Dimensioning of a diaphragm

Task 1

Geometry

SIA 262

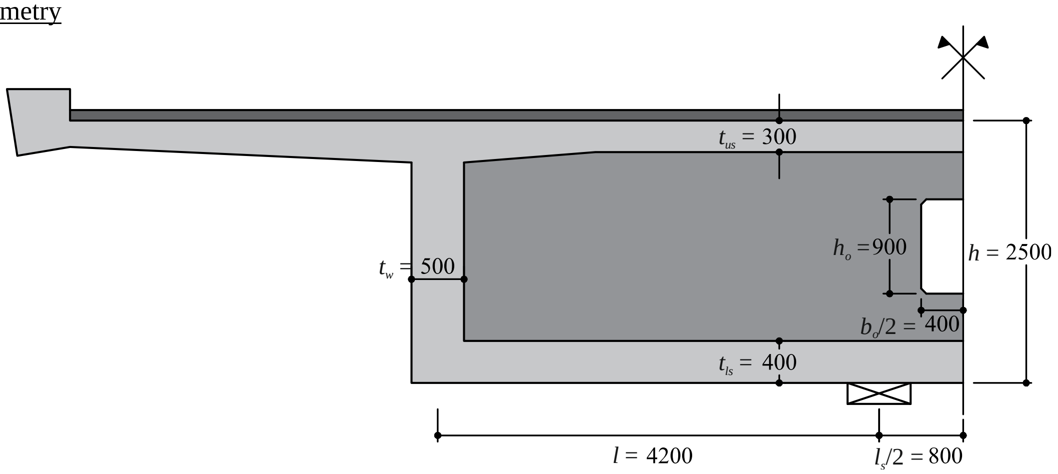


Figure 1: Cross-section of the bridge girder

A diaphragm thickness of t_d = 1 m is chosen.

Terminology

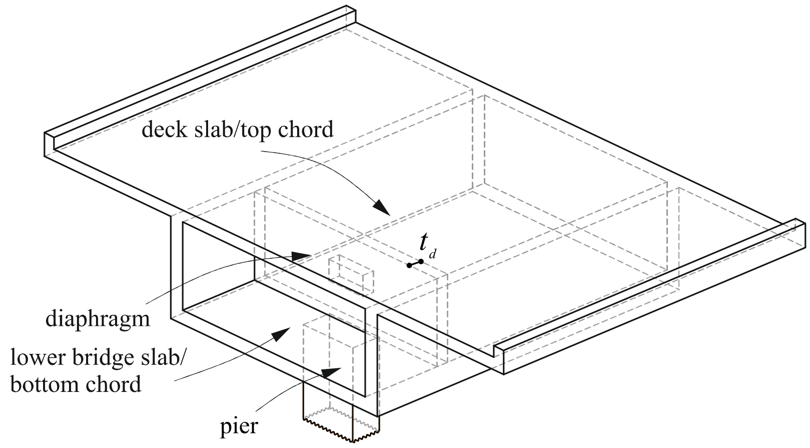


Figure 2: Overview and terminology of bridge girder

Tab. 3

Tab. 8

3.1.2.3.3

Material Properties

Concrete C40/50 $f_{ck} = 40\,\text{MPa};\ f_{ctm} = 3.5\,\text{MPa}$ — Tab. 5/9

$f_{cd} = 24\,\text{MPa};\ \tau_{cd} = 1.25\,\text{MPa}$ — 3.2.2.4

$E_{cm} = k_E \sqrt[3]{f_{cm}} \approx 36.3\text{ GPa};\ D_{\max} = 32\text{ mm}$

Tab. 7/10

Steel B500B $f_{sk} = 500\,\text{MPa};\ f_{sd} = 435\,\text{MPa}$ — 3.3.2.4.2

$E_s = 205\text{ GPa}$

Prestressing Steel Y1860 $f_{pk} = 1860\,\text{MPa};\ f_{pd} = 1390\,\text{MPa}$ — According to task description

$E_p = 195\text{ GPa}$

Loads

The load $F_d = V_{L1} + V_{L2} = V_{R1} + V_{R2} = 6\text{ MN}$ is acting at both ends of the diaphragm.

The deadweight is neglected.

a) Suspension of the entire load on the diaphragm

The loads from the deck slab ($V_{L1} + V_{L2}$) are carried through the web towards the diaphragm (see a) in the bottom figure) to the pier. The load is then introduced through a fan mechanism (b)) into the diaphragm and thus needs to be suspended first (c)).

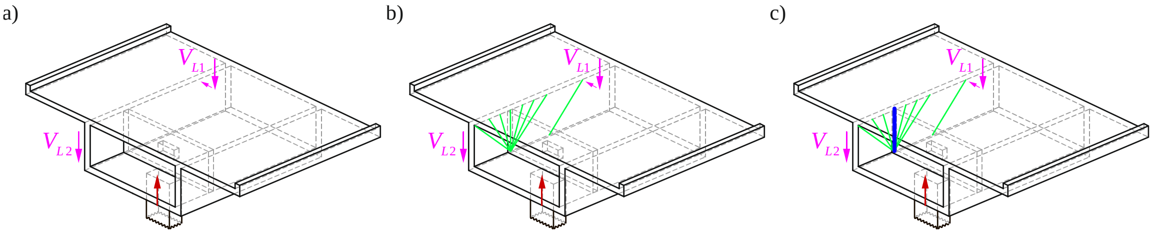


Figure 3: Force flow and fan mechanism for the load introduction into the diaphragm

- Dimensioning of the suspension reinforcement:

$$A_{s,erf} = \frac{F_d}{f_{sd}} = \frac{6\,\text{MN}}{435\,\text{MPa}} = 13793\ \text{mm}^2$$

$\rightarrow$ Choice: 27 x $\varnothing$26 $\quad A_s = 14337\ \text{mm}^2$

$$F_{Rd} = A_s \cdot f_{sd} = 6.2\,\text{MN} > F_d = 6.0\,\text{MN} \quad \text{ok}$$

c_{nom}= 45 mm

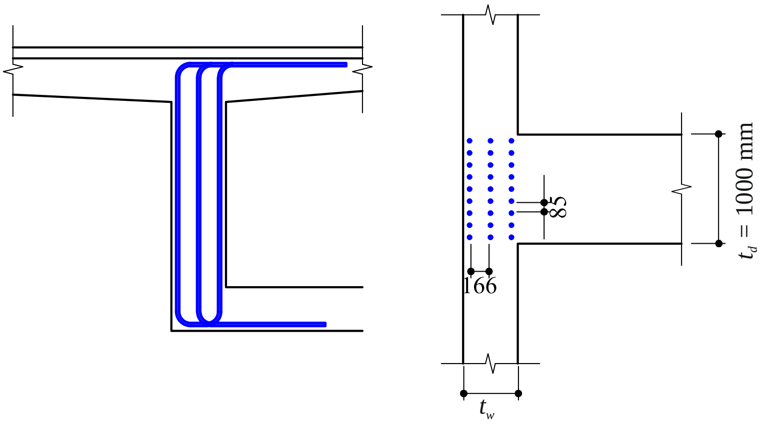


Figure 4: Layout of the suspension reinforcement (concentrated reinforcement)

Distance between rebars

$$a = \frac{(t_w - 3\varnothing - 2c_{nom})}{2} = 166\,\text{mm} > D_{max} \quad \text{ok}$$

$$b = \frac{(t_d - 9\varnothing - 2c_{nom})}{8} = 85\,\text{mm} > D_{max} \quad \text{ok}$$

Due to the large bending radii and anchorage length of Ø26 bars as well as spatial constraints, the possibility of using T-headed bars should be checked (in the sketch, the bending radii are not to scale).

