

# Advanced Structural Concrete

## Colloquium 3

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## Exercise 3

Dimension the slab using **the strip method**.

Dimension the slab using **an elastic FEM-calculation** (e.g. with CEDRUS-7, [4]).

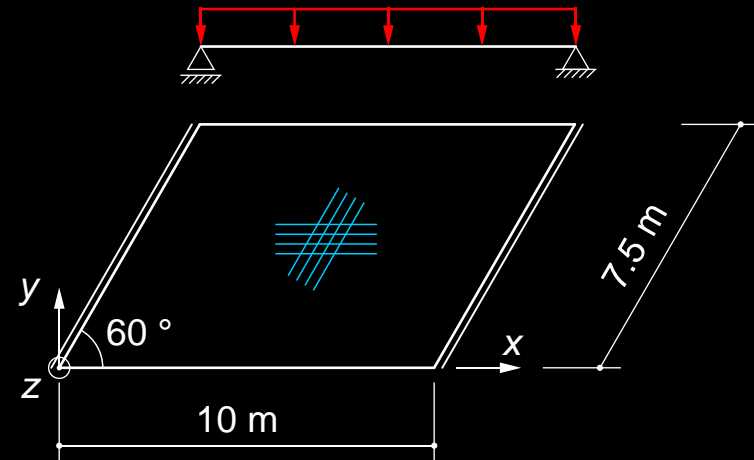
Determine an upper limit value of the ultimate load using the **yield line method**.

### Educational content

- FEM-Calculation:  
Application in practice
- Strip method:  
Pre-dimensioning / Control of FEM-calculations
- Transformation of skew reinforcement

### Derivation of yield conditions

→ Separate document on website



### Legend

- Free edge
- == Simple support

# Colloquium 3

## Dimensioning workflow with FEM

Documentation / check of:

- Geometry (Interaction CAD)
- Loads
- Combination of results and specifications of the limit states
- Deflections (Attention: elastic → Ultimate state: Factor 3-4)
- Reactions
- Internal forces
- Required reinforcement → reinforcement layout

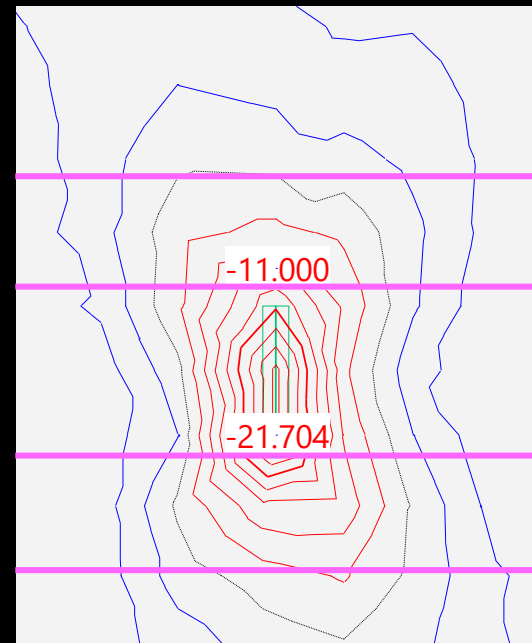
# Colloquium 3

## Dimensioning with FEM

### Reinforcement layout

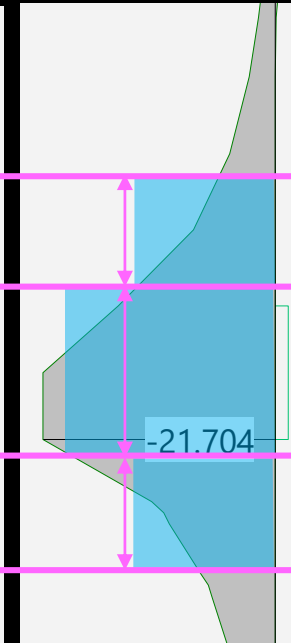
- Representative frames  
→ 2 to 3 gradations for typical slabs
- Look for repetitions
- Peaks do not necessary need to be “covered”, averaging possible, (however, no real lower limit value)
- Caution when averaging over high shear forces (punching)
- Slight over-dimensioning of the field reinforcement is preferred to slight over-dimensioning of the reinforcement above the column

Cedrus-7:  $a_{xt}$  [ $\text{cm}^2/\text{m}$ ]



Reference lines  $5 \text{ cm}^2/\text{m}$ ,  
Equidistance  $2 \text{ cm}^2/\text{m}$

$a_n$  [ $\text{cm}^2/\text{m}$ ]



