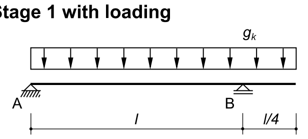
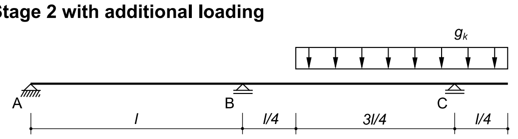
Constructing a three-span beam in stages

Geometry

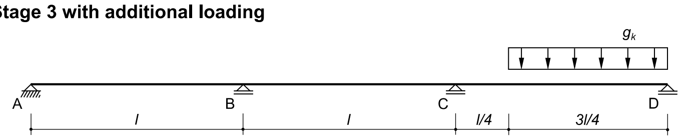
Stage 1 with loading



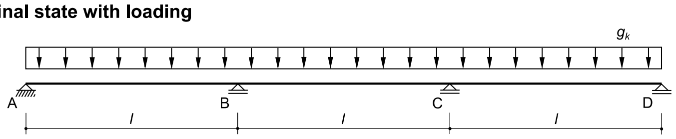
Stage 2 with additional loading



Stage 3 with additional loading



Final state with loading



Material Properties — SIA 262

Concrete C35/45 — Tab. 3

$f_{ck} = 35\,\text{MPa}; \quad f_{ctm} = 3.2\,\text{MPa}$

$f_{cd} = 22\,\text{MPa}; \quad \tau_{cd} = 1.2\,\text{MPa}$ — Tab. 8

$E_{cm} = k_E \sqrt[3]{f_{cm}} \approx 35\text{ GPa with } k_E = 10000$ — 3.1.2.3.3

Steel B500B — Tab. 5/9

$f_{sk} = 500\text{ MPa}; \quad f_{sd} = 435\text{ MPa}$

$E_s = 205\text{ GPa}$ — 3.2.2.4

a) Creep coefficient t_{120} = 120 d, t_{5y} = 5 y

$$\varphi(t,t_0) = \varphi_{RH} \cdot \beta_{fc} \cdot \beta_{\sigma c} \cdot \beta(t_0) \cdot \beta(t-t_0)$$ 3.1.2.6.2

with $\beta_{fc} = 2.6, \quad \beta_{\sigma c} = 1.0$ — Tab. 4

$\varphi_{RH} = 1.25 \;\; (RH = 70\%, \;\; h_0 = 600\,\text{mm})$ — Task sheet, Fig. 2

$\beta(t_0) = 0.45 \;\; (k_T = f(T = 20°\text{C}) = 1.0, \;\; \text{normal hardening cement})$ — Fig. 1

- For t = respective age of concrete = t_{120} - $t_{casting}$: — Fig. 2

$$\beta(t-t_0) = \begin{Bmatrix} \beta(90\,\text{d}) \;\; \text{St. 1} \\ \beta(60\,\text{d}) \;\; \text{St. 2} \\ \beta(30\,\text{d}) \;\; \text{St. 3} \end{Bmatrix} = \begin{Bmatrix} 0.42 \\ 0.38 \\ 0.32 \end{Bmatrix}$$

$$\varphi(t,t_0) = \begin{Bmatrix} \varphi(120\,\text{d}, 30\,\text{d}) \;\; \text{St. 1} \\ \varphi(90\,\text{d}, 30\,\text{d}) \;\; \text{St. 2} \\ \varphi(60\,\text{d}, 30\,\text{d}) \;\; \text{St. 3} \end{Bmatrix} = \begin{Bmatrix} 0.614 \\ 0.556 \\ 0.468 \end{Bmatrix}$$

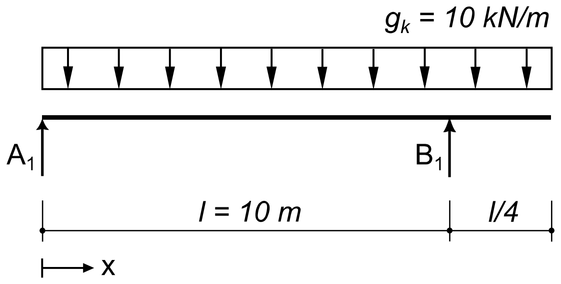
- For $t = t_{5y}$: — Fig.2

$\beta(t-t_0) = 0.85 \quad \text{(approx. for all stages)}$

$$\varphi(t,t_0) = \varphi(5\,\text{y}, 30\,\text{d}) = 1.243$$

b) (i) Moment distribution of each stage

Stage 1:



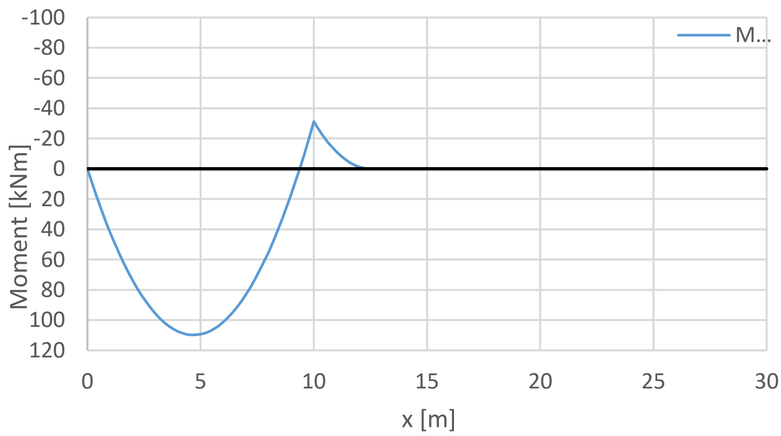
$$\sum F_v = 0: A_1 + B_1 = g_k \cdot \frac{5L}{4}$$

$$\sum M_A = 0: g_k \cdot \left(\frac{5L}{4}\right)^2 \cdot \frac{1}{2} = B_1 \cdot L$$

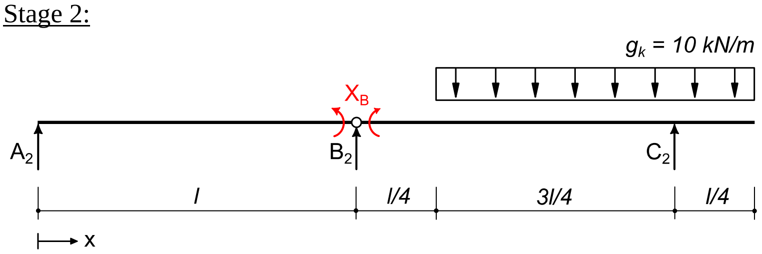
$$\rightarrow B_1 = \frac{25}{32} \cdot g_k \cdot L, \quad A_1 = \frac{15}{32} \cdot g_k \cdot L$$

$$M_B = -g_k \cdot \left(\frac{L}{4}\right)^2 \cdot \frac{1}{2} = -g_k \cdot \frac{L^2}{32}$$

$$M_1 = \begin{cases} A_1 \cdot x - g_k \cdot \dfrac{x^2}{2} & \text{for } 0 \le x \le L \\ A_1 \cdot x + B_1 \cdot (x-L) - g_k \cdot \dfrac{x^2}{2} & \text{for L} < x \le \dfrac{5L}{4} \end{cases}$$

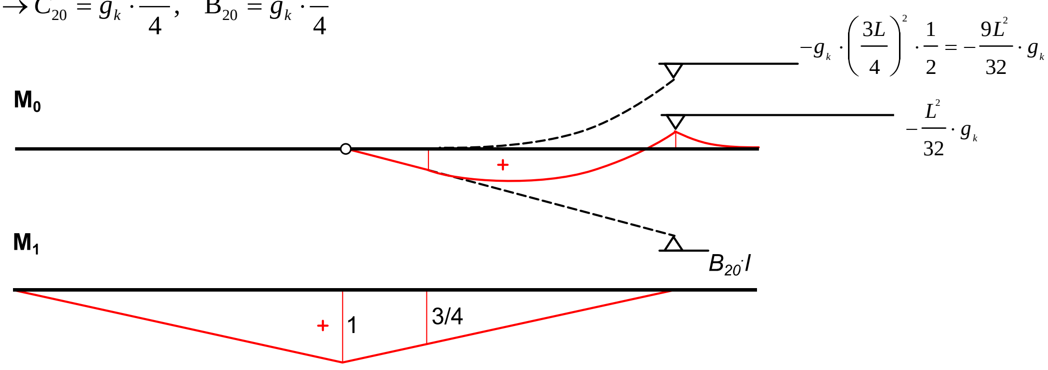


Stage 2:



$$\sum F_V = 0: B_{20} + C_{20} = g_k \cdot L, \quad \sum M_B = 0: C_{20} \cdot L = g_k \cdot L \cdot \left(\frac{L}{4} + \frac{L}{2}\right)$$

$$\rightarrow C_{20} = g_k \cdot \frac{3L}{4}, \quad \mathrm{B}_{20} = g_k \cdot \frac{L}{4}$$



Force method Basic system (BS) and redundant variables (RV)

Dashed: moment curve split for integration with force method

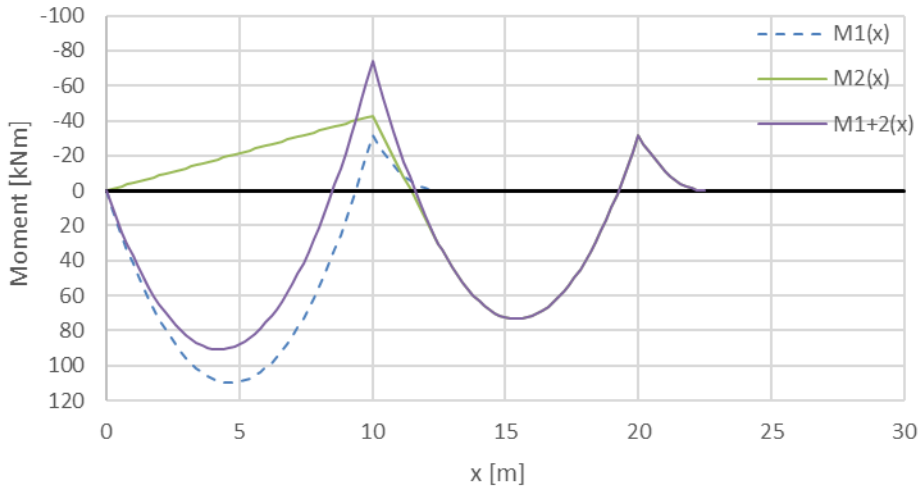
$$\delta_{10} = \frac{1}{6}\cdot\frac{L}{EI}\cdot B_{20}\cdot L\cdot 1 - \frac{1}{12}\cdot\frac{3L}{4EI}\cdot\left(\frac{9}{32}g_k L^2\right)\cdot\frac{3}{4} = \frac{g_k\cdot L^3}{24EI} - \frac{27\cdot g_k\cdot L^3}{2048EI}$$

$$\delta_{11} = \frac{2L}{3EI}$$

$$X_B = -\frac{\delta_{10}}{\delta_{11}} = -\frac{175}{4096}g_k\cdot L^2$$

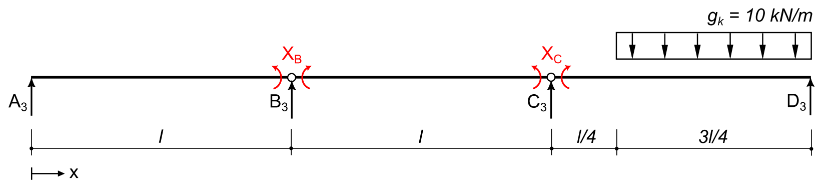
$$\rightarrow M_B = -42.725\,\text{kNm},\quad M_C = -31.25\,\text{kNm}$$

$$M_2(x) = \begin{cases} X_B\cdot\dfrac{x}{L} & 0\le x\le L \\ X_B\cdot\left(1-\dfrac{(x-L)}{L}\right)+B_{20}(x-L) & L< x\le \dfrac{5L}{4} \\ X_B\cdot\left(1-\dfrac{(x-L)}{L}\right)+B_{20}(x-L)-\dfrac{g_k}{2}\left(x-\dfrac{5L}{4}\right)^2 & \dfrac{5L}{4}< x\le 2L \\ -\dfrac{g_kL^2}{32}+\dfrac{g_kL}{4}(x-2L)-\dfrac{g_k}{2}(x-2L)^2 & 2L< x\le \dfrac{9L}{4} \end{cases}$$