- Dimensioning of the diaphragm with a strut-and-tie model

The diaphragm needs to carry the force from the suspension towards the pier. The diaphragm thus acts as a cantilever. The top reinforcement is distributed transversely in the bridge deck (upper slab). For the concrete compressive zone, only the width of the diaphragm itself (membrane element) is considered. In the lower part of the diaphragm, a bi-axial stress state with compressive strength f_{cd} results due to combined longitudinal and transversal bending.

Note:
It is also possible to take part of the bottom slab of the box girder into account when dimensioning the diaphragm. This would, however, require additional reinforcement to spread the compressive force as well as a check of the interaction of the stress field resulting from the load distribution in the longitudinal direction. This is not done in this exercise.

Assessment of the reinforcement in the upper chord:

$$A_{s,erf} \approx \frac{F_d \cdot l}{0.9 \cdot d \cdot f_{sd}} = \frac{6\,\text{MN} \cdot 4.2\,\text{m}}{0.9 \cdot 2.35\,\text{m} \cdot 435\,\text{MPa}} = 27340\ \text{mm}^2 \qquad (d = h - 0.5 \cdot t_{us} = 2.35\,\text{m})$$

$\rightarrow$ Choice: 52 x ∅26@100, in two layers

$$A_s = 52 \cdot \frac{26^2 \pi}{4} = 27608\ \text{mm}^2$$

Concrete compression zone:

$$c = \frac{A_s \cdot f_{sd}}{t_d \cdot f_{cd}} = \frac{27608 \cdot 435}{1000 \cdot 24} = 500\,\text{mm}$$

Inner lever arm:

$$z = d - \frac{c}{2} = 2350 - \frac{500}{2} = 2.1\text{m}$$

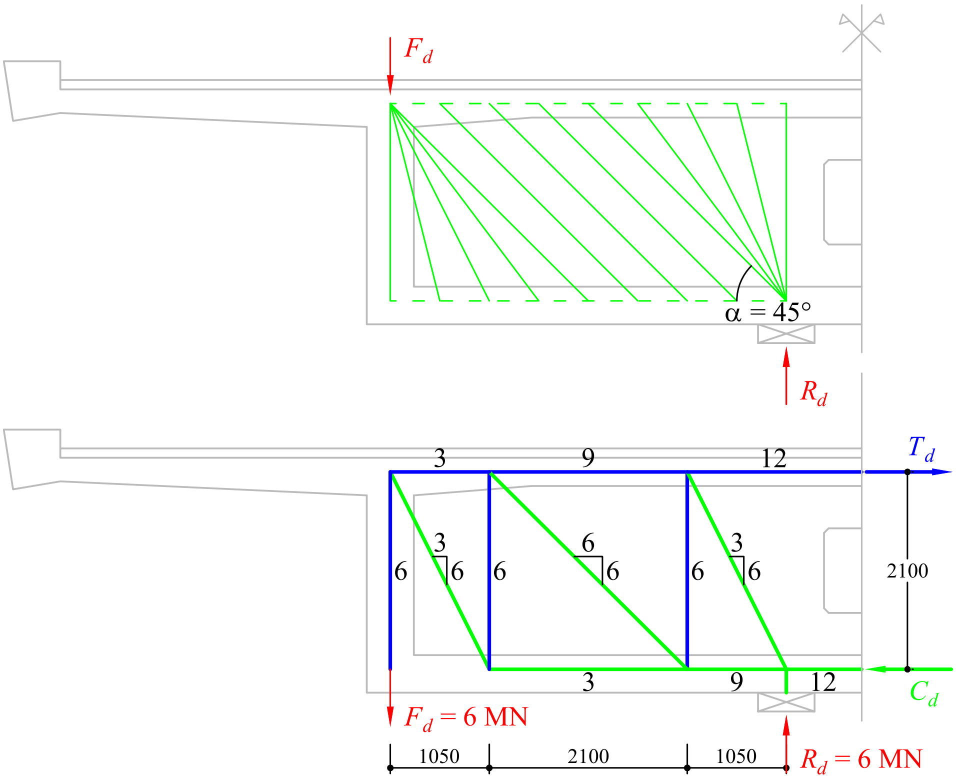


Figure 5: Stress field and strut-and-tie model for the case of the suspension of the entire load

− Reinforcement in the tension chord

The suspension of the load leads to a concentrated load introduction in the upper chord. The horizontal component of the outermost diagonal strut (fan) must be resisted by the reinforcement directly above the diaphragm (web). The shear connection of the webs to the upper flange is ensured by transverse reinforcement that spreads the forces introduced to the upper chord by the inner struts (parallel band and support fan). This distribution reinforcement ensures the activation of the distributed chord reinforcement over the width (in direction of bridge axis) of 2.5 m. The distributed reinforcement needs to be superimposed with the main bending reinforcement in the global longitudinal direction.

