4 Long-term effects

Specialization and additions to Stahlbeton I

4.1 Basics

This chapter examines the long-term (time-dependent) behaviour of concrete structures.

First, the basics are presented (repetition and addition to Stahlbeton I). Subsequently, various methods for investigating the effects of creep will be discussed.

Finally, Trost's Method to account for long-term effects is introduced and illustrated with some examples.

Time-dependent behaviour of concrete

Shrinkage

Volume contraction without load

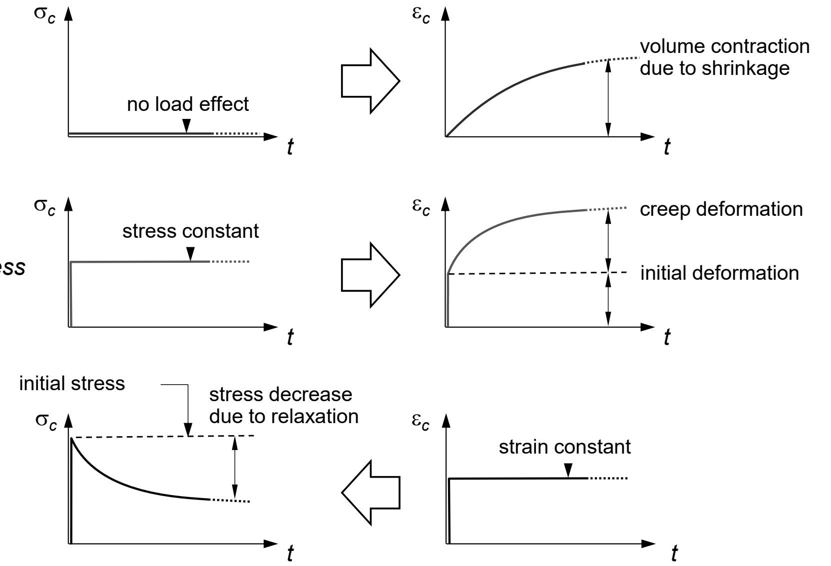
(Figure for free, unrestrained deformations
→ no restraint forces)

Creep

Increase of deformations under constant stress

Relaxation

Decrease of stress under constant strain



Repetition Stahlbeton I:

Concrete is viscous and therefore shows a time-dependent behaviour. This is primarily influenced by the properties of the cement matrix. A distinction is usually made between shrinkage (volume contraction of the concrete without load), creep (increase of deformations under constant stress) and relaxation (decrease in stresses under constant strain).

Creep and relaxation are two aspects of the same phenomenon, also shrinkage is at least partly caused by the same processes. Shrinkage, creep and relaxation occur simultaneously, but are usually considered separately for simplicity, using empirical models calibrated on experimental data.

Due to creep (and shrinkage), prestressing only makes sense with high-performance steel: The steel elongation (prestressing of the tendon) must be large enough so that only a small proportion of the prestress is lost due to creep and shrinkage.

Long-term effects

Shrinkage

Early/capillary (Plastic) shrinkage (up to 4‰ → avoid!)
- Capillary stresses during the evaporation of water from the fresh concrete lead to denser structure of the cement matrix in the first few hours until hardening.
- Avoidance through careful curing (prevention of significant water losses on the fresh concrete surface caused by high concrete or air temperatures, low humidity and wind).

Autogenous and chemical shrinkage (normal concrete up to 0.3‰, UHPC up to 1.2‰)
- Volume contraction during hydration, initially caused by the chemical integration of the water molecules into the hydration products (first days). Afterwards, as soon as the water in the capillary pores is used up, it is mainly caused by capillary tension due to the lower internal relative humidity, leading the hydration to consume water from the gel pores (first weeks).
- Primarily dependent on W/C ratio: The lower the W/C ratio, the greater the autogenous shrinkage (significant effect only for W/C < 0.45 high-performance concrete, UHPC).

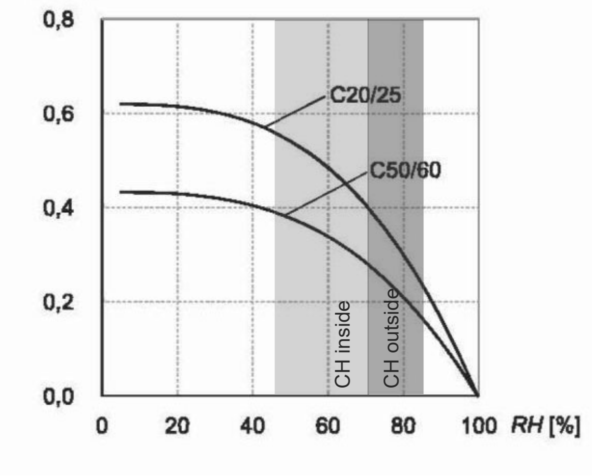
Drying shrinkage (up to approx. 0.3‰ outside at RH=70%, up to approx. 0.5‰ inside at RH=50%)
- Volume contraction in hardened concrete by releasing water into the environment. Begins with formwork stripping or the end of curing and lasts for years.
- Magnitude primarily dependent on cement paste volume (cement, admixtures, entrapped air and water). Faster for high W/C ratios, low air humidity and thin components.

Explanations see slide.

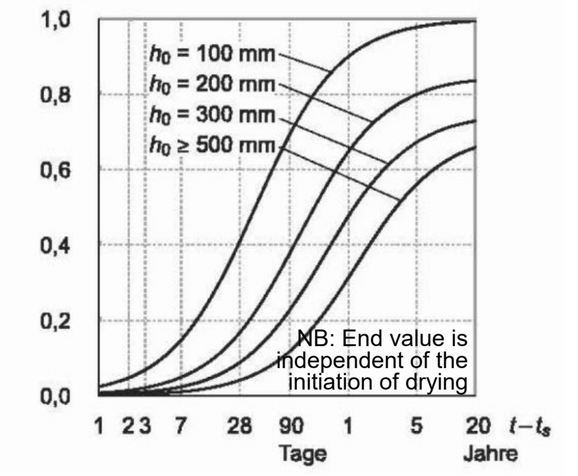
Time-dependent behaviour of concrete

Drying shrinkage ε_{cd}
(according to SIA 262)

Drying shrinkage $\varepsilon_{cd\infty}$ [‰]



Progression $\varepsilon_{cd}(t)\ /\varepsilon_{cd\infty}$



Repetition Stahlbeton I:

The main cause of the shrinkage of normal-strength concrete is drying shrinkage. This is greater in a dry environment than in a humid environment, and it occurs faster in thin components (larger surface area compared to volume) than in thick components. It is sometimes subdivided into further parts, but this is usually not necessary.

In contrast to creep, the time of the "start of loading" (initiation of drying) has no influence on the final value of shrinkage.

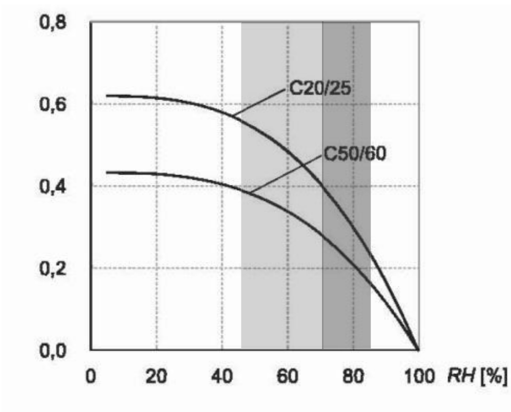
Time-dependent behaviour of concrete

Drying shrinkage ε_{cd}

Autogenous shrinkage ε_{ca} (according to SIA 262)

$$\boxed{\varepsilon_{cs}(t) = \varepsilon_{cd}(t) + \varepsilon_{ca}(t)}$$

Final drying shrinkage value $\varepsilon_{cd\infty}$ [‰]



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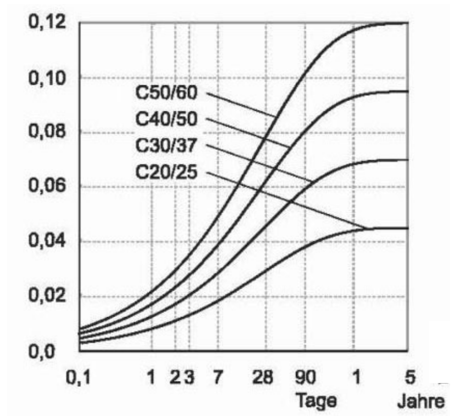
Progression of autogenous shrinkage $\varepsilon_{ca}(t)$ [‰]



Repetition Stahlbeton I:

In the case of high-performance concretes with a very low W/C ratio, autogenous shrinkage must be taken into account in addition to drying shrinkage. This also occurs if the test specimen is stored airtight.

The sum of drying shrinkage and autogenous shrinkage is controlling.

It is very important that early or capillary shrinkage is minimized (which is assumed in the standards). Otherwise, significantly greater shrinkage can occur (up to 4‰!), which can cause large cracks, severely impairing durability in particular. The cause of early or capillary shrinkage is capillary tension during the evaporation of water from the fresh concrete. This leads to denser structure of the cement matrix in the first few hours until hardening. This must be avoided by curing (prevention of significant water losses on the fresh concrete surface caused by high concrete or air temperatures, low humidity and wind).

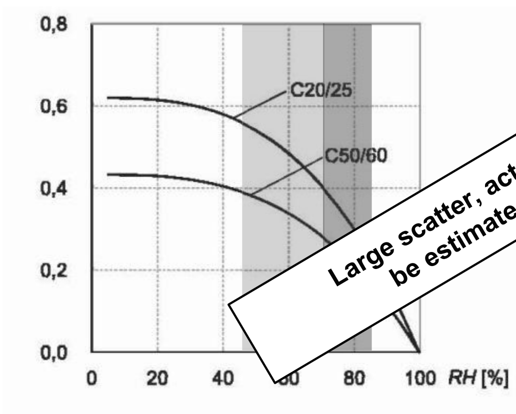
Time-dependent behaviour of concrete

Drying shrinkage ε_{cd}

Autogenous shrinkage ε_{ca} (according to SIA 262)

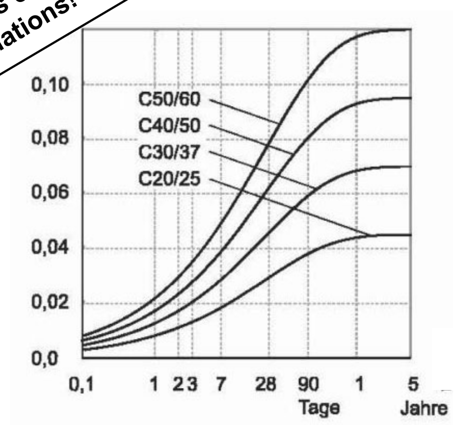
$$\varepsilon_{cs}(t) = \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$$

Final drying shrinkage value $\varepsilon_{cd\infty}$ [‰]



Large scatter, actual shrinkage deformations can only be estimated even with complex calculations!

autogenous shrinkage $\varepsilon_{ca}(t)$ [‰]



Repetition Stahlbeton I:

Shrinkage is subject to large variations (5% fractile values of the shrinkage mass are ± 50...60% in experiments).

