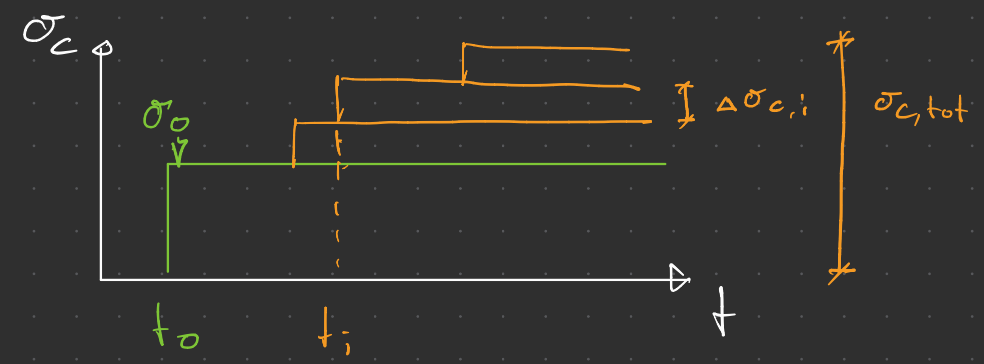
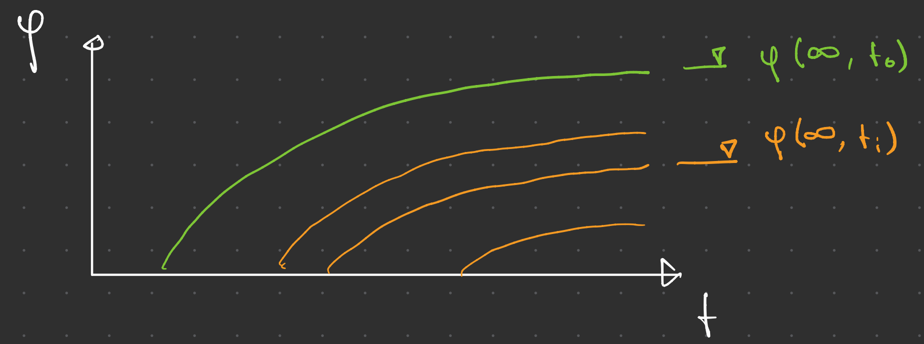
Trost's method



$$\varepsilon_c(t) = \frac{\sigma_0}{E_{co}}\left(1+\varphi(t,t_0)\right) + \sum \frac{\Delta\sigma_{c,i}}{E_{co}}\left(1+\varphi(t,t_i)\right)$$

$$= \underbrace{\frac{\sigma_{co} + \sum \Delta\sigma_{c,i}}{E_{co}}}_{\varepsilon_{c,el}} + \underbrace{\frac{\sigma_{co}}{E_{co}}\varphi(t,t_0) + \underbrace{\sum \frac{\Delta\sigma_{c,i}}{E_{co}}\varphi(t,t_i)}_{(*)}}_{\varepsilon_{cc}\ (\text{creep})}$$

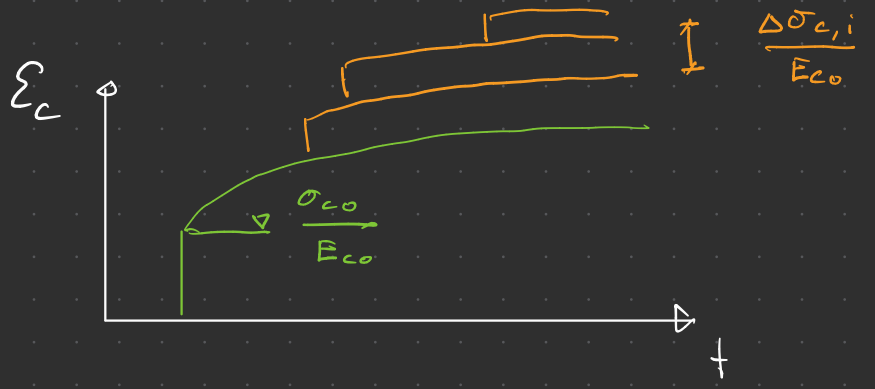
$$\varphi(t,t_i) = \mu(t)\cdot\varphi(t,t_0)$$



