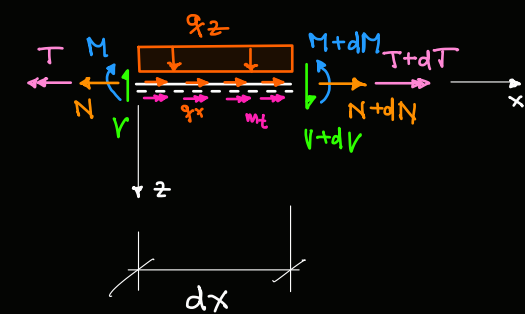


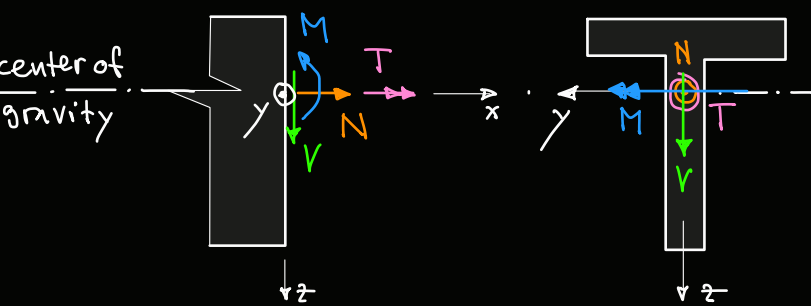
Linear members - Basics ($dF = \frac{\partial F}{\partial x} \cdot dx$)

Equilibrium



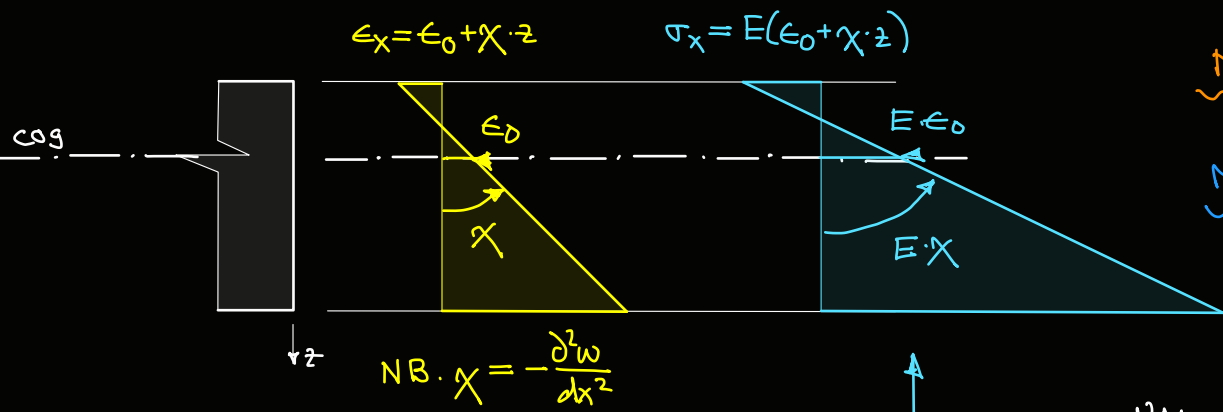
$$\begin{aligned} q_x \cdot dx + dN &= 0 \rightarrow \frac{dN}{dx} = -q_x \\ m_t \cdot dx + dT &= 0 \rightarrow \frac{dT}{dx} = -m_t \\ q_z \cdot dx + dV &= 0 \rightarrow \frac{dV}{dx} = -q_z \\ V \cdot dx - dM &= 0 \rightarrow \frac{dM}{dx} = V \\ \frac{d^2 M}{dx^2} - \frac{dV}{dx} &= \frac{d^2 M}{dx^2} + q_z = 0^* \end{aligned}$$

Stress resultants = internal actions (uniaxial bending) (spannungserresultierende = Schnittkräfte)



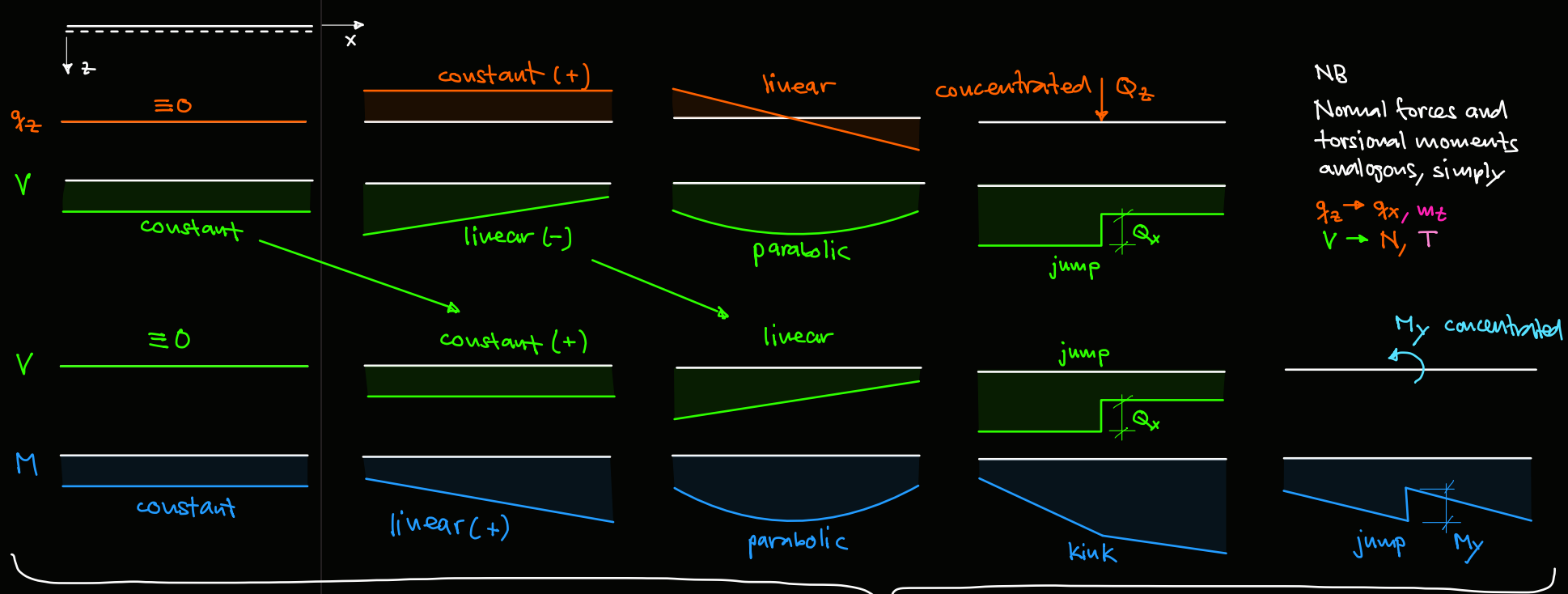
$$\begin{aligned} N &= \iint \sigma_x dA && \text{normal force} \\ T &= \iint (\tau_{zx} \cdot y - \tau_{yx} \cdot z) dA && \text{torsional moment} \\ V &= \iint \tau_{zx} dA && \text{shear force} \\ M &= \iint \sigma_x \cdot z dA && \text{bending moment} \end{aligned}$$

