5 Slabs

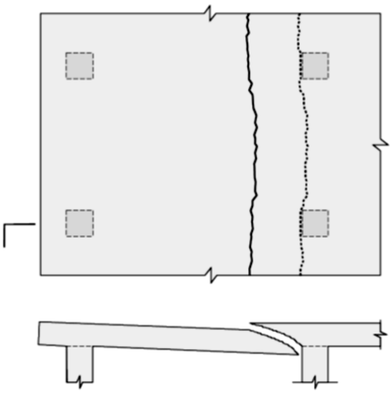
In-depth study and additions to Stahlbeton II

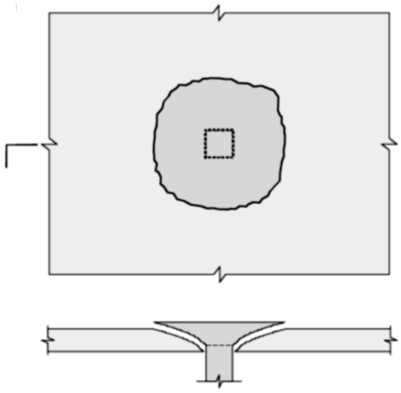
5.6 Punching

Slabs - Influence of shear forces

Slabs without shear reinforcement - Failure mechanisms

- Shear failures as shown in the figure on the left are unlikely in thin slabs. Still, slabs subjected to high loads and primarily carrying in one direction, such as top and bottom slabs of cut-and-cover tunnels may be critical.
- Near concentrated loads (e.g. around columns supporting a flat slab, or supported by a slab on ground), transverse shear forces are often very high. If no shear reinforcement is provided, this can lead to a sudden, very brittle failure (punching).





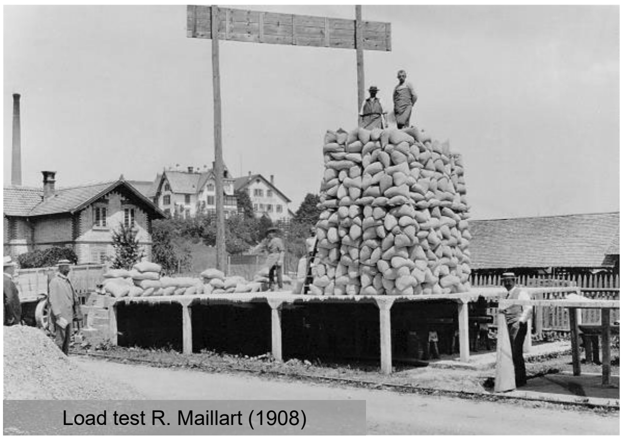
Repetition from Stahlbeton II:

Transverse shear forces transferred in one direction are usually uncritical in slabs (left figure), but may cause brittle failures at point supports and concentrated loads (punching, right figure).

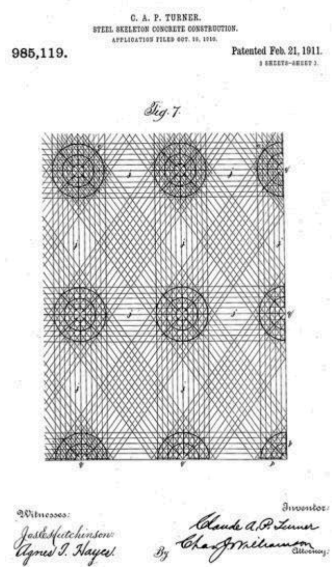
Slabs - Influence of shear forces

Punching

- Flat slabs: load concentration at the supports, maximum v_0 and (m_x, m_y), bending moments with large gradient (elastic solution with point support: m_x and $m_y \rightarrow \infty$)
- With respect to the force flow, mushroom slabs are significantly better
- Early days of concrete construction: Flat slabs as a new type of construction → Mushroom slab systems Maillart / Turner, fully flat slabs only later



Load test R. Maillart (1908)



Patent specification C.A.P. Turner (1911)

Repetition from Stahlbeton II:

The figure shows a load test by Robert Maillart (1872-1940) for the Rorschach filter building (left) and an extract from the patent specification by C.A.P. Turner (1869-1955).

Slabs - Influence of shear forces

Punching

- Flat slabs without shear reinforcement: very brittle failure, progressive collapse possible
- Parking structures are particularly at risk: vehicle fire, corrosion, earth cover exceeding design specification, …
- The punching resistance according to SIA 262 (2003) is significantly reduced with respect to earlier codes (in partial revision 2013 even more strict provisions were introduced)
  → many old buildings are not code-compliant


Gretzenbach (2004)


Wolverhampton (1997)


Bluche (1981)

Repetition from Stahlbeton II:

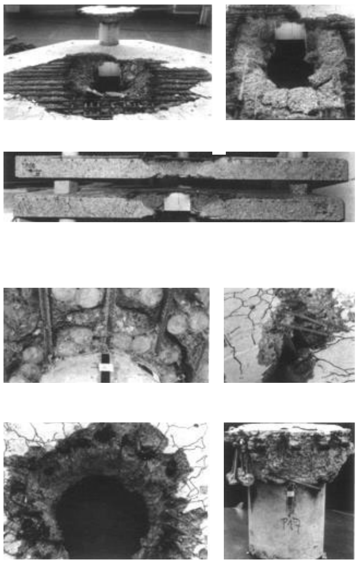
The figure shows punching failures in:

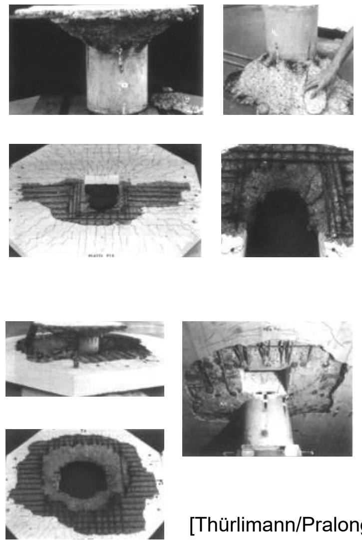
- Wolverhampton (UK, Piper's row car park, built 1965, primary cause corrosion)
- Bluche (CH, Canton VS)
- Gretzenbach (CH, Canton SO, several causes: higher load than designed for (earth cover, garden on top), columns cast too high, fire as final cause triggering the collapse)

Slabs - Influence of shear forces

Punching

Early on many experimental studies worldwide, including ETH Zurich, EMPA





[Thürlimann/Pralong 1979-1984]

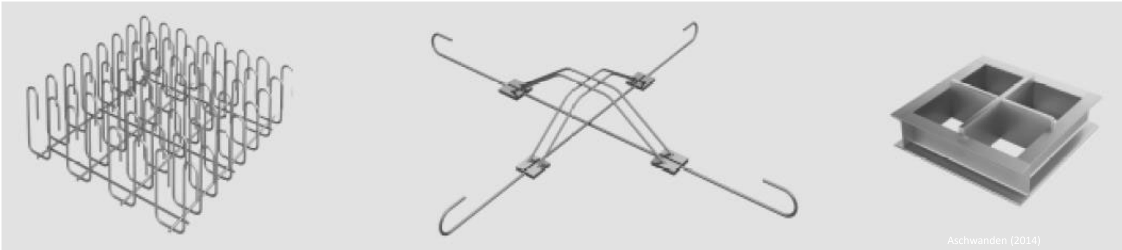
Repetition from Stahlbeton II:

The figure shows punching tests at ETH Zurich (Thürlimann and Pralong, 1979-1984).

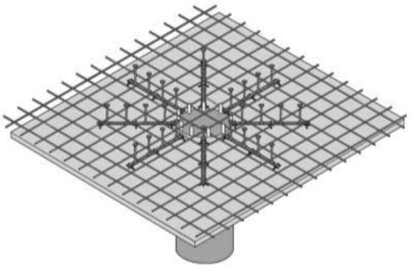
In those decades, many punching tests were also carried out at the EMPA (e.g. Ladner (1977)).