$$(*) \Rightarrow \sum_i \frac{\Delta\sigma_{c,i}}{E_{co}}\varphi(t,t_i) = \mu(t)\,\varphi(t,t_0)\sum_i \frac{\Delta\sigma_{c,i}}{E_{co}}$$

$$= \mu(t)\,\varphi(t,t_0)\,\Delta\sigma_{c,tot}$$

$$= \mu(t)\,\varphi(t,t_0)\left(\sigma_{c,tot} - \sigma_0\right)$$



$$\Rightarrow \boxed{\varepsilon_c(t) = \frac{1}{E_{co}}\Big(\underbrace{\sigma_{co}\left(1+\varphi(t,t_0)\right)}_{\text{creeps fully}} + \underbrace{\Delta\sigma_{c,tot}\left(1+\mu\,\varphi(t,t_0)\right)}_{\text{creeps less}}\Big)}$$

$$\mu(t) \approx \phi \approx 0.8$$

Most important formulas

$$\delta_1 = \delta_{10} + X_1 \delta_{11}$$

$$\delta_1(t) = \delta_{10}(1+\varphi) + X_1 \delta_{11}(1+\varphi) + \Delta X(t)\, \delta_{11}(1+\mu\varphi)$$

System change

before change

$$M(t) = M_0 + \Delta X_1 \cdot M_1 \qquad (X_1 = 0)$$

$$\Delta X_1 = X_{1,oc} \cdot \frac{\varphi}{1+\mu\varphi}$$

$$M(t) = M_0 + X_{1,oc} \cdot M_1 \cdot \frac{\varphi}{1+\mu\varphi}$$

OC- System: $M_{oc} = M_0 + X_{1,oc} \cdot M_1 \Rightarrow X_{1,oc} \cdot M_1 = M_{oc} - M_0$

$$\boxed{M(t) = M_0 + (M_{oc} - M_0) \cdot \frac{\varphi}{1+\mu\varphi}}$$

$\forall$ internal forces

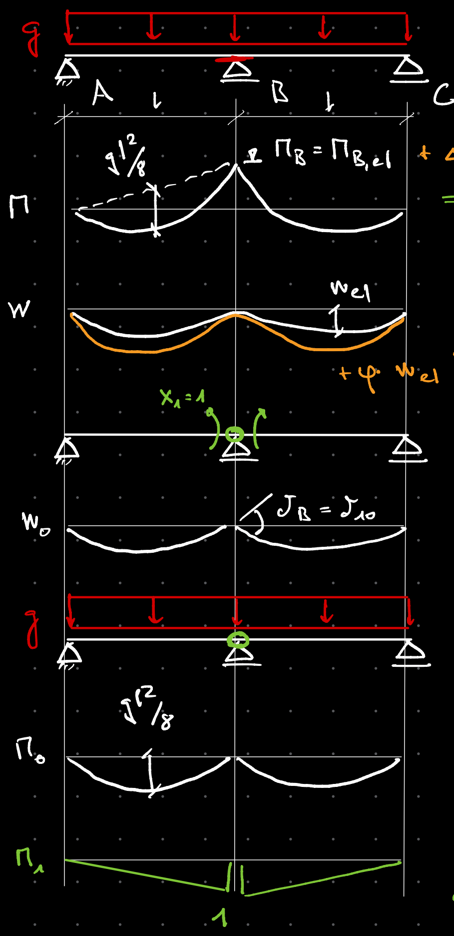
different systems of different ages:

$$\boxed{M(t) = \sum_i M_i \left(1 - \frac{\varphi_i}{1+\mu\varphi_i}\right) + M_{oc} \frac{\varphi_0}{1+\mu\varphi_0}}$$

