Dimensioning of a diaphragm

SIA 262

Task 1

Geometry

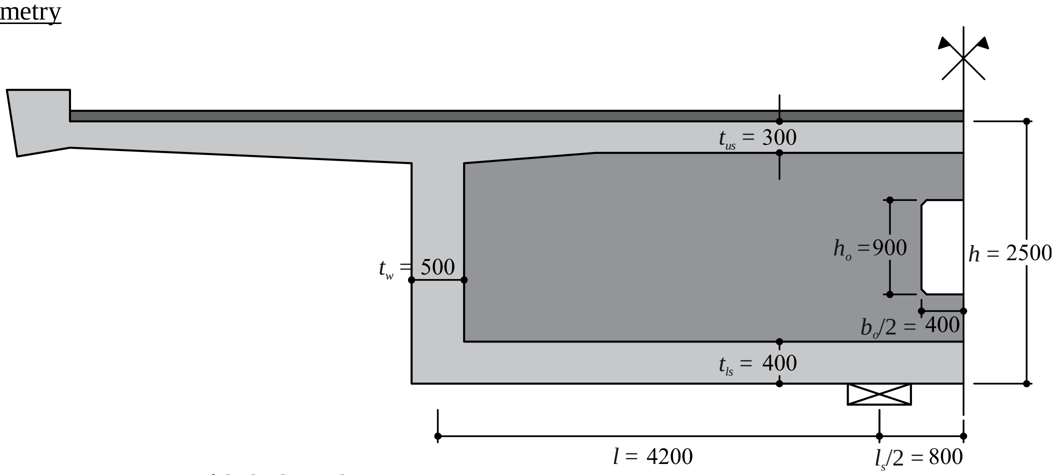


Figure 1: Cross-section of the bridge girder

Material Properties

Concrete C40/50 $f_{ck} = 40\,\text{MPa};\ \ f_{ctm} = 3.5\,\text{MPa}$ (Tab. 3)

$f_{cd} = 24\,\text{MPa};\ \ \tau_{cd} = 1.25\,\text{MPa}$ (Tab. 8)

$E_{cm} = k_E \sqrt[3]{f_{cm}} \approx 36.3\text{ GPa}$ (3.1.2.3.3)

Steel B500B $f_{sk} = 500\,\text{MPa};\ \ f_{sd} = 435\,\text{MPa}$ (Tab. 5/9)

$E_s = 205\text{ GPa}$ (3.2.2.4)

Prestressing Steel Y1860 $f_{pk} = 1860\,\text{MPa};\ \ f_{pd} = 1390\,\text{MPa}$ (Tab. 7/10)

$E_p = 195\text{ GPa}$ (3.3.2.4.2)

Loads

The load $F_d = V_{L1} + V_{L2} = V_{R1} + V_{R2} = 6$ MN is acting at both ends of the diaphragm. (According to task sheet)

The deadweight is neglected.

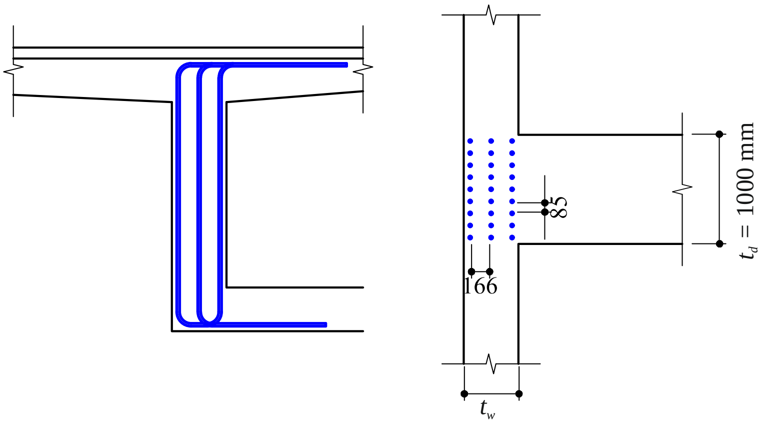
a) Suspension of the entire load on the diaphragm

- Dimensioning of the suspension reinforcement

$$A_{s,erf} = \frac{F_d}{f_{sd}} = \frac{6\,\text{MN}}{435\,\text{MPa}} = 13793\text{ mm}^2$$

$\rightarrow$ Choice: 27 x ∅26 $A_s = 14337\text{ mm}^2$

$F_{Rd} = A_s \cdot f_{sd} = 6.2\,\text{MN} > F_d = 6.0\,\text{MN}$ ok



Distance between rebars

$$a = \frac{(t_w - 3\varnothing - 2c_{nom})}{2} = 166\,\text{mm} > D_{max} \quad \text{ok}$$

$$b = \frac{(t_d - 9\varnothing - 2c_{nom})}{8} = 85\,\text{mm} > D_{max} \quad \text{ok}$$

c_{nom}= 45 mm

Figure 2: Layout of the suspension reinforcement (concentrated reinforcement)

Due to the large bending radii and anchorage length of Ø26 bars as well as spatial constraints, the possibility of using an anchorage plate should be checked (In the sketch, the bending radii are not to scale).

- Dimensioning of the diaphragm with a strut-and-tie model

  The diaphragm acts like a cantilever. The upper reinforcement is distributed transversely in the upper bridge slab. For the concrete compressive zone, only the width of the membrane element is considered. In the lower part of the diaphragm, a bi-axial stress state with the compressive strength f_{cd} results due to longitudinal and transversal bending.

  Note:
  It is also possible to take part of the lower bridge slab into account. This would, however, require additional reinforcement to spread the compressive force as well as a check of the interaction of the stress field resulting from the load distribution in the longitudinal direction. This is not done in this exercise.

