Advanced Structural Concrete

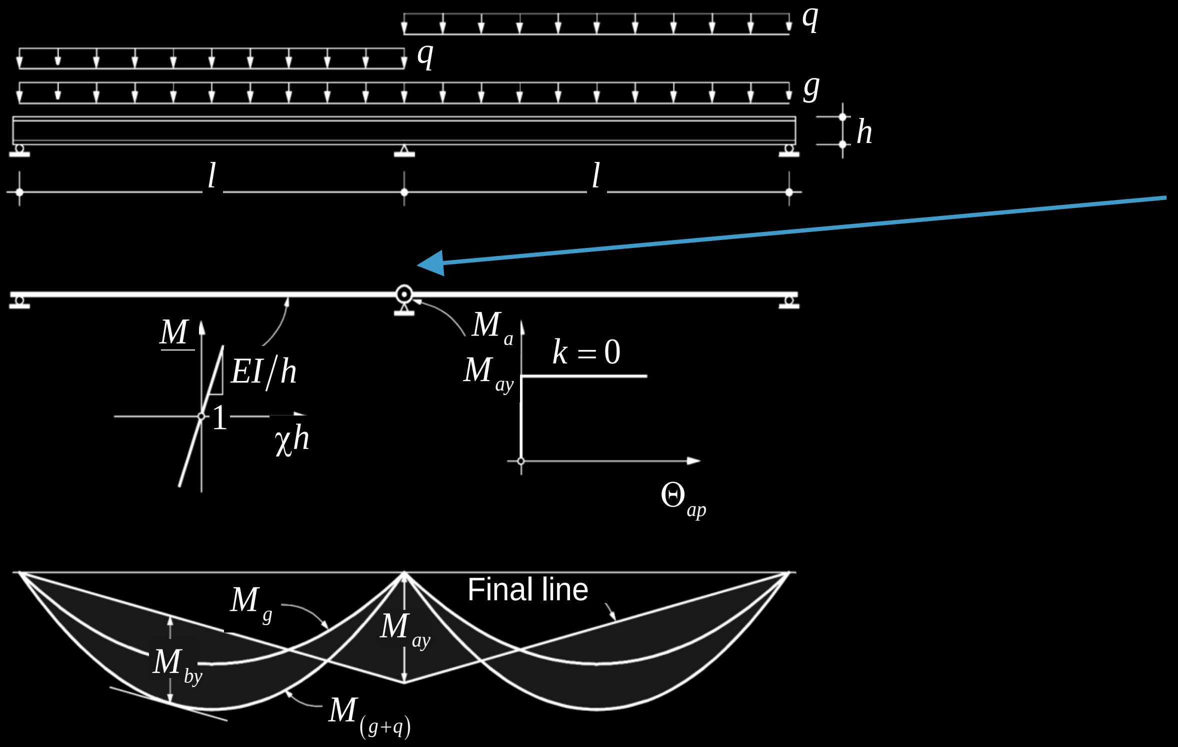
Introduction to Exercise 2

Introduction to Exercise 2

Goal of this exercise:

Learn how to verify the deformation capacity of a reinforced concrete structure by evaluating plastic redistributions of internal forces in hyperstatic systems and calculating the deformation demand.

In general, deformation capacity and deformation demand are coupled. For moderate redistributions of internal forces, this interaction can be neglected.



