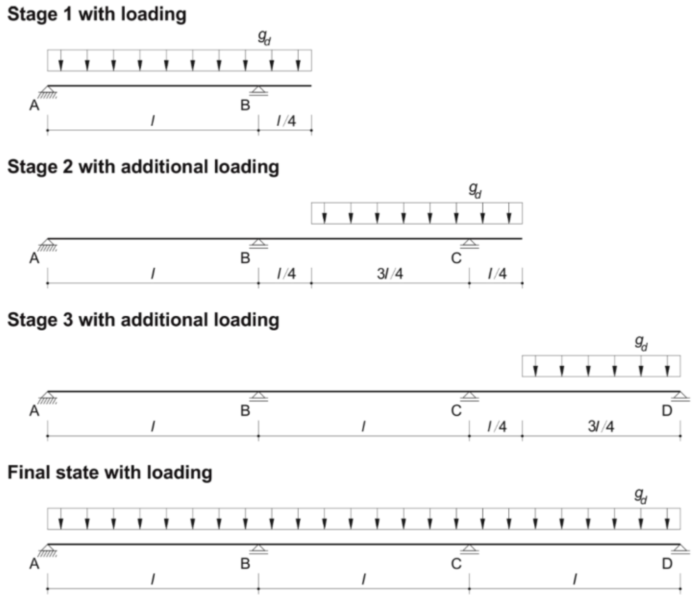
Constructing a three-span beam in stages

<u>Geometry</u>



<u>Material Properties</u>

Concrete — C35/45 — $f_{ck} = 35\,\text{MPa};\ f_{ctm} = 3.2\,\text{MPa}$ — SIA 262 Tab. 3

$f_{cd} = 22\,\text{MPa};\ \tau_{cd} = 1.2\,\text{MPa}$

$E_{cm} = k_E \sqrt[3]{f_{cm}} \approx 35\text{ GPa}$ — Tab. 8

3.1.2.3.3

Steel — B500B — $f_{sk} = 500\,\text{MPa};\ f_{sd} = 435\,\text{MPa}$

$E_s = 205\text{ GPa}$

Tab. 5/9

a) <u>Creep coefficient t_{120} = 120d, t_{5y} = 5y</u> — 3.2.2.4

$$\varphi(t,t_0) = \varphi_{RH} \cdot \beta_{fc} \cdot \beta_{\sigma c} \cdot \beta(t_0) \cdot \beta(t-t_0)$$

with $\beta_{fc} = 2.6, \quad \beta_{\sigma c} = 1.0$

$\varphi_{RH} = 1.25 \ \ (RH = 70\%,\ \ h_0 = 600\,\text{mm})$ — 3.1.2.6.2

$\beta(t_0) = 0.45 \ \ (k_T = f(T = 20°\text{C}) = 1.0,\ \ \text{normal hardening cement})$ — 3.1.2.6.3

Tab. 4

- For $t = t_{120}$: — Task sheet Fig. 2

$$\beta(t-t_0) = \begin{Bmatrix} \beta(90\,\text{d}) & \text{St. 1} \\ \beta(60\,\text{d}) & \text{St. 2} \\ \beta(30\,\text{d}) & \text{St. 3} \end{Bmatrix} = \begin{Bmatrix} 0.42 \\ 0.38 \\ 0.32 \end{Bmatrix}$$

$$\varphi(t,t_0) = \begin{Bmatrix} \varphi(120\,\text{d},30\,\text{d}) & \text{St. 1} \\ \varphi(120\,\text{d},30\,\text{d}) & \text{St. 2} \\ \varphi(120\,\text{d},30\,\text{d}) & \text{St. 3} \end{Bmatrix} = \begin{Bmatrix} 0.614 \\ 0.556 \\ 0.468 \end{Bmatrix}$$

Fig. 2

- For $t = t_{5y}$:

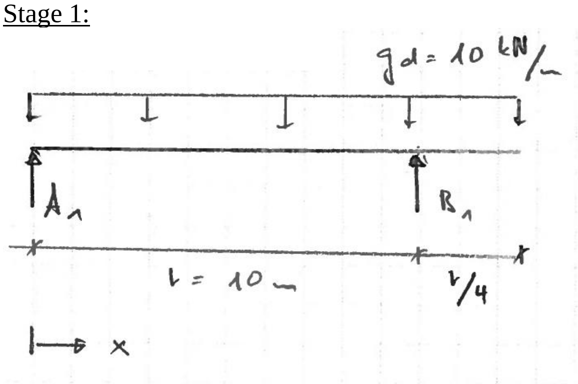
$\beta(t-t_0) = 0.85 \quad \text{(approx. for all stages)}$

$$\varphi(t,t_0) = \varphi(5\,\text{y},30\,\text{d}) = 1.243$$

Fig.2

b) <u>Moment distribution of each stage</u>

<u>Stage 1:</u>



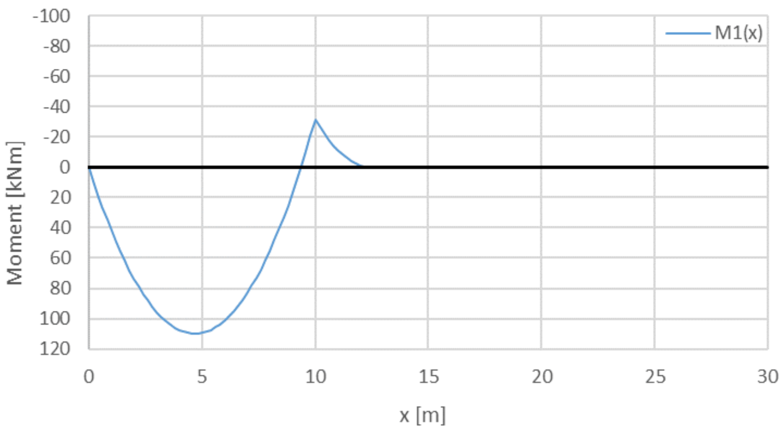
$$\sum F_v = 0: A_1 + B_1 = g_d \cdot \frac{5L}{4}$$

$$\sum M_A = 0: g_d \cdot \left(\frac{5L}{4}\right)^2 \cdot \frac{1}{2} = B_1 \cdot L$$

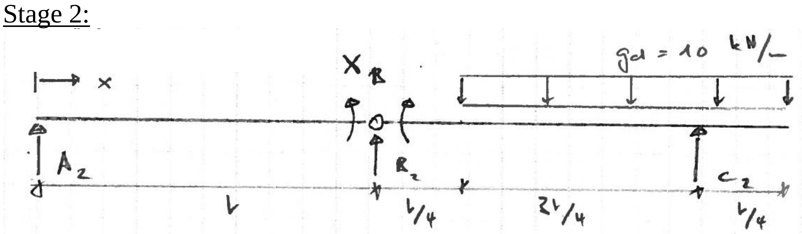
$$\rightarrow B_1 = \frac{25}{32} \cdot g_d \cdot L, \quad A_1 = \frac{15}{32} \cdot g_d \cdot L$$

$$M_B = -g_d \cdot \left(\frac{L}{4}\right)^2 \cdot \frac{1}{2} = -g_d \cdot \frac{L^2}{32}$$

$$M_1 = \begin{cases} A_1 \cdot x - g_d \cdot \dfrac{x^2}{2} & \text{for } 0 \le x \le L \\ A_1 \cdot x + B_1 \cdot (x-L) - g_d \cdot \dfrac{x^2}{2} & \text{for } L \le x \le \dfrac{5L}{4} \end{cases}$$