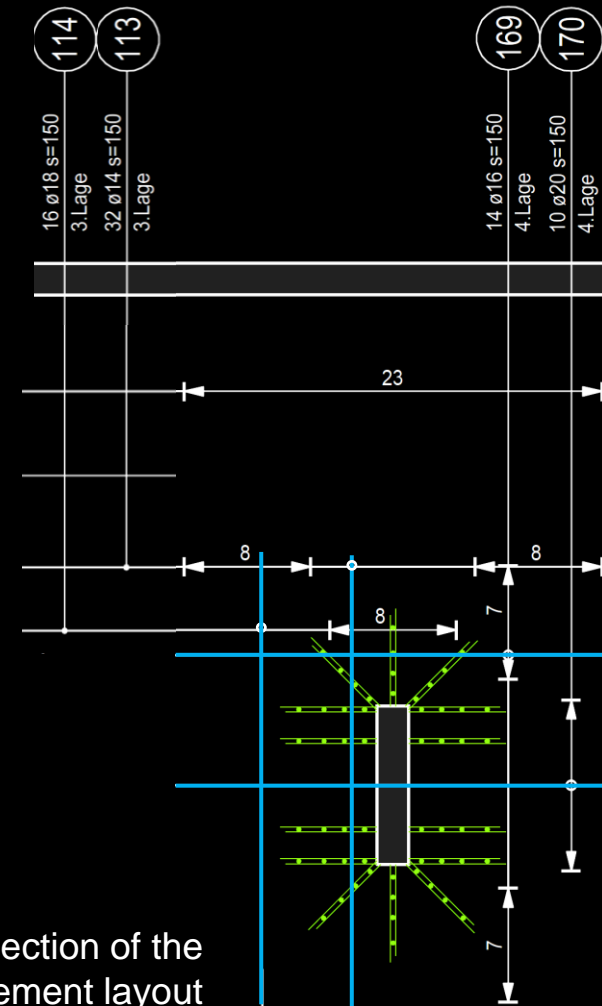
Section

# Colloquium 3

## Dimensioning with FEM

### Reinforcement layout

- Representative frames  
→ 2 to 3 gradations for typical slabs
- Look for repetitions
- Peaks do not necessary need to be “covered”, averaging possible, (however, no real lower limit value)
- Caution when averaging over high shear forces (punching)
- Slight over-dimensioning of the field reinforcement is preferred to slight over-dimensioning of the reinforcement above the column



Edited section of the reinforcement layout

# Colloquium 3

## Colloquium 3

Slab with

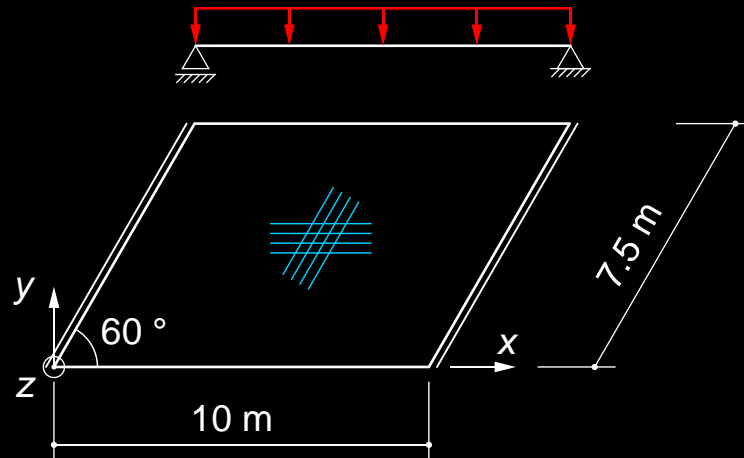
- $h = 0.45 \text{ m}$
- Concrete C30/37  
→  $f_{cd} = 20 \text{ MPa}$
- Steel B500B  
→  $f_{sd} = 435 \text{ MPa}$
- Concrete cover  
 $c_{nom} = 55 \text{ mm}$

Task:

Dimension the slab by using the results from an elastic FEM-calculation

→ here only bending moments

$$q_d = 1.35 \cdot (0.45 \text{ m} \cdot 25 \text{ kN/m}^3 + 3 \text{ kN/m}^2) + 1.5 \cdot 15 \text{ kN/m}^2 = 42 \text{ kN/m}^2$$