- Check assumptions for tensile reinforcement: See page 2

$T_{d,sup} = 12\,\text{MN},\ A_s = 27608\,\text{mm}^2$ from pre-design

$T_{Rd} = 27608 \cdot 435 = 12\,\text{MN} \geq T_{d,sup} = 12\,\text{MN}$ ok

See also Figure 7

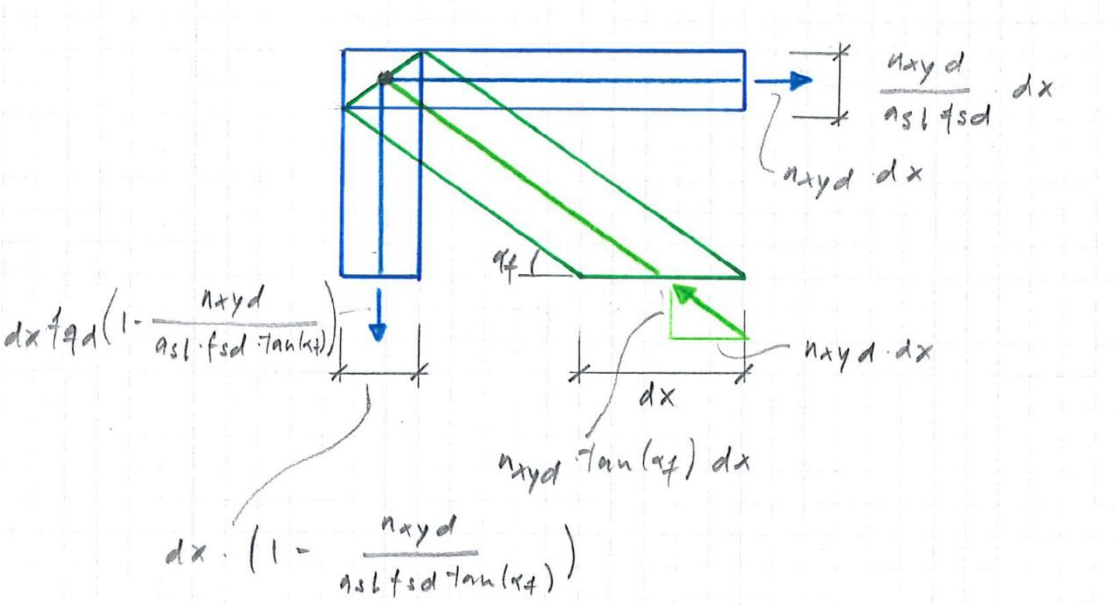


Figure 6: Detail of the spreading of the force in the upper bridge slab.

- Design transverse reinforcement in the bridge deck (upper slab) with shear flow n_{xyd}: See Figure 6

$$n_{xyd} = 0.5\frac{dF_{sup}}{dx} \quad \text{maximum}: n_{xyd} = 0.5\frac{9000\,\text{kN} - 3000\,\text{kN}}{2.1\,\text{m}} = 1429\frac{\text{kN}}{\text{m}}$$

$$n_{xyd} \cdot \tan(\alpha_f) = 1429 \cdot \tan(35°) = 1000\frac{\text{kN}}{\text{m}}$$

$$f_{qd} = n_{xyd} \cdot \tan(\alpha_f) \cdot \left(1 - \frac{n_{xyd}}{a_{sl} \cdot f_{sd} \cdot \tan(\alpha_f)}\right)^{-1}$$ (distributed transverse force in the upper bridge slab)

$$= 1740\frac{\text{kN}}{\text{m}} \qquad \left(a_{sl} = \frac{27608\,\text{mm}^2}{2.5\,\text{m}} = 11043\frac{\text{mm}^2}{\text{m}}\right)$$

$$\rightarrow \varnothing 20@150,\ \text{in 2 layers},\ a_{sq} = 4189\frac{\text{mm}^2}{\text{m}}$$

$$f_{Rd} = 1822\frac{\text{kN}}{\text{m}} > f_{qd} = 1740\frac{\text{kN}}{\text{m}} \quad \text{ok}$$

− Stirrup reinforcement in diaphragm SIA 262 4.3.3.4.3

$$a_{sw,erf} = \frac{V_{Ed}}{z \cdot f_{sd} \cdot \cot(\alpha)} = \frac{6\,\text{MN}}{2100 \cdot 435 \cdot \cot(45°)} = 6568\frac{\text{mm}^2}{\text{m}}$$

$\rightarrow$ Choice $\varnothing 18@150$ 4-legged stirrups

$$a_{sw} = 6785\frac{\text{mm}^2}{\text{m}}$$

- Check of compression diagonal

$$\sigma_c = \frac{V_{Ed}}{t_w \cdot z \cdot \sin(\alpha)\cos(\alpha)} = \frac{6\,\text{MN}}{1000 \cdot 2100 \cdot \sin(45°) \cdot \cos(45°)}$$