Example from lecture slides:
Two-span beam

1st hinge forms at intermediate support

→ rotation demand vs rotation capacity

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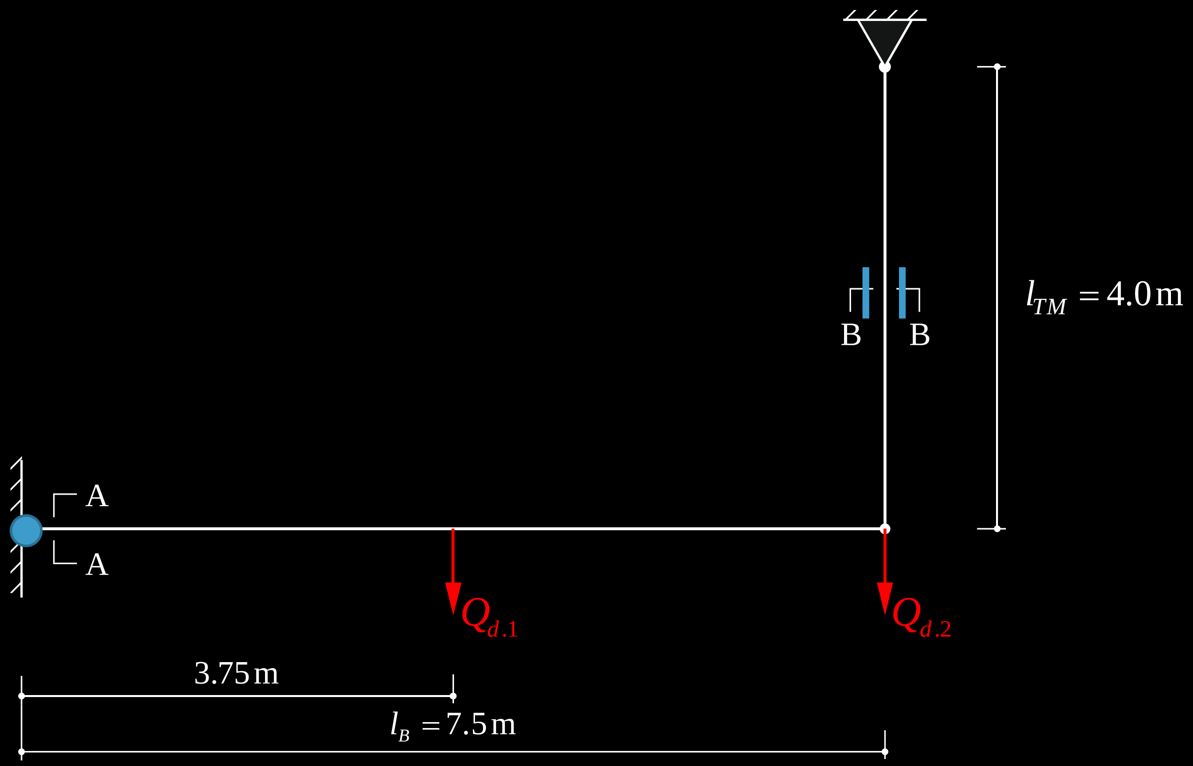
Deformation demand and deformation capacity



Load scenario 1: $Q_{d.1} = 300\,\mathrm{kN},\ Q_{d.2} = 0\,\mathrm{kN}$

Load scenario 2: $Q_{d.1} = 0\,\mathrm{kN},\ Q_{d.2} = 300\,\mathrm{kN}$

Where could plastic redistributions of internal forces take place?
(Assuming constant properties along structural members)