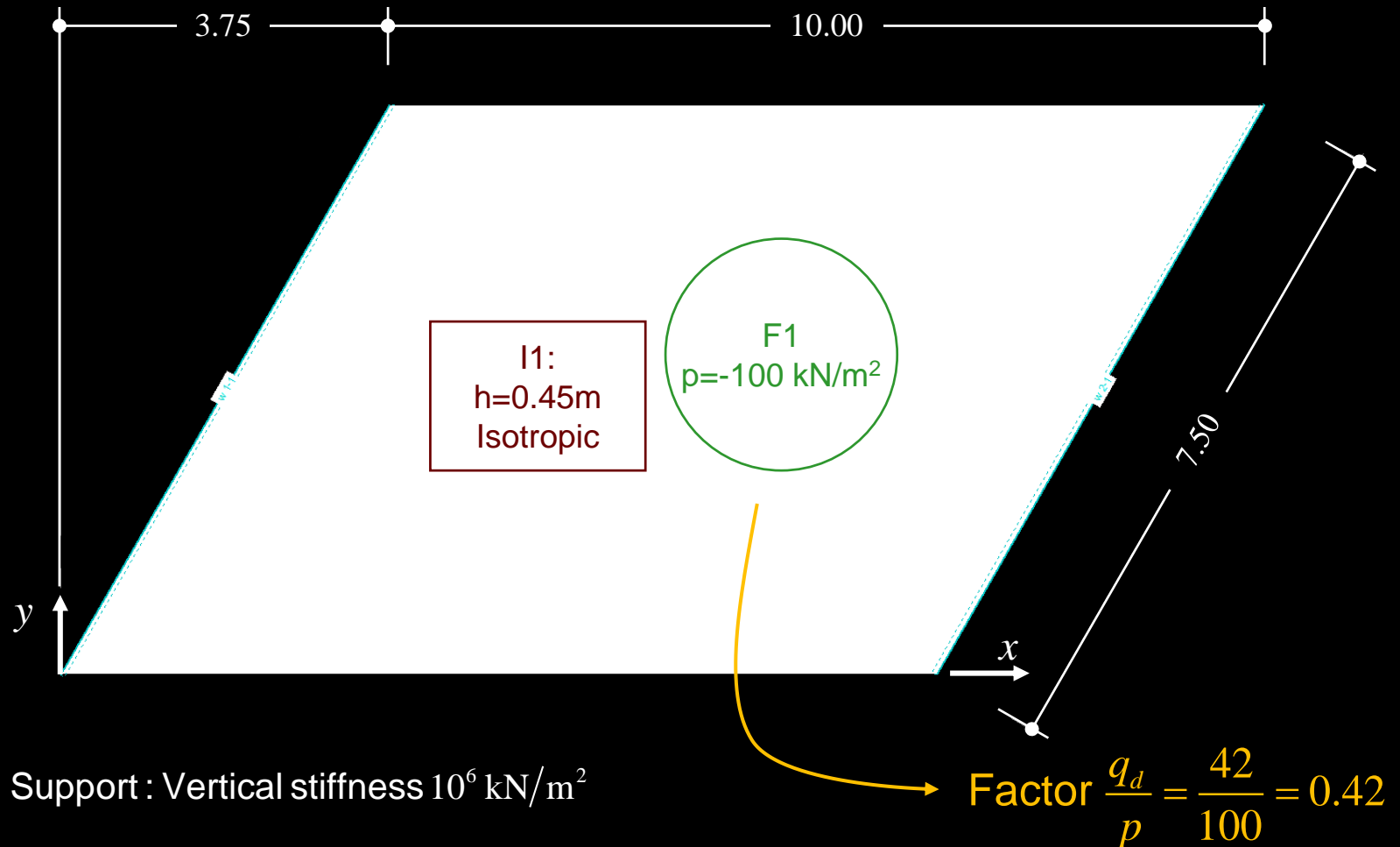


Legend

- Free edge
- == Simple support

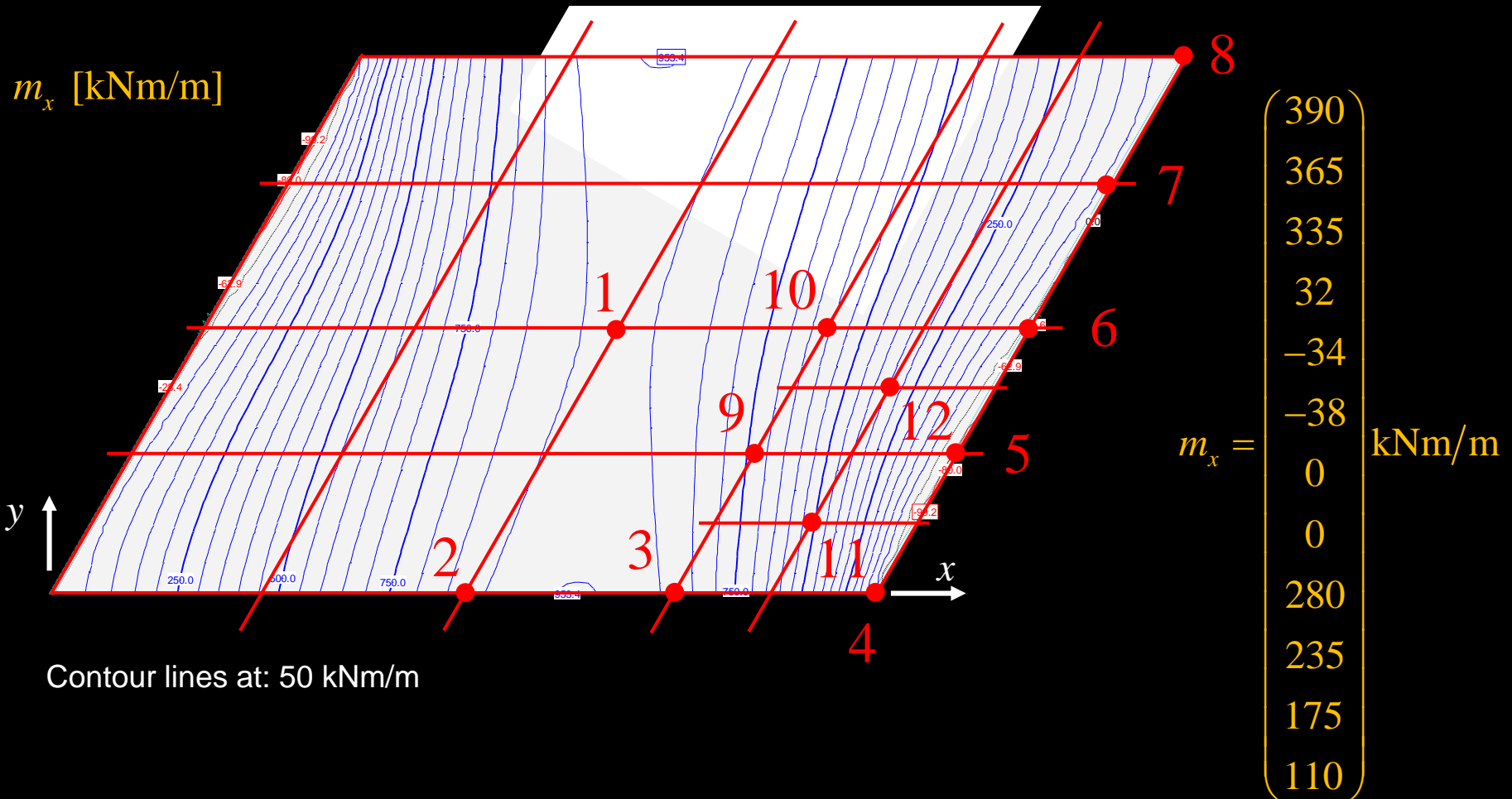
# Colloquium 3

FEM-calculations with reference load



# Colloquium 3

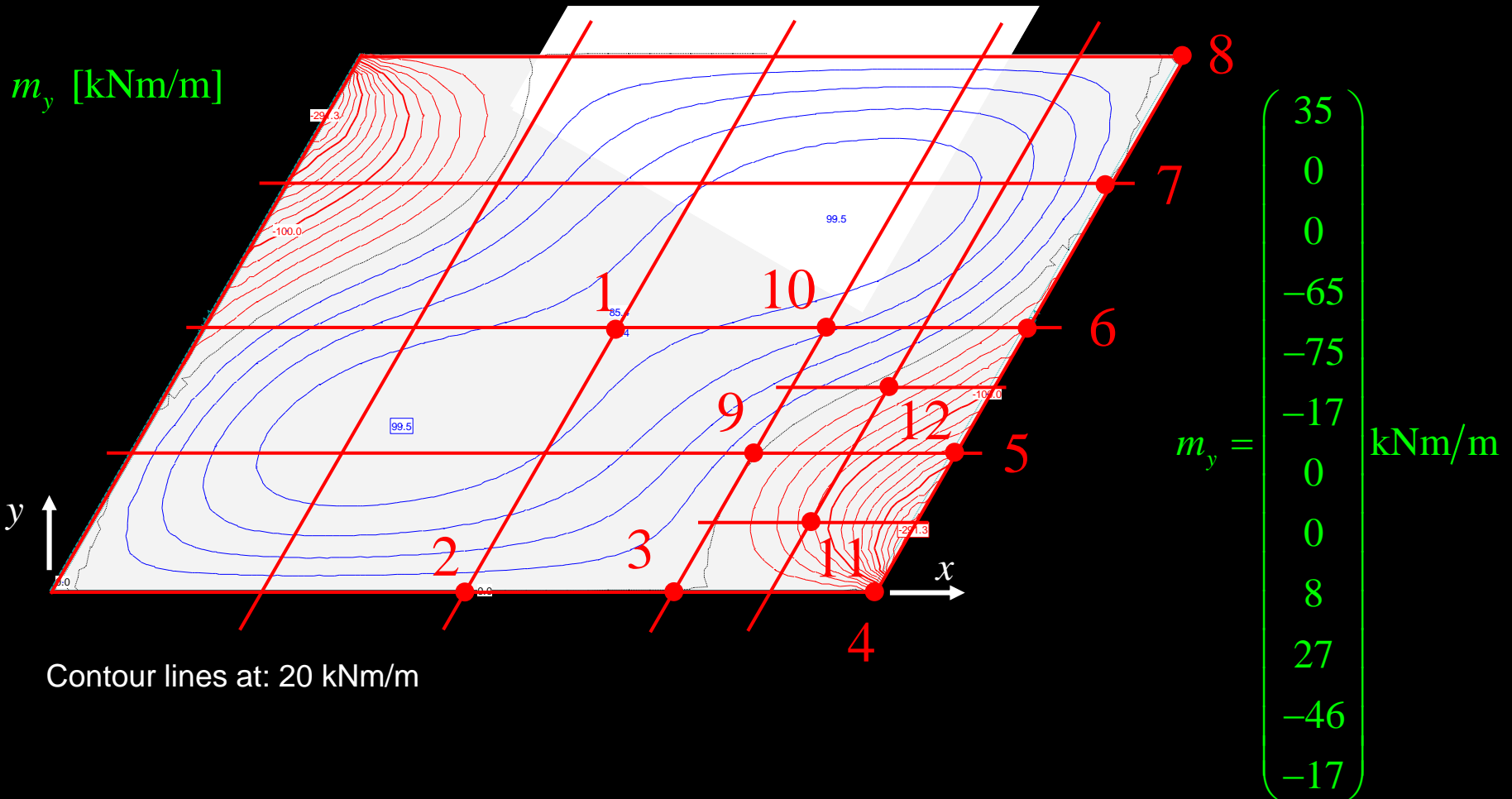
FEM-calculations with reference load





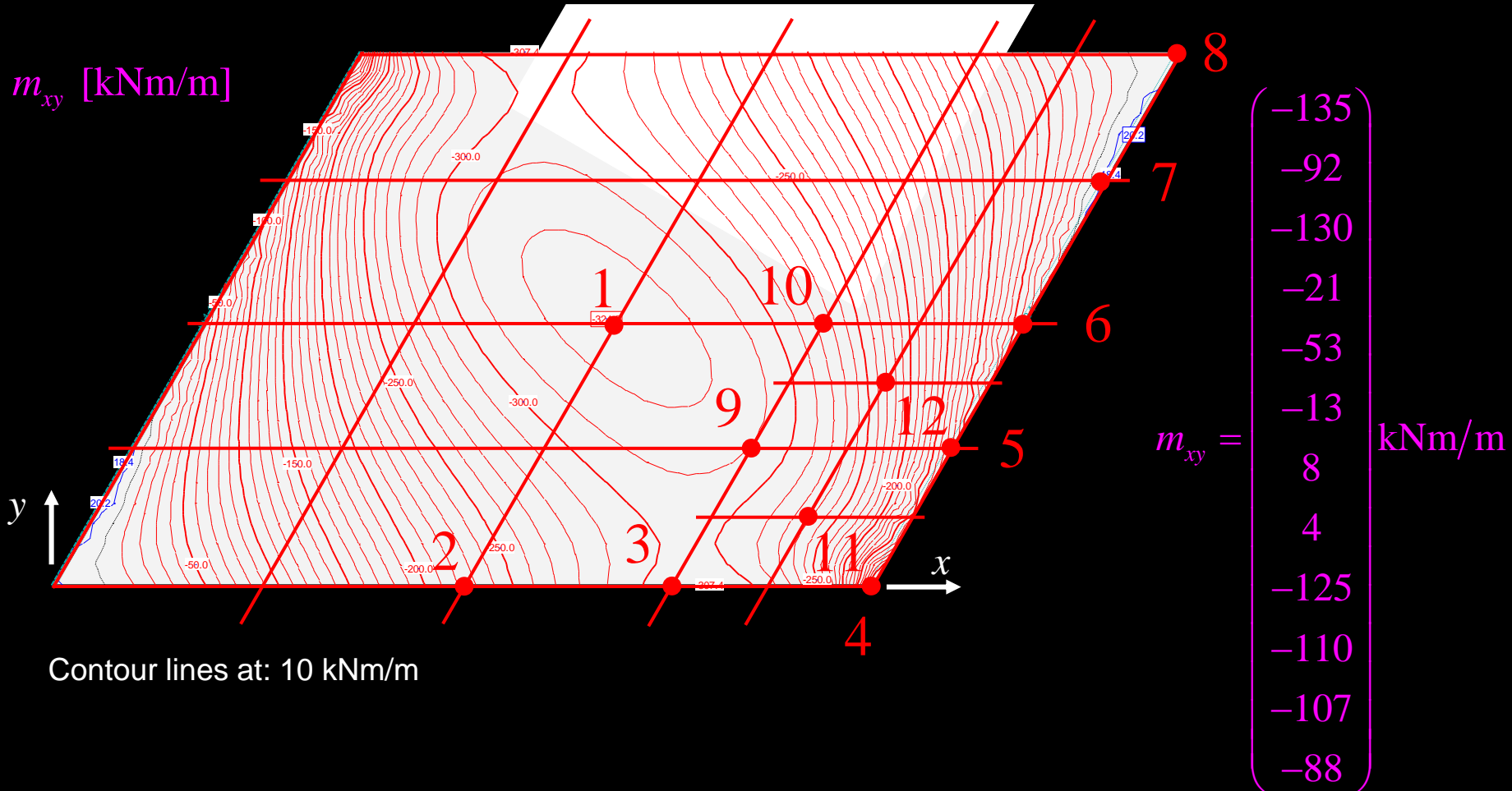
# Colloquium 3

FEM-calculations with reference load



