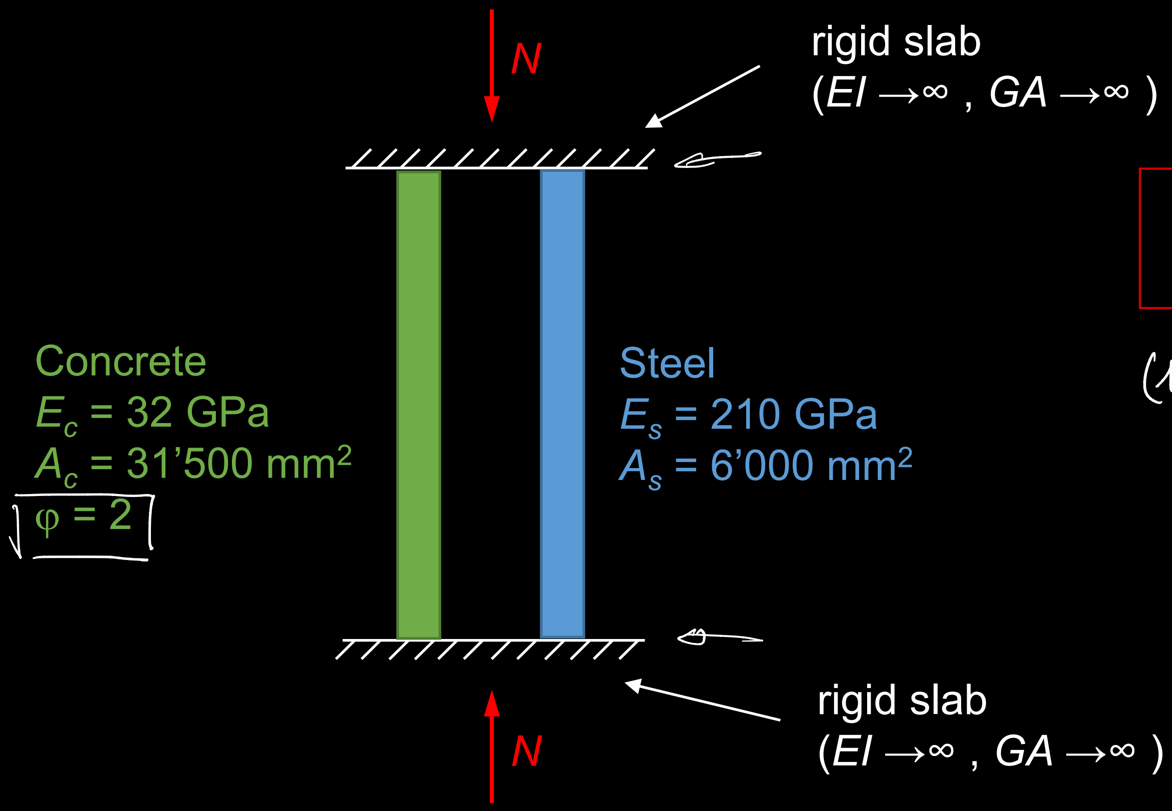
Long-term effects

In-class exercise



How is the load *N* distributed between the two columns?

- at t = 0
- at t $\to \infty$

$$\boxed{(1+\varphi)\cdot \varepsilon_c \overset{!}{=} \varepsilon_s}$$

$$N_c = ?$$
$$N_s = ?$$

$$(1+\varphi)\cdot \frac{N_c}{E_c A_c} = \frac{N_s}{E_s A_s}$$

$$N_s + N_c = N$$

$$\underbrace{\frac{E_s A_s}{E_c A_c}}_{=K}\cdot (1+\varphi)\cdot N_c = N_s = N - N_c$$