  Assessment of the reinforcement in the upper chord:

$$A_{s,erf} \approx \frac{F_d \cdot l}{0.9 \cdot d \cdot f_{sd}} = \frac{6\,\text{MN} \cdot 4.2\,\text{m}}{0.9 \cdot 2.35\,\text{m} \cdot 435\,\text{MPa}} = 27340\ \text{mm}^2 \qquad (d = h - 0.5 \cdot t_{us} = 2.35\,\text{m})$$

$\rightarrow$ Choice: 52 x ∅26@100, in two layers

$$A_s = 52 \cdot \frac{26^2 \pi}{4} = 27608\ \text{mm}^2$$

  Concrete compression zone:

$$c = \frac{A_s \cdot f_{sd}}{t_d \cdot f_{cd}} = \frac{27608 \cdot 435}{1000 \cdot 24} = 500\,\text{mm}$$

  Inner lever arm:

$$z = d - \frac{c}{2} = 2350 - \frac{500}{2} = 2.1\,\text{m}$$

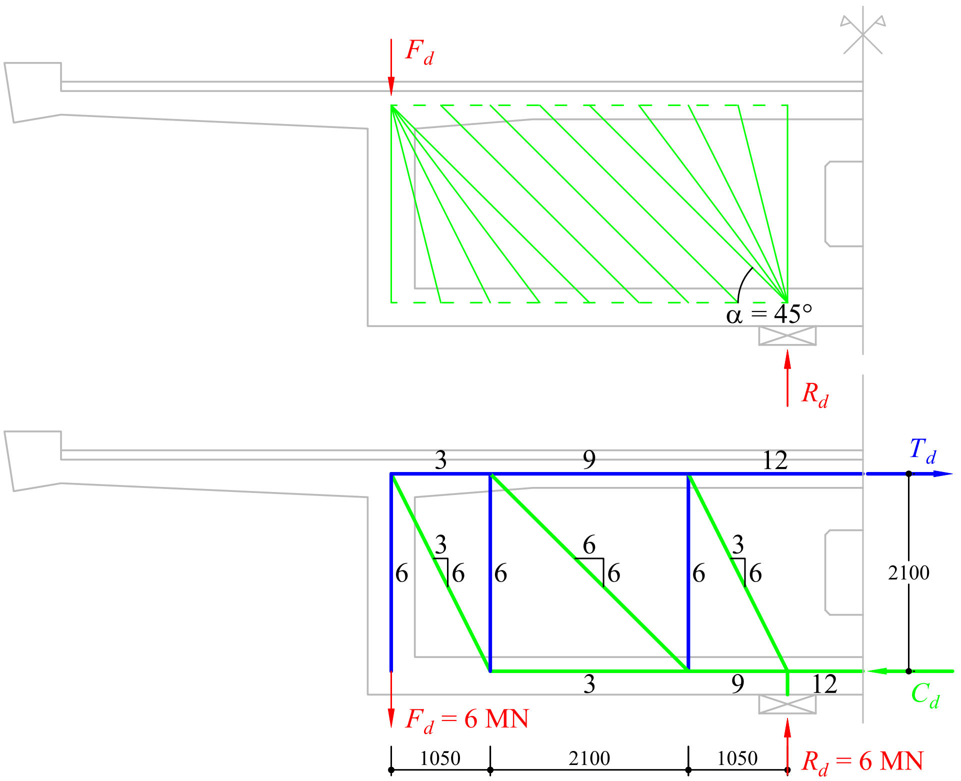


Figure 3: Stress field and strut-and-tie model for the case of the suspension of the entire load

– Reinforcement in the tension chord

The suspension of the load leads to a concentrated load introduction in the upper chord. The load can be carried directly with the reinforcement above the web. The shear connection of the webs to the upper flange is ensured by a reinforcement that spreads the force. This distribution reinforcement ensures the activation of the distributed longitudinal reinforcement over the width of 2.5 m. The distributed reinforcement needs to be superimposed with the main bending reinforcement in the longitudinal direction.

See also Figure 5

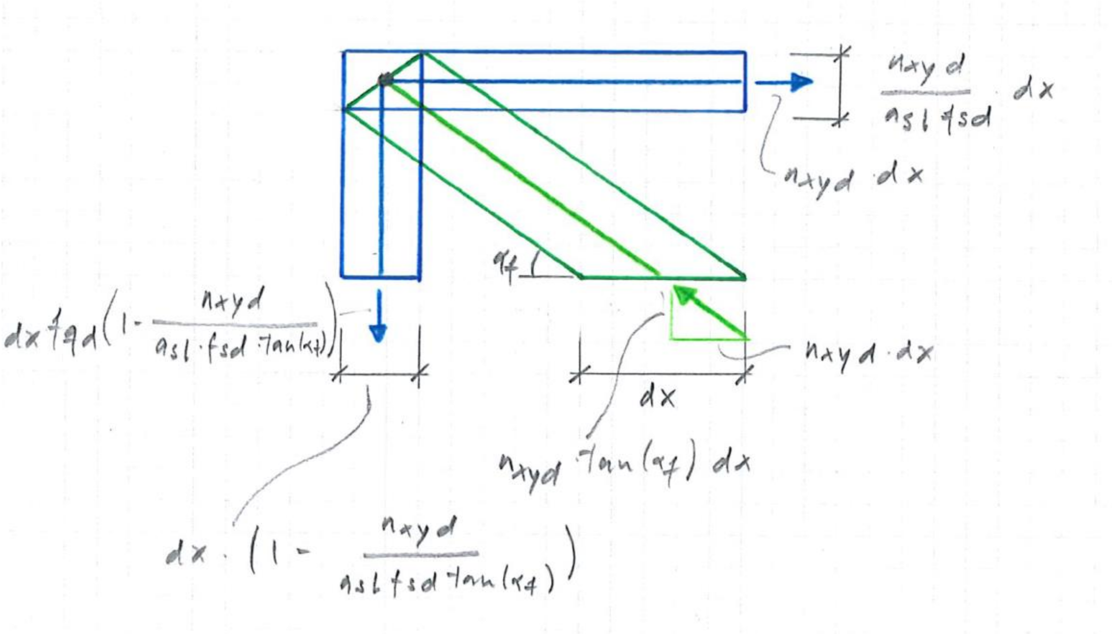


Figure 4: Detail of the spreading of the force in the upper bridge slab

See Figure 4

$$n_{xyd} = 0.5\frac{dF_{sup}}{dx} \qquad \text{maximum}: n_{xyd} = 0.5\frac{9000\,\text{kN} - 3000\,\text{kN}}{2.1\,\text{m}} = 1429\frac{\text{kN}}{\text{m}}$$

$$n_{xyd} \cdot \tan(\alpha_f) = 1429 \cdot \tan(35°) = 1000\frac{\text{kN}}{\text{m}}$$

$$f_{qd} = n_{xyd} \cdot \tan(\alpha_f) \cdot \left(1 - \frac{n_{xyd}}{a_{sl} \cdot f_{sd} \cdot \tan(\alpha_f)}\right)^{-1}$$

$$= 1740\frac{\text{kN}}{\text{m}} \qquad \left(a_{sl} = \frac{27608\,\text{mm}^2}{2.5\,\text{m}} = 11043\frac{\text{mm}^2}{\text{m}}\right)$$

