2 In-plane loading – walls and beams

2.2 Stress fields with prestressing

Learning objectives

Within this chapter, the students are able to:

- recognise the suitability of treating prestressing as equivalent forces for the analysis of 2D and 3D structures.
- create simplified stress fields and strut-and-tie models including prestressing as anchorage and deviation forces.

2 In-plane loading – walls and beams

2.2 Stress fields with prestressing

Basis

Repetition from Stahlbeton II (Vorspannung)

Prestressing of framed structures (SB II)

- Prestressing = controlled application of forces to the structure or building component
- Anchorage, deviation and friction forces act between the prestressing steel and the structural member without prestressing tendon.
- Prestressing generates a residual stress state and causes deformations of the structure.
- In statically indeterminate systems, restraint forces result from restrained deformations.
- The load-bearing behaviour of prestressed beams can be investigated analogously to passively reinforced structures by means of cross-sectional analyses. Note that the strain difference $\Delta\varepsilon$ between prestressing steel and concrete is "frozen" during the injection of the prestressing duct.
- There are two alternative possibilities for treating prestressing:

Residual stress state acting on the entire structure or building component including the prestressing tendon	**Anchorage, deviation and friction forces** acting on the structural member without prestressing tendon
Also referred to as **prestressing treated as resistance**	Also referred to as **prestressing treated as load**

- Both possibilities lead to the same result (with consistent application). The only difference are the boundaries of the system.
- Depending on the specific problem, one or the other option is more convenient.

Repetition from Stahlbeton II:

The maximum spans of reinforced concrete structures are limited due to crack formation and long-term deformations (creep). Deformations become unacceptably large in the case of large slenderness, as required for long-span structures.

With prestressing, these problems can be solved by compensating the stresses caused by permanent loads - or part of them - by the anchorage and deviation forces of the prestressing. Prestressing is thus an essential element of long-span, slender and economical concrete structures. It was essential for the breakthrough and success of concrete structures, especially in bridge construction.

There are two ways of taking prestressing into account in framed structures. They differ in the boundaries of the considered system. Each of these approaches has its advantages and disadvantages, and depending on the particular problem, one or the other approach is more suitable.

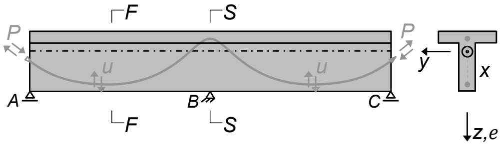
Since the pre-strain (strains at initial prestressing) usually ensures that the prestressing steel yields at ULS in bending, determination of the bending resistance is straightforward when considering prestressing as a **residual stress state**. It makes sense to take into account the residual forces $M_{ps}(P)$, $N_{ps}(P)$, $V_{ps}(P)$ (with P_0 or P_∞) (even if this is not mandatory considering plastic redistribution of internal forces):

Considering the prestressing as **anchorage, deviation and friction forces** is useful for the verification of the stresses in cross-sections. A standard stress calculation (without pre-strain) can be carried out on the reinforced concrete cross-section under the loads incl. $M_c(P)$, $V_c(P)$, $N_c(P)$. It is also advantageous for deflection calculations (incl. creep) and the verification of structural safety in shear. Since the increase in prestressing force is neglected, the resistance of the prestressing is $V_c(P_\infty)$ (included in $V_c(P)$ from static program if prestressing is modelled accordingly). Note: If the verification of the structural safety in bending is performed with the loads including $M_c(P)$, $V_c(P)$, $N_c(P)$, only the increase in prestressing force ($f_{pd} - \sigma_p$) may be taken into account in the bending resistance, not f_{pd} (otherwise prestressing is considered twice).

Prestressing of framed structures (SB II)

Treatment of prestressing / definition of system under consideration (2)

Entire structure / element



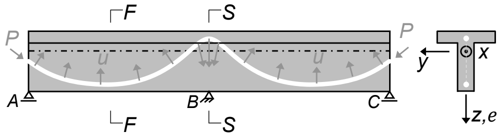
Prestressing causes a residual stress state in the cross-sections: The tensile force in the tendon is in equilibrium with the (compression) forces in the reinforced concrete section. The residual stress state corresponds to strains and curvatures → deformations of the structure.