$$(1 + K(1+\varphi))\cdot N_c = N$$

$$N_c = \frac{1}{1+K(1+\varphi)} N$$

$$K = 1.25$$

$t = 0$: $\varphi = 0$ $N_c = 0.44 \cdot N$

$t \to \infty$: $\varphi = 2$ $N_c = 0.21 \cdot N$

effects of creep on

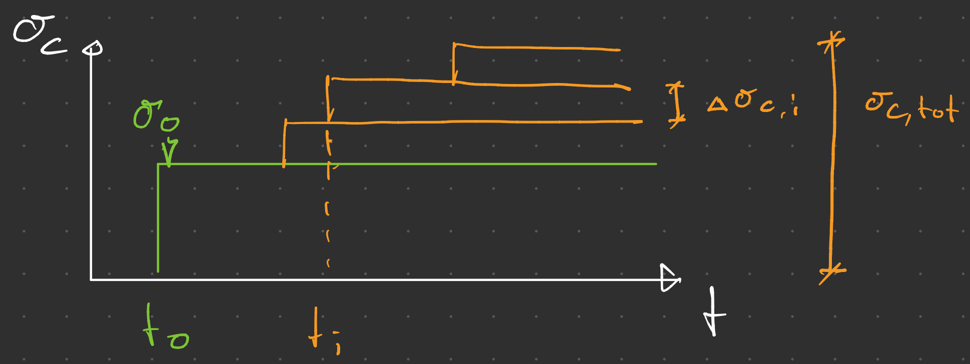
deformations

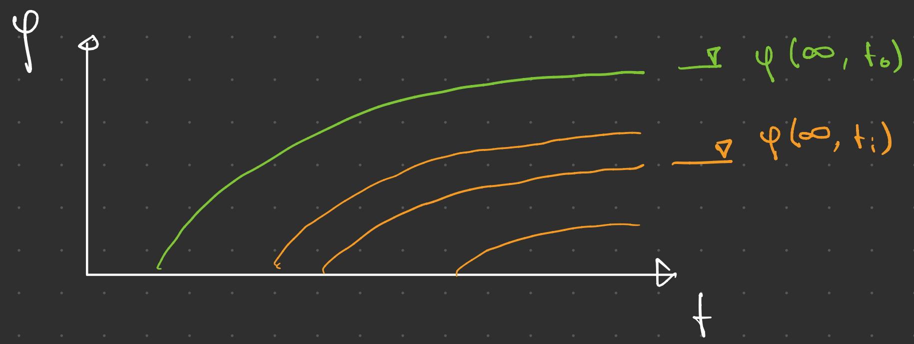
- ↳ permanent loads
- ↳ uncracked concrete EI^{I} (minor importance for cracked concrete EI^{II})

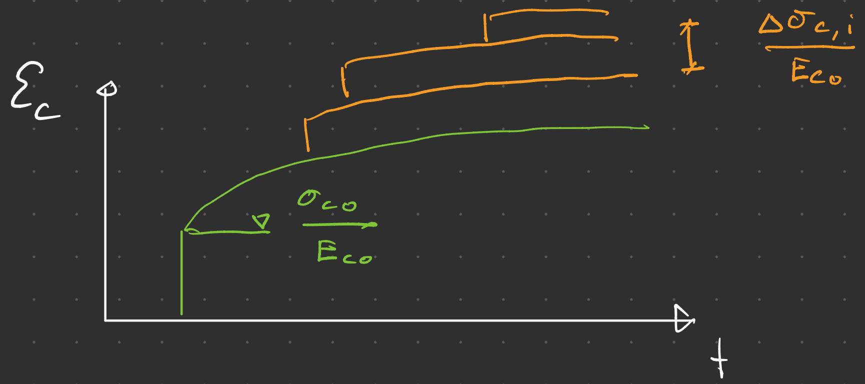
internal forces

- isostatic system
  - → no effect
- hyperstatic system
  - uniform creep properties → no effect
  - non-uniform creep properties → load redistribution
  - change of static system → load redistribution

Trost's method







$$\varepsilon_c(t) = \frac{\sigma_0}{E_{c0}}\left(1+\varphi(t,t_0)\right) + \sum \frac{\Delta\sigma_{c,i}}{E_{c0}}\left(1+\varphi(t,t_i)\right)$$

$$= \underbrace{\frac{\sigma_{c0} + \sum \Delta\sigma_{c,i}}{E_{c0}}}_{\varepsilon_{c,el}} + \underbrace{\frac{\sigma_{c0}}{E_{c0}}\varphi(t,t_0) + \underbrace{\sum \frac{\Delta\sigma_{c,i}}{E_{c0}}\varphi(t,t_i)}_{(*)}}_{\varepsilon_{cc}\ \text{(creep)}}$$

$$\varphi(t,t_i) = \mu(t)\cdot\varphi(t,t_0)$$

$$(*) \Rightarrow \sum_i \frac{\Delta\sigma_{c,i}}{E_{c0}}\varphi(t,t_i) = \mu(t)\,\varphi(t,t_0)\sum_i \frac{\Delta\sigma_{c,i}}{E_{c0}}$$

