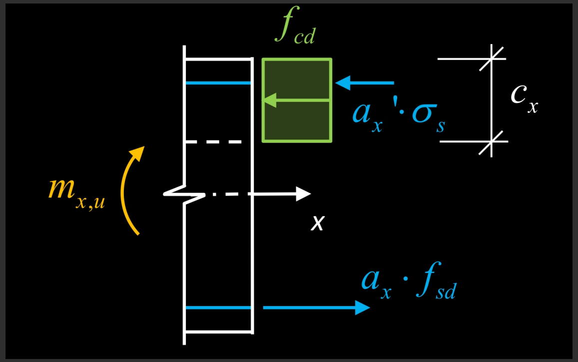
Normal Moment Yield Condition

! membrane actions cannot be considered

$$\underbrace{m_{x,u}\cdot\cos^2\varphi_u + m_{y,u}\cdot\sin^2\varphi_u}_{①\ \text{resistance}} = m_{n,u} \overset{!}{=} m_n = \underbrace{m_x\cdot\cos^2\varphi_u + m_y\cdot\sin^2\varphi_u + 2m_{xy}\cdot\sin\varphi_u\cos\varphi_u}_{②\ \text{load}}$$

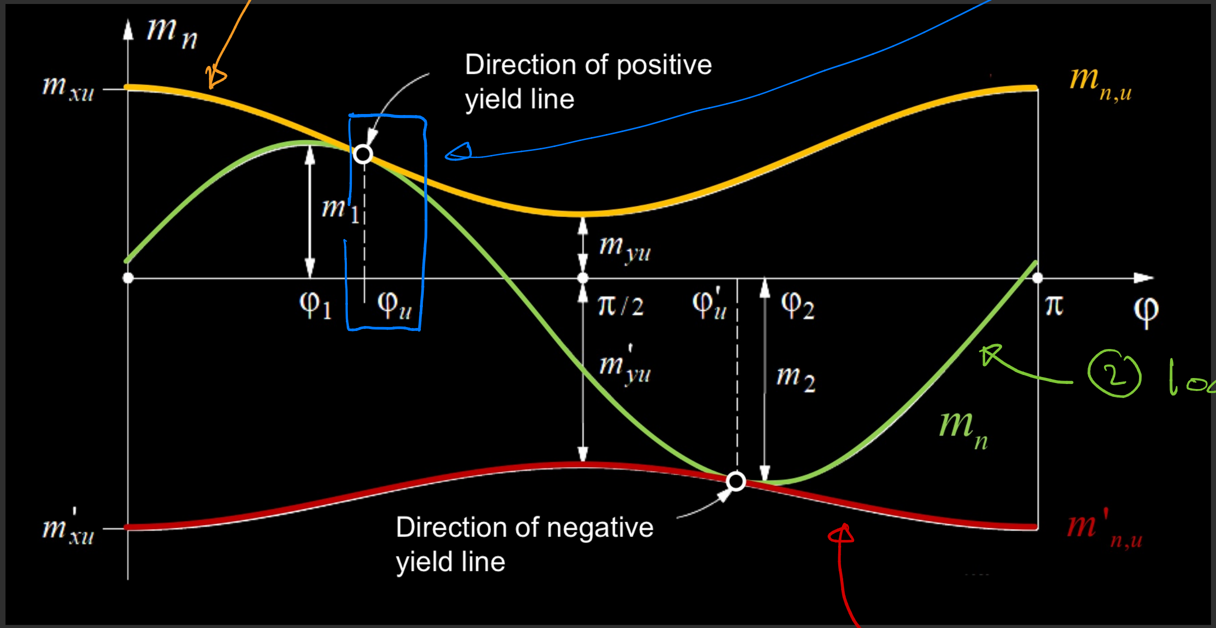
no twisting term in x/y-direction

① resistance:



$m_{n,u}(\varphi) - m_n(\varphi)$
↳ min

① resistance (pos)



② load

① resistance (neg)

$$|\tan\varphi_u| = \sqrt{\frac{(m_{x,u}-m_x)}{(m_{y,u}-m_y)}}$$

$$\boxed{\begin{aligned} \underbrace{m_{x,u}}_{\text{resistance}} &= \underbrace{m_x + m_{xy}\cdot\tan\varphi_u}_{\text{actions}} \\ m_{y,u} &= m_y + m_{xy}\cdot\cot\varphi_u \end{aligned}}$$