Slabs - Influence of shear forces

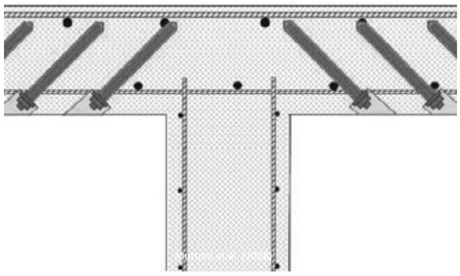
Conceptual solution to the problem: Punching shear reinforcement (or mushroom slabs!)



stirrup cage | bent reinforcement | Steel Forms



Dowels (Studs)



Reinforcing (post-installed) anchors

Repetition from Stahlbeton II:

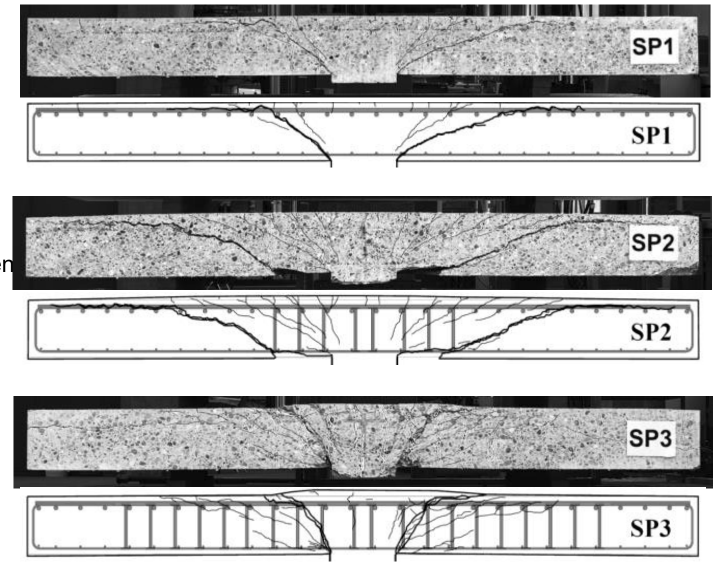
The figure shows various punching reinforcements (bottom right: to reinforce existing structures).

Slabs - Influence of shear forces

Punching : Types of failure

Example: experiments by Etter, Heinzmann, Jäger, Marti (2009)
IBK report 324

- failure at «inner perimeter»
  (here without punching reinforcement)
- failure at «outer perimeter»
  (section defined by the extent of punching reinforcemen
- «compression strut» failure
  (with high amount and extent of punching reinforcement)



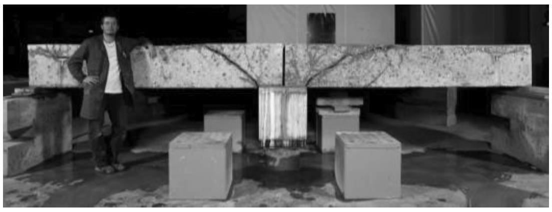
Repetition from Stahlbeton II:

The figure shows sections from test specimens by Etter, Heinzmann, Jäger and Marti (2009) with the typical types of failure that occur in flat slabs.

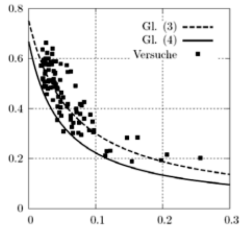
Slabs - Influence of shear forces

Punching: Mechanical model implemented in SIA 262

- Research focus of Prof. Muttoni at ETH Lausanne: Since 2000 various series of experiments (among others with Fernández Ruiz, Guandalini, Guidotti, Lips, Kunz)

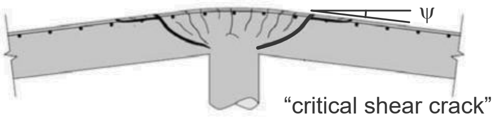


[EPFL - ibeton]



→ Governing parameter: State of strain in the support area (→ bending deformations, as already identified e.g. by Kinnunen / Nylander in 1960 and considered in SIA 162/1968 ("Guideline 18"), but not included in standard SIA 162/1989 to avoid complicating the design by using deformation-dependent strength criteria).

→ Model for slabs without shear reinforcement (basis of the design according to SIA 262 and *fib* Model Code 2010): Failure occurs when a critical shear crack is too wide to be able to transfer the shear (hyperbolic failure criterion closely related to relationships for compression softening):



If curvatures due to bending are neglected:
(crack opening) ~ (slab rotation ψ) · (static depth d)

Repetition from Stahlbeton II:

The figure shows a specimen at the EPFL and the basic assumptions of the model for punching used in SIA 262.

These design specifications for punching, are based on a mechanical model, but as in all current standards, are based on (semi-)empirical relationships calibrated on experiments.

The figure at the top right shows the comparison of test results with the predictions according to the SIA 262 model (normalised slab rotation on abscissa, normalised nominal shear stress at failure on ordinate). The agreement is good for the tests considered (rotationally symmetrical inner supports). As only few tests with non-symmetrical loading (or even on edge and corner supports) have been carried out to date, reliable calibration is hardly possible for these cases.

Slabs - Influence of shear forces

Punching resistance of slabs according to SIA 262

Conceptual provisions

- The deformation capacity of slabs subjected to concentrated loads shall be achieved by the following measures:

  → Either ensure a nominal slab rotation (capacity) ψ > 0.02 under the design load V_d
  (i.e. do not overdimension bending reinforcement, choose a sufficiently large supporting area and slab thickness)

  → Or provide a punching reinforcement with $V_{Rd,s} \geq V_d /2$ (*)

  Otherwise, imposed deformations must be taken into account in the design (constraint forces due to restrained temperature changes, differential settlements, shrinkage, etc.).
  → May cause strong variation (increase) of the load V_d, very difficult to quantify: avoid!

(*) according to fib Model Code 2010: $V_{Rd,s} \geq V_d /2$ with $\sigma_{sd} = f_{sd}$ (SIA 262: not specified)

Repetition from Stahlbeton II:

In addition to the design model, the SIA 262 contains various conceptual provisions (see slide).

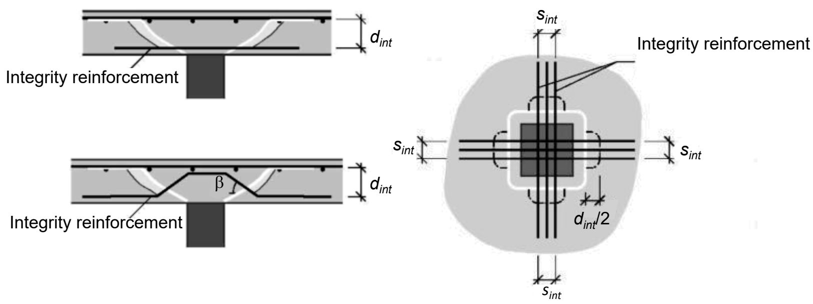
Slabs - Influence of shear forces

Punching resistance of slabs according to SIA 262

Conceptual provisions

- To avoid a progressive collapse (due to punching in spite of a code-compliant design), at least one of the following measures shall be taken:
  → Provide a punching reinforcement with $V_{d,s} \geq V_d/2$ (*)
  → Provide integrity reinforcement preventing a collapse in case of punching (details see SIA 262, 4.3.6.7)

(*) according to fib Model Code 2010: $V_{Rd,s} \geq V_d/2$ with $\sigma_{sd} = f_{sd}$ (SIA 262: not specified)



Repetition from Stahlbeton II:

In addition to the design model, the SIA 262 contains various conceptual specifications (see slide).