$$= 5.7\,\text{MPa} < k_c \cdot f_{cd} = 13.2\,\text{MPa} \qquad \text{ok}$$

SIA 262 4.3.3.4.6
$k_c = 0.55$

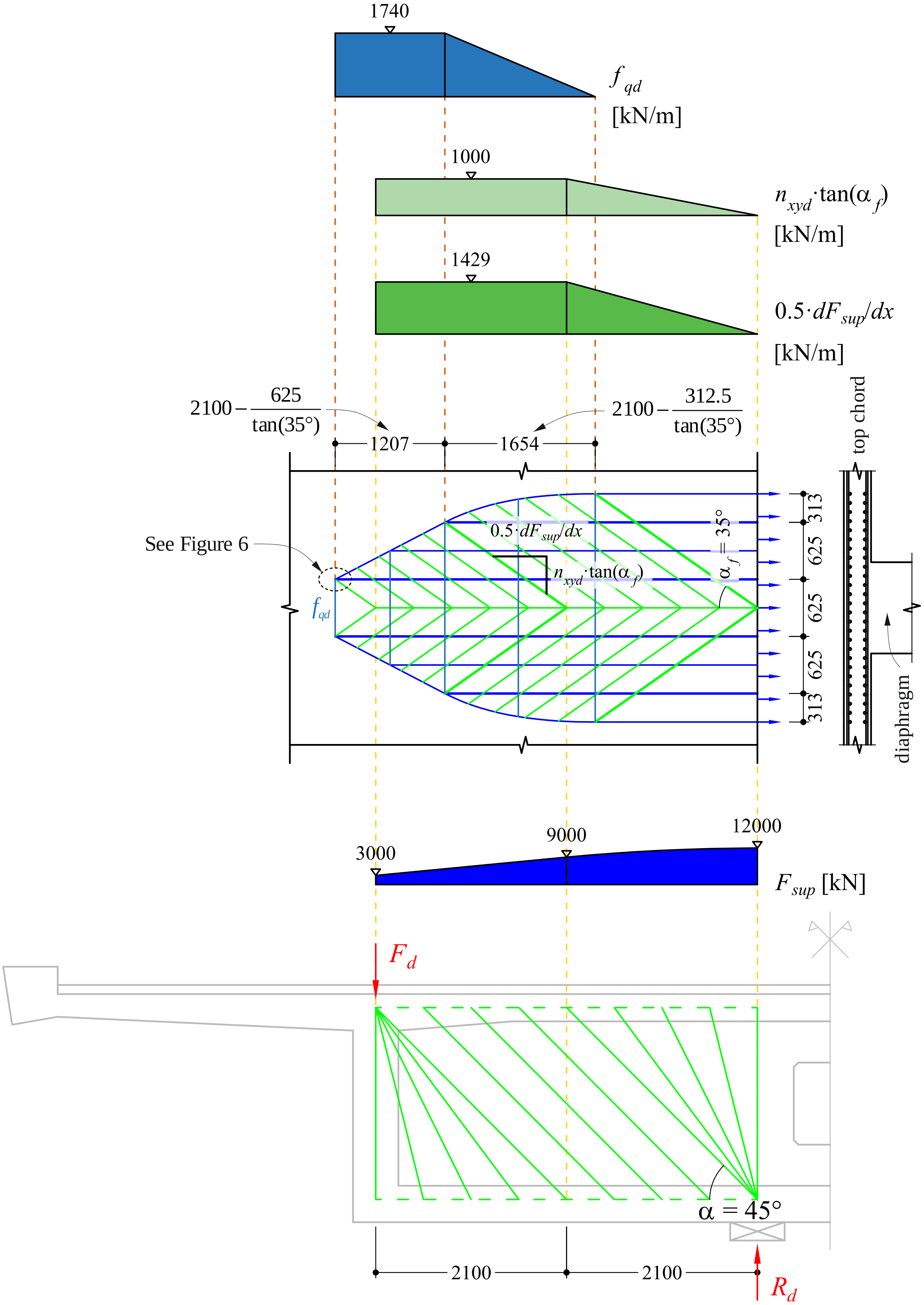


Figure 7: Spreading of the force in the upper bridge slab

b) Load transfer without suspension reinforcement

- Global stress field

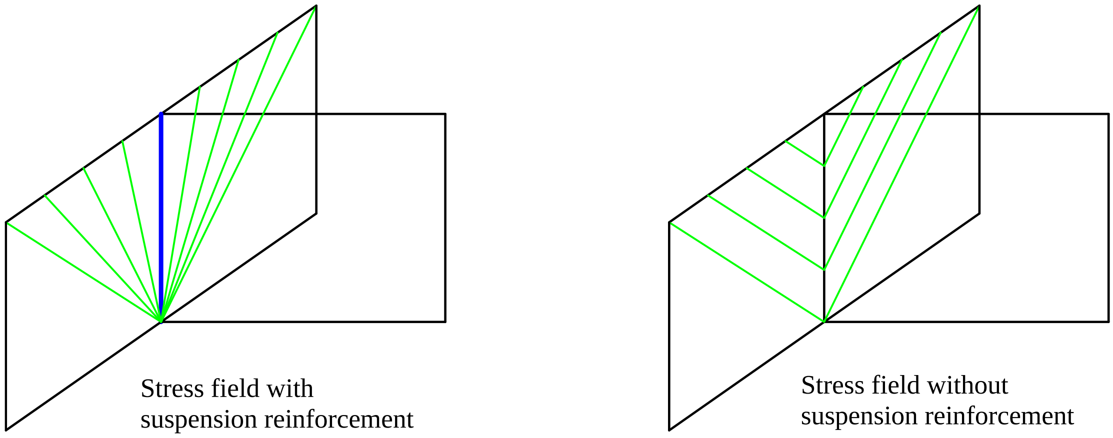


Figure 8: Global stress field (web and diaphragm)

- Dimensioning of the diaphragm with a strut-and-tie model

The upper reinforcement is distributed in the upper bridge slab and the compression zone is limited to the width of the diaphragm just as in task 1a). The load is introduced over the height of the diaphragm. This requires horizontal reinforcement in the diaphragm, which resists these forces.

It should be noted that the strut-and-tie model only works because the dead load of the diaphragm is neglected. The additional load must be guided around the opening, which is shown in Figure 9.
As an alternative to the load deviation, the shear force could be guided directly to the support (dashed line). In that case, the stress fields of the fans and the parallel fields would overlap.

– Reinforcement in the tension chord

$T_{d,sup} = 12\,\text{MN}$

$\rightarrow$ Choice: 52 x ∅26@100, in 2 layers

$$A_s = 52 \cdot \frac{26^2 \pi}{4} = 27608\,\text{mm}^2$$

$T_{Rd} = 27608 \cdot 435 = 12\,\text{MN} \geq T_d = 12\,\text{MN}$ ok

The shear connection is ensured with distribution reinforcement just as in task 1a). The same required maximum reinforcement content results, grading of the reinforcement content is possible according to the flow of forces in Figure 8.

– Distributed transverse reinforcement in the tension chord

$\rightarrow$ ∅20@150, in 2 layers, $a_{sq} = 4189\,\frac{\text{mm}^2}{\text{m}}$

$f_{Rd} = 1822\,\frac{\text{kN}}{\text{m}} > f_{qd} = 1740\,\frac{\text{kN}}{\text{m}}$ ok

– Distributed longitudinal reinforcement in the diaphragm

$$a_{s,erf} = \frac{T_d}{z \cdot f_{sd}} = \frac{6}{2.1 \cdot 435} = 6568\,\frac{\text{mm}^2}{\text{m}}$$

$\rightarrow$ Choice: ∅18@150, in 4 layers $a_s = 4 \cdot \frac{18^2 \cdot \pi}{4 \cdot 0.15} = 6768\,\frac{\text{mm}^2}{\text{m}}$

