3 Slabs

In-depth study and additions to Stahlbeton II

3.5 Influence of shear forces

Slabs - Influence of shear forces

Shear resistance of slabs - General remarks (→ Stahlbeton II)

- Slabs, especially those with shear reinforcement (three-dimensionally reinforced), are generally very ductile structures.
- On the other hand, a shear failure of slabs without shear reinforcement is very brittle → practically impossible to redistribute the internal forces (therefore no stress relief of the affected areas by internal force redistribution)!
- Often slabs are designed according to the lower bound theorem of the theory of plasticity. In doing so the maximum shear forces occurring in the course of the load history can deviate significantly from the shear load in the calculated (bending) failure state (*).
  For a safe design, the shear force at each point of the slab should therefore, strictly speaking, be checked during the entire load history (internal force redistribution under the same external loads).
- In practice, shear structural safety is usually only checked in the state of maximum internal force redistribution, which is also the basis for the bending design. This is associated with considerable uncertainties, especially since the shear forces resulting from FE calculations scatter strongly (they are determined numerically as derivatives of the bending moments, one order of magnitude less accurate).
  In case of doubt, a ductile behaviour must be ensured by arranging a shear reinforcement!

(*) also applies to a design based on linear elastic FE calculations (= equilibrium state), since crack formation, residual stress states due to settlements, construction process, etc. can never be completely recorded or correctly modelled!

The subject of this chapter is the influence of shear forces on the behaviour of slabs.

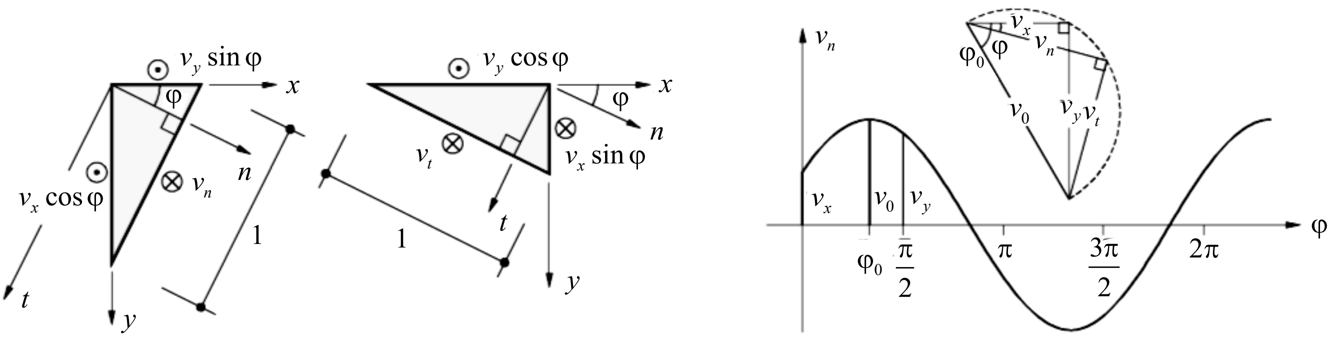
This is essentially a repetition from the lecture Stahlbeton II with selective additions.

Slabs - Influence of shear forces

Shear resistance of slabs - General remarks

- In a slab, the principal shear force $v(\varphi_0) = v_0$ is carried in the direction φ_0 at every point. Perpendicular to it the shear force is zero: $v = v(\varphi_0 \pm \pi/2) = 0$.

→ Measure for shear stress: nominal shear stress $\tau_{nom} = v_0 / z$
(with z = lever arm of the internal forces).



$$v_n = v_x \cos\varphi + v_y \sin\varphi \qquad v_t = -v_x \sin\varphi + v_y \cos\varphi \qquad v_0 = \sqrt{v_x^2 + v_y^2} \qquad \tan\varphi_0 = \frac{v_y}{v_x}$$

Repetition from Stahlbeton II:

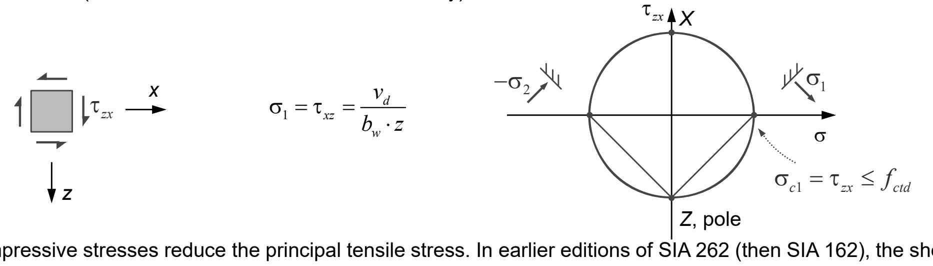
Principal shear force and associated direction.

Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement

- Shear stresses in the uncracked (isotropic) state correspond to a principal tensile stress of the same amount, $\sigma_{c1} = |\tau_{zx}|$ (elastic shear flow: $\tau_{max} = 1.5 \cdot \tau_{nom} = 1.5 \cdot v_0 / z$)
- In the case of thin slabs, which according to SIA 262 may be designed without shear reinforcement, the tensile strength of the concrete is implicitly taken into account (which is usually even slightly higher than the permissible value for insignificant components). This can be justified on the following reasons:
  - Higher redundancy than beam structures (biaxial load-bearing, beneficial compressive membrane forces neglected in the design)
  - Shear stress generally lower (except in the vicinity of concentrated loads and supports)
  - No failure at first shear crack formation under moderate shear stress (if crack roughness is sufficient and longitudinal reinforcement has reserves)

→ In contrast to beam structures (minimum shear reinforcement mandatory), shear reinforcement can often be omitted in thin slabs.



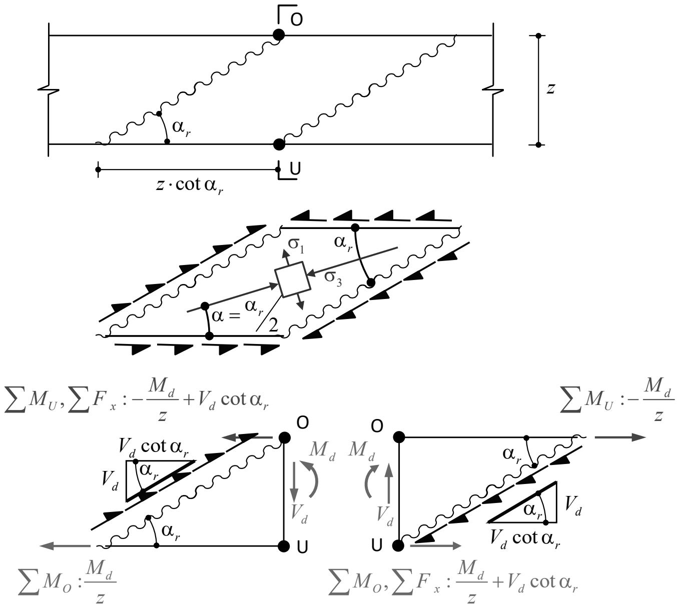
- NB: Longitudinal compressive stresses reduce the principal tensile stress. In earlier editions of SIA 262 (then SIA 162), the shear resistance of prestressed beams was verified on this basis.

Repetition from Stahlbeton II:

"Nominal shear stresses" in the uncracked state

Slabs - Influence of shear forces

Web tension failure - Component without shear reinforcement



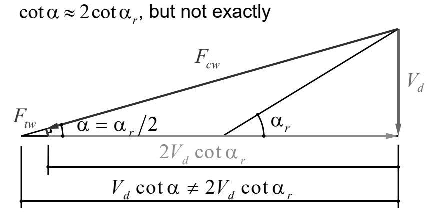
In thin slabs, no failure occurs at first shear crack formation under moderate shear stress, provided that the crack roughness (aggregate interlock) is sufficient and the longitudinal reinforcement has reserves.

(The additional tensile forces in the longitudinal reinforcement due to shear are twice as large as with shear reinforcement!)

Forces acting on a vertical cut:

$\alpha = \alpha_r/2$

$\cot\alpha \approx 2\cot\alpha_r$, but not exactly



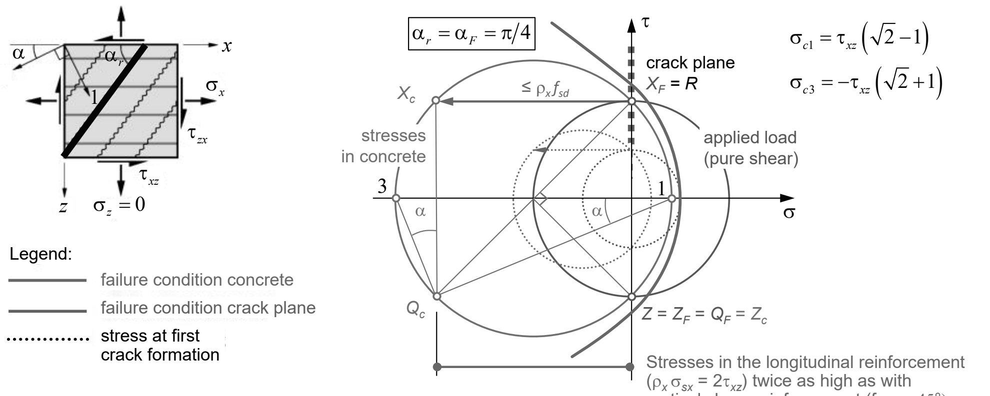
Repetition from Stahlbeton II:

Model for beams without shear reinforcement with a set of parallel, rough cracks transferring pure shear stresses. The longitudinal reinforcement must be able to absorb additional tensile forces. These are twice as high as those in a beam with shear reinforcement and a parallel compression field of inclination $\alpha = \alpha_r$ in the web.

Influence of shear forces

Shear resistance of slabs without shear reinforcement

- Simple model for shear transmission through aggregate interlock in the first cracks under 45° (pure shear stress in the first cracks) → longitudinal reinforcement needs to resist double of the additional tensile force due to V:



NB1: Aggregate interlock depends on crack opening, not strains → Scale effect

NB2: The load-bearing capacity due to aggregate interlock is not necessarily sufficient in regions subjected to high shear stress (slabs in the support area) to avoid brittle failure in the event of initial shear cracking!

The simple model shown on the previous slide can be applied to membrane elements under uniform loading and extended for general crack failure conditions (shear and normal stresses).

On this slide the stress states in the longitudinal reinforcement and in the concrete for the case of cracks with an inclination of 45° are shown by means of a Mohr's circle. The cracks transmit pure shear stresses (without compressive stress). In the concrete between the cracks there is a biaxial state of stress with principal stresses $-\tau_{xz}\cdot(\sqrt{2}+1)$ (compression) and $\tau_{xz}\cdot(\sqrt{2}-1)$ (tension).

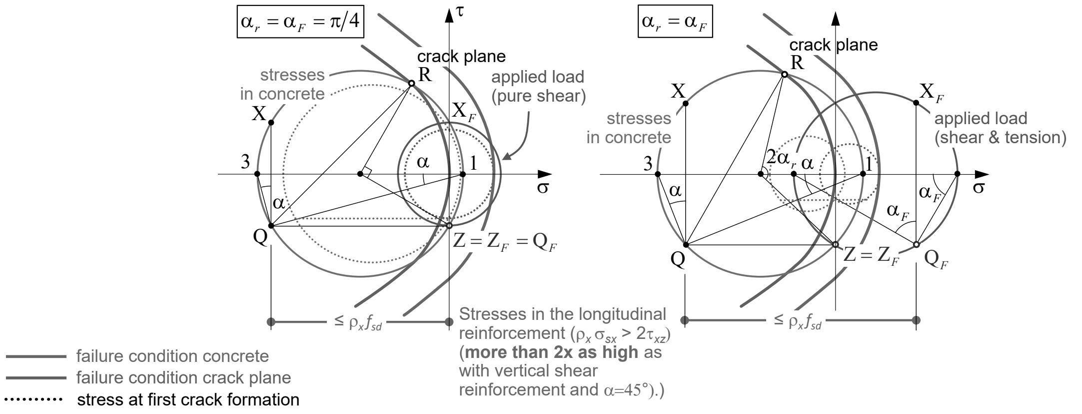
It can be seen that, just as in the structural model for beams without shear reinforcement (previous slide), the resulting equivalent reinforcement stresses are twice as high as in a compression field with an inclination of 45° in orthogonally reinforced elements.

Note: The figure on the left shows a more general case with initial crack inclination > 45°, the Mohr's circles on the right are valid for an initial crack direction of 45°.

Influence of shear forces

Shear resistance of slabs without shear reinforcement

- Consideration of more realistic failure criteria for shear transmission by aggregate interlock, i.e. Mohr's envelope. Shear can only be transmitted with compressive stress → even more longitudinal reinforcement required!