$i = \#$ system change

Effect of creep on two-span girder – time dependent force method

hs/30.11.2023



one casting system
"Eingusssystem"

$+ \Delta M_B$? $= 0$

$+ \varphi \cdot w_{el}$? ✓

Basic system (isostatic) + redundant variable
BS + RV

Relative rotation at B

- due to g:

$$\vartheta_{10} = \int \frac{M_0 \cdot M_1}{EI} dx = \frac{g l^2}{8} \cdot \frac{L}{EI} \cdot \frac{1}{3} \cdot 2 = \frac{g l^2}{12\, EI}$$

- due to $X_1 = 1$:

$$\vartheta_{11} = \int \frac{M_1 \cdot M_1}{EI} dx = \frac{1 \cdot 1}{EI} \cdot l \cdot \frac{1}{3} \cdot 2 = \frac{2L}{3EI}$$

Short-term compatibility ($t=0$)

$\varphi = 0, \Delta X(t) = 0$

$$\vartheta_1 = \vartheta_{10}(1 + \cancel{\varphi}^{=0}) + X_1 \vartheta_{11}(1 + \cancel{\varphi}^{=0}) + \cancel{\Delta X}^{=0} \vartheta_{11}(1 + \mu\varphi) \overset{!}{=} 0$$

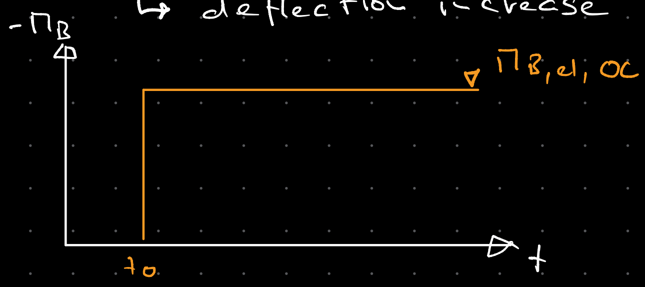
$$X_1 = M_B = -\frac{\vartheta_{10}}{\vartheta_{11}} = -\frac{g l^2}{8}$$

Time-dependent compatibility

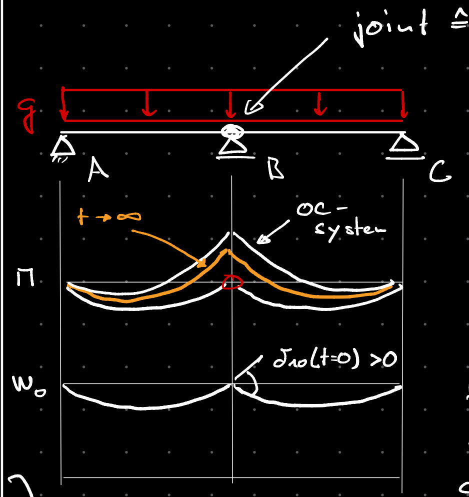
$$\vartheta_1 = \vartheta_{10}(1+\varphi) + X_1 \vartheta_{11}(1+\varphi) + \Delta X \vartheta_{11}(1+\mu\varphi) \overset{!}{=} 0$$

$$\boxed{\Delta X = \Delta M_B = 0}$$

⇒ No redistribution of internal forces due to creep
↳ deflection increase



here: for uncracked structure



staged construction
"Bauzustände"

for systems of same age

two simply supported girders connected

≙ made continuous after applying g ($t = t_0$)

same result as for OC system

$$\vartheta_{10} = \frac{g l^2}{12\, EI}$$ ← relative rotation ("kink") at B, frozen for $t > t_0$

$$\vartheta_{11} = \frac{2L}{3EI}$$

"stc" ($t = t_0$)