– Shear reinforcement

SIA 262 4.3.3.4.3

$$a_{sw,erf} = \frac{V_{Ed}}{z \cdot f_{sd} \cdot \cot(\alpha)} = \frac{6\text{MN}}{2100 \cdot 435 \cdot \cot(45°)} = 6568\frac{\text{mm}^2}{\text{m}}$$

$\rightarrow$ Choice $\varnothing 18@150$ 4-legged stirrups

$$a_{sw} = 6785\frac{\text{mm}^2}{\text{m}}$$

– Check of compression diagonal

SIA 262 4.3.3.4.6 $k_c = 0.55$

$$\sigma_c = \frac{V_{Ed}}{t_w \cdot z \cdot \sin(\alpha)\cos(\alpha)} = \frac{6\,\text{MN}}{1000 \cdot 2100 \cdot \sin(45°) \cdot \cos(45°)}$$

$$= 5.7\,\text{MPa} < k_c \cdot f_{cd} = 13.2\,\text{MPa} \qquad \text{ok}$$

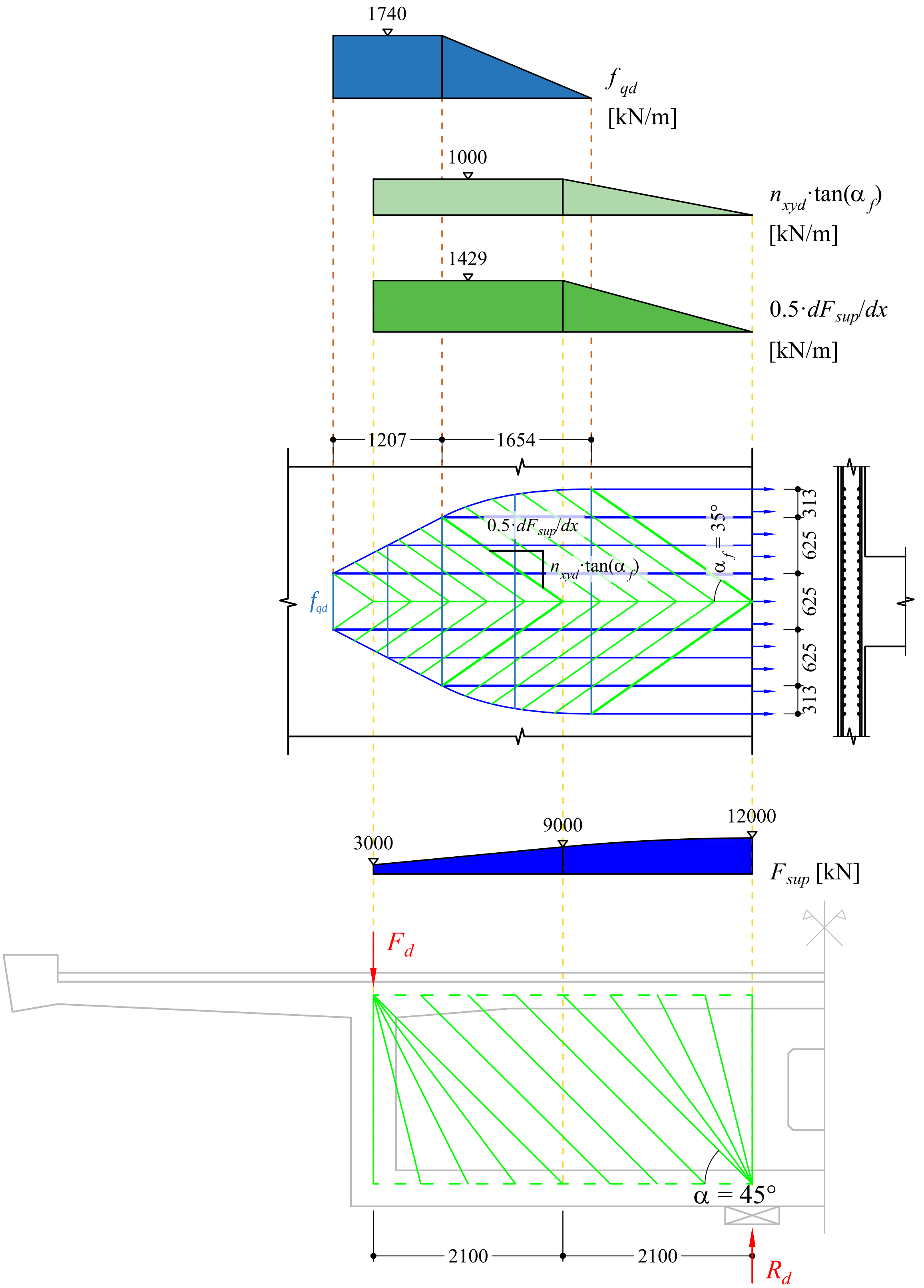


Figure 5: Spreading of the force in the upper bridge slab

b) Load transfer without suspension reinforcement

- Global stress field

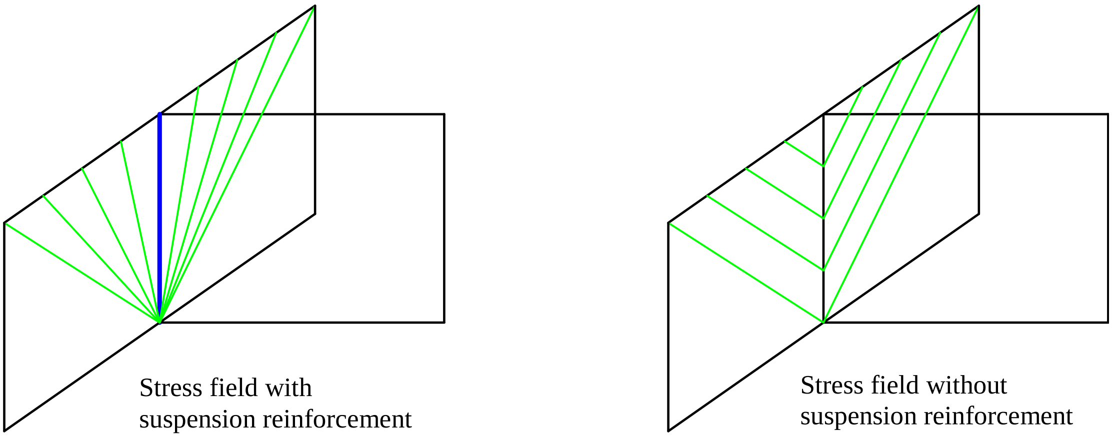


Figure 6: Global stress field (web and diaphragm)

- Dimensioning of the diaphragm with a strut-and-tie model

The upper reinforcement is distributed in the upper bridge slab and the compression zone is limited to the width of the diaphragm like in task 1a). The load is introduced over the height of the diaphragm. This requires a longitudinal reinforcement in the diaphragm, which resists the longitudinal forces.