The internal actions contain only the restraint actions $M_{ps}(P)$, $V_{ps}(P)$, $N_{ps}(P)$. Actions on the total cross-section :

$$\begin{aligned} M &= M_{g,q} + M_{ps} \\ V &= V_{g,q} + V_{ps} \\ N &= N_{g,q} + N_{ps} \end{aligned}$$

$$\begin{Bmatrix} -P\cos\beta_p \cdot e \approx -P \cdot e \\ -P\sin\beta_p \\ -P\cos\beta_p \approx -P \end{Bmatrix}$$

Structure / element without prestressing tendon



The prestressing corresponds to anchorage, deviation and friction forces acting on the structure without the tendon. These loads result in the so-called internal actions due to prestressing $M_c(P)$, $V_c(P)$, $N_c(P)$ and deformations (compatible with the arrangement of supports).

The internal actions contain the total internal forces due to prestressing $M_c(P)$, $V_c(P)$, $N_c(P)$. Actions on the cross-section without the prestressing the tendon:

$$\begin{aligned} M_c &= M_{g,q} + M_c(P) &&= M_{g,q} + M_{ps} - P\cos\beta_p \cdot e \\ V_c &= V_{g,q} + V_c(P) &&= V_{g,q} + V_{ps} - P\sin\beta_p \\ N_c &= N_{g,q} + N_c(P) &&= N_{g,q} + N_{ps} - P\cos\beta_p \end{aligned}$$

Repetition from Stahlbeton II:

If the entire structure is considered, including the prestressing tendon, the prestressing causes a residual stress state in each section. In statically indeterminate systems, the deformations corresponding to this residual stress state are not necessarily compatible with the configuration of boundary conditions and residual forces $M_{ps}(P)$, $V_{ps}(P)$, $N_{ps}(P)$ result from the prestressing. These forces together with the external loads (dead load, live loads, ...) should be taken into account as loads. The cross-sectional resistance corresponds to the resistance of the entire cross-section including prestressing. This is why we speak of "prestressing treated as resistance".

Alternatively, one can consider the structure without the prestressing tendon. Anchorage, deviation and friction forces act as a result of prestressing. This is why the term "prestressing treated as load" is used in this context. The restraint forces acting on the entire structure are accounted for directly by determining the internal actions due to anchorage, deviation and friction forces and are thus included in the internal actions due to prestressing $M_c(P)$, $V_c(P)$, $N_c(P)$.

2 In-plane loading – walls and beams

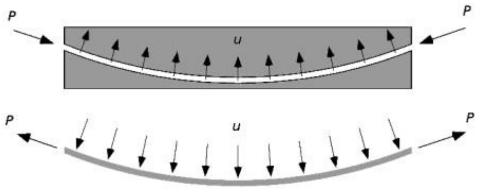
2.2 Stress fields with prestressing

Particularities in membrane, slab and shell structures
Additions to Stahlbeton II (Vorspannung)

Prestressing of membrane, slab and shell structures

Treatment of prestressing in membrane, slab and shell structures
The treatment of prestressing as a residual stress state in the total system is deemed to fail in two-dimensional or three-dimensional structures because the residual stress state due to prestressing cannot be uniquely determined (internal static indeterminacy, unknown spreading of compressive force, reference cross-section unclear, etc.).

The treatment of the prestressing as anchorage, deviation and friction forces on the subsystem "reinforced concrete structure without prestressing", on the other hand, is possible without any problems. This also allows to visualise the force flow (using stress fields, strut-and-tie models).