<u>Stage 2:</u>



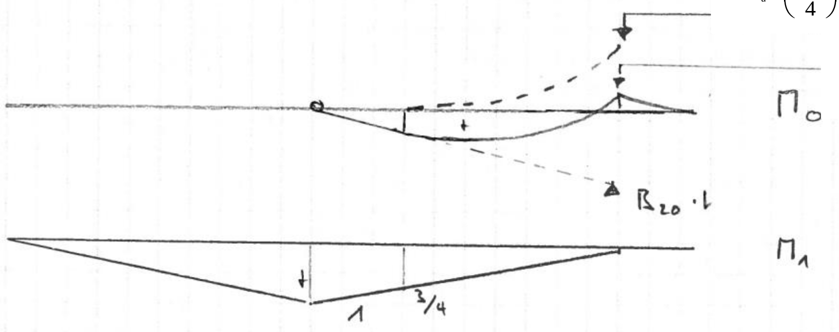
Force method BS+RV

$$\sum F_V = 0: B_{20} + C_{20} = g_d \cdot L, \quad \sum M_B = 0: C_{20} \cdot L = g_d \cdot L \cdot \left(\frac{L}{4} + \frac{L}{2}\right)$$

$$\rightarrow C_{20} = g_d \cdot \frac{3L}{4}, \quad \mathrm{B}_{20} = g_d \cdot \frac{L}{4}$$

$$-g_d \cdot \left(\frac{3L}{4}\right)^2 \cdot \frac{1}{2} = -\frac{9L^2}{32} \cdot g_d$$

$$-\frac{L^2}{32} \cdot g_d$$



Dashed: moment curve, split for integration with force method

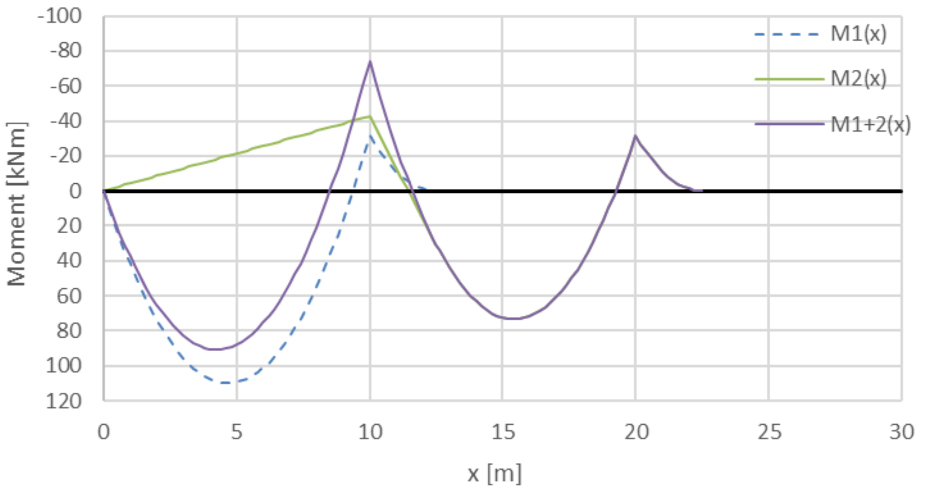
$$\delta_{10} = \frac{1}{6}\cdot\frac{L}{EI}\cdot B_{20}\cdot L\cdot 1 - \frac{1}{12}\cdot\frac{3L}{4EI}\cdot\left(\frac{9}{32}g_d L^2\right)\cdot\frac{3}{4} = \frac{g_d\cdot L^3}{24EI} - \frac{27\cdot g_d\cdot L^3}{2048EI}$$

$$\delta_{11} = \frac{2L}{3EI}$$

$$X_B = -\frac{\delta_{10}}{\delta_{11}} = -\frac{175}{4096}g_d\cdot L^2$$

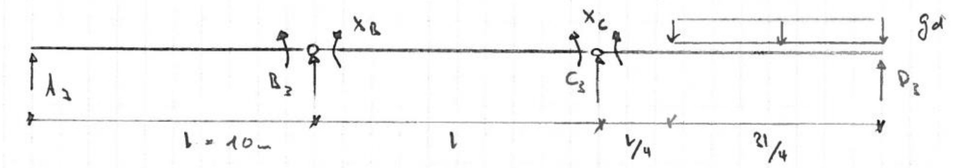
$$\rightarrow M_B = -42.725\,\text{kNm},\quad M_C = -31.25\,\text{kNm}$$

$$M_2(x) = \begin{cases} X_B\cdot\dfrac{x}{L} & 0\le x\le L \\ X_B\cdot\left(1-\dfrac{(x-L)}{L}\right)+B_{20}(x-L) & L\le x\le\dfrac{5L}{4} \\ X_B\cdot\left(1-\dfrac{(x-L)}{L}\right)+B_{20}(x-L)-\dfrac{g_d}{2}\left(x-\dfrac{5L}{4}\right)^2 & \dfrac{5L}{4}\le x\le 2L \\ -\dfrac{g_d L^2}{32}+-\dfrac{g_d L}{4}(x-2L) & 2L\le x\le\dfrac{9L}{4} \end{cases}$$