$T_{Rd} = 6786 \cdot 435 \cdot 2.1 = 6.2\,\text{MN} > T_d = 6\,\text{MN}$ ok

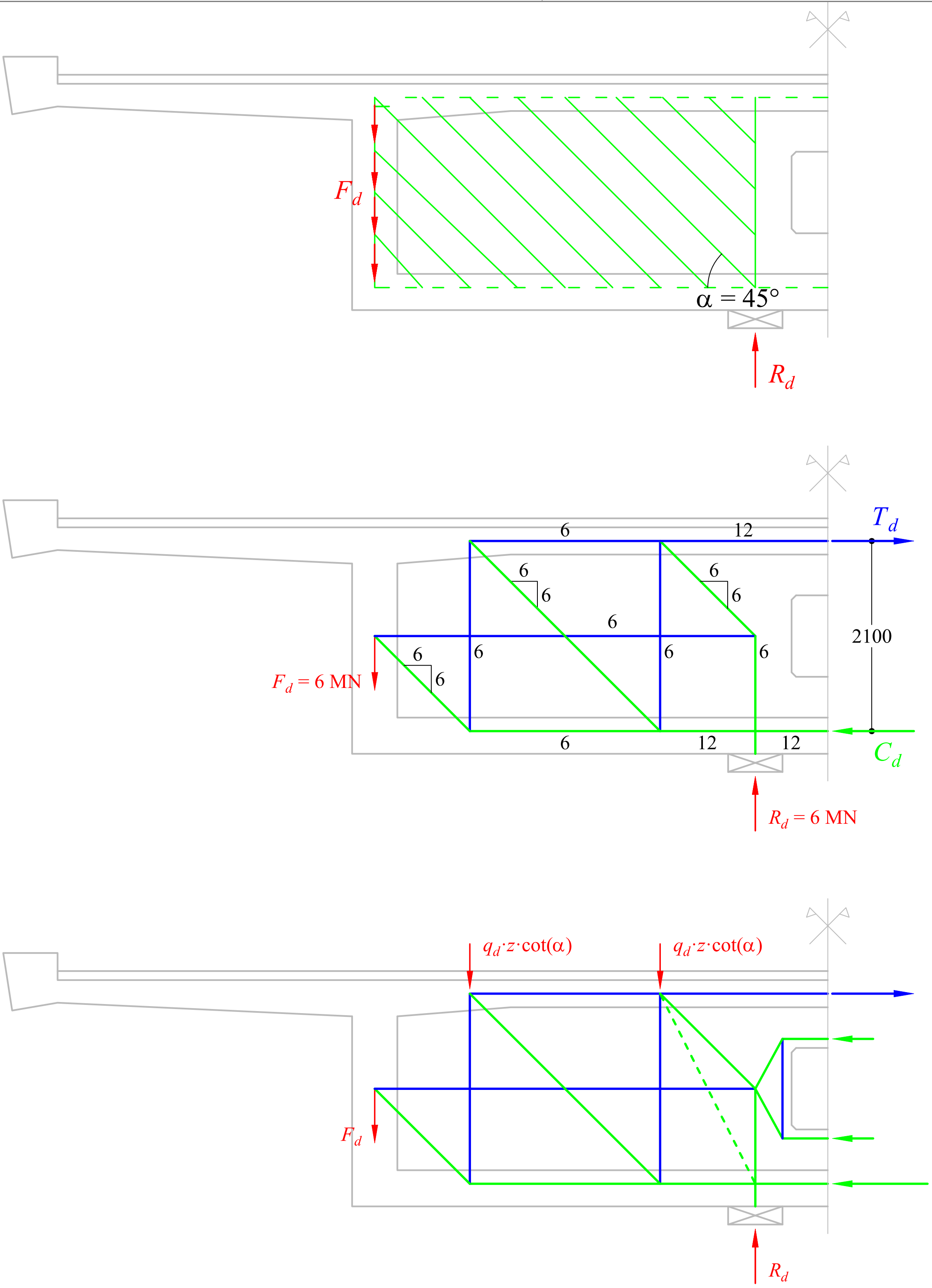


Figure 9: Stress field and strut-and-tie model for the case without suspension reinforcement

– Check of compression diagonal (parallel field)

$$\sigma_c = \frac{V_d}{t_d \cdot z \cdot \sin(\alpha)\cos(\alpha)} = \frac{6}{1000 \cdot 2.1 \cdot \sin(45°)\cos(45°)} = 5.7\,\text{MPa}$$

$$\sigma_c = 5.7\,\text{MPa} < k_c \cdot f_{cd} = 13.2\,\text{MPa} \qquad \text{ok}$$

– Stirrup reinforcement same as is task 1a)

→ ∅18@150, 4-legged

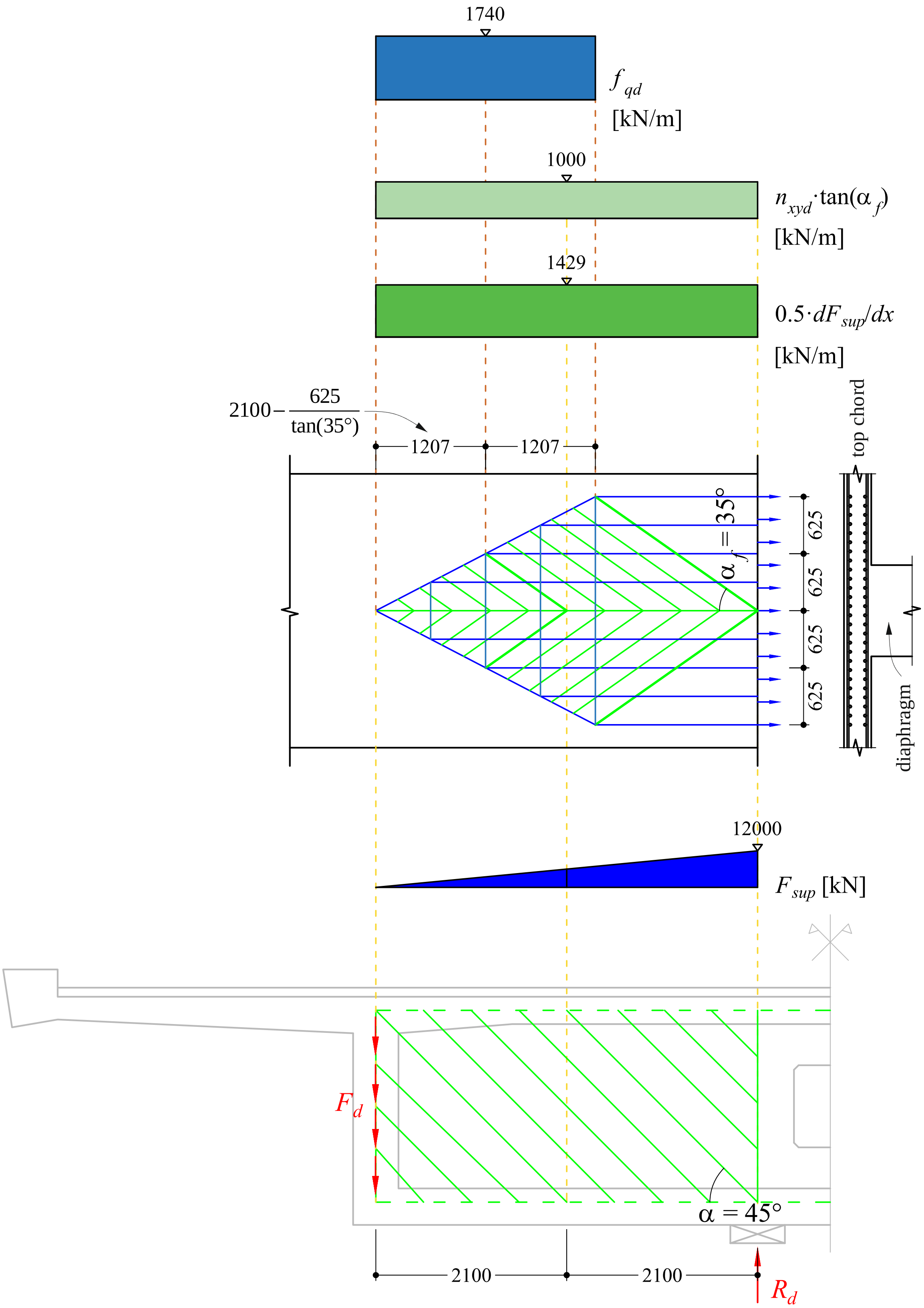


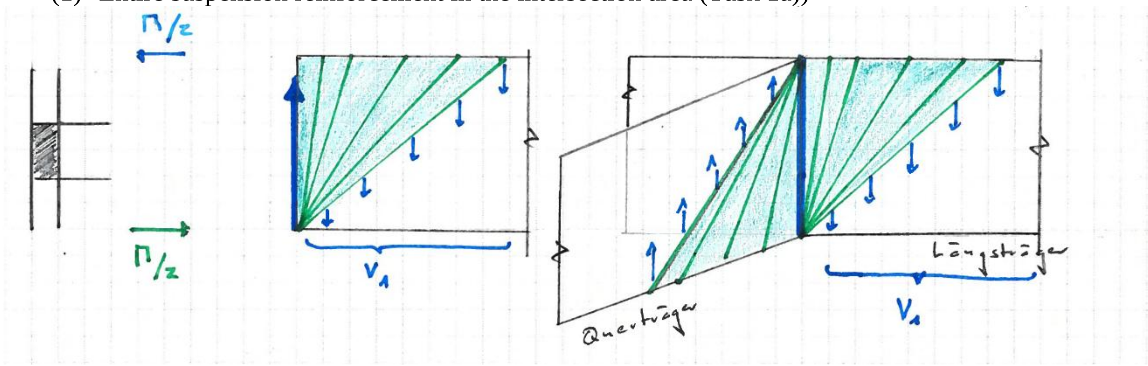
Figure 10: Spreading of the force in the bridge deck (upper slab)

