Rotation demand

statically indeterminate system (1x)

linear elastic behaviour

static system + loading (no initial restraint)

basic system "BS" + redundant variable "RV" e.g. hinge in B

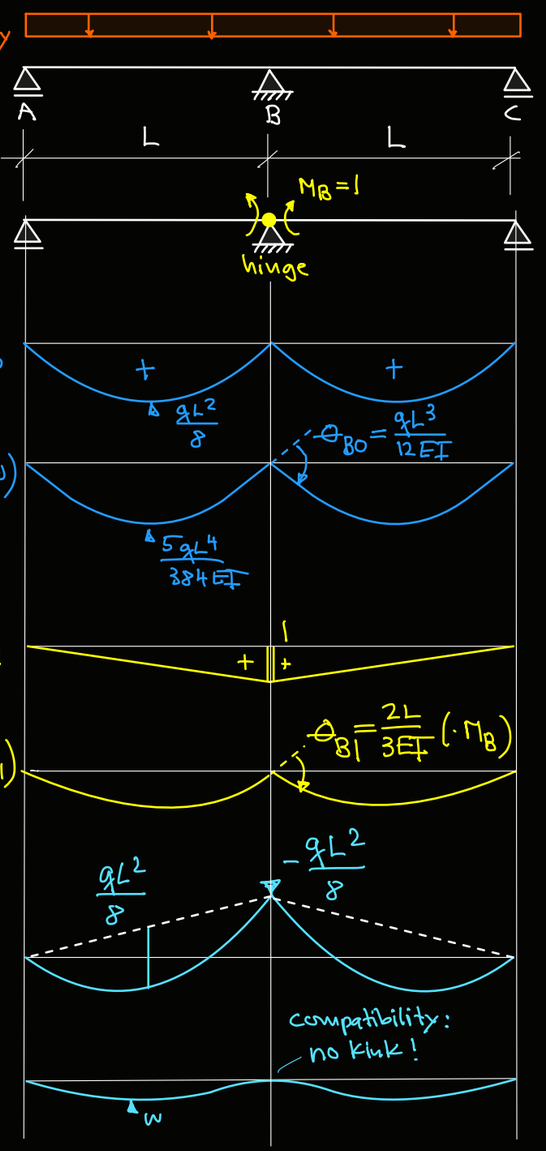
$M_0 = M_0(q)$ in BS and corresponding deflection w_0

$M_1 = M(M_B = 1)$ in BS and corresponding deflection w_1

$M = M_0 + M_B \cdot M_1$

and resulting deflection

$w = w_0 + M_B \cdot w_1$



* Many assumptions:
- EI = const. → not correct: $EI_{II} \ll EI_I$ and $EI_{II}^- > EI_{II}^+$ (cracking) ($EI_{II} \approx E_s A_s (d-x)(d-\frac{x}{3})$ ν A_s)
- no initial restraint
- no strain hardening in B

Force method

- release constraints until system is isostatic
- compatibility of deformations at released constraints
- deformations determined using work theorem

→ deformation state (real) + virtual force state

→ external work = internal work

$$1 \cdot \theta_{B0} = \int \chi_0 M_1 = \int \frac{M_0 M_1}{EI} dx = 2 \cdot \frac{qL^2}{8EI} \cdot 1 \cdot \frac{L}{3} = \frac{qL^3}{12EI}$$

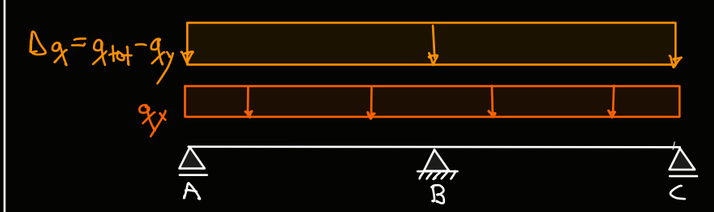
$$1 \cdot \theta_{B1} = \int \chi_1 M_1 = \int \frac{M_1 \cdot M_1}{EI} dx = 2 \cdot \frac{1}{EI} \cdot 1 \cdot \frac{L}{3} = \frac{2L}{3EI}$$