Kinematic restriction: Navier-Bernoulli (Euler)



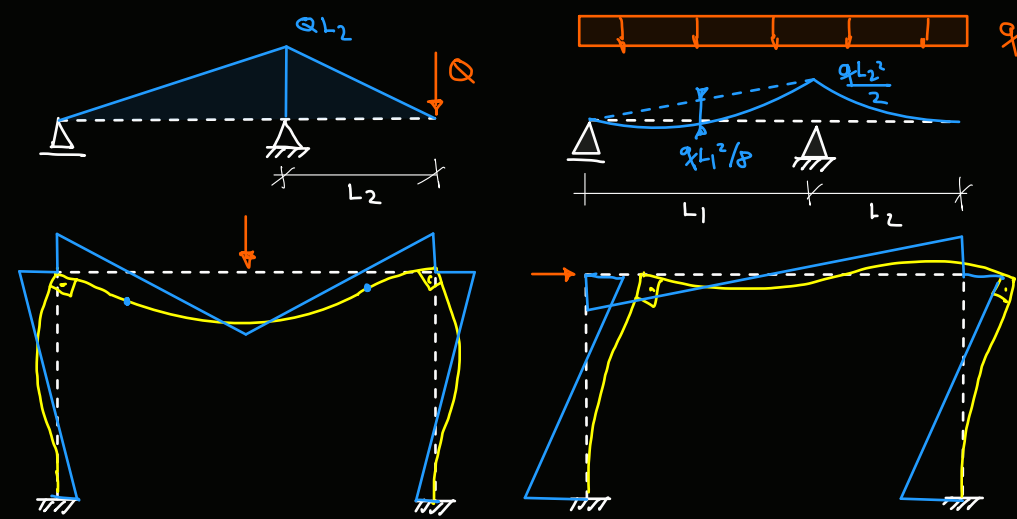
Material characteristics

linear elastic behaviour
(common in analysis, ≠ design)



basic function analysis
+ knowing that $M \sim x$
(deflections!)

intuitive understanding



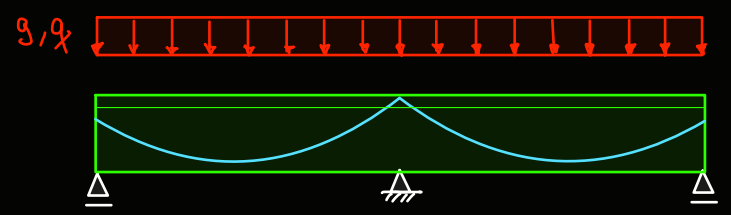
$$\begin{aligned} N &= \iint \sigma_x dA = E \epsilon_0 \cdot \underbrace{\iint dA}_{\text{area } A} + E \chi \cdot \underbrace{\iint z dA}_{\text{first moment of area } \approx 0 \text{ wrt cog}} = EA \epsilon_0 \\ M &= \iint \sigma_x z dA = E \epsilon_0 \cdot \underbrace{\iint z dA}_{=0, \text{ see } N} + E \chi \cdot \underbrace{\iint z^2 dA}_{\text{moment of inertia } I} = EI \chi \end{aligned}$$

$$\text{NB1. } \frac{d^2 M}{dx^2} = -q_z = EI \frac{d^2 \chi}{dx^2} = -\frac{d^4 w}{dx^4} \rightarrow \frac{d^4 w}{dx^4} = q_z$$

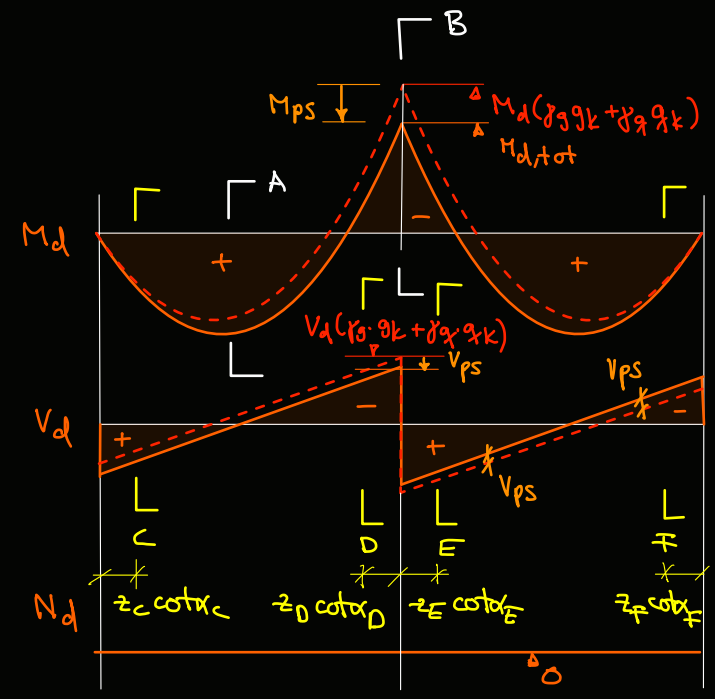
NB2. $\begin{Bmatrix} N \\ M \end{Bmatrix} \begin{Bmatrix} \epsilon_0 \\ \chi \end{Bmatrix} = \text{generalised stresses/strains} \rightarrow \text{theorems of limit analysis apply in terms of these as well}$

Design of linear members

(example with prestressing, considered as resistance)