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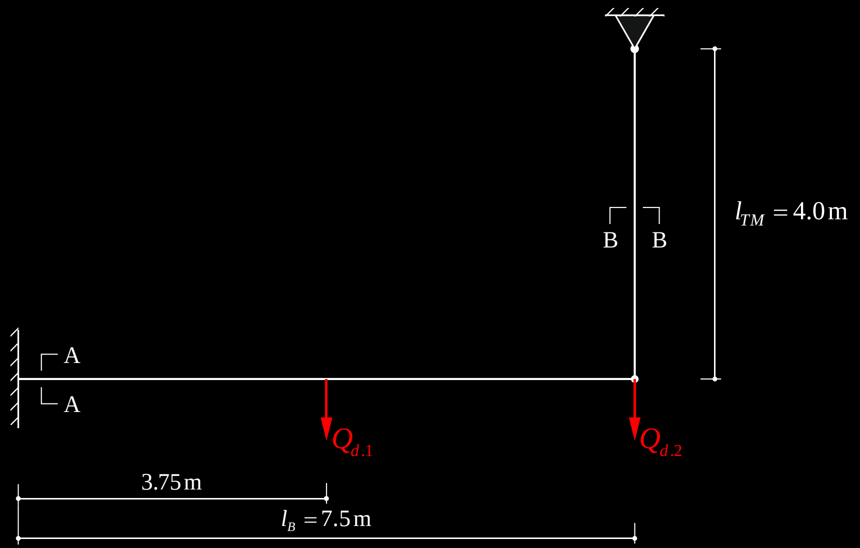
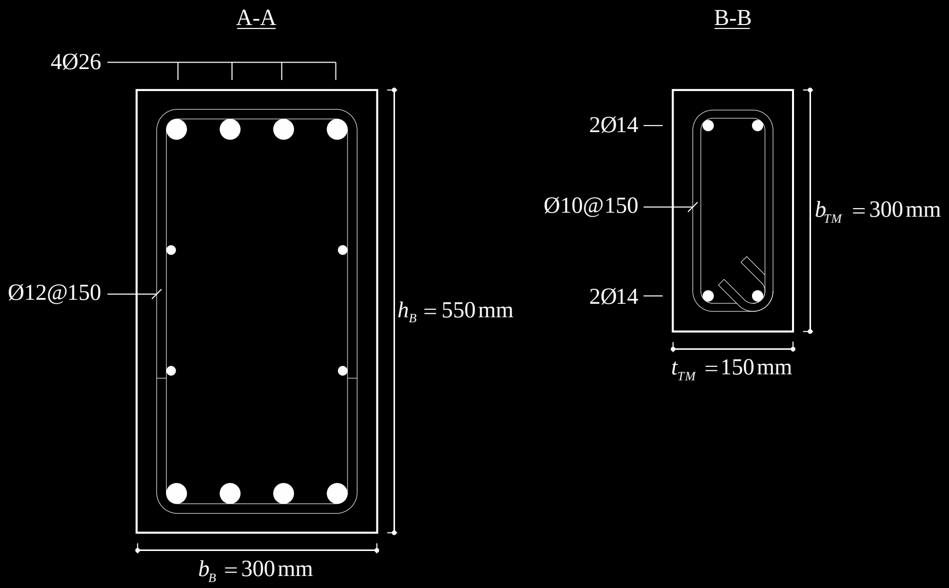
Deformation demand and deformation capacity





Load scenario 1: $Q_{d.1} = 300\,kN,\ Q_{d.2} = 0\,kN$

Load scenario 2: $Q_{d.1} = 0\,kN,\ Q_{d.2} = 300\,kN$

Assume fully cracked state: EI^{II}, EA^{II}

Task: Verify bearing capacity of the structure considering the deformation demand and the deformation capacity

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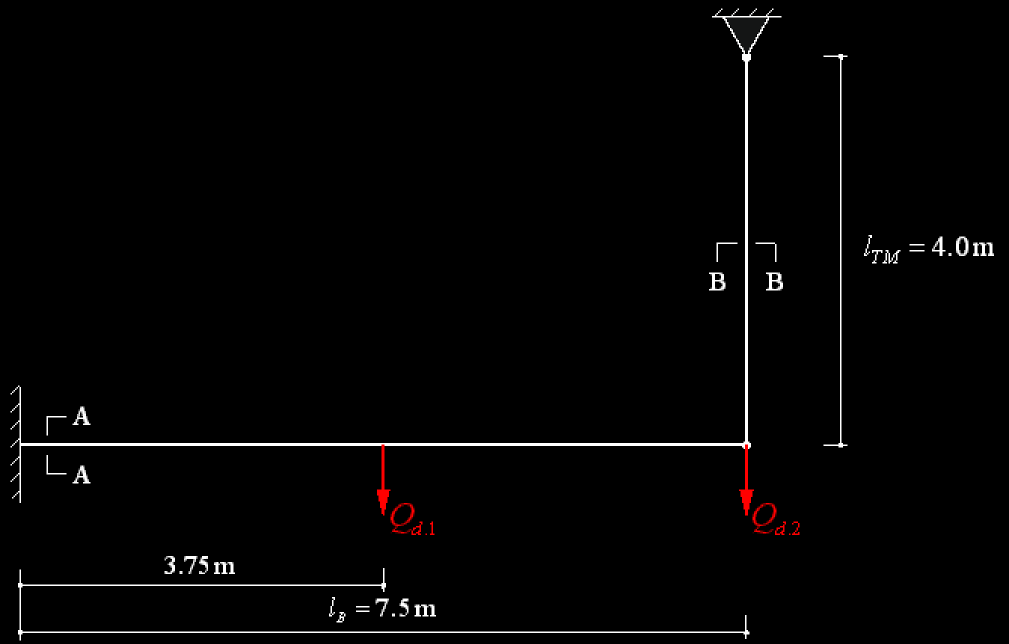
Deformation demand and deformation capacity



