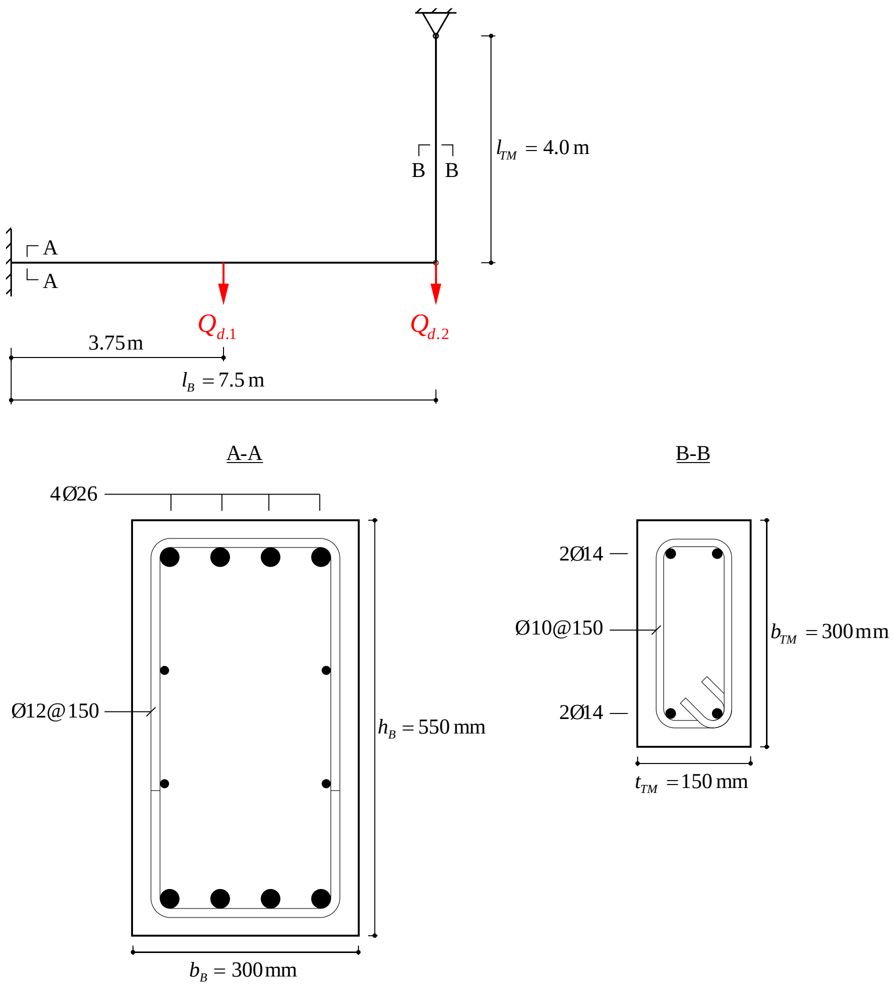
Deformation capacity and demand

SIA 262

Geometry



Material Properites

Concrete C25/30

$f_{cd} = 16.5\,\text{MPa};\ f_{cm} = 33\,\text{MPa}$ — Table 8 and 3

$\varepsilon_{cud} = 0.3\%$

$E_{cm} = k_e \sqrt[3]{f_{cm}} \approx 30.1\,\text{GPa}$ — 3.1.2.3.3

$f_{ctm} = 2.6\,\text{MPa}$ — Table 3

Steel B500B

$f_s = 500\,\text{MPa};\ f_{sd} = 435\,\text{MPa};\ f_t = 540\,\text{MPa}$ — Table 5 and 9

$\varepsilon_{ud} = 4.5\%$ — 4.2.2.6

$$E_s = 205\,\text{GPa};\ E_{sh} = \frac{f_t - f_s}{\varepsilon_{ud} - \frac{f_s}{E_s}} = 0.94\,\text{GPa}$$

$\tau_{b0} = 2 \cdot f_{ctm};\ \tau_{b1} = f_{ctm}$

$c_{nom} = 25\,\text{mm}$

Load

Load Scenario 1: $Q_{d.1} = 300\,\text{kN},\ Q_{d.2} = 0\,\text{kN}$

Load Scenario 2: $Q_{d.1} = 0\,\text{kN},\ Q_{d.2} = 300\,\text{kN}$

(no residual stresses nor restraint, as would result e.g. due to support settlements)