Task 2

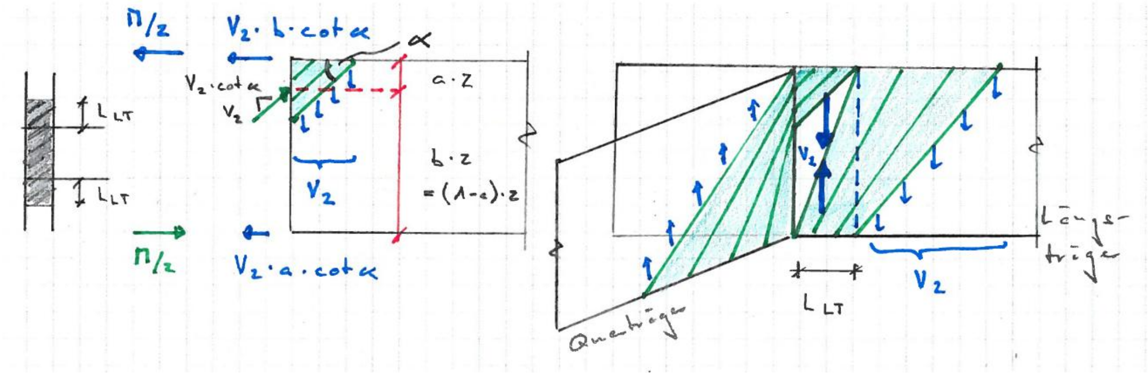
Längsträger = web

Querträger = diaphragm

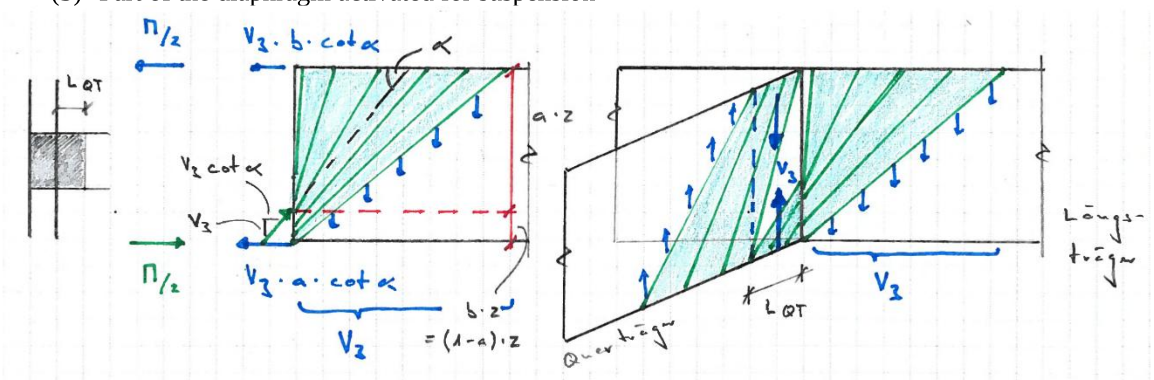
(1) Entire suspension reinforcement in the intersection area (Task 1a))



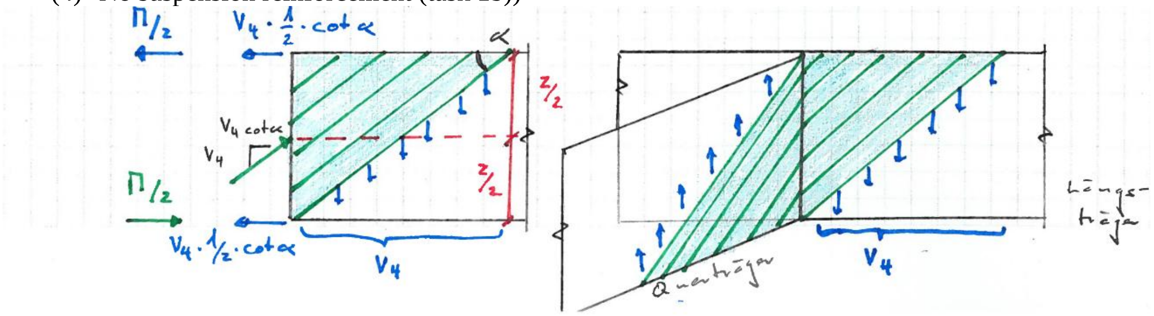
(2) Part of the web activated for suspension



(3) Part of the diaphragm activated for suspension



(4) No suspension reinforcement (task 1b))



Task 3

- General
  - Load is directly suspended in the intersection area
  - It turns out that the tension chord reinforcement as well as the width of the compression zone can only be kept within the width of the diaphragm
  - Treatment of prestressing as anchorage and deviation forces

- Prestressing forces

$$P_{0,1} = 27 \cdot A_p \cdot 0.7 f_{pk} = 5273\,\text{kN} \qquad (A_p = 150\,\text{mm}^2)$$

$$P_{\infty,1} = 0.85 P_{0,1} = 4482\,\text{kN}$$

$$\beta_1 = \tan^{-1}\left(2\frac{f_1}{L/2}\right) = 7.0° \qquad (L = 10\,\text{m},\ f_1 = 305\,\text{mm})$$

$$\rightarrow \begin{Bmatrix} P_{\infty,1,x} \\ P_{\infty,1,z} \end{Bmatrix} = \begin{Bmatrix} \cos\beta_1 \\ \sin\beta_1 \end{Bmatrix} \cdot P_{\infty,1} = \begin{Bmatrix} 4449 \\ 543 \end{Bmatrix} \text{kN}$$

Deviation forces (simplification: acting vertically):

$$u_{\infty,1} = \frac{8 \cdot P_{\infty,1,x} \cdot f}{l^2} = \frac{8 \cdot 4449 \cdot 0.305}{10^2} = 108.5\frac{\text{kN}}{\text{m}}$$

$$P_{0,2} = 15 \cdot A_p \cdot 0.7 f_{pk} = 2930\,\text{kN}$$

$$P_{\infty,2} = 0.85 P_{0,2} = 2491\,\text{kN}$$

$$\beta_2 = 15.2° \quad \text{with } f_2 = 680\,\text{mm} \rightarrow \begin{Bmatrix} P_{\infty,2,x} \\ P_{\infty,2,z} \end{Bmatrix} = \begin{Bmatrix} 2404 \\ 654 \end{Bmatrix} \text{kN}$$

Deviation forces: $u_{\infty,2} = 130.7\frac{\text{kN}}{\text{m}}$

The deviation forces of 108.5+130.7 = 239.2 kN/m are lumped to the truss nodes by multiplying with the respective lengths, i.e., 239.2 kN/m·2.10 m = 503 kN and 239.2 kN/m·0.80 m = 191 kN.