Load scenario 1: $Q_{d.1} = 300\,\text{kN},\ Q_{d.2} = 0\,\text{kN}$

Load scenario 2: $Q_{d.1} = 0\,\text{kN},\ Q_{d.2} = 300\,\text{kN}$

1. Determine section forces and compare them with the resistance of the structure. Are redistributions of forces necessary?

2. Calculate deformation demand

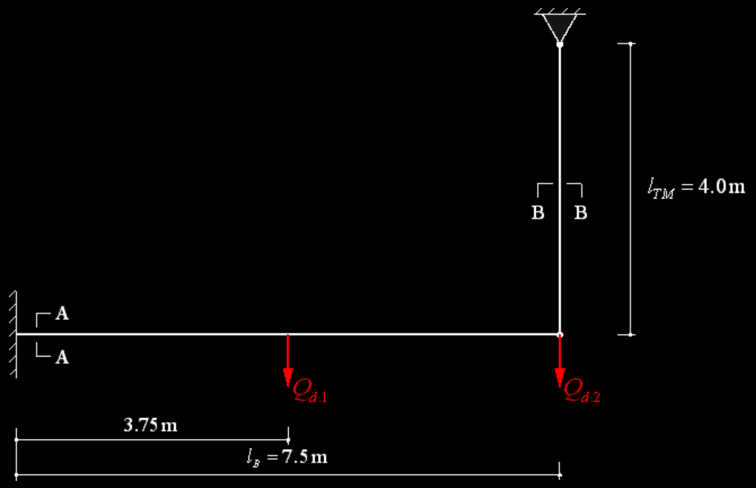
3. Calculate deformation capacity

4. Compare deformation demand vs deformation capacity

Introduction to Exercise 2

Determine section forces and compare them with the resistance of the structure.
Are redistributions of forces necessary?

Load scenario 1: $Q_{d.1} = 300\,\text{kN},\ Q_{d.2} = 0\,\text{kN}$



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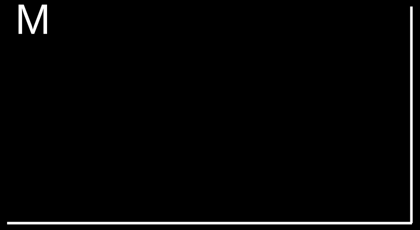

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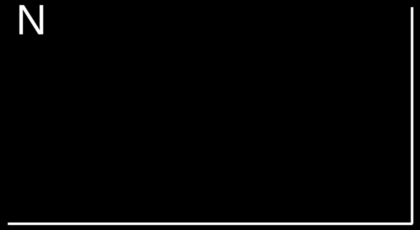
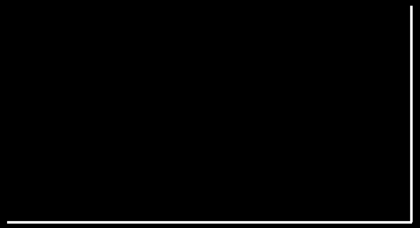


Static systems



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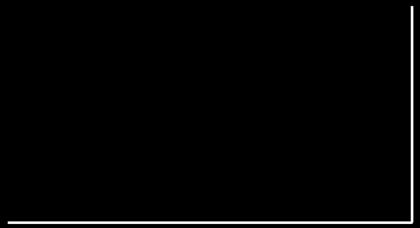