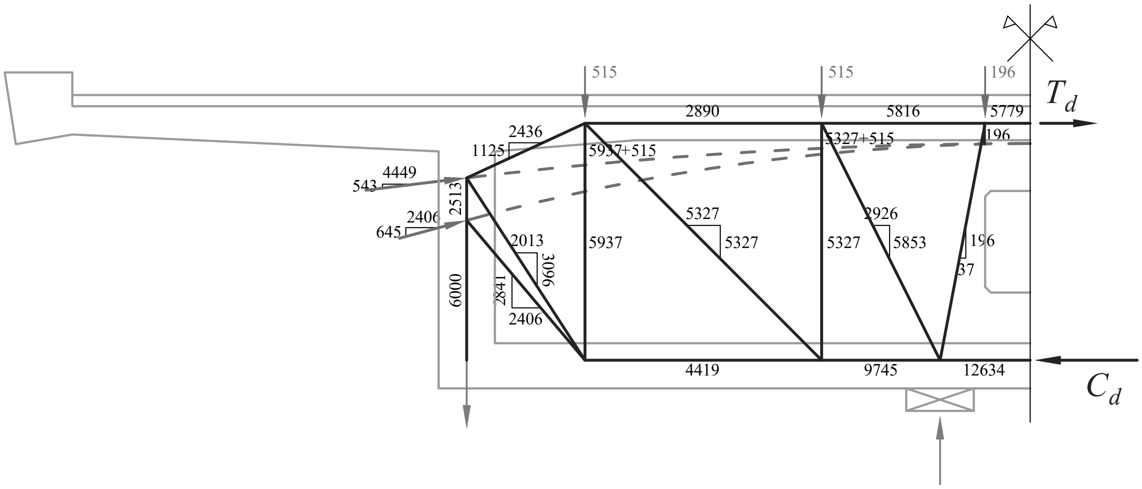
In design practice, the anchorage, deviation and friction forces are usually determined considering the prestressing force without any increase. The increase in the prestressing force at ULS could theoretically be investigated with suitable considerations (e.g. stress fields), but the effort is not worthwhile usually (small influence, since the initial preload $0.7f_{pk}$ is only slightly (approx. 3-7%) lower than the design value of the yield stress $f_{p0.1k}$ /1.15). It is more relevant to estimate the influence of long-term losses on the prestressing force.

Explanations see slide.

The consideration of prestressing as anchorage and deviation forces is excellently suited for the design and verification of structures using strut-and-tie models and stress fields.

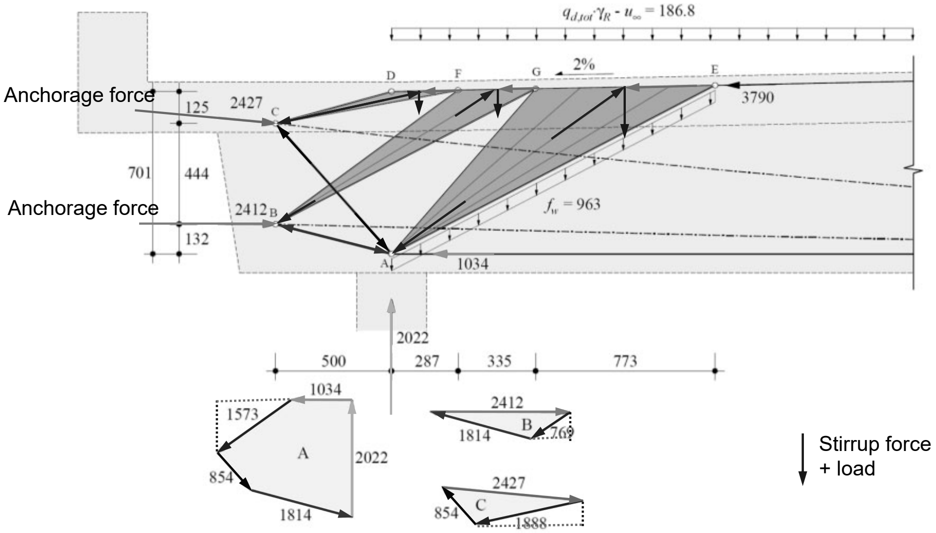
Prestressing of membrane, slab and shell structures

Treatment of prestressing in membrane structures



Prestressing of membrane, slab and shell structures

Treatment of prestressing in membrane structures
Example neglecting the increase in prestressing force [Marti und Stoffel, 1999]



Anchorage force

Anchorage force

Stirrup force
+ load

In detailed verifications of beams, the web is often idealised as a membrane.