# Colloquium 3

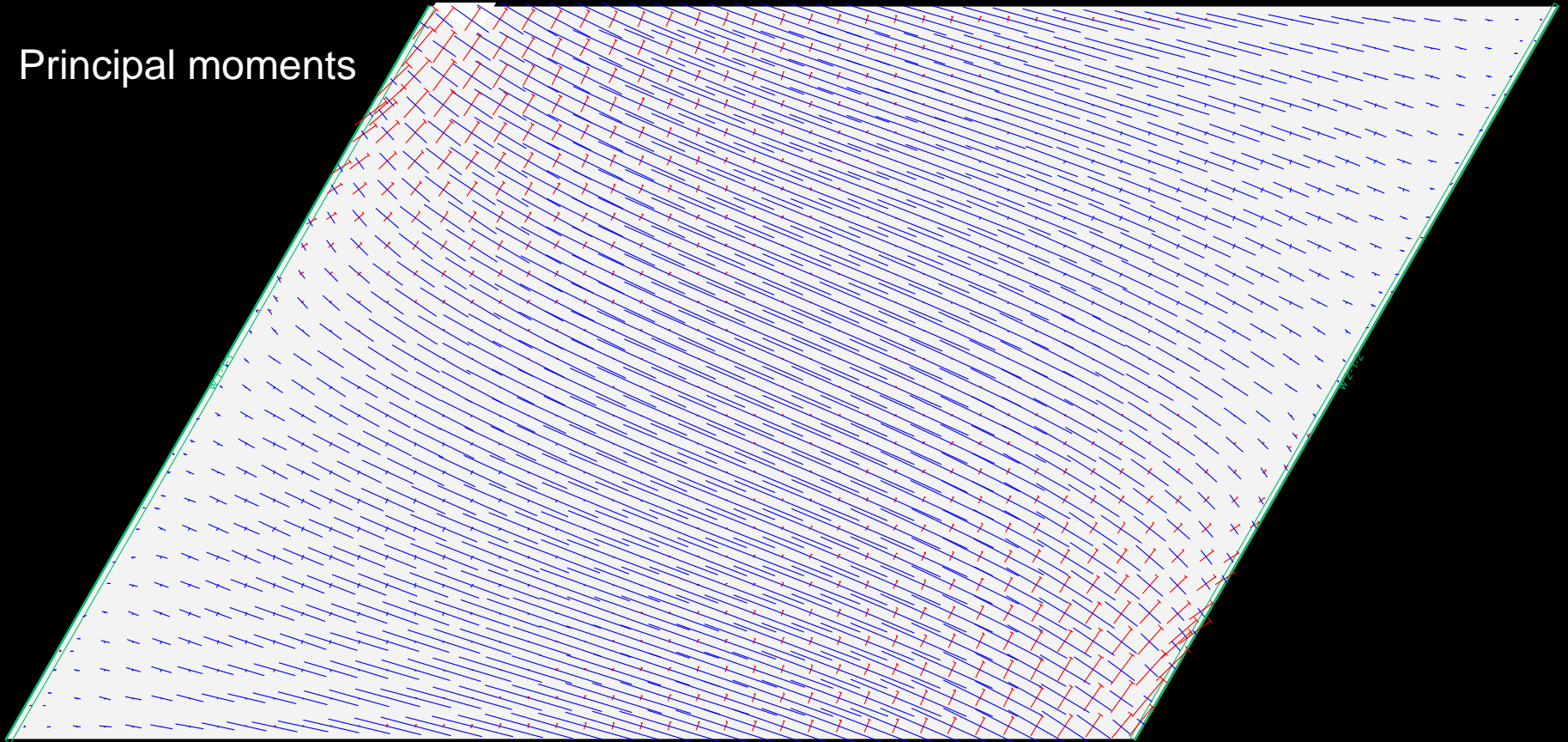
FEM-calculations with reference load



# Colloquium 3

FEM-calculations with reference load

Principal moments



# Colloquium 3

## Reinforcement 1./2. layer

$\varnothing_{\xi inf} =$	26 ( $s = 100 \text{ mm}$ )	mm
	26 ( $s = 100 \text{ mm}$ )	
	26 ( $s = 100 \text{ mm}$ )	