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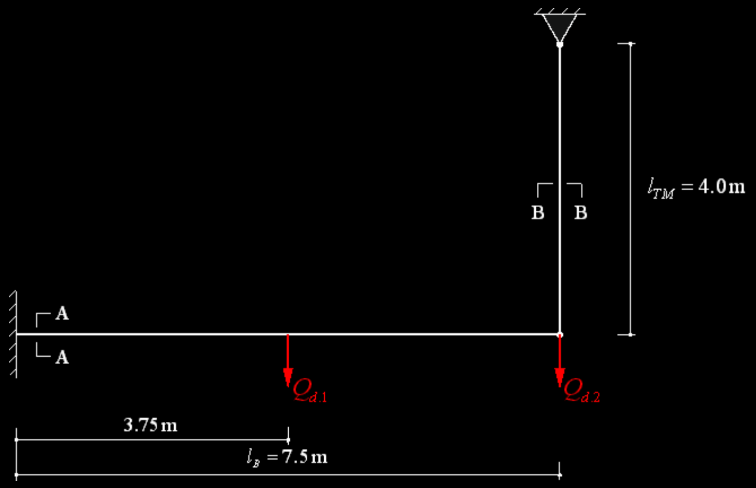
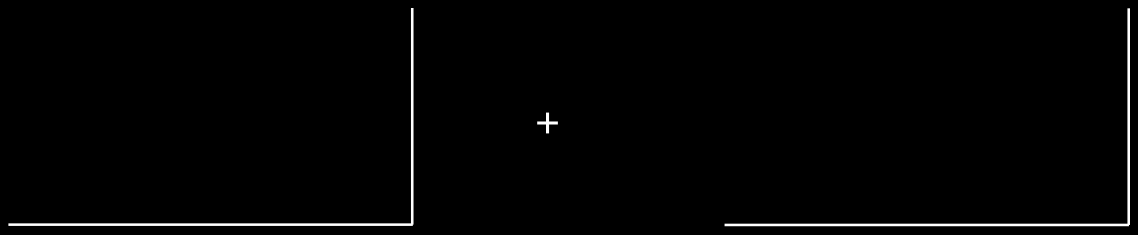
Deformation demand

Load scenario 1: $Q_{d.1} = 300\,\text{kN},\ Q_{d.2} = 0\,\text{kN}$



Static systems



Introduction to Exercise 2

Deformation capacity

Rotation capacity:

- Limitation of the plastic rotation by the reinforcing steel (rupture of the reinforcement):

$$\Theta_{pus} = L_{pl} \cdot \left(\frac{\varepsilon_{smu}}{d-x} - \frac{\varepsilon_{smy}}{d-x} \right)$$

Curvature at onset of yielding

Curvature at rupture of the reinforcement

- Limitation of the plastic rotation by the concrete (compressive failure):

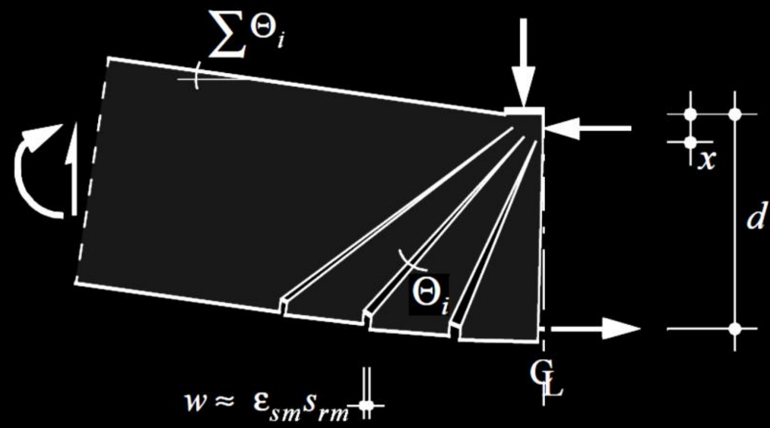
$$\Theta_{puc} = L_{pl} \left(\frac{\varepsilon_{c2d}}{x} - \frac{\varepsilon_{smy}}{d-x} \right)$$

Curvature at onset of yielding

Curvature at concrete crushing

Rotation per crack: $\Theta_i \approx \frac{\varepsilon_{sm} s_{rm}}{d-x}$

Plastic hinge rotation = sum of the plastic rotations of all cracks from the onset of yielding