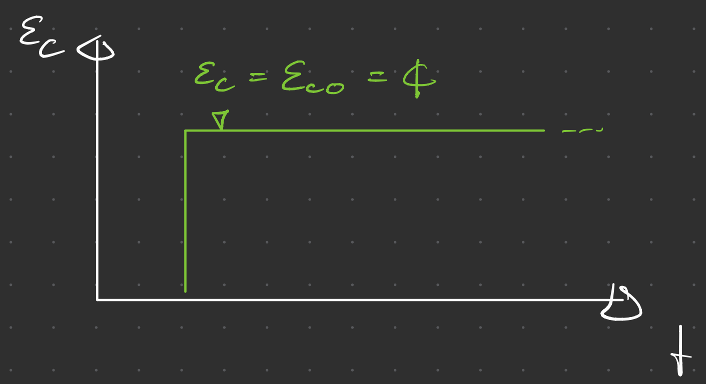
$$= \mu(t)\,\varphi(t,t_0)\,\Delta\sigma_{c,tot}$$

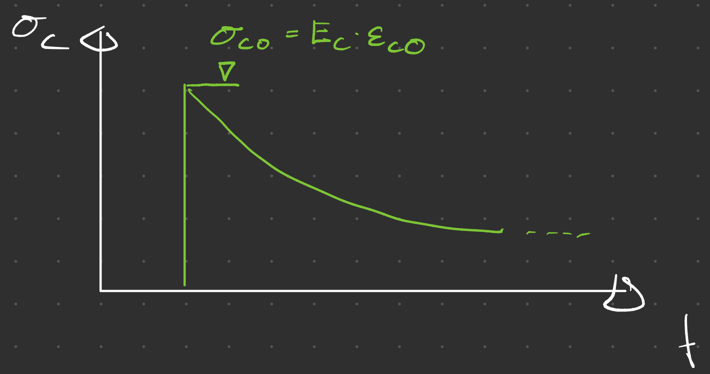
$$= \mu(t)\,\varphi(t,t_0)\left(\sigma_{c,tot} - \sigma_0\right)$$

$$\Rightarrow \boxed{\varepsilon_c(t) = \frac{1}{E_{c0}}\Big(\underbrace{\sigma_{c0}\left(1+\varphi(t,t_0)\right)}_{\text{creeps fully}} + \underbrace{\Delta\sigma_{c,tot}\left(1+\mu\,\varphi(t,t_0)\right)}_{\text{creeps less}}\Big)}$$

$\mu(t) \approx$ ¢ ≈ 0.8

Trost's method – Relaxation





$$\varepsilon_c(t) = \frac{\sigma_{c0}}{E_c} = \text{const} \quad , \quad \text{"}\varphi\text{"} = \varphi(t, t_0)$$

$$\varepsilon_c(t) = \underbrace{\frac{1}{\cancel{E_c}}\left(\sigma_{c0}\left(\cancel{1}+\varphi\right) + \Delta\sigma_c(t)\left(1+\mu\varphi\right)\right)}_{\text{Trost's formula}} \overset{!}{=} \frac{\cancel{\sigma_{c0}}}{\cancel{E_c}} = \text{const}$$

$$\Rightarrow \sigma_{c0}\,\varphi + \Delta\sigma_c(t)\left(1+\mu\varphi\right) = 0$$

$$\Rightarrow \Delta\sigma_c(t) = -\sigma_{c0}\frac{\varphi}{1+\mu\varphi}$$

$$\boxed{\sigma_c(t)} = \sigma_{c0} + \Delta\sigma_c(t) = \boxed{\sigma_{c0}\left(1 - \frac{\varphi}{1+\mu\varphi}\right)}$$

1) Considering effects of creep in force method
2) Considering a system change with parts of different ages
hs, 28.11.2024

Most important formulas

$$\delta_1 = \delta_{10} + X_1 \delta_{11}$$

$$\boxed{\delta_1(t) = \delta_{10}(1+\varphi) + X_1 \delta_{11}(1+\varphi) + \Delta X(t)\, \delta_{11}(1+\mu\varphi)}$$

