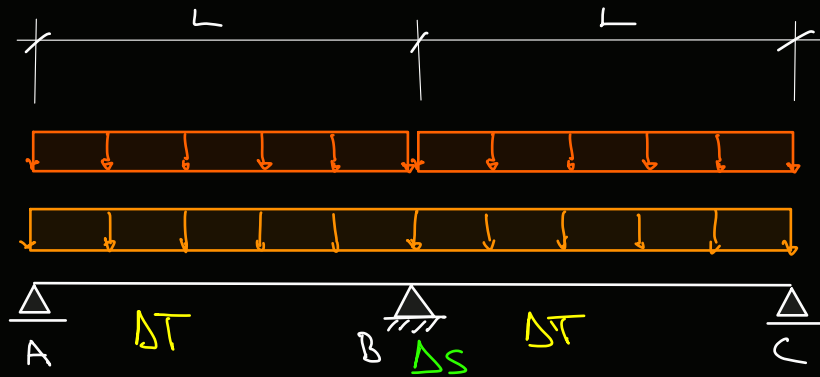


Elastic design



$$q_d = \gamma_g q_k$$

$$q_d = \gamma_g q_k$$

Loads to be considered

$q_d = \gamma_g q_k$: variable load, where unfavourable only

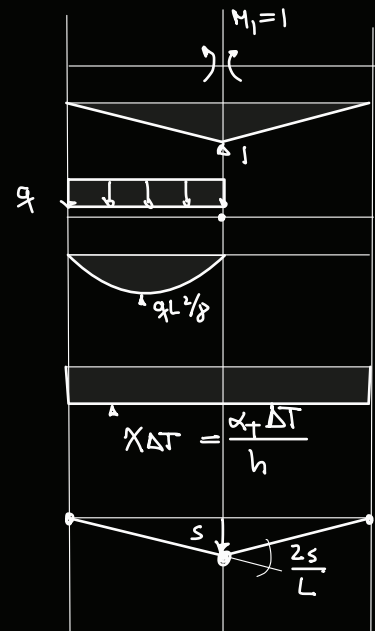
$q_d = \gamma_g q_k$: permanent load, entire structure

$\Delta T^{\pm}$: Temperature gradient $\pm$

Δs : Differential settlement $\pm$

Where unfavourable

NB. Force method:

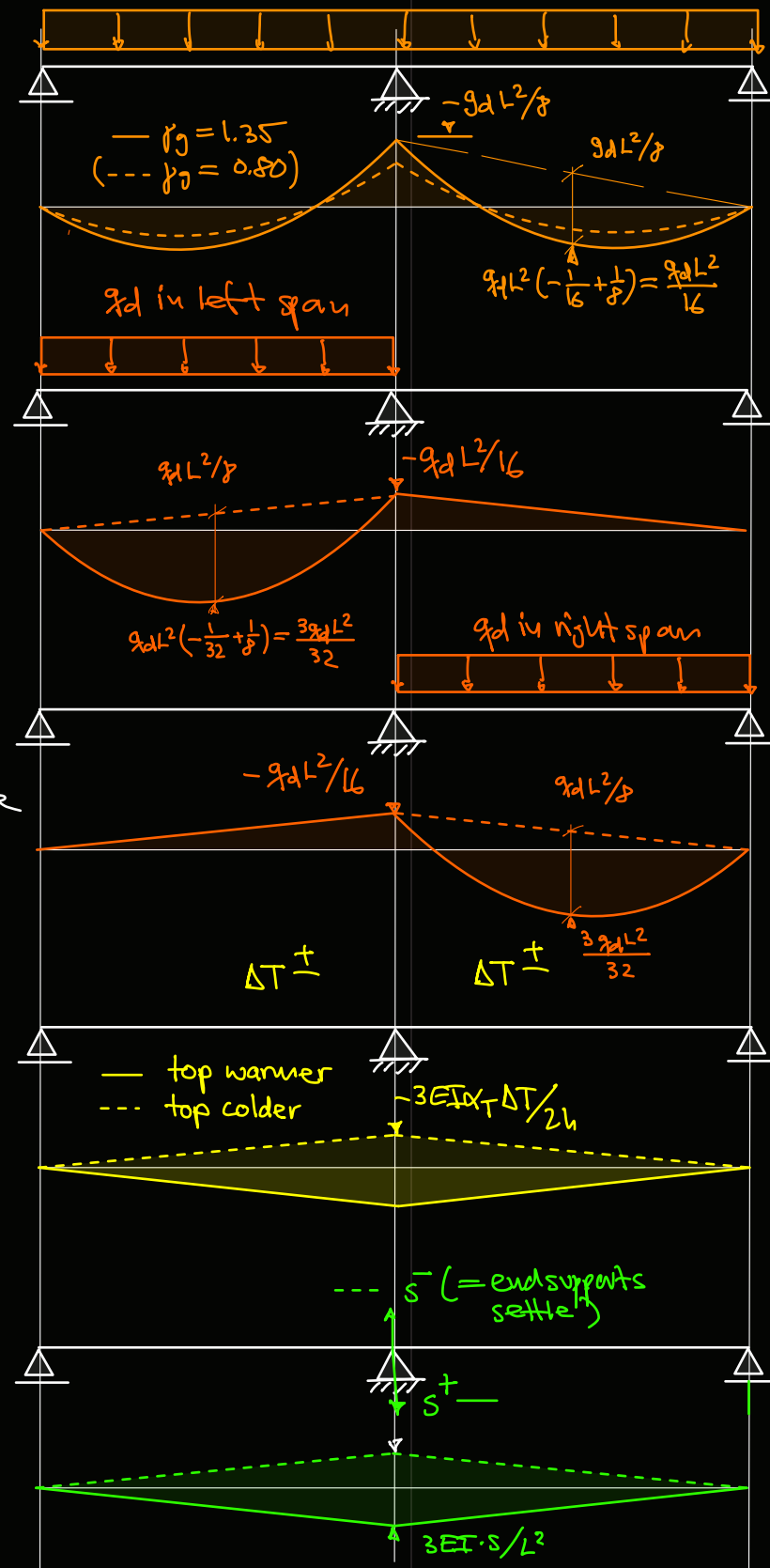


$$\varphi_{11} = 2 \cdot 1 \cdot \frac{L}{3EI} = \frac{2L}{3EI}$$

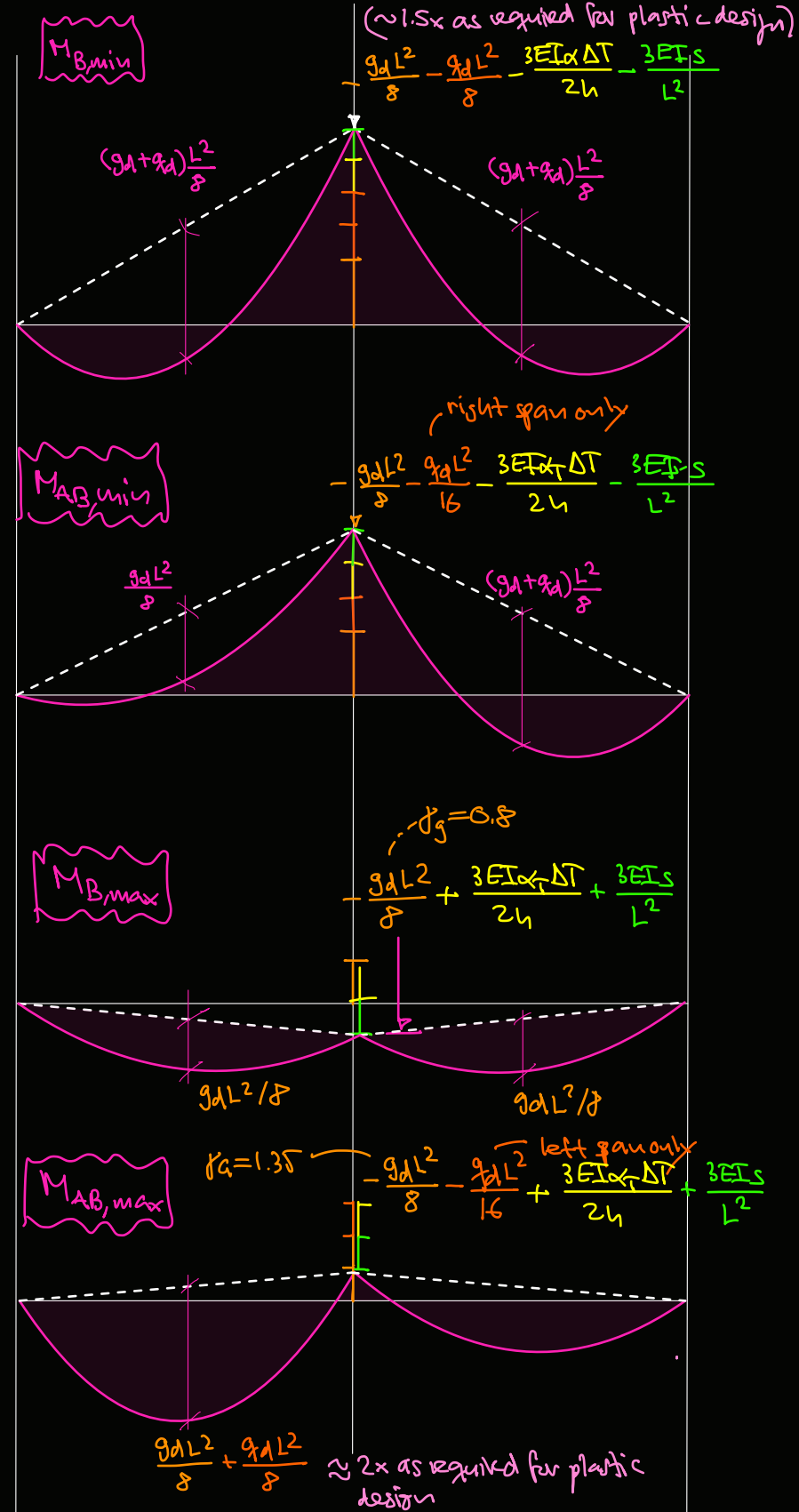
$$\varphi_{10} = \frac{qL^2}{8} \cdot \frac{L}{3EI} = \frac{qL^3}{24EI}, M_1 = -\frac{\varphi_{10}}{\varphi_{11}} = -\frac{qL^2}{16}$$