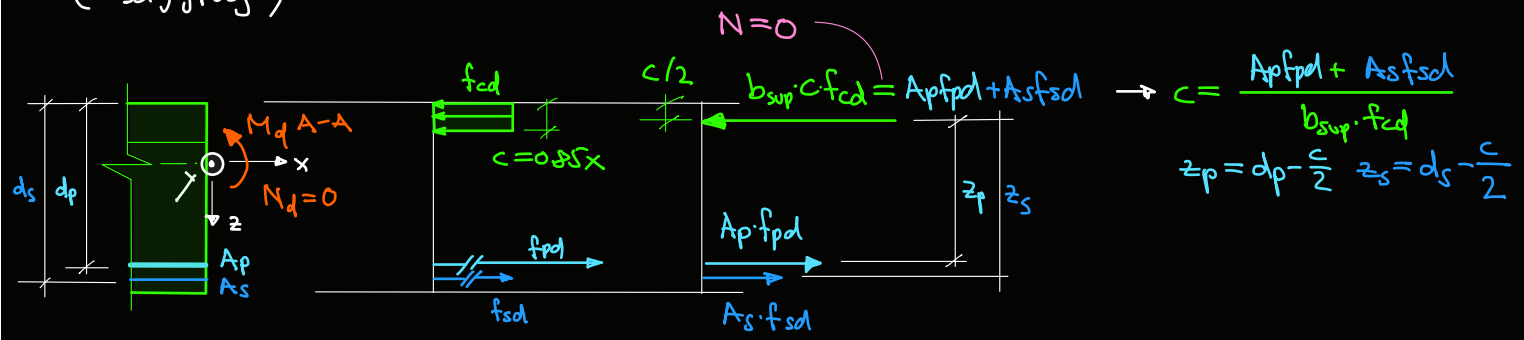


relevant internal actions:
 • $g, q \rightarrow M_d(\gamma_g g_k + \gamma_q q_k)$ ---
 + secondary moments $M_{ps}(P, e_p)$
 (usually with $\gamma_p = 1$)
 L (prestressing = part of system, -P and -P.e $\neq$ int. actions)



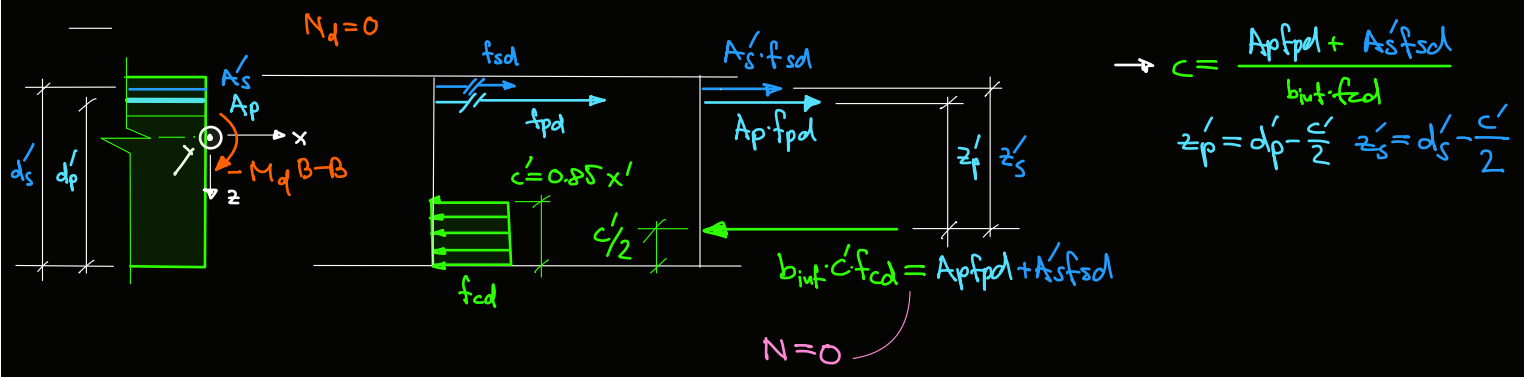
section A-A: positive bending resistance ("sagging")

$$M_{Rd} = A_p \cdot f_{pd} \cdot z_p + A_s \cdot f_{sd} \cdot z_s \geq M_d^{A-A}$$



section B-B: negative bending resistance ("hogging")

$$M_{Rd} = A_p \cdot f_{pd} \cdot z_p + A_s \cdot f_{sd} \cdot z_s \geq -M_d^{B-B}$$



Sections D-D...F-F: "shear strength" (P = P_{00})

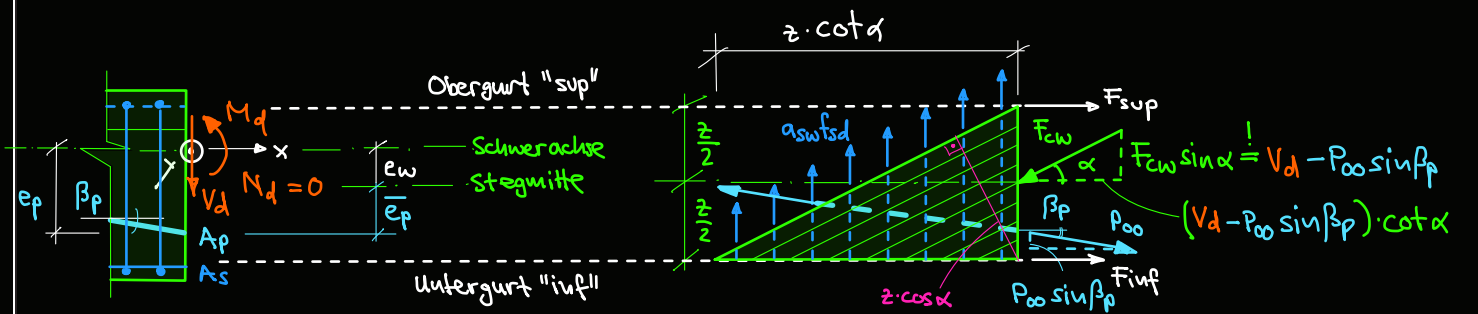
- no "sectional design" (length $z \cot \alpha$)
- interdependence $V \rightleftharpoons M$

$$V_{Rd,c} = k_c b_w n_{min} f_{cd} \cdot z \cdot \sin \alpha \cos \alpha + P_{00} \sin \beta_p$$

$$V_{Rd,s} = a_{sw} f_{sd} z \cot \alpha + P_{00} \sin \beta_p$$

$$V_{Rd} = \min \{ V_{Rd,c}; V_{Rd,s} \}$$

+ tension chord force including contribution by V_d



concrete crushing: $k_c b_w n_{min} f_{cd} \cdot z \cdot \cos \alpha \geq \frac{V_d - P_{00} \sin \beta_p}{\sin \alpha} \leftrightarrow V_{Rd,c}$

stirrup yielding: $a_{sw} f_{sd} \cdot z \cdot \cot \alpha \geq V_d - P_{00} \sin \beta_p \leftrightarrow V_{Rd,s}$