The nodal zones are dimensioned according to the loads resulting from the load introduction in the anchorage zone. The resulting stresses inside the fan can be determined from the geometry of the nodes. The stress field is idealised as point-centred fans at the supports.

The point-centred fan intersects with the opening of the bridge (Figure 11). Hence, additional horizontal reinforcement is needed above the opening due to the deviation forces. The loads in this area are generally small, so the additional reinforcement that would anyway be placed around the opening is usually sufficient to resist them (needs to be checked, however).

- Reinforcement in the tension chord

The required passive reinforcement is reduced due to the prestressing. Passive reinforcement is only placed directly above the web such that no distribution reinforcement is needed.

See Figure 11

$$T_d = 5770\,\text{kN (from STM model)} \qquad A_{s,erf} = \frac{5770}{435} = 13264\,\text{mm}^2$$

$\rightarrow$ Choice: $20\ \varnothing 30 @ 100$, in two layers $A_s = 20 \cdot 707 = 14140\,\text{mm}^2$

$T_{Rd} = 435 \cdot 14140 = 6151\,\text{kN} > T_d$ ok

- Check of the compression chord

Height of concrete compression zone: $c = \dfrac{12589}{24 \cdot 1000} = 524\,\text{mm}$

The resulting depth of the compression zone is slightly higher than assumed (500 mm). The effective depth of the compression zone would have to be determined iteratively (compression zone depth depends on inner lever arm and vice versa). Due to the small deviation, this step is not carried out here.

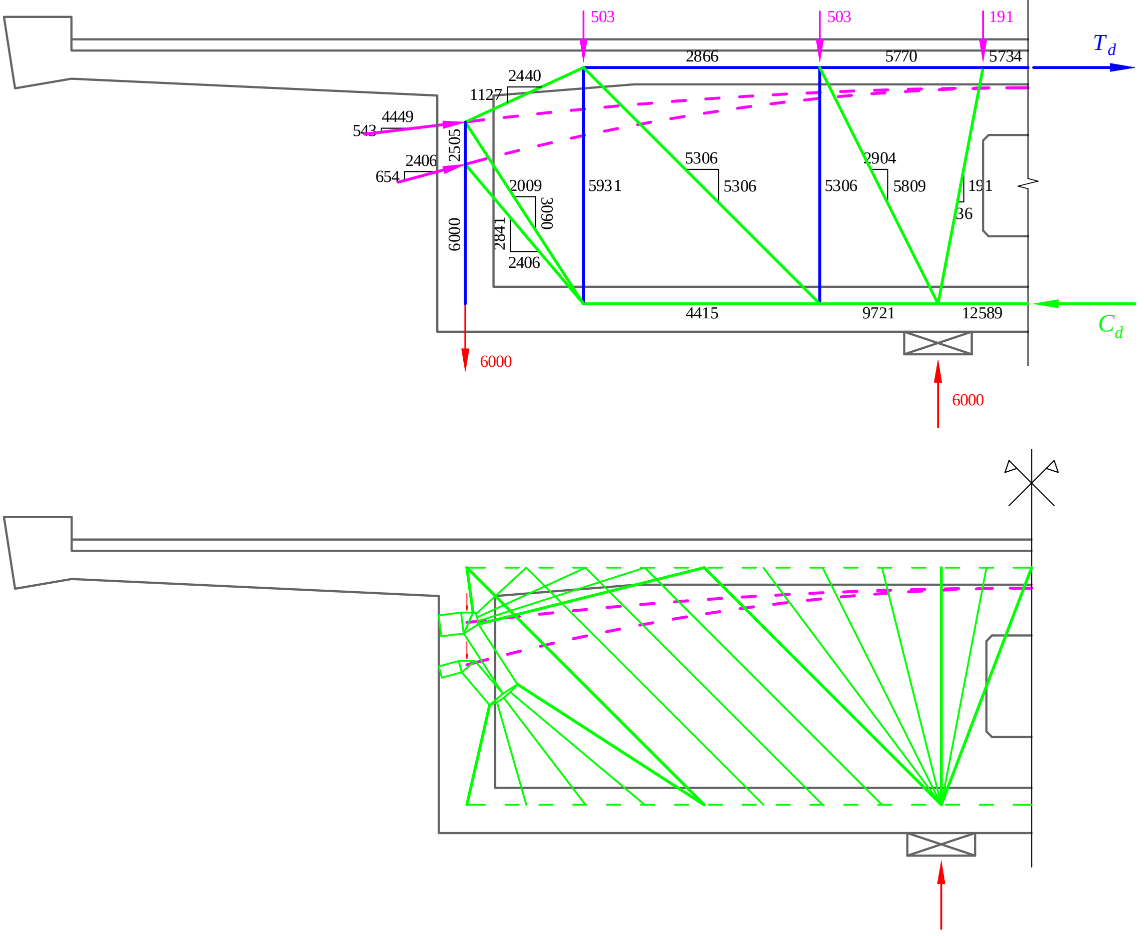


Figure 11: Stress field and strut-and-tie model for the case with prestressing

- Shear reinforcement in the diaphragm: same as in tasks 1a) and b)

  Choice: $\varnothing 18@150$, 4-legged $a_{sw} = 6786 \dfrac{\text{mm}^2}{\text{m}}$

  $V_{Rd} = 6.2\,\text{MN} > V_d = 5.9\,\text{MN}$ ok

- Concrete compression diagonals (parallel field)

$$\sigma_{cd} = \frac{V_d}{b_w \cdot z \cdot \sin(\alpha) \cdot \cos(\alpha)} = \frac{5306}{1000 \cdot 2100 \cdot \sin(45°) \cdot \cos(45°)} = 5.1\,\text{MPa}$$

$$\sigma_{cd} = 5.1\,\text{MPa} < k_c \cdot f_{cd} = 0.55 \cdot 24 = 13.2\,\text{MPa} \quad \text{ok}$$

- Superposition of a fan and a parallel field

A superposition of a fan and a parallel field is necessary in the area of the upper anchorage. The controlling point lies at the intersection of the flattest element of the fan with the parallel field, since the stresses in the fan originate from the nodal zone. The stress inside the fan decreases inversely proportional to the distance from the nodal zone.