Slabs - Influence of shear forces

Punching resistance of slabs according to SIA 262

Verification format

The punching resistance is determined on the basis of nominal transverse shear stresses as follows:

$$\boxed{V_{Rd,c} = k_r \cdot \tau_{cd} \cdot d_v \cdot u} \qquad \text{with} \quad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

k_r Coefficient for static depth of the slab, slab rotation, and maximum aggregate size

d_v Effective static depth in mm

u control perimeter

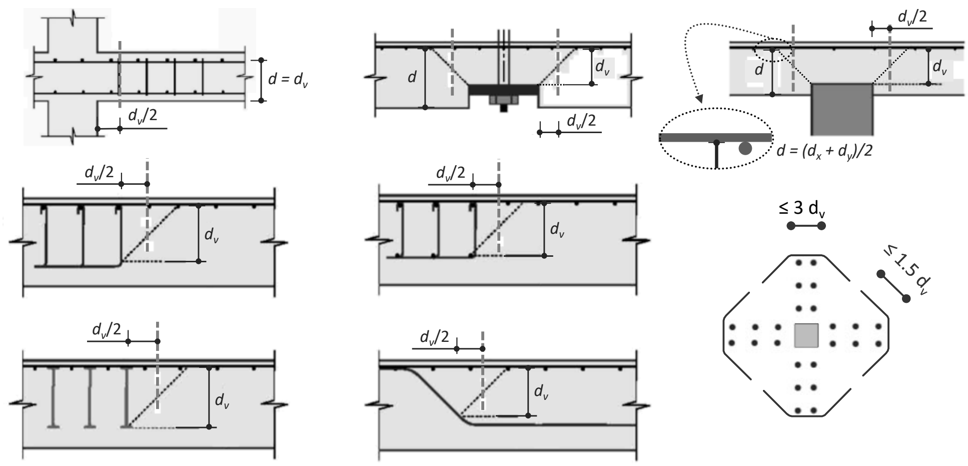
The coefficient k_r depends primarily on the utilisation of the bending reinforcement over the support, which is determined over the width b_s of a nominal "support strip" in each reinforcement direction.

In the following, first the geometrical parameters (effective static depth d_v, control perimeter u, width of the support strip b_s) and then the coefficient k_r are explained.

Slabs - Influence of shear forces

Punching: Control perimeter and support strip

- Effective static depth d_v according to figures below
- Effective static depth d_v to be taken into account when determining the location of the control perimeter



Repetition from Stahlbeton II:

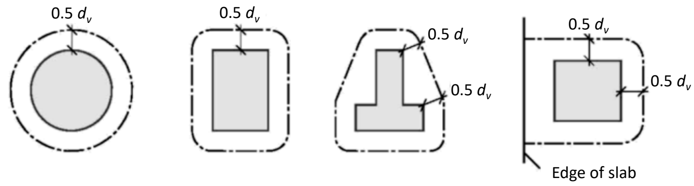
The behaviour is very complex and the semi-empirical resistance model is calibrated on tests. Therefore, all punching specifications in standards require numerous definitions and limitations of the field of application. The slide shows some of them from SIA 262.

Slabs - Influence of shear forces

Punching: Control perimeter and support strip

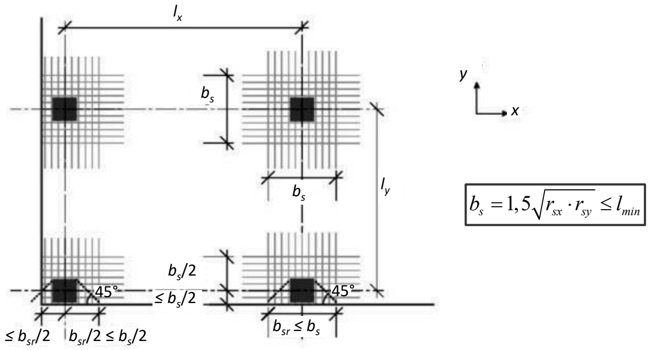
Control perimeter (convex line at distance ≥ $d_v/2$ from support edge → length u)

NB: Actions within the control perimeter may be deducted from the design value of the punching load (self weight, foundation stresses, deviation forces from prestressing, etc.)



Support strip (width b_s)

NB: Bending demand m_{sd} and bending resistance m_{Rd} to be used in formulas for k_r (see following slides): mean values over the width of the support strip



Repetition from Stahlbeton II:

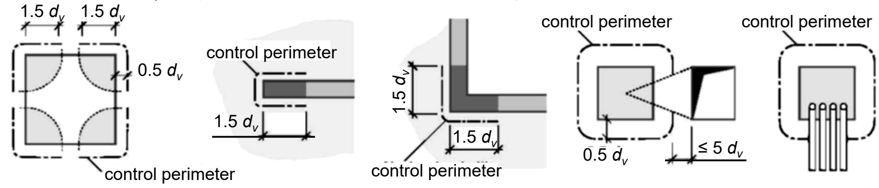
Further definitions and limitations of the field of application.

Slabs - Influence of shear forces

Punching: Reference section

Reduction of the length of the control perimeter to account for non-constant distribution of the shear forces along the perimeter

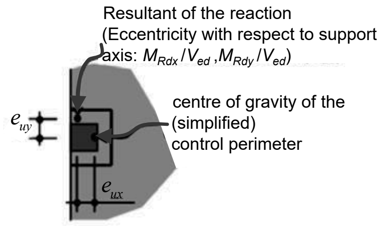
- Consideration of load concentrations in corners, recesses, pipes / ducts, etc.
  (pipes / ducts at a distance < $5d_v$ only permissible in radial direction)!



- Additional reduction of the control perimeter for moment transmission column-slab by the coefficient k_e (simplifying the curvatures of the control perimeter as corners):

$$k_e = \frac{1}{1 + \frac{e_u}{b}}$$

$$e_u = \sqrt{e_{ux}^2 + e_{uy}^2}$$



Approximation for regularly supported flat slabs, supports rigidly connected, supports do not carry horizontal actions:

- k_e = 0.90 Interior supports
- k_e = 0.75 Wall ends, wall corners
- k_e = 0.70 Edge supports, interior supports with large recesses near the columns
- k_e = 0.65 Corner supports

Repetition from Stahlbeton II:

Further definitions and limitations of the field of application.

5 Slabs

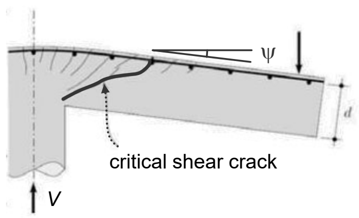
In-depth study and additions to Stahlbeton II

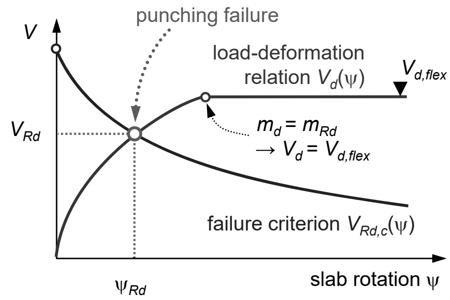
5.6.1 Behaviour without punching reinforcement

Slabs - Influence of shear forces

Punching of slabs without punching reinforcement according to SIA 262

- Basic model: critical shear crack fails if it has opened too much and can no longer transfer the load
- Opening of the critical shear crack (and hence, the punching resistance) is related to the slab rotation ψ through a relationship derived from mechanical considerations and calibrated on experiments → failure criterion $V_{Rd} = V_{Rd}(\psi)$
- An analytical relationship $\psi = \psi(m_{sd}/m_{Rd})$ is established, based on the model, between the slab rotation ψ and the ratio of applied bending moment to bending resistance (m_{sd}/m_{Rd}) in a nominal support strip
- By linking m_{sd} to the support reaction V_d (see following slides) one obtains the load-deformation relationship $\psi = \psi(V_d)$ and hence, $V_d = V_d(\psi)$





Repetition from Stahlbeton II:

Model for the punching resistance according to SIA 262.

Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262

$$\boxed{V_{Rd,c}(\psi) = k_r \cdot \tau_{cd} \cdot d_v \cdot u} \quad \text{with} \quad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

$$k_r = \frac{1}{0.45 + 0.18 \cdot \psi \cdot d \cdot k_g} \leq 2 \quad \text{with} \quad k_g = \frac{48}{16 + D_{\max}}$$

$$\psi = 1.5 \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd}}{m_{Rd}} \right)^{3/2}$$

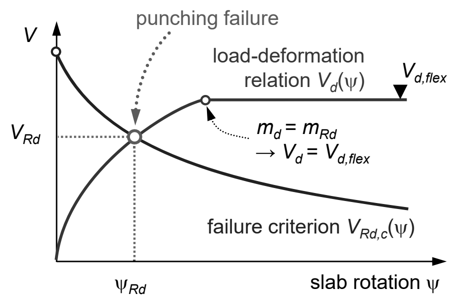
(determine m_{sd}, m_{Rd} und r_s for directions x, y separately, larger value of ψ controls)

- k_r Coefficient for static depth of the slab, slab rotation, and maximum aggregate size
- d_v Effective static depth in mm
- u Control perimeter
- r_s Distance of the point of zero moment (radial moment = 0) from support axis
- m_{sd} Average bending moment in the support strip
- m_{Rd} Average bending resistance in the support strip

Note: The load-deformation relationship does not have to be determined in design (i.e., in the verification whether punching reinforcement is required for a given action V_d).

However, it is needed to calculate the actual punching resistance according to the code.

See the following slides for more details.



Repetition from Stahlbeton II:

Punching resistance according to SIA 262.

Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262

Dimensioning (only governing direction shown (determine ψ_d for m_{sd}, $m_{Rd,}$ and r_s in directions *x*, *y* separately, smaller value of V_{Rd} governs)

Given: V_d, support dimensions, static depth (and thus *u*)

Question: Is punching resistance sufficient without shear reinforcement / are the slab thickness and bending reinforcement sufficient?

Procedure

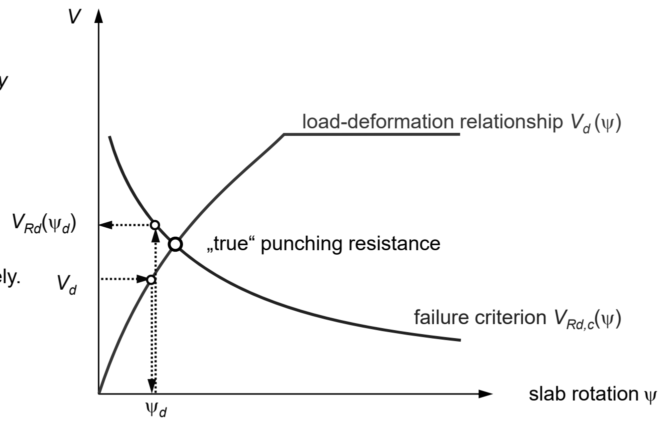
1. Assume *d* and m_{Rd} (select a reasonable reinforcement)
2. Determine of r_s and m_{sd} (V_d) → ψ_d → V_{Rd} (ψ_d) per direction *x, y* (different levels of approximation, see following slides)
3. Increase *d* and / or m_{Rd}, until $V_{Rd}(\psi_d) > V_d$ (or provide punching reinforcement)

NB: The resulting value V_{Rd} (ψ_d) is greater than the actual punching resistance V_{Rd}.

The «true» value of V_{Rd} would have to be determined iteratively. (intersection of the curves $V_{Rd,c}$ (ψ) and V_d (ψ)).

This is unnecessary in design, which can be done without the determination of the load-deformation relationship V_d (ψ).

The determination of the actual punching resistance is explained in more detail in the following slide.



Repetition from Stahlbeton II:

Design procedure (for punching) according to SIA 262.

Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262

Check / Verification of existing structures (only the governing direction is shown!)

Given: Support dimension (and thus *u*), *d*, m_{Rd}
Question: What is the punching resistance (without shear reinforcement)?

$V_{flex,sd}$ Support reaction at which the bending reinforcement yields (in the considered direction)

ψ_{sd} Slab rotation when reaching $V_{flex,sd}$

Procedure

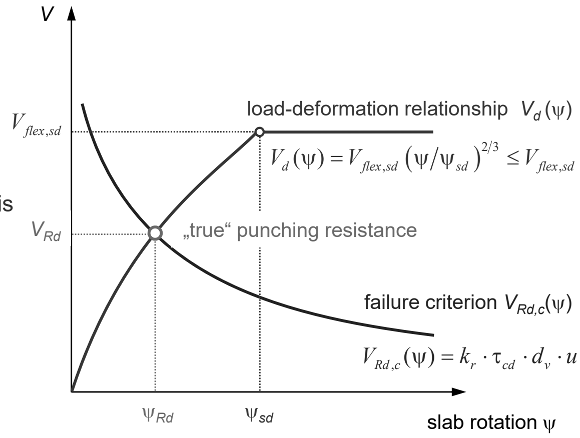
1. Determine the load-deformation relationship $V_d(\psi)$ per direction *x, y*
   (for level of approximation 3: factor 1.5 may be reduced to 1.2)

$$\psi = 1.5\frac{r_s}{d}\frac{f_{sd}}{E_s}\left(\frac{m_{sd}}{m_{Rd}}\right)^{\frac{3}{2}} = \psi_{sd}\left(\frac{m_{sd}(\psi)}{m_{Rd}}\right)^{\frac{3}{2}} \quad \text{with} \quad \psi_{sd} = 1.5\frac{r_s}{d}\frac{f_{sd}}{E_s}$$

$$V_{flex,sd} = \frac{V_d(\psi)}{m_{sd}(\psi)}m_{Rd} \rightarrow \frac{m_{sd}(\psi)}{m_{Rd}} = \frac{V_d(\psi)}{V_{flex,sd}} \quad \text{with} \quad \frac{V_d(\psi)}{m_{sd}(\psi)} \text{ from FE slab analysis}$$

$$\rightarrow \psi = \psi_{sd}\left(\frac{V_d(\psi)}{V_{flex,sd}}\right)^{\frac{3}{2}} \qquad \rightarrow V_d(\psi) = V_{flex,sd}\left(\frac{\psi}{\psi_{sd}}\right)^{\frac{2}{3}} \le V_{flex,sd}$$

2. Equating $V_{Rd,c}(\psi) = V_d(\psi) \rightarrow \psi_{Rd}$, $V_{Rd}(\psi_{Rd}) = V_d(\psi_{Rd})$
   (direction with smaller value of V_{Rd} controls)



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The slide explains the procedure for determining the punching resistance for given conditions (slab thickness, reinforcement) according to SIA 262.

Assuming that the support reaction V_d is proportional to the bending moment m_{sd} in the support strip (applicable to linear elastic behaviour), the load-deformation relationship $V_d(\psi)$ is correlated to the relationship $m_{sd}(\psi)$. Thus, the support reaction V_d is proportional to $\psi^{2/3}$, with an upper limit of $V_{flex,sd}$ (support reaction where the bending reinforcement yields), which is achieved with a rotation ψ_{sd}.

Thus, the relationship $V_d(\psi)$ is known. The punching resistance is the intersection of this relationship with the failure criterion $V_{Rd,c}(\psi)$.

The value of V_d/m_{sd} can be determined by a slab calculation, or approximated according to the following slides depending on the level of approximation used.

Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262: Levels of approximation (LoA)

(a) Continuously supported flat slabs $0.5 \le l_x / l_y \le 2$, no (small) plastic redistribution ("normal" slab in building construction):

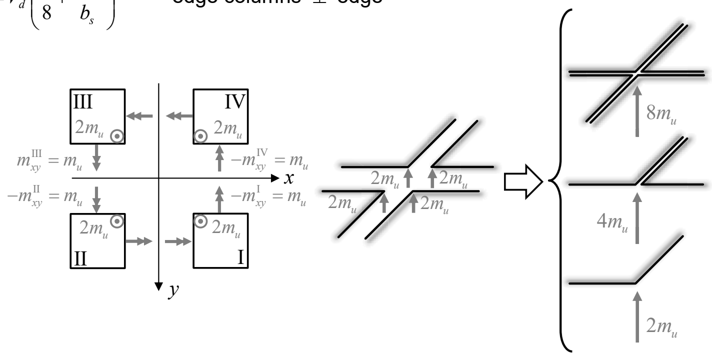
- Level of approximation 1: $r_{sx} = 0.22 \cdot l_x$, $r_{sy} = 0.22 \cdot l_y$ and $m_{sd} / m_{Rd} = 1.0$
- Level of approximation 2: $r_{sx} = 0.22 \cdot l_x$, $r_{sy} = 0.22 \cdot l_y$, estimated bending moments:

$$m_{sd} = V_d \left(\frac{1}{8} + \frac{|e_{u,i}|}{2b_s} \right) \quad \text{interior colmns} \qquad m_{sd} = V_d \left(\frac{1}{8} + \frac{|e_{u,i}|}{2b_s} \right) \ge \frac{V_d}{4} \quad \text{edge columns} \parallel \text{edge}$$

$$m_{sd} = V_d \left(\frac{1}{8} + \frac{|e_{u,i}|}{b_s} \right) \ge \frac{V_d}{2} \quad \text{corner columns} \qquad m_{sd} = V_d \left(\frac{1}{8} + \frac{|e_{u,i}|}{b_s} \right) \quad \text{edge columns} \perp \text{edge}$$

The corresponding minimum values result directly from the consideration of the combination of individual slab segments with discontinuous twisting moment fields.