Using the rather complicated formulae from the standards, the actual shrinkage deformations can still only be estimated.

Long-term effects

Creep and relaxation

Cause / Phenomena

- Stress leads to rearrangement or evaporation of water in the cement paste. The associated sliding and compaction processes lead to volume contraction.
- The standards assume that creep deformations cease after some decades (asymptotically approaching the long-term creep coefficient φ_∞). This is controversial today, there are e.g. older cantilever bridges indicating that creep deformations might keep increasing continuously. Yet only few experiments are available.

Influences on the magnitude of creep deformations

- Load level (creep deformations approximately proportional to the load)
- Cement paste volume (high cement paste volume = larger creep deformations)
- Concrete compressive strength (high compressive strength = smaller creep deformations)
- Age of the concrete (loading at a young age causes larger creep deformations)

Influences on the course of time

- Creep is faster in smaller elements (thin components)
- Creep is faster at low relative humidity (dry environment)

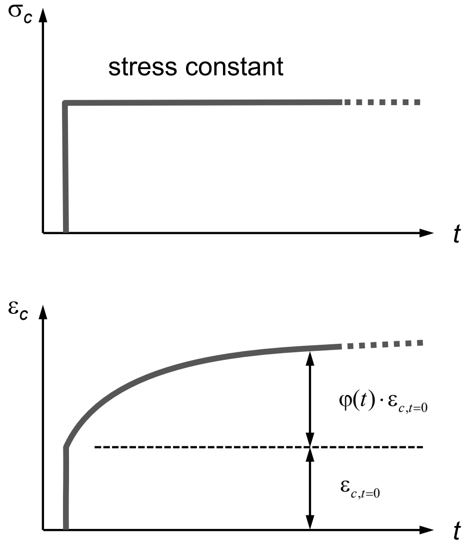
Relaxation

- Creep and relaxation are related phenomena
- Relaxation behaviour is influenced by the same variables as creep

Explanations see slide.

Long-term effects

Creep



- Increase in deformation under constant stress
- $\varepsilon_c(t) = \varepsilon_{c,el} + \varepsilon_{cc}(t)$
- $\varepsilon_{cc}(t) = \varphi(t, t_0) \cdot \varepsilon_{c,el}$ resp.
  $\varepsilon_c(t) = \varepsilon_{c,el}\left(1 + \varphi(t, t_0)\right)$
- $\varphi(t,t_0)$ creep index
- Normal case: $\varphi_{t=\infty} \approx 1.5 \ldots 2.5$ i.e. increase of deformations by factor 2.5...3.5
- Analogous behaviour under tension (non-cracked concrete)

Repetition Stahlbeton I:

The creep deformations of the concrete are determined by the creep coefficient φ (φ = ratio creep deformation / elastic deformation).

Uncracked concrete creeps (and relaxes) also under tensile stress. However, considerably less test data is available for this than for compressive stress.

Long-term effects

Relaxation (≈ creep at ε = const.)

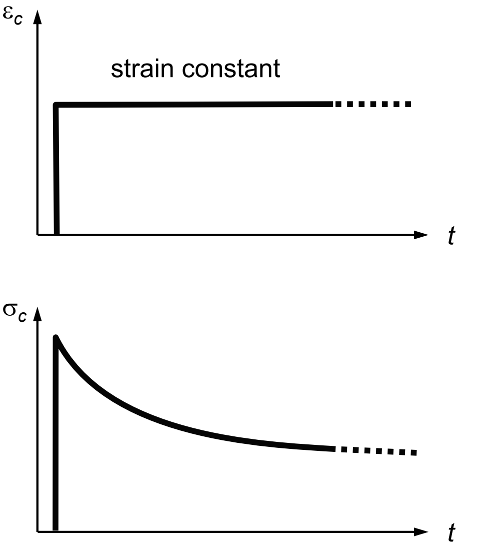
- Decrease in stress under constant strain
- Approximation (fictitious modulus of elasticity):

$$\sigma_c(t) = \sigma_{c,t=0} \cdot \frac{1}{1+\varphi}$$

- Better approximation (derived e.g. using Trost method)

$$\sigma_c(t) = \sigma_{c,t=0} \cdot \left(1 - \frac{\varphi(t)}{1+\mu\varphi}\right)$$

- Normal case: $\varphi_{t=\infty} \approx 1.5 \ldots 2.5$, μ = ca. 0.75, i.e. reduction of the initial stress to approx. 25%.
- Decrease is less pronounced (to approx. 40%) if deformation is imposed slowly



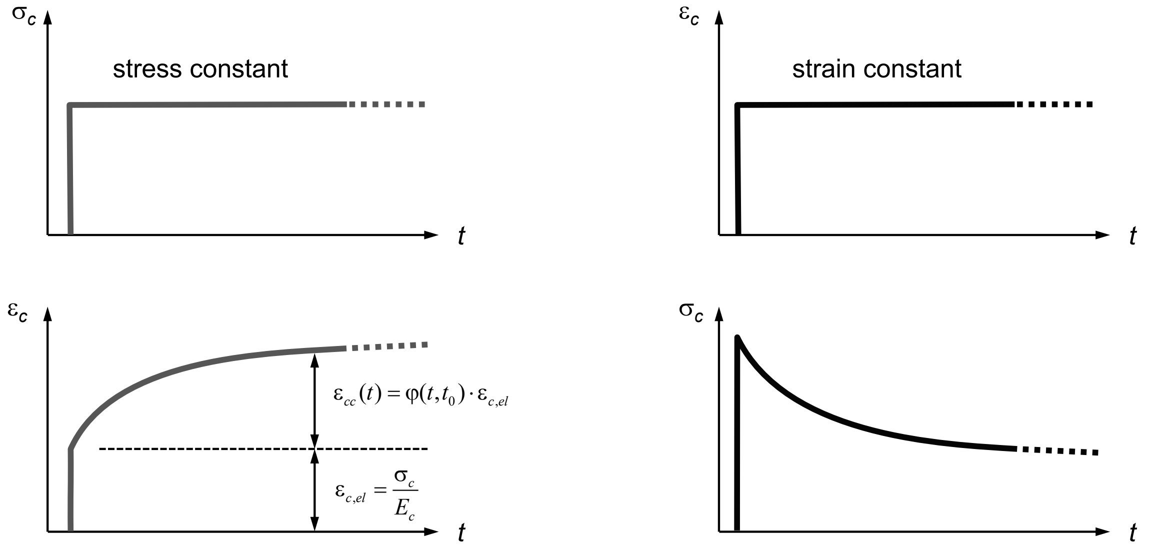
Repetition Stahlbeton I:

Creep and relaxation, as already mentioned, are consequences of the same phenomena. The relaxation function can thus also be recorded with the creep coefficient φ or determined from the creep function (requires the solution of a linear inhomogeneous Volterra integral equation (see Marti, structural analysis)).

The approximation of the relaxation function via a fictitious modulus of elasticity underestimates the reduction of stresses due to relaxation. A better approximation results from Trost's method (exact if the coefficient μ is determined from the creep function as described above).

Long-term effects

Creep | **Relaxation** (= creep)



Repetition Stahlbeton I:

Creep and relaxation are subject to similarly large scatter as shrinkage (5% fractile values of the creep coefficient are ± 30...40% in experiments).

The actual creep deformations (or relaxation) can therefore only be estimated with the rather complex formulae contained in the standards.

Long-term effects

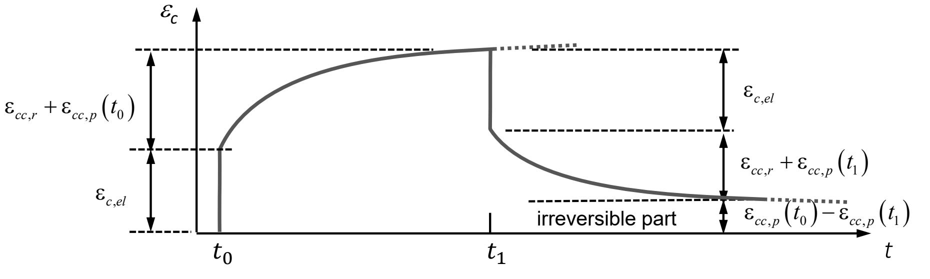
Creep - reversible and plastic part

- The deformations of the concrete under load are composed of the elastic deformations $\varepsilon_{c,el}$ and the time-dependent creep deformations ε_{cc}
- The creep deformations ε_{cc} consist of a reversible component $\varepsilon_{cc,r}$ (reversal sets in relatively quickly, half-life approx. 30 days) and an irreversible (plastic) component $\varepsilon_{cc,p}$:

$$\varepsilon_c(t) = \varepsilon_{c,el} + \boxed{\varepsilon_{cc,r}(t) + \varepsilon_{cc,p}(t)} = \varepsilon_{c,el} \boxed{+\varepsilon_{cc}(t)}$$

The irreversible part $\varepsilon_{cc,p}$ depends on the time of loading = concrete age at load application (old concrete is less prone to creep) and occurs much slower than the reversible component.

- Example: Loading and complete unloading after a longer period of time (permanent stretching):

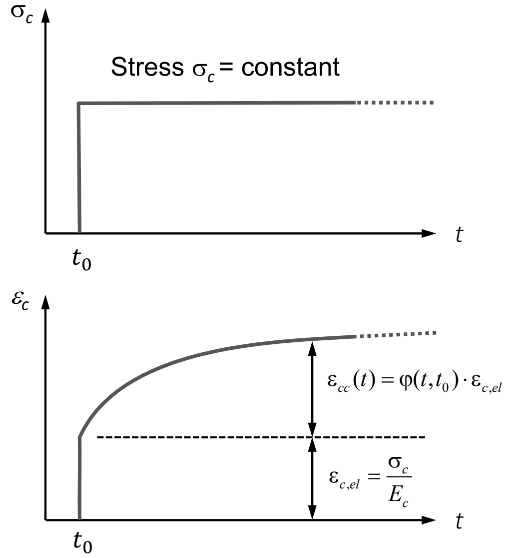


For simplicity, no distinction is made between the two components in the following slides , and no unloading is considered.

The creep deformations consist of a reversible and an irreversible part.

Long-term effects

Creep – Final value and progression
(see also SIA 262, 3.1.2.6)



- Increase in deformation under constant stress
- $$\begin{aligned}\varepsilon_c(t) &= \varepsilon_{c,el} + \varphi(t,t_0)\cdot\varepsilon_{c,el} \\ &= \left(1+\varphi(t,t_0)\right)\cdot\varepsilon_{c,el}\end{aligned}$$

  with

$\varphi(t,t_0)$	creep coefficient
t	time
t_0	age of the concrete at the start of exposure
$t-t_0$	load duration

- Normal case: $\varphi_{t=\infty} \approx 1.5 \ldots 2.5$, i.e. increase of deformations by a factor of 2.5...3.5
- Analogous behaviour under tension (in uncracked concrete)

On this and the following slides the determination of the creep progression according to SIA 262 is explained (essentially corresponds to fib Model Code 1990 and Eurocode 2).

