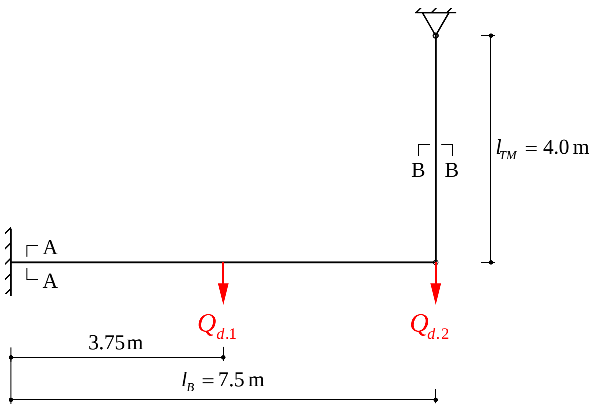
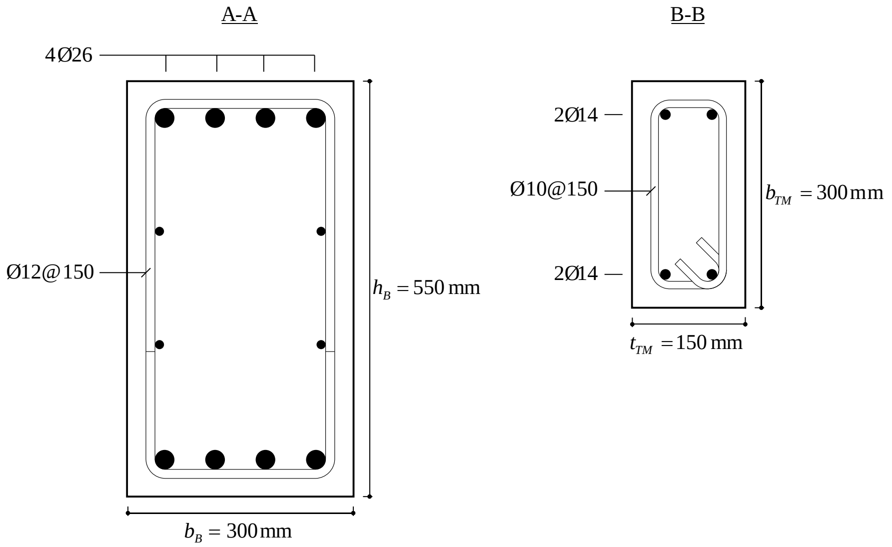
Deformation capacity and demand

SIA 262

Geometry





Material Properites

Concrete C25/30

$f_{cd} = 16.5\,\text{MPa};\ f_{cm} = 33\,\text{MPa}$ (Table 8 and 3)

$E_{cm} = k_e \sqrt[3]{f_{cm}} \approx 30.1\,\text{GPa}$ (3.1.2.3.3)

$f_{ctm} = 2.6\,\text{MPa}$ (Table 3)

Steel B500B

$f_s = 500\,\text{MPa};\ f_{sd} = 435\,\text{MPa};\ f_t = 540\,\text{MPa}$ (Table 5 and 9)

$\varepsilon_{uk} = 5\%$ (Table 5)

$$E_s = 205\,\text{GPa};\ E_{sh} = \frac{f_t - f_s}{\varepsilon_{uk} - \frac{f_s}{E_s}} = 0.84\,\text{GPa}$$

$\tau_{b0} = 2 \cdot f_{ctm};\ \tau_{b1} = f_{ctm}$

$c_{nom} = 25\,\text{mm}$

Load

Load scenario 1: $Q_{d.1} = 300\,\text{kN},\ Q_{d.2} = 0\,\text{kN}$

Load scenario 2: $Q_{d.1} = 0\,\text{kN},\ Q_{d.2} = 300\,\text{kN}$

<u>Resistance</u>

Beam:

$\varnothing_w = 12\,\text{mm}$
$\varnothing = 26\,\text{mm}$
$A_{s.B} = 2122\,\text{mm}^2$

$$d = h_B - c_{nom} - \varnothing_w - \frac{\varnothing}{2} = 500\,\text{mm}$$

$$M_{Rd} = A_{s.B} \cdot f_{sd} \left(d - \frac{A_{s.B} \cdot f_{sd}}{2 \cdot b_B \cdot f_{cd}} \right) = 375.7\,\text{kNm}$$

$$x = \frac{A_{s.B} \cdot f_{sd}}{0.85 \cdot b_b \cdot f_{cd}} = 219.6\,\text{mm}$$

$$\frac{x}{d} = 0.44 > 0.35 \rightarrow \text{verification of deformation capacity required!}$$

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$$z = d - \frac{0.85 \cdot x}{2} = 406.7\,\text{mm}$$

Tension member:

$A_{s.TM} = 616\,\text{mm}^2$

$$N_{Rd} = A_{s.TM} \cdot f_{sd} = 267.9\,\text{kN}$$

<u>Stiffness</u>

Beam:

$$\rho_B = \frac{A_{s.B}}{b_B \cdot d} = 1.42\%$$

$$n = \frac{E_s}{E_{cm}} = 6.81$$

$$x^{II} = d \cdot \left(\sqrt{(\rho_B \cdot n)^2 + 2 \cdot \rho_B \cdot n} - \rho_B \cdot n \right) = 176.6\,\text{mm}$$

$$EI^{II} = A_{s.B} \cdot E_s \cdot (d - x^{II}) \cdot \left(d - \frac{x^{II}}{3} \right) = 62.1\,\text{MNm}^2$$

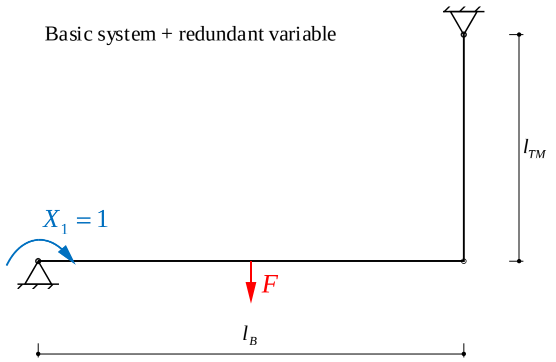
Tension member:

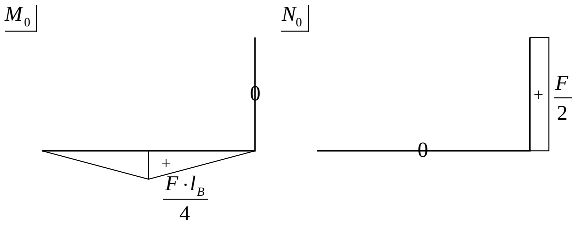
$$EA^{II} = E_s \cdot A_{s.TM} = 126.2\,\text{MN}$$

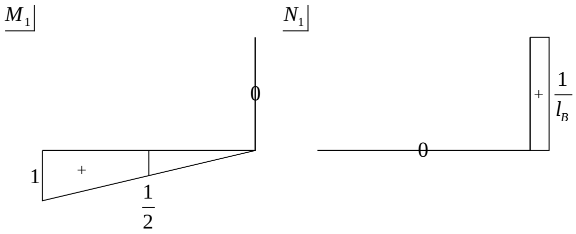
a) <u>Verification of structure for load scenario 1</u>

Elastic internal forces:

Basic system + redundant variable





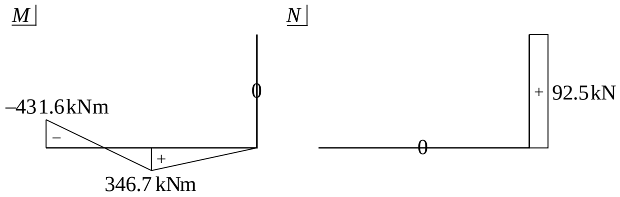


$$\delta_{10} = \underbrace{\frac{1}{3} \cdot \frac{Fl_B}{4} \cdot \frac{1}{2} \cdot \frac{l_B}{2EI^{II}} + \frac{1}{6} \cdot \frac{Fl_B}{4} \cdot \left(2 \cdot \frac{1}{2} + 1\right) \cdot \frac{l_B}{2EI^{II}}}_{\frac{3Fl_B^2}{48EI^{II}}} + 1 \cdot \frac{F}{2} \cdot \frac{1}{l_B} \cdot \frac{l_{TM}}{EA^{II}}$$