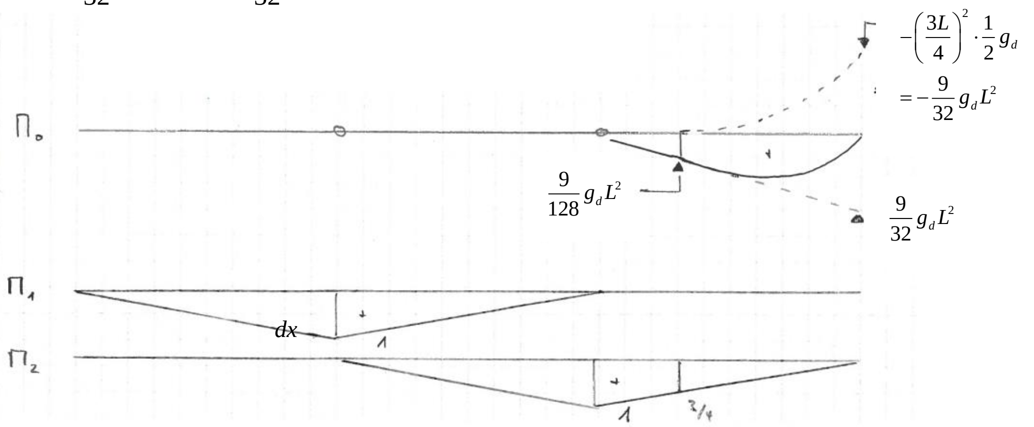
<u>Stage 3:</u>

Force method BS+RV



$$\sum F_V = 0: C_{30}+D_{30} = g_d\cdot\frac{3L}{4},\qquad \sum M_C = 0: D_{30}\cdot L = \frac{3L}{4}\cdot g_d\cdot\left(\frac{L}{4}+\frac{3L}{8}\right)$$

$$\rightarrow D_{30} = \frac{15}{32}g_d L,\quad C_{30} = \frac{9}{32}g_d L$$



$$\delta_{10} = 0$$

$$\delta_{20} = \frac{1}{6}\cdot\frac{L}{EI}\cdot 1\cdot\frac{9L^2}{32}g_d - \frac{1}{12}\cdot\frac{3L}{4EI}\cdot\frac{9L^2}{32}g_d\cdot\frac{3}{4} = \frac{69}{2048}\frac{g_d L^3}{EI}$$

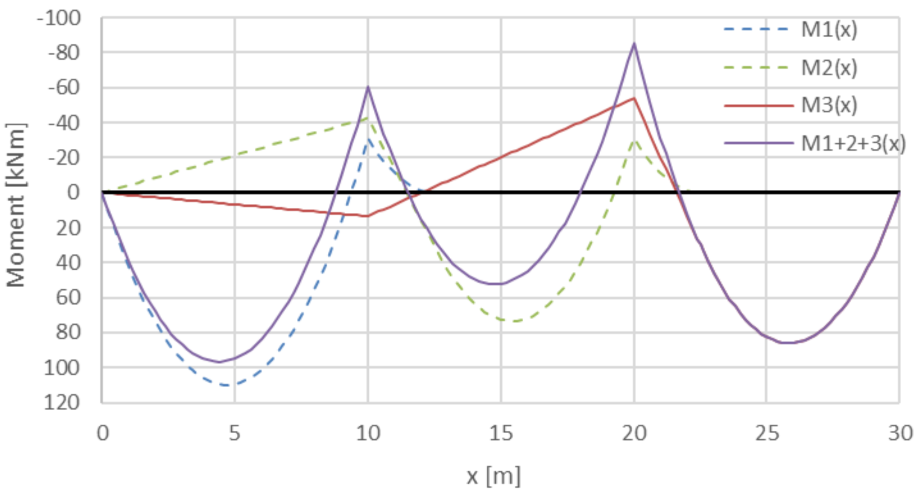
$$\delta_{11} = \delta_{22} = \frac{2}{3}\cdot\frac{L}{EI}$$