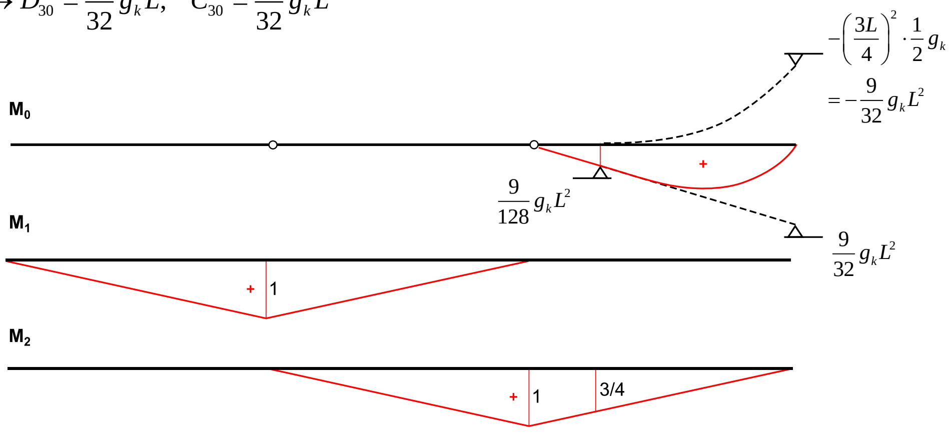
Stage 3:

Force method BS+RV



$$\sum F_V = 0: C_{30}+D_{30} = g_k\cdot\frac{3L}{4},\qquad \sum M_C = 0: D_{30}\cdot L = \frac{3L}{4}\cdot g_k\cdot\left(\frac{L}{4}+\frac{3L}{8}\right)$$

$$\rightarrow D_{30} = \frac{15}{32}g_k L,\quad C_{30} = \frac{9}{32}g_k L$$



Dashed: moment curve split for integration with force method

$$\delta_{10} = 0$$

$$\delta_{20} = \frac{1}{6}\cdot\frac{L}{EI}\cdot 1\cdot\frac{9L^2}{32}g_k - \frac{1}{12}\cdot\frac{3L}{4EI}\cdot\frac{9L^2}{32}g_k\cdot\frac{3}{4} = \frac{69}{2048}\frac{g_k L^3}{EI}$$

$$\delta_{11} = \delta_{22} = \frac{2}{3}\cdot\frac{L}{EI}$$

$$\delta_{12} = \delta_{12} = \frac{1}{6}\cdot\frac{L}{EI}$$

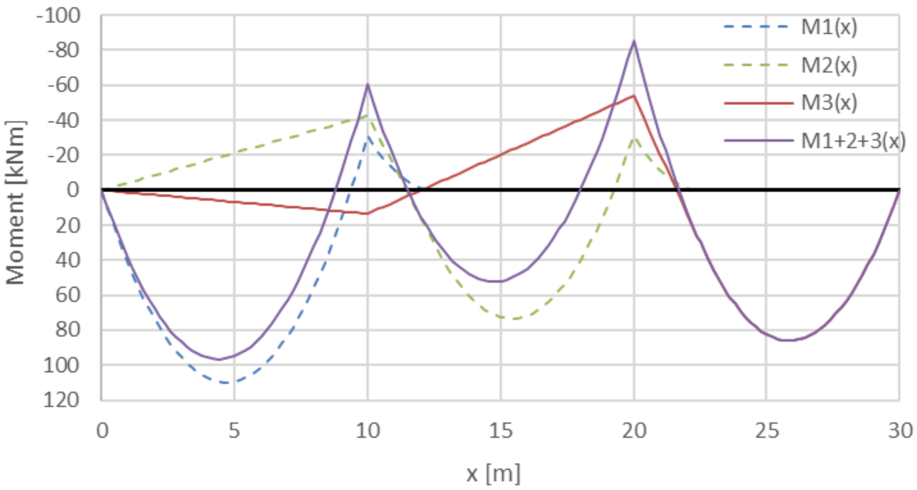
$$\rightarrow \begin{bmatrix}\delta_{10}\\ \delta_{20}\end{bmatrix} + \begin{bmatrix}\delta_{11} & \delta_{12}\\ \delta_{21} & \delta_{22}\end{bmatrix}\cdot\begin{bmatrix}X_B\\ X_C\end{bmatrix} = \underline{0}$$