It should be noted that the strut-and-tie model only works because the dead weight of the diaphragm is neglected. The additional load must be guided around the opening, which is shown in the lower figure. As an alternative to the load deviation, the shear force could be guided directly into the support (dashed line). In that case, the stress fields of the fans and the parallel fields overlap.

– Reinforcement in the tension chord

$T_{d,sup} = 12\,\text{MN}$

$\rightarrow$ Choice: 52 x $\varnothing$26@100, in 2 layers

$$A_s = 52 \cdot \frac{26^2 \pi}{4} = 27608\,\text{mm}^2$$

$$T_{Rd} = 27608 \cdot 435 = 12\,\text{MN} \geq T_d = 12\,\text{MN} \qquad \text{ok}$$

The shear connection is ensured with distribution reinforcement like in task 1a). The same required maximum reinforcement content results, grading of the reinforcement content is possible according to the flow of forces in Figure 8.

– Distribution reinforcement

$\rightarrow \varnothing$26@150, in 2 layers

$$f_{Rd} = 1822\,\frac{\text{kN}}{\text{m}} > f_{qd} = 1740\,\frac{\text{kN}}{\text{m}} \qquad \text{ok}$$

– Distributed longitudinal reinforcement in the diaphragm

$$a_{s,erf} = \frac{T_d}{z \cdot f_{sd}} = \frac{6}{2.1 \cdot 435} = 6568\,\frac{\text{mm}^2}{\text{m}}$$

$\rightarrow$ Choice: $\varnothing$18@150, in 4 layers $\qquad a_s = 4 \cdot \frac{18^2 \cdot \pi}{4 \cdot 0.15} = 6768\,\frac{\text{mm}^2}{\text{m}}$

$$T_{Rd} = 6786 \cdot 435 \cdot 2.1 = 6.2\,\text{MN} > T_d = 6\,\text{MN} \qquad \text{ok}$$

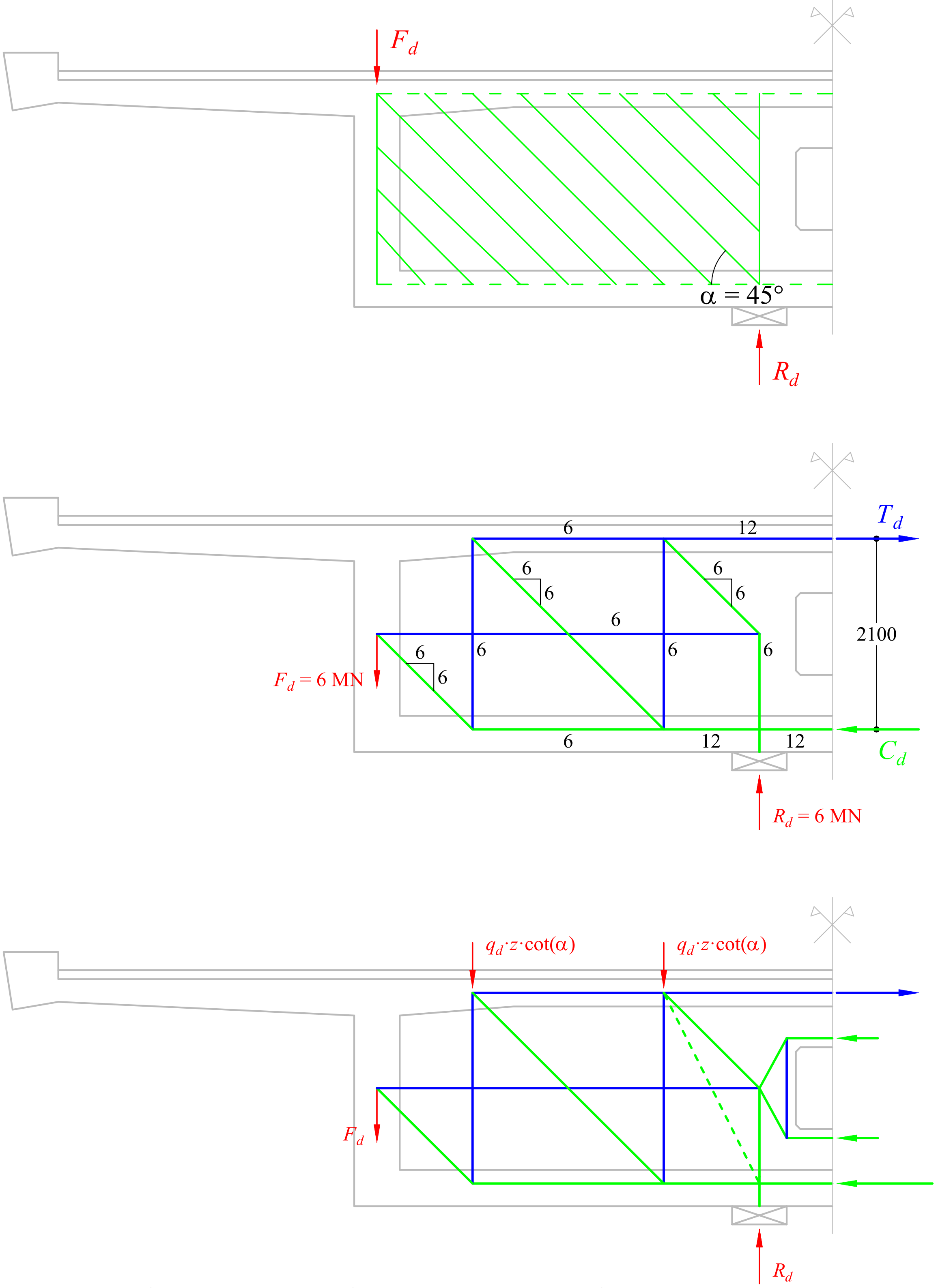


Figure 7: Stress field and strut-and-tie model for the case without suspension reinforcement