NB: There is a scale effect and the validity is limited to moderate shear stresses!

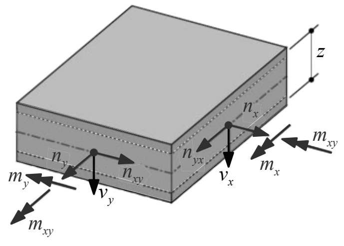
The structural model can be extended by considering realistic relationships for the possible shear and normal stresses at the cracks (aggregate interlock).

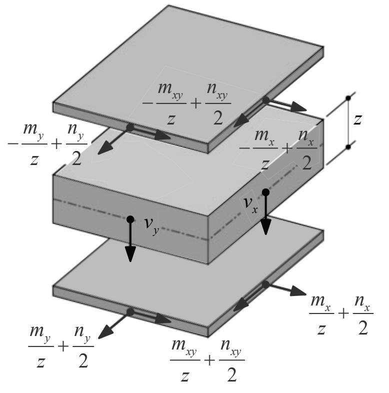
In the figure on the left, a pure shear load and an inclination of the cracks of 45° is still assumed. However, the cracks cannot transmit pure shear stresses. A compressive stress acting simultaneously is required. It can be seen that with this model even more longitudinal reinforcement is required than in the case of pure shear stress in the crack planes.

In the figure on the right, not a pure shear, but a general load is applied (shear and normal stresses). It is assumed that the cracks run in the direction of the applied load (principal stress direction). The required force in the longitudinal reinforcement can be determined analogously to pure shear.

Slabs - Influence of shear forces

Sandwich model





Equilibrium solution (general shell loading):

- Sandwich covers carry bending and twisting moments as well as possible membrane forces
  → plane loading, treatment as membrane elements with corresponding reinforcement
  (→ see yield conditions for membrane elements)
- Sandwich core absorbs shear forces
  → Sandwich core absorbs principal shear force v_0 in direction φ_0 and can be treated like the web of a beam in this direction

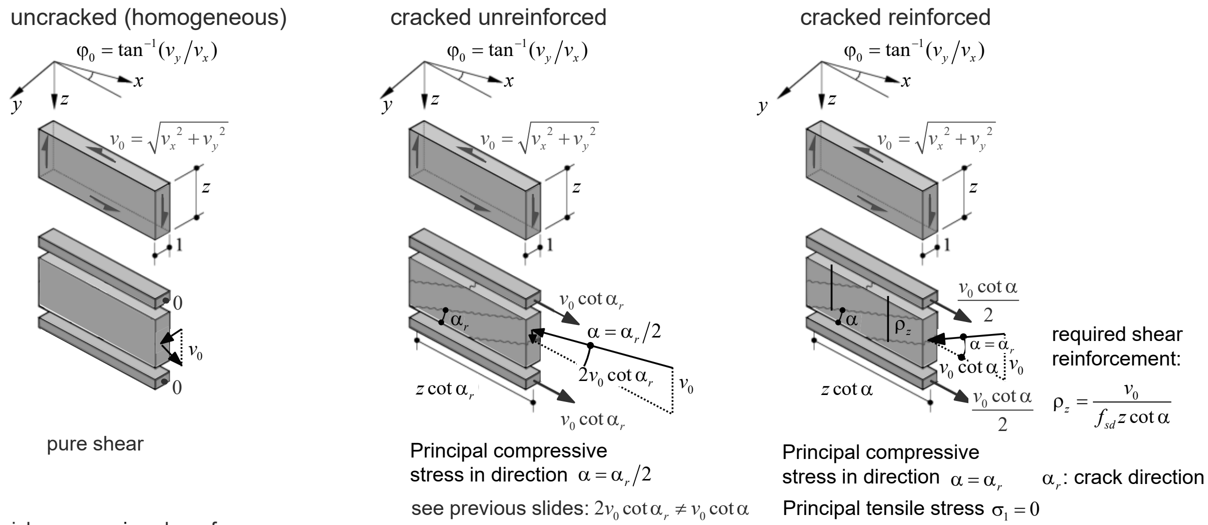
NB: High membrane (compression) forces: core can also be used for this (take into account interaction with v)

Repetition from Stahlbeton II:

The loading of a shell element can be divided between the sandwich covers and the core through statically equivalent forces. The core carries only the transverse (=slab) shear force.

Slabs - Influence of shear forces

Sandwich model - Core



- Sandwich core carries shear forces
  → Sandwich core carries principal shear force v_0 in the direction φ_0 and can be treated like the web of a beam in this direction.
  Tensile forces in the slab plane are to be carried by the sandwich covers (additional membrane loading).

Repetition from Stahlbeton II:

The figure shows three possible model concepts for carrying slab shear forces in the core of the sandwich model. In all three cases it is taken into account that the principal shear force is transferred in the direction φ_0 at every point of the slab (perpendicular shear force = 0).

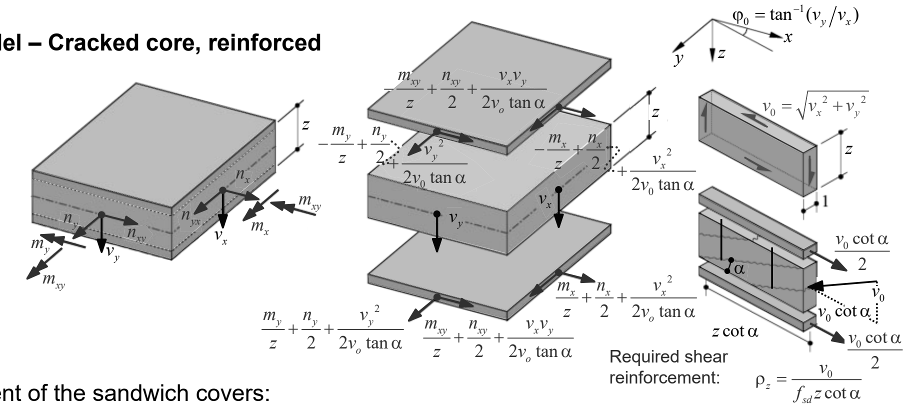
The figure on the left shows the transfer of the shear force in an uncracked core. In this case there there a pure shear stress state (tensile and compressive stresses of the same magnitude under ±45°).

The middle figure shows the transfer of the shear force in a cracked core without shear reinforcement. The load-bearing capacity corresponds to the model shown on the previous slides. The sandwich covers («chords») must absorb twice as much additional force as in the case of shear reinforcement.

The figure on the right shows the transfer of the shear force in a cracked core with vertical shear reinforcement. The load-bearing effect corresponds to a web of a beam with shear reinforcement (see next slide).

Slabs - Influence of shear forces

Sandwich model – Cracked core, reinforced



→ Reinforcement of the sandwich covers:

$$a_{sx}f_{sd} \ge \frac{m_x}{z}+\frac{n_x}{2}+\frac{v_x^2}{2v_0\tan\alpha}+k\left|\frac{m_{xy}}{z}+\frac{n_{xy}}{2}+\frac{v_xv_y}{2v_0\tan\alpha}\right| \qquad a'_{sx}f_{sd} \ge -\frac{m_x}{z}+\frac{n_x}{2}+\frac{v_x^2}{2v_0\tan\alpha}+k'\left|-\frac{m_{xy}}{z}+\frac{n_{xy}}{2}+\frac{v_xv_y}{2v_0\tan\alpha}\right|$$

$$a_{sy}f_{sd} \ge \frac{m_y}{z}+\frac{n_y}{2}+\frac{v_y^2}{2v_0\tan\alpha}+\frac{1}{k}\left|\frac{m_{xy}}{z}+\frac{n_{xy}}{2}+\frac{v_xv_y}{2v_0\tan\alpha}\right| \qquad a'_{sy}f_{sd} \ge -\frac{m_y}{z}+\frac{n_y}{2}+\frac{v_y^2}{2v_0\tan\alpha}\frac{1}{k'}\left|-\frac{m_{xy}}{z}+\frac{n_{xy}}{2}+\frac{v_xv_y}{2v_0\tan\alpha}\right|$$

(The factors k, k' can in principle be selected differently at each point of the slab (avoid abrupt changes or anchor differential reinforcement forces). Selection of the compression field inclination α: Analogous considerations as with beams. Often $k = k' = \cot\alpha = 1$ is chosen)

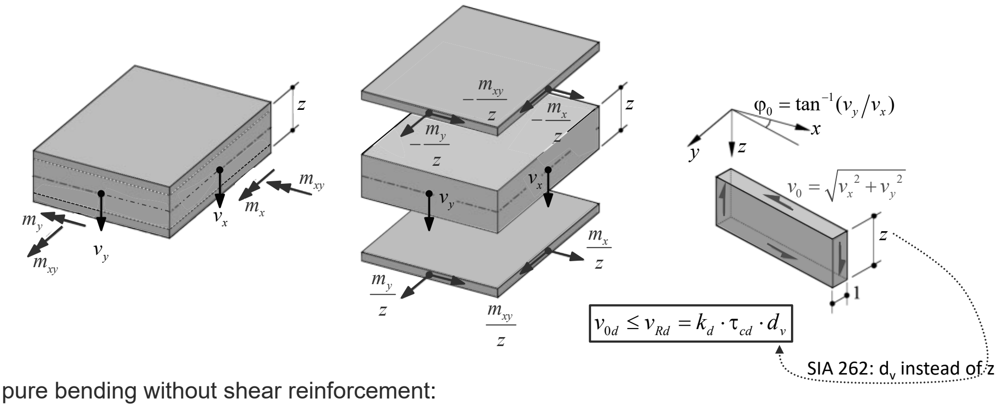
Repetition from Stahlbeton II:

The longitudinal tensile forces due to shear forces, which are to be absorbed by the sandwich covers, result in additional membrane forces in the covers (transformation of the additional «longitudinal» tensile force due to shear in the direction φ_0 in *x*- and y-direction). The last terms of the sandwich cover forces and required resistances of the reinforcements shown in the slide correspond to the components of these «chord tensile forces» (see formulas on slide 8 for the components of v_0).

The reinforcement of the sandwich covers can be designed for the resulting forces on the basis of the yield conditions for membrane elements.

Slabs - Influence of shear forces

Sandwich model - Pure bending, uncracked core



→ Slabs under pure bending without shear reinforcement:
$n_x = n_y = n_{xy} = 0$, $v_{0d} \le v_{Rd} = k_d \tau_{cd} d_v$

→ Terms with n_x, n_y, n_{xy} disappear

→ Terms with v_x, v_y disappear if an uncracked core is assumed.

→ With aggregate interlock according to slide 4, at least twice the longitudinal reinforcement (2·terms with v_x, v_y) is required as a result of shear force → Reinforcement in slabs without shear reinforcement should not be graded / curtailed too early!

Repetition from Stahlbeton II:

In the case of an uncracked core, there are no longitudinal tensile forces as a result of shear force. However, in the case of a cracked core without shear reinforcement the longitudinal tensile forces would be twice as high as in the case of shear reinforcement. For this reason, the bending reinforcement should not be graded too early for slabs without shear reinforcement.

The reinforcement of the sandwich covers can also be designed for the resulting forces on the basis of the yield conditions for membrane elements.

Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement according to SIA 262

Nominal shear resistance without shear reinforcement

$$v_{Rd} = k_d \cdot \tau_{cd} \cdot d_v \qquad \text{with} \qquad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

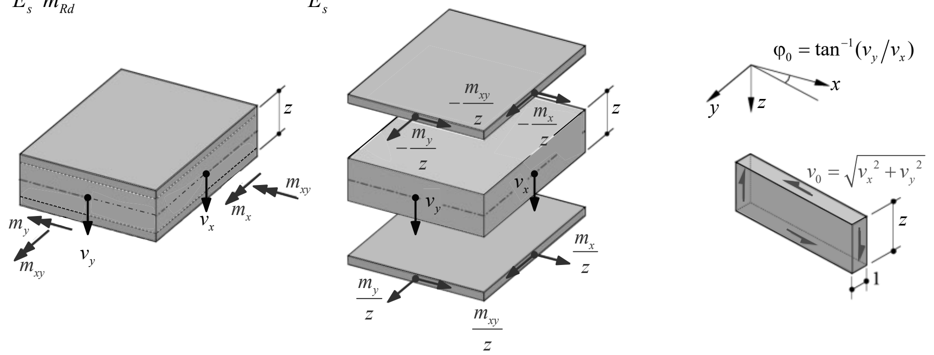
$$k_d = \frac{1}{1+\varepsilon_v \cdot d \cdot k_g} \qquad \text{with} \qquad k_g = \frac{48}{16+D_{max}}$$

$$\varepsilon_v = \frac{f_{sd}}{E_s}\frac{m_d}{m_{Rd}} \qquad \text{resp.} \qquad \varepsilon_v = 1.5\frac{f_{sd}}{E_s}$$

k_d: Reduction factor for static depth of the slab, utilization of longitudinal reinforcement and maximum aggregate size

d_v: Effective static depth taking into account cross-section discontinuities

ε_v: Strain of bending reinforcement (1.5 f_{sd}/E_s applies to plastic deformations, +50% in case of graded longitudinal reinforcement)



Repetition from Stahlbeton II:

The nominal shear resistance without shear reinforcement is determined according to SIA 262 on the basis of the specified relationships. These are based on the concept that a shear failure occurs when a critical shear crack has opened to such an extent that it can no longer transmit the shear stresses required for the transmission of the shear force (see slides 5-6). Therefore, the shear resistance decreases with increasing use of the bending reinforcement (which is accompanied by greater chord elongation and thus larger crack openings).