NB: For interior supports an explanation with the moment field (transforming a concentrated load to a uniformly distributed one) is even simpler



(Partial) repetition from Stahlbeton II:

The slide shows the assumptions for r_s and m_{sd} according to the levels of approximation 1-2 of SIA 262. It also shows a derivation of the values for m_{sd} according to LoA 2.

Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262:
Levels of approximation (LoA)

(b) Flat slabs with $l_x / l_y < 0.5$ or $l_x / l_y > 2$, slabs with complex geometry or detailed examination required:

- Level of approximation 3: Determination of r_s (distance of the point of zero moment, i.e. radial moment = 0, from support axis) and m_{sd} (mean value of the bending moments including twisting moments in the support strip) from an elastic (usually linear elastic FE) slab calculation. Factor 1.2 instead of 1.5 in formula for ψ:

$$\psi = \cancel{1.5}\ 1.2 \cdot \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd}}{m_{Rd}} \right)^{3/2}$$

The slide shows the assumptions according to the level of approximation 3 of SIA 262.

Slabs - Influence of shear forces

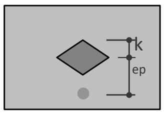
Punching resistance of slabs without punching reinforcement according to SIA 262:
Selected additional provisions (for detailing provisions see SIA 262, 5.5.3)

- Bending resistance m_{Rd} = mean value over support strip, taking prestressing into account.
  (reinforcement must generally be fully anchored at a distance of 2.5·d_v from the control perimeter, but at most at the point of zero bending moment in the respective direction. In the case of edge and corner supports, the reinforcement perpendicular to the edge must be fully anchored → hairpin shaped reinforcement).
- Prestressed slabs with decompr ssion moment m_{Dd}:

$$\psi = (1.5 \text{ or } 1.2)\frac{r_s}{d}\frac{f_{sd}}{E_s}\left(\frac{m_{sd}-m_{Dd}}{m_{Rd}-m_{Dd}}\right)^{3/2}$$

... m_{Dd} = long-term value (shrinkage, creep, relaxation) under consideration of normal forces due to restraints
(for m_{Dd}, only the part of the compressive force that is effective in the support strip may be taken into account)
... m_{sd} = incl. constraints (e.g. secondary moments due to prestressing)
... prestress with unfavourable effect must be taken into account where applicable
... use signs of m_{sd}, m_{Rd}, and m_{Dd} consistently, otherwise might obtain completely wrong results!

NB1: The decompression moment is generally: $m_{Dd} = P·(e_p+k)$. If the prestressing is considered as anchorage and deviation forces ("on the load side"), the contribution $P·e_p$ to m_{Dd} *is* already considered in the correspondingly reduced bending moments m_{sd}. The bending resistance m_{Rd} is also smaller by the amount $P·e_p$ (only the increase in prestressing force as resistance) → only the portion $P·k$ can be subtracted in the numerator and the denominator, taking into account the distribution of P over the slab width and, if necessary, the reduction of P by normal forces due to restraints.



NB2: In addition, the contribution of inclined prestressing forces to the punching resistance may be taken into account (even if prestressing is considered on the load side; the support reaction V_d does not reflect the isostatic effect of prestressing).

Repetition from Stahlbeton II:

Further definitions for the application of the specifications of SIA 262. For the punching resistance of prestressed slabs, see the following slide.

Slabs - Influence of shear forces

Punching resistance of prestressed slabs without punching reinforcement according to SIA 262

Check / Verification of existing structures (only the governing direction is shown!)

Given: Support dimension (and thus u), d, m_{Rd}

Question: What is the punching resistance (without shear reinforcement)?

$V_{flex,sd}$ Support reaction at which the bending reinforcement yields (in the considered direction)

ψ_{sd} Slab rotation when reaching $V_{flex,sd}$

m_{dec} Decompression moment (see slides 37 and 39)

Procedure

1. Determination of the load-deformation relationship $V_d(\psi)$ per direction x, y (for LoA 3 replace factor 1.5 by 1.2)

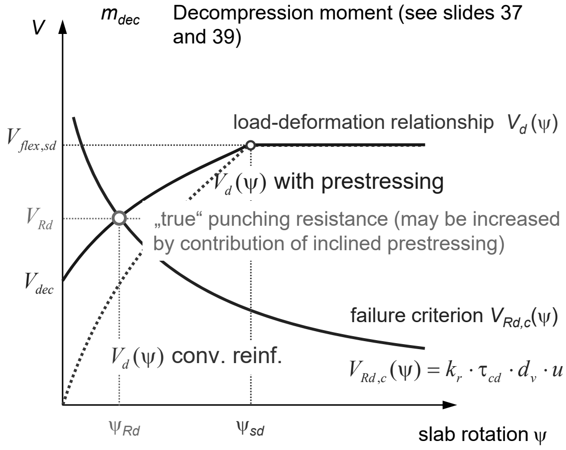
$$\psi = 1.5 \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd} - m_{dec}}{m_{Rd} - m_{dec}} \right)^{3/2} = \psi_{sd} \left(\frac{m_{sd}(\psi) - m_{dec}}{m_{Rd} - m_{dec}} \right)^{3/2} \quad \text{mit } \psi_{sd} = 1.5 \frac{r_s}{d} \frac{f_{sd}}{E_s}$$

$$V_{flex,sd} = \frac{V_{d,gq}(\psi)}{m_{sd,gq}(\psi)} m_{Rd}, V_{dec,sd} = \frac{V_{d,gq}(\psi)}{m_{sd,gq}(\psi)} m_{dec} \rightarrow \frac{m_{sd}(\psi) - m_{dec}}{m_{Rd} - m_{dec}} = \frac{V_d(\psi) - V_{dec}}{V_{flex,sd} - V_{dec}}$$

(with $\frac{V_{d,gq}(\psi)}{m_{sd,gq}(\psi)}$ from FE slab analysis)

$$\rightarrow \psi = \psi_{sd} \left(\frac{V_d(\psi) - V_{dec}}{V_{flex,sd} - V_{dec}} \right)^{3/2} \quad \rightarrow V_d(\psi) = V_{dec} + (V_{flex,sd} - V_{dec}) \left(\frac{\psi}{\psi_{sd}} \right)^{2/3} \le V_{flex,sd}$$

2. Equating $V_{Rd,c}(\psi) = V_d(\psi) \rightarrow \psi_{Rd}$, $V_{Rd}(\psi_{Rd}) = V_d(\psi_{Rd})$ (direction with smaller value of V_{Rd} controls)



When determining the punching resistance of prestressed slabs, it is assumed that the slab rotation can be neglected before the decompression moment is reached. As with conventionally reinforced slabs, it is assumed that the support reaction V_d and the bending moment m_{sd} in the support strip - both due to external load (not prestressing) - are proportional to each other, which applies to linear elastic behaviour.

Under these conditions, $(V_d - V_{dec})/(V_{flex,sd} - V_{dec})$ and $(m_{sd} - m_{dec})/(m_{Rd} - m_{dec})$ are correlated, from which the load-deformation relationship $V_d(\psi)$ can be determined. The punching resistance is the intersection of this relationship with the failure criterion $V_{Rd,c}(\psi)$.