$$\delta_{11} = \frac{1}{3} \cdot 1 \cdot 1 \cdot \frac{l_B}{EI^{II}} + 1 \cdot \frac{1}{l_B} \cdot \frac{1}{l_B} \cdot \frac{l_{TM}}{EA^{II}}$$

$$\delta_{10} + X_1 \cdot \delta_{11} \overset{!}{=} 0 \rightarrow X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{\frac{Fl_{TM}}{2l_B EA^{II}} - \frac{3Fl_B^2}{48EI^{II}}}{\frac{l_B}{3EI^{II}} + \frac{l_{TM}}{l_B^2 EA^{II}}}$$

$$M = M_0 + X_1 \left(F = Q_{d.1}\right) \cdot M_1$$
$$N = N_0 + X_1 \left(F = Q_{d.1}\right) \cdot N_1$$

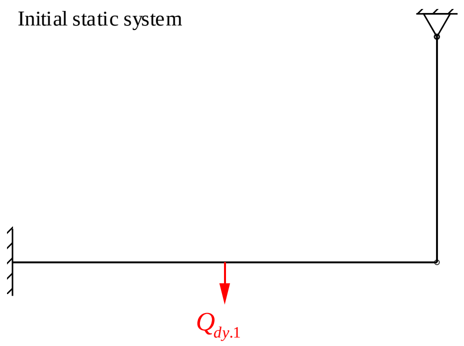


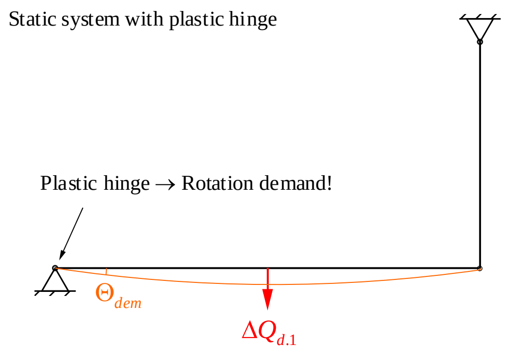
$$M_{B.clamp} = -431.6\,\text{kNm} > M_{Rd}$$
$$M_{B.field} = 346.7\,\text{kNm} < M_{Rd}$$
$$N_{TM} = 92.5\,\text{kN} < N_{Rd}$$

The clamping moment exceeds the moment resistance. Reaching the moment resistance, a plastic hinge at the clamp is formed and the static system of the beam changes to a simply supported beam, which has further load bearing capacity ($M_{B.field} < M_{Rd}$).The rotation capacity of the plastic hinge has to be compared with its deformation demand resulting from the system change.

$$Q_{dy.1} = Q_{d.1} \cdot \frac{M_{Rd}}{M_{B.clamp}} = 261.2\,\text{kN}$$
$$\Delta Q_{d.1} = Q_{d.1} - Q_{dy.1} = 38.8\,\text{kN}$$

$Q_{dy.1}$ corresponds to the load at which a plastic hinge forms at the fixed end of the beam.
$\Delta Q_{d.1}$ corresponds to the load that has to be carried by the simply supported beam.





Internal forces after redistribution:

$$M_{B.clamp.red} = M_{Rd} = 375.7\,\text{kNm} \rightarrow \text{OK}$$

$$M_{B.field.red} = \frac{Q_{dy.1} \cdot l_B}{4} + \frac{X_1\left(F = Q_{dy.1}\right)}{2} + \frac{\Delta Q_{d.1} \cdot l_B}{4} = 374.6\,\text{kNm} < M_{Rd} \rightarrow \text{OK}$$

$$N_{TM.red} = \frac{Q_{dy.1}}{2} + \frac{X_1\left(F = Q_{dy.1}\right)}{l_B} + \frac{\Delta Q_{d.1}}{2} = 99.9\,\text{kN} < N_{Rd} \rightarrow \text{OK}$$

$$A_{clamp} = V_{B.max} = Q_{d.1} - N_{TM.red} = 200.1\,\text{kN}$$

Verification shear force:

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$$\alpha = 30°$$

$$V_{Rd,s} = a_{sw} \cdot f_{sd} \cdot z \cdot \cot(\alpha) = 461.5\,\text{kN} > V_{B.max} \rightarrow \text{OK}$$ 4.3.3.4.3

$$a_{sw} = 2 \cdot 753\frac{\text{mm}^2}{\text{m}}$$

$$V_{Rd,c} = b_B \cdot f_{cd} \cdot z \cdot k_c \cdot \sin(\alpha) \cdot \cos(\alpha) = 479.4\,\text{kN} > V_{B.max} \rightarrow \text{OK}$$ 4.3.3.4.5

$$k_c = 0.55$$

Rotation demand:

$$\Theta_{dem} = \delta_{10}\left(F = \Delta Q_{d.1}\right) = \frac{\Delta Q_{d.1} \cdot l_{TM}}{2 \cdot l_B \cdot EA^{II}} + \frac{3 \cdot \Delta Q_{d.1} \cdot l_B^2}{48EI^{II}} = 2.28\,\text{mrad}$$ Work theorem

Rotation capacity:

$$\zeta = 275\,\text{mm}$$ Centroid

$$I_y = \frac{h_b^3 \cdot b_B}{12} + 2 \cdot A_{s.B} \cdot (n-1) \cdot \left(\zeta - \left(c_{nom} + \varnothing_w + \frac{\varnothing}{2}\right)\right)^2 = 0.005\,\text{m}^4$$

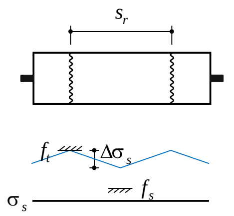
$$M_r = \frac{f_{ctm} \cdot I_y}{\zeta} = 51.1\,\text{kNm}$$

$$\rho_{eff} = \frac{1}{\dfrac{M_r \cdot (d - x_{II}) \cdot E_s}{f_{ctm} \cdot EI^{II}} + 1 - n} = 6.6\%$$

$$s_{r0} = \frac{Ø}{4}\cdot\left(\frac{1}{\rho_{eff}}-1\right) = 92\,\text{mm}$$

$$s_r = s_{r0}\,(\lambda = 1)$$

$$\Delta\sigma_s = \frac{2\cdot\tau_{b1}\cdot s_r}{Ø} = 18.4\,\text{MPa} < f_t - f_s = 40\,\text{MPa}$$



The reinforcement yields over the entire crack element when failure is reached (Regime 3).

$$x_{P1} = \sqrt{\frac{2\cdot A_{s.B}\cdot\left(f_t - f_s - \frac{2\cdot\tau_{b1}\cdot s_r}{Ø}\right)\cdot z}{f_{wd}}} = 362\,\text{mm}$$

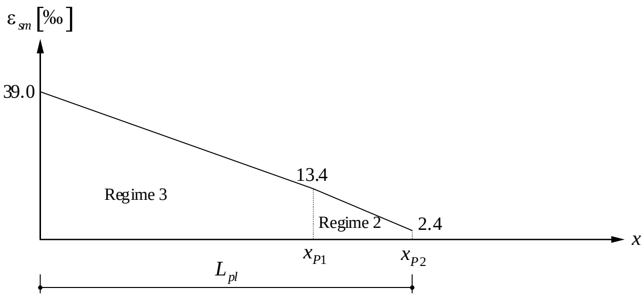
$$x_{P2} = \sqrt{\frac{2\cdot A_{s.B}\cdot(f_t - f_s)\cdot z}{f_{wd}}} = 493\,\text{mm} = L_{pl} < z\cdot\cot(\alpha) = 704\,\text{mm}$$