Resistance

Beam:

$$d = h_B - c_{nom} - Ø_w - \frac{Ø}{2} = 500\,\text{mm}$$

$$M_{Rd} = A_{s.B} \cdot f_{sd} \left(d - \frac{A_{s.B} \cdot f_{sd}}{2 \cdot b_B \cdot f_{cd}} \right) = 375.7\,\text{kNm}$$

$$x = \frac{A_{s.B} \cdot f_{sd}}{0.85 \cdot b_B \cdot f_{cd}} = 219.6\,\text{mm}$$

$$\frac{x}{d} = 0.44 > 0.35 \rightarrow \text{verification of deformation capacity required!}$$

$$z = d - \frac{0.85 \cdot x}{2} = 406.7\,\text{mm}$$

$Ø_w = 12\,\text{mm}$
$Ø = 26\,\text{mm}$
$A_{s.B} = 2124\,\text{mm}^2$

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Tension member:

$$N_{Rd} = A_{s.TM} \cdot f_{sd} = 267.9\,\text{kN}$$

$A_{s.TM} = 616\,\text{mm}^2$

Stiffnesses (cracked elastic State II)

Beam:

$$\rho_B = \frac{A_{s.B}}{b_B \cdot d} = 1.42\%$$

$$n = \frac{E_s}{E_{cm}} = 6.81$$

$$x^{II} = d \cdot \left(\sqrt{(\rho_B \cdot n)^2 + 2 \cdot \rho_B \cdot n} - \rho_B \cdot n \right) = 176.6\,\text{mm}$$

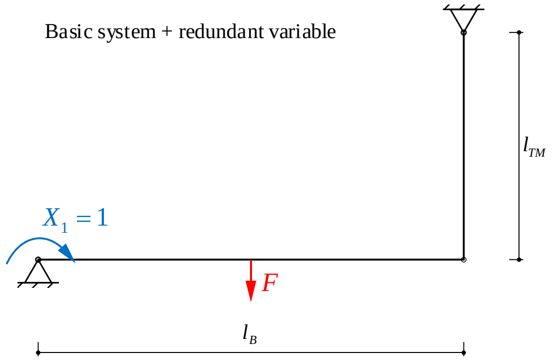
$$EI^{II} = A_{s.B} \cdot E_s \cdot (d - x^{II}) \cdot \left(d - \frac{x^{II}}{3} \right) = 62.1\,\text{MNm}^2$$

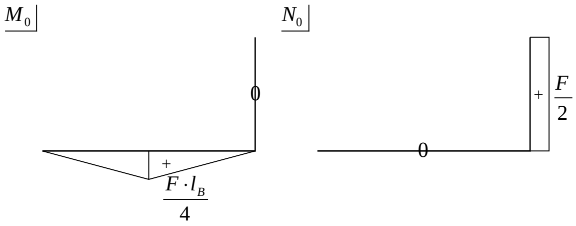
Tension member:

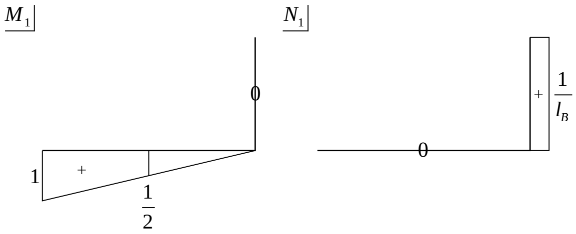
$$EA^{II} = E_s \cdot A_{s.TM} = 126.2\,\text{MN}$$

a) Verification of structure for Load Scenario 1

Linear elastic internal forces using the force method:





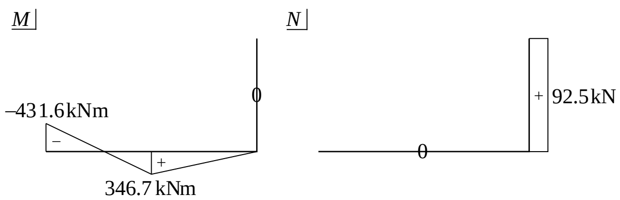


$$\delta_{10} = \underbrace{\frac{1}{3}\cdot\frac{Fl_B}{4}\cdot\frac{1}{2}\cdot\frac{l_B}{2EI^{II}} + \frac{1}{6}\cdot\frac{Fl_B}{4}\cdot\left(2\cdot\frac{1}{2}+1\right)\cdot\frac{l_B}{2EI^{II}}}_{\frac{3Fl_B^2}{48EI^{II}}=\frac{Fl_B^2}{16EI^{II}}} + 1\cdot\frac{F}{2}\cdot\frac{1}{l_B}\cdot\frac{l_{TM}}{EA^{II}}$$

$$\delta_{11} = \frac{1}{3}\cdot 1\cdot 1\cdot\frac{l_B}{EI^{II}} + 1\cdot\frac{1}{l_B}\cdot\frac{1}{l_B}\cdot\frac{l_{TM}}{EA^{II}}$$

$$\delta_{10} + X_1\cdot\delta_{11} \overset{!}{=} 0 \rightarrow X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{\dfrac{Fl_{TM}}{2l_B EA^{II}} + \dfrac{Fl_B^2}{16EI^{II}}}{\dfrac{l_B}{3EI^{II}} + \dfrac{l_{TM}}{l_B^2 EA^{II}}}$$

$$M = M_0 + X_1\left(F = Q_{d.1}\right)\cdot M_1$$
$$N = N_0 + X_1\left(F = Q_{d.1}\right)\cdot N_1$$



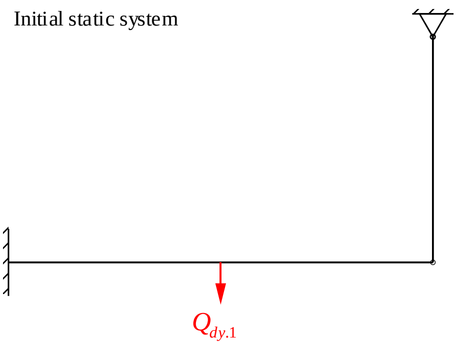
$$\left|M_{B.clamp} = -431.6\,\text{kNm}\right| > M_{Rd}$$
$$M_{B.field} = 346.7\,\text{kNm} < M_{Rd}$$
$$N_{TM} = 92.5\,\text{kN} < N_{Rd}$$

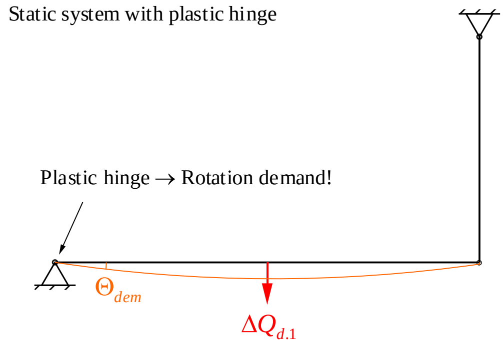
The clamping moment exceeds the moment resistance. Upon reaching the moment resistance, a plastic hinge at the clamp is formed and the static system of the beam changes to a simply supported beam, which has further load-bearing capacity ($M_{B.span} < M_{Rd}$). The rotation capacity of the plastic hinge has to be compared to its deformation demand resulting from the system change.

$$Q_{dy.1} = Q_{d.1}\cdot\frac{M_{Rd}}{M_{B.clamp}} = 261.2\,\text{kN}$$

$$\Delta Q_{d.1} = Q_{d.1} - Q_{dy.1} = 38.8\,\text{kN}$$

$Q_{dy.1}$ corresponds to the load at which a plastic hinge forms at the fixed end of the beam.
$\Delta Q_{d.1}$ corresponds to the load that has to be carried by the simply supported beam.





Internal forces after redistribution:

$$M_{B.clamp.red} = M_{Rd} = 375.7\,\text{kNm} \rightarrow \text{OK}$$

$$M_{B.field.red} = \frac{Q_{dy.1} \cdot l_B}{4} + \frac{X_1\left(F = Q_{dy.1}\right)}{2} + \frac{\Delta Q_{d.1} \cdot l_B}{4} = 374.6\,\text{kNm} < M_{Rd} \rightarrow \text{OK}$$

$$N_{TM.red} = \frac{Q_{dy.1}}{2} + \frac{X_1\left(F = Q_{dy.1}\right)}{l_B} + \frac{\Delta Q_{d.1}}{2} = 99.9\,\text{kN} < N_{Rd} \rightarrow \text{OK}$$

$$A_{clamp} = V_{B.max} = Q_{d.1} - N_{TM.red} = 200.1\,\text{kN}$$

Verification shear force:

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$$\alpha = 30°$$

4.3.3.4.3

$$V_{Rd,s} = a_{sw} \cdot f_{sd} \cdot z \cdot \cot(\alpha) = 461.5\,\text{kN} > V_{B.max} \rightarrow \text{OK}$$

$$a_{sw} = 2 \cdot 753 \frac{\text{mm}^2}{\text{m}}$$

4.3.3.4.6

$$V_{Rd,c} = b_B \cdot f_{cd} \cdot z \cdot k_c \cdot \sin(\alpha) \cdot \cos(\alpha) = 479.4\,\text{kN} > V_{B.max} \rightarrow \text{OK}$$

$$k_c = 0.55$$

Rotation demand:

Work theorem

$$\Theta_{dem} = \delta_{10}\left(F = \Delta Q_{d.1}\right) = \frac{\Delta Q_{d.1} \cdot l_{TM}}{2 \cdot l_B \cdot EA^{II}} + \frac{\Delta Q_{d.1} \cdot l_B^2}{16 EI^{II}} = 2.28\,\text{mrad}$$

Rotation capacity:

Centroid

$$\zeta = 275\,\text{mm}$$

$$I_y = \frac{h_b^3 \cdot b_B}{12} + 2 \cdot A_{s.B} \cdot (n-1) \cdot \left(\zeta - \left(c_{nom} + \varnothing_w + \frac{\varnothing}{2}\right)\right)^2 = 0.005\,\text{m}^4$$

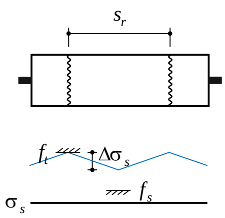
$$M_r = \frac{f_{ctm} \cdot I_y}{\zeta} = 51.1\,\text{kNm}$$

$$\rho_{eff} = \frac{1}{\dfrac{M_r \cdot (d - x_{II}) \cdot E_s}{f_{ctm} \cdot EI^{II}} + 1 - n} = 6.6\%$$

$$s_{r0} = \frac{Ø}{4} \cdot \left(\frac{1}{\rho_{eff}} - 1 \right) = 92\,\text{mm}$$

$$s_r = s_{r0}\left(\lambda = 1\right)$$

$$\Delta\sigma_s = \frac{2 \cdot \tau_{b1} \cdot s_r}{Ø} = 18.4\,\text{MPa} < f_t - f_s = 40\,\text{MPa}$$



The reinforcement yields over the entire crack element when failure is reached (Regime 3).

$$x_{P1} = \sqrt{\frac{2 \cdot A_{s.B} \cdot \left(f_t - f_s - \frac{2 \cdot \tau_{b1} \cdot s_r}{Ø} \right) \cdot z}{f_{wd}}} = 362\,\text{mm}$$

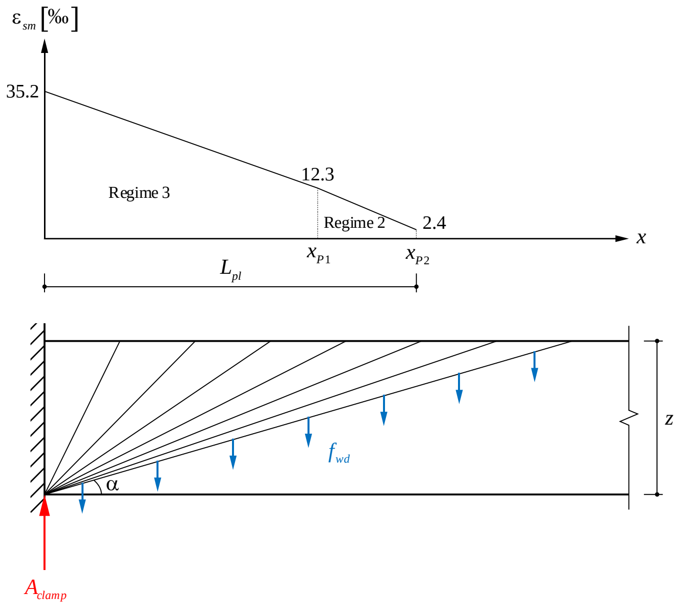
$$x_{P2} = \sqrt{\frac{2 \cdot A_{s.B} \cdot \left(f_t - f_s \right) \cdot z}{f_{wd}}} = 493\,\text{mm} = L_{pl} < z \cdot \cot(\alpha) = 704\,\text{mm}$$