Normal Moment Yield Condition

$$\underbrace{m_{x,u}\cdot\cos^2\varphi_u + m_{y,u}\cdot\sin^2\varphi_u}_{\text{① resistance}} = m_{n,u} \overset{!}{=} m_n = \underbrace{m_x\cdot\cos^2\varphi_u + m_y\cdot\sin^2\varphi_u + 2m_{xy}\cdot\sin\varphi_u\cos\varphi_u}_{\text{② load}}$$

no twisting term in x/y-direction

$m_{n,u}(\varphi) - m_n(\varphi)$
↳ min

$$\boxed{\begin{aligned} Y &= m_{xy}^2 - \overbrace{\left(m_{x,u}-m_x\right)}^{\geq 0}\overbrace{\left(m_{y,u}-m_y\right)}^{\geq 0} = 0 \\ Y' &= m_{xy}^2 - \underbrace{\left(m'_{x,u}+m_x\right)}_{\geq 0}\underbrace{\left(m'_{y,u}+m_y\right)}_{\geq 0} = 0 \end{aligned}}$$

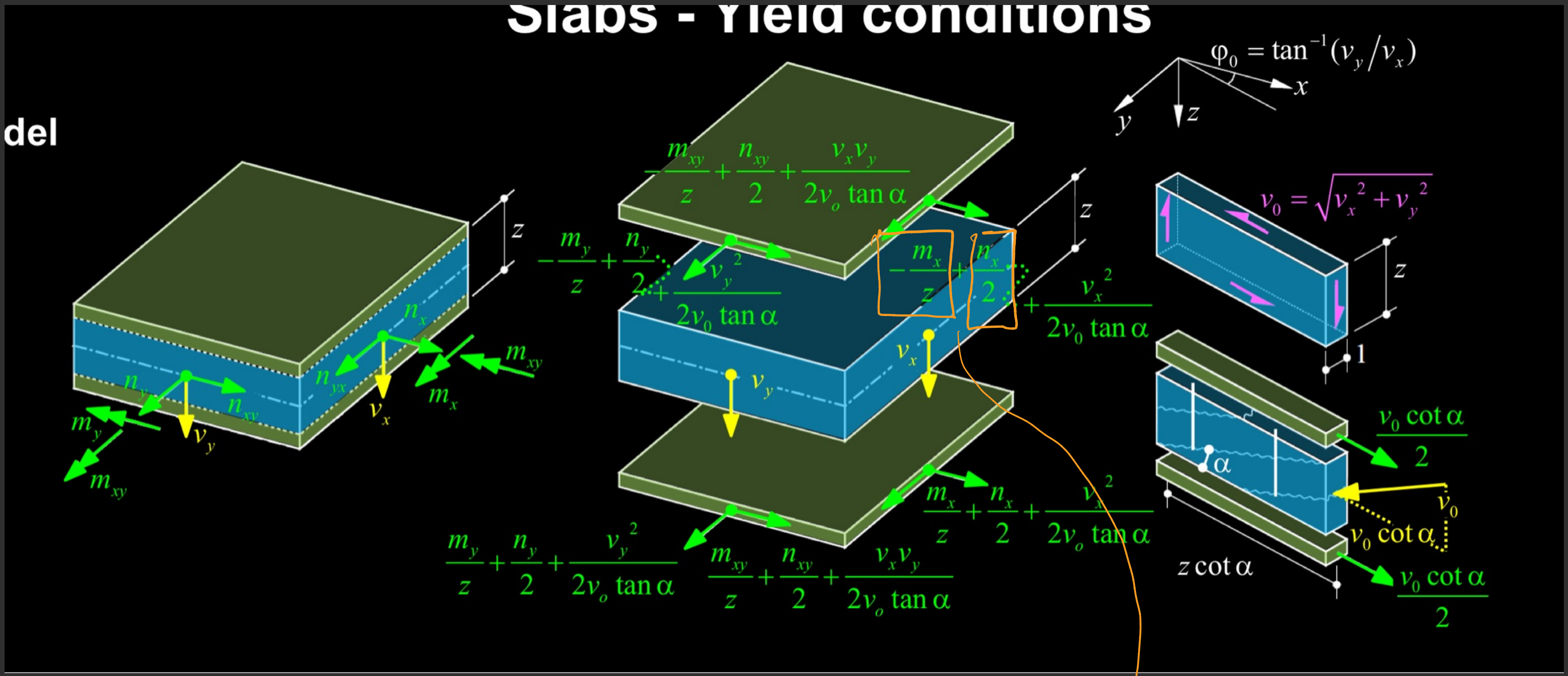
$$\left|\tan\varphi_u\right| = \sqrt{\frac{\left(m_{x,u}-m_x\right)}{\left(m_{y,u}-m_y\right)}}$$

$$\boxed{\begin{aligned} \underbrace{m_{x,u}}_{\text{resistance}} &= \underbrace{m_x + m_{xy}\cdot\tan\varphi_u}_{\text{actions}} \\ m_{y,u} &= m_y + m_{xy}\cdot\cot\varphi_u \end{aligned}}$$

$$\boxed{\begin{aligned} m_{x,u} &\geq m_x + k\cdot\left|m_{xy}\right| \\ m_{y,u} &\geq m_y + \frac{1}{k}\cdot\left|m_{xy}\right| \end{aligned}}$$