$$\rightarrow \begin{bmatrix}0\\ \frac{69}{2048}\end{bmatrix}\cdot\frac{g_k L^3}{EI} + \begin{bmatrix}\frac{2}{3} & \frac{1}{6}\\ \frac{1}{6} & \frac{2}{3}\end{bmatrix}\cdot\frac{L}{EI}\cdot\begin{bmatrix}X_B\\ X_C\end{bmatrix} = \underline{0}$$

$$\rightarrow X_B = 0.013477\cdot g_k L^2, \quad X_c = -\frac{207}{3840}g_k L^2 = -0.053906\cdot g_k L^2$$

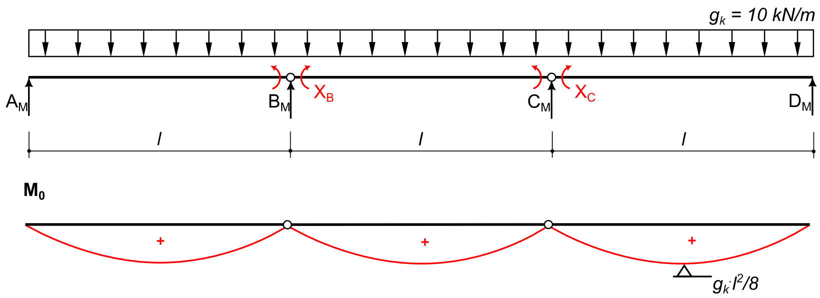
$$\rightarrow M_B = 13.477 \text{ kNm}, M_C = -53.906 \text{ kNm}$$

$$M_3(x) = \begin{cases} X_B\cdot\frac{x}{L} & 0 \le x \le L \\ X_B\cdot\left(2-\frac{x}{L}\right) + X_C\frac{(x-L)}{L} & L < x \le 2L \\ X_C\cdot\left(3-\frac{x}{L}\right) + C_{30}(x-2L) & 2L < x \le \frac{9L}{4} \\ X_C\cdot\left(3-\frac{x}{L}\right) + C_{30}(x-2L) - g_k\frac{\left(x-\frac{9L}{4}\right)^2}{2} & \frac{9L}{4} < x \le 3L \end{cases}$$



(ii) Monolithic structure:

Force method BS+RV



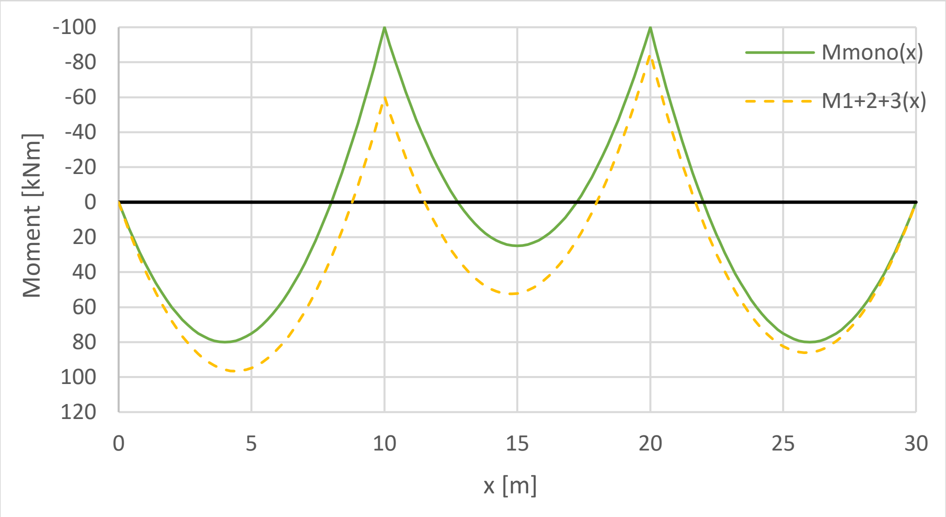
M_1 and M_2 analogues to Stage 3. From symmetry: $X_B = X_C$.