L_{pl} Plastic hinge length, region in which the chord reinforcement yields

ε_{smu} Mean steel elongation when reaching $\sigma_{sr} = f_t$

ε_{smy} Mean steel elongation when reaching $\sigma_{sr} = f_s$

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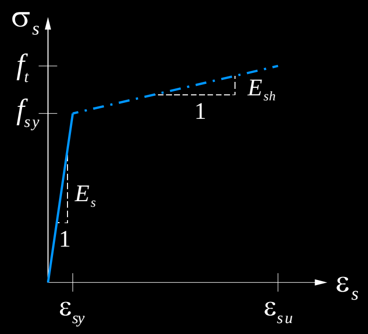
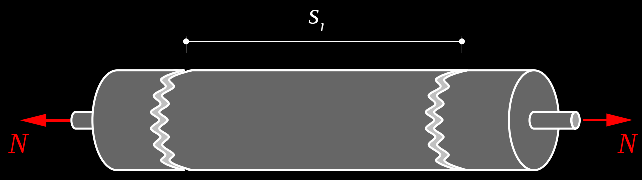
Deformation capacity

Rotation capacity:

$$\rho_{eff} = \frac{1}{\frac{M_r(d-x)E_s}{f_{ct}EI^{II}} + 1 - n}$$





① Elastic reinforcement over entire crack element:

$$\varepsilon_{sm} = \frac{\sigma_{sr}}{E_s} - \frac{\tau_{b0}s_r}{E_s\varnothing} = \frac{\sigma_{sr}}{E_s} - \lambda\frac{f_{ctm}(1-\rho)}{2E_s\rho}$$

$$\sigma_{sr} \le f_{sy}$$

② Reinforcement yields near cracks, elastic between cracks:

$$\varepsilon_{sm} = \frac{(\sigma_{sr}-f_{sy})^2\varnothing}{4E_{sh}\tau_{b1}s_r}\left(1-\frac{E_{sh}\tau_{b0}}{E_s\tau_{b1}}\right) + \frac{(\sigma_{sr}-f_{sy})}{E_s}\frac{\tau_{b0}}{\tau_{b1}} + \left(\varepsilon_{sy} - \frac{\tau_{b0}s_r}{E_s\varnothing}\right)$$

$$f_{sy} \le \sigma_{sr} \le \left(f_{sy} + \frac{2\tau_{b1}s_r}{\varnothing}\right)$$

③ Reinforcement yields over entire crack element:

$$\varepsilon_{sm} = \varepsilon_{sy} + \frac{(\sigma_{sr}-f_{sy})}{E_{sh}} - \frac{\tau_{b1}s_r}{E_{sh}\varnothing}$$

$$\left(f_{sy} + \frac{2\tau_{b1}s_r}{\varnothing}\right) \le \sigma_{sr} \le f_{su}$$

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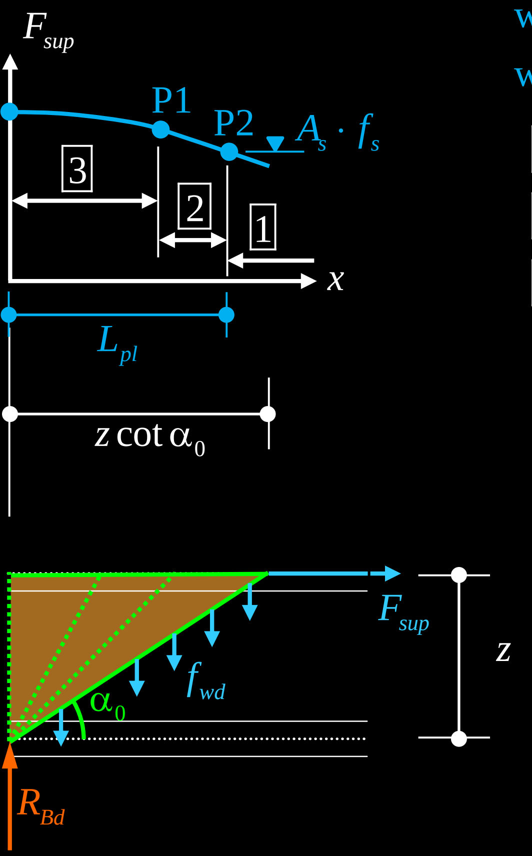
Deformation capacity