System change

$$\boxed{M(t) = M_0 + (M_{0c} - M_0) \cdot \frac{\varphi}{1+\mu\varphi}}$$

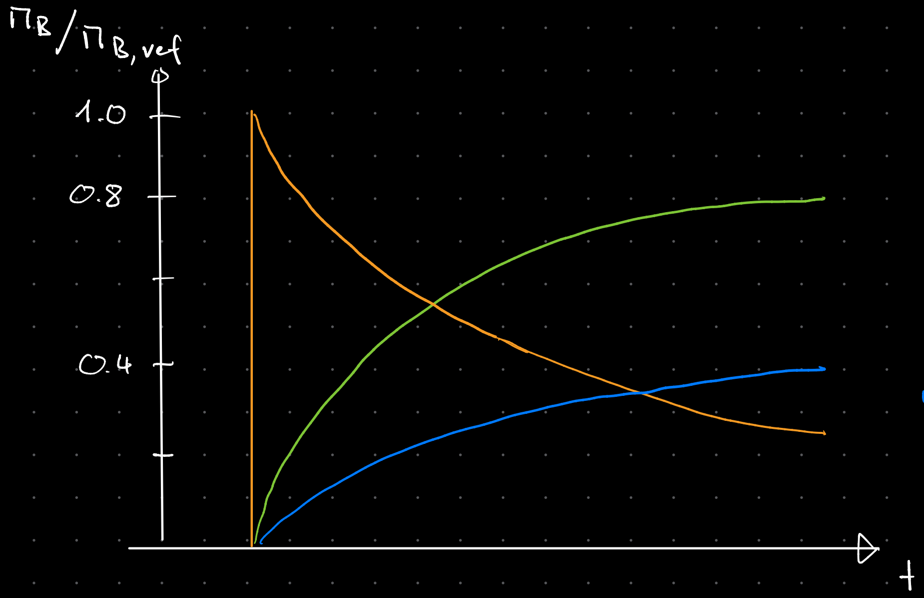
$\forall$ internal forces

different systems of different ages:

$$\boxed{M(t) = \sum_i M_i \left(1 - \frac{\varphi_i}{1+\mu\varphi_i}\right) + M_{0c} \frac{\varphi_0}{1+\mu\varphi_0}}$$

$i = \#$ system change

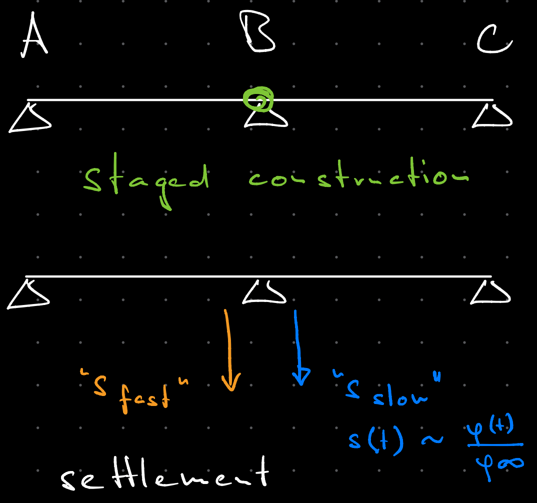
Comparison



$$\frac{\varphi_\infty}{1+\mu\,\varphi_\infty} \approx 0.75 \dots 0.8$$

$$\frac{\varphi(t)}{\varphi_\infty\,(1+\mu\,\varphi(t))} \xrightarrow{t\to\infty} \frac{1}{1+\mu\,\varphi_\infty} \approx 0.4$$

$$1-\frac{\varphi_\infty}{1+\mu\cdot\varphi_\infty} \approx 0.25 \dots 0.33$$



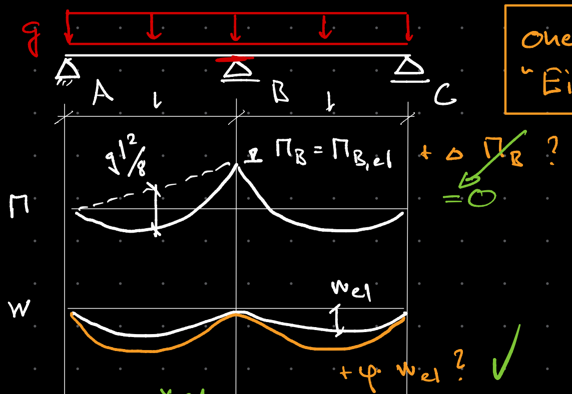
System change ($M_{B,ref} \hat{=}$ one casting system)

