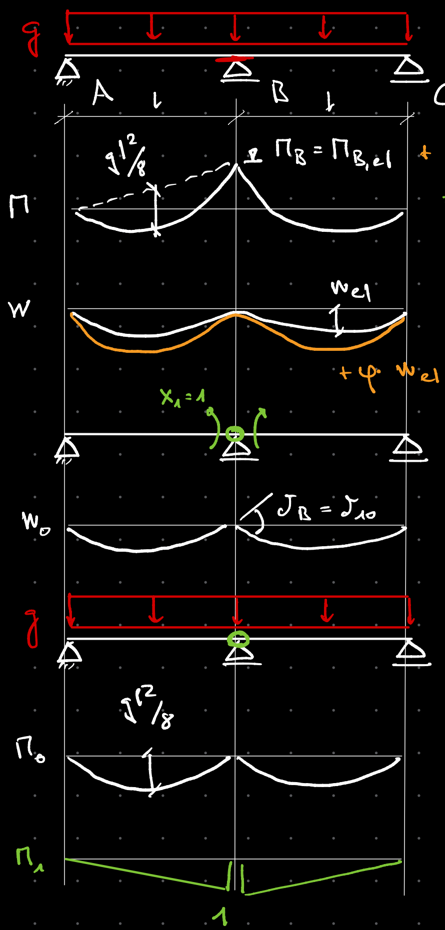
hs, 18 & 25.11.2025

# Effect of creep on two-span girder – time dependent force method

one casting system "Eingusssystem"




Basic system (isostatic) + redundant variable
BS + RV

Relative rotation at B

- due to g:

$$\vartheta_{10} = \int \frac{M_0 \cdot M_1}{EI}\,dx = \frac{gl^2}{8} \cdot \frac{l}{EI} \cdot \frac{1}{3} \cdot 2 = \frac{gl^2}{12\,EI}$$

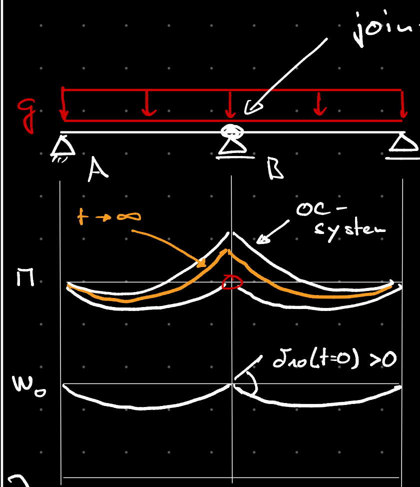
- due to $X_1 = 1$:

$$\vartheta_{11} = \int \frac{M_1 \cdot M_1}{EI}\,dx = \frac{1 \cdot 1}{EI} \cdot l \cdot \frac{1}{3} \cdot 2 = \frac{2l}{3EI}$$

here: for uncracked structure

staged construction "Bauzustände"

for systems of same age




two simply supported girders connected

≙ made continuous after applying g $(t = t_0)$

same result as for OC system

$$\vartheta_{10} = \frac{gl^2}{12\,EI}$$ ← relative rotation ("kink") at B, frozen for $t > t_0$

$$\vartheta_{11} = \frac{2l}{3EI}$$

## Short-term compatibility (t=0)

$\varphi = 0, \; \Delta X_1 = 0$

$$\vartheta_1 = \vartheta_{10}(1+\cancel{\varphi}) + X_1 \vartheta_{11}(1+\cancel{\varphi}) + \cancel{\Delta X_1} \vartheta_{11}(1+\mu\varphi) \overset{!}{=} 0$$

$$X_1 = -\frac{\vartheta_{10}}{\vartheta_{11}} = \frac{gl^2}{8} \quad ; \quad M_B = X_1 \underbrace{M_1}_{=1} = X_1$$