$$\delta_{10} = \delta_{20} = \frac{1}{3} \cdot \frac{L}{EI} \cdot \frac{g_k L^2}{8} \cdot 1 \cdot 2 = \frac{1}{12} \cdot \frac{g_k L^3}{EI}$$

$$\delta_{11} = \delta_{22} = \frac{2}{3} \cdot \frac{L}{EI}, \quad \delta_{12} = \delta_{21} = \frac{1}{6} \cdot \frac{L}{EI} \quad \text{(from stage 3)}$$

$$\rightarrow \delta_1 = \delta_{10} + \delta_{11} X_B + \delta_{12} X_B \rightarrow X_B = -\frac{g_k L^2}{10} = X_C$$

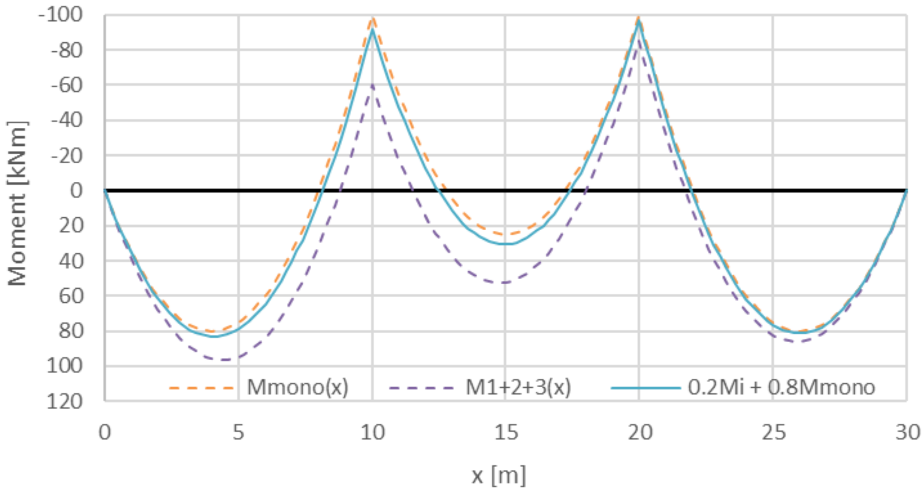
$$M_{Mono}(x) = \begin{cases} \dfrac{g_k L}{2} \cdot x - g_k \dfrac{x^2}{2} - \dfrac{g_k L^2}{10} \dfrac{x}{L} & 0 \le x \le L \\ \dfrac{g_k L}{2} \cdot (x-L) - g_k \dfrac{(x-L)^2}{2} - \dfrac{g_k L^2}{10} & L < x \le 2L \\ \dfrac{g_k L}{2} \cdot (x-2L) - g_k \dfrac{(x-2L)^2}{2} - \dfrac{g_k L^2}{10}\left(1 - \dfrac{x-2L}{L}\right) & 2L < x \le 3L \end{cases}$$



c) Approximation of the moment distribution for t→ ∞

Superposition of 20% of the bending moments from the construction stages and 80% of the bending moments from the monolithically constructed structure:

$$M_{t\to\infty}(x) = 0.2\cdot(M_1(x)+M_2(x)+M_3(x))+0.8\cdot M_{Mono}(x)$$



d) Moment distribution with the Trost Method

$$M_t(x) = M_0(x) + \left(M_{Mono}(x) - M_0(x)\right)\cdot\frac{\varphi(t,t_0)}{1+\mu\cdot\varphi(t,t_0)}$$

$$M_t(x) = M_0(x)\cdot\left(1-\frac{\varphi(t,t_0)}{1+\mu\cdot\varphi(t,t_0)}\right) + M_{Mono}(x)\cdot\frac{\varphi(t,t_0)}{1+\mu\cdot\varphi(t,t_0)}$$