The dimensioning or checking is then carried out using strut-and-tie models and stress fields, analogously to passively reinforced building components. The figure shows an example of a practical application.

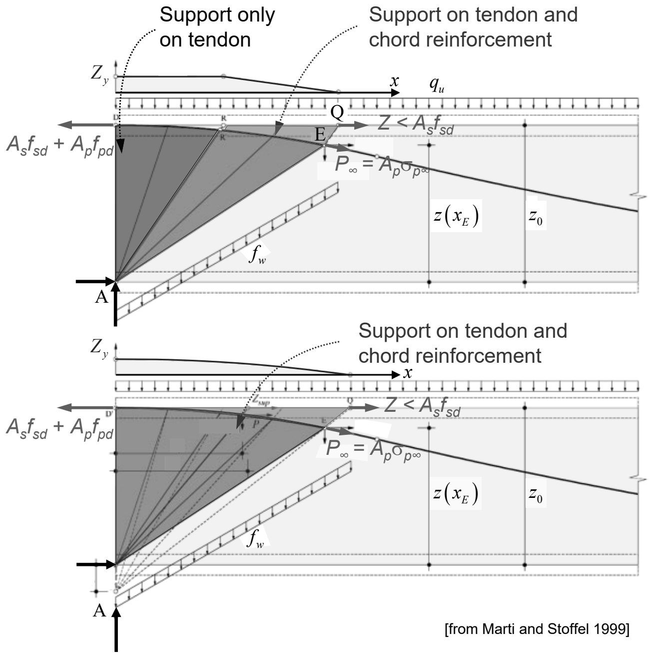
Additional remarks:

- For individual load cases, the examination can be carried out by means of manual calculations. In suitable programs (CAD), the force flow can be traced by means of graphic statics together with the development of the strut-and-tie models and stress fields.
- For the investigation of complex geometries and for the consideration of multiple load cases, automated tools for the development of strut-and-tie models or stress fields, such as the Compatibility-based Stress Field Method presented later in the course are very useful.

Prestressing of membrane, slab and shell structures

Chord force distribution for centred fan with prestressing

- → The distribution of the chord forces between conventional reinforcement (*Z*) and tendon (*P*) is not directly determined from equilibrium.
- → Plausible assumption of force distribution:
  - Increase in prestressing only in the decompression region (the assumption of P_∞ at the edge of the fan is reasonable, i.e. increase of the prestressing only in the fan area)
  - In normal conditions, the fan can never be supported only by the tendon, but it is partially supported by the tendon and the conventional reinforcement.
- → Possible solution see figure on the top: Assumption that in the first area the fan is supported only by the tendon; in the second area it is supported by the tendon and the conventional reinforcement of the tension chord. The position of the points E, Q and the value of f_w can be determined from equilibrium.
- → Alternative solution in the lower figure: fictitious fan for determining the tension chord force (parabolic); the fan is supported over its entire length both on tendon and on the conventional reinforcement. The geometry can also be determined from equilibrium. The stirrup forces are different above and below the tendon.



[from Marti and Stoffel 1999]

In the case of prestressed beams, the chord force distribution cannot be unequivocally determined assuming a certain inclination of the compression field because the distribution of the chord forces cannot be determined from equilibrium conditions.

The figure shows two plausible solutions for the force distribution. In the upper figure, the first part of the fan (yellow) is supported only by the tendon. To the right, the (orange) fan is supported both on the tendon and on the conventional reinforcement, in which a linear distribution of the force is assumed. The position of the transition from the yellow to the orange fan follows from the condition that the force in the tendon should not decrease in the direction of the support. At the end of the fan, the initial prestressing (minus long-term losses) acts.

In the lower figure, the fan is supported along its entire length by the tendon and the conventional reinforcement. The distribution is chosen assuming a fictitious, point-centred fan for the support on the tension chord so that at the end of the fan, the initial tension force (minus long-term losses) acts again.