Long-term effects

Creep – Final value and progression
(see also SIA 262, 3.1.2.6)

$$\varphi\left(t_0, t\right) = \varphi_{RH} \cdot \beta_{\sigma c} \cdot \beta_{fc} \cdot \beta(t_0) \cdot \beta(t - t_0) \qquad (\approx 1.5 \ldots 2.5)$$

t_0: concrete age at time of loading

t: time at which the creep coefficient φ is determined

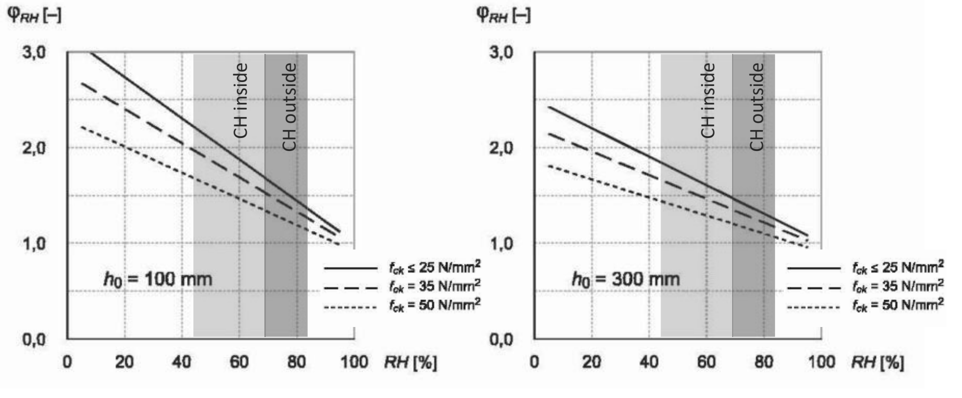
- Relative humidity: $\varphi_{RH}(\text{CH}) \approx 1.25 \ldots 1.5 \;\; (RH \approx 80 \ldots 65\%)$
- Stress level: $\beta_{\sigma c} = e^{1.5\left(\frac{\sigma_c}{f_{ck}} - 0.45\right)}$ (für $\sigma_c > 0.45 f_{ck}$, sonst $\beta_{\sigma c} = 1$)
- Concrete compressive strength: $\beta_{fc} = \begin{matrix} \ldots & C25/30 & C30/37 & C35/45 & \ldots \\ \ldots & 2.9 & 2.7 & 2.6 & \ldots \end{matrix}$
- Concrete age at loading: (corrected for the influence of the temperature: $t_{0,\text{eff}} \rightarrow k_T t_0$) $\beta(t_0) \approx 1.2 \ldots 0.2 \quad \beta(t_0 = 28\,\text{d}) = 0.5$
- Load duration (→ progression): $\beta\left((t = \infty) - t_0\right) \approx 1$

Long-term effects

Creep – Final value and progression
(see also SIA 262, 3.1.2.6)

$$\varphi(t, t_0) = \varphi_{RH} \cdot \beta_{\sigma c} \cdot \beta_{fc} \cdot \beta(t_0) \cdot \beta(t - t_0) \qquad (\approx 1.5 \ldots 2.5)$$

φ_{RH}: Coefficient for relative humidity (RH: usually the annual mean)



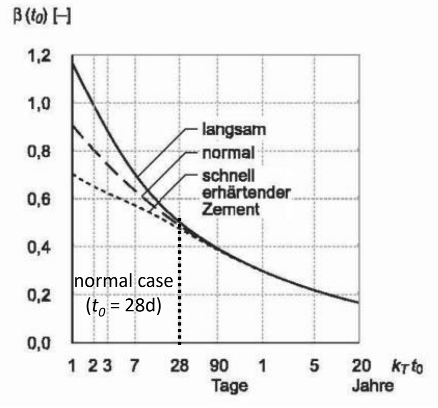
Long-term effects

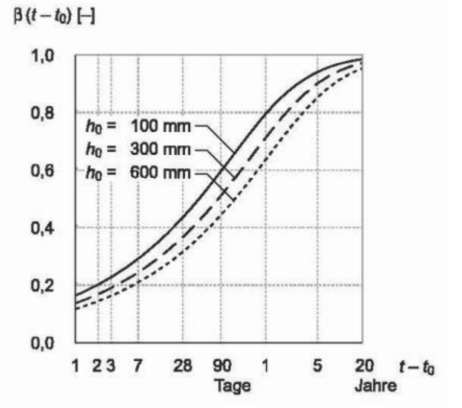
Creep – Final value and progression
(see also SIA 262, 3.1.2.6)

$$\varphi\left(t,t_0\right)=\varphi_{RH}\cdot\beta_{\sigma c}\cdot\beta_{fc}\cdot\beta(t_0)\cdot\beta(t-t_0)\qquad(\approx 1.5\ldots2.5)$$

$\beta(t_0)$ Concrete age at loading

φ_{RH} : Load duration (→ progression)





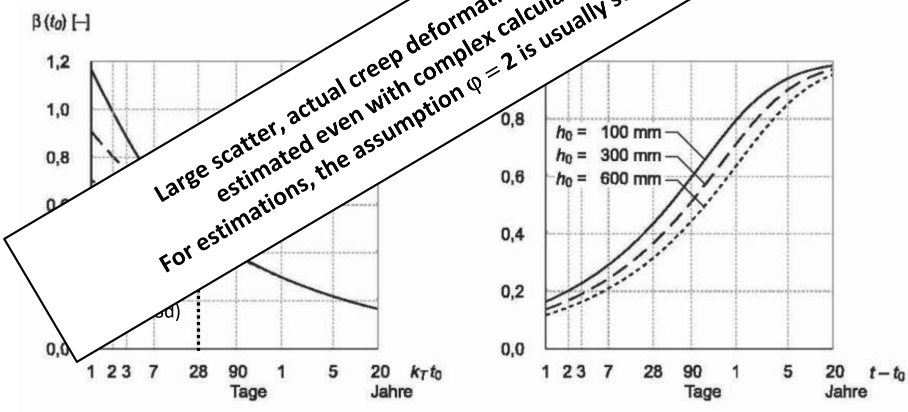
Long-term effects

Creep – Final value and progression
(see also SIA 262, 3.1.2.6)

$$\varphi\left(t,t_0\right)=\varphi_{RH}\cdot\beta_{\sigma c}\cdot\beta_{fc}\cdot\beta(t_0)\cdot\beta \ldots \quad \ldots 1.5\ldots2.5)$$

Large scatter, actual creep deformations can only be estimated even with complex calculations!
For estimations, the assumption $\varphi = 2$ is usually sufficient.

$\beta(t_0)$: Concrete age at loading (→ progression)



4 Long-term effects

4.2 Effect of creep on the load-bearing and deformation behaviour

Long-term effects

Effect of creep on deformations of a structure

- The effect of creep must always be taken into account when determining deformations due to permanent loads. The increase in deformation due to creep is considerably smaller in the cracked stage II than in the non-cracked stage I (see Stahlbeton I).
- Deformations are often governing the design, for example in the case of:
  - passively reinforced, slender girders (above $h/L \approx 1/12$)
  - passively reinforced slabs (flat slabs, canopies, slabs near facade area, non-load-bearing walls)
  - prestressed bridge girders, whose stresses in construction and final state differ strongly (cantilever construction, continuous beams cast span by span)

Effect of creep on internal forces and stresses

- Restraint and residual stresses are partially relieved due to creep over time (relaxation).
- For statically determinate (= isostatic) systems and for statically indeterminate systems with uniform creep properties, creep has no effect on the internal forces
- Significant internal force redistributions occur in statically indeterminate (= hyperstatic) systems as a result of changes of the static system and non-uniform creep properties.
  The calculation of creep effects is complicated by the interdependence (creep depends on the level of stress and vice versa).

Approaches for the calculation of creep and shrinkage problems

- Method with age adjusted modulus of elasticity
- Unit creep curve method (Dischinger method)
- Rüsch Method (improved method Dischinger)
- Creep step method
- Trost Method (sufficiently accurate and suitable for manual calculations)

Explanations see slide.

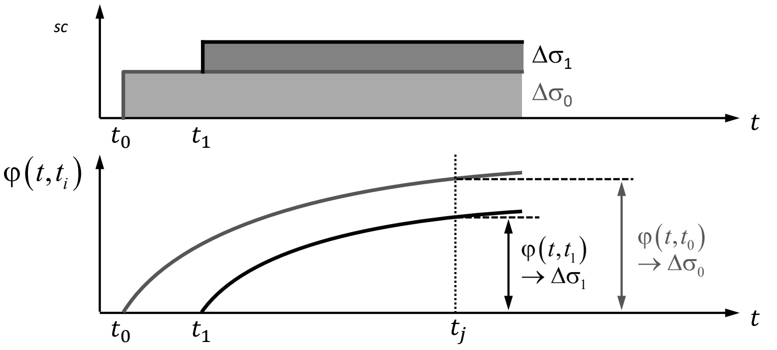
Long-term effects

Creep – Boltzmann superposition principle

- The creep strain due to any stress development σ(t) can generally be expressed as follows:

$$\varepsilon_{cc}(t) = \frac{1}{E_{c0}} \int_{\tau=0}^{\tau=t} \frac{\partial \sigma}{\partial \tau} \varphi(t,\tau)\, d\tau$$

- For discrete stress increments (steps) $\Delta\sigma_i$, which are applied at time t_i results: $\varepsilon_{cc}(t) = \frac{1}{E_{c0}} \sum_{i=0}^{n} \Delta\sigma_i \cdot \varphi(t,t_i)$



When determining the creep strain as a result of any stress development, the creep curve corresponding to the time of loading must be used for each infinitesimal stress increase (for each infinitesimal stress increase from the time of its occurrence to the considered end time).

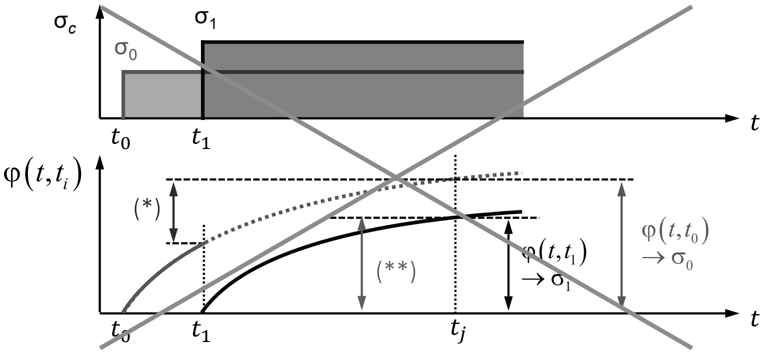
If the stress curve is discretized in steps, the creep curve corresponding to the time of loading times the respective stress change can be applied for each step (in the time interval from the stress change to the considered end time).