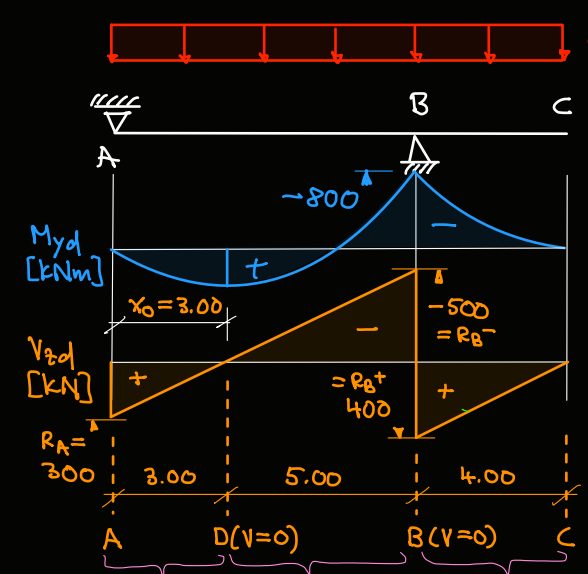
chord forces: $F_{sup} = -\frac{M_d}{z} + N_d \left(\frac{1}{2} + \frac{e_w}{z} \right) - P_{00} \cos \beta_p \left(\frac{1}{2} - \frac{\bar{e}_p}{z} \right) + (V_d - P_{00} \sin \beta_p) \cdot \frac{\cot \alpha}{2}$
 $F_{inf} = +\frac{M_d}{z} + N_d \left(\frac{1}{2} - \frac{e_w}{z} \right) - P_{00} \cos \beta_p \left(\frac{1}{2} + \frac{\bar{e}_p}{z} \right) + (V_d - P_{00} \sin \beta_p) \cdot \frac{\cot \alpha}{2}$

$A_p = 0, N = 0$: $F_{inf, sup} = \pm \frac{M_d}{z} + \frac{V_d \cdot \cot \alpha}{2}$
 pure bending | "tension shift"

Spannungsfelder für Träger und stat. äquivalente Fachwerkmodelle [m, kN]

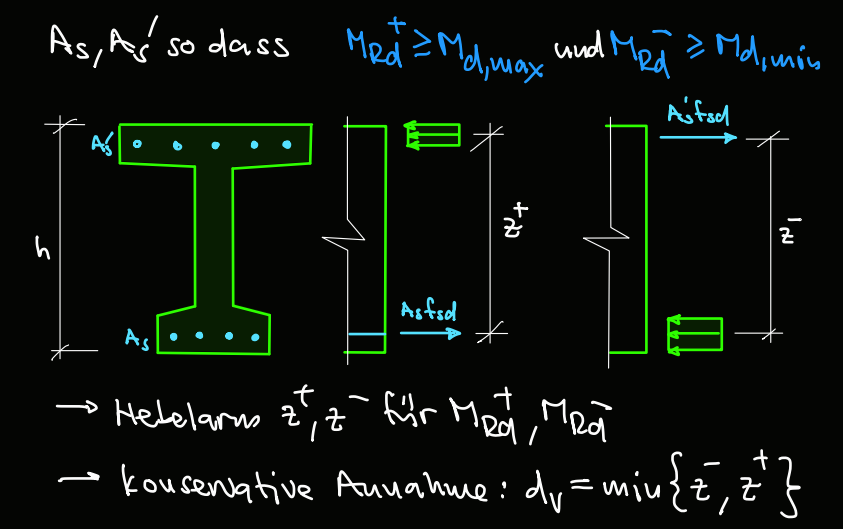
1. System und Belastung (inkl. g)
2. Schnittkräfte
 - Biegebemessung → d_v
 - Querkraftnullpunkte = Unterteilung in Schnittkörper SK
3. Sinuvolle Einteilung pro Schnittkörper (Druckfeldneigung $0.5 \leq \alpha \leq 1$) $\alpha = \frac{\pi}{4}$ selten sinuvoll
4. Fachwerkmodell:
 - stat. äquivalente Einzellasten
5. Bemessung

Reaktionen und Schnittkräfte → Querkraftnullpunkte (1, 2)



$$\begin{aligned} \sum M_A = 0: R_B &= \frac{q_d \cdot 12m \cdot 6m}{8m} = 900 \text{ kN} \\ \sum F_z = 0: R_A &= q_d \cdot 12m - R_B = 300 \text{ kN} \\ x_0 &= \frac{R_A}{q_d} = 3.0 \text{ m} \\ M_{d, \min} = M_B &= -\frac{q_d (4m)^2}{2} = -800 \text{ kNm} \\ M_{d, \max} = M_D &= R_A \cdot x_0 - q_d \frac{x_0^2}{2} = 300 \cdot 3 - 100 \cdot 4.5 = 450 \text{ kNm} \end{aligned}$$