	14 ( $s = 200 \text{ mm}$ )	
	14 ( $s = 200 \text{ mm}$ )	
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	14 ( $s = 200 \text{ mm}$ )	
	14 ( $s = 200 \text{ mm}$ )	
	14 ( $s = 200 \text{ mm}$ )	
	26 ( $s = 100 \text{ mm}$ )	
	26 ( $s = 100 \text{ mm}$ )	
	26 ( $s = 100 \text{ mm}$ )	
	26 ( $s = 100 \text{ mm}$ )	

$\varnothing_{\eta inf} =$	20	mm
	14	
	14	
	14	
	14	
	14	
	14	
	14	
	14	
	14	
	14	
	14	

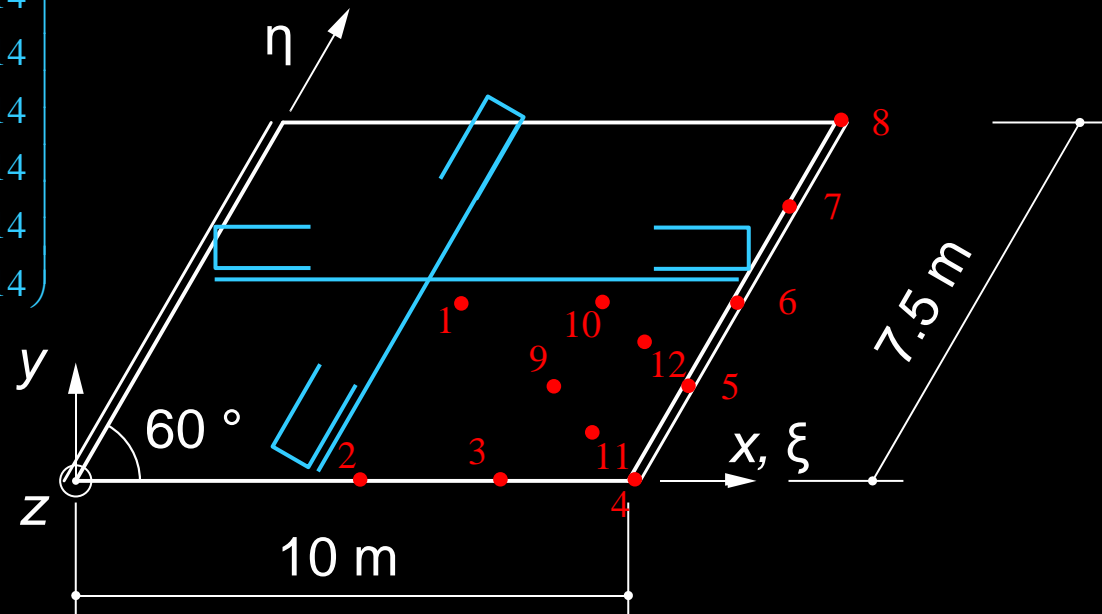
$s_{\eta} = 200 \text{ mm}$

1. Layer:  $\xi$ -Direction:

$$d_1 = 450 \text{ mm} - c_{nom} - \varnothing_1/2 = 382 \text{ mm}$$

2. Layer:  $\eta$ -Direction:

$$d_2 = 450 \text{ mm} - c_{nom} - \varnothing_1 - \varnothing_2/2 = 359 \text{ mm}$$