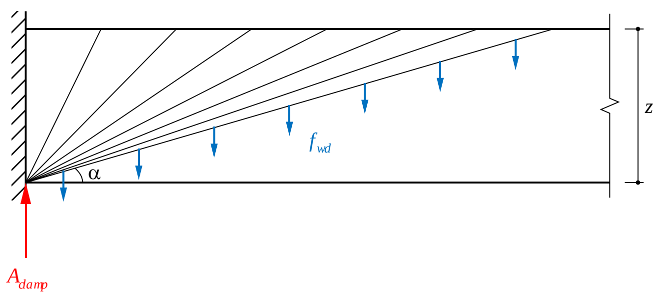
$$f_{wd} = \frac{V_{B.max}}{z\cdot\cot(\alpha)} = 284.1\frac{\text{kN}}{\text{m}}$$

$$\varepsilon_{sm}(\sigma_{sr} = f_t) = \frac{f_s}{E_s} + \frac{f_t - f_s}{E_{sh}} - \frac{\tau_{b1}\cdot s_r}{E_{sh}\cdot Ø} = 39.0‰$$

$$\varepsilon_{sm}(\sigma_{sr} = f_s + \Delta\sigma) = \frac{f_s}{E_s} + \frac{\Delta\sigma_s}{E_{sh}} - \frac{\tau_{b1}\cdot s_r}{E_{sh}\cdot Ø} = 13.4‰$$

$$\varepsilon_{sm}(\sigma_{sr} = f_s) = \frac{f_s}{E_s} - \frac{\tau_{b0}\cdot s_r}{E_s\cdot Ø} = 2.4‰$$





$$\varepsilon_{smu} = \frac{\frac{\varepsilon_{sm}(\sigma_{sr} = f_t) + \varepsilon_{sm}(\sigma_{sr} = f_s + \Delta\sigma)}{2}\cdot x_{P1} + \frac{\varepsilon_{sm}(\sigma_{sr} = f_s + \Delta\sigma) + \varepsilon_{sm}(\sigma_{sr} = f_s)}{2}\cdot(x_{P2} - x_{P1})}{x_{P2}} = 21.3‰$$

$$\varepsilon_{smy} = \varepsilon_{sm}(\sigma_{sr} = f_s) = 2.4‰$$

$$\Theta_{puc} = L_{pl} \cdot \left(\frac{\varepsilon_{cu}}{x} - \frac{\varepsilon_{smy}}{d-x} \right) = 2.61\,\text{mrad} > \Theta_{dem} = 2.28\,\text{mrad} \rightarrow \text{OK} \qquad \text{(concrete crushing)}$$

$$\Theta_{pus} = L_{pl} \cdot \left(\frac{\varepsilon_{smu}}{d-x} - \frac{\varepsilon_{smy}}{d-x} \right) = 33.40\,\text{mrad} > \Theta_{dem} = 2.28\,\text{mrad} \rightarrow \text{OK} \qquad \text{(steel rupture)}$$

The rotation capacity of the plastic hinge is satisfactory.

Remark:
Rotation capacity simplified:

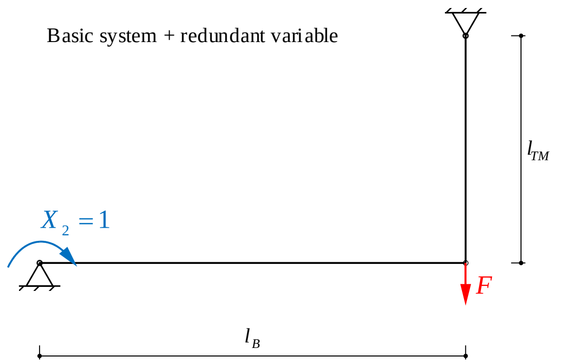
$$\Theta_{puc} = d \cdot \left(\frac{\varepsilon_{cu}}{x} - \frac{\varepsilon_{smy}}{d-x} \right) = 2.64\,\text{mrad} > \Theta_{dem} = 2.28\,\text{mrad} \rightarrow \text{OK} \qquad \text{(concrete crushing)}$$