$$\delta_{12} = \delta_{12} = \frac{1}{6}\cdot\frac{L}{EI}$$

$$\rightarrow \begin{bmatrix} 0 \\ \frac{69}{2048} \end{bmatrix}\cdot\frac{g_d L^3}{EI} + \begin{bmatrix} \frac{2}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{2}{3} \end{bmatrix}\cdot\frac{L}{EI}\cdot\begin{pmatrix} X_B \\ X_C \end{pmatrix} = \underline{0}$$

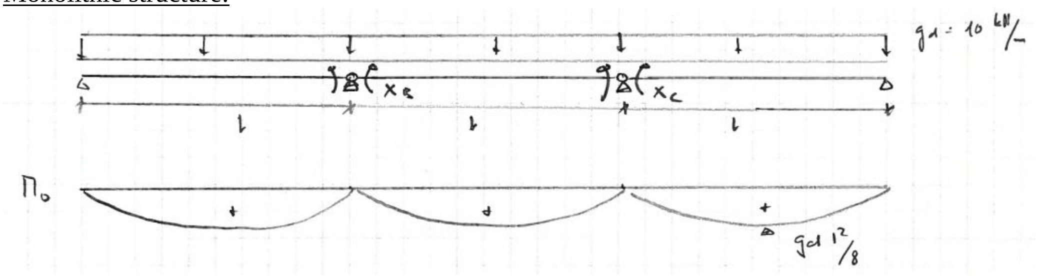
$$\rightarrow X_B = 0.013477\cdot g_d L^2, \quad X_c = -\frac{207}{3840}g_d L^2 = -0.053906\cdot g_d L^2$$

$$M_3(x) = \begin{cases} X_B\cdot\frac{x}{L} & 0 \le x \le L \\ X_B\cdot\left(1-\frac{(x-L)}{L}\right) + X_C\frac{(x-L)}{L} & L < x \le 2L \\ X_C\cdot\left(1-\frac{(x-2L)}{L}\right) + C_{30}(x-2L) & 2L < x \le \frac{9L}{4} \\ X_C\cdot\left(1-\frac{(x-2L)}{L}\right) + C_{30}(x-2L) - g_d\frac{\left(x-\frac{9L}{4}\right)^2}{2} & \frac{9L}{4} < x \le 3L \end{cases}$$



Force method BS+RV

Monolithic structure:



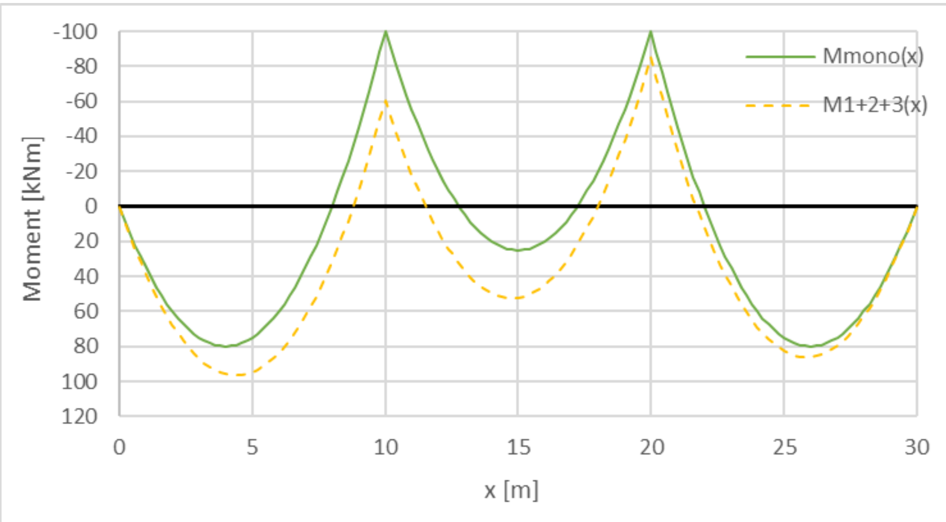
M_1 and M_2 analogues to Stage 3. From symmetry: $X_B = X_C$.