## Time-dependent compatibility

$$\vartheta_1 = \cancel{\vartheta_{10}(1+\varphi)} + \cancel{X_1 \vartheta_{11}(1+\varphi)} + \Delta X_1 \vartheta_{11}(1+\mu\varphi) \overset{!}{=} 0$$

$$\boxed{\Delta X_1 = 0 \quad ; \quad \Delta M_B = \Delta X_1 M_1 = 0}$$

⇒ No redistribution of internal forces due to creep
↳ deflection increase


## "stc" $(t = t_0)$

$\varphi = 0, \; \Delta X_1 = 0$

$$X_1 = 0 \quad ; \quad M_B(t=t_0) = X_1 \cdot M_1 = 0$$

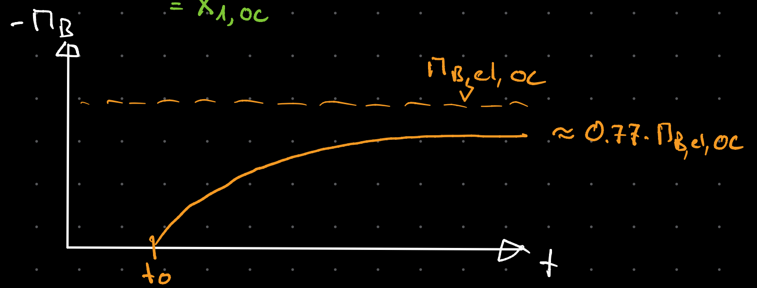
$$\vartheta_1 = \vartheta_{10}$$

## "tdc" $(t > t_0)$

$$\vartheta_1 = \vartheta_{10}(\cancel{1}+\varphi) + \underbrace{X_1 \vartheta_{11}(1+\varphi)}_{=0} + \Delta X_1 \vartheta_{11}(1+\mu\varphi) \overset{!}{=} \cancel{\vartheta_{10}}$$

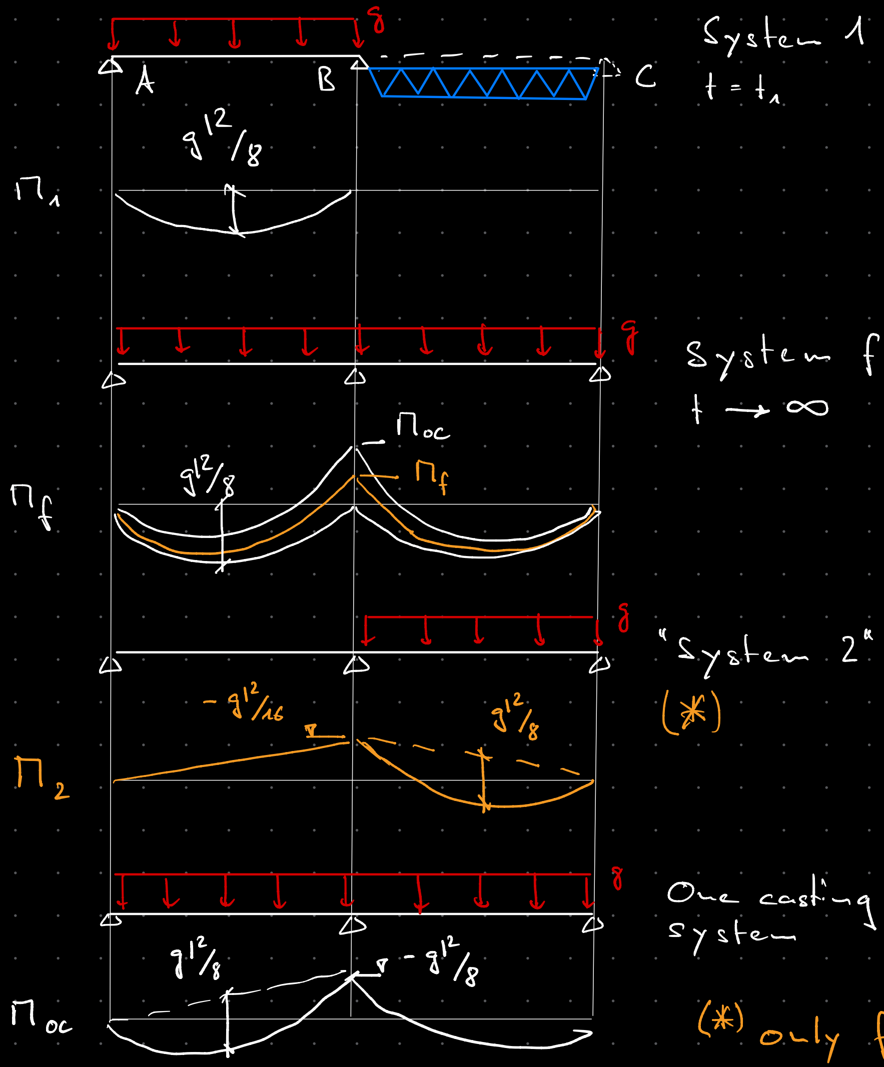
$$\vartheta_{10}\,\varphi + \Delta X_1 \vartheta_{11}(1+\mu\varphi) = 0$$