Long-term effects

Creep – Boltzmann superposition principle

Incorrect procedure for determining creep deformations (creep from the respective load level for the entire load with new creep coefficient):

- (*) Effective = correct portion of creep caused by σ_0 in time interval $t_1 \ldots t_j$ $\quad \Delta\varphi(t_j, t_1) \rightarrow \sigma_0$ (right)
- (**) Incorrectly determined portion of creep caused by σ_0 in time interval $t_1 \ldots t_j$ $\quad \Delta\varphi(t_j, t_1) \rightarrow \sigma_0$ (wrong)



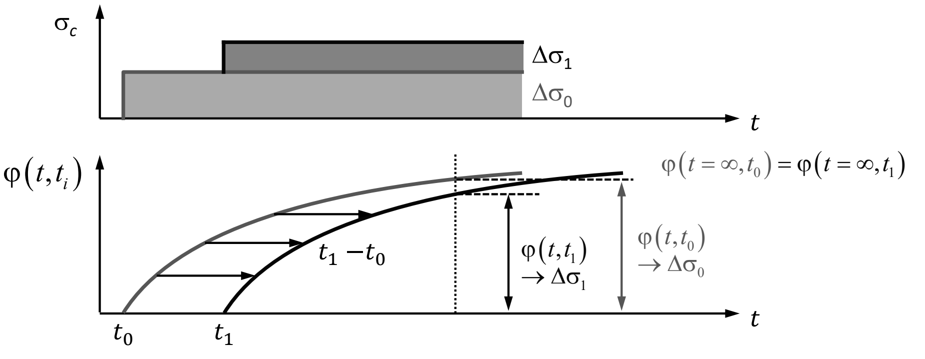
An incorrect development of the creep strains is obtained, if the creep curve corresponding to the time of loading times the respective total stress is used (for the time interval until the next stress change).

Long-term effects

Approaches for the calculation of creep and shrinkage problems

Method with age adjusted modulus of elasticity

- Effect of concrete age at loading neglected
  → same creep curve for all loads, shifted along abscissa (horizontal)
- Unrealistic (overestimates ability of old concrete to creep)



Historically, there have been various approaches to determine creep deformations for a given stress curve. Due to limited computing capacity, attempts were made to solve the problem with simple approaches. From today's perspective, these approaches are interesting as they show which effects are taken into account or neglected.

The simplest procedure is that of the age adjusted modulus of elasticity. The same creep curve is used for each stress change, independent of the time of loading.

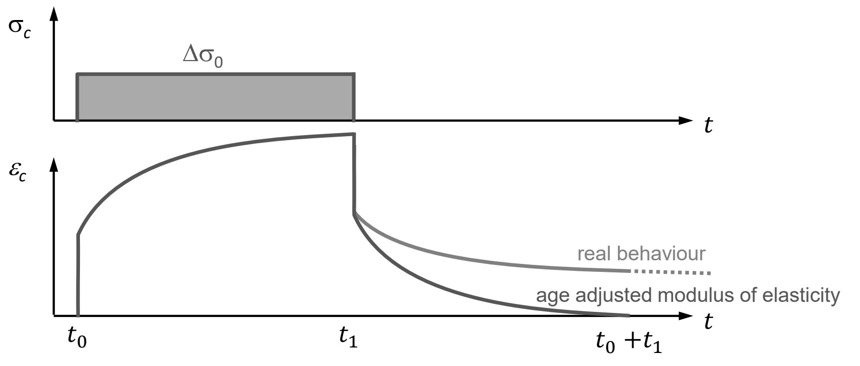
This method overestimates the creep properties of the old concrete.

Long-term effects

Approaches for the calculation of creep and shrinkage problems

Method with age adjusted modulus of elasticity

- Effect of concrete age at loading neglected
  → same creep curve for all loads, shifted along abscissa (horizontal)
- Unrealistic (overestimates ability of old concrete to creep)
- Unrealistic: corresponds to assumption of viscoelastic, i.e. fully reversible behaviour



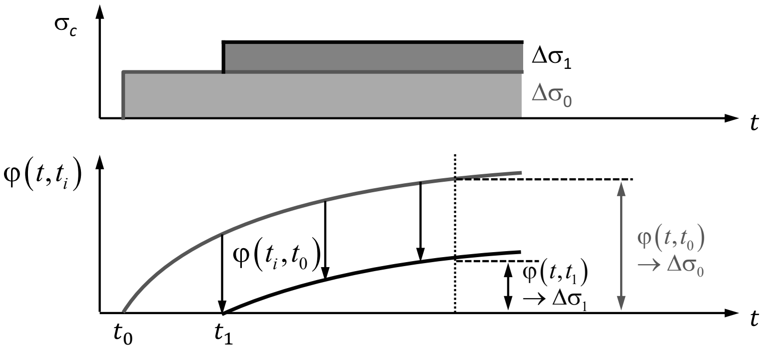
Since the same creep curve is assumed for the method with an age adjusted modulus of elasticity, independent of the concrete age, the irreversible part of the creep deformations cannot be explained or modelled with it.

Long-term effects

Approaches for the calculation of creep and shrinkage problems

Unit creep curve method (Dischinger method)

- Same creep curve for all loads, shifted along ordinate (i.e. vertically)
- Advantage: Representation in recursion formulae possible
- Unrealistic: underestimates creep of old concrete



Dischinger proposed to use a unit creep curve for all stress levels, starting from the time of application of the first load. Whereby for each stress level only the portion of creep which occurred after its application was taken into account. This corresponds to the use of creep curves shifted along the ordinate. The advantage is that the creep curve could be approximated using recursion formulae.

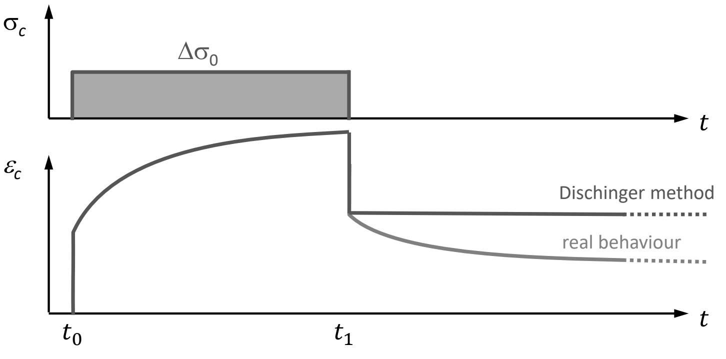
This method underestimates the creep of old concrete.

Long-term effects

Approaches for the calculation of creep and shrinkage problems

Unit creep curve method (Dischinger method)

- Same creep curve for all loads, shifted along ordinate (i.e. vertically)
- Advantage: Representation in recursion formulae possible
- Unrealistic: underestimates creep of old concrete
- unrealistic: neglects viscoelastic behaviour (no reversible part)



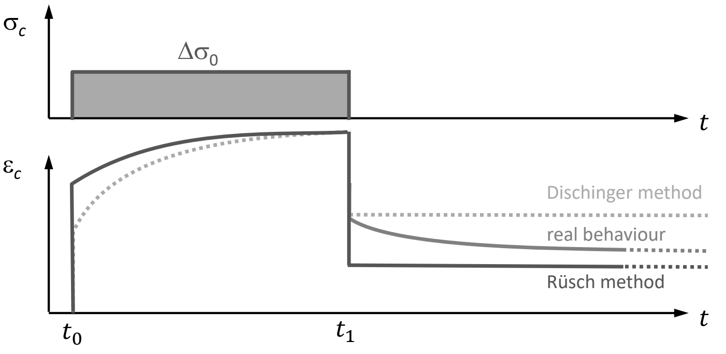
In the Dischinger method (unit creep curve) a complete relief causes creep deformations of the same magnitude as the creep deformations due to the original load. Therefore the reversible portion of the creep deformations cannot be explained or modelled.

Long-term effects

Approaches for the calculation of creep and shrinkage problems

Rüsch method (improved Dischinger method)

- Basically the same assumptions as Dischinger method
- Superposition of the entire reversible part of creep deformations (neglected in the Dischinger method) with the elastic elongation
- Reasonably realistic, since the reversible portion of creep deformations occurs relatively quickly



In order to eliminate the disadvantages of the Dischinger method (unit creep curve) without losing its advantages (recursive formulae), Rüsch corrected the creep curves by a reversible proportion occurring simultaneously with the elastic elongation. This allows the real behaviour to be modelled reasonably accurately, especially in the final state.

Long-term effects

Approaches for the calculation of creep and shrinkage problems

Creep step method

- The stress history is only known in advance in simple cases (this was assumed in the previous considerations). In general, it depends on the creep behaviour. The solution therefore usually requires an iterative or step-by-step approach.
- Based on the Dischinger method a differential equation for creep behaviour can be formulated (also possible with the Rüsch method). For numerical solutions, the creep step method can be used, which is based on a subdivision of the load history into time intervals or into "creep steps" (subdivision of the creep coefficient $\varphi(t=\infty, t_0)$ in equal creep intervals $\Delta\varphi$, usually more appropriate).
- Linearisation of the creep and stress function per interval results in the increase of creep deformation in the time interval. $\Delta t_i = t_i - t_{i-1}$ (note that since Dischinger's Method is used, the reversible part of creep is not accounted for):

$$\Delta\varepsilon_{cc,\mathrm{i}} = \frac{\sigma_{i-1}}{E_{c0}}\Delta\varphi_i + \frac{\Delta\sigma_i}{E_{c0}}\frac{\Delta\varphi_i}{2} = \frac{\sigma_{i-1}+\Delta\sigma_i/2}{E_{c0}}\Delta\varphi_i;$$

$\Delta\varphi_i = \varphi_i - \varphi_{i-1}$: Change of the creep function during Δt_i

$\Delta\sigma_i = \sigma_i - \sigma_{i-1}$: Change of the concrete stress during Δt_i

- Total strain increase in time interval $\Delta t_i = t_i - t_{i-1}$:

$$\Delta\varepsilon_{c,\mathrm{i}} = \frac{\Delta\sigma_i}{E_{c0}} + \Delta\varepsilon_{cc,\mathrm{i}} + \Delta\varepsilon_{cs,\mathrm{i}} = \frac{\Delta\sigma_i}{E_{c0}} + \frac{\sigma_{i-1}}{E_{c0}}\Delta\varphi_i + \frac{1}{2}\frac{\Delta\sigma_i}{E_{c0}}\Delta\varphi_i + \Delta\varepsilon_{cs,\mathrm{i}} = \frac{\Delta\sigma_i}{E_{c0}} + \frac{\sigma_{i-1}+\Delta\sigma_i/2}{E_{c0}}\Delta\varphi_i + \Delta\varepsilon_{cs,\mathrm{i}}$$

Additional remarks:

- Numerically, time-dependent behaviour can be investigated using general creep curves. This is implemented in many software packages that take "long-term effects" into account.

- In FE programs, the creep strains that would occur under the given stresses (at the beginning of the time step) can be applied as a load to the system in each time step. The stress changes and the actual creep strains at the end of the time step can be determined from this. The reduced creep capacity of the old concrete has to be taken into account. This can be done using Trost's method, which is explained on the following slides.