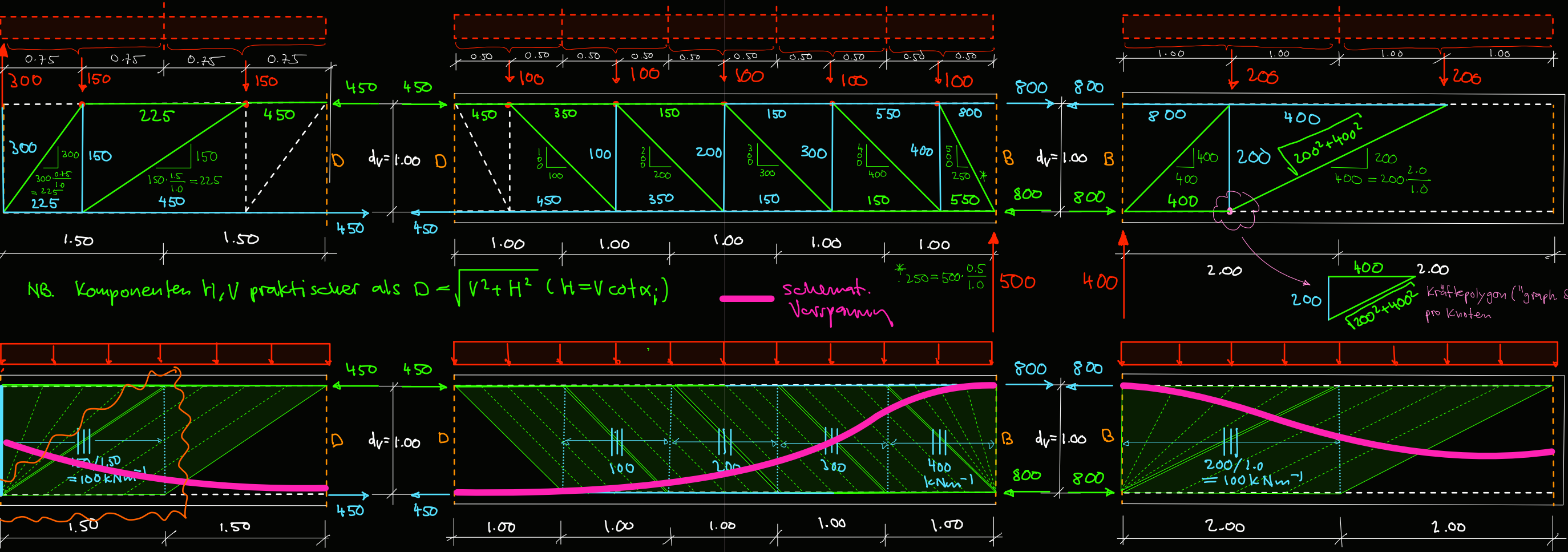
Biegebemessung → d_v (2)



stat. äquivalente Einzellasten

– Beton = Druck
– Bewehrung = Zug

konzentrierte Aufladungsbewehrung $A_{sfsl} \geq R_A$



NB. Komponenten H, V praktischer als $D = \sqrt{V^2 + H^2}$ ($H = V \cot \alpha_i$)

Schemat. Verspannung

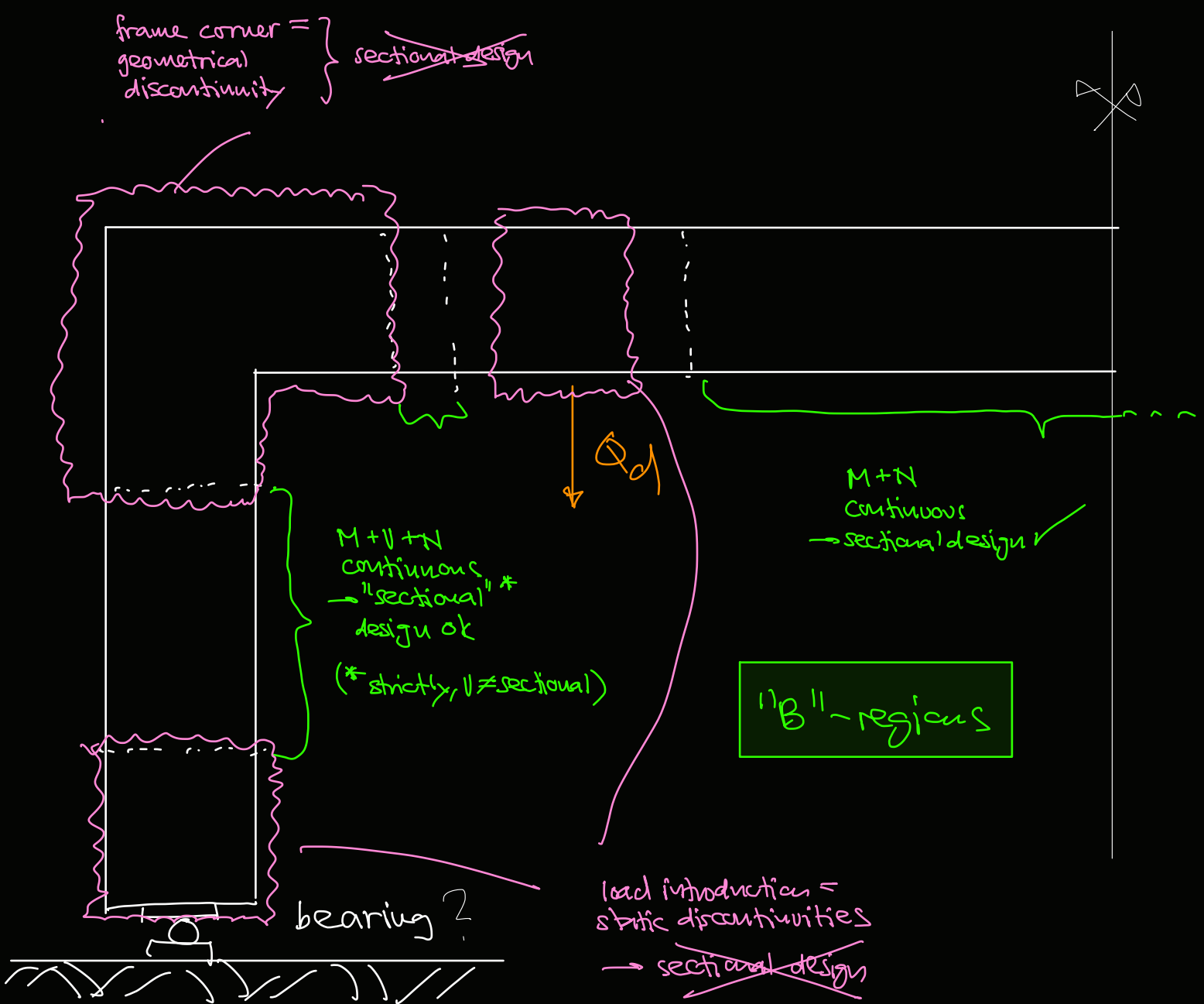
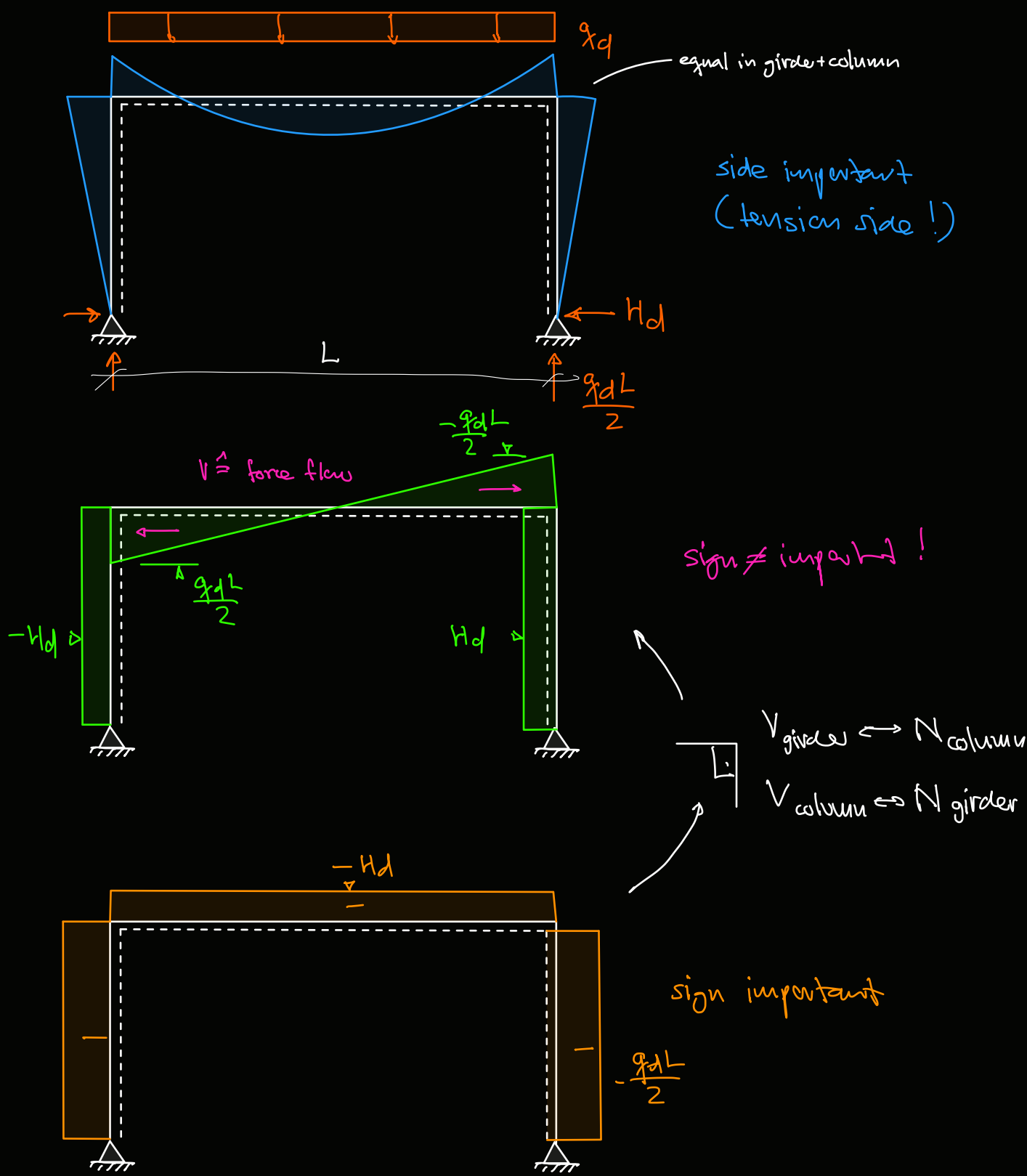
Kräftepolygon ("graph. statik") pro Knoten