$$f_{wd} = \frac{V_{B.max}}{z \cdot \cot(\alpha)} = 284.1 \frac{\text{kN}}{\text{m}}$$

$$\varepsilon_{sm}(\sigma_{sr} = f_t) = \frac{f_s}{E_s} + \frac{f_t - f_s}{E_{sh}} - \frac{\tau_{b1} \cdot s_r}{E_{sh} \cdot Ø} = 35.2‰$$

$$\varepsilon_{sm}(\sigma_{sr} = f_s + \Delta\sigma_s) = \frac{f_s}{E_s} + \frac{\Delta\sigma_s}{E_{sh}} - \frac{\tau_{b1} \cdot s_r}{E_{sh} \cdot Ø} = 12.3‰$$

$$\varepsilon_{sm}(\sigma_{sr} = f_s) = \frac{f_s}{E_s} - \frac{\tau_{b0} \cdot s_r}{E_s \cdot Ø} = 2.4‰$$



To determine the mean (over the plastic length) strain $\overline{\varepsilon_{smu}}$ of the tension chord at rupture of the reinforcement, the strain distribution of the reinforcement is assumed as linear within Regimes 2 and 3.

$$\overline{\varepsilon_{smu}} = \frac{\frac{\varepsilon_{sm}(\sigma_{sr} = f_t) + \varepsilon_{sm}(\sigma_{sr} = f_s + \Delta\sigma)}{2} \cdot x_{P1} + \frac{\varepsilon_{sm}(\sigma_{sr} = f_s + \Delta\sigma) + \varepsilon_{sm}(\sigma_{sr} = f_s)}{2} \cdot \left(x_{P2} - x_{P1} \right)}{x_{P2}} = 19.4‰$$

$$\varepsilon_{smy} = \varepsilon_{sm}(\sigma_{sr} = f_s) = 2.4‰$$

$$\Theta_{puc} = L_{pl} \cdot \left(\frac{\varepsilon_{cud}}{x} - \frac{\varepsilon_{smy}}{d-x} \right) = 2.61\,\text{mrad} > \Theta_{dem} = 2.28\,\text{mrad} \rightarrow \text{OK} \qquad \text{(concrete crushing)}$$

$$\Theta_{pus} = L_{pl} \cdot \left(\frac{\varepsilon_{smu}}{d-x} - \frac{\varepsilon_{smy}}{d-x} \right) = 29.91\,\text{mrad} > \Theta_{dem} = 2.28\,\text{mrad} \rightarrow \text{OK} \qquad \text{(steel rupture)}$$

The rotation capacity of the plastic hinge is thus sufficient.

Remark:
Rotation capacity simplified:

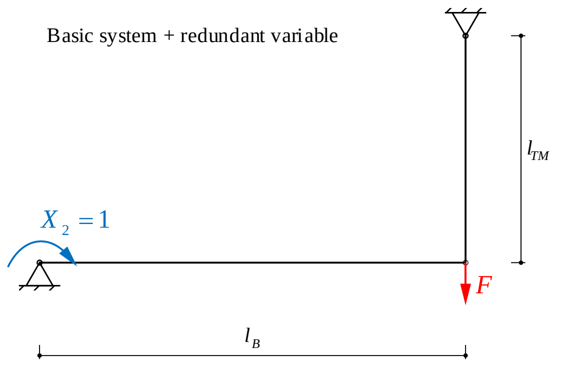
$$\Theta_{puc} = d \cdot \left(\frac{\varepsilon_{cud}}{x} - \frac{\varepsilon_{smy}}{d-x} \right) = 2.64\,\text{mrad} > \Theta_{dem} = 2.28\,\text{mrad} \rightarrow \text{OK} \qquad \text{(concrete crushing)}$$

