Advanced Structural Concrete

Colloquium 4

Colloquium 4

Goal of this exercise and colloquium:

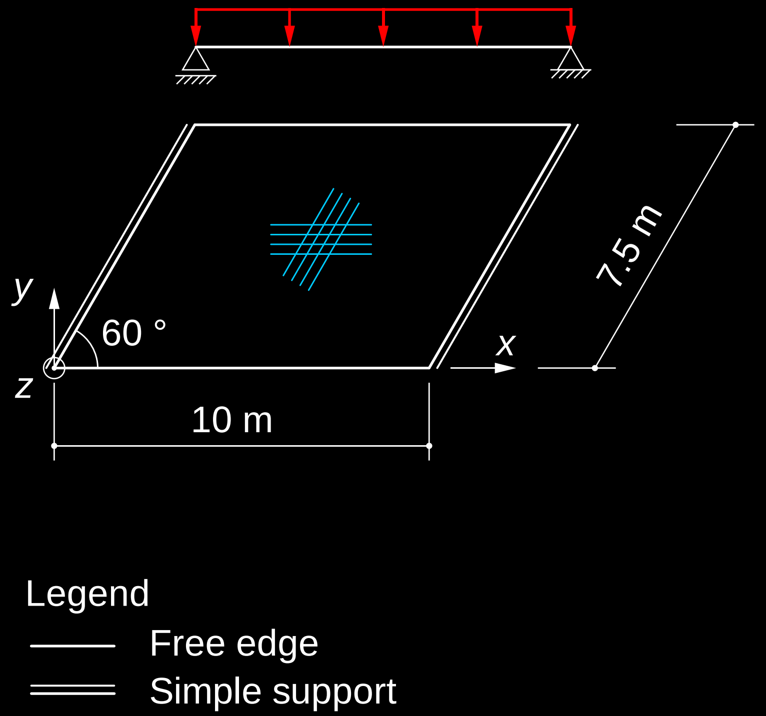
Dimension the slab using the strip method.

Dimension the slab using an elastic FEM-calculation (e.g. with CEDRUS-7, [4]).

Determine an upper limit value of the ultimate load using the yield line method.

Learn:

- FEM-Calculation: Application in practice
- Strip method: Pre-dimensioning / Control of FEM-calculations
- Transformation of skew reinforcement



For the derivation of yield conditions, check out the document on the website.

Colloquium 4

Dimensioning workflow with FEM-calculations

Documentation / check of:

- Geometry (Interaction CAD)
- Loads
- Combination of results and specifications of the limit states
- Deflections (Attention: elastic → Ultimate state: Factor 3-4 than serviceability)
- Reaction forces
- Internal forces
- Required reinforcement → reinforcement layout

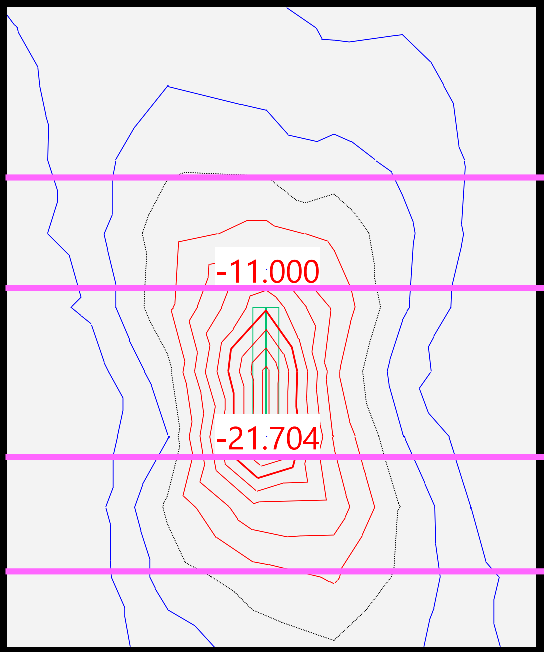
Colloquium 4

Dimensioning with FEM

Reinforcement layout

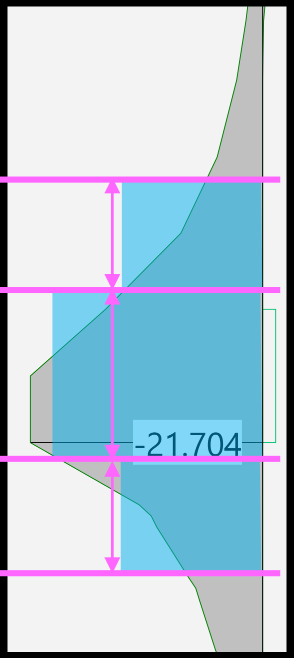
- Representative frames
  → 2 to 3 gradations for typical slabs
- Look for repetitions
- Peaks do not necessary need to be “covered”, averaging possible, (however, no real lower limit value)
- Caution when averaging over high shear forces (punching)
- Slight over-dimensioning of the field reinforcement is preferred to slight over-dimensioning of the reinforcement above the column

Cedrus-7: a_{xt} [cm^2/m]



Reference lines 5 cm^2/m,
Equidistance 2 cm^2/m

a_n [cm^2/m]