$\varphi = 0, \Delta X = 0$

$X_1 = M_B(t = t_0) = 0$

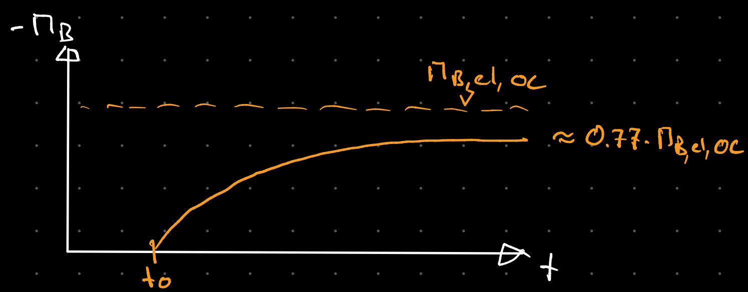
$\vartheta_1 = \vartheta_{10}$

"tdc" ($t > t_0$)

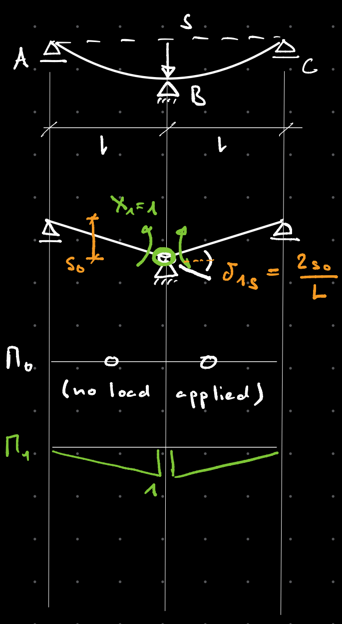
$$\vartheta_1 = \vartheta_{10}(\cancel{1}+\varphi) + \underbrace{X_1 \vartheta_{11}(1+\varphi)}_{= M_B(t=t_0) = 0} + \Delta X \vartheta_{11}(1+\mu\varphi) \overset{!}{=} \cancel{\vartheta_{10}}$$

$$\vartheta_{10}\, \varphi + \Delta X\, \vartheta_{11}(1+\mu\varphi) = 0$$

$$\boxed{\Delta X = \Delta M_B = -\frac{\vartheta_{10}}{\vartheta_{11}} \cdot \frac{\varphi}{1+\mu\varphi} = M_{B,oc,el} \cdot \frac{\varphi}{1+\mu\varphi}}$$



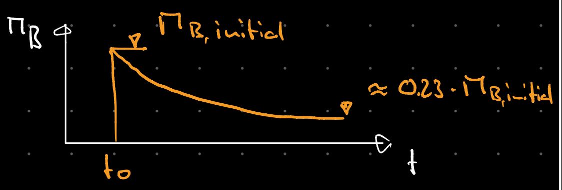
Support settlement
Effect of creep for fast / slow settlement



$$\delta_{10} = \int \frac{M_0 M_1}{EI} dx = \underline{0}$$

$$\delta_{11} = \int \frac{M_1 \cdot M_1}{EI} dx = \frac{2L}{3EI}$$

time-independent settlement ("fast" restraint)



"stc", $t = t_0 \Rightarrow \varphi = 0,\ \Delta X = 0$

$$\delta_1 = \delta_{10}(1+\cancel{\varphi}^{\,0}) + X_1 \delta_{11}(1+\cancel{\varphi}^{\,0}) + \cancel{\Delta X \delta_{11}(1+\mu\varphi)}^{\,=0} \overset{!}{=} \delta_{1s}$$

$$\Rightarrow X_1 = \frac{\delta_{1s}}{\delta_{11}} \ (*) = M_{B,initial}$$

$$= \frac{3EI}{L^2} \cdot s_0$$

"tdc", $t > t_0$

$$\delta_1 = \cancel{\delta_{10}}^{\,=0}(1+\varphi) + X_1 \delta_{11}(1+\varphi) + \Delta X \delta_{11}(1+\mu\varphi) \overset{!}{=} \delta_{1s}$$