Additional remark:

- In the sandwich model, z was used instead of d_v. Both d and d_v appear in the formulae of SIA 262.

Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement according to SIA 262

Nominal shear resistance without shear reinforcement

$$v_{Rd} = k_d \cdot \tau_{cd} \cdot d_v \quad \text{with} \quad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

$$k_d = \frac{1}{1+\varepsilon_v \cdot d \cdot k_g} \quad \text{with} \quad k_g = \frac{48}{16+D_{\max}}$$

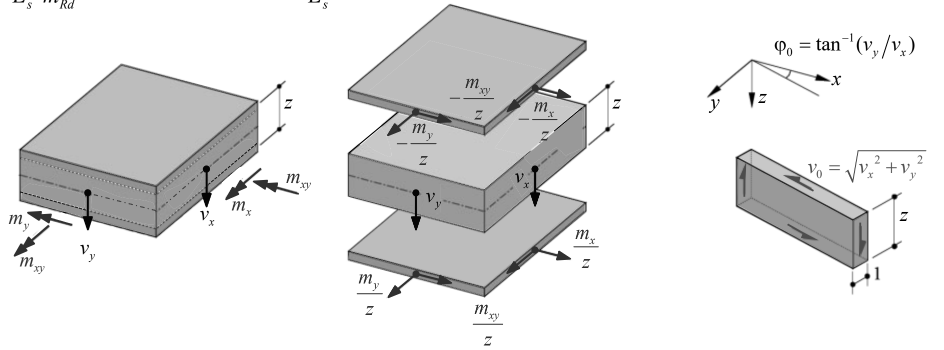
$$\varepsilon_v = \frac{f_{sd}}{E_s}\frac{m_d}{m_{Rd}} \quad \text{resp.} \quad \varepsilon_v = 1.5\frac{f_{sd}}{E_s}$$

(Pre-)dimensioning, B500B, D_{max} = 32 mm:

k_g = 1.0; m_d / m_{Rd} = 1.0 (no plastic redistribution)

$\rightarrow \varepsilon_v = f_{sd} / E_s$ = 2.12‰

$$\rightarrow v_{Rd} = \frac{\tau_{cd} \cdot d_v}{1+\dfrac{d}{471\text{ mm}}}$$



Repetition from Stahlbeton II:

For preliminary design, the specified simplifications can be used.

Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement according to SIA 262

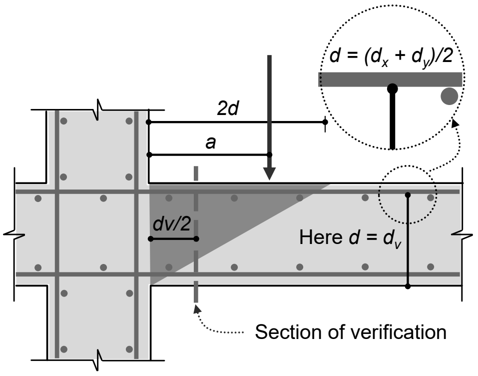
Nominal shear resistance without shear reinforcement

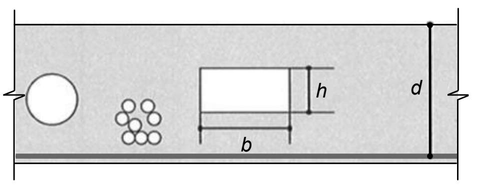
$$v_{Rd} = k_d \cdot \tau_{cd} \cdot d_v \quad \text{with} \quad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

$$k_d = \frac{1}{1+\varepsilon_v \cdot d \cdot k_g} \quad \text{with} \quad k_g = \frac{48}{16+D_{\max}}$$

$$\varepsilon_v = \frac{f_{sd}}{E_s}\frac{m_d}{m_{Rd}} \quad \text{resp.} \quad \varepsilon_v = 1.5\frac{f_{sd}}{E_s}$$

- Control section at a distance d_v /2 from the support edge or edge of the load, if necessary at reinforcement gradations
- Reduction of concentrated loads at distance a < 2d from bearing edge with factor $a/(2d)$ permissible
- Ducts, pipes:
  Diameter / width / height > d/6
  (for cable bundles: dimension of the entire bundle)
  Reduction of d_v by the largest dimension of the inlay or pipe ($d_v = d - \max(b, h)$)





Repetition from Stahlbeton II:

Additional specifications on shear resistance according to SIA 262.

Slabs - Influence of shear forces

Shear resistance of slabs without shear reinforcement according to SIA 262

Nominal shear resistance without shear reinforcement

$$v_{Rd} = k_d \cdot \tau_{cd} \cdot d_v \quad \text{with} \quad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

$$k_d = \frac{1}{1+\varepsilon_v \cdot d \cdot k_g} \quad \text{with} \quad k_g = \frac{48}{16 + D_{\max}}$$

$$\varepsilon_v = \frac{f_{sd}}{E_s}\frac{m_d}{m_{Rd}} \quad \text{resp.} \quad \varepsilon_v = 1.5\frac{f_{sd}}{E_s}$$

- slab with prestress or normal force, with decompression moment m_{Dd} : $\varepsilon_v = \frac{f_{sd}}{E_s}\frac{m_d - m_{Dd}}{m_{Rd} - m_{Dd}}$
  ... m_{Dd} = long-term value of the decompression moment (see chapter punching) accounting for normal forces (e.g. due to restraint by stiff supports)
  ... m_d = incl. Moments due to restraint and imposed deformations (e.g. secondary moments from prestressing)
- Concrete compressive strength f_{ck} > 70 MPa: $D_{\max} = 0$, d.h. $k_g = 3$ ($\rightarrow v_{Rd}(f_{ck})$ is discontinuous at 70 MPa)
- Clear deviation of the principal direction φ_0 of the shear force from the direction of the principal reinforcement by angle ϑ: increase of elongation ε_v with factor $\frac{1}{\sin^4\vartheta + \cos^4\vartheta}$ (i.e. in the worst case, ϑ = 45°: factor 2)

Repetition from Stahlbeton II:

Additional specifications on shear resistance according to SIA 262.

Additional remark:

- The influence of the decompression moment will be explained later (punching)

- The discontinuity of the shear resistance at 70 MPa accounts for the fact that cracks in high strength concrete tend to pass through the aggregates and are therefore smoother than in normal strength concrete, but the chosen value of 70 MPa for the limit cannot be mechanically justified (depending on the strength and shape of the aggregates used and other parameters).

Influence of shear forces

Derivation of the factor for deviation of the principal direction φ_0 of the shear force from the direction of the principal reinforcement (compression field model for sandwich cover)

Compatibility assuming linear elastic behaviour & stress-free cracks
$\rightarrow \varphi_{1\varepsilon} =$ principal strain direction with $\varphi_{1\varepsilon} = \varphi_{1c}$ resp. $\alpha_r + \varphi_{1\varepsilon} = \pi/2$

$$\varepsilon_x = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)\sin^2\alpha_r \qquad = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)\cos^2\varphi_{1\varepsilon}$$

$$\varepsilon_z = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)\cos^2\alpha_r \qquad = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)\sin^2\varphi_{1\varepsilon}$$

$$\sigma_{sxr} = E_s\varepsilon_x + \frac{\tau_{b0}s_{rmx}}{\varnothing} \qquad \sigma_{szr} = E_s\varepsilon_z + \frac{\tau_{b0}s_{rmz}}{\varnothing}$$

Neglecting concrete strains and tension stiffening (i.e. $\varepsilon_3 = 0, \tau_{b0} = 0$):

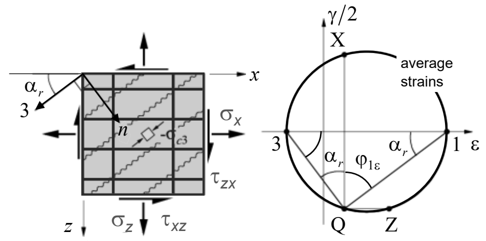
$$\sigma_{sxr} \approx E_s\varepsilon_1\cos^2\varphi_{1\varepsilon} \qquad \sigma_{szr} \approx E_s\varepsilon_1\sin^2\varphi_{1\varepsilon}$$

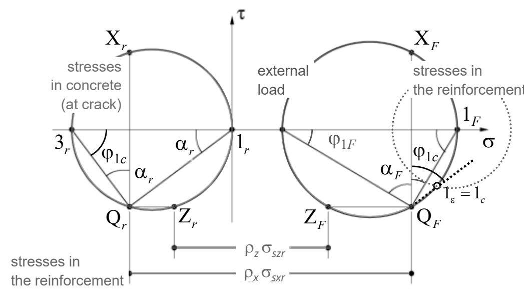
By equilibrium at the cracks, stresses in direction $1\varepsilon = F$ follow as:
(for stress-free cracks only reinforcement stresses act in this direction; note that generally $\varphi_{1c} \neq \varphi_{1F}$ resp. $\alpha_r \neq \alpha_F$)

$$\sigma_F(\varphi_{1\varepsilon} = \varphi_{1c}) = \rho_x\sigma_{sxr}\sin^2\alpha_r + \rho_z\sigma_{szr}\cos^2\alpha_r$$
$$= \rho_x\sigma_{sxr}\cos^2\varphi_{1\varepsilon} + \rho_z\sigma_{szr}\sin^2\varphi_{1\varepsilon}$$

$$\rightarrow \sigma_F(\varphi_{1\varepsilon} = \varphi_{1c}) = \rho_x E_s\varepsilon_1\cos^4\varphi_{1\varepsilon} + \rho_z E_s\varepsilon_1\sin^4\varphi_{1\varepsilon}$$

$$\rightarrow \varepsilon_1 = \frac{\sigma_F(\varphi_{1\varepsilon} = \varphi_{1c})}{E_s}\frac{1}{\left(\rho_x\cos^4\varphi_{1\varepsilon} + \rho_z\sin^4\varphi_{1\varepsilon}\right)}$$





The magnification factor $(\sin^4\vartheta + \cos^4\vartheta)^{-1}$ can be derived by considering the deformations of the «sandwich cover» on the flexural tension side using a stress field model.

The tensile force perpendicular to the principal compressive stress direction (= perpendicular to the cracks) can easily be determined from the forces in the reinforcement that cross the crack (assuming that cracks are stress free). On the other hand, the stress in the reinforcement can be determined from the principal strain. This results in a relationship between the principal tensile elongation and the tensile force in the corresponding direction. It can be seen that the principal distortion in the isotropic reinforcement is greater by the factor $(\sin^4\vartheta + \cos^4\vartheta)^{-1}$ than it would be the case in reinforcement in the direction of the principal strains.

Additional remark:

- Only in special cases the principal strain direction corresponds to the principal stress direction of the applied load. This means the reinforcement in the crack corresponds to a normal and shear force with respect to the crack direction (as shown above). Thus, the given relation does not link the principal elongation with the applied principal tensile stress (but with the tensile force perpendicular to the principal elongation).