4 Long-term effects

4.3 Simplified method for the investigation of long-term effects

Long-term effects

Approaches for the calculation of creep and shrinkage problems

Trost method

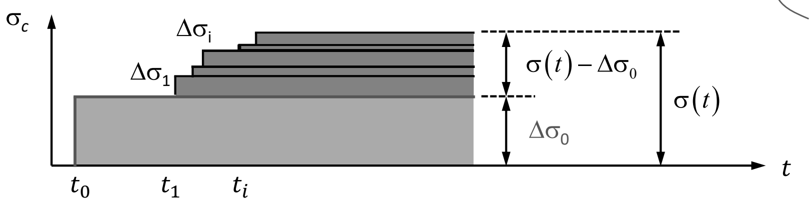
In practice, a relatively large proportion of the total stress is applied at a time t_0 followed by smaller stress increments $\Delta\sigma_i$ (additional loads, but also internal force redistributions). The Trost method takes advantage of this to avoid an iterative or step-by-step approach.

The creep function for the stress increments ($\sigma(t) - \Delta\sigma_0 = \sum_{i=1}^{n} \Delta\sigma_i$) occurring at the time period $t_i > t_0$ (resp. $t_0 < t_i \leq \infty$) is reduced with an ageing coefficient $\mu(t)$ (sometimes also called «relaxation factor»).

The creep deformation due to the total change in stress according to Boltzmann's superposition principle is:

$$\varepsilon_{cc}(t) = \frac{\sigma_0}{E_{c0}} \cdot \varphi(t, t_0) + \sum_{i=1}^{n} \frac{\Delta\sigma_i}{E_{c0}} \cdot \varphi(t, t_i) = \frac{\sigma_0}{E_{c0}} \cdot \varphi(t, t_0) + \frac{\sigma(t) - \sigma_0}{E_{c0}} \cdot \mu(t) \cdot \varphi(t, t_0)$$

Ageing coefficient (simplified by Trost to be identical for all load levels, $t_i > t_0$)
General derivation see Marti, Theory of Structures, Chapter 7.4.2



On this and the following slides the Trost method is explained. It is well suited for manual calculations (especially in combination with the force method) and is usually sufficiently accurate for practical applications.

Long-term effects

Approaches for the calculation of creep and shrinkage problems

Trost method

The ageing coefficient results from the equation on the previous slide:

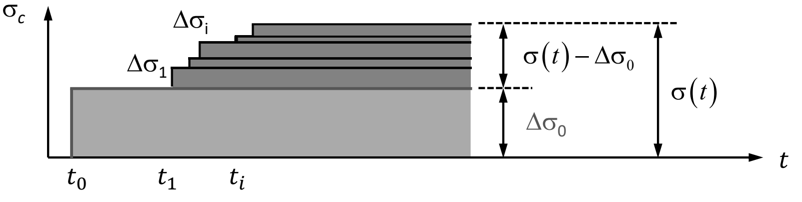
$$\sum_{i=1}^{n} \frac{\Delta\sigma_i}{E_{c0}} \cdot \varphi\left(t, t_i\right) = \frac{\sigma\left(t\right) - \sigma_0}{E_{c0}} \cdot \mu(t) \cdot \varphi\left(t, t_0\right) \rightarrow \mu(t) = \frac{\sum_{i=1}^{n} \Delta\sigma_i \cdot \varphi\left(t, t_i\right)}{\left(\sigma\left(t\right) - \sigma_0\right) \cdot \varphi\left(t, t_0\right)}$$

The total deformations at time *t* thus amount to:

$$\varepsilon_c\left(t\right) = \frac{\sigma_0}{E_{c0}}\left(1 + \varphi\left(t, t_0\right)\right) + \frac{\sigma\left(t\right) - \sigma_0}{E_{c0}}\left(1 + \mu(t) \cdot \varphi\left(t, t_0\right)\right) + \varepsilon_{cs}\left(t\right)$$

elastic deformations

shrinkage



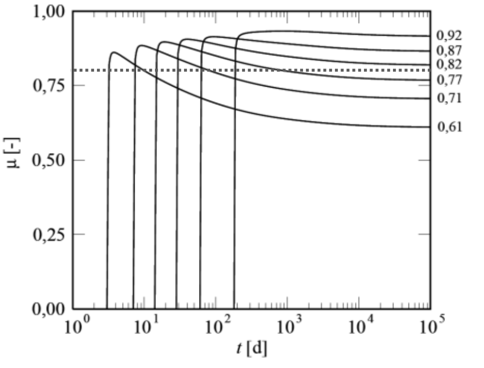
Explanations see slide.

Long-term effects

Approaches for the calculation of creep and shrinkage problems

Trost method

- The stress curve is generally not known → $\mu(t)$ cannot be calculated directly in the way outlined on the previous slides
- If the relaxation function is determined from the creep function (solution of a linear, inhomogeneous Volterra integral equation), the corresponding ageing coefficient can be determined numerically [see Seelhofer 2009 or Marti, Theory of Structures]:



- The evaluation sows that τ $\mu(t)$ varies only slightly

  → Ageing coefficient μ independent of time sufficiently accurate for practical applications

  → for usual conditions (φ = 1.5...4) approximately $\mu \approx 0.80$

Explanations see slide.

Long-term effects

Approaches for the calculation of creep and shrinkage problems

Trost method

- With this approximation the total deformation at time t is:

$$\varepsilon_c(t) = \frac{1}{E_{c0}}\Big[\underbrace{\sigma_0(1+\varphi)}_{\text{initial stress}} + \underbrace{\Delta\sigma(1+\mu\cdot\varphi)}_{\text{stresses added over time}}\Big] + \varepsilon_{cs}(t)$$

with $\sigma_0 = \Delta\sigma_0 = \sigma(t=t_0), \quad \Delta\sigma = \sigma(t) - \sigma_0, \quad \varphi = \varphi(t,t_0), \quad t > t_{0,} \quad \mu \approx 0.8$

- Alternative formulation using fictitious («refined age adjusted») moduli of elasticity for long-term influences:

$$\varepsilon_c(t) = \frac{\sigma_0}{\dfrac{E_{c0}}{1+\varphi(t,t_0)}} + \frac{\Delta\sigma(t)}{\dfrac{E_{c0}}{1+\mu\cdot\varphi(t,t_0)}} + \varepsilon_{cs}(t) = \frac{\sigma_0}{E_c'} + \frac{\Delta\sigma(t)}{E_c''} + \varepsilon_{cs}(t): \underbrace{E_c' = \frac{E_{c0}}{1+\varphi(t,t_0)}}_{\text{initial stress}}, \quad \underbrace{E'' = \frac{E_{c0}}{1+\mu\cdot\varphi(t,t_0)}}_{\text{stresses added over time}}$$

Explanations see slide.

Calculation of relaxation function based on creep coefficient and ageing factor

- Relaxation function = stress curve for constant (imposed) initial strain, i.e. initial strains $\varepsilon_{c0} = \dfrac{\sigma_0}{E_{c0}}$ remain constant

Method with age adjusted modulus of elasticity $(\varphi = \varphi(t, t_0))$

$$\varepsilon_c(t) = \frac{\sigma_0}{E_{c0}}(1+\varphi) + \frac{\Delta\sigma(t)}{E_{c0}}(1+\varphi) = \frac{\sigma_0}{E_{c0}}$$

$$\rightarrow \Delta\sigma(t) = -\sigma_0 \frac{\varphi}{1+\varphi}$$

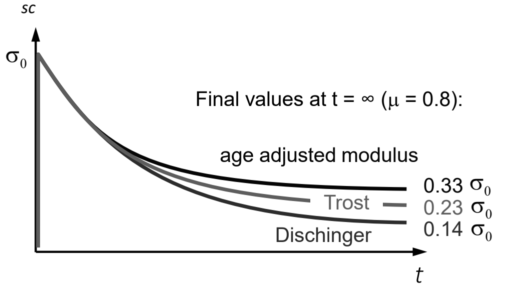
$$\rightarrow \sigma(t) = \sigma_0 + \Delta\sigma(t) = \sigma_0\left(1 - \frac{\varphi}{1+\varphi}\right) = \sigma_0 \frac{1}{1+\varphi}$$

Trost method $(\varphi = \varphi(t, t_0))$

$$\varepsilon_c(t) = \frac{\sigma_0}{E_{c0}}(1+\varphi) + \frac{\Delta\sigma(t)}{E_{c0}}(1+\mu\cdot\varphi) = \frac{\sigma_0}{E_{c0}}$$

$$\rightarrow \Delta\sigma(t) = -\sigma_0 \frac{\varphi}{1+\mu\cdot\varphi}$$

$$\rightarrow \sigma(t) = \sigma_0 + \Delta\sigma(t) = \sigma_0\left(1 - \frac{\varphi}{1+\mu\cdot\varphi}\right)$$



The Trost method is simple and agrees well with experiments (better than more complicated procedures) → only this procedure is used in the following!

Explanations see slide.

Long-term effects

Terminology and generalisation of the Force Method → Time-dependent Force Method

- In the following, Trost's Method is used in combination with the Force Method, known from «Baustatik I/II»
- To account for long-term effects, compatibility conditions are expressed here at different moments in time
- Further information on the Force Method: Peter Marti, «Theory of Structures» resp. «Baustatik», Chapter 16. The following summaries are taken from this book (p. 254 and p. 257, respectively):

1. Determine the degree n of static indeterminacy.
2. Select a stable, statically determinate *basic system* by releasing n constraints and introducing corresponding *redundant variables* X_i.
3. Determine the support force variables and stress resultants C_0, S_0 and C_i, S_i for the basic system as a result of loads or as a result of unit force variables $X_i = 1$.
4. Determine the deformations (incompatibilities) δ_{i0} or δ_{ij} at the position and in the direction of X_i as a result of the external actions (loads and imposed deformations) or as a result of the unit force variables $X_j = 1$.
5. Set up and solve the following *compatibility conditions*:

$$\delta_i = \delta_{i0} + \sum_{j=1}^{n} \delta_{ij} X_j = 0 \qquad (i = 1, 2, \ldots, n) \tag{16.8}$$

6. Determine the support force variables and stress resultants for the statically indeterminate system by *superposing* the corresponding variables on the basic system:

$$C = C_0 + \sum_{i=1}^{n} C_i X_i \quad , \quad S = S_0 + \sum_{i=1}^{n} S_i X_i \tag{16.9}$$

1. Bestimmen des Grads n der statischen Unbestimmtheit.
2. Wahl eines stabilen, statisch bestimmten *Grundsystems* durch Lösen von n Bindungen und Einführen entsprechender *überzähliger Grössen* X_i.
3. Ermitteln der Lagerkraft- und Schnittgrössen C_0, S_0 bzw. C_i, S_i am Grundsystem infolge der Lasten bzw. infolge der Einheitskraftgrössen $X_i = 1$.
4. Ermitteln der Verformungen (Klaffungen) δ_{i0} bzw. δ_{ij} an der Stelle und in der Richtung von X_i infolge der äusseren Einwirkungen (Lasten und eingeprägte Verformungen) bzw. infolge der Einheitskraftgrössen $X_j = 1$.
5. Aufstellen und Lösen der *Kompatibilitätsbedingungen*

$$\delta_i = \delta_{i0} + \sum_{j=1}^{n} \delta_{ij} X_j = 0 \qquad (i = 1, 2, \ldots, n) \tag{16.8}$$

6. Bestimmen der Lagerkraft- und Schnittgrössen des statisch unbestimmten Systems durch *Superposition* der entsprechenden Grössen am Grundsystem

$$C = C_0 + \sum_{i=1}^{n} C_i X_i \quad , \quad S = S_0 + \sum_{i=1}^{n} S_i X_i \tag{16.9}$$

Example 1 shows that in systems with uniform creep properties, there are no internal force redistributions due to creep.