$$\boxed{\Delta X_1 = \underbrace{-\frac{\vartheta_{10}}{\vartheta_{11}}}_{= X_{1,OC}} \cdot \frac{\varphi}{1+\mu\varphi} \quad ; \quad M_B = M_1 \Delta X_1 = M_{B,oc,el} \cdot \frac{\varphi}{1+\mu\varphi}}$$



# staged construction

## for systems of different age




$$M_{tot} = M_0 + \underbrace{X_1 M_1}_{=0} + \Delta X_1 \cdot M_1, \qquad \Delta X_1 = X_{1,oc} \cdot \frac{\varphi}{1+\mu\varphi}$$

see previous slides

$$= M_0 + \underbrace{X_{1,oc} \cdot M_1}\ \frac{\varphi}{1+\mu\varphi}$$

$$M_{oc} = M_0 + X_{1,oc} \cdot M_1$$

$$\hookrightarrow \boxed{X_{1,oc} \cdot M_1} = M_{oc} - M_0$$

$$\Rightarrow M_{tot} = M_0 + (M_{oc} - M_0)\frac{\varphi}{1+\mu\varphi}$$

$$\boxed{M_{tot} = M_0\left(1 - \frac{\varphi}{1+\mu\varphi}\right) + M_{oc}\frac{\varphi}{1+\mu\varphi}}$$

Moment of different systems

Moment of system cast at once

(*) only forces on "new" part considered
$t = t_s$ (time when new part is loaded)

## Approximation:

$$M(t) = \sum M_i \left(1 - \frac{\varphi_i}{1+\mu\varphi_i}\right) + M_{oc}\frac{\varphi_0}{1+\mu\varphi_0}$$

$$= M_1\left(1 + \frac{\varphi_1}{1\ \ \mu\varphi_1}\right) + M_2\left(1 + \frac{\varphi_2}{1\ \ \mu\varphi_2}\right) + M_{oc}\frac{\varphi_0}{1+\mu\varphi_0}$$

for $t = t_s$: $\varphi = 0$, $M_B = 0.5\, M_{oc}$

$t \to \infty$: $\varphi = 2$, $M_B = 0.88 \cdot M_{oc}$

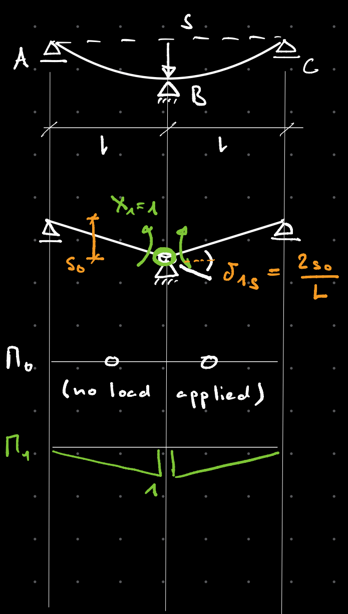
## Approximation "20 : 80"

$$M(t) = 0.2 \sum M_i + 0.8\, M_{oc} = 0.2(M_1 + M_2) + 0.8\, M_{oc}$$

for $t = t_s$: $\varphi = 0$, $M_B = 0.5\, M_{oc}$

$t \to \infty$: $\varphi = 2$, $M_B = 0.9 \cdot M_{oc}$

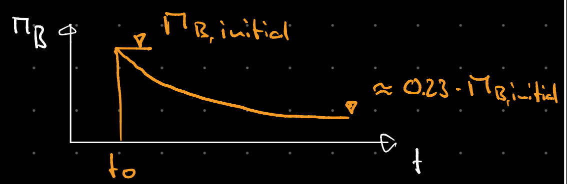
# Support settlement
# Effect of creep for fast / slow settlement