Fachwerkmodell

Spannungsfeld

→ Querkraftverlauf

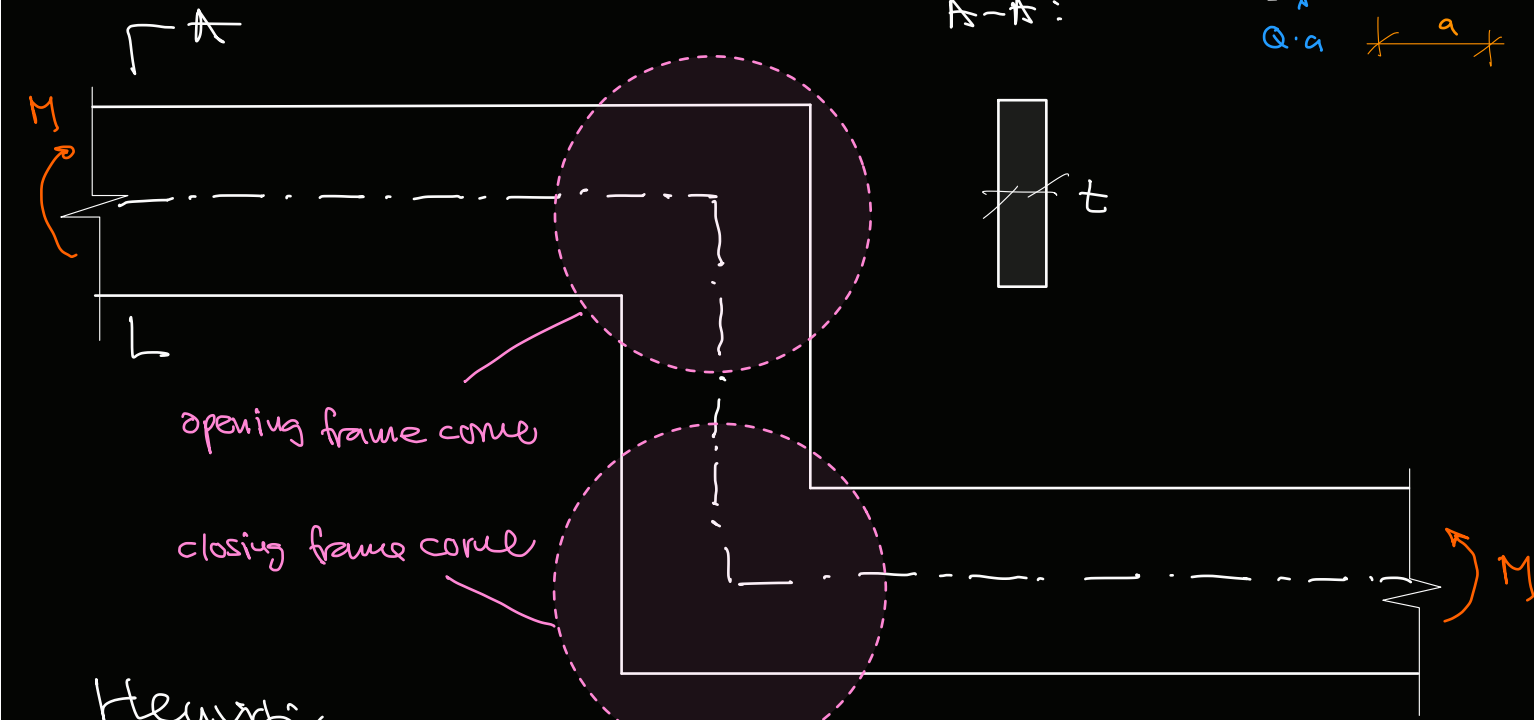
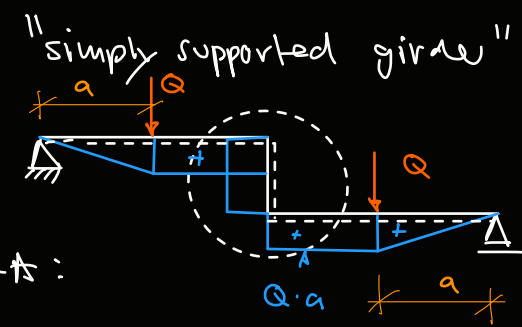
Stress fields - Introduction