$$\delta_{1s}(1+\varphi) + \Delta X \delta_{11}(1+\mu\varphi) = \delta_{1s}$$

$$\Rightarrow \Delta X(t) = \Delta M_B(t) = -\underbrace{\frac{\delta_{1s}}{\delta_{11}}}_{= M_{B,initial}} \cdot \frac{\varphi}{1+\mu\varphi}$$

$$\boxed{M_B(t) = M_{B,initial} + \Delta M_B(t) = M_{B,initial}\left(1 - \frac{\varphi}{1+\mu\varphi}\right)}$$

for $t \to \infty$: $\mu = 0.8$, $\varphi = 2$

$$M_B(t) = M_{B,initial} \cdot 0.23$$

$\underbrace{\text{full initial restraint is reduced to 23\%}}_{\text{relaxation}}$

time-dependent settlement ("slow" restraint)

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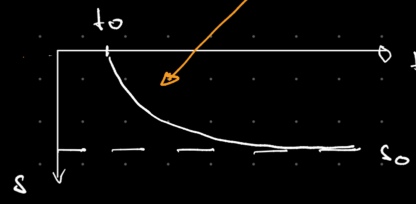
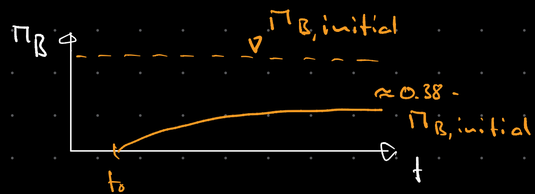
$$s(t) = s_0 \cdot \frac{\varphi(t)}{\varphi_\infty}$$





"stc", $t = t_0$

$\delta_1 = \ldots = 0$ since $s(t_0) = 0$

$$\Rightarrow X_1 = M_B(t_0) = 0$$

"tdc", $t > t_0$

$$\delta_1 = \cancel{\delta_{10}}^{\,=0}(1+\varphi) + \cancel{X_1}^{\,=0}\delta_{11}(1+\varphi) + \Delta X(t)\,\delta_{11}(1+\mu\varphi) \overset{!}{=} \delta_{1s}(t)$$

$$\Delta X(t) = \Delta M_B(t) = \underbrace{\frac{\delta_{1s}}{\delta_{11}}}_{= M_{B,initial}} (*) \cdot \frac{\varphi}{\varphi_\infty(1+\mu\varphi)}$$

$$\Rightarrow \boxed{M_B(t) = M_{B,initial} \cdot \frac{\varphi(t)}{\varphi_\infty(1+\mu\varphi(t))}}$$

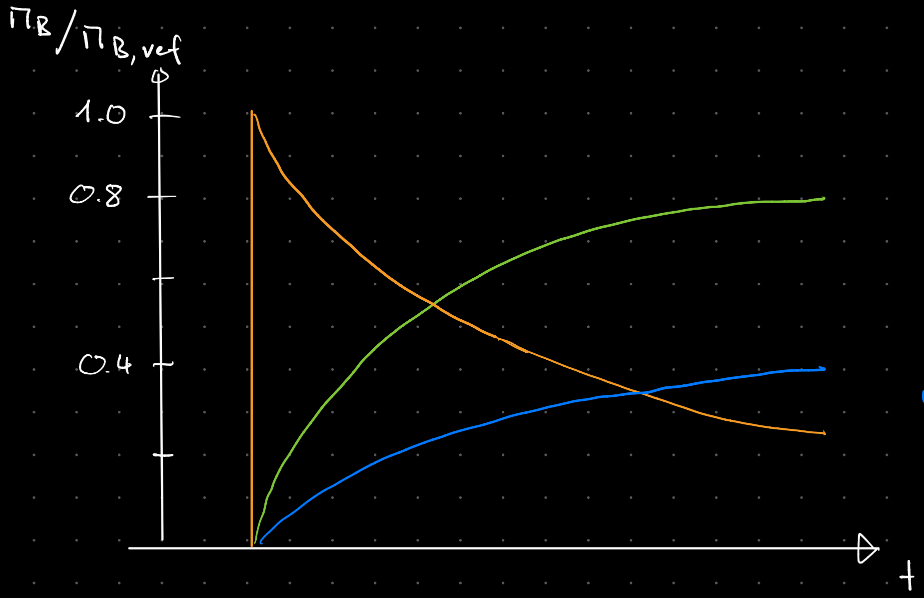
for $t \to \infty$: $\varphi = 2$, $\mu = 0.8$