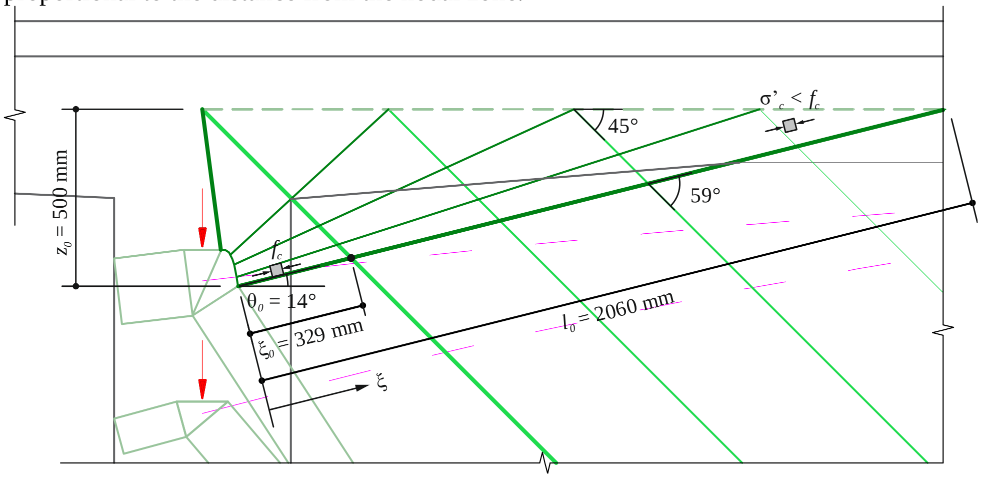


Figure 12: Superposition of a fan and a parallel field

On the upper fan boundary, the stress is:

$$\sigma_c' = \frac{V_d}{b_w \cdot z_0 \cdot \sin(\theta_0)\cos(\theta_0)} = \frac{1127}{1000 \cdot 500 \cdot \sin(14°)\cos(14°)} = 9.6\,\text{MPa}$$

Due to the widening of the fan, the stress decreases inversely proportional:

Approach: $\sigma_c(\xi) = \dfrac{f_c}{A + B \cdot \xi}$ with boundary conditions: $\sigma_c(\xi = 0) = f_c$ and $\sigma_c(\xi = l_0) = \sigma'_c$

$$\sigma_c(\xi) = f_c \left(1 + \frac{\frac{f_c}{\sigma'_c} - 1}{l_0} \cdot \xi\right)^{-1} \rightarrow \sigma_c(\xi_0 = 329\text{ mm}) = 24\left(1 + \frac{\frac{24}{9.6} - 1}{2060} \cdot 329\right)^{-1} = 19.4\,\text{MPa}$$

The two compression states intersect at an angle $\alpha_0 = 59°$. The superposition can be displayed with Mohr's Circle.

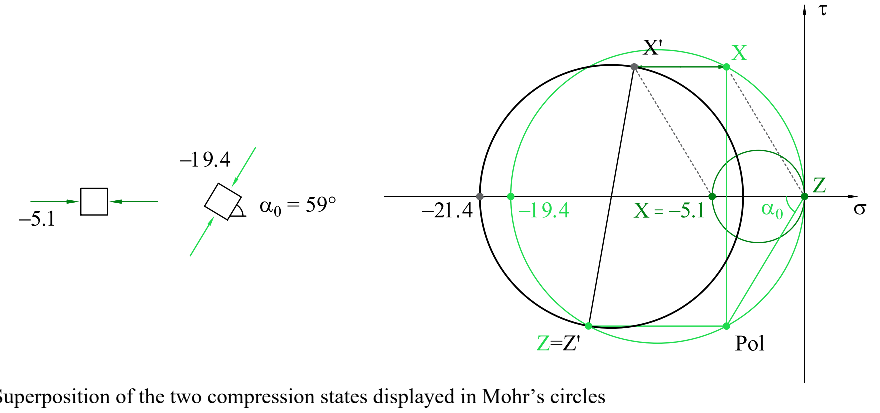
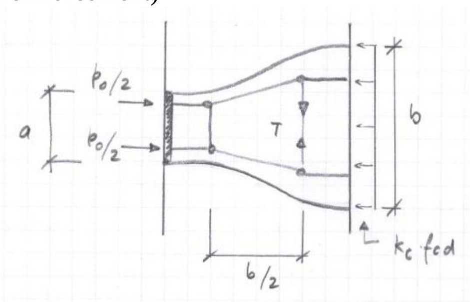


Figure 13: Superposition of the two compression states displayed in Mohr's circles

The resulting main stress resultant is $\sigma_{cd} = 21.4\,\text{MPa}$. The check of the concrete strength can therefore not be satisfied with a reduction factor of $k_c = 0.55$. The concrete strength can be increased with confinement reinforcement according to SIA 262 4.2.1.8. It is assumed that the spiral reinforcement of the anchorage is sufficient to increase the concrete strength.
The same generally applies to the areas of the boundary of the nodal zones of the fan since f_{cd} is assumed there.

- Reinforcement against splitting of the load introduction zone of the prestressing reinforcement (spiral reinforcement)



Assuming a squared zone:

$$P_0 = b^2 \cdot k_c \cdot f_{cd}$$

$$b_{erf} = \sqrt{\frac{P_0}{k_c \cdot f_{cd}}} = \{632\ ;\ 471\}\,\text{mm}$$

Assumption: $a = 350\,\text{mm}$ (anchor plate)

Transversal tensile force:

$$T_1 = \left(\frac{b-a}{4}\right) \cdot \left(\frac{b}{2}\right)^{-1} \cdot \frac{P_{0,1}}{2} = 588\,\text{kN}$$

$$T_2 = 188\,\text{kN}$$

Commonly, the spiral reinforcement is taken into consideration with $\sigma_s = 250\,\text{MPa}$ (deemed sufficiently low to ensure acceptable crack widths, i.e., avoid the calculation of crack widths).

$$A_{s,erf} = \frac{T_1}{250\,\text{MPa}} = 2352\,\text{mm}^2$$

→ Spiral with ∅18 every 100 mm (= s)

$$n = 2 \cdot \left\lfloor \frac{b}{s} \right\rfloor = 2 \cdot \left\lfloor \frac{632}{100} \right\rfloor = 2 \cdot \lfloor 6.32 \rfloor = 12 \text{ cut surfaces of the spiral}$$

$\lfloor\ \rfloor$: symbol for flooring

$$A_s = \frac{18^2 \cdot \pi}{4} \cdot 12 = 3054\,\text{mm}^2 \ > A_{s,erf} \qquad \text{ok}$$

- Reinforcement against spalling

To avoid the spalling of the concrete around the anchorage zone, the following reinforcement is suggested (practitioner’s rule – cf. VSL documentations):

$$A_{sp} = 0.03 \frac{P_0}{f_{sd}} \gamma_p = 436\,\text{mm}^2$$

→ 2 ∅ 18 in every direction $\left(\frac{18^2 \cdot \pi}{4} \cdot 2 = 509\,\text{mm}^2\right)$