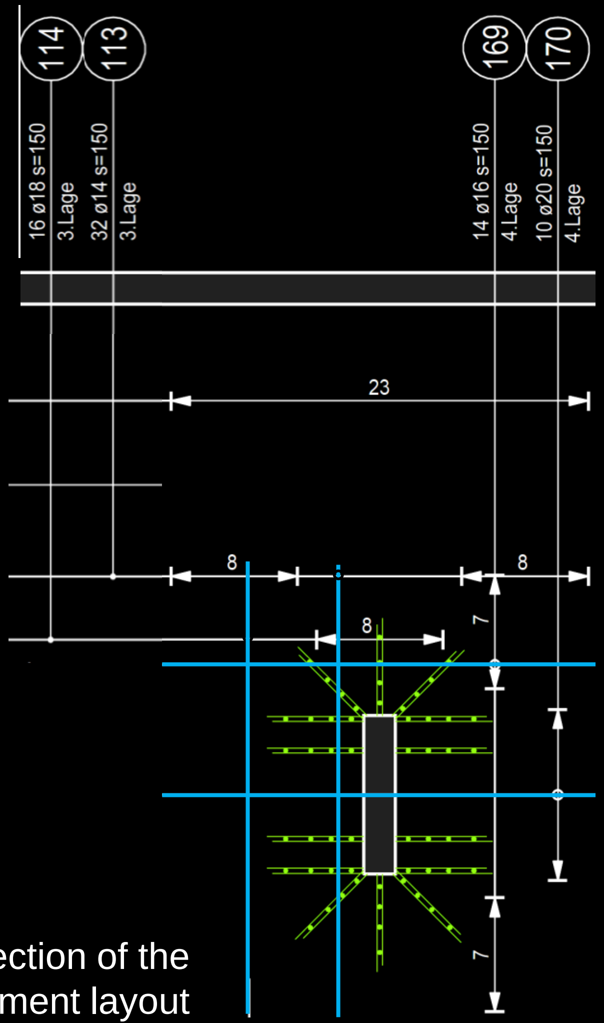
Section

Colloquium 4

Dimensioning with FEM

Reinforcement layout

- Representative frames
  → 2 to 3 gradations for typical slabs
- Look for repetitions
- Peaks do not necessary need to be “covered”, averaging possible, (however, no real lower limit value)
- Caution when averaging over high shear forces (punching)
- Slight over-dimensioning of the field reinforcement is preferred to slight over-dimensioning of the reinforcement above the column



Edited section of the reinforcement layout

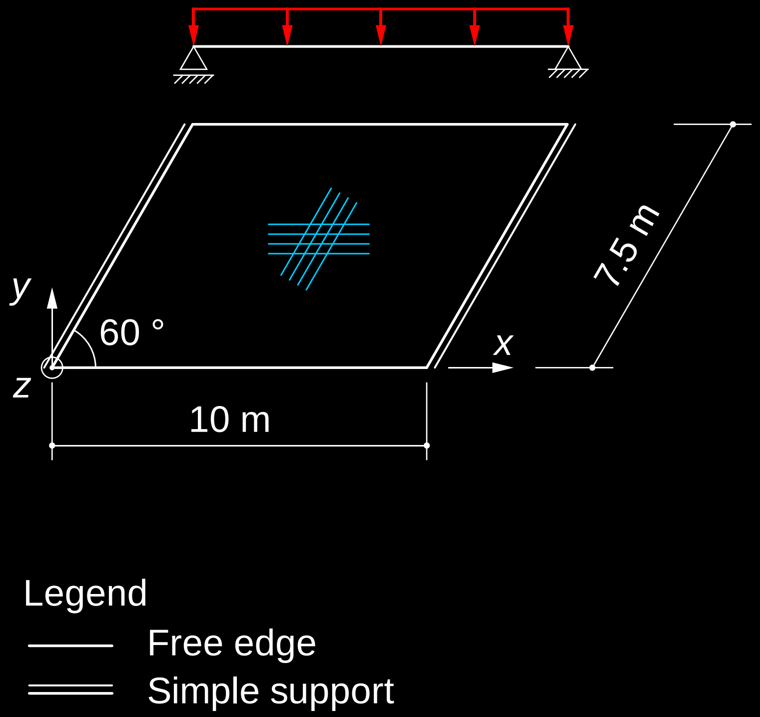
Colloquium 4

Colloquium 4

Slab with

- Thickness: h = 0.45 m
- Concrete C30/37
  → f_{cd} = 20 MPa
- Steel B500B
  → f_{sd} = 435 MPa
- Concrete cover
  c_{nom} = 55 mm

$$q_d = 1.35 \cdot \left(0.45\ \text{m} \cdot 25\ \text{kN/m}^3 + 3\ \text{kN/m}^2\right) + 1.5 \cdot 15\ \text{kN/m}^2 = 42\ \text{kN/m}^2$$