$k = \tan\varphi_u$
(in FE, often $k = 1$)

Sandwich Model

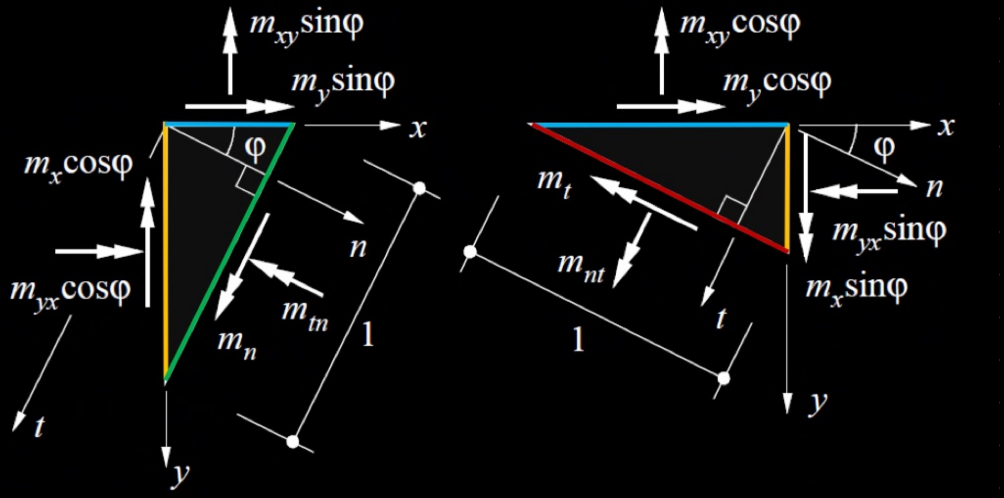


$$n(m_x) = \frac{m_x}{z}$$

in case of:

- membrane actions (n_x, n_y)
- high twisting moments (m_{xy})

Slabs



Bending and twisting moments in any direction φ:

$$m_n = m_x \cos^2\varphi + m_y \sin^2\varphi + m_{xy} \sin 2\varphi$$

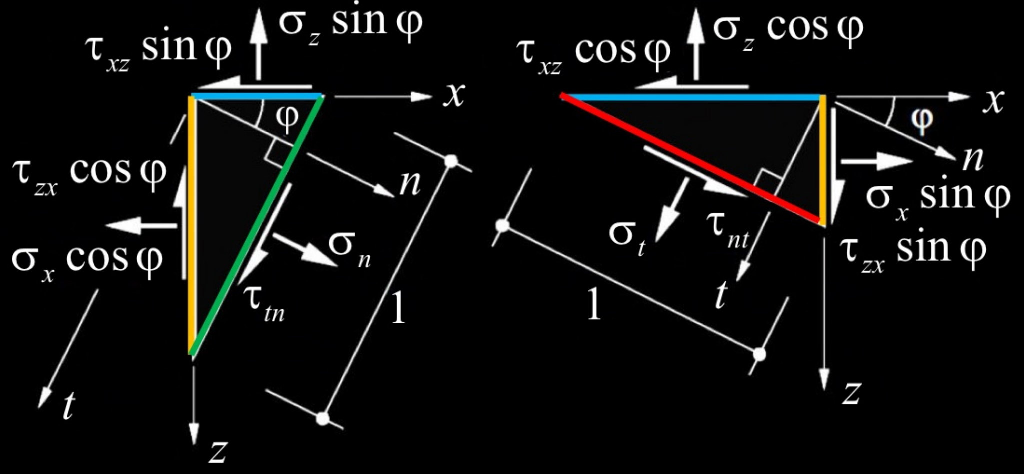
$$m_t = m_x \sin^2\varphi + m_y \cos^2\varphi - m_{xy} \sin 2\varphi$$

$$m_{tn} = \left(m_y - m_x\right)\sin\varphi\cos\varphi + m_{xy} \cos 2\varphi$$

NB: $\sin 2\varphi = 2\sin\varphi\cos\varphi,\ \cos 2\varphi = \cos^2\varphi - \sin^2\varphi$