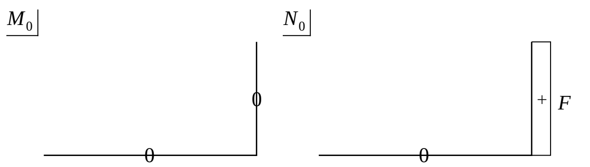
$$\Theta_{pus} = d \cdot \left(\frac{0.5 \cdot \varepsilon_{ud}}{d-x} - \frac{\varepsilon_{smy}}{d-x} \right) = 35.93\,\text{mrad} > \Theta_{dem} = 2.28\,\text{mrad} \rightarrow \text{OK} \qquad \text{(steel rupture)}$$

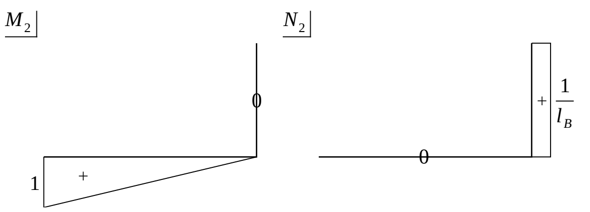
The rotation capacity determined with the simplified method is in good agreement with the more refined method considering concrete crushing. For the given geometry (rectangular cross-section and one clamped side) assuming $L_{pl} = d$ is justified. The simplified approach overestimates the mean elongations at failure of the reinforcement. For situations with steel rupture being the governing failure mode, the more refined calculation method is recommended.

b) Verification of structure for load scenario 2

Elastic internal forces using force method:





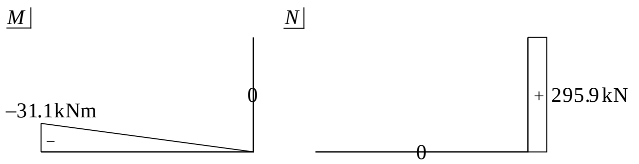


$$\delta_{20} = 1 \cdot F \cdot \frac{1}{l_B} \cdot \frac{l_{TM}}{EA^{II}}$$

$$\delta_{22} = \frac{1}{3} \cdot 1 \cdot 1 \cdot \frac{l_B}{EI^{II}} + 1 \cdot \frac{1}{l_B} \cdot \frac{1}{l_B} \cdot \frac{l_{TM}}{EA^{II}}$$

$$\delta_{20} + X_2 \cdot \delta_{22} \overset{!}{=} 0 \rightarrow X_2 = -\frac{\delta_{20}}{\delta_{22}} = -\frac{\dfrac{F l_{TM}}{l_B EA^{II}}}{\dfrac{l_B}{3EI^{II}} + \dfrac{l_{TM}}{l_B^2 EA^{II}}}$$

$$M = M_0 + X_2 \left(F = Q_{d.2}\right) \cdot M_1$$
$$N = N_0 + X_2 \left(F = Q_{d.2}\right) \cdot N_1$$



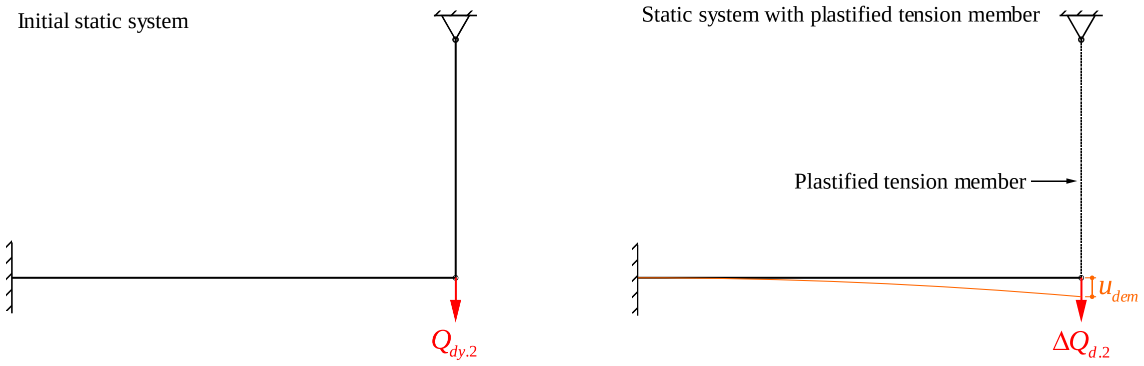
$$M_{B.clamp} = -31.1\,\text{kNm} < M_{Rd}$$
$$N_{TM} = 295.9\,\text{kN} > N_{Rd} = 267.9\,\text{kN}$$