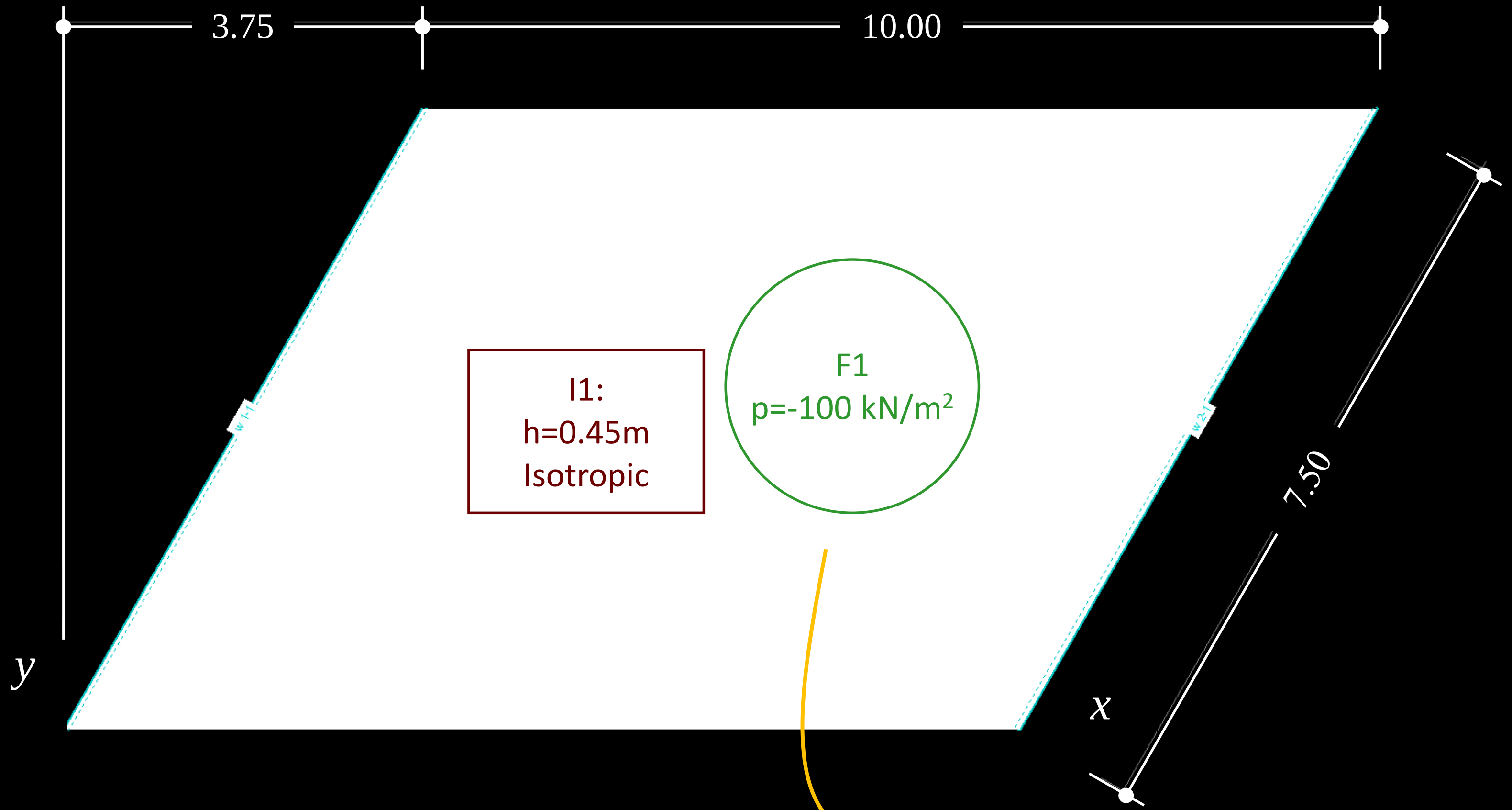
Task:

Dimension the slab by using the results from an elastic FEM-calculation

→ here only bending moments

Colloquium 4

FEM-calculations with reference load

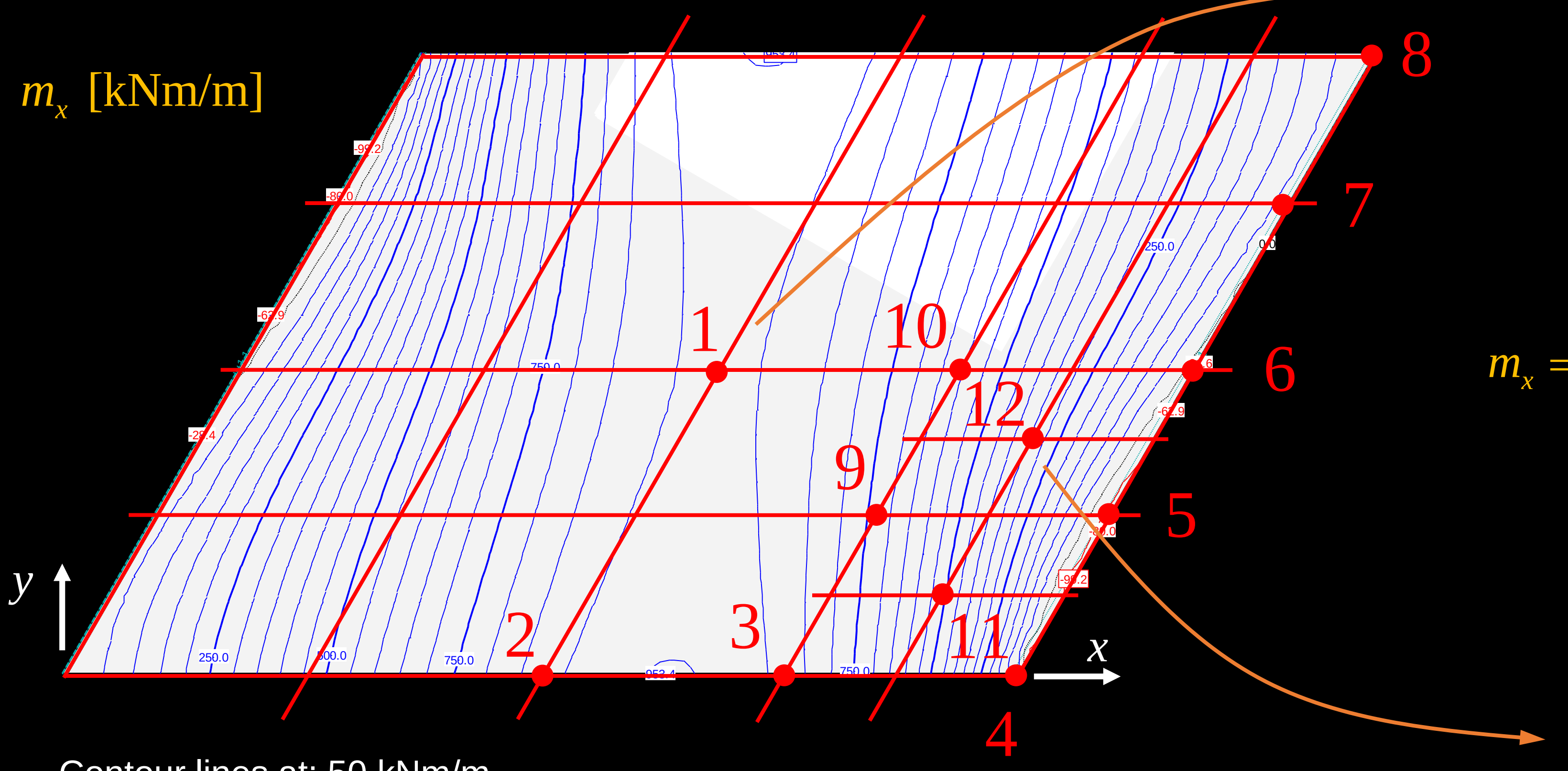


Support : Vertical stiffness $10^6 \ \mathrm{kN/m^2}$

Factor $\frac{q_d}{p} = \frac{42}{100} = 0.42$

Colloquium 4

FEM-calculations with reference load

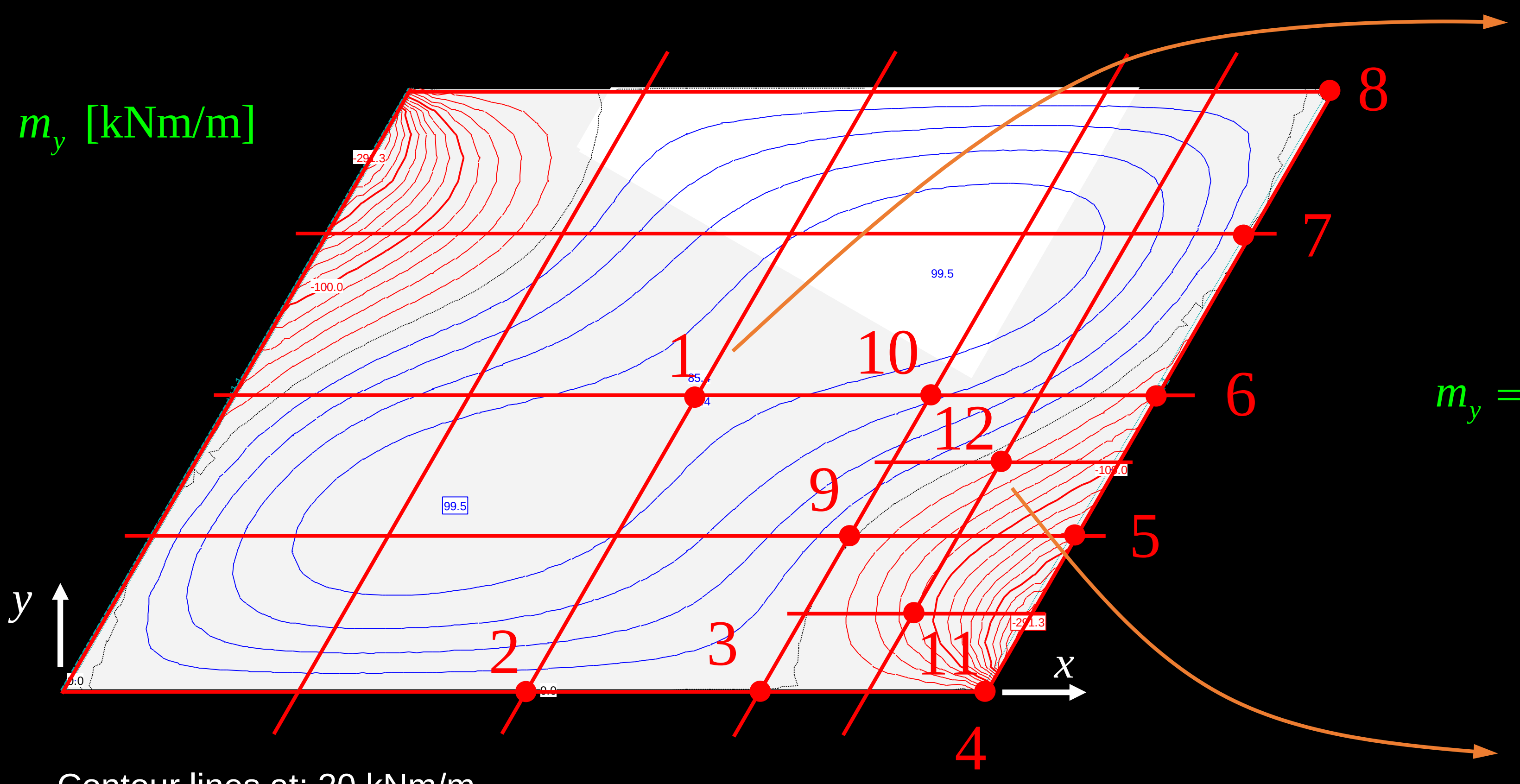


$$m_x = \begin{pmatrix} 390 \\ 365 \\ 335 \\ 32 \\ -34 \\ -38 \\ 0 \\ 0 \\ 280 \\ 235 \\ 175 \\ 110 \end{pmatrix} \text{kNm/m}$$