$$\varphi_{10} = 2 \cdot \frac{\alpha_T \Delta T}{h} \cdot \frac{L}{2}, M_1 = -\frac{3EI \alpha_T \Delta T}{2h}$$

$$\varphi_1 = M_1 \cdot \frac{2L}{3EI} = \frac{2s}{L} \rightarrow M_1 = \frac{3EI \cdot s}{L^2}$$



Design for envelope of possible combinations $\rightarrow$ inefficient design in ULS STR



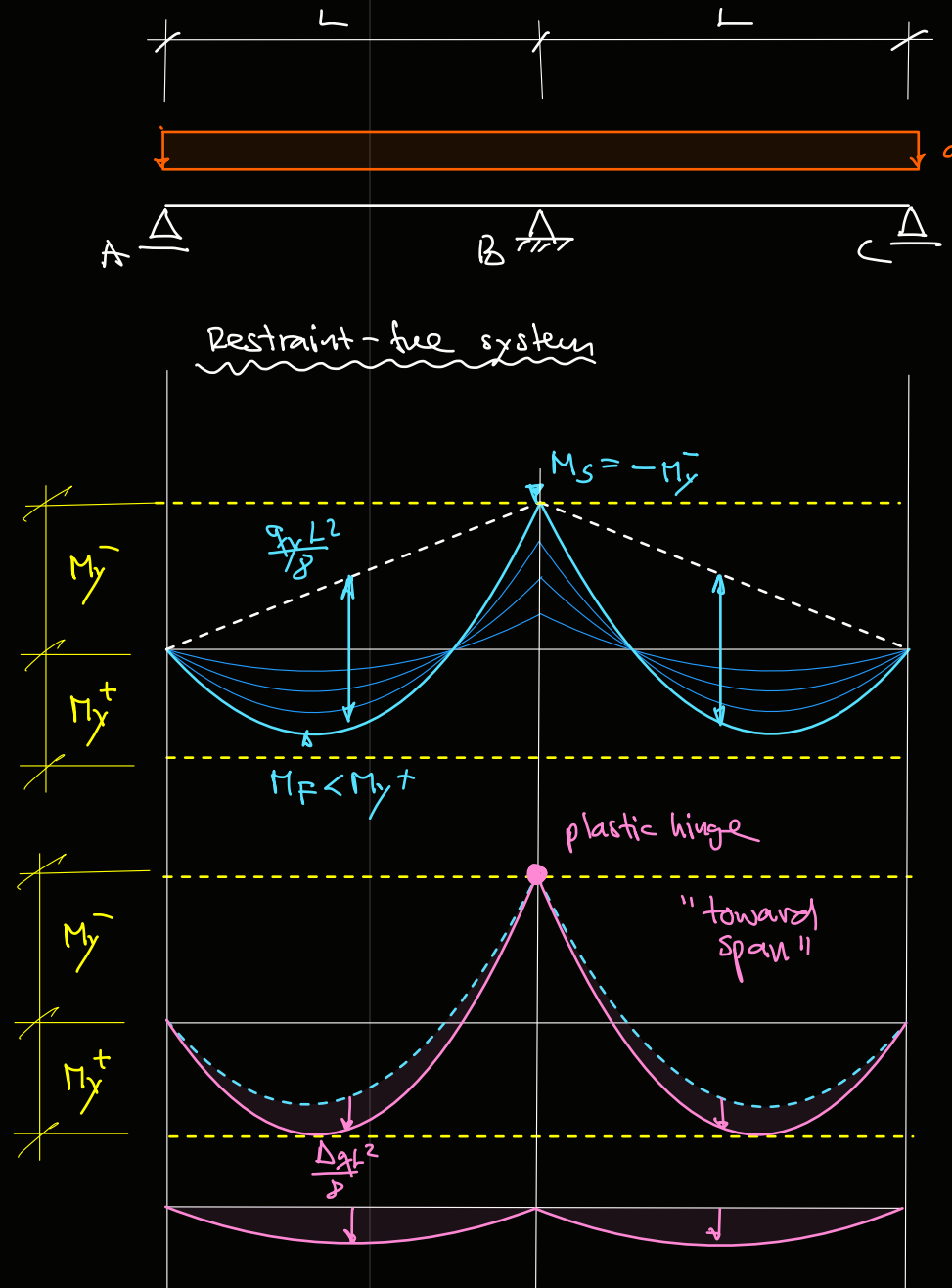
Moment redistribution

$[V = \frac{dM}{dx} \text{ changes accordingly}]$

Plastic design
for $q = q_u$:

Elastic bending
moment profiles
valid up to $q = q_y$

Moment
redistribution
for $q_y \rightarrow q_u$



$$EI = \text{const}: M_S = -\frac{qL^2}{8} \rightarrow \frac{q_y L^2}{8} = M_y^-$$

$$q_y = \frac{8M_y^-}{L^2}$$

- q can be increased until either
- M_y^+ is reached in the span
→ limit analysis load achieved
 - rotation capacity over support is reached → failure at lower load

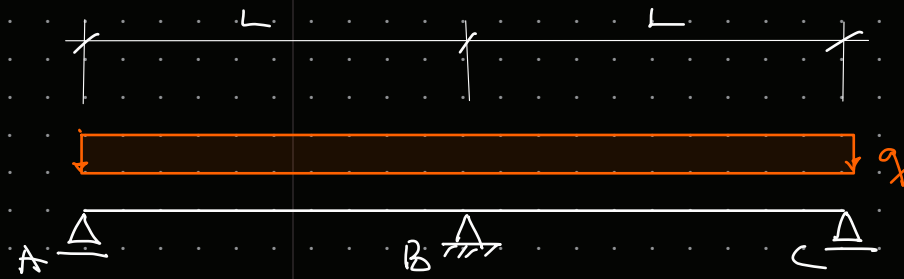
Alternative: select $\frac{M_y^-}{M_y^+} \sim \frac{M_S(q_u)}{M_F(q_u)} \rightarrow$ "elastic design"

* $\Delta T, \pm$, support settlements A, C or B,
creep, shrinkage → envelope

→ possible, but: initial restraint (envelope)*
(see next slide) must be included
in $M_F, M_S \rightarrow$ inefficient design!