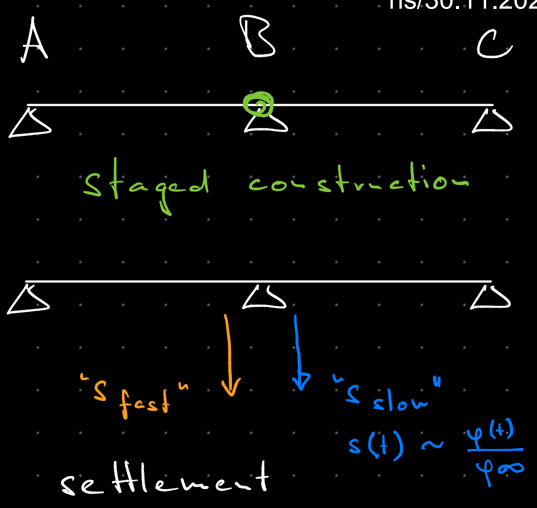
$$\varphi(t) = \varphi_\infty$$

$$M_B(t \to \infty) = M_{B,initial} \cdot 0.38$$

initially zero restraint builds up to 38% of full elastic restraint

Comparison





$$\frac{\varphi_\infty}{1+\mu\,\varphi_\infty} \approx 0.75 \dots 0.8$$

$$\frac{\varphi(t)}{\varphi_\infty\,(1+\mu\,\varphi(t))} \xrightarrow{t \to \infty} \frac{1}{1+\mu\,\varphi_\infty} \approx 0.4$$

$$1 - \frac{\varphi_\infty}{1+\mu \cdot \varphi_\infty} \approx 0.25 \dots 0.33$$

System change ($M_{B,ref} \,\hat{=}\,$ one casting system)