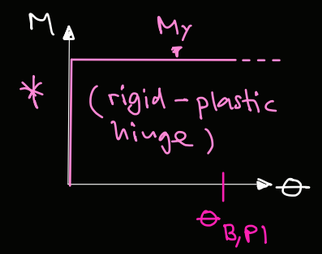
compatibility: no relative rotation ("kink") at B:

$$\theta_B = \theta_{B0} + M_B \cdot \theta_{B1} \overset{!}{=} 0 \quad (M_B = 1)$$

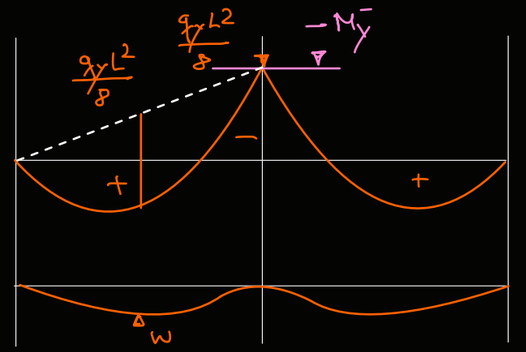
$$\rightarrow M_B = -\frac{\theta_{B0}}{\theta_{B1}} = -\frac{qL^3}{12EI} \cdot \frac{3EI}{2L} = -\frac{qL^2}{8}$$

elastic-plastic behaviour (assume first plastic hinge at B)





Stage 1: Linear elastic behaviour for $q_x \leq q_y$

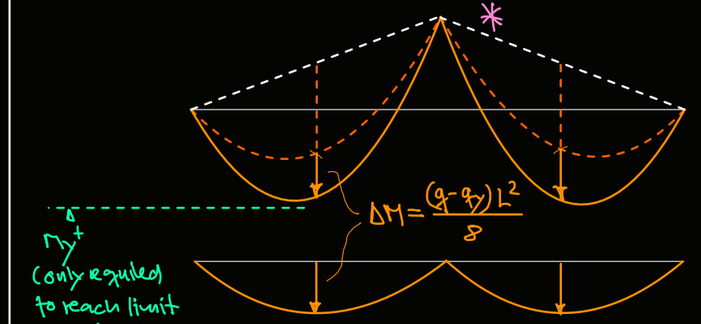


plastic hinge in B when

$$\frac{qL^2}{8} = M_y^-$$

$$\rightarrow q_y = \frac{8M_y^-}{L^2}$$

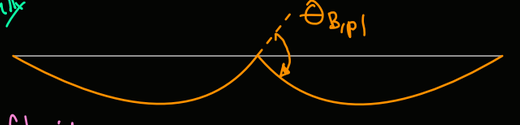
Stage 2: Moment redistribution i.e. plastic hinge in B → additional load carried as two simply supported beams



additional load $\Delta q = q_{tot} - q_y$ carried as two simply supported beams

↓

plastic hinge rotation $\theta_{B,pl}$ as in BS on left side:

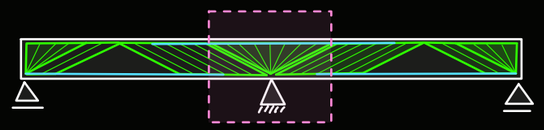


$$\theta_{B,pl} = \frac{\Delta q L^3}{12EI} = \frac{(q_{tot} - q_y)L^3}{12EI} = \left(q_{tot} - \frac{8M_y^-}{L^2}\right)\frac{L^3}{12EI}$$

* see left side: this is merely an estimate!

rotation demand in B to achieve q_{tot}

Plastic hinge length ⟷ Tension chord force profile $F_{sup}(x)$



"plastic hinge length" L_{pl} = ?
mean strain at steel rupture ε_{smu} = ?