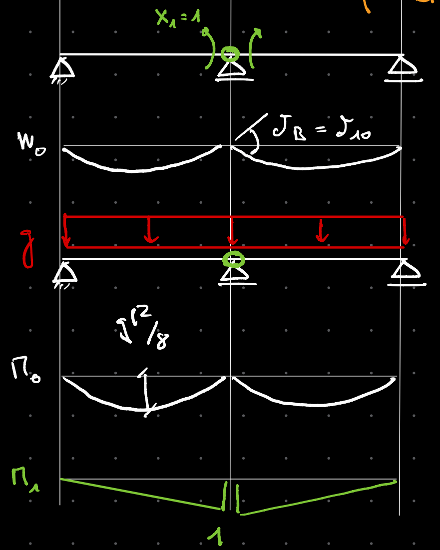
fast restraint ($M_{B,ref} \hat{=}$ full elastic restraint)

slow restraint ($M_{B,ref} \hat{=}$ full elastic restraint (fictitious))

Effect of creep on two-span girder – time dependent force method

One casting system
"Eingusssystem"





Basic system (isostatic) + redundant variable
BS + RV

Relative rotation at B

- due to g:

$$\vartheta_{10} = \int \frac{M_o \cdot M_1}{EI}\,dx = \frac{gl^2}{8} \cdot \frac{L}{EI} \cdot \frac{1}{3} \cdot 2 = \frac{gl^2}{12\,EI}$$

- due to $X_1 = 1$:

$$\vartheta_{11} = \int \frac{M_1 \cdot M_1}{EI}\,dx = \frac{1 \cdot 1}{EI} \cdot l \cdot \frac{1}{3} \cdot 2 = \frac{2L}{3EI}$$

Short-term compatibility ($t=0$)

$\varphi = 0, \; \Delta X_1 = 0$

$$\vartheta_1 = \vartheta_{10}(1+\cancel{\varphi}^{0}) + X_1 \vartheta_{11}(1+\cancel{\varphi}^{0}) + \cancel{\Delta X_1}^{0} \vartheta_{11}(1+\mu\varphi) \overset{!}{=} 0$$

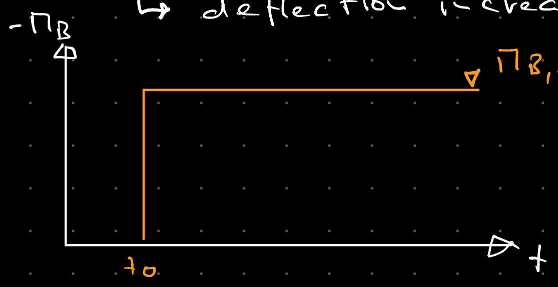
$$X_1 = -\frac{\vartheta_{10}}{\vartheta_{11}} = \frac{gl^2}{8} \; ; \quad M_B = X_1 \underbrace{M_1}_{=1} = X_1$$

Time-dependent compatibility

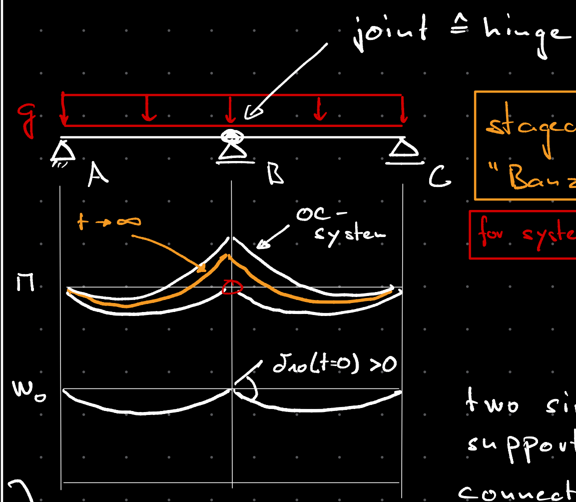
$$\vartheta_1 = \underline{\vartheta_{10}\cancel{(1+\varphi)}} + \underline{X_1 \vartheta_{11}\cancel{(1+\varphi)}} + \Delta X_1 \vartheta_{11}(1+\mu\varphi) \overset{!}{=} 0$$

$$\boxed{\Delta X_1 = 0 \; ; \quad \Delta M_B = \Delta X_1 M_1 = 0}$$

⇒ No redistribution of internal forces due to creep
↳ deflection increase



here: for uncracked structure



staged construction
"Bauzustände"

for systems of same age

two simply supported girders connected

≙ made continuous after applying g ($t = t_o$)

same result as for OC system

$$\vartheta_{10} = \frac{gl^2}{12\,EI} \;\leftarrow\; \text{relative rotation ("kink") at B, frozen for } t > t_o$$

$$\vartheta_{11} = \frac{2L}{3EI}$$

"stc" ($t = t_o$)