Important terms used with the Force Method

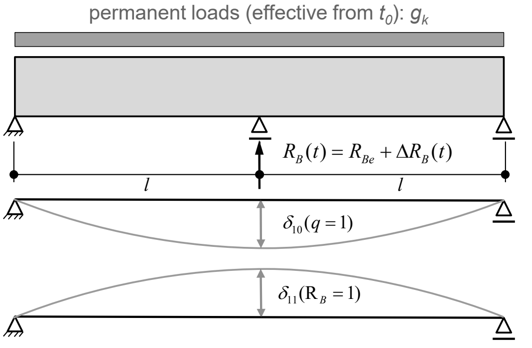
English term	*Deutscher Begriff*
statically determinate (= isostatic):	statisch bestimmt
statically indeterminate (= hyperstatic):	statisch unbestimmt
degree of static indeterminacy:	Grad der statischen Unbestimmtheit
internal action (= stress resultant):	Schnittgrösse (= Spannungsresultierende)
constraint:	Bindung
redundant variable:	überzählige Grösse
basic system:	Grundsystem
compatibility condition:	Verträglichkeitsbedingung

Long-term effects

Redistribution of internal forces in statically indeterminate systems

Systems with uniform creep properties

Example 1: Two-span beam, solution with force method



BS (basic system): Intermediate bearing removed
RV: (redundant variable): Reaction at intermediate support

Displacements in the basic system (elastic, $t = t_0$):

$$\delta_{10} = \frac{5}{384}\frac{g_k(2l)^4}{EI} \qquad \delta_{11} = \frac{1}{48}\frac{(2l)^3}{EI}$$

Compatibility condition at time t_0 :

$$\delta = \delta_{10} + R_{Be}\cdot\delta_{11} = 0$$

Time-dependent compatibility condition with the Trost method:

$$\delta = \delta_{10}\cdot(1+\varphi) + R_{Be}\cdot\delta_{11}\cdot(1+\varphi) + \Delta R_B(t)\cdot\delta_{11}\cdot(1+\mu\varphi) = 0$$

$$\underbrace{\delta_{10} + R_{Be}\cdot\delta_{11}}_{=\,0\text{ (compatibility at time }t_0)} + \Delta R_B(t)\cdot\delta_{11}\frac{1+\mu\varphi}{1+\varphi} = 0 \quad \boxed{\rightarrow \Delta R_B(t) = 0}$$

→ Generalization to general systems is possible → With uniform creep properties, the redundant variables of stat. indeterminate systems do not change!

Example 1 shows that in systems with uniform creep properties, there are no internal force redistributions due to creep.

Long-term effects

Redistribution of internal forces in statically indeterminate systems

Systems with uniform creep properties

Generalization to general systems

Compatibility at $t = t_0$:
$$\begin{pmatrix}\delta_{10}\\ \vdots \\ \delta_{i0}\end{pmatrix} + \begin{pmatrix}\delta_{11}&\cdots&\delta_{1i}\\ \vdots&\ddots&\vdots\\ \delta_{i1}&\cdots&\delta_{ii}\end{pmatrix}\cdot\begin{pmatrix}X_{1e}\\ \vdots\\ X_{ie}\end{pmatrix}=\begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}\;(**)$$

$\rightarrow$ RV at $t = t_0$:
$$\begin{pmatrix}X_{1e}\\ \vdots\\ X_{ie}\end{pmatrix}=\begin{pmatrix}\delta_{11}&\cdots&\delta_{1i}\\ \vdots&\ddots&\vdots\\ \delta_{i1}&\cdots&\delta_{ii}\end{pmatrix}^{-1}\cdot\begin{pmatrix}\delta_{10}\\ \vdots \\ \delta_{i0}\end{pmatrix}$$

Cange of redundant variables using the Trost method: $X_i(t) = X_{ie} + \Delta X_i(t)$

All coefficients multiplied by the same factor!

$\rightarrow$ Compatibility for $t > t_0$:
$$\overbrace{(1+\varphi)\begin{pmatrix}\delta_{10}\\ \vdots \\ \delta_{i0}\end{pmatrix} + \begin{pmatrix}\delta_{11}&\cdots&\delta_{1i}\\ \vdots&\ddots&\vdots\\ \delta_{i1}&\cdots&\delta_{ii}\end{pmatrix}\cdot\left[(1+\varphi)\begin{pmatrix}X_{1e}\\ \vdots\\ X_{ie}\end{pmatrix}\right.}^{\text{All coefficients multiplied by the same factor!}}\left. + (1+\mu\varphi)\begin{pmatrix}\Delta X_1\\ \vdots\\ \Delta X_i\end{pmatrix}\right]=\begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}$$

$\rightarrow$ using (**):
$$\begin{pmatrix}\delta_{11}&\cdots&\delta_{1i}\\ \vdots&\ddots&\vdots\\ \delta_{i1}&\cdots&\delta_{ii}\end{pmatrix}\cdot\begin{pmatrix}\Delta X_1\\ \vdots\\ \Delta X_i\end{pmatrix}\frac{(1+\mu\varphi)}{(1+\varphi)}=\begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}\quad\text{where}\quad\left\|\begin{pmatrix}\delta_{11}&\cdots&\delta_{1i}\\ \vdots&\ddots&\vdots\\ \delta_{i1}&\cdots&\delta_{ii}\end{pmatrix}\right\|\neq 0\quad\rightarrow\begin{pmatrix}\Delta X_1\\ \vdots\\ \Delta X_i\end{pmatrix}=\begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}$$

If the creep properties are uniform, redundant variables do not change in statically indeterminate systems!

It can also be proven for general (*n*-fold statically indeterminate) systems that with uniform creep properties no internal force redistributions due to creep occur:

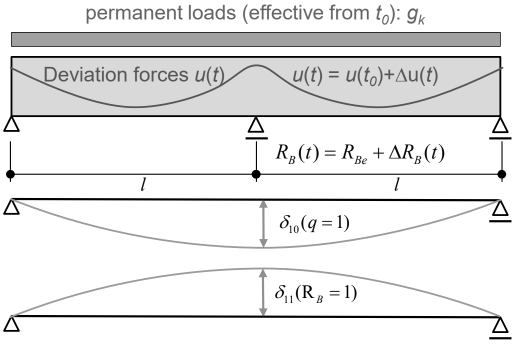
The time-dependent compatibility conditions for an *n*-fold statically indeterminate system are investigated. It becomes apparent that in the system of linear equations for the changes of the redundant variables {DX} the determinant [δ] of the coefficient matrix is different from zero (under consideration of the compatibility conditions at the time $t = 0$). But the constant vector is a zero vector, [δ] {DX} $= \{0\}$. Thus only the trivial solution is possible, i.e. the vector of the changes of the redundant variables {DX} must also be a zero vector, {DX} = {0}. The redundant variables therefore do not change.

Long-term effects

Redistribution of internal forces in statically indeterminate systems

Systems with uniform creep properties

Example 2: Prestressed two-span beam, solution with force method



BS: Intermediate bearing removed
RV: Reaction intermediate support

Displacements in the basic system (elastic, $t = t_0$):

$$\delta_{10} = \frac{5}{384}\frac{(2l)^4}{EI} \qquad \delta_{11} = \frac{1}{48}\frac{(2l)^3}{EI}$$

Compatibility condition at time t_0 :

$$\delta = (g_k + u(t_0)) \cdot \delta_{10} + R_{Be} \cdot \delta_{11} = 0$$

Time-dependent compatibility condition with the Trost method:

$$\delta_B = \left(g_k + u(t_0)\right) \cdot \delta_{10}\left(1+\varphi\right) + R_{Be} \cdot \delta_{11}\left(1+\varphi\right) + \Delta u(t) \cdot \delta_{10}\left(1+\mu\varphi\right) + \Delta R_B(t) \cdot \delta_{11}\left(1+\mu\varphi\right) = 0$$

$$\underbrace{\left(g_k + u(t_0)\right) \cdot \delta_{10} + R_{Be} \cdot \delta_{11}}_{=0\text{ (compatibility at time } t_0\text{)}} + \Delta u(t) \cdot \delta_{10}\frac{1+\mu\varphi}{1+\varphi} + \Delta R_B(t) \cdot \delta_{11}\frac{1+\mu\varphi}{1+\varphi} = 0$$

$$\boxed{\rightarrow \Delta R_B(t) = -\Delta u(t)\frac{\delta_{10}}{\delta_{11}}}$$

→ Support reactions change due to time-dependent prestressing losses (RV proportional to prestressing force = deviation force)

Example 2 shows that in prestressed systems with uniform creep properties, the redundant variables change over time only as a result of prestressing losses. The restraint moments are proportional to the (time-dependent) prestressing force (i.e. to the corresponding deviation forces).