$$\delta_{10} = \delta_{20} = \frac{1}{3} \cdot \frac{L}{EI} \cdot \frac{g_d L^2}{8} \cdot 1 \cdot 2 = \frac{1}{12} \cdot \frac{g_d L^3}{EI}$$

$$\delta_{11} = \delta_{22} = \frac{2}{3} \cdot \frac{L}{EI}, \quad \delta_{12} = \delta_{21} = \frac{1}{6} \cdot \frac{L}{EI} \quad \text{(from stage 3)}$$

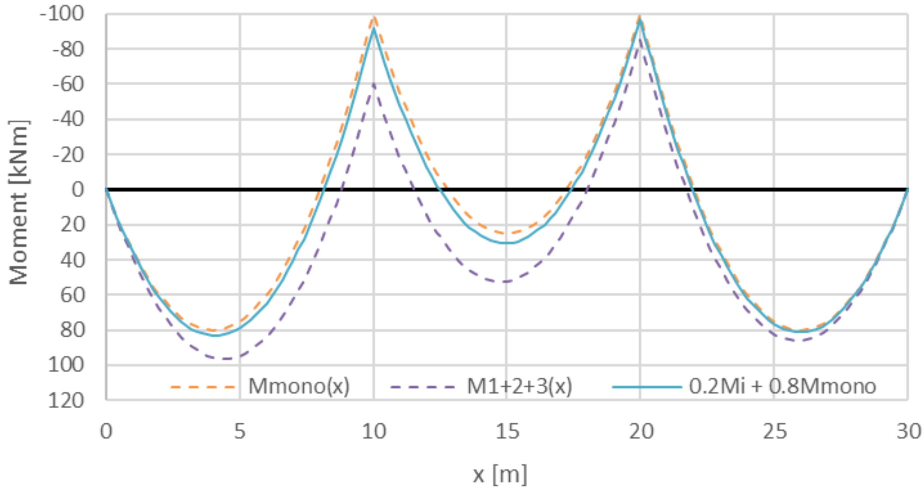
$$\rightarrow \delta_1 = \delta_{10} + \delta_{11} X_B + \delta_{12} X_B \rightarrow X_B = -\frac{g_d L^2}{10} = X_C$$

$$M_{Mono}(x) = \left\{ \begin{array}{lr} \frac{g_d L}{2} \cdot x - g_d \frac{x^2}{2} - \frac{g_d L^2}{10} \frac{x}{L} & 0 \le x \le L \\ \frac{g_d L}{2} \cdot (x-L) - g_d \frac{(x-L)^2}{2} - \frac{g_d L^2}{10} & L \le x \le 2L \\ \frac{g_d L}{2} \cdot (x-2L) - g_d \frac{(x-2L)^2}{2} - \frac{g_d L^2}{10} \left(1 - \frac{x-2L}{L}\right) & 2L \le x \le 3L \end{array} \right\}$$



c) Approximation of the moment distribution for $t \to \infty$

$$M_{t\to\infty}(x) = 0.2\cdot(M_1(x)+M_2(x)+M_3(x)) + 0.8\cdot M_{Mono}(x)$$



d) Moment distribution with the Trost Method

$$M_t(x) = M_0(x) + \left(M_{Mono}(x) - M_0(x)\right)\cdot\frac{\varphi(t,t_0)}{1+\mu\cdot\varphi(t,t_0)}$$

:

$$M_t(x) = M_0(x)\cdot\left(1-\frac{\varphi(t,t_0)}{1+\mu\cdot\varphi(t,t_0)}\right) + M_{Mono}(x)\cdot\frac{\varphi(t,t_0)}{1+\mu\cdot\varphi(t,t_0)}$$