$$\delta_{10} = \int \frac{M_0 M_1}{EI} dx = \underline{0}$$

$$\delta_{11} = \int \frac{M_1 \cdot M_1}{EI} dx = \frac{2L}{3EI}$$

## time-independent settlement ("fast" restraint)


"stc", $t = t_0 \Rightarrow \varphi = 0,\ \Delta X_1 = 0$

$$\delta_1 = \delta_{10}(1+\cancel{\varphi})^{0} + X_1 \delta_{11}(1+\cancel{\varphi})^{0} + \underbrace{\Delta X_1 \delta_{11}(1+\mu\varphi)}_{=0} \overset{!}{=} \delta_{1s}$$

$$\Rightarrow X_1 = \boxed{\frac{\delta_{1s}}{\delta_{11}}}\ (*)\ ; \quad M_{B,initial} = X_1 \cdot M_1$$

$$= \frac{3EI}{L^2} \cdot s_0$$

"tdc", $t > t_0$

$$\delta_1 = \cancel{\delta_{10}}^{=0}(1+\varphi) + X_1 \delta_{11}(1+\varphi) + \Delta X_1 \delta_{11}(1+\mu\varphi) \overset{!}{=} \delta_{1s}$$

$$\delta_{1s}(1+\varphi) + \Delta X_1 \delta_{11}(1+\mu\varphi) = \delta_{1s}$$

$$\Rightarrow \Delta X_1(t) = -\underbrace{\boxed{\frac{\delta_{1s}}{\delta_{11}}}}_{= X_{1,stc}}\ (*) \cdot \frac{\varphi}{1+\mu\varphi}\ ; \quad \Delta M_B(t) = \Delta X_1 M_1$$

$$M_B(t) = M_{B,initial} + \Delta M_B(t)$$

$$= M_{B,initial}\left(1 - \frac{\varphi}{1+\mu\varphi}\right)$$

for $t \to \infty$: $\mu = 0.8,\ \varphi = 2$

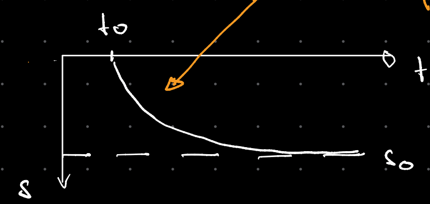
$$M_B(t) = M_{B,initial} \cdot 0.23$$

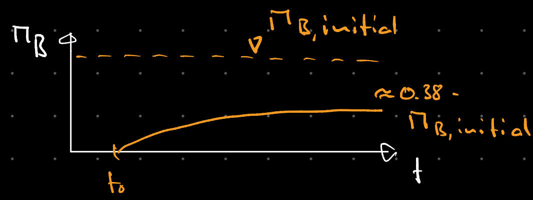
full initial restraint is reduced to 23%

relaxation

## time-dependent settlement ("slow" restraint)

$$s(t) = s_0 \cdot \frac{\varphi(t)}{\varphi_\infty}$$




"stc", $t = t_0$

$\delta_1 = \dots = 0$ since $s(t_0) = 0$

$\Rightarrow X_1 = 0$

$\Rightarrow M_B(t_0) = X_1 M_1 = 0$

"tdc", $t > t_0$

$$\delta_1 = \cancel{\delta_{10}}^{=0}(1+\varphi) + \cancel{X_1}^{=0} \delta_{11}(1+\varphi) + \Delta X_1(t) \delta_{11}(1+\mu\varphi) \overset{!}{=} \delta_{1s}(t)$$

$$\Delta X_1(t) = \underbrace{\boxed{\frac{\delta_{1s}}{\delta_{11}}}}_{= X_{1,stc}}\ (*) \cdot \frac{\varphi}{\varphi_\infty(1+\mu\varphi)}$$

$$\Delta M_B(t) = \Delta X_1 M_1$$

$$\Rightarrow M_B(t) = M_{B,initial} \cdot \frac{\varphi(t)}{\varphi_\infty(1+\mu\varphi(t))}$$

for $t \to \infty$: $\varphi = 2,\ \mu = 0.8$

$\varphi(t) = \varphi_\infty$

$$M_B(t \to \infty) = M_{B,initial} \cdot 0.38$$

initially zero restraint builds up to 38% of full elastic restraint

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