The assumption that the initial prestressing load (minus long-term losses) acts at the end of the fan makes sense in most cases: increases in prestressing outside this range are rarely plausible. A more detailed study of the distribution of the chord forces on conventional reinforcement and tendons is rarely necessary.

2 In-plane loading – walls and beams

2.2 Stress fields with prestressing

Puente del Tercer Milenio, Zaragoza

Puente del Tercer Milenio, Zaragoza

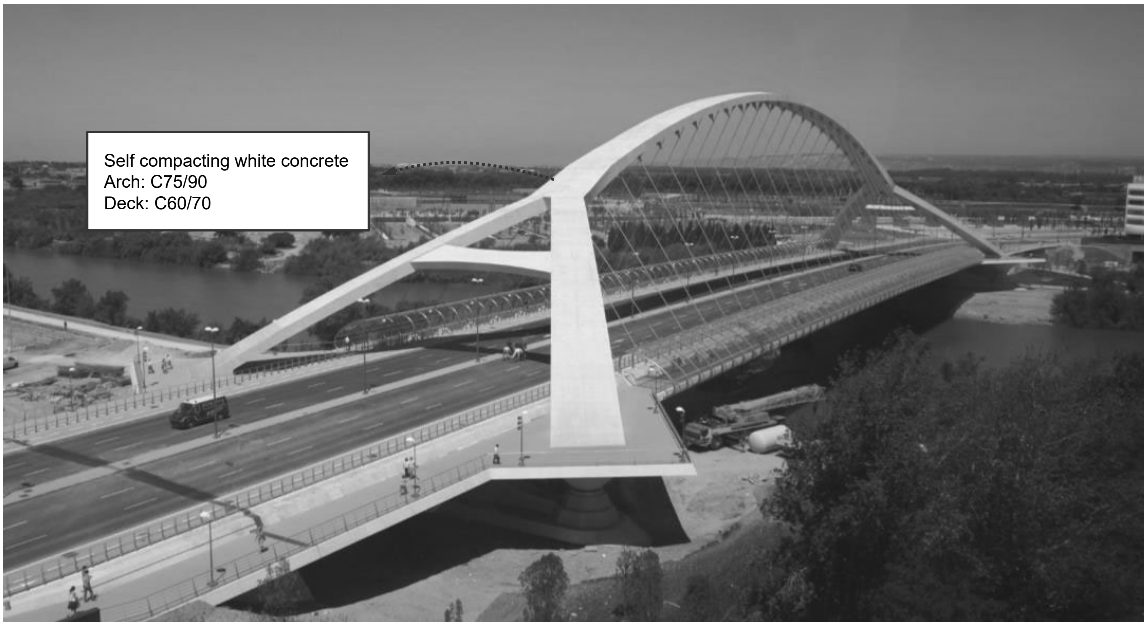
Project / Photos: Arenas & Asociados / Juan José Arenas de Pablo

The application of prestressing to membrane, slab and shell structures is illustrated using the tying slabs («lajas de amarre») of the Puente del Tercer Milenio in Zaragoza.

This bridge was designed (detailed design for tender) in 2002 (conceptual design by Prof. Juan José Arenas, W. Kaufmann as project manager).

It was built only some years later, for the Expo 2008.