Long-term effects

Redistribution of internal forces in statically indeterminate systems

Systems with non-uniform creep properties

Example 3: Hinged frame with concrete beam and steel columns, solution with force method

Displacements in the basic system (elastic, $t = t_0$):

$$\delta^R_{10} = -\frac{gl^2}{8}\cdot\frac{l}{4}\cdot\frac{2}{3}\cdot\frac{l}{EI_R} = -\frac{gl^4}{48EI} \qquad \delta^S_{10} = 0 \qquad \delta^R_{11} = -\left(\frac{l}{4}\right)^2\cdot\frac{l}{EI_R} = \frac{l^3}{16EI} \qquad \delta^S_{11} = 2\cdot\left(\frac{l}{4}\right)^2\cdot\frac{1}{3}\cdot\frac{h}{EI_S} = \frac{l^3}{16EI}$$

Compatibility condition at time t_0:

$$\delta_1(t_0) = \delta^S_{10} + \delta^R_{10} + X_{1e}\left(\delta^S_{11} + \delta^R_{11}\right) = 0 \rightarrow X_{1e} = -\frac{\delta^S_{10} + \delta^R_{10}}{\delta^S_{11} + \delta^R_{11}} = \frac{g_k l}{6}$$

Time-dependent compatibility condition with the Trost method (support does not creep), taking into account the compatibility at t_0:

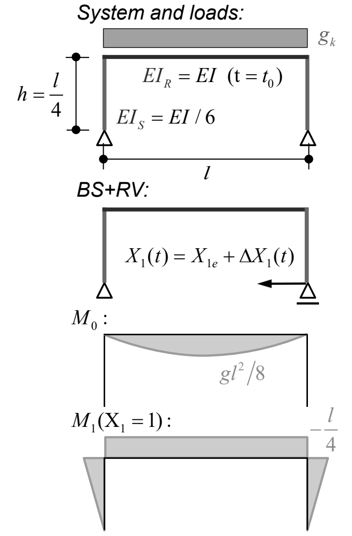
$$\delta_1(t) = \delta^S_{10} + \delta^R_{10}\left(1+\varphi\right) + X_{1e}\left[\delta^S_{11} + \delta^R_{11}\left(1+\varphi\right)\right] + \Delta X_1\left[\delta^S_{11} + \delta^R_{11}\left(1+\mu\varphi\right)\right] = 0$$

$$\delta^S_{10} + \delta^R_{10} + \delta^R_{10}\cdot\varphi + X_{1e}\left(\delta^S_{11} + \delta^R_{11}\right) + X_{1e}\cdot\delta^R_{11}\cdot\varphi + \Delta X_1\left[\delta^S_{11} + \delta^R_{11}\left(1+\mu\varphi\right)\right] = 0$$

$$\delta^R_{10}\varphi + X_{1e}\cdot\delta^R_{11}\varphi + \Delta X_1\left[\delta^S_{11} + \delta^R_{11}\left(1+\mu\varphi\right)\right] = 0 \rightarrow \Delta X_1(t) = -\varphi\frac{\delta^R_{10} + X_{1e}\cdot\delta^R_{11}}{\delta^S_{11} + \delta^R_{11}\left(1+\mu\varphi\right)}$$

$$\rightarrow \Delta X_1(t) = \frac{g_k l}{6}\frac{\varphi}{2+\mu\varphi} = X_{1e}\frac{\varphi}{2+\mu\varphi}, \qquad \boxed{X_1(t) = \frac{g_k l}{6}\left(1 + \frac{\varphi}{2+\mu\varphi}\right)}$$

Formula applies only for this example (system-dependent)



→ In the case of non-uniform creep properties internal forces are redistributed due to creep

Example 3 shows that in systems with non-uniform creep properties, internal force redistributions due to creep occur. In the example, the frame corner moment increases by approximately 55% over time, as the steel columns do not creep, unlike the concrete beam.

These rearrangements must be taken into account when calculating deflections. In the ultimate limit state of structural safety they can be neglected in ductile structures (residual stress states irrelevant, lower-bound theorem of limit analysis).

Long-term effects

Redistribution of internal forces in statically indeterminate systems

Systems with non-uniform creep properties - Generalization to general systems

Compatibility at $t = t_0$:
$$\begin{pmatrix} \delta_{10} \\ \vdots \\ \delta_{i0} \end{pmatrix} + \begin{pmatrix} \delta_{11} & \dots & \delta_{1i} \\ \vdots & \ddots & \vdots \\ \delta_{i1} & \cdots & \delta_{ii} \end{pmatrix} \cdot \begin{pmatrix} X_{1e} \\ \vdots \\ X_{ie} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \; (**)$$

$\rightarrow$ RV at $t = t_0$:
$$\begin{pmatrix} X_{1e} \\ \vdots \\ X_{ie} \end{pmatrix} = \begin{pmatrix} \delta_{11} & \dots & \delta_{1i} \\ \vdots & \ddots & \vdots \\ \delta_{i1} & \cdots & \delta_{ii} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \delta_{10} \\ \vdots \\ \delta_{i0} \end{pmatrix}$$

no constant factor for creep influence
(each RV has a different factor in general)
$\rightarrow$ redundant variables must change!

Cange of redundant variables using the Trost method: $X_i(t) = X_{ie} + \Delta X_i(t)$

$\rightarrow$ Compatibility for $t > t_0$:
$$\overbrace{(1+\varphi)\begin{pmatrix} \delta_{10} \\ \vdots \\ \delta_{i0} \end{pmatrix} + \begin{pmatrix} \delta_{11} & \dots & \delta_{1i} \\ \vdots & \ddots & \vdots \\ \delta_{i1} & \cdots & \delta_{ii} \end{pmatrix} \cdot \left[(1+\varphi)\begin{pmatrix} X_{1e} \\ \vdots \\ X_{ie} \end{pmatrix}\right.} \left. + (1+\mu\varphi)\begin{pmatrix} \Delta X_1 \\ \vdots \\ \Delta X_i \end{pmatrix} \right] = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$$

$\rightarrow$ using (**):
$$\begin{pmatrix} \delta_{11} & \dots & \delta_{1i} \\ \vdots & \ddots & \vdots \\ \delta_{i1} & \cdots & \delta_{ii} \end{pmatrix} \cdot \begin{pmatrix} \Delta X_1 \\ \vdots \\ \Delta X_i \end{pmatrix} \frac{(1+\mu\varphi)}{(1+\varphi)} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{where} \quad \begin{vmatrix} \delta_{11} & \dots & \delta_{1i} \\ \vdots & \ddots & \vdots \\ \delta_{i1} & \cdots & \delta_{ii} \end{vmatrix} \neq 0 \quad \rightarrow \begin{pmatrix} \Delta X_1 \\ \vdots \\ \Delta X_i \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$$

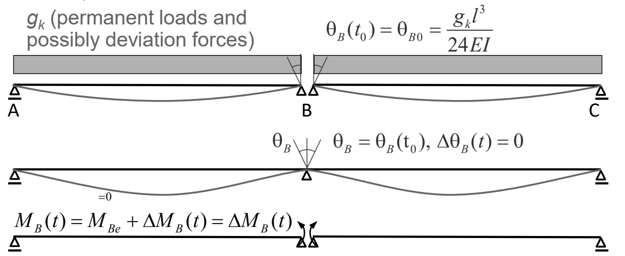
$\rightarrow$ In the case of non-uniform creep properties, internal force redistributions occur as a result of creep

The comparison with the *n*-fold statically indeterminate system with the same creep properties shows that redistributions are required in case of non-uniform creep properties.

Long-term effects

Influence of creep for system changes

Example 4 - Connection of two simple beams with the same creep behaviour



System, load relevant for creep:

Construction sequence:

1. Two single span girders are positioned (lifted in)
2. *t*=*t*$_0$: Monolithic connection at B

BS+RV: $\theta_{B0} = \frac{g_k l^3}{24EI}, \quad \theta_{B1} = \frac{l}{3EI}$

Bending moment and girder end rotation at B (per side) at t_0: $M_B(t_0) = M_{Be} = 0, \quad \theta_B(t_0) = \theta_{B0} = \frac{g_k l^3}{24EI}$

Comparison: Bending moment at B on a single two-span beam: («OC»: one casting) $M_{B,OC} = -\frac{\theta_{B0}}{\theta_{B1}} = -\frac{g_k l^2}{8}$

Compatibility condition (relative rotation of girder ends at B remains constant after = t_0):

$$\Delta\theta_B(t) = \theta_B(t) = \theta_{B0} \cdot \varphi + \Delta M_B(t) \cdot \theta_{B1}\left(1+\mu\varphi\right) = 0$$

$$\rightarrow \Delta M_B(t) = -\frac{\theta_{B0}}{\theta_{B1}} \frac{\varphi}{1+\mu\varphi} = \boxed{M_{B,OC} \cdot \frac{\varphi}{1+\mu\varphi} = M_B(t)}$$

At the intermediate support, a moment of approx. 80% of the two-span beam built in one casting «OC» develops due to creep.

Example 4 illustrates the internal force redistributions during a system change.

Two beams are built as single span beams (for example prefabricated beams lifted in). Afterwards they are monolithically connected at the intermediate support to form a two-span beam.

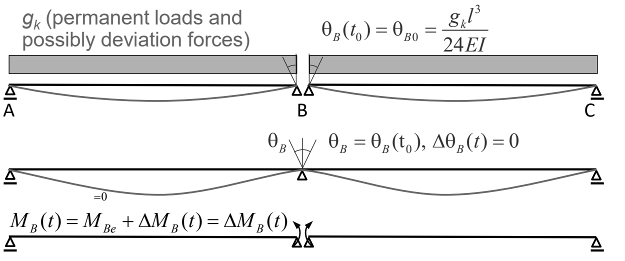
In the course of time, the internal force distribution approaches more and more the conditions of a two-span beam (= those if the system would have been constructed in one casting).

Long-term effects

Influence of creep in system changes

The ratio of the moment at B to the moment of the system built in one casting «OC» for various points in time and creep coefficients:

	56 days	180 days	1 year	5 years
φ(t)	1.00	1.75	2.00	2.50
$M_B(t)/M_{B,OC}$	0.56	0.73	0.77	0.83



As a general rule, in system changes, creep largely builds up the stress state of the system built in one casting σ_{OC}. The higher the creep coefficient, the closer it approximates the state of the system built in one casting («Einguss-System» in German).

As an approximation one may use:

φ ≈ 1 φ ≈ 2

$S_{t=\infty}$	$\approx$	$S_A + (0.6\ldots0.8)(S_{OC} - S_A)$
S_A		Internal forces before system change (initial state)
S_{OC}		Internal forces of system built in one casting "OC"

The observation, that the distribution of the internal forces approaches the state of a system cast at once (one casting “OC”) more and more over the course of time, also applies to other systems.

For normal conditions, the final state results in a distribution of the internal forces which is much closer to the state of a system built in one casting than to the distribution of the internal forces immediately after the system change.

The distribution of the internal forces at time t = ∞ can therefore be estimated approximately by using the redundant variables with approx. 80% (for φ = 2) or 60% (for φ = 1) of their value in the system built in one casting.