3 Slabs

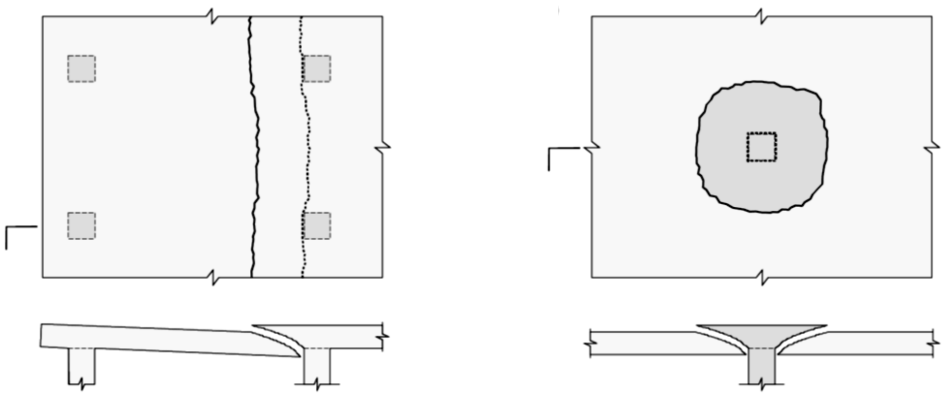
In-depth study and additions to Stahlbeton II

3.6 Punching

Slabs - Influence of shear forces

Slabs without shear reinforcement - Failure mechanisms

- Shear failures as shown in the figure on the left are unlikely in thin slabs. Still, slabs subjected to high loads and primarily carrying in one direction, such as top and bottom slabs of cut-and-cover tunnels may be critical.
- Near concentrated loads (e.g. around columns supporting a flat slab, or supported by a slab on ground), transverse shear forces are often very high. If no shear reinforcement is provided, this can lead to a sudden, very brittle failure (punching).



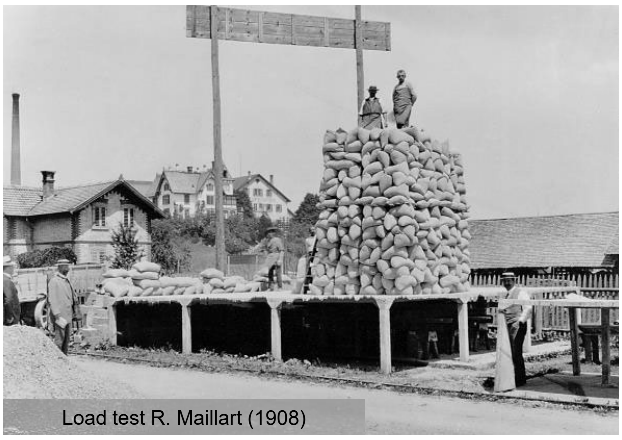
Repetition from Stahlbeton II:

Transverse shear forces transferred in one direction are usually uncritical in slabs (left figure), but may cause brittle failures at point supports and concentrated loads (punching, right figure).

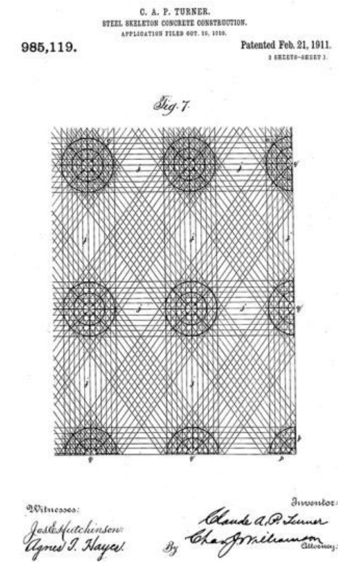
Slabs - Influence of shear forces

Punching

- Flat slabs: load concentration at the supports, maximum v_0 and (m_x, m_y), bending moments with large gradient (elastic solution with point support: m_x *and* $m_y \rightarrow \infty$)
- With respect to the force flow, mushroom slabs are significantly better
- Early days of concrete construction: Flat slabs as a new type of construction
  → Mushroom slab systems Maillart / Turner, fully flat slabs only later



Load test R. Maillart (1908)



Patent specification C.A.P. Turner (1911)

Repetition from Stahlbeton II:

The figure shows a load test by Robert Maillart (1872-1940) for the Rorschach filter building (left) and an extract from the patent specification by C.A.P. Turner (1869-1955).

Slabs - Influence of shear forces

Punching

- Flat slabs without shear reinforcement: very brittle failure, progressive collapse possible
- Parking structures are particularly at risk: Vehicle fire, corrosion, earth cover exceeding design specification, …
- The punching resistance according to SIA 262 (2003) is significantly reduced with respect to earlier codes (in partial revision 2013 even more strict provisions were introduced)
  → many old buildings are not code-compliant

Repetition from Stahlbeton II:

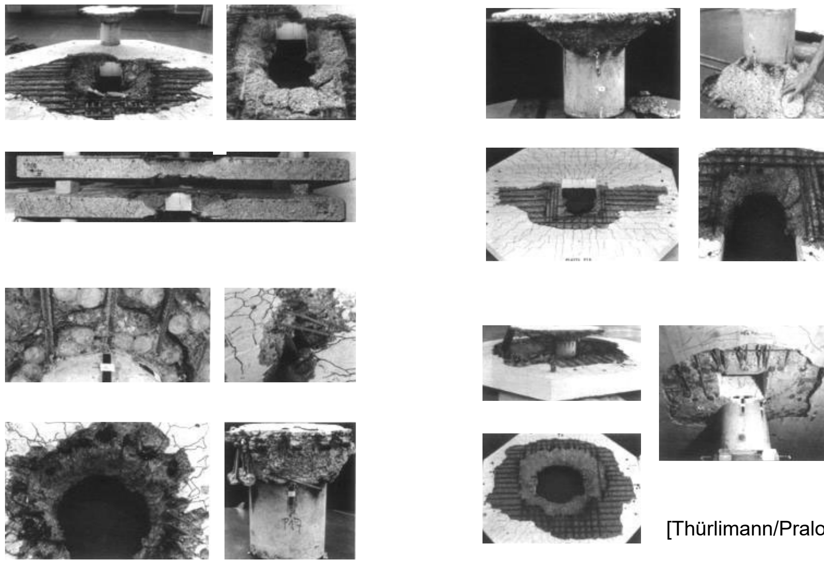
The figure shows punching failures in:

- Wolverhampton (UK, Piper's row car park, built 1965, primary cause corrosion)
- Bluche (CH, Canton VS)
- Gretzenbach (CH, Canton SO, several causes: higher load than designed for (earth cover, garden on top), columns cast too high, fire as final cause triggering the collapse)

Slabs - Influence of shear forces

Punching

Early on many experimental studies worldwide, including ETH Zurich, EMPA



[Thürlimann/Pralong 1979-1984]

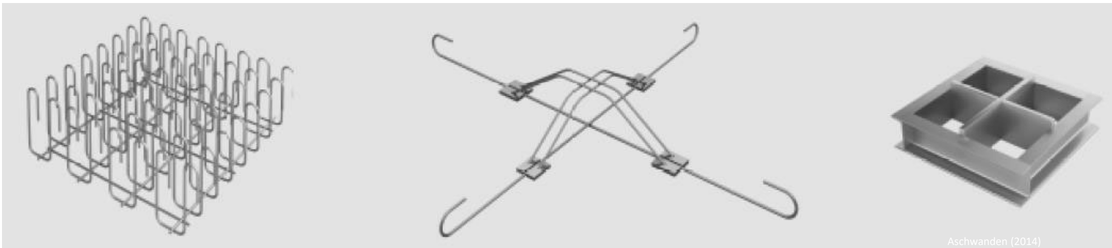
Repetition from Stahlbeton II:

The figure shows punching tests at ETH Zurich (Thürlimann and Pralong, 1979-1984).

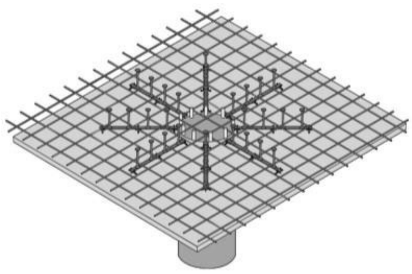
In those decades, many punching tests were also carried out at the EMPA (e.g. Ladner (1977).

Slabs - Influence of shear forces

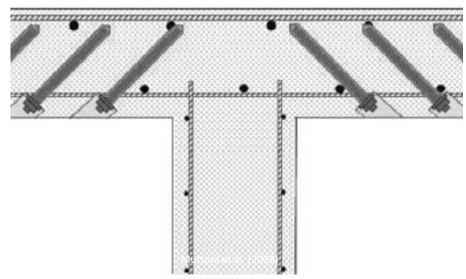
Conceptual solution to the problem: Punching shear reinforcement (or mushroom slabs!)



stirrup cage | bent reinforcement | Steel Forms



Dowels (Studs)



Reinforcing (post-installed) anchors

Repetition from Stahlbeton II:

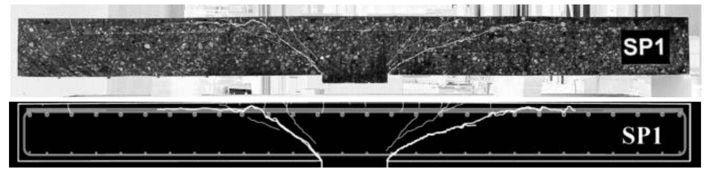
The figure shows various punching reinforcements (bottom right: to reinforce existing structures).

Slabs - Influence of shear forces

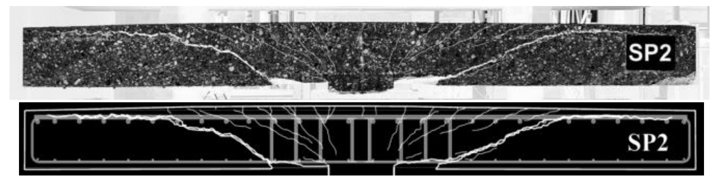
Punching : Types of failure

Example: experiments by Etter, Heinzmann, Jäger, Marti (2009)
IBK report 324

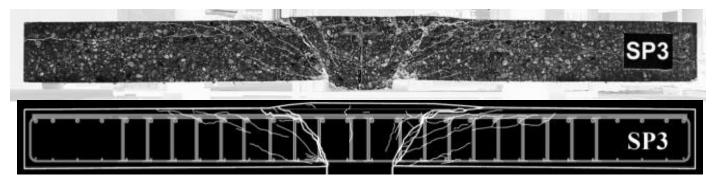
- failure at «inner perimeter»
  (here without punching reinforcement)



- failure at «outer perimeter»
  (section defined by extent of
  punching reinforcement)



- «compression strut» failure
  (with high amount and extent of
  punching reinforcement)



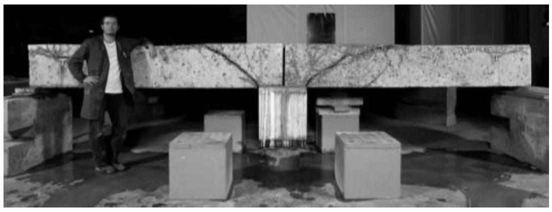
Repetition from Stahlbeton II:

The figure shows sections from test specimens by Etter, Heinzmann, Jäger and Marti (2009) with the typical types of failure that occur in flat slabs.

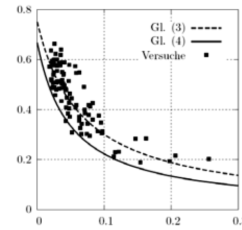
Slabs - Influence of shear forces

Punching: Mechanical model implemented in SIA 262

- Research focus of Prof. Muttoni at ETH Lausanne: Since 2000 various series of experiments (among others with Fernández Ruiz, Guandalini, Guidotti, Lips, Kunz)

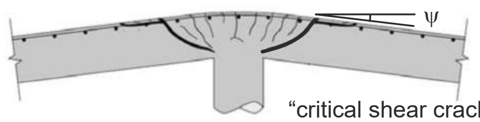


[EPFL - ibeton]



→ Governing parameter: State of strain in the support area (→ bending deformations, as already identified e.g. by Kinnunen / Nylander in 1960 and considered in SIA 162/1968 ("Guideline 18"), but not included in standard SIA 162/1989 to avoid complicating the design by using deformation-dependent strength criteria).

→ Model for slabs without shear reinforcement (basis of the design according to SIA 262 and *fib* Model Code 2010): Failure occurs when a critical shear crack is too wide to be able to transfer the shear (hyperbolic failure criterion closely related to relationships for compression softening):



If curvatures due to bending are neglected:
(crack opening) ~ (slab rotation ψ) · (static depth d)

Repetition from Stahlbeton II:

The figure shows a specimen at the EPFL and the basic assumptions of the model for punching used in SIA 262.

These design specifications for punching, are based on a mechanical model, but as in all current standards, are based on (semi-)empirical relationships calibrated on experiments.

The figure at the top right shows the comparison of test results with the predictions according to the SIA 262 model (normalised slab rotation on abscissa, normalised nominal shear stress at failure on ordinate). The agreement is good for the tests considered (rotationally symmetrical inner supports). As only few tests with non-symmetrical loading (or even on edge and corner supports) have been carried out to date, reliable calibration is hardly possible for these cases.