Contour lines at: 50 kNm/m

Colloquium 4

FEM-calculations with reference load

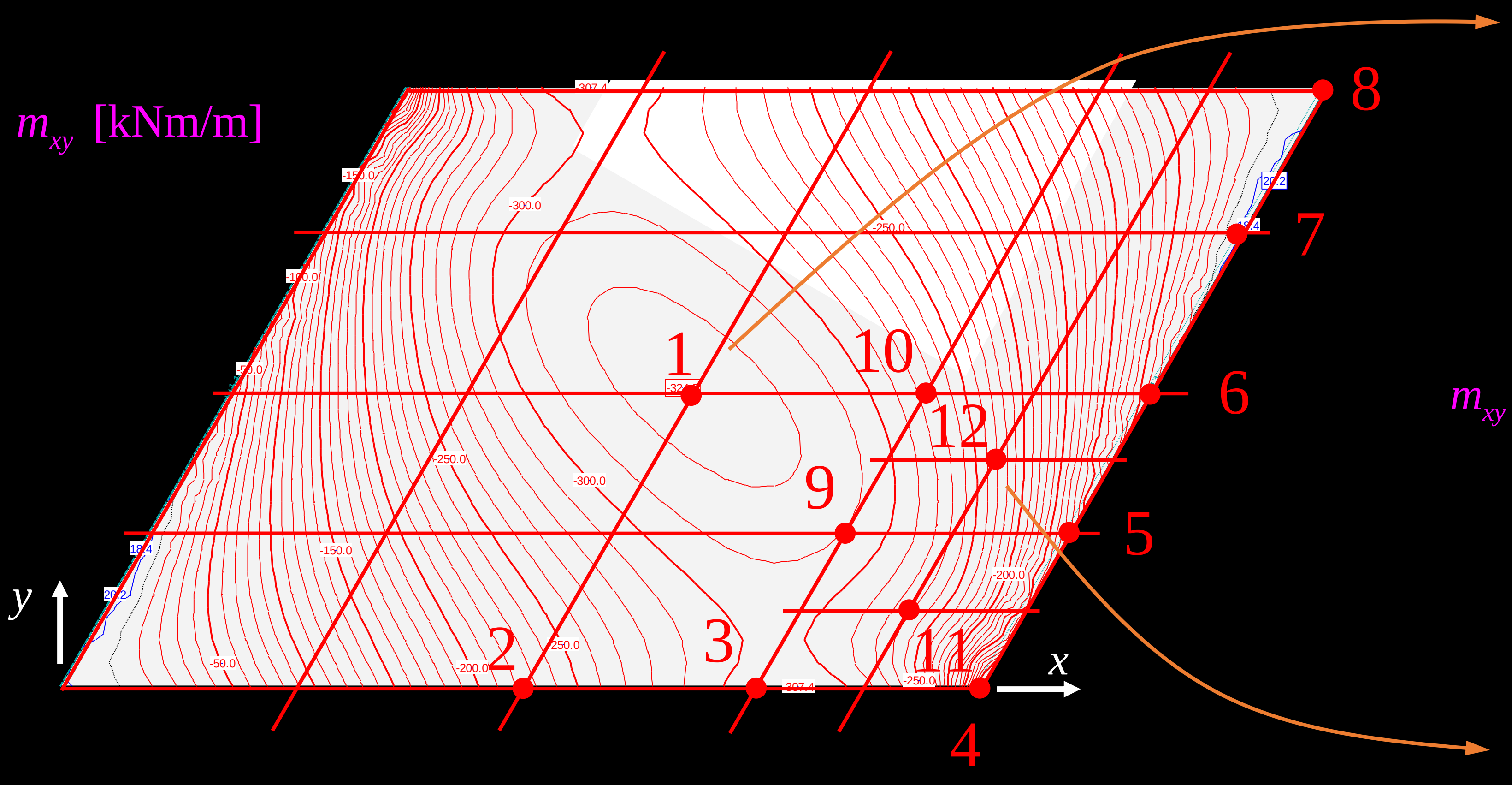


$$m_y = \begin{pmatrix} 35 \\ 0 \\ 0 \\ -65 \\ -75 \\ -17 \\ 0 \\ 0 \\ 8 \\ 27 \\ -46 \\ -17 \end{pmatrix} \text{kNm/m}$$