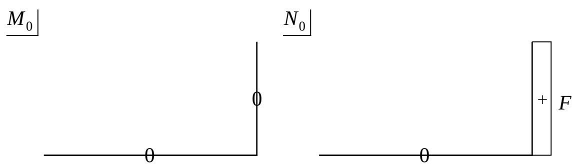
$$\Theta_{pus} = d \cdot \left(\frac{0.5 \cdot \varepsilon_{u}}{d-x} - \frac{\varepsilon_{smy}}{d-x} \right) = 40.39\,\text{mrad} > \Theta_{dem} = 2.28\,\text{mrad} \rightarrow \text{OK} \qquad \text{(steel rupture)}$$

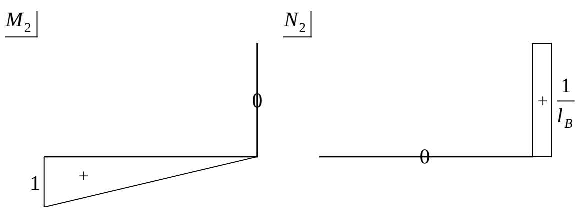
The rotation capacity determinded with the simplified method is in good agreement with the more refined method taking into account concrete crushing. For the given geometry (rectangular cross section) assuming $L_{pl} = d$ is justified. The simplified approch overestimates the mean elongations at failure of the reinforcement. For situations with steel rupture being the governing failure mode, the more refined calculation method is recommended.

b) <u>Verification of structure for load scenario 2</u>

Elastic internal forces:







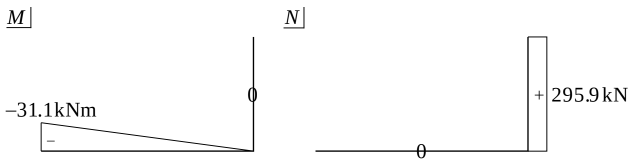
$$\delta_{10} = 1 \cdot F \cdot \frac{1}{l_B} \cdot \frac{l_{TM}}{EA^{II}}$$

$$\delta_{11} = \frac{1}{3} \cdot 1 \cdot 1 \cdot \frac{l_B}{EI^{II}} + 1 \cdot \frac{1}{l_B} \cdot \frac{1}{l_B} \cdot \frac{l_{TM}}{EA^{II}}$$

$$\delta_{10} + X_2 \cdot \delta_{11} \overset{!}{=} 0 \rightarrow X_2 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{\dfrac{F l_{TM}}{l_B EA^{II}}}{\dfrac{l_B}{3EI^{II}} + \dfrac{l_{TM}}{l_B^2 EA^{II}}}$$

$$M = M_0 + X_2\left(F = Q_{d.2}\right) \cdot M_1$$

$$N = N_0 + X_2\left(F = Q_{d.2}\right) \cdot N_1$$



$$M_{B.clamp} = -31.1\text{kNm} < M_{Rd}$$

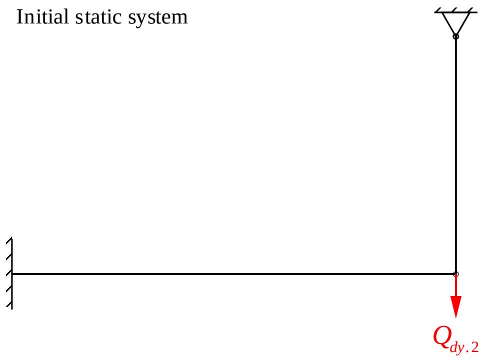
$$N_{TM} = 295.9\,\text{kN} > N_{Rd} = 267.9\,\text{kN}$$

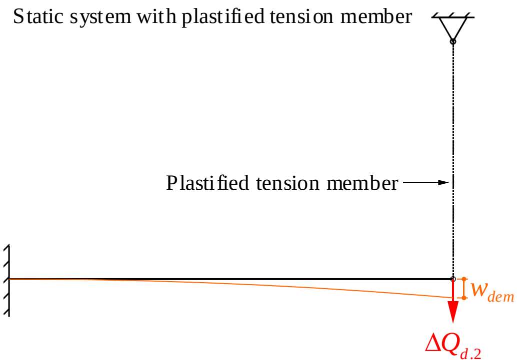
The normal force in the tension member exceeds the normal resistance. Reaching the normal resistance, the tension member plastifies and a cantilever beam remains as the static system, which has further load bearing capacity ($M_{B.clamp} < M_{Rd}$). The deformation capacity of the tension member has to be compared with the deformation demand resulting from the system change.