Puente del Tercer Milenio, Zaragoza

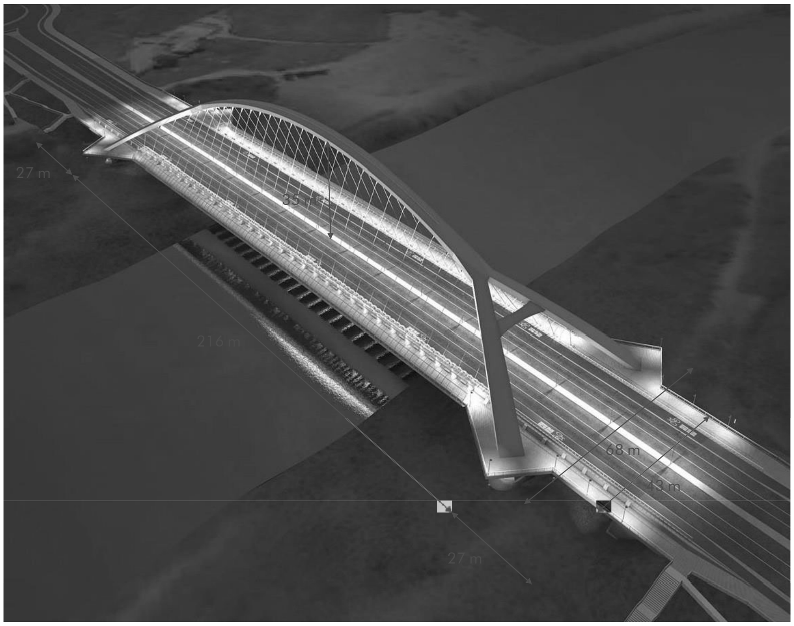


Self compacting white concrete
Arch: C75/90
Deck: C60/70

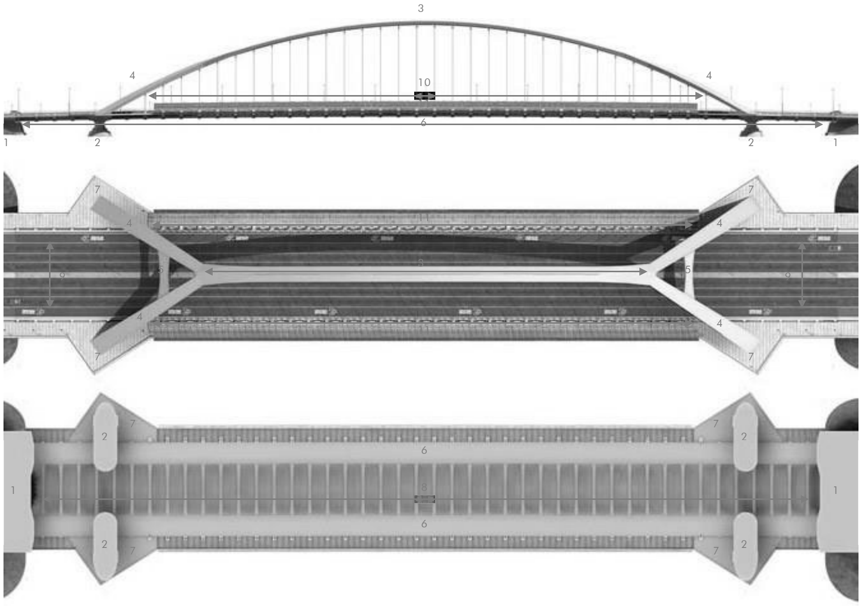
It is a very wide bridge, carrying 6+2 traffic lanes plus separate sidewalks with wind barriers on each side.

Among other technical challenges, the design of the tying slabs (equilibrating the huge horizontal thrust of the arch, but also acting as slabs under traffic load) was a demanding task. They were heavily prestressed and dimensioned using the sandwich model (partly modified to carry compression in the core), treating the prestressing as load (anchor and deviation forces).