– Check of compression diagonal (parallel field)

$$\sigma_c = \frac{V_d}{t_d \cdot z \cdot \sin(\alpha)\cos(\alpha)} = \frac{6}{1000 \cdot 2.1 \cdot \sin(45°)\cos(45°)} = 5.7\,\text{MPa}$$

$$\sigma_c = 5.7\,\text{MPa} < k_c \cdot f_{cd} = 13.2\,\text{MPa} \qquad \text{ok}$$

– Stirrup reinforcement same as is task 1a)

→ ∅18@150, 4-legged

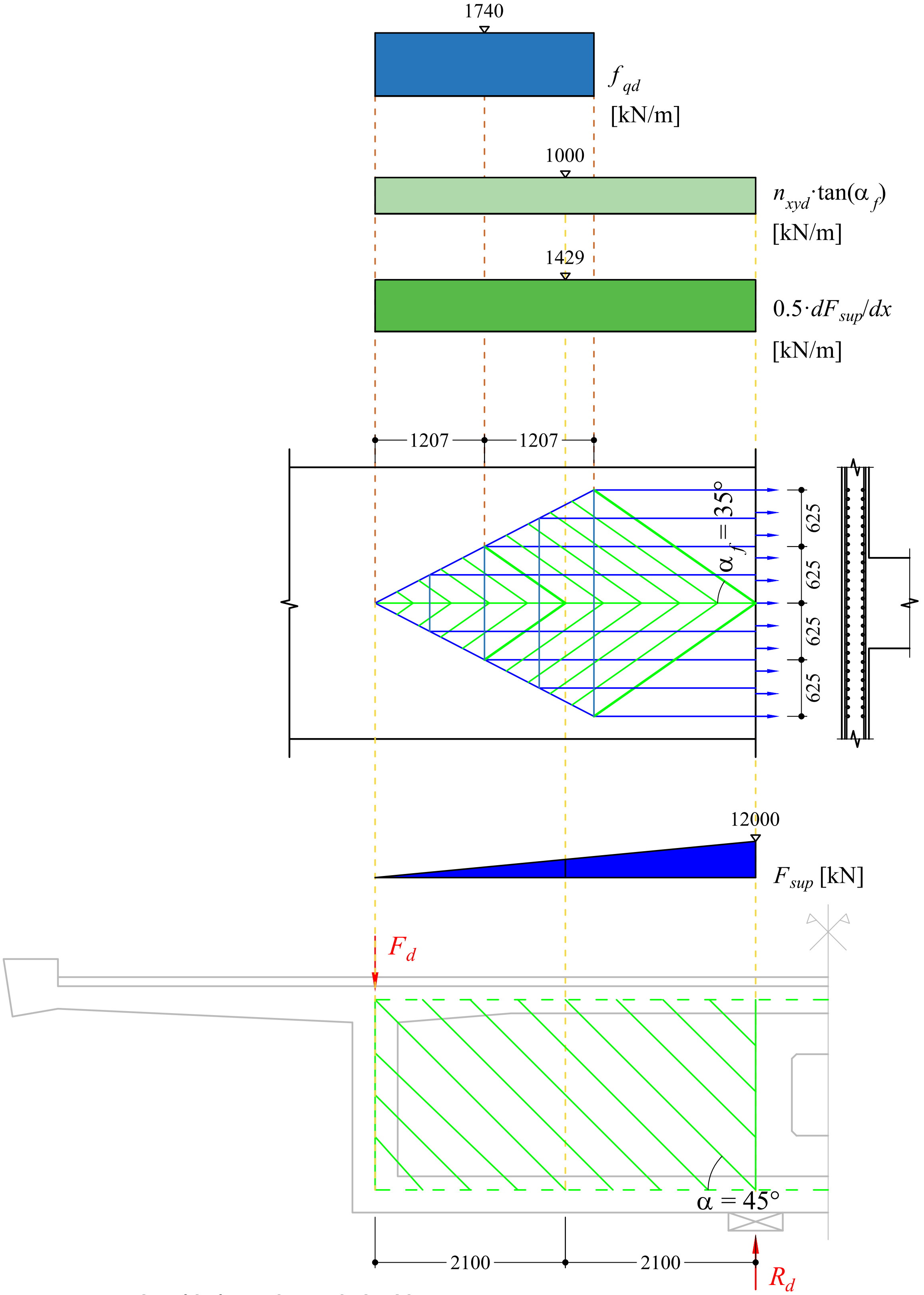


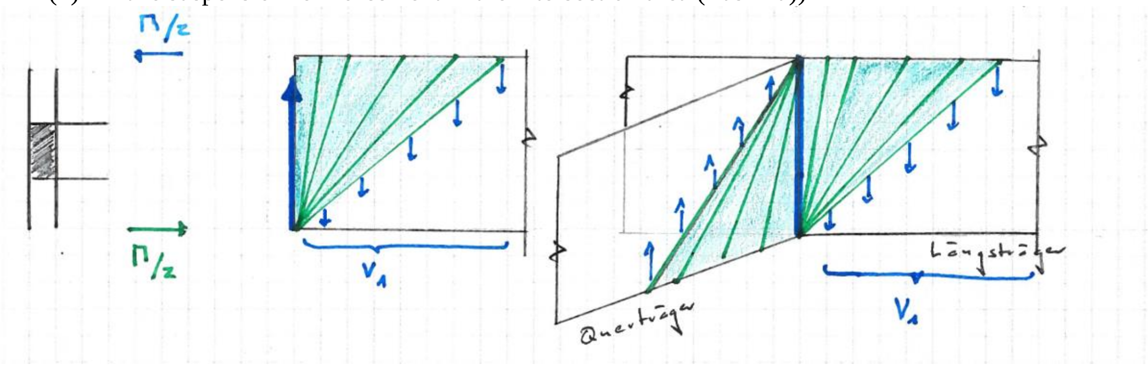
Figure 8: Spreading of the force in the upper bridge slab