As with conventionally reinforced slabs, $V_{flex,sd}$ denotes the column reaction at which the bending reinforcement (of the considered direction) yields, and ψ_{sd} the slab rotation when $V_{flex,sd}$ is reached. The value of V_d/m_{sd} can be determined with a slab calculation (or approximated for conventionally reinforced slabs).

Additional remark:

- Simplified representation (constraint moments due to prestress not taken into account).

Slabs - Influence of shear forces

Punching resistance of prestressed slabs without punching reinforcement according to SIA 262

Prestress taken into account on the resistance side:

$$\psi = 1.5 \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd} - m_{dec}}{m_{Rd} - m_{dec}} \right)^{3/2}$$

mit $m_{sd} = m_{gq,d} + m_{ps}$: Design value of the bending moment in the support strip (negative)

$m_{gq,d}$: Design value of the bending moment due to vertical loads (negative)

m_{ps} : Secondary moment due to prestress (usually positive)

$m_{dec} = -P_\infty (e_p + k)$: Decompression moment (negative)

P_∞ : Prestressing force at t=∞ (positive) (reduce if normal force does not fully act in the support strip!)

e_p : Eccentricity of prestress (in the support strip), here positive upwards (upper side of slab)

k : extent of core (positive, usually = h/6)

m_{Rd} : Design value of the bending resistance $-(A_s f_{sd} \cdot z_s + A_p f_{pd} \cdot z_p)$ (negative)

(All moments used with the usual sign convention, i.e. tension at bottom = positive -> support moments negative)

If the prestressing is taken into account on the resistance side, the proportion of the inclined prestressing force (sum of the vertical components on the decisive circumference) can either be added to the punching resistance or subtracted from the design value of the column reaction = reduced punching load ($V_{d,red} = V_d - \Delta V_d(P)$, $\Delta V_d(P) = \Sigma(P_\infty \sin\alpha_p)$), but **not** both!

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When determining the punching resistance of prestressed slabs, it is important that the signs of the various terms are handled consistently. Otherwise, a completely wrong value of the quotient (m_{sd} – m_{dec})/(m_{Rd} –m_{dec}) results and thus a completely wrong value for the punching resistance.

In addition, the decompression moment must be determined as accurately as possible, since the calculated punching resistance is sensitive to its magnitude. Here it must be taken into account that under certain circumstances not the entire normal force due to prestressing acts on the slab or in the support strip. In such cases, the magnitude of the decompression moment must be reduced accordingly.

Slabs - Influence of shear forces

Punching resistance of prestressed slabs without punching reinforcement according to SIA 262

Prestress taken into account on the load side (as anchor and deviation forces):

$$\psi = 1.5 \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd} - m_{dec}}{m_{Rd} - m_{dec}} \right)^{3/2}$$

mit $m_{sd} = m_{gq,d} + m_p$: Design value of the bending moment in the support strip (negative)

$m_{gq,d}$: Design value of the bending moment due to vertical loads (negative)

m_p : Bending moment due to prestress (Long-term stresses $P_\infty e_p$ and secondary moments, positive)

$m_{dec} = -P_\infty k$: Decompression moment (negative)

P_∞ : Prestressing force at t=∞ (positive) (reduce if normal force does not fully act in the support strip!)

k : extent of core (positive, usually = h/6)

(the part $P_\infty e_p$ of prestressing is already included in m_{sd}, do not use here a second time!)

m_{Rd} : Design value of the bending resistance $-(A_s f_{sd} \cdot z_s + A_p f_{pd} \cdot z_p - P_\infty e_p)$ (negative)

(prestressing contribution reduced by $P_\infty e_p = A_p \sigma_{p\infty} e_p$, since $P_\infty e_p$ is already considered in m_p)

(All moments used with the usual sign convention, i.e. tension at bottom = positive -> support moments negative)

m_{sd}, m_{Rd} and m_{dec} differ all by the same value $P_\infty e_p$, compared to considering prestress as resistance side ⇨ same result!

Even if the prestress is introduced on the load side, the proportion of the inclined prestressing force to the punching resistance can be taken into account: The column reaction, which is used as punching load, does not reflect the isostatic effect of prestress (would be different if the integral of the shear forces along the control perimeter was used as load).

Explanations / remarks see slide 21.

5 Slabs

In-depth study and additions to Stahlbeton II

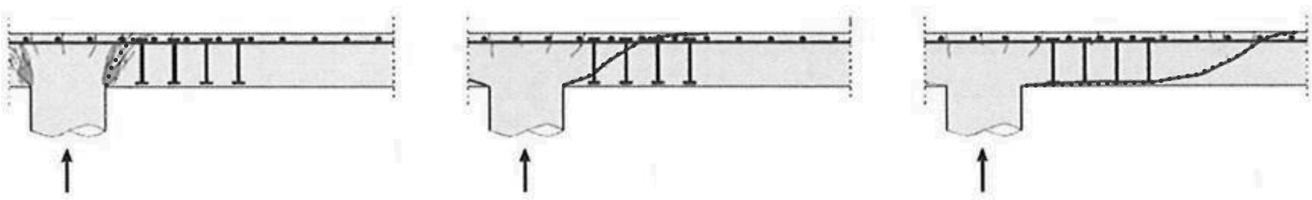
5.6.2 Behaviour with punching reinforcement

Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

The following verifications must be carried out for slabs with punching reinforcement:

- Resistance of the first concrete compression strut next to the supported area
- Resistance of the punching reinforcement (reinforced zone)
- Punching verification (without punching reinforcement) outside the reinforced zone



Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

Minimum required resistance of punching reinforcement:

$$V_{d,s} \geq V_d - V_{Rd,c}$$

... resp. in order to neglect imposed deformations in the design and / or avoid the necessity of an integrity reinforcement

$$V_{d,s} \geq \max \begin{Bmatrix} V_d - V_{Rd,c} \\ V_d / 2 \end{Bmatrix}$$

Resistance of punching reinforcement (normal: inclination β = 90°):
(A_{sw}: only punching reinforcement within distance 0.35...1.0 $\cdot d_v$ of the supported area is taken into account)

$$V_{Rd,s} = \sum A_{sw} k_e \sigma_{sd} \sin\beta$$

Nominal stress in the punching reinforcement:
(f_{bd}: design value of the bond stress)
(NB: according to fib Model Code 2010: $V_{d,s} \geq V_d/2$ with $\sigma_{sd} = f_{sd}$)

$$\sigma_{sd} = \frac{E_s \psi}{6}\left(1 + \frac{f_{bd}}{f_{sd}} \frac{d}{\varnothing_{sw}}\right) \leq f_{sd}$$

Repetition from Stahlbeton II:

Punching resistance according to SIA 262 with punching reinforcement.

Slabs - Influence of shear forces

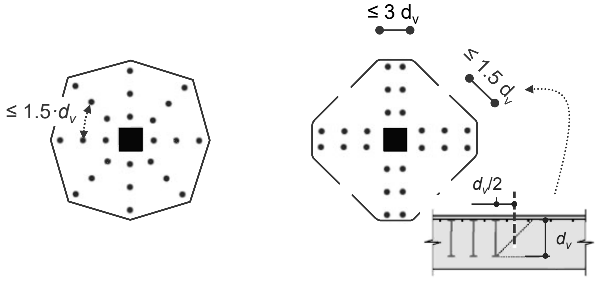
Punching resistance of slabs with punching reinforcement according to SIA 262

Resistance of the first concrete compression strut:
(Factors > 2 and according to SIA 262 > 3.5 are admissible, provided that the effectiveness of the reinforcement is experimentally proven)

$$V_{Rd,\max} = 2 \cdot k_r \tau_{cd} d_v u \leq 3.5 \cdot \tau_{cd} d_v u$$
$$= 2 \cdot V_{Rd,c} \text{ mit } k_r \leq 1.75$$

Punching verification (without punching reinforcement) outside the reinforced zone
(supported surface defined by out reinforcement, control perimeter according to figure)

$$V_{Rd,c} = k_r \tau_{cd} d_v u$$

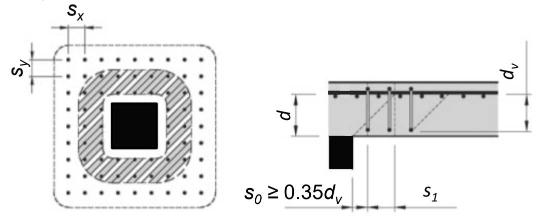


Repetition from Stahlbeton II:

Punching resistance according to SIA 262 with punching reinforcement.

Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262:
Selected additional provisions (for detailing provisions see SIA 262, 5.5.3)



Resistance of the punching reinforcement:
(A_{sw}: punching shear reinforcement only at a distance of 0.35…1.0·d_v from the supported surface)

$$V_{Rd,s} = \sum A_{sw} k_e \sigma_{sd} \sin\beta$$

SIA 262 5.5.3.8: At least two legs in radial direction

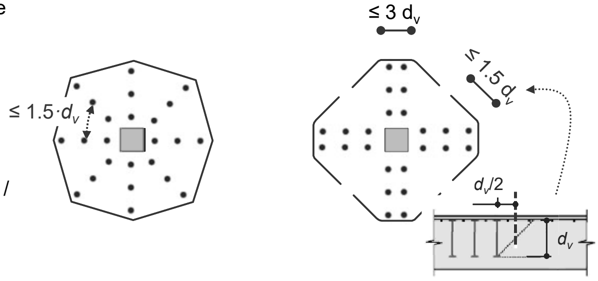
SIA 262 5.5.3.10: Full anchorage in compression and tension zone

Arrangement of the punching reinforcement within the distance $s_0 < s_1$ from the supported surface:

- radial distance and maximum Ø, see SIA 262, Tab. 20 and Fig. 39
- tangential distance in the second ring ≤ 1.5·d_v

Generally provide the same cross-section A_{sw} per «ring» (rings geometrically similar to control perimeter)

Punching reinforcement in straight radial rows: same radial distance of dowels / vertical reinforcement satisfies the condition of equal A_{sw} per ring



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Repetition from Stahlbeton II:

Further remarks for the application of the specifications of SIA 262.

Additional remark:

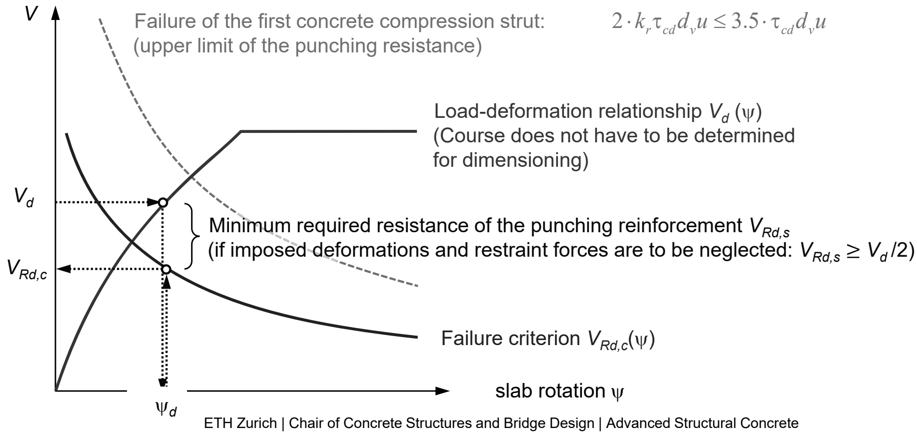
- Experiments show that the arrangement at the bottom right is less effective than the star-shaped arrangement.

Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

Dimensioning (only governing direction shown (determine ψ_d for m_{sd}, m_{Rd} and r_s per directions x, y, smaller value of V_{Rd} is governing)

1. Determination of $V_{Rd,c}$ (= same as determination V_{Rd} without punching reinforcement, see slides above)
2. Required resistance $V_{Rd,s} \geq V_{d,s} = V_d - V_{Rd,c}$ ($\geq V_d/2$ if constraint forces are to be neglected)
3. Check that failure of the first compression strut is not governing $V_{Rd} \leq 2 \cdot V_{Rd,c}$
4. Definition of the size of the reinforced area (such that outside, $V_{Rd,c}$ alone is sufficient)



Repetition Stahlbeton II:

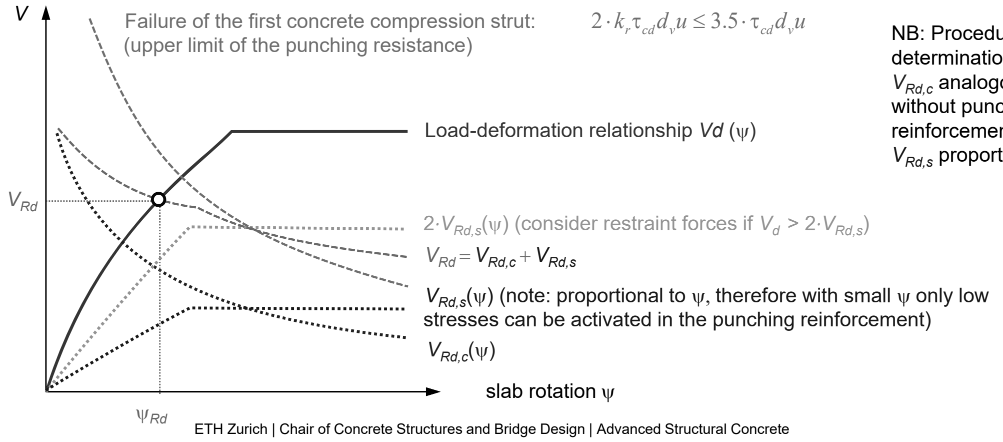
Procedure for dimensioning the punching reinforcement according to SIA 262.

Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

Check / Verification of existing structures

1. Determination of load-deformation relationship $V_d\ (\psi)$ and punching resistance $V_{Rd}\ (\psi) = V_{Rd,c} + V_{Rd,s}$ → equate, intersection = V_{Rd}
2. Check $V_{Rd} \geq V_d$ (V_d incl. Imposed deformations and restraint forces, if $V_d > 2 \cdot V_{Rd,s}$)
3. Check that failure of the first compression strut is not governing $V_{Rd} \leq 2 \cdot V_{Rd,c}$
4. Verify the size of the reinforced area with separate verification



NB: Procedure for the determination of $V_d\ (\psi)$ and $V_{Rd,c}$ analogous to that without punching reinforcement; $V_{Rd,s}$ proportional to ψ)

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The slide explains the procedure for determining the punching resistance with punching reinforcement for given conditions (slab thickness, reinforcement) according to SIA 262.

The load-deformation relationship $V_d\ (\psi)$ and the failure criterion $V_{Rd,c}\ (\psi)$ can be determined in the same way as for slabs without shear reinforcement. In addition, the resistance of the punching reinforcement has to be determined according to the relationships:

$$V_{Rd,s} = \sum A_{sw} k_e \sigma_{sd} \sin\beta; \qquad \sigma_{sd} = \frac{E_s \psi}{6}\left(1 + \frac{f_{bd}}{f_{sd}} \frac{d}{\varnothing_{sw}}\right) \leq f_{sd}$$

The resistance of the punching reinforcement increases linearly with the slab rotation ψ up to a maximum when the reinforcement yields.

The punching resistance corresponds to the intersection of the relationship $V_d(\psi)$ with the curve corresponding to the sum $V_{Rd,c}(\psi) + V_{Rd,s}(\psi)$ (limited by the upper limit 2 $V_{Rd,c}(\psi)$).

In practice, often a very low stress results in the punching reinforcement. Alternatively, in such cases a design can be made on the basis of a truss model in which the punching shear reinforcement is fully activated ($V_{Rd,s}$ mit $\sigma_{sd} = f_{sd}$) but the resistance of the concrete is neglected ($V_{Rd,c} = 0$). The upper limit of 2 $V_{Rd,c}\ (\psi)$ must also be considered in this case.