Rotation capacity:



with P1: $\sigma_{smin} = f_s$

with P2: $\sigma_{sr} = f_s$

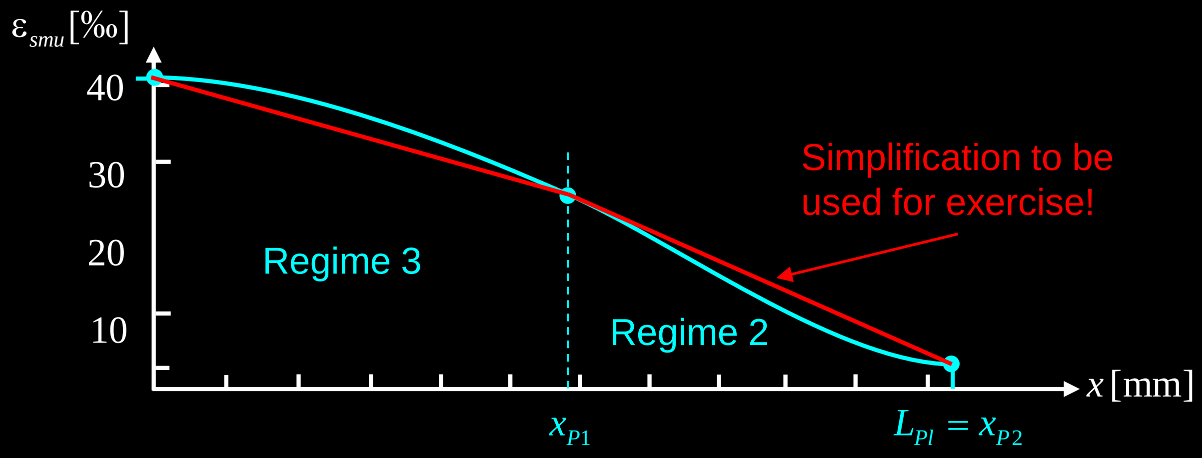
3 fully yielded

2 partially yielded

1 elastic

$$x_{P2} = \sqrt{\frac{2A_s\left(f_t - f_s\right)z}{f_{wd}}}$$

$$x_{P1} = \sqrt{\frac{2A_s\left(f_t - f_s - \frac{2\tau_{b1}s_r}{\varnothing}\right)z}{f_{wd}}}$$



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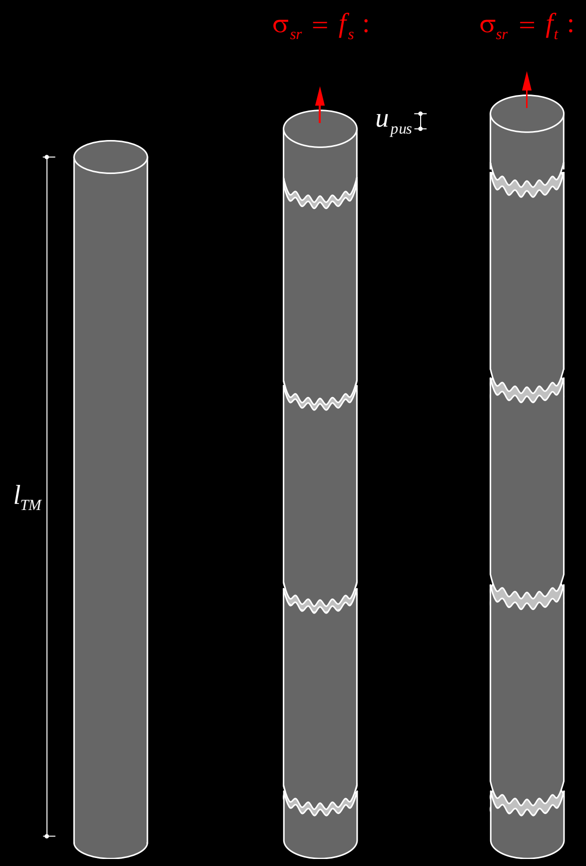
Deformation capacity

Tension member:



The deformation capacity of a tension member can be assessed as follows:

$$u_{pus} = \left(\varepsilon_{sm}\left(\sigma_{sr} = f_t\right) - \varepsilon_{sm}\left(\sigma_{sr} = f_s\right)\right) \cdot l_{TM}$$

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Exercise 2: Organisation

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