Slabs - Influence of shear forces

Punching resistance of slabs according to SIA 262

Conceptual provisions

- The deformation capacity of slabs subjected to concentrated loads shall be achieved by the following measures:

  → Either ensure a nominal slab rotation (capacity) $\psi > 0.02$ under the design load V_d
  (i.e. do not overdimension bending reinforcement, choose a sufficiently large supporting area and slab thickness)

  → Or provide a punching reinforcement with $V_{Rd,s} \geq V_d/2$ (*)

  Otherwise, imposed deformations must be taken into account in the design (constraint forces due to restrained temperature changes, differential settlements, shrinkage, etc.).
  → May cause strong variation (increase) of the load V_d, very difficult to quantify: avoid!

(*) according to fib Model Code 2010: $V_{Rd,s} \geq V_d/2$ with $\sigma_{sd} = f_{sd}$ (SIA 262: not specified)

Repetition from Stahlbeton II:

In addition to the design model, the SIA 262 contains various conceptual provisions (see slide).

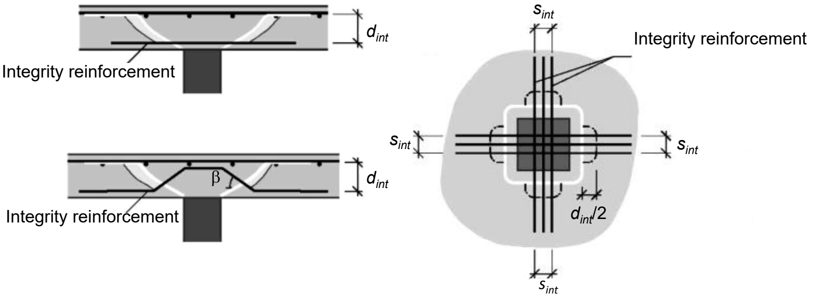
Slabs - Influence of shear forces

Punching resistance of slabs according to SIA 262

Conceptual provisions

- To avoid a progressive collapse (due to punching in spite of a code-compliant design), at least one of the following measures shall be taken:
  → Provide a punching reinforcement with $V_{d,s} \geq V_d/2$ (*)
  → Provide integrity reinforcement preventing a collapse in case of punching (details see SIA 262, 4.3.6.7)

(*) according to fib Model Code 2010: $V_{Rd,s} \geq V_d/2$ with $\sigma_{sd} = f_{sd}$ (SIA 262: not specified)



Repetition from Stahlbeton II:

In addition to the design model, the SIA 262 contains various conceptual specifications (see slide).

Slabs - Influence of shear forces

Punching resistance of slabs according to SIA 262

Verification format

The punching resistance is determined on the basis of nominal transverse shear stresses as follows:

$$\boxed{V_{Rd,c} = k_r \cdot \tau_{cd} \cdot d_v \cdot u} \qquad \text{with} \quad \tau_{cd} = \frac{0.3\eta_t\sqrt{f_{ck}}}{\gamma_c}$$

k_r Coefficient for static depth of the slab, slab rotation and maximum aggregate size

d_v Effective static depht in mm

u control perimeter

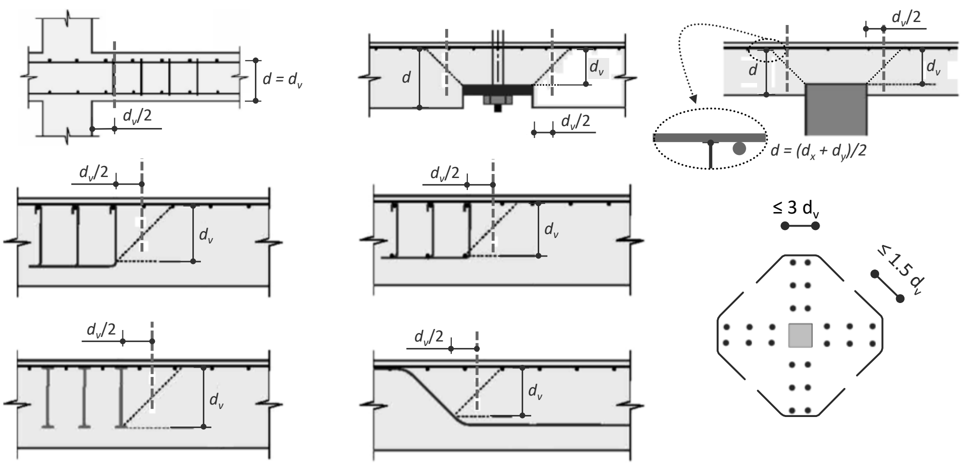
The coefficient k_r depends primarily on the utilisation of the bending reinforcement over the support, which is determined over the width b_s of a nominal "support strip" in each reinforcement direction.

In the following, first the geometrical parameters (effective static depth d_v, control perimeter u, width of the support strip b_s) and then the coefficient k_r are explained.

Slabs - Influence of shear forces

Punching: Control perimeter and support strip

- Effective static depth d_v according to figures below
- Effective static depth d_v to be taken into account when determining the location of the control perimeter



Repetition from Stahlbeton II:

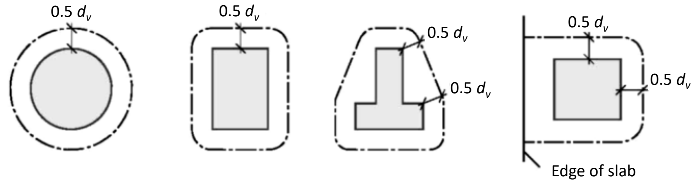
The behaviour is very complex and the semi-empirical resistance model is calibrated on tests. Therefore all punching specifications in standards require numerous definitions and limitations of the field of application. The slide shows some of them from SIA 262.

Slabs - Influence of shear forces

Punching: Control perimeter and support strip

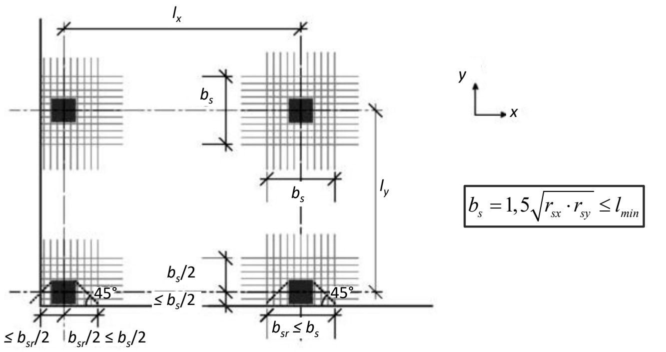
Control perimeter (convex line at distance ≥ $d_v/2$ from support edge → length u)

NB: Actions within the control perimeter may be deducted from the design value of the punching load (self weight, foundation stresses, deviation forces from prestressing, etc.)



Support strip (width b_s)

NB: Bending demand m_{sd} and bending resistance m_{Rd} to be used in formulas for k_r (see following slides): mean values over the width of the support strip



Repetition from Stahlbeton II:

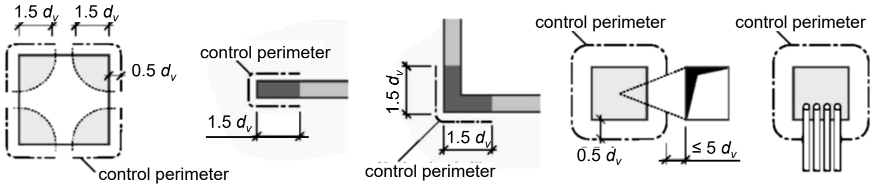
Further definitions and limitations of the field of application.

Slabs - Influence of shear forces

Punching: Reference section

Reduction of the length of the control perimeter to account for non-constant distribution of the shear forces along the perimeter

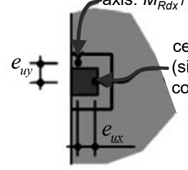
- Consideration of load concentrations in corners, recesses, pipes / ducts, etc.
  (pipes / ducts at a distance < $5d_v$ only permissible in radial direction)!



- Additional reduction of the control perimeter for moment transmission column-slab by the Coefficient k_e (Simplifying the curvatures of the control perimeter as corners):

$$k_e = \frac{1}{1+\frac{e_u}{b}}$$

$$e_u = \sqrt{e_{ux}^2 + e_{uy}^2}$$



Approximation for regularly supported flat slabs, supports rigidly connected, supports do not carry horizontal actions:

- k_e = 0.90 Interior supports
- k_e = 0.75 Wall ends, wall corners
- k_e = 0.70 Edge supports, interior supports with large recesses near the columns
- k_e = 0.65 Corner supports

Repetition from Stahlbeton II:

Further definitions and limitations of the field of application.

3 Slabs

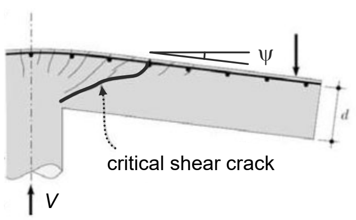
In-depth study and additions to Stahlbeton II

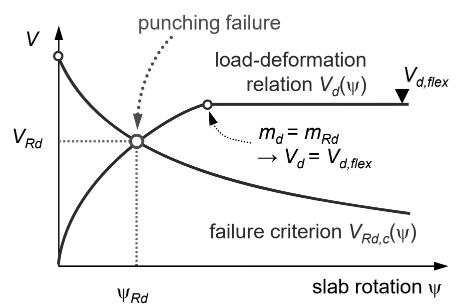
3.6.1 Behaviour without punching reinforcement

Slabs - Influence of shear forces

Punching of slabs without punching reinforcement according to SIA 262

- Basic model: critical shear crack fails if it has opened too much and can no longer transfer the load
- Opening of the critical shear crack (and hence, the punching resistance) is related to the slab rotation ψ through a relationship derived from mechanical considerations and calibrated on experiments → failure criterion $V_{Rd} = V_{Rd}(\psi)$
- An analytical relationship $\psi = \psi(m_{sd}/m_{Rd})$ is established, based on the model, between the slab rotation ψ and the ratio of applied bending moment to bending resistance (m_{sd}/m_{Rd}) in a nominal support strip
- By linking m_{sd} to the support reaction V_d (see following slides) one obtains the load-deformation relationship $\psi = \psi(V_d)$ and hence, $V_d = V_d(\psi)$





Repetition from Stahlbeton II:

Model for the punching resistance according to SIA 262.

Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262

$$\boxed{V_{Rd,c}(\psi) = k_r \cdot \tau_{cd} \cdot d_v \cdot u} \qquad \text{with} \quad \tau_{cd} = \frac{0.3\eta_t \sqrt{f_{ck}}}{\gamma_c}$$

$$k_r = \frac{1}{0.45 + 0.18 \cdot \psi \cdot d \cdot k_g} \leq 2 \qquad \text{with} \quad k_g = \frac{48}{16 + D_{\max}}$$

$$\psi = 1.5 \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd}}{m_{Rd}} \right)^{3/2}$$

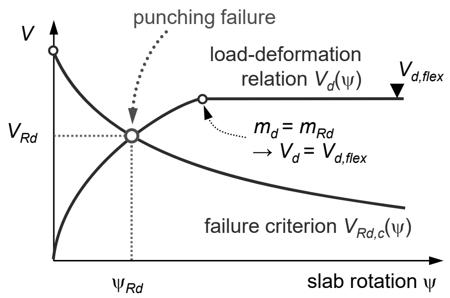
(determine m_{sd}, m_{Rd} und r_s for directions x, y separately, larger value of ψ controls)

- k_r Coefficient for static depth of the slab, slab rotation, and maximum aggregate size
- d_v Effective static depth in mm
- u Control perimeter
- r_s Distance of the point of zero moment (radial moment = 0) from support axis
- m_{sd} Average bending moment in the support strip
- m_{Rd} Average bending resistance in the support strip

Note: The load-deformation relationship does not have to be determined in design (i.e., in the verification whether punching reinforcement is required for a given action V_d).

However, it is needed to calculate the actual punching resistance according to the code.

See the following slides for more details.



Repetition from Stahlbeton II:

Punching resistance according to SIA 262.

Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262

Dimensioning (only governing direction shown (determine ψ_d for m_{sd}, m_{Rd} und r_s in directions *x*, *y* separately, smaller value of V_{Rd} governs)

Given: V_d, support dimensions, static depth (and thus *u*)

Question: Is punching resistance sufficient without shear reinforcement / are the slab thickness and bending reinforcement sufficient?