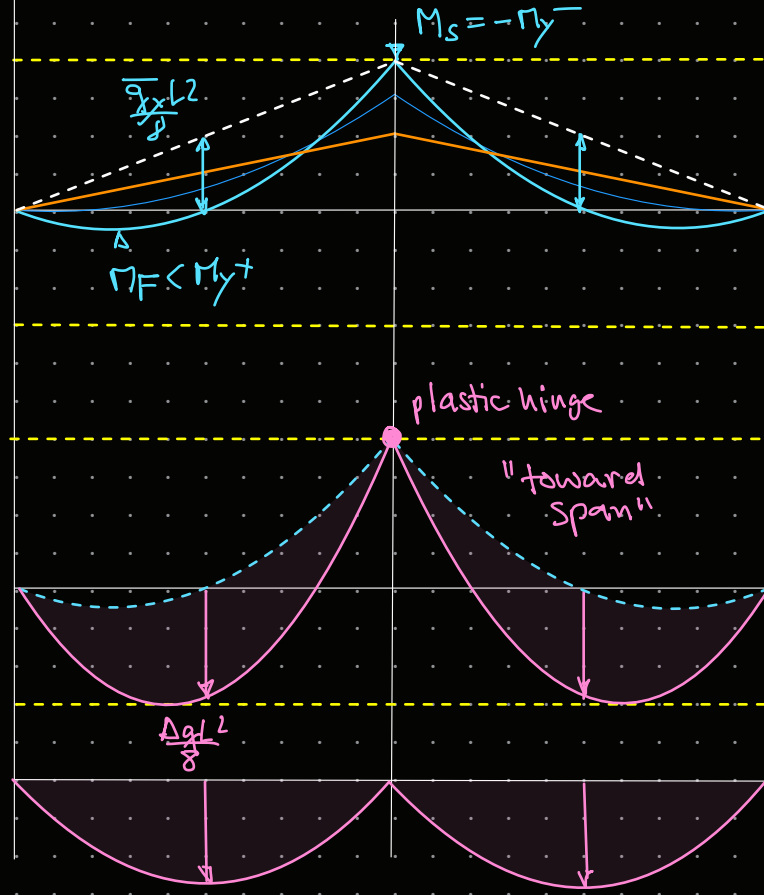
Moment redistribution

$[V = \frac{dM}{dx} \text{ changes accordingly}]$



System with **negative initial restraint**

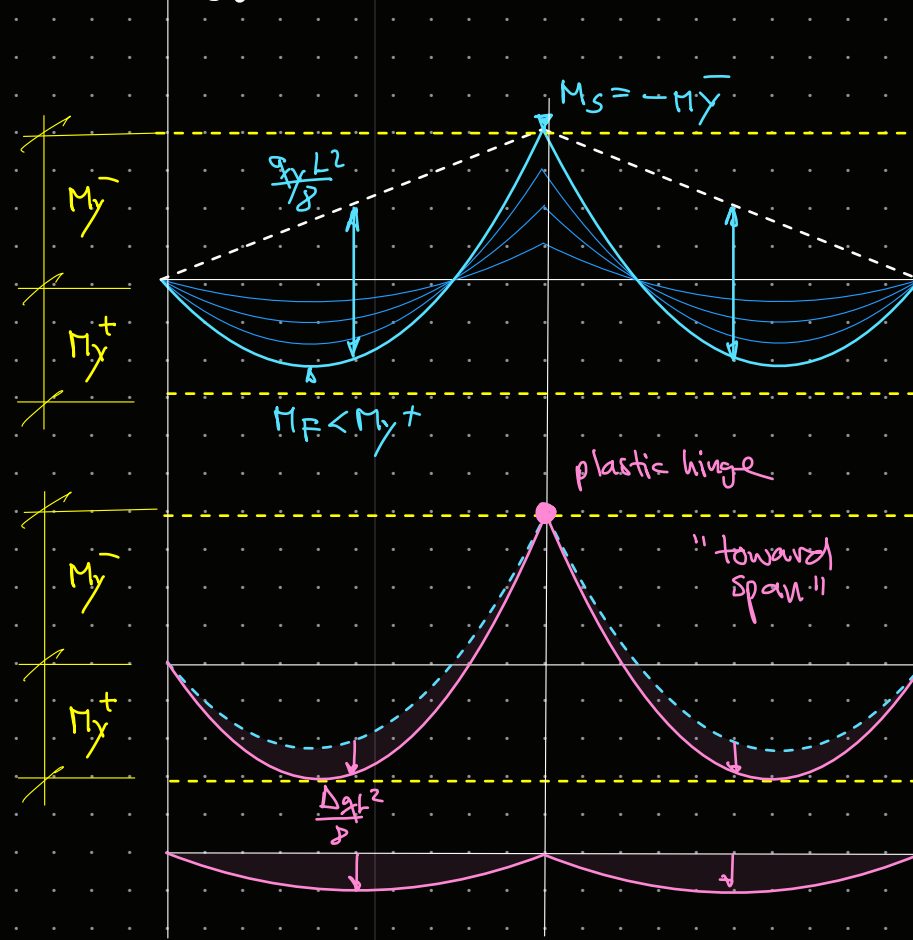
e.g. settlement of A, C



Elastic bending moment profiles valid up to $q = q_y$

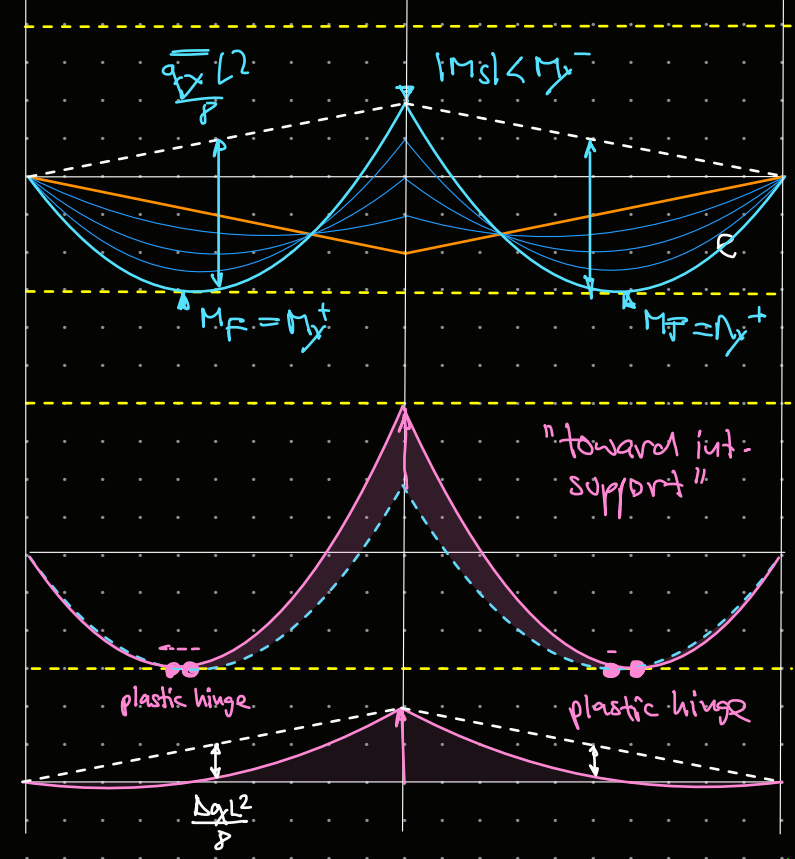
Moment redistribution for $q_y \rightarrow q_u$

Restraint-free system



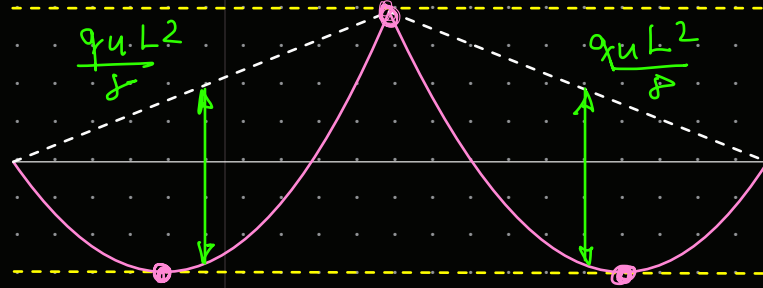
System with **positive initial restraint**

e.g. settlement of B or secondary moments due to prestressing



Limit analysis:

$q_u (= q_y + \Delta q)$ independent of initial restraint

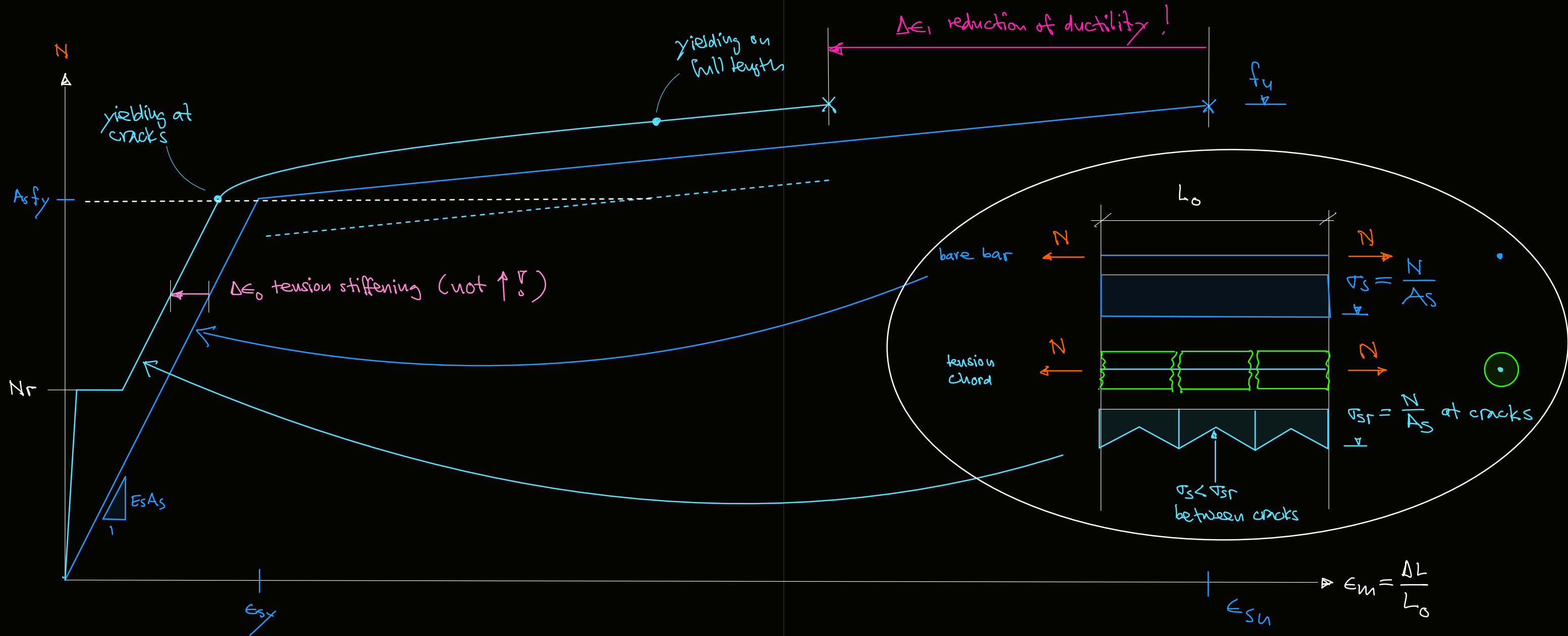


if the plastic hinges have sufficient ROTATION CAPACITY