Task 2

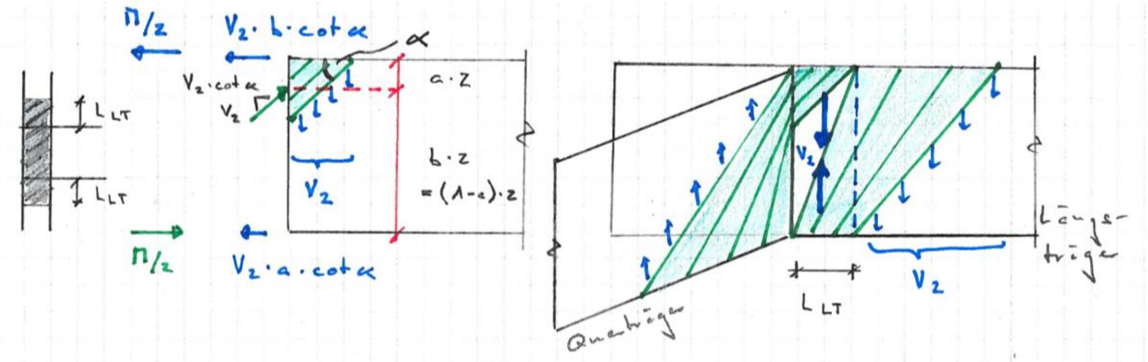
Längsträger = web

Querträger = diaphragm

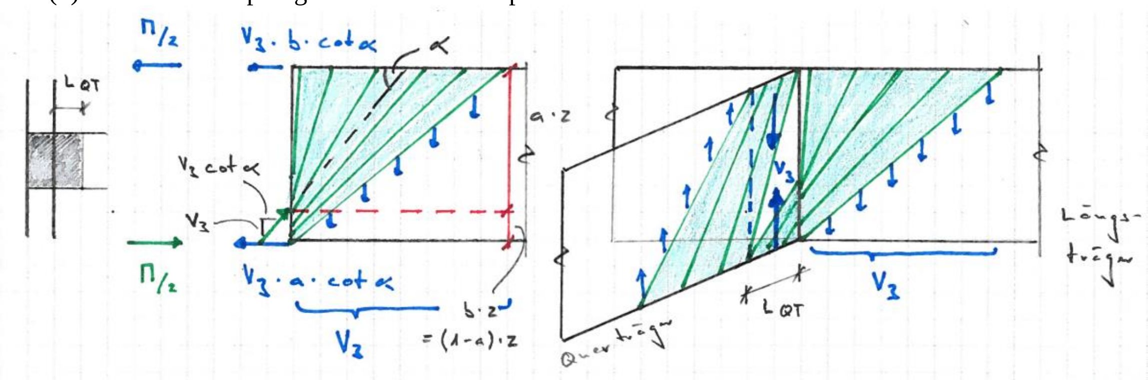
(1) Entire suspension reinforcement in the intersection area (Task 1a))



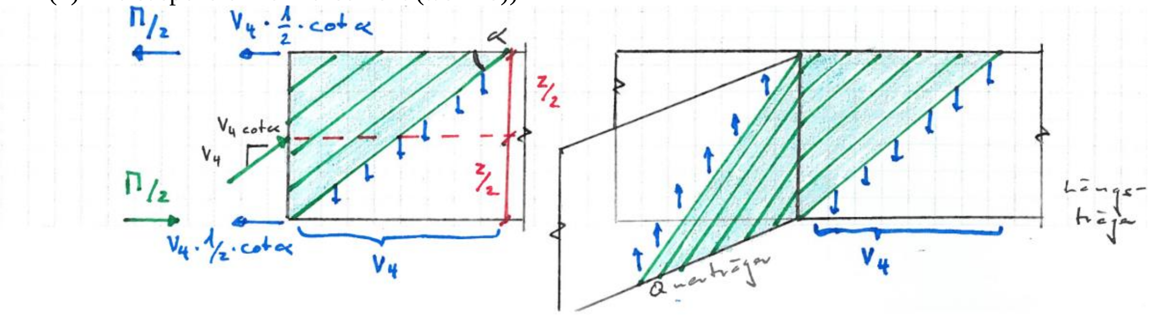
(2) Part of the web activated for suspension



(3) Part of the diaphragm activated for suspension



(4) No suspension reinforcement (task 1b))



Task 3

- General
  - Load is directly suspended in the intersection area
  - It turns out that the tension chord reinforcement as well as the width of the compression zone can only be kept within the width of the diaphragm
  - Treatment of the prestressing reinforcement as anchorage and deviation forces

- Prestressing forces

$$P_{0,1} = 27 \cdot A_p \cdot 0.7 f_{pk} = 5273\,\text{kN} \qquad (A_p = 150\,\text{mm}^2)$$

$$P_{\infty,1} = 0.85 P_{0,1} = 4482\,\text{kN}$$

$$\beta_1 = \tan^{-1}\left(2\frac{f_1}{L/2}\right) = 7.0° \qquad (L = 10\,\text{m},\ f_1 = 305\,\text{mm})$$

$$\rightarrow \begin{Bmatrix} P_{\infty,1,x} \\ P_{\infty,1,z} \end{Bmatrix} = \begin{Bmatrix} \cos\beta_1 \\ \sin\beta_1 \end{Bmatrix} \cdot P_{\infty,1} = \begin{Bmatrix} 4449 \\ 543 \end{Bmatrix} \text{kN}$$

Deviation forces (simplification: acting vertically):

$$u_{\infty,1} = \frac{8}{l^2}\left(P_{\infty,1,z} \cdot \frac{l}{2} - P_{\infty,1,x} \cdot f_1\right) = \frac{8}{10^2}(543 \cdot 5 - 4449 \cdot 0.305) = 108.5\frac{\text{kN}}{\text{m}}$$

$$P_{0,2} = 15 \cdot A_p \cdot 0.7 f_{pk} = 2930\,\text{kN}$$

$$P_{\infty,2} = 0.85 P_{0,2} = 2491\,\text{kN}$$

$$\beta_2 = 15.2° \quad \text{with } f_2 = 680\,\text{mm} \rightarrow \begin{Bmatrix} P_{\infty,2,x} \\ P_{\infty,2,z} \end{Bmatrix} = \begin{Bmatrix} 2404 \\ 654 \end{Bmatrix} \text{kN}$$

Deviation forces: $u_{\infty,2} = 130.7\frac{\text{kN}}{\text{m}}$

The nodal zones are dimensioned according to the loads resulting from the load introduction in the anchorage zone. The resulting stresses inside the fan can be determined from the geometry of the nodes. The stress field is idealized as point-centred fans at the supports.