Contour lines at: 20 kNm/m

Colloquium 4

FEM-calculations with reference load

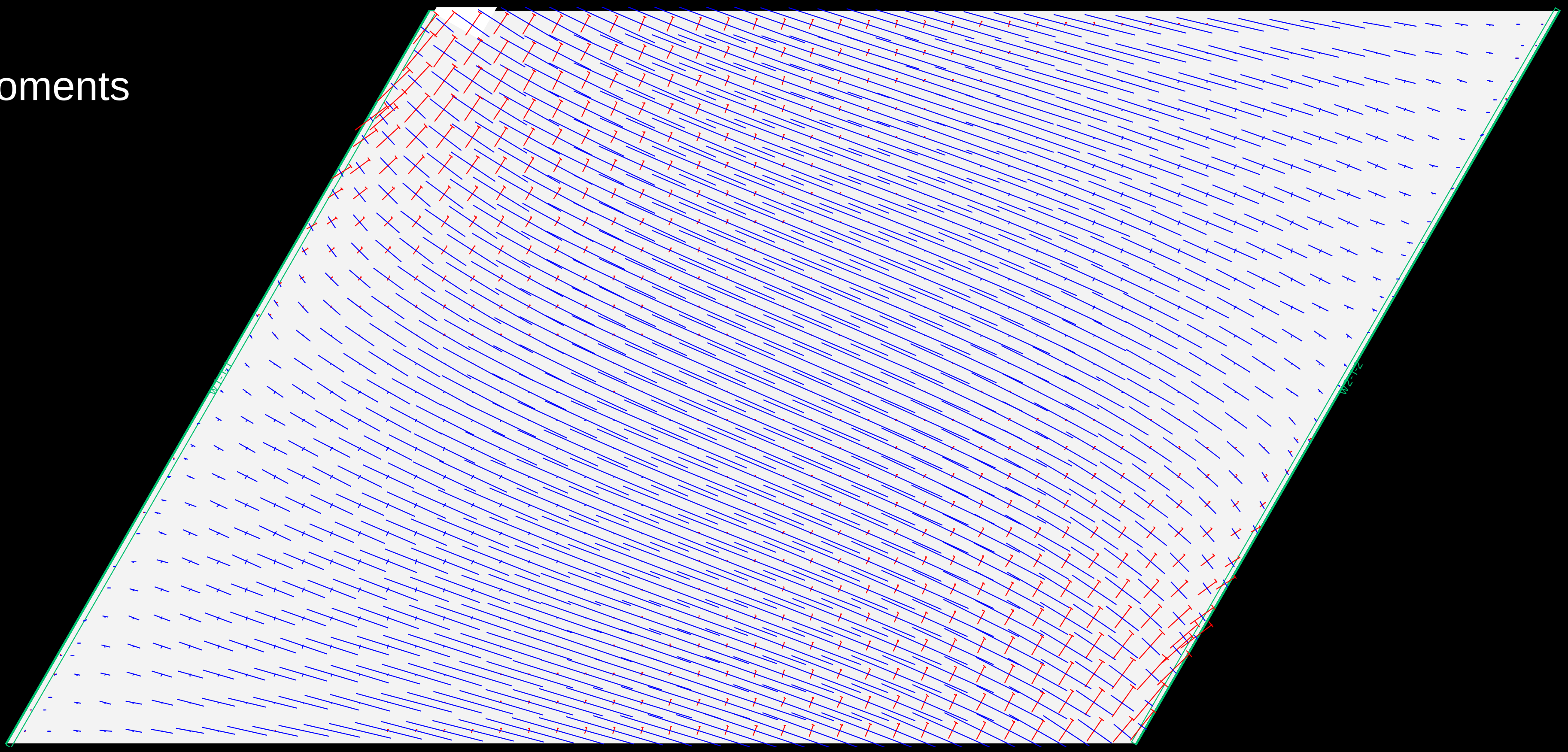


$$m_{xy} = \begin{pmatrix} -135 \\ -92 \\ -130 \\ -21 \\ -53 \\ -13 \\ 8 \\ 4 \\ -125 \\ -110 \\ -107 \\ -88 \end{pmatrix} \text{kNm/m}$$

Colloquium 4

FEM-calculations with reference load

Principal moments



Colloquium 4

Reinforcement 1./2. layer

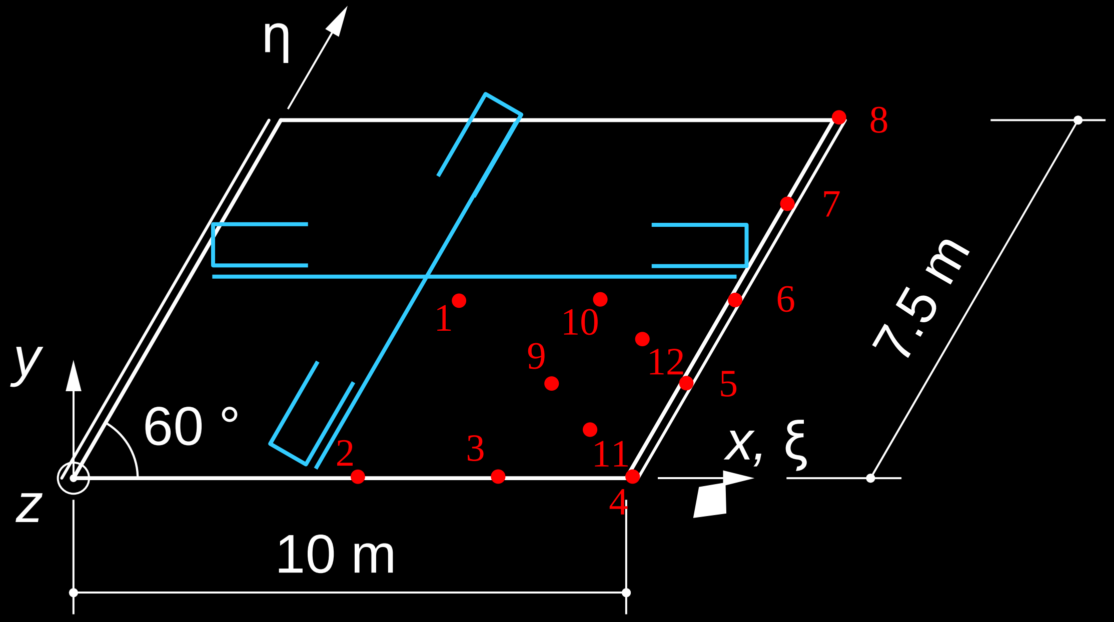
$$\varnothing_{\xi inf} = \begin{pmatrix} 26\ (s=100\ \text{mm}) \\ 26\ (s=100\ \text{mm}) \\ 26\ (s=100\ \text{mm}) \\ 14\ (s=200\ \text{mm}) \\ 14\ (s=200\ \text{mm}) \\ 14\ (s=200\ \text{mm}) \\ 14\ (s=200\ \text{mm}) \\ 14\ (s=200\ \text{mm}) \\ 26\ (s=100\ \text{mm}) \\ 26\ (s=100\ \text{mm}) \\ 26\ (s=100\ \text{mm}) \\ 26\ (s=100\ \text{mm}) \end{pmatrix} \begin{matrix} 1 \\ 2 \\ \vdots \\ \\ \\ \\ \\ \\ \\ \\ \\ 12 \end{matrix} \text{mm}$$

$$\varnothing_{\eta inf} = \begin{pmatrix} 20 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \end{pmatrix} \begin{matrix} 1 \\ 2 \\ \vdots \\ \\ \\ \\ \\ \\ \\ \\ \\ 12 \end{matrix} \text{mm}$$

$s_\eta = 200$ mm

1. Layer: ξ-Direction:
   $d_1 = 450\ \text{mm} - c_{nom} - \varnothing_1/2 = 382\ \text{mm}$

2. Layer: η-Direction:
   $d_2 = 450\ \text{mm} - c_{nom} - \varnothing_1 - \varnothing_2/2 = 359\ \text{mm}$



Colloquium 4

Reinforcement 3./4. layer

$$\varnothing_{\xi sup} = \begin{pmatrix} 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \end{pmatrix} \text{mm} \quad \begin{matrix} 1 \\ 2 \\ \vdots \\ \\ \\ \\ \\ \\ \\ \\ \\ 12 \end{matrix}$$

$s_\xi = 200$ mm

$$\varnothing_{\eta sup} = \begin{pmatrix} 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \\ 14 \end{pmatrix} \text{mm} \quad \begin{matrix} 1 \\ 2 \\ \vdots \\ \\ \\ \\ \\ \\ \\ \\ \\ 12 \end{matrix}$$