Pr
m

Membranes



$$\sigma_n = \sigma_x \cos^2\varphi + \sigma_z \sin^2\varphi + 2\tau_{xz} \sin\varphi\cos\varphi$$

$$\sigma_t = \sigma_x \sin^2\varphi + \sigma_z \cos^2\varphi - 2\tau_{xz} \sin\varphi\cos\varphi$$

$$\tau_{nt} = (\sigma_z - \sigma_x)\sin\varphi\cos\varphi + \tau_{xz}(\cos^2\varphi - \sin^2\varphi)$$

$$\sin(2\varphi) = 2\sin\varphi\cdot\cos\varphi$$

Slabs

Membranes

φ in slabs' corresponds to α in membranes

$$|\tan\varphi_u| = \sqrt{\frac{(m_{x,u}-m_x)}{(m_{y,u}-m_y)}}$$

$$\tan\varphi_u = \frac{1}{\sqrt{\cot^2\varphi_u}}$$

$$\cot^2\alpha = \frac{n_{xc}}{n_{zc}} = \frac{a_{sx} f_{sd} - n_x}{a_{sz} f_{sd} - n_z}$$

$$\boxed{\begin{aligned} m_{x,u} &= m_x + m_{xy}\cdot\tan\varphi_u \\ m_{y,u} &= m_y + m_{xy}\cdot\cot\varphi_u \end{aligned}}$$

resistance actions

$$Y_1 = n_{xz}^2 - (a_{sx} f_{sx} - n_x)(a_{sz} f_{sz} - n_z) = 0$$

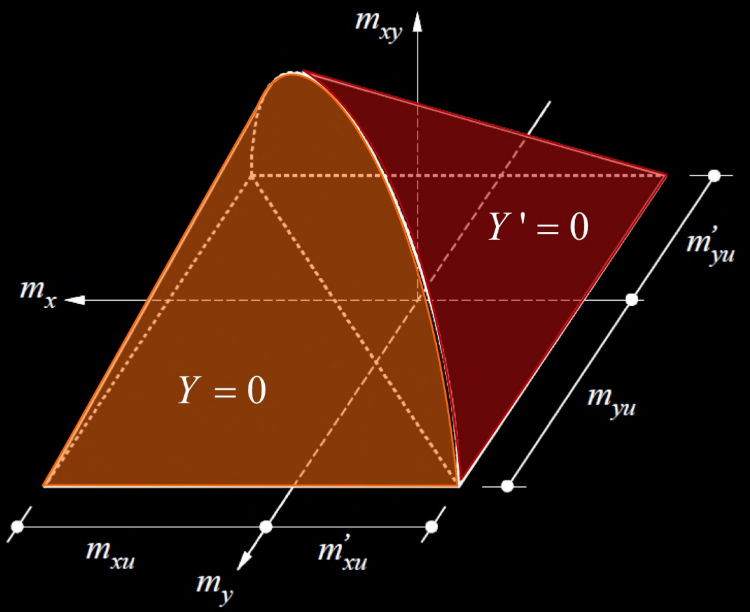
$$\boxed{k = \cot\alpha} \longrightarrow \begin{aligned} a_{sx} f_{sx} &\geq n_x + k|n_{xz}| \\ a_{sz} f_{sz} &\geq n_z + k^{-1}|n_{xz}| \end{aligned}$$

resistance load

Slabs

similar to n_{xy}^2

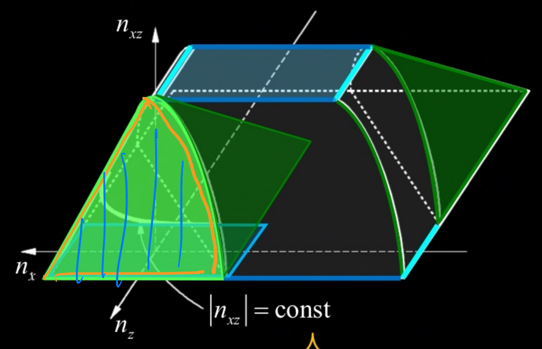
$$\boxed{\begin{aligned} Y &= m_{xy}^2 - \overbrace{\left(m_{x,u} - m_x\right)}^{\geq 0}\overbrace{\left(m_{y,u} - m_y\right)}^{\geq 0} = 0 \\ Y' &= m_{xy}^2 - \underbrace{\left(m'_{x,u} + m_x\right)}_{\geq 0}\underbrace{\left(m'_{y,u} + m_y\right)}_{\geq 0} = 0 \end{aligned}}$$

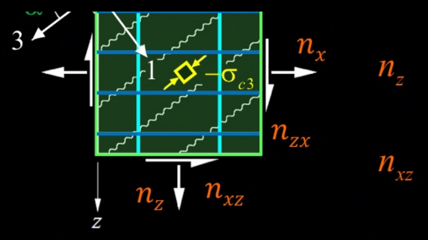


Membranes

resistance in x

$$Y_1 = n_{xz}^2 - \overbrace{(a_{sx} f_{sx} - n_x)}(a_{sz} f_{sz} - n_z) \qquad = 0$$





Proce
Move
origin
(or vi