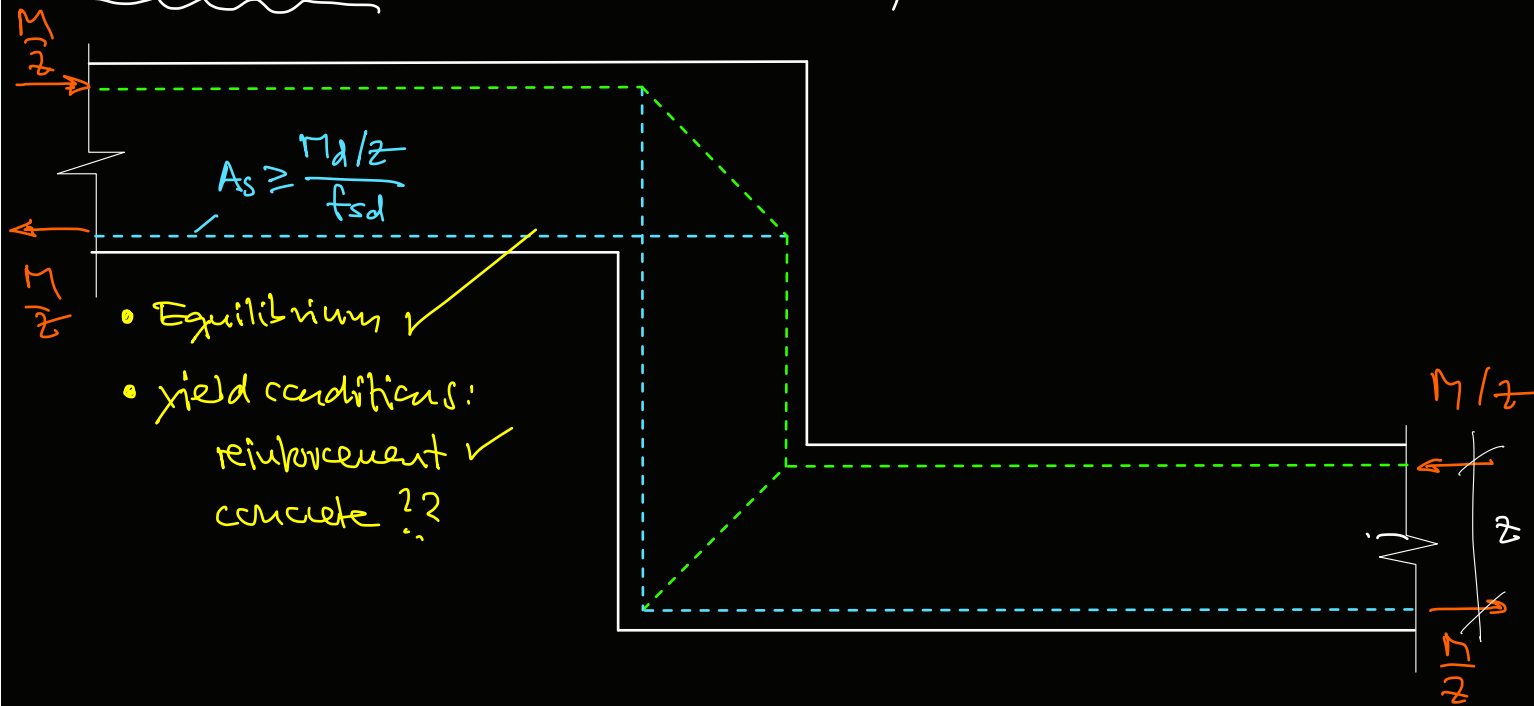
sectional design fails in most relevant regions of the structure (i.e. prone to non-ductile failure)
 → stress fields solve the problem