M_0 : Moment before system change

M_{Mono} : Moment of monolithic system

Generalized:

$$M_t(x) = \sum_{i=1}^{n} M_{0,i}(x)\cdot\left(1-\frac{\varphi(t_i,t_0)}{1+\mu\cdot\varphi(t_i,t_0)}\right) + M_{Mono}(x)\cdot\frac{\varphi(t,t_0)}{1+\mu\cdot\varphi(t,t_0)}$$

$$M_{120\,\mathrm{d}}(x) = M_1(x)\cdot\left(1-\frac{\varphi(90\,\mathrm{d},30\,\mathrm{d})}{1+\mu\cdot\varphi(90\,\mathrm{d},30\,\mathrm{d})}\right) + M_2(x)\cdot\left(1-\frac{\varphi(60\,\mathrm{d},30\,\mathrm{d})}{1+\mu\cdot\varphi(60\,\mathrm{d},30\,\mathrm{d})}\right)$$

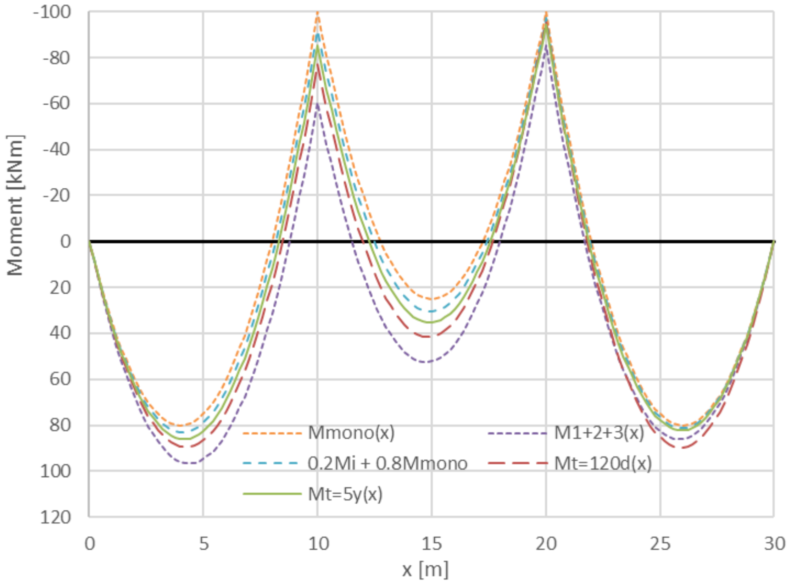
$\mu = 0.8$

$$+M_3(x)\cdot\left(1-\frac{\varphi(30\,\mathrm{d},30\,\mathrm{d})}{1+\mu\cdot\varphi(30\,\mathrm{d},30\,\mathrm{d})}\right) + M_{Mono}(x)\cdot\frac{\varphi(90\,\mathrm{d},30\,\mathrm{d})}{1+\mu\cdot\varphi(90\,\mathrm{d},30\,\mathrm{d})}$$

$$M_{120\,\mathrm{d}}(x) = 0.588M_1(x) + 0.615M_2(x) + 0.659M_3(x) + 0.412M_{Mono}(x)$$

$$M_{5\,\mathrm{y}}(x) = \left(M_1(x)+M_2(x)+M_3(x)\right)\cdot\left(1-\frac{\varphi(5\,\mathrm{y},30\,\mathrm{d})}{1+\mu\cdot\varphi(5\,\mathrm{y},30\,\mathrm{d})}\right) + M_{Mono}(x)\cdot\frac{\varphi(5\,\mathrm{y},30\,\mathrm{d})}{1+\mu\cdot\varphi(5\,\mathrm{y},30\,\mathrm{d})}$$

$$M_{5\,\mathrm{y}}(x) = 0.377\cdot\left(M_1(x)+M_2(x)+M_3(x)\right) + 0.623M_{Mono}$$



Remark:

As the age of the structure increases, the distribution of moments increasingly approaches that of the monolithic structure due to creep. The approximation "80% monolithic structure, 20% sum of the individual construction stages" approximates the long-term behaviour relatively well. Since the determination of the creep coefficient is also subject to certain uncertainty and the bending stiffness over the beam length is by no means constant (crack formation), the calculation with the Trost method can also only be regarded as an approximation of the real stress state.