# Colloquium 3

## Reinforcement 3./4. layer

$$\begin{matrix} \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \end{matrix} \text{ mm}$$

$s_{\xi} = 200 \text{ mm}$

$$\begin{matrix} \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \\ \textcircled{14} \end{matrix} \text{ mm}$$

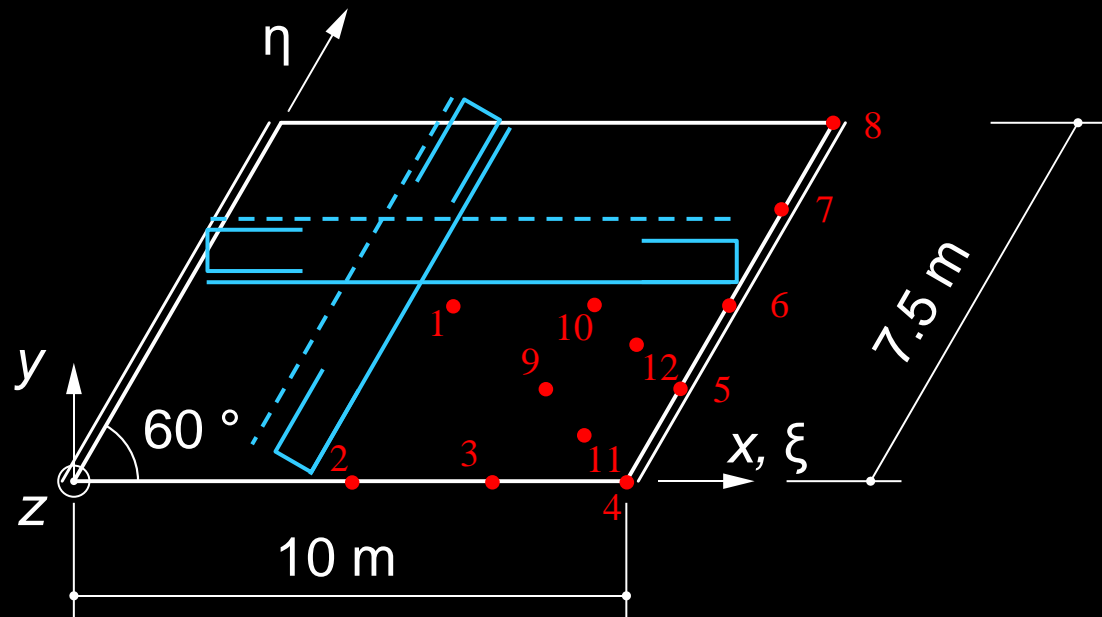
$s_{\eta} = 200 \text{ mm}$

4. Layer:  $\xi$ -Direction:

$$d_4 = 450 \text{ mm} - c_{nom} - \textcircled{4}/2 = 388 \text{ mm}$$

3. Layer:  $\eta$ -Direction:

$$d_3 = 450 \text{ mm} - c_{nom} - \textcircled{4} - \textcircled{3}/2 = 374 \text{ mm}$$



# Colloquium 3

## Bending resistance in skewed reinforcement directions

Lower reinforcement :

$$m_{\xi u, inf} = a_{s\xi, inf} \cdot f_{sd} \left( d_1 - \frac{a_{s\xi, inf} \cdot f_{sd}}{2 \cdot f_{cd}} \right)$$

$$m_{\eta u, inf} = a_{s\eta, inf} \cdot f_{sd} \left( d_2 - \frac{a_{s\eta, inf} \cdot f_{sd}}{2 \cdot f_{cd}} \right)$$

$$m_{\xi u, inf} = \begin{pmatrix} 749 \\ 749 \\ 749 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 749 \\ 749 \\ 749 \\ 749 \end{pmatrix} \text{ kNm/m} \quad m_{\eta u, inf} = \begin{pmatrix} 234 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \\ 118 \end{pmatrix} \text{ kNm/m}$$

Upper reinforcement :

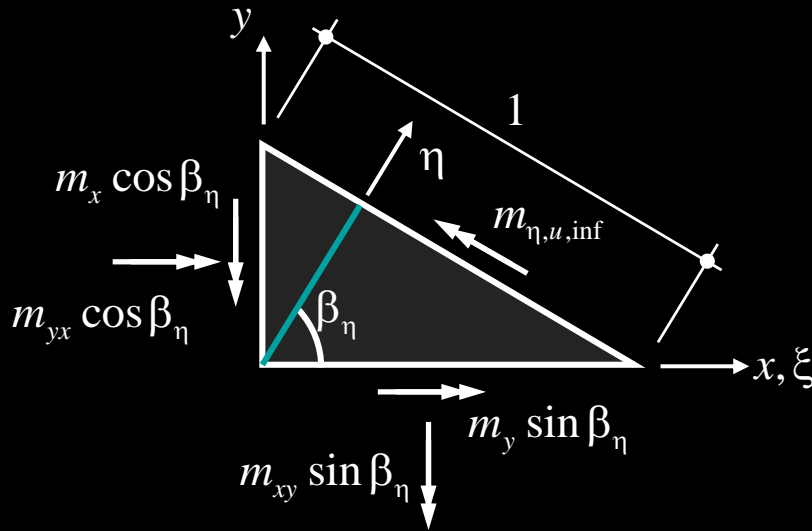
$$m_{\xi u, sup} = a_{s\xi, sup} \cdot f_{sd} \left( d_4 - \frac{a_{s\xi, sup} \cdot f_{sd}}{2 \cdot f_{cd}} \right)$$

$$m_{\eta u, sup} = a_{s\eta, sup} \cdot f_{sd} \left( d_3 - \frac{a_{s\eta, sup} \cdot f_{sd}}{2 \cdot f_{cd}} \right)$$

$$m_{\xi u, sup} = \begin{pmatrix} 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \\ 127 \end{pmatrix} \text{ kNm/m} \quad m_{\eta u, sup} = \begin{pmatrix} 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \\ 122 \end{pmatrix} \text{ kNm/m}$$