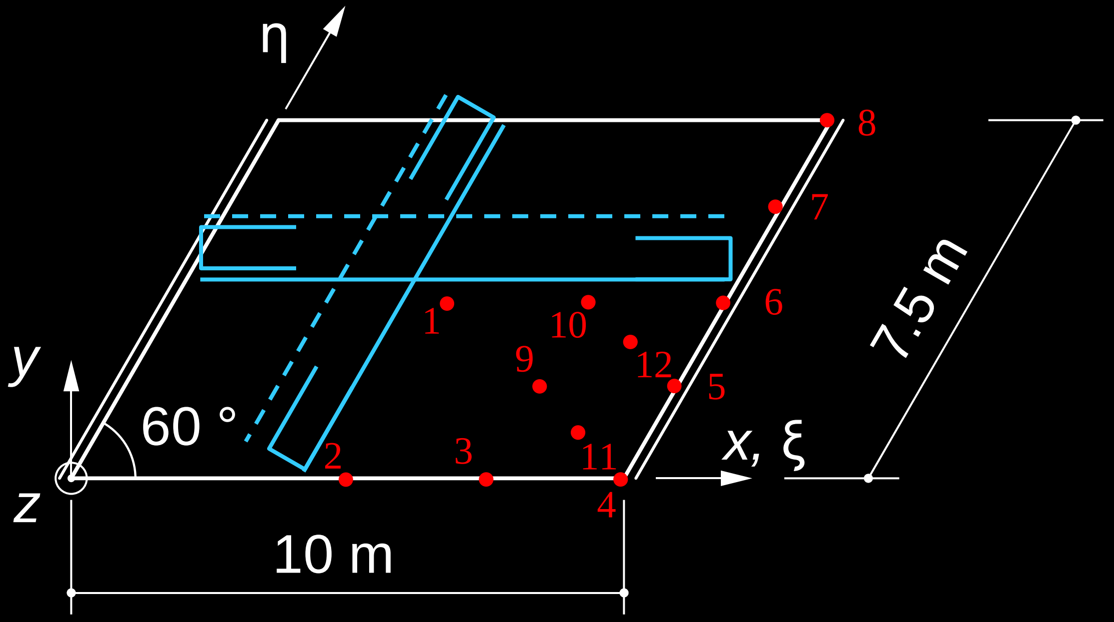
$s_\eta = 200$ mm

4. Layer: ξ-Direction:

$$d_4 = 450 \text{ mm} - c_{nom} - \varnothing_4/2 = 388 \text{ mm}$$

3. Layer: η-Direction:

$$d_3 = 450 \text{ mm} - c_{nom} - \varnothing_4 - \varnothing_3/2 = 374 \text{ mm}$$



Colloquium 4

Bending resistance in skewed reinforcement directions

Lower reinforcement:

$$m_{\xi u,inf} = a_{s\xi,inf} \cdot f_{sd} \left(d_1 - \frac{a_{s\xi,inf} \cdot f_{sd}}{2 \cdot f_{cd}} \right)$$

$$m_{\eta u,inf} = a_{s\eta,inf} \cdot f_{sd} \left(d_2 - \frac{a_{s\eta,inf} \cdot f_{sd}}{2 \cdot f_{cd}} \right)$$

$$m_{\xi u,inf} = \begin{pmatrix} 749 \\ 749 \\ 749 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 749 \\ 749 \\ 749 \\ 749 \end{pmatrix} \text{kNm/m} \qquad m_{\eta u,inf} = \begin{pmatrix} 234 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \end{pmatrix} \text{kNm/m}$$

Upper reinforcement:

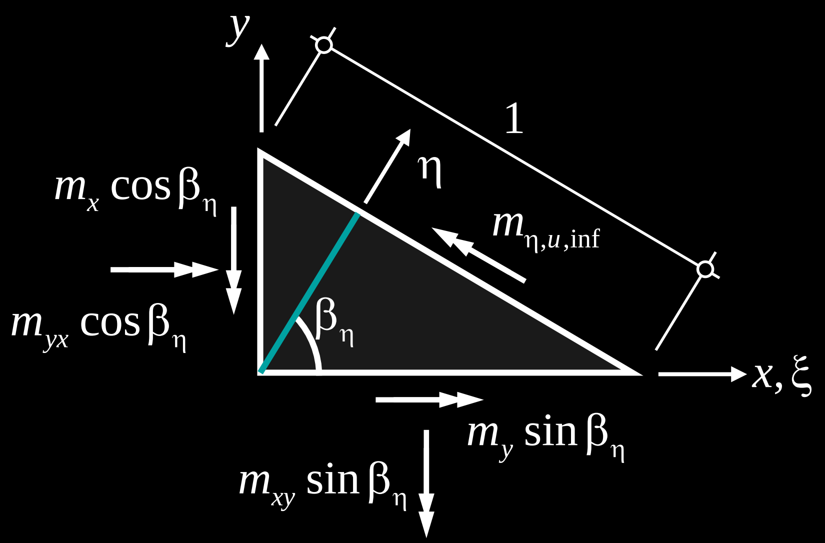
$$m_{\xi u,sup} = a_{s\xi,sup} \cdot f_{sd} \left(d_4 - \frac{a_{s\xi,sup} \cdot f_{sd}}{2 \cdot f_{cd}} \right)$$

$$m_{\eta u,sup} = a_{s\eta,sup} \cdot f_{sd} \left(d_3 - \frac{a_{s\eta,sup} \cdot f_{sd}}{2 \cdot f_{cd}} \right)$$

$$m_{\xi u,sup} = \begin{pmatrix} 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \end{pmatrix} \text{kNm/m} \qquad m_{\eta u,sup} = \begin{pmatrix} 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \end{pmatrix} \text{kNm/m}$$