The normal force in the tension member exceeds the normal resistance. Reaching the normal resistance, the tension member plastifies and a cantilever beam remains as the static system, which has further load bearing capacity ($M_{B.clamp} < M_{Rd}$). The deformation capacity of the tension member has to be compared with the deformation demand resulting from the system change.

$$Q_{dy.2} = Q_{d.2} \cdot \frac{N_{Rd}}{N_{TM}} = 271.6\,\text{kN}$$

$$\Delta Q_{d.2} = Q_{d.2} - Q_{dy.2} = 28.4\,\text{kN}$$

$Q_{dy.2}$ corresponds to the load at which the tension member yields and cannot take any more load. $\Delta Q_{d.2}$ corresponds to the load that has to be carried by the cantilever beam alone.



Internal forces after redistribution:

$$N_{TM.red} = N_{Rd} = 267.9\,\text{kN} \rightarrow \text{OK}$$

$$M_{B.clamp.red} = X_2\left(F = Q_{dy.2}\right) - \Delta Q_{d.2} \cdot l_B = -241.1\,\text{kNm} < M_{Rd} \rightarrow \text{OK}$$

$$V_{B.max} = Q_{d.2} - N_{TM.red} = 32.1\,\text{kN}$$

Verification shear force:

$$V_{B.max} = 32.1\,\text{kN} < \min\left(V_{Rd,s}, V_{Rd,c}\right) = 461.5\,\text{kN} \rightarrow \text{OK}$$

Deformation demand tension member:

$$u_{dem} = \frac{\Delta Q_{d.2} \cdot l_B^3}{3EI^{II}} = 64.3\,\text{mm}$$

Deformation capacity tension member:

$$\rho_{TM} = \frac{A_{s.TM}}{b_{TM} \cdot t_{TM}} = 1.37\%$$

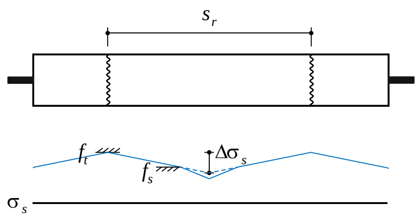
$$s_{r0} = \frac{Ø}{4} \cdot \left(\frac{1}{\rho_{TM}} - 1\right) = 252\,\text{mm}$$ $Ø = 14\,\text{mm}$

Stirrups Ø10@150 $\rightarrow s_r = 150\,\text{mm}$

The stirrups weaken the concrete cross-section. Therefore, the cracks are likely to form next to the stirrups.

$$\lambda = \frac{s_r}{s_{r0}} = 0.6$$

$$\Delta\sigma_s = \frac{2 \cdot \tau_{b1} \cdot s_r}{Ø} = 55.7\,\text{MPa} > f_t - f_s = 40\,\text{MPa}$$



The reinforcement does not yield over the entire crack element when failure is reached (Regime 2).

$\varnothing = 14\,\text{mm}$

$$\varepsilon_{sm}(\sigma_{sr} = f_t) = \frac{(f_t - f_s)^2 \cdot \varnothing}{4 \cdot E_{sh} \cdot \tau_{b1} \cdot s_r} \cdot \left(1 - \frac{E_{sh} \cdot \tau_{b0}}{E_s \cdot \tau_{b1}}\right) + \frac{(f_t - f_s)}{E_s} \cdot \frac{\tau_{b0}}{\tau_{b1}} + \left(\frac{f_s}{E_s} - \frac{\tau_{b0} \cdot s_r}{E_s \cdot \varnothing}\right) = 17.7‰$$

$$\varepsilon_{smy} = \frac{f_s}{E_s} - \frac{\tau_{b0} \cdot s_r}{E_s \cdot \varnothing} = 2.2‰$$

$$u_{pus} = \left(\varepsilon_{sm}(\sigma_{sr} = f_t) - \varepsilon_{smy}\right) \cdot l_{TM} = 62.1\,\text{mm} < u_{dem} = 64.3\,\text{mm} \rightarrow \text{not OK}$$

The deformation capacity of the tension member is not satisfactory. This shows that the expected increase in load is slightly too large and hence the deformation demand cannot be met.