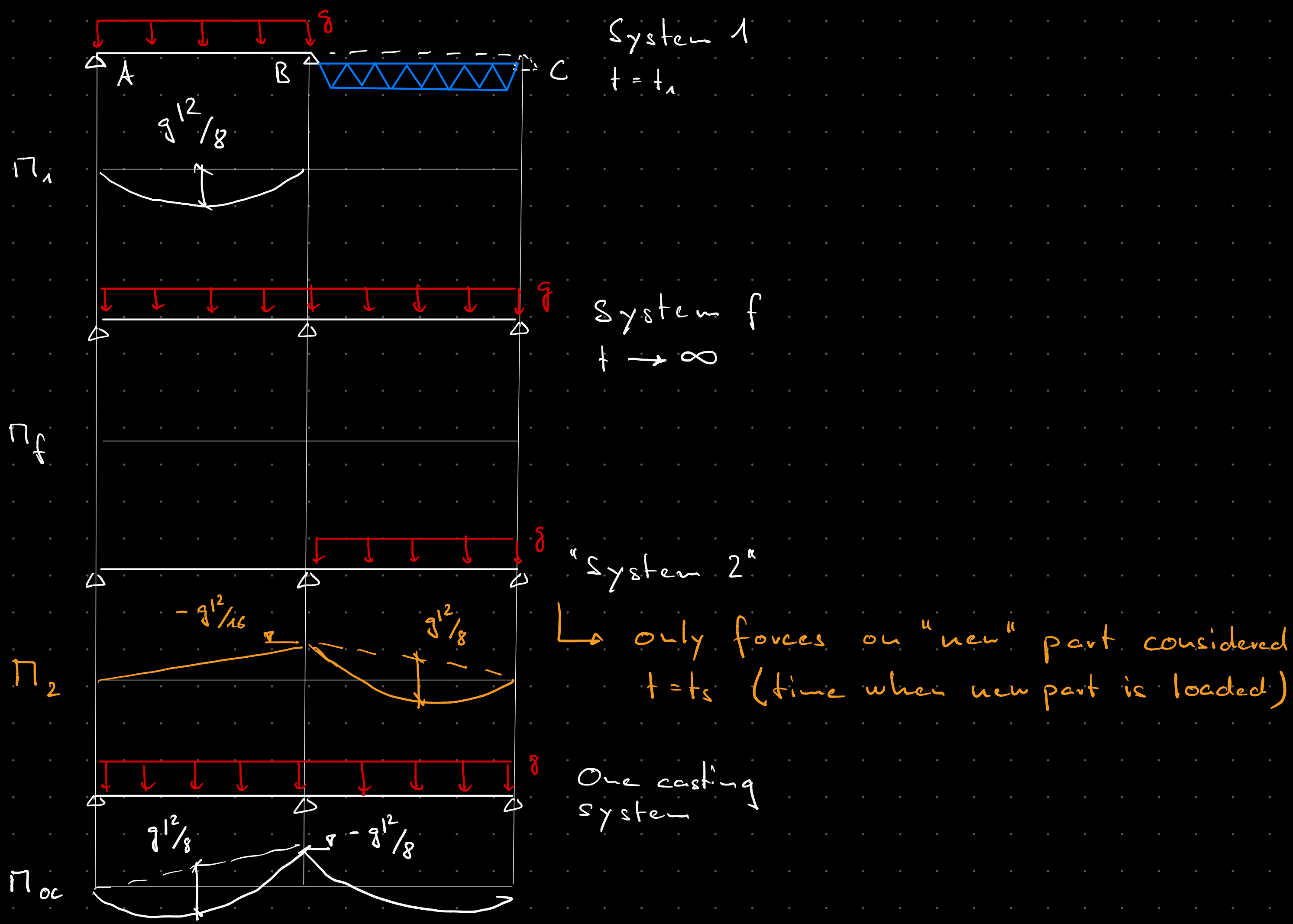
fast restraint ($M_{B,ref} \,\hat{=}\,$ full elastic restraint)

slow restraint ($M_{B,ref} \,\hat{=}\,$ full elastic restraint (fictitious))

hs/30.11.2023
rev. hs/15.01.2024

staged construction

for systems of different age



Approximation:

$$M(t) = \sum M_i \left(1 - \frac{\varphi_i}{1+\mu\varphi_i}\right) + M_{oc}\,\frac{\varphi_0}{1+\mu\varphi_0}$$

$$= M_1\left(1 - \frac{\varphi_1}{1+\mu\varphi_1}\right) + M_2\left(1 - \frac{\varphi_2}{1+\mu\varphi_2}\right) + M_{oc}\,\frac{\varphi_0}{1+\mu\varphi_0}$$

for $t = t_s$: $\varphi = 0$, $M_B = 0.5\, M_{oc}$

$t \to \infty$: $\varphi = 2$, $M_B = 0.88 \cdot M_{oc}$

Approximation "20 : 80"

$$M(t) = 0.2 \sum M_i + 0.8\, M_{oc} = 0.2\,(M_1 + M_2) + 0.8\, M_{oc}$$

for $t = t_s$: $\varphi = 0$, $M_B = 0.5\, M_{oc}$

$t \to \infty$: $\varphi = 2$, $M_B = 0.9 \cdot M_{oc}$