# Colloquium 3

## Transformation of the bending resistance in the global x,y coordinates



Lower reinforcement :

$$\beta_{\xi} = 0^{\circ}, \quad \beta_{\eta} = 60^{\circ}$$

$$m_{xu,inf} = m_{\xi u,inf} \cos^2 \beta_{\xi} + m_{\eta u,inf} \cos^2 \beta_{\eta}$$

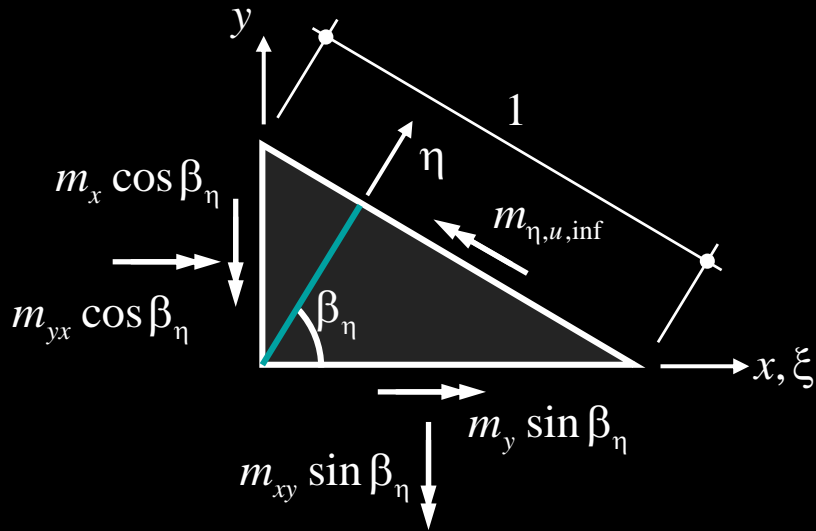
$$m_{yu,inf} = m_{\xi u,inf} \sin^2 \beta_{\xi} + m_{\eta u,inf} \sin^2 \beta_{\eta}$$

$$m_{xyu,inf} = m_{\xi u,inf} \sin \beta_{\xi} \cos \beta_{\xi} + m_{\eta u,inf} \sin \beta_{\eta} \cos \beta_{\eta}$$

$$m_{xu,inf} = \begin{pmatrix} 807 \\ 778 \\ 778 \\ 157 \\ 157 \\ 157 \\ 157 \\ 157 \\ 778 \\ 778 \\ 778 \\ 778 \end{pmatrix} \text{ kNm/m} \quad m_{yu,inf} = \begin{pmatrix} 175 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \\ 89 \end{pmatrix} \text{ kNm/m} \quad m_{xyu,inf} = \begin{pmatrix} 101 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \\ 51 \end{pmatrix} \text{ kNm/m}$$

# Colloquium 3

## Transformation of the bending resistance in the global x,y coordinates



Upper reinforcement :

$$\beta_{\xi} = 0^{\circ}, \quad \beta_{\eta} = 60^{\circ}$$

$$m_{xu,sup} = m_{\xi u,sup} \cos^2 \beta_{\xi} + m_{\eta u,sup} \cos^2 \beta_{\eta}$$

$$m_{yu,inf} = m_{\xi u,sup} \sin^2 \beta_{\xi} + m_{\eta u,sup} \sin^2 \beta_{\eta}$$

$$m_{xyu,inf} = m_{\xi u,sup} \sin \beta_{\xi} \cos \beta_{\xi} + m_{\eta u,sup} \sin \beta_{\eta} \cos \beta_{\eta}$$

$$m_{xu,sup} = \begin{pmatrix} 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \\ 158 \end{pmatrix} \text{ kNm/m} \quad m_{yu,sup} = \begin{pmatrix} 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \\ 92 \end{pmatrix} \text{ kNm/m} \quad m_{xyu,sup} = \begin{pmatrix} 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \\ 53 \end{pmatrix} \text{ kNm/m}$$



# Colloquium 3

## Control of the yield condition

for positive bending resistance :

$$Y_{inf} = (m_{xyu,inf} - m_{xy})^2 - (m_{xu,inf} - m_x)(m_{yu,inf} - m_y) = \begin{pmatrix} -2742 \\ -16193 \\ -6524 \\ -13957 \\ -20366 \\ -16469 \\ -12043 \\ -11681 \\ -9208 \\ -7581 \\ -56302 \\ -51331 \end{pmatrix} (\text{kNm/m})^2 \leq 0$$

for negative bending resistance :

$$Y_{sup} = (m_{xyu,sup} + m_{xy})^2 - (m_{xu,sup} + m_x)(m_{yu,sup} + m_y) = \dots$$