Colloquium 4

Transformation of the bending resistance in the global *x,y* coordinates



Lower reinforcement: $\beta_\xi = 0°, \quad \beta_\eta = 60°$

$$m_{xu,inf} = m_{\xi u,inf} \cos^2\beta_\xi + m_{\eta u,inf} \cos^2\beta_\eta$$

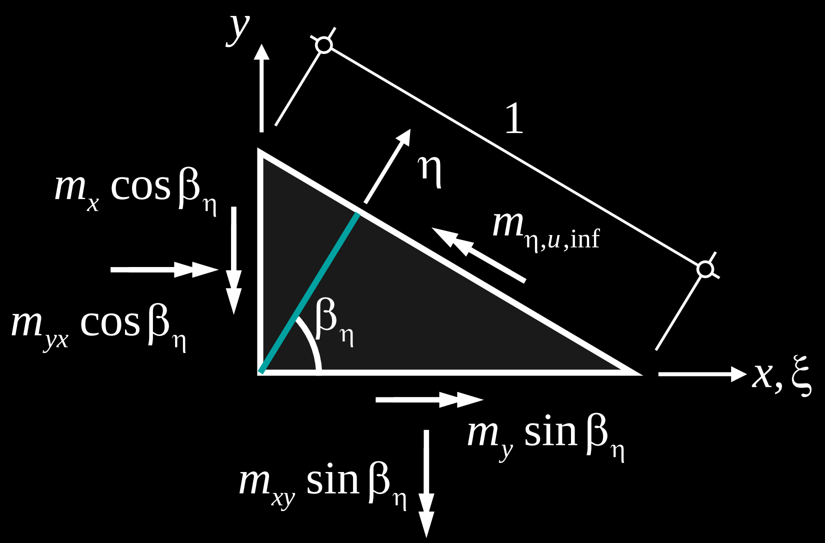
$$m_{yu,inf} = m_{\xi u,inf} \sin^2\beta_\xi + m_{\eta u,inf} \sin^2\beta_\eta$$

$$m_{xyu,inf} = m_{\xi u,inf} \sin\beta_\xi \cos\beta_\xi + m_{\eta u,inf} \sin\beta_\eta \cos\beta_\eta$$

$$m_{xu,inf} = \begin{pmatrix} 807 \\ 778 \\ 778 \\ 157 \\ 157 \\ 157 \\ 157 \\ 157 \\ 778 \\ 778 \\ 778 \\ 778 \end{pmatrix} \text{kNm/m} \qquad m_{yu,inf} = \begin{pmatrix} 175 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \end{pmatrix} \text{kNm/m} \qquad m_{xyu,inf} = \begin{pmatrix} 101 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \end{pmatrix} \text{kNm/m}$$

Colloquium 4

Transformation of the bending resistance in the global *x,y* coordinates



Upper reinforcement:

$$\beta_\xi = 0°, \quad \beta_\eta = 60°$$

$$m_{xu,sup} = m_{\xi us,up} \cos^2\beta_\xi + m_{\eta u,sup} \cos^2\beta_\eta$$

$$m_{yu,inf} = m_{\xi u,sup} \sin^2\beta_\xi + m_{\eta u,sup} \sin^2\beta_\eta$$

$$m_{xyu,inf} = m_{\xi u,sup} \sin\beta_\xi \cos\beta_\xi + m_{\eta u,sup} \sin\beta_\eta \cos\beta_\eta$$

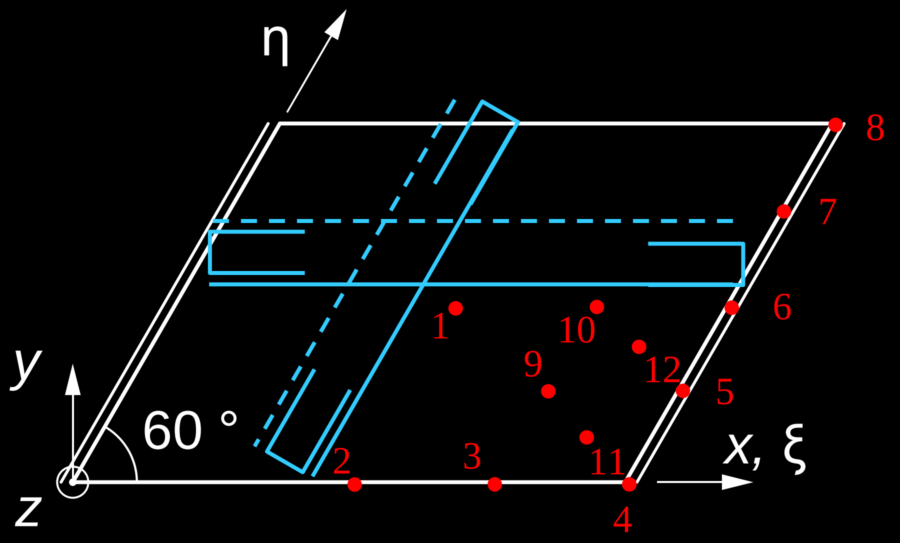
$$m_{xu,sup} = \begin{pmatrix} 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \end{pmatrix} \text{kNm/m} \qquad m_{yu,sup} = \begin{pmatrix} 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \end{pmatrix} \text{kNm/m} \qquad m_{xyu,sup} = \begin{pmatrix} 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \end{pmatrix} \text{kNm/m}$$

Colloquium 4

Control of the yield condition

for positive bending resistance :

$$Y_{inf} = \left(m_{xyu,inf} - m_{xy}\right)^2 - \left(m_{xu,inf} - m_x\right)\left(m_{yu,inf} - m_y\right) = \begin{pmatrix} -2742 \\ -16193 \\ -6524 \\ -13957 \\ -20366 \\ -16469 \\ -12043 \\ -11681 \\ -9208 \\ -7581 \\ -56302 \\ -51331 \end{pmatrix} \begin{matrix} 1 \\ 2 \\ \vdots \\ \\ \\ \\ \\ \\ \\ \\ \\ 12 \end{matrix} \left(\text{kNm/m}\right)^2 \leq 0$$



for negative bending resistance :

$$Y_{sup} = \left(m_{xyu,sup} + m_{xy}\right)^2 - \left(m_{xu,sup} + m_x\right)\left(m_{yu,sup} + m_y\right) = \ldots$$

Introduction to Exercise 4

Exercise 4: Organisation

Handout: 14.12.2023

Voluntary submission for correction: 20.12.2023

Publication solution: 21.12.2023