The point-centred fan intersects with the opening of the bridge (Figure 9). Hence, an additional horizontal reinforcement is needed above the opening due to the deviation forces. The loads in this area are generally small, so the additional reinforcement that would anyways be placed around the opening is probably sufficient to withstand them (needs to be checked).

- Reinforcement in the tension chord

The required passive reinforcement is reduced due to the prestressing. Passive reinforcement is only placed directly above the web so that no distribution reinforcement is needed.

$$T_d = 5734\,\text{kN} \qquad A_{s,erf} = \frac{5734}{435} = 13182\,\text{mm}^2$$

$\rightarrow$ Choice: $20\ \varnothing 30 @ 100$, in two layers $A_s = 20 \cdot 707 = 14140\,\text{mm}^2$

$$T_{Rd} = 435 \cdot 14140 = 6151\,\text{kN}$$

- Check of the compression chord

Height of concrete compression zone: $c = \dfrac{12589}{24 \cdot 1000} = 524\,\text{mm}$

The resulting height of the compression zone is slightly higher than the assumed height (500 mm). The effective height of the compression zone can be determined by an iterative process (inner lever arm influences the height of the compression zone). Due to the small deviation, this step is not necessary here.

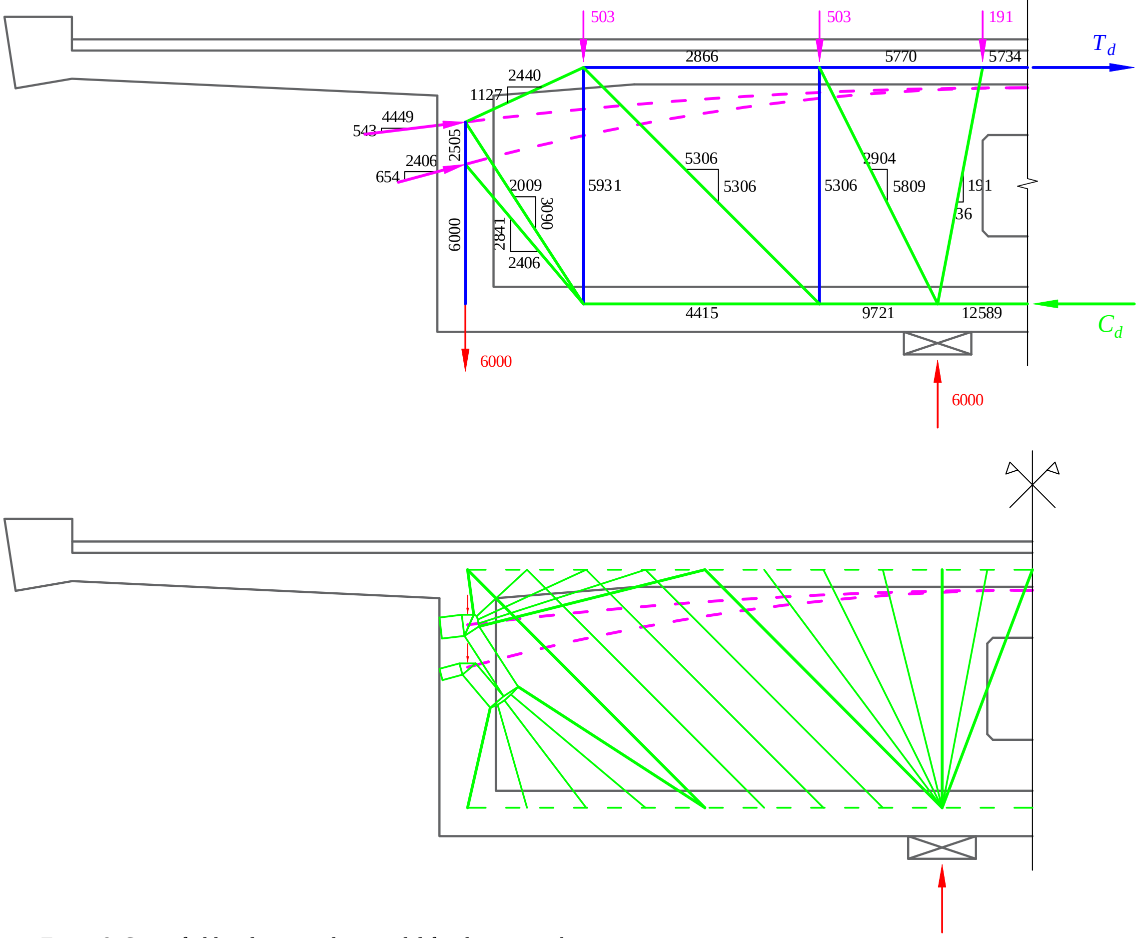


Figure 9: Stress field and strut-and-tie model for the case with prestressing

- Shear reinforcement in the diaphragm: same as in 1a) and b)

  Choice: $\varnothing 18@150$, 4-legged $a_{sw} = 6786 \frac{\text{mm}^2}{\text{m}}$

  $V_{Rd} = 6.2\,\text{MN} > V_d = 5.9\,\text{MN}$ ok

- Concrete compression diagonals (parallel field)

$$\sigma_{cd} = \frac{V_d}{b_w \cdot z \cdot \sin(\alpha) \cdot \cos(\alpha)} = \frac{5306}{1000 \cdot 2100 \cdot \sin(45°) \cdot \cos(45°)} = 5.1\,\text{MPa}$$

$$\sigma_{cd} = 5.1\,\text{MPa} < k_c \cdot f_{cd} = 0.55 \cdot 24 = 13.2\,\text{MPa}$$

- Superposition of a fan and a parallel field

A superposition of a fan and a parallel field is necessary in the area of the upper anchorage. The controlling point lies at the intersection of the flattest element of the fan with the parallel field, since the stresses in the fan originate from the nodal zone. The stress inside the fan decreases inversely proportional to the distance from the nodal zone.