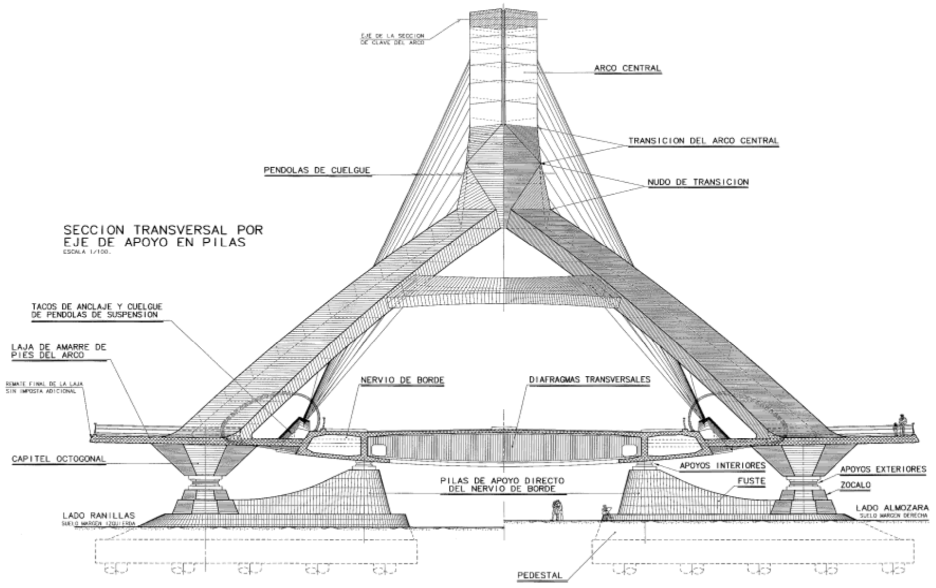
Puente del Tercer Milenio, Zaragoza



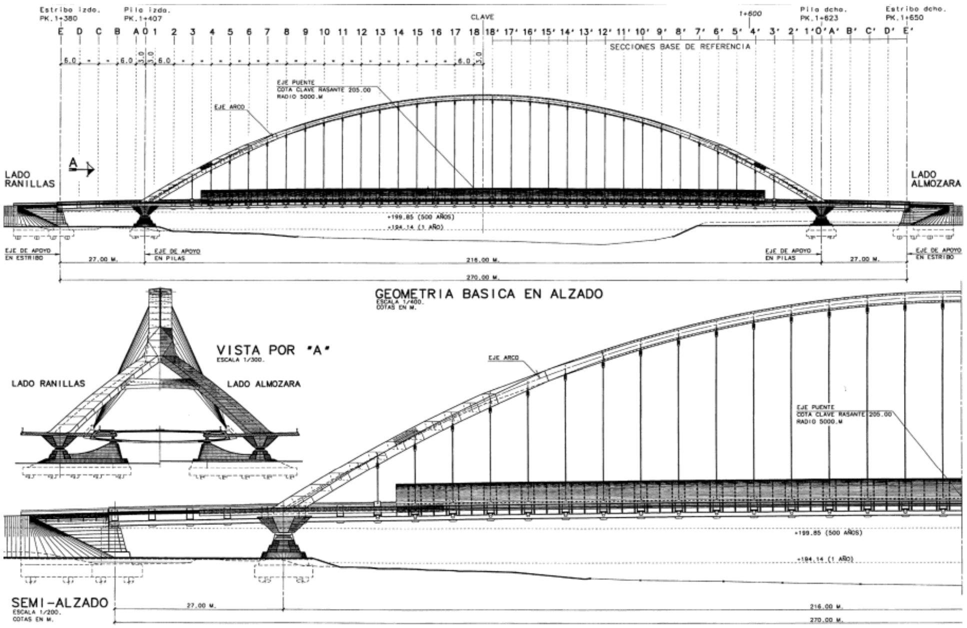
Puente del Tercer Milenio, Zaragoza



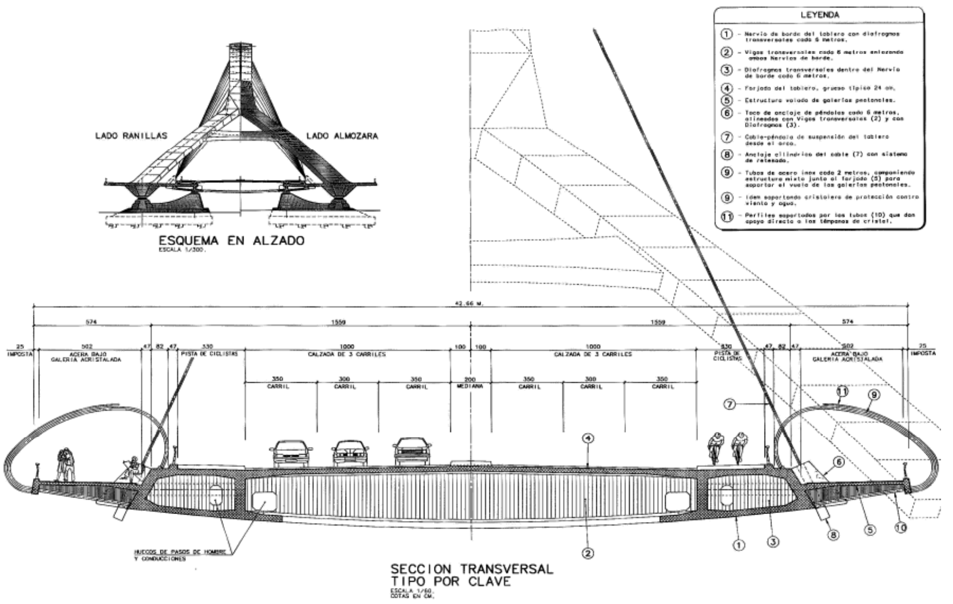
Puente del Tercer Milenio, Zaragoza



Puente del Tercer Milenio, Zaragoza



Puente del Tercer Milenio, Zaragoza



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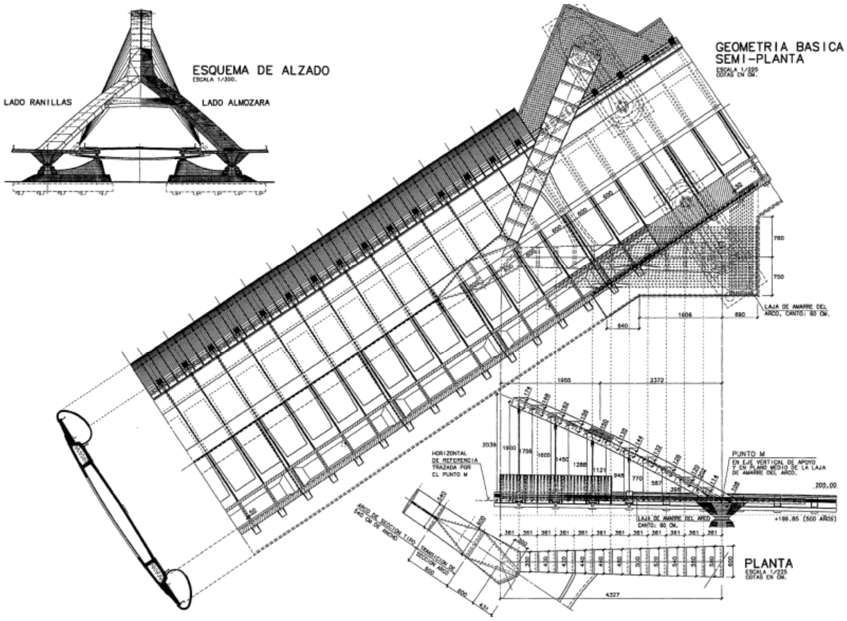


2 In-plane loading – walls and beams

2.2 Stress fields with prestressing

In-class exercise

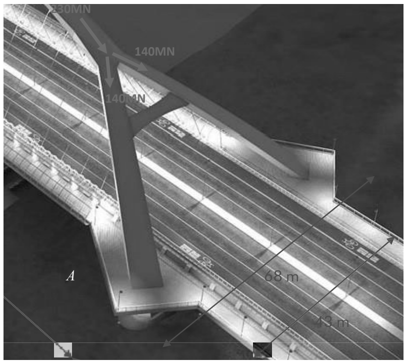
In-class exercise

1) Determine the global force flow and the magnitude of the forces.

   Note that the supports at the arch abutments directly resist the vertical component of the arch force, but are free to move horizontally
   (→ full horizontal arch thrust carried by the bridge deck)

2) Determine the in-plane force flow in the tying slabs in more detail, using a strut-and-tie model.

3) Determine the amount of prestressing needed and pre-dimension the tying slabs for membrane elements.



Materials:

Concrete:	C60/70
Steel:	B500B
Prestressing steel:	Y1860