Procedure

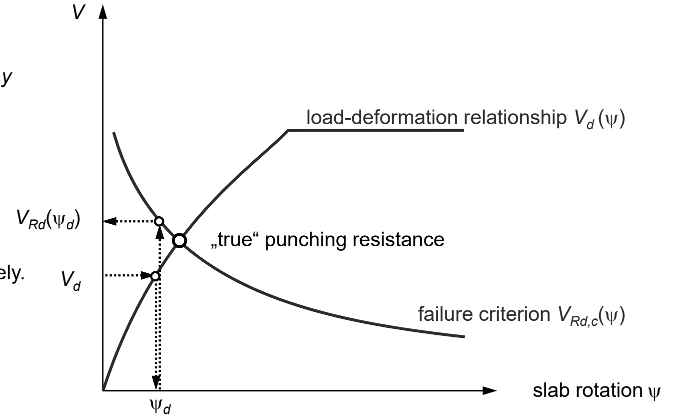
1. Assume *d* and m_{Rd} (select a reasonable reinforcement)
2. Determine of r_s and m_{sd} (V_d) $\rightarrow \psi_d \rightarrow V_{Rd}(\psi_d)$ per direction *x, y* (different levels of approximation, see following slides)
3. Increase *d* and / or m_{Rd}, until $V_{Rd}(\psi_d) > V_d$ (or provide punching reinforcement)

NB: The resulting value $V_{Rd}(\psi_d)$ is greater than the actual punching resistance V_{Rd}.

The «true» value of V_{Rd} would have to be determined iteratively. (intersection of the curves $V_{Rd}(\psi)$ and $V_d(\psi)$).

This is unnecessary in design, which can be done without the determination of the load-deformation relationship $V_d(\psi)$.

The determination of the actual punching resistance is explained in more detail in the following slide.



Repetition from Stahlbeton II:

Design procedure (for punching) according to SIA 262.

Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262

Check / Verification of existing structures (only the governing direction is shown!)

Given: Support dimension (and thus u), d, m_{Rd}
Question: What is the punching resistance (without shear reinforcement)?

$V_{flex,\,sd}$ Support reaction at which the bending reinforcement yields (in the considered direction)

ψ_{sd} Slab rotation when reaching $V_{flex,\,sd}$

Procedure

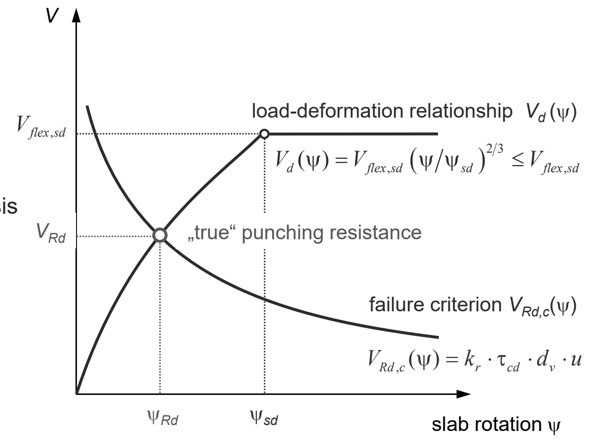
1. Determine the load-deformation relationship $V_d\,(\psi)$ per direction *x, y* (for level of approximation 3: factor 1.5 may be reduced to 1.2)

$$\psi = 1.5\frac{r_s}{d}\frac{f_{sd}}{E_s}\left(\frac{m_{sd}}{m_{Rd}}\right)^{\frac{3}{2}} = \psi_{sd}\left(\frac{m_{sd}(\psi)}{m_{Rd}}\right)^{\frac{3}{2}} \qquad \text{with } \psi_{sd} = 1.5\frac{r_s}{d}\frac{f_{sd}}{E_s}$$

$$V_{flex,sd} = \frac{V_d(\psi)}{m_{sd}(\psi)}m_{Rd} \rightarrow \frac{m_{sd}(\psi)}{m_{Rd}} = \frac{V_d(\psi)}{V_{flex,sd}} \qquad \text{with } \frac{V_d(\psi)}{m_{sd}(\psi)} \text{ from FE slab analysis}$$

$$\rightarrow \psi = \psi_{sd}\left(\frac{V_d(\psi)}{V_{flex,sd}}\right)^{\frac{3}{2}} \qquad \rightarrow V_d(\psi) = V_{flex,sd}\left(\frac{\psi}{\psi_{sd}}\right)^{\frac{2}{3}} \le V_{flex,sd}$$

2. Equating $V_{Rd,c}\,(\psi) = V_d\,(\psi) \rightarrow \psi_{Rd}$, $V_{Rd}\,(\psi_{Rd}) = V_d\,(\psi_{Rd})$ (direction with smaller value of V_{Rd} controls)



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The slide explains the procedure for determining the punching resistance for given conditions (slab thickness, reinforcement) according to SIA 262.

Assuming that the support reaction V_d is proportional to the bending moment m_{sd} in the support strip (applicable to linear elastic behaviour), the load-deformation relationship $V_d(\psi)$ is correlated to the relationship $m_{sd}(\psi)$. Thus, the support reaction V_d is proportional to $\psi^{2/3}$, with an upper limit of $V_{flex,sd}$ (support reaction where the bending reinforcement yields), which is achieved with a rotation ψ_{sd}.

Thus, the relationship $V_d(\psi)$ is known. The punching resistance is the intersection of this relationship with the failure criterion $V_{Rd,c}\,(\psi)$.

The value of $V_d\,/m_{sd}$ can be determined by a slab calculation, or approximated according to the following slides depending on the level of approximation used.

Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262: Levels of approximation (LoA)

(a) Continuously supported flat slabs $0.5 \le l_x / l_y \le 2$, no (small) plastic redistribution ("normal" slab in building construction):

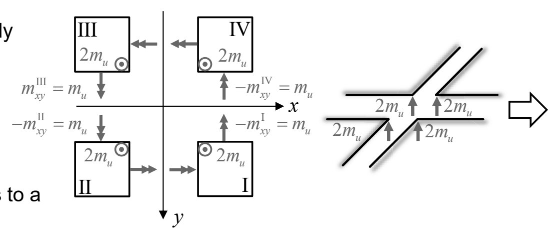
- Level of approximation 1: $r_{sx} = 0.22 \cdot l_x$, $r_{sy} = 0.22 \cdot l_y$ and $m_{sd} / m_{Rd} = 1.0$
- Level of approximation 2: $r_{sx} = 0.22 \cdot l_x$, $r_{sy} = 0.22 \cdot l_y$, estimated bending moments:

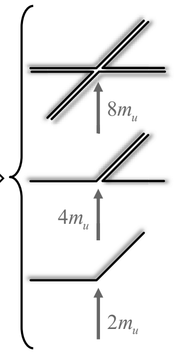
$$m_{sd} = V_d \left(\frac{1}{8} + \frac{|e_{u,i}|}{2b_s} \right) \quad \text{interior colmns} \qquad m_{sd} = V_d \left(\frac{1}{8} + \frac{|e_{u,i}|}{2b_s} \right) \ge \frac{V_d}{4} \quad \text{edge columns} \parallel \text{edge}$$

$$m_{sd} = V_d \left(\frac{1}{8} + \frac{|e_{u,i}|}{b_s} \right) \ge \frac{V_d}{2} \quad \text{corner columns} \qquad m_{sd} = V_d \left(\frac{1}{8} + \frac{|e_{u,i}|}{b_s} \right) \quad \text{edge columns} \perp \text{edge}$$

The corresponding minimum values result directly from the consideration of the combination of individual slab segments with discontinuous twisting moment fields.

NB: For interior supports an explanation with the moment field (transforming a concentrated loads to a uniformly distributed one) is even simpler





(Partial) repetition from Stahlbeton II:

The slide shows the assumptions for r_s and m_{sd} according to the levels of approximation 1-2 of SIA 262. It also shows a derivation of the values for m_{sd} according to LoA 2.

Slabs - Influence of shear forces

Punching resistance of slabs without punching reinforcement according to SIA 262: approximation levels

(b) Flat slabs with $l_x / l_y < 0.5$ or $l_x / l_y > 2$, slabs with complex geometry or detailed examination required:

- Level of approximation 3: Determination of r_s (distance of the point of zero moment, i.e. radial moment = 0, from support axis) and m_{sd} (mean value of the bending moments in the support strip) from an elastic (usually linear elastic FE) slab calculation. Factor 1.2 instead of 1.5 in formula for ψ:

$$\psi = \cancel{1.5}\ 1.2 \cdot \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd}}{m_{Rd}} \right)^{3/2}$$

The slide shows the assumptions according to LoA 3 of SIA 262.

Slabs - Influence of shear forces

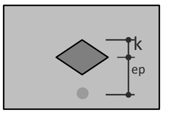
Punching resistance of slabs without punching reinforcement according to SIA 262:
Selected additional provisions (for detailing provisions see SIA 262, 5.5.3)

- Bending resistance m_{Rd} = mean value over support strip, taking prestressing into account.
  (reinforcement must generally be fully anchored at a distance of $2.5 \cdot d_v$ from the control perimeter, but at most at the point of zero bending moment in the respective direction. In the case of edge and corner supports, the reinforcement perpendicular to the edge must be fully ancored → hairpin shaped reinforcement).
- Prestressed slabs with decompression moment m_{Dd}:

$$\psi = (1.5 \text{ or } 1.2)\frac{r_s}{d}\frac{f_{sd}}{E_s}\left(\frac{m_{sd}-m_{Dd}}{m_{Rd}-m_{Dd}}\right)^{3/2}$$

... m_{Dd} = long-term value (shrinkage, creep, relaxation) under consideration of normal forces due to restraints (for m_{Dd}, only the part of the compressive force that is effective in the support strip may be taken into account)
... m_{sd} = incl. constraints (e.g. secondary moments due to prestressing)
... prestress with unfavourable effect must be taken into account where applicable
... use signs of m_{sd} , m_{Rd} and m_{Dd} consistently, otherwise completely wrong results!

NB1: The decompression moment is generally: $m_{Dd} = P \cdot (e_p + k)$. If the prestressing is considered as anchor and deviation forces ("on the load side"), the contribution $P \cdot e_p$ to m_{Dd} *is* already considered in the correspondingly reduced bending moments m_{sd}. The bending resistance m_{Rd} is also smaller by the amount $P \cdot e_p$ (only the increase in prestressing force as resistance) → only the portion $P \cdot k$ can be subtracted in the numerator and the denominator, taking into account the distribution of P over the slab width and, if necessary, the reduction of P by normal forces due to restraints.



NB2: In addition, the contribution of inclined prestressing forces to the punching resistance may be taken into account (even if prestressing is considered on the load side; the support reaction V_d does not reflect the isostatic effect of prestressing).

Repetition from Stahlbeton II:

Further definitions for the application of the specifications of SIA 262. For the punching resistance of prestressed slabs, see the following slide.

Slabs - Influence of shear forces

Punching resistance of prestressed slabs without punching reinforcement according to SIA 262

Check / Verification of existing structures (only the governing direction is shown!)

Given: Support dimension (and thus u), d, m_{Rd}

Question: What is the punching resistance (without shear reinforcement)?

$V_{flex,sd}$ Support reaction at which the bending reinforcement yields (in the considered direction)

ψ_{sd} Slab rotation when reaching $V_{flex,sd}$

m_{dec} Decompression moment (see slides 37 and 39)

Procedure

1. Determination of the load-deformation relationship $V_d(\psi)$ per direction x, y (for LoA 3 replace factor 1.5 by 1.2)

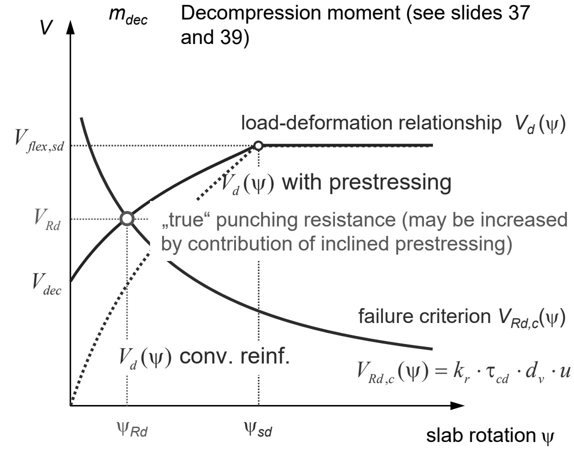
$$\psi = 1.5\frac{r_s}{d}\frac{f_{sd}}{E_s}\left(\frac{m_{sd}-m_{dec}}{m_{Rd}-m_{dec}}\right)^{3/2} = \psi_{sd}\left(\frac{m_{sd}(\psi)-m_{dec}}{m_{Rd}-m_{dec}}\right)^{3/2} \quad \text{mit } \psi_{sd} = 1.5\frac{r_s}{d}\frac{f_{sd}}{E_s}$$

$$V_{flex,sd} = \frac{V_{d,gq}(\psi)}{m_{sd,gq}(\psi)} m_{Rd}, V_{dec,sd} = \frac{V_{d,gq}(\psi)}{m_{sd,gq}(\psi)} m_{dec} \rightarrow \frac{m_{sd}(\psi)-m_{dec}}{m_{Rd}-m_{dec}} = \frac{V_d(\psi)-V_{dec}}{V_{flex,sd}-V_{dec}}$$

(with $\frac{V_{d,gq}(\psi)}{m_{sd,gq}(\psi)}$ from FE slab analysis)

$$\rightarrow \psi = \psi_{sd}\left(\frac{V_d(\psi)-V_{dec}}{V_{flex,sd}-V_{dec}}\right)^{3/2} \qquad \rightarrow V_d(\psi) = V_{dec} + (V_{flex,sd}-V_{dec})\left(\frac{\psi}{\psi_{sd}}\right)^{2/3} \leq V_{flex,sd}$$

2. Equating $V_{Rd,c}(\psi) = V_d(\psi) \rightarrow \psi_{Rd}$, $V_{Rd}(\psi_{Rd}) = V_d(\psi_{Rd})$ (direction with smaller value of V_{Rd} controls)



When determining the punching resistance of prestressed slabs, it is assumed that the slab rotation can be neglected before the decompression moment is reached. As with conventionally reinforced slabs, it is assumed that the support reaction V_d and the bending moment m_{sd} in the support strip - both due to external load (not prestressing) - are proportional to each other, which applies to linear elastic behaviour.