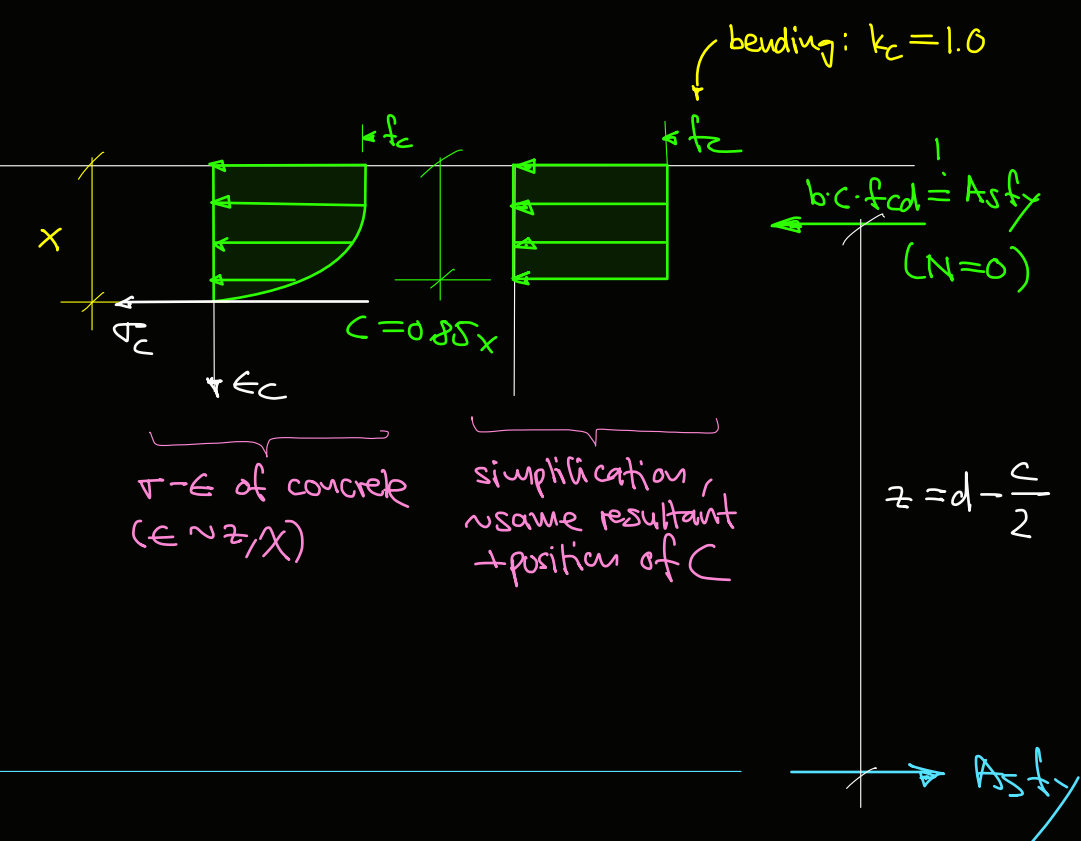
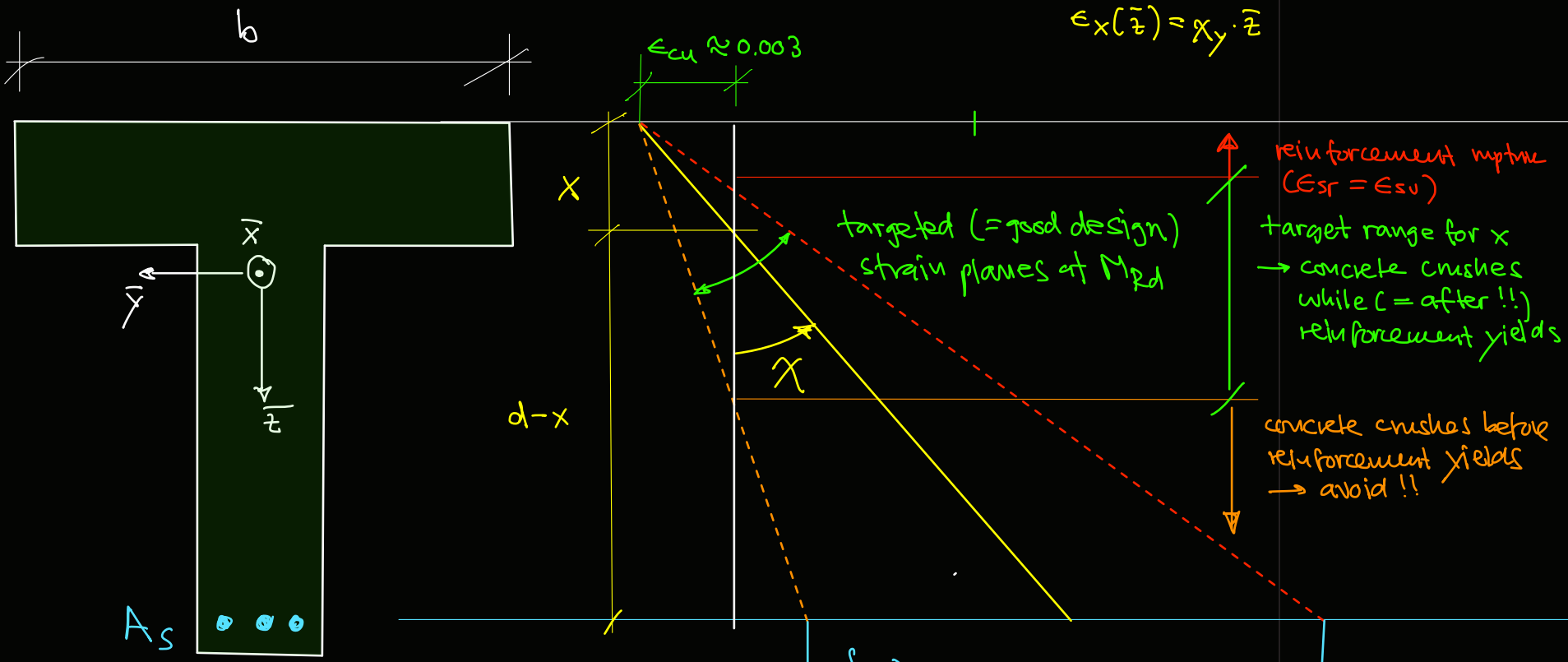
otherwise: premature failure at $q_y < q < q_u$ due to
- concrete crushing or
- steel rupture