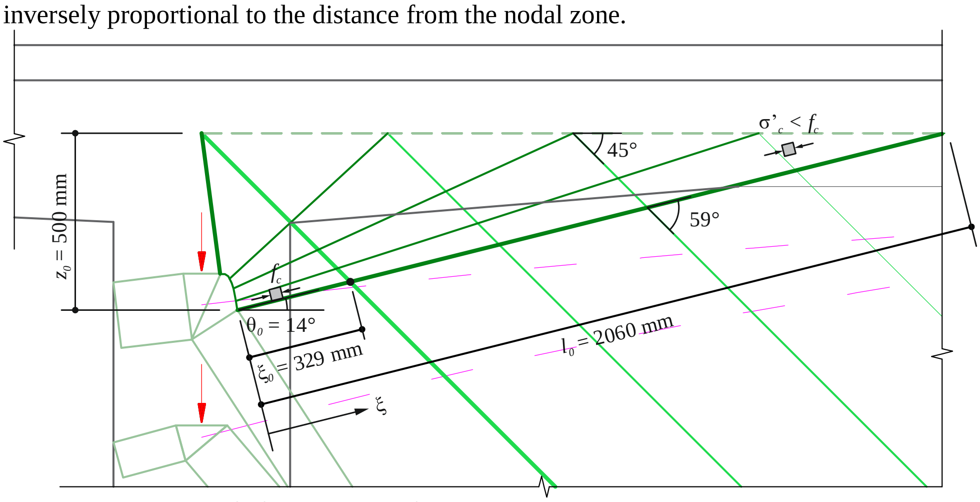


Figure 11: Superposition of a fan and a parallel field

On the upper fan boundary, the stress is:

$$\sigma_c' = \frac{V_d}{b_w \cdot z_0 \cdot \sin(\theta_0)\cos(\theta_0)} = \frac{1127}{1000 \cdot 500 \cdot \sin(14°)\cos(14°)} = 9.6\,\text{MPa}$$

Due to the widening of the fan, the stress decreases inversely proportional:

$$\sigma_c(\xi) = f_c \left(1 + \frac{\frac{f_c}{\sigma'_c} - 1}{l_0} \cdot \xi\right)^{-1}$$

$$\sigma_c(\xi_0 = 329) = 24\left(1 + \frac{\frac{24}{9.6} - 1}{2060} \cdot 329\right)^{-1} = 19.4\,\text{MPa}$$

The two compression states intersect in an angle $\alpha_0 = 59°$. The superposition can be displayed with the Mohr Circle.

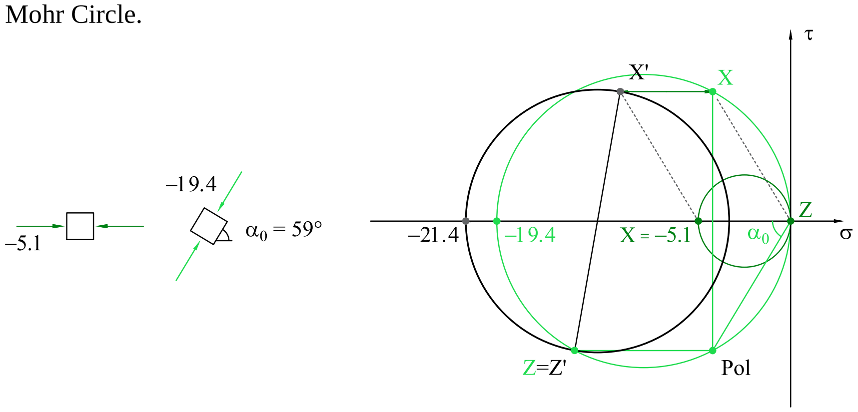
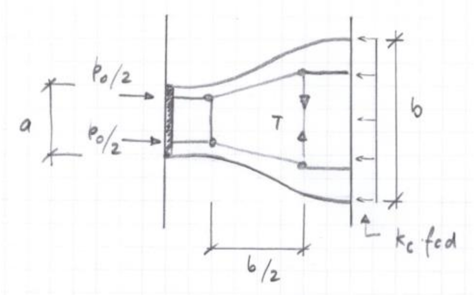


Figure 11: Superposition of the two compression states displayed in Mohr's circles

The resulting main stress resultant is $\sigma_{cd} = 21.4\,\text{MPa}$. The check of the concrete strength can therefore not be fulfilled with a reduction factor of $k_c = 0.55$. The concrete strength can be increased with confinement reinforcement according to SIA 262 4.2.1.8. It is assumed that the spiral reinforcement of the anchorage is sufficient to increase the concrete strength.
The same generally applies to the areas of the boundary of the nodal zones of the fan since f_{cd} is assumed there.

- Reinforcement against splitting of the load introduction zone of the prestressing reinforcement (spiral reinforcement)



Assuming a squared zone:

$$P_0 = b^2 \cdot k_c \cdot f_{cd}$$

$$b_{erf} = \sqrt{\frac{P_0}{k_c \cdot f_{cd}}} = \{632\ ;\ 471\}\,\text{mm}$$

Assumption: $a = 350\,\text{mm}$ (anchorage plate)

Transversal tensile force:

$$T_1 = \left(\frac{b-a}{4}\right) \cdot \left(\frac{b}{2}\right)^{-1} \cdot \frac{P_{0,1}}{2} = 588\,\text{kN}$$

$$T_2 = 188\,\text{kN}$$

Commonly, the spiral reinforcement is taken into consideration with $\sigma_s = 250\,\text{MPa}$ to avoid the calculation of crack widths.

$$A_{s,erf} = \frac{T_1}{250\,\text{MPa}} = 2352\,\text{mm}^2$$

$\rightarrow$ Spiral with $\varnothing 18$ every $100\,\text{mm}$ (= s)

$$n = 2 \cdot \left\lfloor \frac{b}{s} \right\rfloor = 2 \cdot \left\lfloor \frac{632}{100} \right\rfloor = 2 \cdot \lfloor 6.32 \rfloor = 12 \text{ cut surfaces of the spiral}$$

$\lfloor\ \rfloor$: symbol for flooring

$$A_s = \frac{18^2 \cdot \pi}{4} \cdot 12 = 3054\,\text{mm}^2 \qquad \text{ok}$$

- Reinforcement against spalling

To avoid the spalling of the concrete around the anchorage zone, the following reinforcement is suggested by Eurocode Annex J:

$$A_{sp} = 0.03 \frac{P_0}{f_{sd}} \gamma_p = 436\,\text{mm}^2$$

$\rightarrow$ 2 $\varnothing$ 18 in every direction