Under these conditions, $(V_d - V_{dec})/(V_{flex,sd} - V_{dec})$ and $(m_{sd} - m_{dec})/(m_{Rd} - m_{dec})$ are correlated, from which the load-deformation relationship $V_d(\psi)$ can be determined. The punching resistance is the intersection of this relationship with the failure criterion $V_{Rdc}(\psi)$.

As with conventionally reinforced slabs, $V_{flex,sd}$ denotes the column reaction at which the bending reinforcement (of the considered direction) yields, and ψ_{sd} the slab rotation when $V_{flex,sd}$ is reached. The value of V_d/m_{sd} can be determined with a slab calculation (or approximated with conventionally reinforced slabs).

Additional remark:

- Simplified representation (constraint moments due to prestress not taken into account).

Slabs - Influence of shear forces

Punching resistance of prestressed slabs without punching reinforcement according to SIA 262

Prestress taken into account on the resistance side:

$$\psi = 1.5\frac{r_s}{d}\frac{f_{sd}}{E_s}\left(\frac{m_{sd}-m_{dec}}{m_{Rd}-m_{dec}}\right)^{3/2}$$

mit $m_{sd} = m_{gq,d} + m_{ps}$: Design value of the bending moment in the support strip (negative)

$m_{gq,d}$: Design value of the bending moment due to vertical loads (negative)

m_{ps} : Secondary moment due to prestress (usually positive)

$m_{dec} = -P_\infty (e_p + k)$: Decompression moment (negative)

P_∞ : Prestressing force at t=∞ (positive) (reduce if normal force does not fully act in the support strip!)

e_p : Eccentricity of prestress (in the support strip), here positive upwards (upper side of slab)

k : extent of core (positive, usually = h/6)

m_{Rd} : Design value of the bending resistance $-(A_s f_{sd} \cdot z_s + A_p f_{pd} \cdot z_p)$ (negative)

(All moments used with the usual sign convention, i.e. tension at bottom = positive -> support moments negative)

If the prestressing is taken into account on the resistance side, the proportion of the inclined prestressing force (sum of the vertical components on the decisive circumference) can either be added to the punching resistance or subtracted from the design value of the column reaction = reduced punching load ($V_{d,red} = V_d - \Delta V_d(P)$, $\Delta V_d(P) = \Sigma(P_\infty \sin\alpha_p)$), but not both!

When determining the punching resistance of prestressed slabs, it is important that the signs of the various terms are handled consistently. Otherwise, a completely wrong value of the quotient $(m_{sd} - m_{dec})/(m_{Rd} - m_{dec})$ results and thus a completely wrong value of the punching resistance.

In addition, the decompression moment must be determined as accurately as possible, since the calculated punching resistance is sensitive to its magnitude. Here it must be taken into account that under certain circumstances not the entire normal force due to prestressing acts on the slab or in the support strip. In such cases, the magnitude of the decompression moment must be reduced accordingly.

Slabs - Influence of shear forces

Punching resistance of prestressed slabs without punching reinforcement according to SIA 262

Prestress taken into account on the load side (as anchor and deviation forces):

$$\psi = 1.5 \frac{r_s}{d} \frac{f_{sd}}{E_s} \left(\frac{m_{sd} - m_{dec}}{m_{Rd} - m_{dec}} \right)^{3/2}$$

mit $m_{sd} = m_{gq,d} + m_p$: Design value of the bending moment in the support strip (negative)

$m_{gq,d}$: Design value of the bending moment due to vertical loads (negative)

m_p : Bending moment due to prestress (Long-term stresses $P_\infty e_p$ and secondary moments, positive)

$m_{dec} = -P_\infty k$: Decompression moment (negative)

P_∞ : Prestressing force at t=∞ (positive) (reduce if normal force does not fully act in the support strip!)

k : extent of core (positive, usually = h/6)

(the part $P_\infty e_p$ of prestressing is already included in m_{sd}, do not use here a second time!)

m_{Rd} : Design value of the bending resistance $-(A_s f_{sd} \cdot z_s + A_p f_{pd} \cdot z_p - P_\infty e_p)$ (negative)

(prestressing contribution reduced by $P_\infty e_p = A_p \sigma_{p\infty} e_p$, since $P_\infty e_p$ is already considered in m_p)

(All moments used with the usual sign convention, i.e. tension at bottom = positive -> support moments negative)

m_{sd}, m_{Rd} and m_{dec} differ all by the same value $P_\infty e_p$, compared to considering prestress as resistance side ⇨ same result!

Even if the prestress is introduced on the load side, the proportion of the inclined prestressing force to the punching resistance can be taken into account: The column reaction, which is used as punching load, does not reflect the isostatic effect of prestress (would be different if the integral of the shear forces along the control perimeter was used as load).

Explanations / remarks see slide 38.

3 Slabs

In-depth study and additions to Stahlbeton II

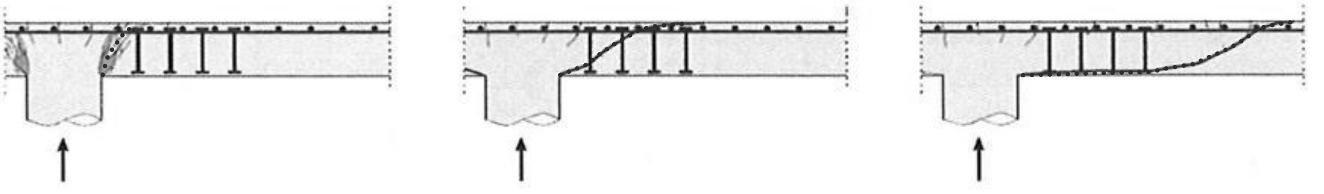
3.6.2 Behaviour with punching reinforcement

Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

The following verifications must be carried out for slabs with punching reinforcement:

- Resistance of the first concrete compression strut next to the supported area
- Resistance of the punching reinforcement (reinforced zone)
- Punching verification (without punching reinforcement) outside the reinforced zone



Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

Minimum required resistance of punching reinforcement:

$$V_{d,s} \geq V_d - V_{Rd,c}$$

... resp. in order to neglect imposed deformations in the design and / or avoid the necessity of an integrity reinforcement

$$V_{d,s} \geq \max \begin{Bmatrix} V_d - V_{Rd,c} \\ V_d / 2 \end{Bmatrix}$$

Resistance of punching reinforcement (normal: inclination β = 90°):
(A_{sw}: only punching reinforcement within distance 0.35...1.0 $\cdot d_v$ of the supported area is taken into account)

$$V_{Rd,s} = \sum A_{sw} k_e \sigma_{sd} \sin\beta$$

Nominal stress in the punching reinforcement:
(f_{bd}: design value of the bond stress)
(NB: according to fib Model Code 2010: $V_{d,s} \geq V_d/2$ with $\sigma_{sd} = f_{sd}$)

$$\sigma_{sd} = \frac{E_s \psi}{6}\left(1 + \frac{f_{bd}}{f_{sd}} \frac{d}{\varnothing_{sw}}\right) \leq f_{sd}$$

Repetition from Stahlbeton II:

Punching resistance according to SIA 262 with punching reinforcement.

Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

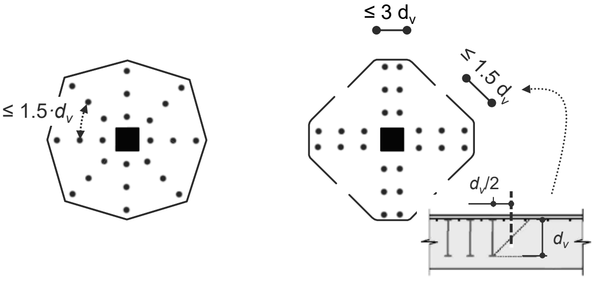
Resistance of the first concrete compression strut:
(Factors > 2 and according to SIA 262 > 3.5 admissible, provided that the effectiveness of the reinforcement is experimentally proven)

$$V_{Rd,\max} = 2 \cdot k_r \tau_{cd} d_v u \leq 3.5 \cdot \tau_{cd} d_v u$$
$$= 2 \cdot V_{Rd,c} \text{ mit } k_r \leq 1.75$$

Punching verification (without punching reinforcement) outside the reinforced zone

(supported surface defined by out reinforcement, control perimeter according to figure)

$$V_{Rd,c} = k_r \tau_{cd} d_v u$$

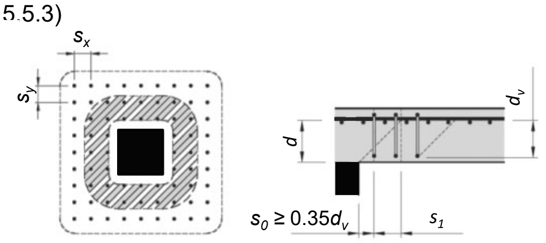


Repetition from Stahlbeton II:

Punching resistance according to SIA 262 with punching reinforcement.

Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262:
Selected additional provisions (for detailing provisions see SIA 262, 5.5.3)



Resistance of the punching reinforcement:
(A_{sw}: punching shear reinforcement only at a distance of 0.35…1.0$\cdot d_v$ from the supported surface)

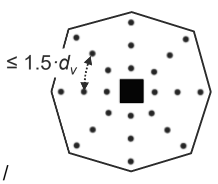
$$V_{Rd,s} = \sum A_{sw} k_e \sigma_{sd} \sin\beta$$

SIA 262 5.5.3.8: At least two legs in radial direction

SIA 262 5.5.3.10: Full anchorage in compression and tension zone

Arrangement of the punching reinforcement within the distance $s_0 < s_1$ from the supported surface:

- radial distance and maximum Ø, see SIA 262, Tab. 20 and Fig. 39
- tangential distance in the second ring ≤ 1.5$\cdot d_v$



Generally provide the same cross-section A_{sw} per «ring»
(rings geometrically similar to control perimeter)

Punching reinforcement in straight radial rows: same radial distance of dowels / vertical reinforcement satisfies the condition of equal A_{sw} per ring

Repetition from Stahlbeton II:

Further definitions for the application of the specifications of SIA 262.

Additional remark:

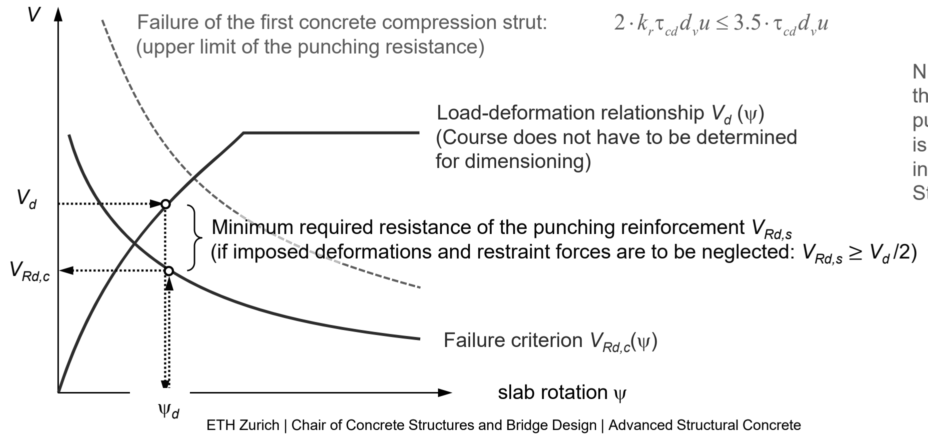
- Experiments show that the arrangement at the bottom right is less effective than the star shaped arrangement.

Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

Dimensioning (only governing direction shown (determine ψ_d for m_{sd}, m_{Rd} and r_s per directions x, y, smaller value of V_{Rd} is governing)

1. Determination of $V_{Rd,c}$ (= same as determination V_{Rd} without punching reinforcement, see slides above)
2. Required resistance $V_{Rd,s} \geq V_{d,s} = V_d - V_{Rd,c}$ ($\geq V_d/2$ if constraint forces are to be neglected)
3. Check that failure of the first compression strut is not governing $V_{Rd} \leq 2 \cdot V_{Rd,c}$
4. Definition of the size of the reinforced area (such that outside, $V_{Rd,c}$ alone is sufficient)



NB: The determination of the effectively existing punching resistance is explained in more detail in the lecture Advanced Structural Concrete.

Repetition Stahlbeton II:

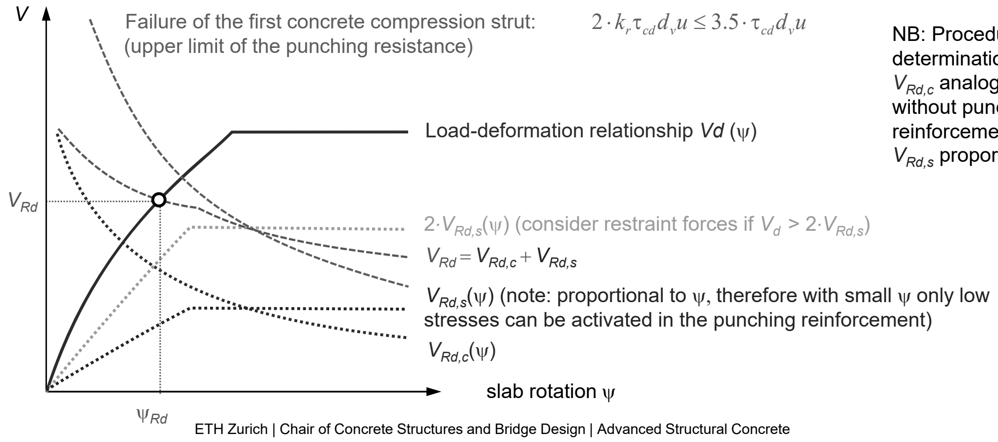
Procedure for dimensioning the punching reinforcement according to SIA 262.

Slabs - Influence of shear forces

Punching resistance of slabs with punching reinforcement according to SIA 262

Check / Verification of existing structures

1. Determination of load-deformation relationship $V_d\ (\psi)$ and punching resistance $V_{Rd}\ (\psi) = V_{Rd,c} + V_{Rd,s} \rightarrow$ equate, intersection = V_{Rd}
2. Check $V_{Rd} \geq V_d$ (V_d incl. Imposed deformations and restraint forces, if $V_d > 2 \cdot V_{Rd,s}$)
3. Check that failure of the first compression strut is not governing $V_{Rd} \leq 2 \cdot V_{Rd,c}$
4. Verify the size of the reinforced area with separate verification



NB: Procedure for the determination of $V_d\ (\psi)$ and $V_{Rd,c}$ analogous to that without punching reinforcement; $V_{Rd,s}$ proportional to ψ)

The slide explains the procedure for determining the punching resistance with punching reinforcement for given conditions (slab thickness, reinforcement) according to SIA 262.

The load-deformation relationship $V_d\ (\psi)$ and the failure criterion $V_{Rd,c}\ (\psi)$ can be determined in the same way as for slabs without shear reinforcement. In addition, the resistance of the punching reinforcement has to be determined according to the relationships:

$$V_{Rd,s} = \sum A_{sw} k_e \sigma_{sd} \sin\beta; \qquad \sigma_{sd} = \frac{E_s \psi}{6}\left(1 + \frac{f_{bd}}{f_{sd}} \frac{d}{\varnothing_{sw}}\right) \leq f_{sd}$$

The resistance of the punching reinforcement increases linearly with the slab rotation ψ up to a maximum when the reinforcement yields.

The punching resistance corresponds to the intersection of the relationship $V_d(\psi)$ with the curve corresponding to the sum $V_{Rd,c}(\psi) + V_{Rd,s}(\psi)$ (limited by the upper limit 2 $V_{Rd,c}(\psi)$).

In practice, often a very low stress results in the punching reinforcement. Alternatively, in such cases a design can be made on the basis of a truss model in which the punching shear reinforcement is fully activated ($V_{Rd,s}$ mit $\sigma_{sd} = f_{sd}$) but the resistance of the concrete is neglected ($V_{Rd,c} = 0$). The upper limit of 2 $V_{Rd,c}\ (\psi)$ must also be considered in this case.

3 Slabs

In-depth study and additions to Stahlbeton II

3.7 Additions

Additions - Elastic sheets

Kirchhoff's slab theory
(rigid linear elastic slabs with small deflections)

The fourth-order differential equation results from the equilibrium and compatibility conditions for linear elastic behaviour (inhomogeneous bipotential equation) :

$$\boxed{\underbrace{\frac{\partial^4 w}{\partial x^4}}_{\substack{\text{beam in}\\ \text{x-direction}}} + \underbrace{2\frac{\partial^4 w}{\partial x^2 \partial y^2}}_{\substack{\text{additional}\\ \text{term}}} + \underbrace{\frac{\partial^4 w}{\partial y^4}}_{\substack{\text{beam in}\\ \text{y-direction}}} = \Delta\Delta w = \frac{q}{D} \quad mit \quad D = \frac{Eh^3}{12(1-\nu^2)}}$$

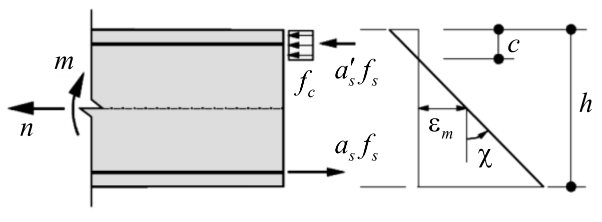
Only two boundary conditions can be adapted to the solution, but there are three variables at the boundary (moments m_n, m_{tn} and shear force v_t) → Support force (see slabs part 1), thus the following boundary conditions:

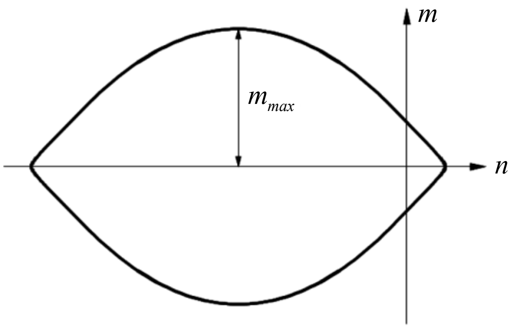
- clamped slab edge: $w = 0$ $\frac{\partial w}{\partial x} = 0$ thus $\frac{\partial^2 w}{\partial x \partial y} = 0$ and thus $m_{xy} = 0$. m_n and v_n are the support reations.
- simply supported slab edge: $m_x = 0$ $\Delta w = 0$ resulting support force $v_n + m_{tn,t} = m_{n,n} + 2m_{nt,t}$
- free slab edge: $m_n = 0$ disappearing support force $v_n + m_{tn,t} = m_{n,n} + 2m_{nt,t} = 0$

Additions - Membrane action

Development of membrane forces

- Cracking leads to deformations in the middle plane of the slab already in the serviceability limit state (dilatancy)
- The resulting deformations are rarely possible without constraint
  → Compressive membrane forces in cracked areas
  → Usually increase of bending resistance
- Membrane force can usually only be roughly estimated (depending on geometry, deformations of the slab middle plane, stiffness of the membrane support).
- Therefore, this effect is typically neglected in design.





Membrane action has a great influence on the behaviour of slabs. The (usually) favourable effect of compressive membrane forces due to crack formation is usually neglected in the dimensioning.

This is often very much on the safe side, especially when it comes to proving fatigue safety. In fatigue tests on slabs under concentrated loads, for example, it is found that significantly smaller stress differences result in the reinforcement than would be expected according to bending theory. If the membrane effect is taken into account, these results can be explained. North American design provisions for bridge decks semi-empirically account for this.

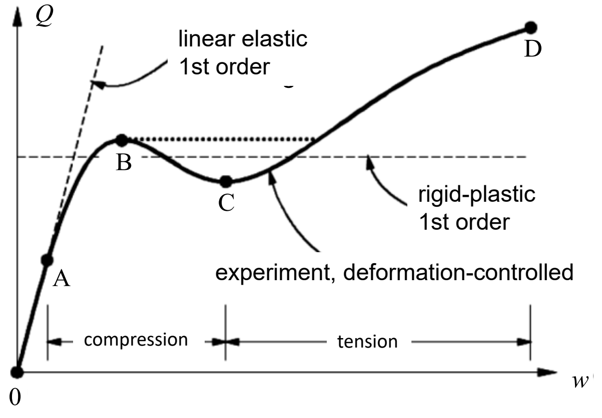
The effect of membrane action can e.g. be estimated with nonlinear finite element calculations using a mechanically consistent model (layered formulation of Cracked Membrane Model as shell element), as implemented by Prof. Karel Thoma at HSLU Lucerne.

Additions - Membrane action

Development of membrane forces

Behaviour (qualitative)

1. Linear elastic (OA)
2. Crack formation, build-up of compressive membrane forces (AB)
3. Maximum load (B) > Load capacity for rigid-ideally plastic behaviour without membrane action (M-N interaction)
4. Load decreases if deformation controlled, compressive membrane forces are reduced (BC);
(Load-controlled: «snapthrough" of the slab)
5. With external membrane support, build-up of tensile membrane forces with increasing deflection. Failure load often >> first maximum (with large deformations, can only be measured with a corresponding calculation)

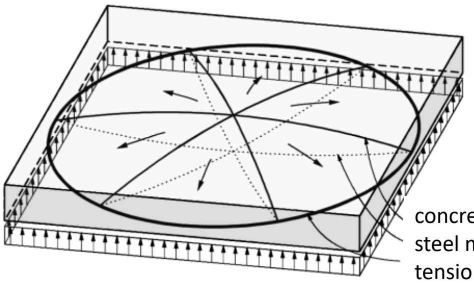


While a consideration of compressive membrane forces would be useful in many cases, tensile membrane forces (which only occur with large deformations) in slabs should only be activated in exceptional cases.

Additions - Membrane effect

Spatial model for load-bearing capacity

- Membrane support not by bearing, but by tension ring (uncracked area of the slab)
- Load transfer: Compression membrane (concrete) and tension membrane (reinforcement: conventional or e.g. prestressing without bond)
- Without horizontal membrane support (external or by a tension ring), the membrane forces of the concrete and reinforcement membranes are in equilibrium → not an actual membrane effect.
- Membrane action can be used to explain the load-bearing capacity of an unreinforced slab (at the location of the membrane support, horizontal **and** vertical components of the membrane forces need to be resisted)



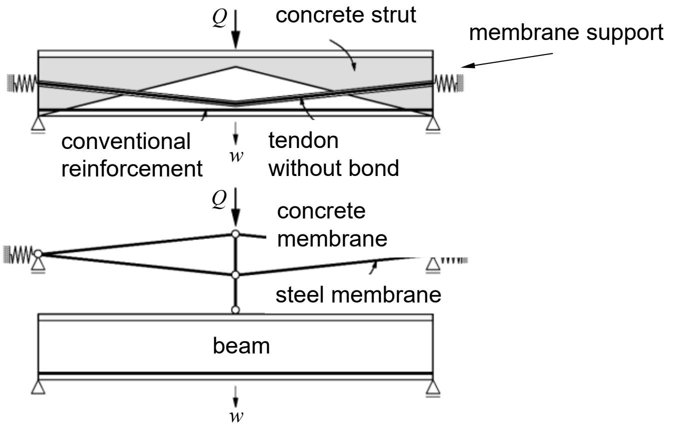
In contrast to beams, a membrane effect in slabs does not necessarily require horizontally restrained supports. In the integral over the entire slab, the membrane forces must disappear in the case of a statically defined support arrangement in the slab plane (for horizontal forces). But, as shown in the figure, a tension ring can form in the outer area on which a compression membrane is supported in the inner area.

This simple observation shows that membrane forces can also occur in slabs with horizontally sliding supports.

Additions - Membrane effect

Model for the load-bearing capacity (Ritz, 1978)

- Model for load-bearing behaviour of slab strips prestressed without bond with membrane effect
- Load carried by bending or membrane effect (of concrete and steel membrane),depending on stiffness ratios (if membrane support is missing, no actual membrane effect)



The slide shows a simple model for the investigation of the load-bearing behaviour with a membrane support. This can also be described analytically for simple cases. Despite the high relevance of this effect, not much meaningful research has been carried out since the time of the work indicated in the slide.