$$Q_{dy.2} = Q_{d.2} \cdot \frac{N_{Rd}}{N_{TM}} = 271.6\,\text{kN}$$

$$\Delta Q_{d.2} = Q_{d.2} - Q_{dy.2} = 28.4\,\text{kN}$$

$Q_{dy.2}$ corresponds to the load at which the tension member yields and can not take any more load. $\Delta Q_{d.2}$ corresponds to the load that has to be carried by the cantilever beam alone.





Internal forces after redistribution:

$$N_{TM.red} = N_{Rd} = 267.9\,\text{kN} \rightarrow \text{OK}$$

$$M_{B.clamp.red} = X_2\left(F = Q_{dy.2}\right) - \Delta Q_{d.2} \cdot l_B = -241.1\,\text{kNm} < M_{Rd} \rightarrow \text{OK}$$

$$V_{B.max} = Q_{d.2} - N_{TM.red} = 32.1\,\text{kN}$$

Verification shear force:

$$V_{B.max} = 32.1\,\text{kN} < \min\left(V_{Rd,s}, V_{Rd,c}\right) = 230.7\,\text{kN} \rightarrow \text{OK}$$

Deformation demand tension member:

$$w_{dem} = \frac{\Delta Q_{d.2} \cdot l_B^3}{3EI^{II}} = 64.3\,\text{mm}$$

Deformation capacity tension member:

$$\rho_{TM} = \frac{A_{s.TM}}{b_{TM} \cdot t_{TM}} = 1.37\%$$

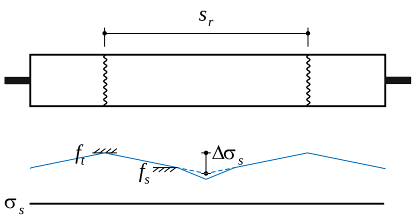
$$s_{r0} = \frac{Ø}{4} \cdot \left(\frac{1}{\rho_{TM}} - 1\right) = 252\,\text{mm}$$ Ø=14mm

Stirrups Ø10@150 $\rightarrow s_r = 150\,\text{mm}$

The stirrups weaken the concrete cross-section. Therefore, the cracks are likely to form next to the stirrups.

$$\lambda = \frac{s_r}{s_{r0}} = 0.6$$

$$\Delta\sigma_s = \frac{2 \cdot \tau_{b1} \cdot s_r}{Ø} = 55.7\,\text{MPa} > f_t - f_s = 40\,\text{MPa}$$



The reinforcement does not yield over the entire crack element when failure is reached (Regime 2).

$$\varepsilon_{sm}(\sigma_{sr} = f_t) = \frac{(f_t - f_s)^2 \cdot Ø}{4 \cdot E_{sh} \cdot \tau_{b1} \cdot s_{r0}} \cdot \left(1 - \frac{E_{sh} \cdot \tau_{b0}}{E_s \cdot \tau_{b1}}\right) + \frac{(f_t - f_s)}{E_s} \cdot \frac{\tau_{b0}}{\tau_{b1}} + \left(\frac{f_s}{E_s} - \frac{\tau_{b0} \cdot s_r}{E_s \cdot Ø}\right) = 19.5‰$$

$$\varepsilon_{smy} = \frac{f_s}{E_s} - \frac{\tau_{b0} \cdot s_r}{E_s \cdot Ø} = 2.2‰$$

$$w_{pus} = \left(\varepsilon_{sm}(\sigma_{sr} = f_t) - \varepsilon_{smy}\right) \cdot l_{TM} = 69.3\,\text{mm} > w_{dem} = 64.3\,\text{mm} \rightarrow \text{Ok}$$

The deformation capacity of the tension member is satisfactory. However, assuming the effective crack width would be $s_r = s_{ro}$ (conservative), the deformation capacity was $w_{pus} = 41.1$mm and not sufficient.