5 Slabs

In-depth study and additions to Stahlbeton II

5.7 Additions

Additions - Elastic sheets

Kirchhoff's slab theory
(rigid linear elastic slabs with small deflections)

The fourth-order differential equation results from the equilibrium and compatibility conditions for linear elastic behaviour (inhomogeneous bipotential equation) :

$$\underbrace{\frac{\partial^4 w}{\partial x^4}}_{\text{beam in x-direction}} + 2\underbrace{\frac{\partial^4 w}{\partial x^2 \partial y^2}}_{\text{additional term}} + \underbrace{\frac{\partial^4 w}{\partial y^4}}_{\text{beam in y-direction}} = \Delta\Delta w = \frac{q}{D} \quad mit \quad D = \frac{Eh^3}{12(1-\nu^2)}$$

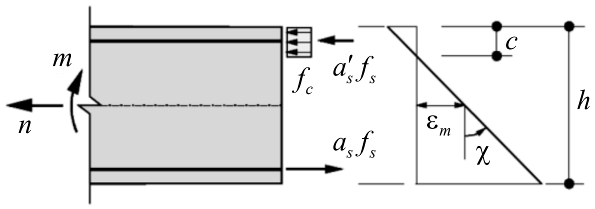
Only two boundary conditions can be adapted to the solution, but there are three variables at the boundary (moments m_n, m_{tn} and shear force v_t) → Support force (see slabs part 1), thus the following boundary conditions:

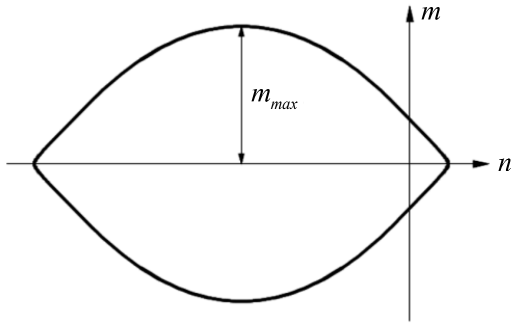
- clamped slab edge: $w = 0$ $\frac{\partial w}{\partial x} = 0$ thus $\frac{\partial^2 w}{\partial x \partial y} = 0$ and thus $m_{xy} = 0$. m_n and v_n are the support reations.
- simply supported slab edge: $m_x = 0$ $\Delta w = 0$ resulting support force $v_n + m_{tn,t} = m_{n,n} + 2m_{nt,t}$
- free slab edge: $m_n = 0$ disappearing support force $v_n + m_{tn,t} = m_{n,n} + 2m_{nt,t} = 0$

Additions - Membrane action

Development of membrane forces

- Cracking leads to deformations in the middle plane of the slab already in the serviceability limit state (dilatancy)
- The resulting deformations are rarely possible without constraint
  → Compressive membrane forces in cracked areas
  → Usually increase of bending resistance
- Membrane force can usually only be roughly estimated (depending on geometry, deformations of the slab middle plane, stiffness of the membrane support).
- Therefore, this effect is typically neglected in design.





Membrane action has a great influence on the behaviour of slabs. The (usually) favourable effect of compressive membrane forces due to crack formation is usually neglected in the dimensioning.

This is often very much on the safe side, especially when it comes to proving fatigue safety. In fatigue tests on slabs under concentrated loads, for example, it is found that significantly smaller stress differences result in the reinforcement than would be expected according to the bending theory. If the membrane effect is taken into account, these results can be explained. North American design provisions for bridge decks semi-empirically account for this.

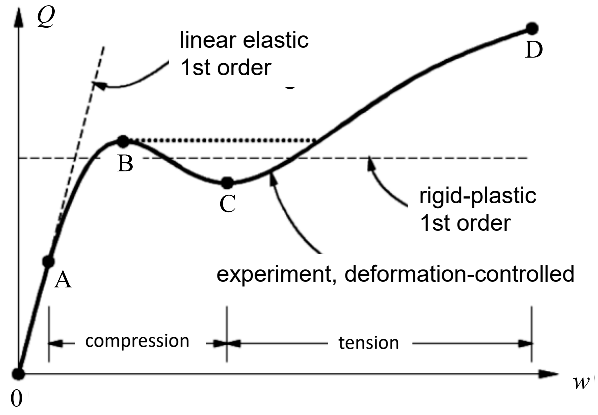
The effect of membrane action can e.g. be estimated with nonlinear finite element calculations using a mechanically consistent model (layered formulation of Cracked Membrane Model as shell element), as implemented by Prof. Karel Thoma at HSLU Lucerne.

Additions - Membrane action

Development of membrane forces

Behaviour (qualitative)

1. Linear elastic (OA)
2. Crack formation, build-up of compressive membrane forces (AB)
3. Maximum load (B) > Load capacity for rigid-ideally plastic behaviour without membrane action (M-N interaction)
4. Load decreases if deformation controlled, compressive membrane forces are reduced (BC);
   (Load-controlled: «snap-through" effect of the slab)
5. With external membrane support, build-up of tensile membrane forces with increasing deflection. Failure load often >> first maximum (with large deformations, can only be measured with a corresponding calculation)

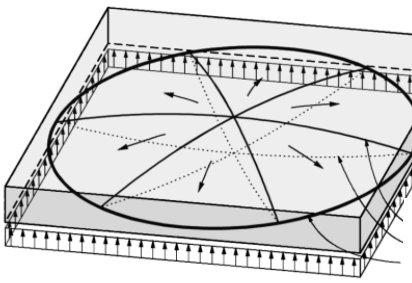


While a consideration of compressive membrane forces would be useful in many cases, tensile membrane forces (which only occur with large deformations) in slabs should only be activated in exceptional cases.

Additions - Membrane effect

Spatial model for load-bearing capacity

- Membrane support not by bearing, but by tension ring (uncracked area of the slab)
- Load transfer: Compression membrane (concrete) and tension membrane (reinforcement: conventional or e.g. prestressing without bond)
- Without horizontal membrane support (external or by a tension ring), the membrane forces of the concrete and reinforcement membranes are in equilibrium → not an actual membrane effect.
- Membrane action can be used to explain the load-bearing capacity of an unreinforced slab (at the location of the membrane support, horizontal **and** vertical components of the membrane forces need to be resisted)



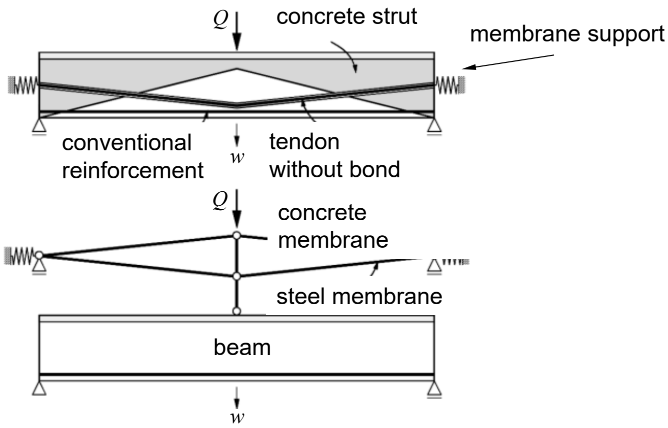
In contrast to beams, a membrane effect in slabs does not necessarily require horizontally restrained supports. In the integral over the entire slab, the membrane forces must disappear in the case of a statically defined support arrangement in the slab plane (for horizontal forces). But, as shown in the figure, a tension ring can form in the outer area on which a compression membrane is supported in the inner area.

This simple observation shows that membrane forces can also occur in slabs with horizontally sliding supports.

Additions - Membrane effect

Model for the load-bearing capacity (Ritz, 1978)

- Model for load-bearing behaviour of slab strips prestressed without bond with membrane effect
- Load carried by bending or membrane effect (of concrete and steel membrane), depending on stiffness ratios (if membrane support is missing, no actual membrane effect)



The slide shows a simple model for the investigation of the load-bearing behaviour with a membrane support. This can also be described analytically for simple cases. Despite the high relevance of this effect, not much meaningful research has been carried out since the time of the work indicated in the slide.