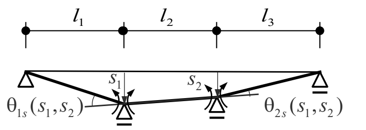
Long-term effects

Influence of creep on restraint internal forces (caused by imposed deformations)

Example 5a - three-span beam, time-independent («fast») support displacements s_1, s_2

Compatibility condition at time $t = t_0$:

$$\begin{aligned} X_{1A}\theta_{11} + X_{2A}\theta_{12} &= \theta_{1s} \\ X_{2A}\theta_{21} + X_{2A}\theta_{22} &= \theta_{2s} \end{aligned} \rightarrow \begin{pmatrix} X_{1A} \\ X_{2A} \end{pmatrix} = \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix}^{-1} \begin{pmatrix} \theta_{1s} \\ \theta_{2s} \end{pmatrix}$$



$X_1(t) = X_{1A} + \Delta X_1(t)$ $\quad$ $X_2(t) = X_{2A} + \Delta X_2(t)$

Time-dependent compatibility condition (Trost method):

$$\Delta\theta_1(t) = X_{1A}\theta_{11}\cdot\varphi + \Delta X_1(t)\theta_{11}\cdot(1+\mu\varphi) + X_{2A}\theta_{12}\cdot\varphi + \Delta X_2(t)\theta_{12}\cdot(1+\mu\varphi) = 0$$
$$\Delta\theta_2(t) = X_{1A}\theta_{21}\cdot\varphi + \Delta X_1(t)\theta_{21}\cdot(1+\mu\varphi) + X_{2A}\theta_{22}\cdot\varphi + \Delta X_2(t)\theta_{22}\cdot(1+\mu\varphi) = 0$$

	56 days	5 years
φ(t)	1.00	2.00
$X_i(t)/X_{iA}(t)$	0.44	0.23

...ditto, inversely:

$$\Delta X_1(t)\theta_{11} + \Delta X_2(t)\theta_{12} = -\overbrace{\left[X_{1A}\theta_{11} + X_{2A}\theta_{12}\right]}^{\theta_{1s}}\frac{\varphi}{1+\mu\varphi}$$
$$\Delta X_1(t)\theta_{21} + \Delta X_2(t)\theta_{22} = -\underbrace{\left[X_{1A}\theta_{21} + X_{2A}\theta_{22}\right]}_{\theta_{2s}}\frac{\varphi}{1+\mu\varphi}$$

$$\rightarrow \begin{pmatrix} \Delta X_1(t) \\ \Delta X_2(t) \end{pmatrix} = -\frac{\varphi}{1+\mu\varphi}\begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix}^{-1}\begin{pmatrix} \theta_{1s} \\ \theta_{2s} \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} \Delta X_1(t) \\ \Delta X_2(t) \end{pmatrix} = -\frac{\varphi}{1+\mu\varphi}\begin{pmatrix} X_{1A} \\ X_{2A} \end{pmatrix} \quad \text{resp.} \quad \boxed{X_i(t) = X_{iA}\left(1 - \frac{\varphi}{1+\mu\varphi}\right)}$$

(analogous to relaxation function)

→ Time-independent restraint forces ("fast imposed deformation") are reduced by creep (or relaxation) to 1/3...1/4 of the initial value

Example 5 illustrates the influence of creep on restraint internal forces due to imposed deformations.

In the case of a time-independent (fast) support displacement, the restraint internal forces are strongly reduced analogous to the relaxation function (see slide 32). At time t = ∞ they amount to only 25...30% of the initial value X_A immediately after the bearing displacements.

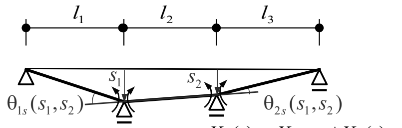
Long-term effects

Influence of creep on restraint internal forces (caused by imposed deformations)

Example 5b - three-span beam, time-dependent («slow») support displacements s_1, s_2

Assumption: Settlement process (s_1, s_2) proportional to creep function:

$$s_i(t) = s_i(t=\infty)\frac{\varphi(t,t_0)}{\varphi(t=\infty,t_0)} = s_{i,\infty}\frac{\varphi}{\varphi_\infty} \qquad t = t_0: \begin{matrix} s_i = 0 \\ X_i = 0 \end{matrix}$$



$$X_1(t) = \underbrace{X_{1A}}_{=0} + \Delta X_1(t) \qquad X_2(t) = \underbrace{X_{2A}}_{=0} + \Delta X_2(t)$$

	56 days	180 days	5 years
φ(t)	1.00	1.75	2.00
$X_i(t)/X_{iE,el}(t)$	0.44	0.36	0.38

Time-dependent compatibility condition (Trost method):

$$\Delta\theta_1(t) = \Delta X_1(t)\theta_{11}\cdot(1+\mu\varphi) + \Delta X_2(t)\theta_{12}\cdot(1+\mu\varphi) = \theta_{1s,\infty}\frac{\varphi}{\varphi_\infty}$$

$$\Delta\theta_2(t) = \Delta X_1(t)\theta_{21}\cdot(1+\mu\varphi) + \Delta X_2(t)\theta_{22}\cdot(1+\mu\varphi) = \theta_{2s,\infty}\frac{\varphi}{\varphi_\infty}$$

...ditto, inversely:

$$\begin{matrix} \Delta X_1(t)\theta_{11} + \Delta X_2(t)\theta_{12} = \theta_{1s,\infty}\dfrac{\varphi}{\varphi_\infty(1+\mu\varphi)} \\ \Delta X_1(t)\theta_{21} + \Delta X_2(t)\theta_{22} = \theta_{2s,\infty}\dfrac{\varphi}{\varphi_\infty(1+\mu\varphi)} \end{matrix} \rightarrow \begin{pmatrix} \Delta X_1(t) \\ \Delta X_2(t) \end{pmatrix} = \frac{\varphi}{\varphi_\infty(1+\mu\varphi)}\begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix}^{-1}\begin{pmatrix} \theta_{1s,\infty} \\ \theta_{2s,\infty} \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} = \frac{\varphi}{\varphi_\infty(1+\mu\varphi)}\begin{pmatrix} X_{1E,el} \\ X_{2E,el} \end{pmatrix} \text{ resp. } \boxed{X_i(t) = X_{iE,el}\frac{\varphi}{\varphi_\infty(1+\mu\varphi)}}$$

$X_{iE,el}$: value in elastic system subjected to $\theta_{is,\infty}$ without creep

→ Due to creep (or relaxation) time-dependent restraint forces ("slow imposed deformation") reach only approx. 40% of the elastic (short-term) value

In the case of a time-dependent (slow) bearing displacement (example 5b), as a result of relaxation the restraint internal forces only build up to about 40% of the value $X_{E.el}$, which would occur in the case of a fast bearing displacement of the same magnitude.

Long-term effects

Aspects not covered in the lecture:

Composite cross-sections of concrete and steel or precast concrete components and in-situ concrete

→ Residual stresses or force redistributions due to shrinkage and creep of the concrete
(steel does neither creep nor shrink, prefabricated components creep less than in-situ concrete)

→ Determination of the force redistributions from the compatibility condition (plane cross-sections remain plane)

→ Consideration of creep due to time-dependent residual stresses with the Trost method

Effect of crack formation on creep behaviour

→ In all previous slides, uncracked behaviour was assumed (results valid e.g. for girders fully prestressed under permanent loads)

→ Crack formation and long-term effects influence one another

→ Approximate calculation analogous to the non-cracked state with ficticious creep coefficient φ':
- Determination of cracked elastic stiffness $EI^{II}_{t=0}$ with E_{c0} resp. $EI^{II}_{t=\infty}$ with $E_{c0}/(1+\varphi)$ (see Stahlbeton I)
- Calculation with $EI^{II}_{t=0}$ using the ficticious creep coefficient $\varphi' = EI^{II}_{t=0} / EI^{II}_{t=\infty} - 1$.

Effect of creep on prestressed systems

→ Prestress losses due to shrinkage, creep and relaxation of the prestressing steel see Stahlbeton II.

→ Internal forces due to prestressing are to be taken into account when determining the creep-generating stresses. Treatment as anchor, deviation and friction forces (prestressing on the load side) is advisable → Creep caused by sum of permanent loads and anchor and deviation forces due to prestressing.

→ For highly prestressed, deformation-sensitive systems, such as cantilever bridges during the construction stage (*), the long-term effects must be carefully investigated and upper/lower limit values must be used.

(*) large deformations due to dead weight (+) and prestressing (-), resulting deformation = difference, sensitive to assumptions made (there is no "safe side" when determining camber = «Überhöhung» in German)

Long-term effects

Summary

- The term "long-term effects" covers shrinkage, creep and relaxation. Creep and relaxation of concrete are related phenomena.
- Due to the large variability of the material properties, the shrinkage and creep behaviour can only be determined approximately, even with complex calculations.
- All permanent actions (dead weight, superimposed loads, prestressing) cause creep.
- The stress history usually depends on the creep behaviour. The solution of creep problems therefore requires an iterative / step-by-step approach. For manual calculations, the Trost method (with an ageing coefficient of $\mu \approx 0.8$ for stresses that do not act from the beginning) is appropriate.
- In statically indeterminate systems with uniform creep properties, the restraint forces due to creep change exclusively as a result of time-dependent prestress losses (RV due to prestressing is proportional to the prestressing force resp. the deviation forces).
- In statically indeterminate systems with non-uniform creep properties, the redundant variables change as a result of creep.
- After system changes, creep largely builds up the stress state of the system built in one casting. The more prone to creep the system components are, the closer it approximates the system built in one casting. For normal conditions ($\varphi \approx 2$) approx. 80% of the latter is reached.
- Time-independent restraint forces ("fast imposed deformation") are reduced by creep (resp. relaxation) to 1/3...1/4 of the initial value. The reduction of the initial restraint forces is larger, the more prone to creep the system components are.

$$X_i(t) = X_{iA}\left(1 - \frac{\varphi}{1+\mu\varphi}\right)$$

- Time-dependent restraint forces ("slow imposed deformation") achieve as a result of creep (resp. relaxation) only approx. 40% of the elastic (short-term) value. The restraint forces never act in full-size and the more prone to creep the system parts are, the less they build up.

$$X_i(t) = X_{iE,el}\,\frac{\varphi}{\varphi_\infty(1+\mu\varphi)}$$

- Relaxation reduces the restraint forces, but not the deformations!

As an example for the last three bullet points, consider a bridge with a stiff deck and monolithically connected piers.

- Prestressing, creep and shrinkage as well as temperature reductions cause a shortening of the deck, which generates restraint forces in the piers.
- *Restraint forces* in the piers due to initial prestressing (elastic shortening of the deck) would reduce over time according to «time-independent restraint», i.e., to 20..30% of their initial value. However, since the shortening of the deck increases due to creep under prestressing (which would build up restraints according to «time-dependent restraint» if considered on its own), restraint forces due to prestressing remain approximately constant over time (if piers and deck have the same creep properties and the longitudinal stiffness of the deck is much higher than the longitudinal restraint caused by the piers).
- Over time, *restraint forces* due to shrinkage of the deck imposed to the piers will build up according to «time-dependent restraint», i.e., to about 40% of the value obtained without considering relaxation.
- *Restraint forces* caused by daily as well as seasonal temperature changes (length change of deck) needs to be accounted for with almost their full elastic value, since they will also occur after many years when the concrete's ability to creep is much reduced.
- In contrast to the *restraint forces*, the bridge end displacements are NOT reduced by relaxation nor creep, and only marginally by the restraint caused by the piers (the deck is typically about 2 orders of magnitude stiffer than the piers). Usually, the full unrestrained bridge deck displacements are therefore used.
- These bridge end displacements need to be calculated with the full values, i.e., shortening of the deck due to initial prestressing, multiplied by (1+φ), plus shrinkage and temperature changes without reductions. These full displacements (multiplied by appropriate partial factors) are governing the design of bridge bearings and expansion joints at the abutments.

Note that in the above calculations, reduced stiffness of the piers (due to cracking) may be considered (this will substantially reduce the *restraint forces* but only marginally increase the bridge end displacements).