M_0 : Moment before system change

M_{Mono} : Moment of monolithic system

Generalized:

$$M_t(x) = \sum_{i=1}^{n}\left[M_{0,i}(x)\cdot\left(1-\frac{\varphi(t_i,t_0)}{1+\mu\cdot\varphi(t_i,t_0)}\right)\right] + M_{Mono}(x)\cdot\frac{\varphi(t,t_0)}{1+\mu\cdot\varphi(t,t_0)}$$

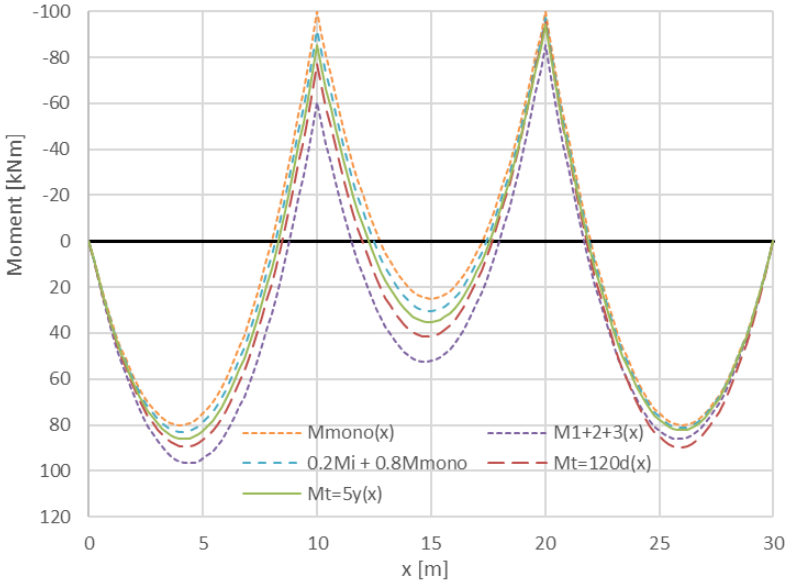
$\mu = 0.8$

$$M_{120\,\mathrm{d}}(x) = M_1(x)\cdot\left(1-\frac{\varphi(120\,\mathrm{d},30\,\mathrm{d})}{1+\mu\cdot\varphi(120\,\mathrm{d},30\,\mathrm{d})}\right) + M_2(x)\cdot\left(1-\frac{\varphi(90\,\mathrm{d},30\,\mathrm{d})}{1+\mu\cdot\varphi(90\,\mathrm{d},30\,\mathrm{d})}\right)$$
$$+M_3(x)\cdot\left(1-\frac{\varphi(60\,\mathrm{d},30\,\mathrm{d})}{1+\mu\cdot\varphi(60\,\mathrm{d},30\,\mathrm{d})}\right) + M_{Mono}(x)\cdot\frac{\varphi(120\,\mathrm{d},30\,\mathrm{d})}{1+\mu\cdot\varphi(120\,\mathrm{d},30\,\mathrm{d})}$$

$$M_{120\,\mathrm{d}}(x) = 0.588M_1(x)+0.615M_2(x)+0.659M_3(x)+0.412M_{Mono}(x)$$

$$M_{5\,\mathrm{y}}(x) = \left(M_1(x)+M_2(x)+M_3(x)\right)\cdot\left(1-\frac{\varphi(5\,\mathrm{y},30\,\mathrm{d})}{1+\mu\cdot\varphi(5\,\mathrm{y},30\,\mathrm{d})}\right) + M_{Mono}(x)\cdot\frac{\varphi(5\,\mathrm{y},30\,\mathrm{d})}{1+\mu\cdot\varphi(5\,\mathrm{y},30\,\mathrm{d})}$$

$$M_{5\,\mathrm{y}}(x) = 0.377\cdot\left(M_1(x)+M_2(x)+M_3(x)\right)+0.623M_{Mono}(x)$$



<u>Remark:</u>
As the age of the structure increases, the distribution of moments increasingly approaches that of the monolithic structure due to creep. The approximation "80% monolithic structure, 20% sum of the individual construction stages" approximates the long-term behaviour relatively well. Since the determination of the creep coefficient is also subject to some uncertainty and the bending stiffness over the length of the beam is by no means constant (crack formation), the calculation using the Trost method can only be regarded as an approximation of the real stress state.