Tension stiffening

$r = \text{crack} / \text{cracking}$



Bending (recap) → rotation capacity (Navier-Bernoulli → ϵ_x linear in z)

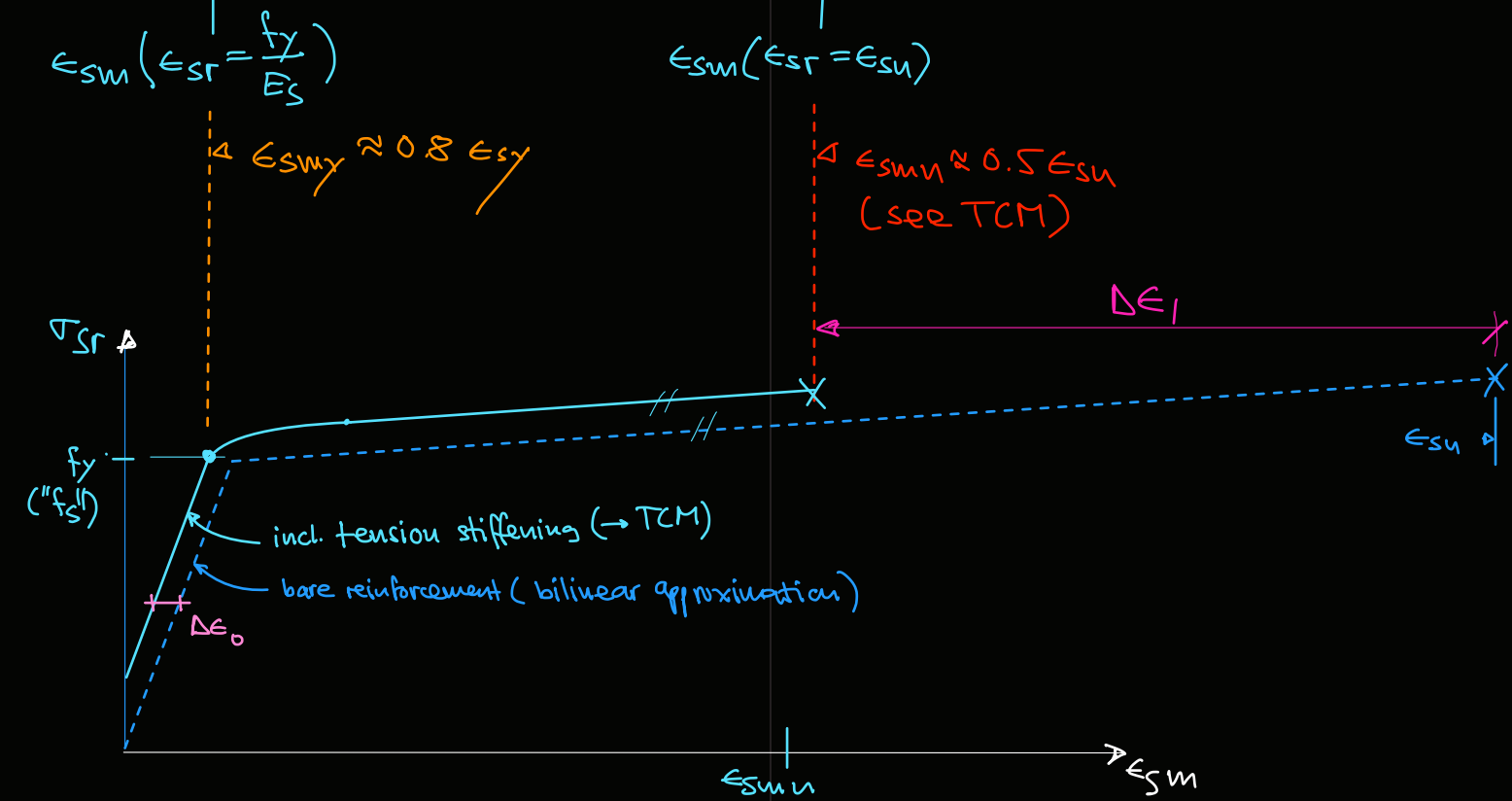


d : static depth
 x : depth of neutral axis
 χ : curvature

$$\chi = \frac{\epsilon_{sup}}{x} \leq \frac{\epsilon_{cu}}{x}$$

$$\chi = \frac{\epsilon_s}{d-x} \leq \frac{\epsilon_{smu}}{d-x}$$

concrete crushing
 steel rupture



$$c = \frac{A_s f_y}{b \cdot f_c}$$

$$M_y = A_s \cdot f_y \cdot z = A_s f_y \left(d - \frac{c}{2} \right)$$

or with $g = \frac{A_s}{b \cdot d}$, $\omega = g \cdot \frac{f_y}{f_c}$:

$$M_y = b d^2 f_c \omega \left(1 - \frac{\omega}{2} \right)$$

NB: Curvature = $\frac{\text{rotation}}{\text{length}}$ $\neq$ rotation
 χ $\theta = \int \chi dx$