Truss models and stress fields

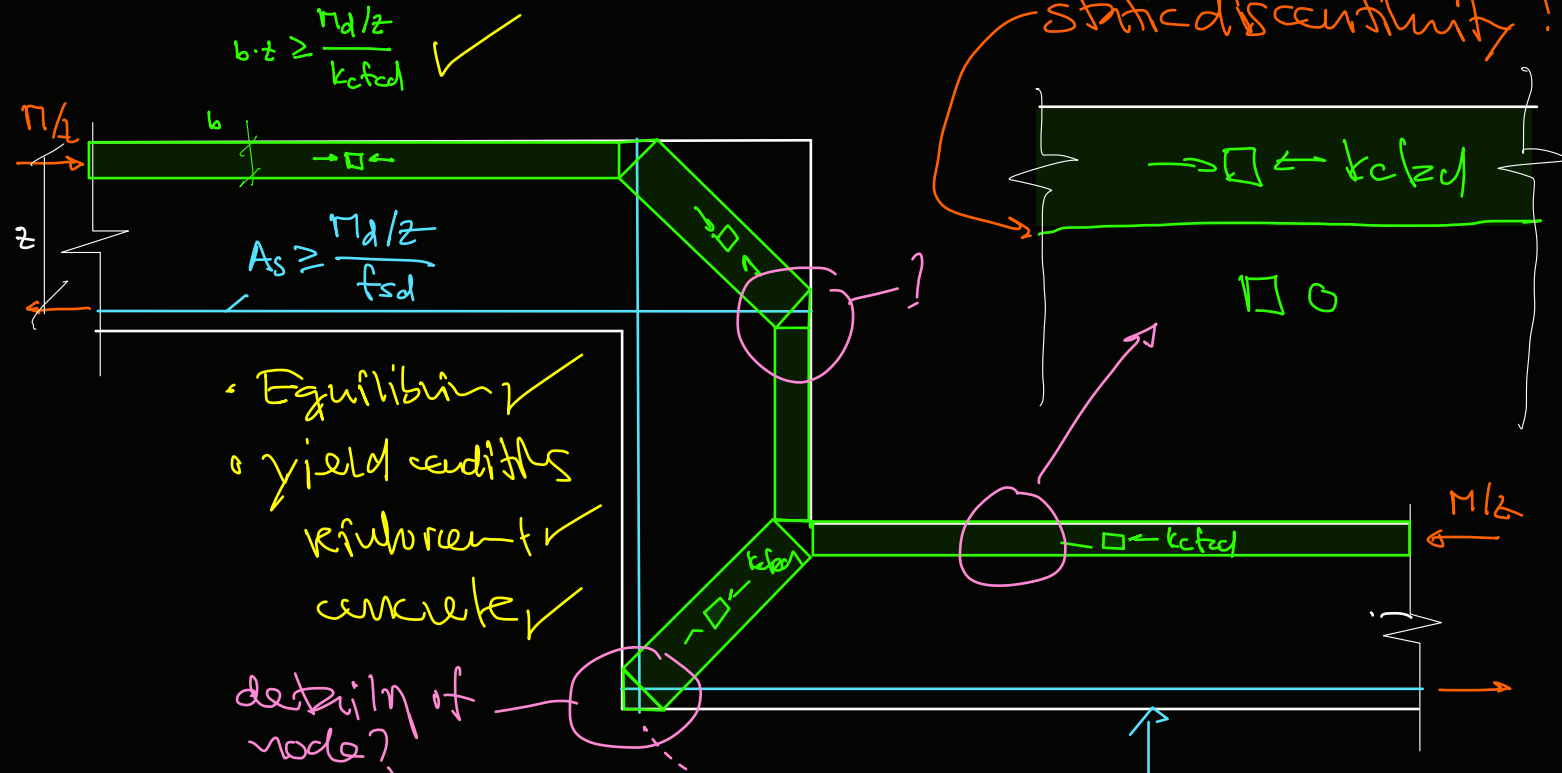
Geometry + loads (static boundary conditions)



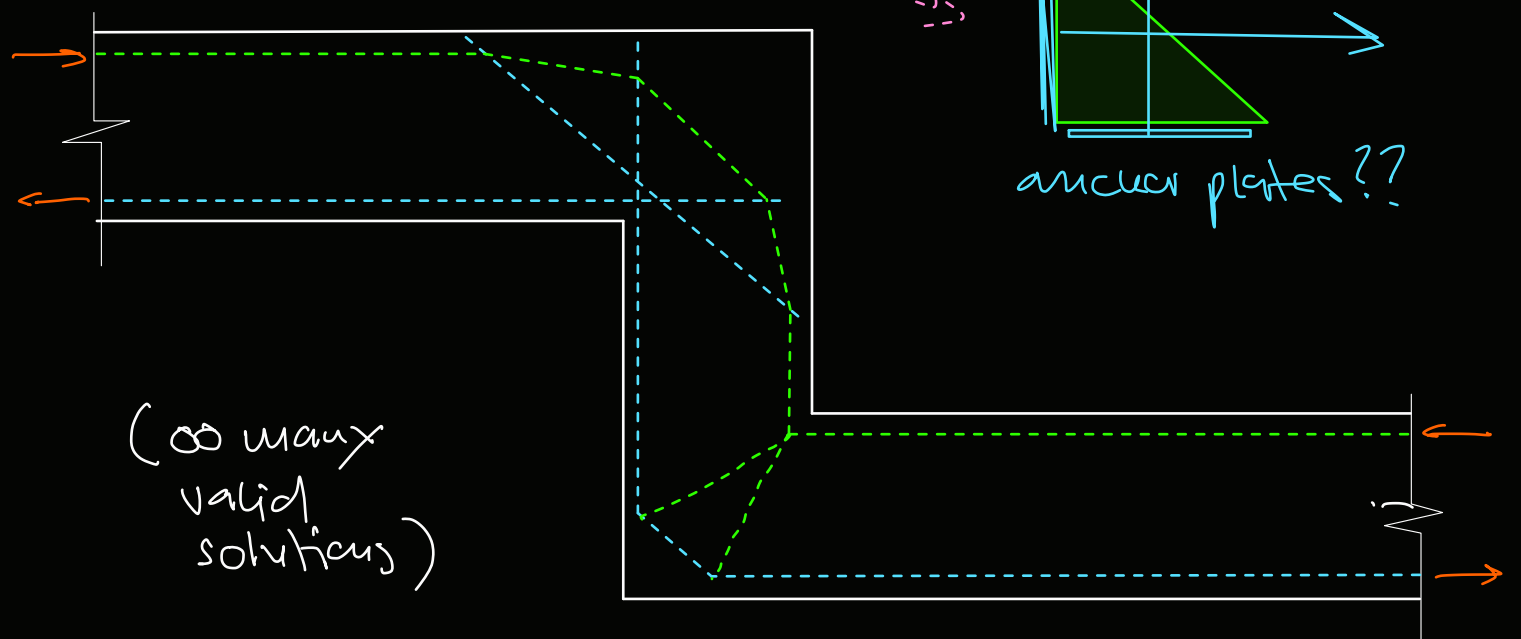
Heuristic Truss model ("Stabwerkmodell")



Strut-and-tie model (discontinuous stress field)



Refined truss model



Stress fields – Basics

In-class exercise