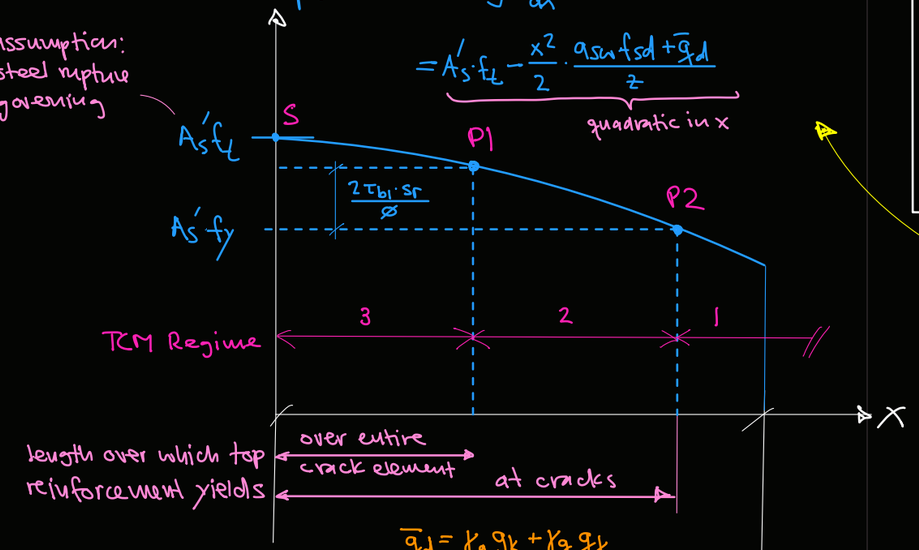
fan region = detailed study

assumption: steel rupture governing

$F_{sup}(x) = A_s' f_t + \int \frac{dF_{sup}}{dx} dx$

$= A_s' f_t - \frac{x^2}{2} \cdot \frac{a_{sw} f_{sd} + \bar{q}_d}{z}$

quadratic in x

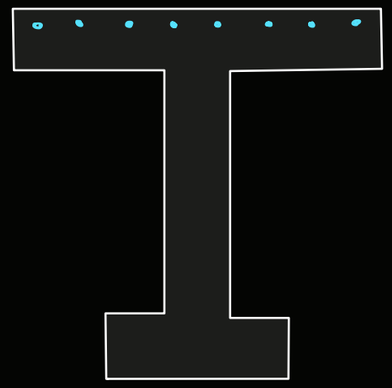


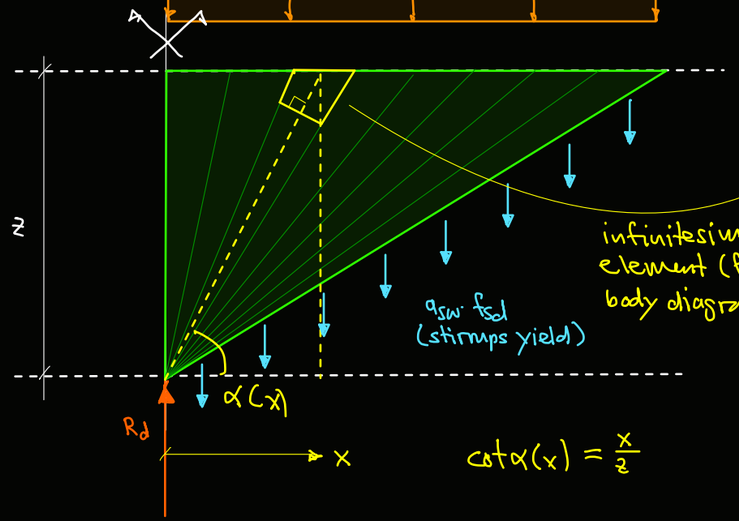
Simplified determination of θ_u:

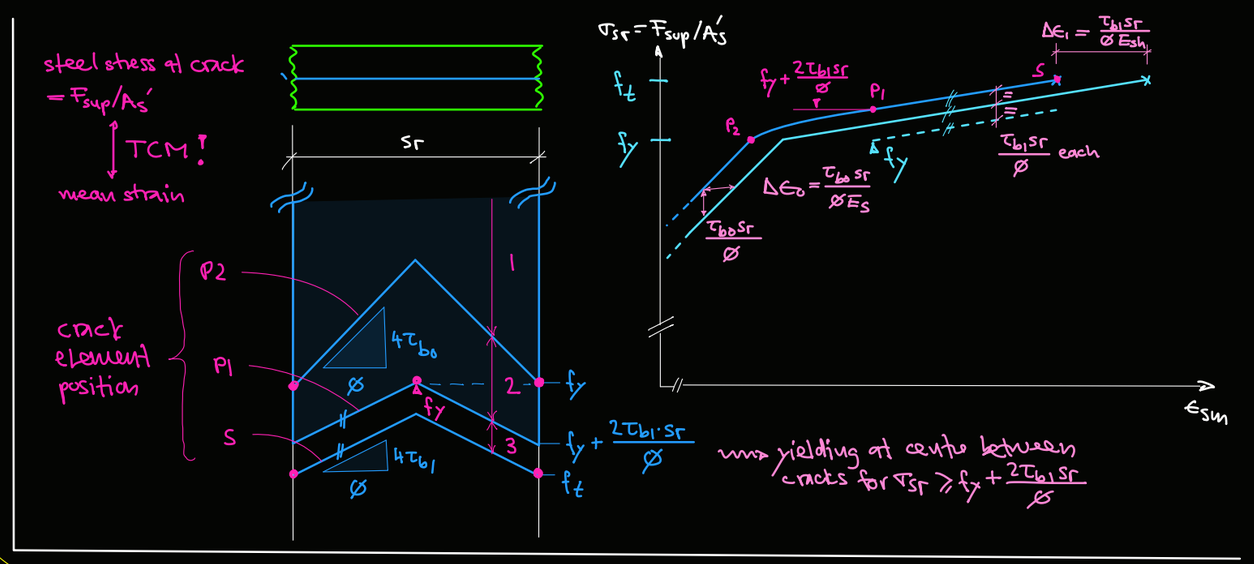
$\theta_u = L_{pl} \cdot \frac{\varepsilon_{smu}}{d - x}$

"Plastic hinge length" L_{pl} chosen such that

$\theta_u = \int_{-x(P2)}^{x(P2)} \frac{\varepsilon_s(x)}{d-x} dx \overset{!}{=} L_{pl} \cdot \frac{\varepsilon_{smu}}{d-x}$

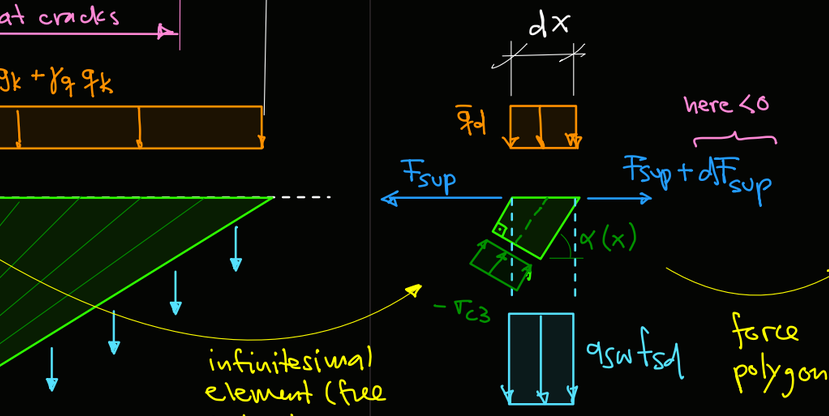




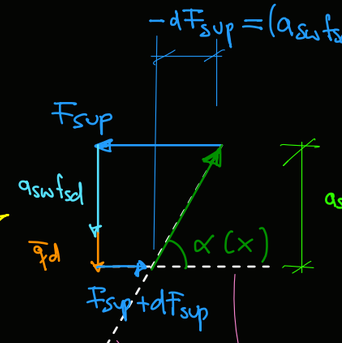


integrate (with $F_{sup}(0) = A_s f_t$)

$\frac{dF_{sup}}{dx} = (a_{sw} \cdot f_{sd} + \bar{q}_d) \cdot \cot\alpha = \underbrace{(a_{sw} \cdot f_{sd} + \bar{q}_d) \cdot \frac{x}{z}}_{\text{linear in x}}$



force polygon



known directions → Force polygon must be closed = equilibrium