$\varphi = 0, \; \Delta X_1 = 0$

$X_1 = 0 \; ; \quad M_B(t=t_o) = X_1 \cdot M_1 = 0$

$\vartheta_1 = \vartheta_{10}$

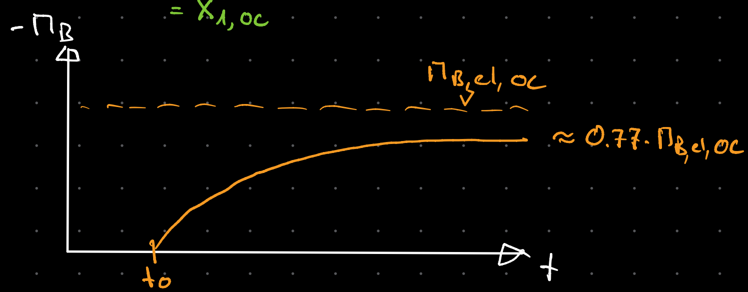
"tdc" ($t > t_o$)

$$\vartheta_1 = \vartheta_{10}(\cancel{1}+\varphi) + \underbrace{X_1 \vartheta_{11}(1+\varphi)}_{=0} + \Delta X_1 \vartheta_{11}(1+\mu\varphi)$$

$\overset{!}{=} \cancel{\vartheta_{10}}$

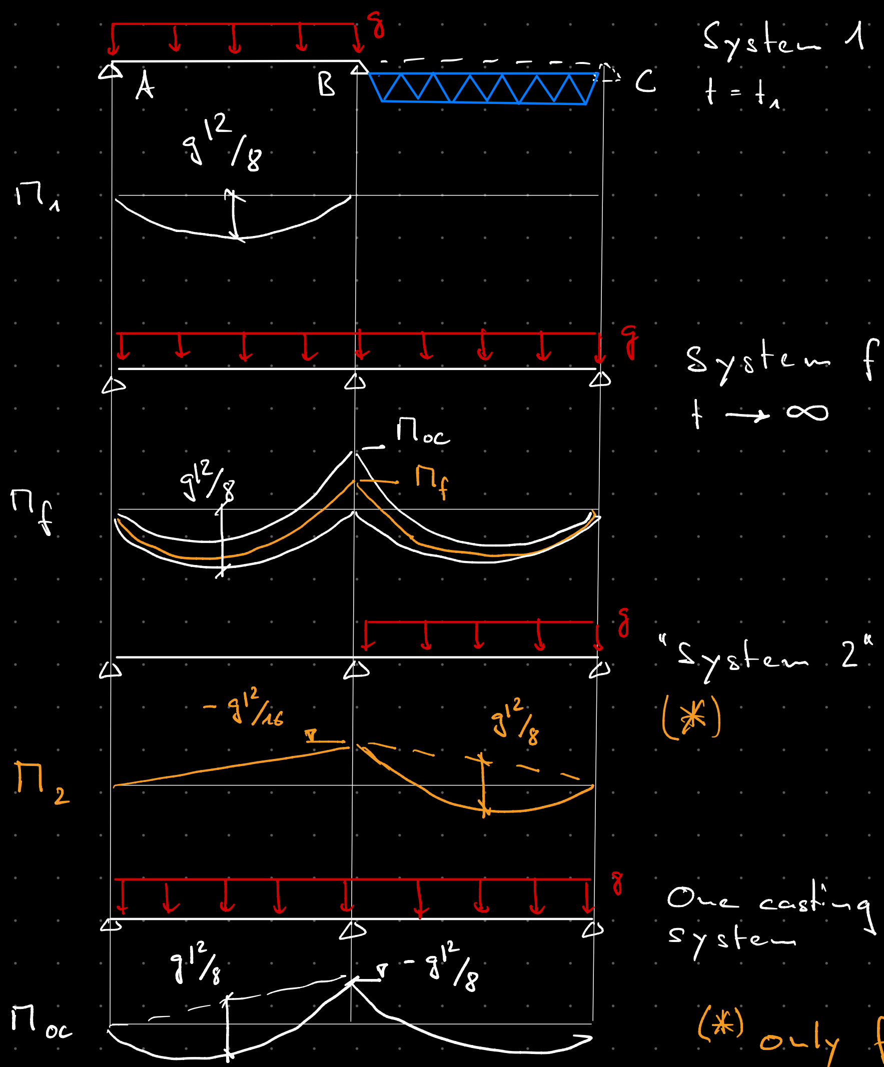
$$\vartheta_{10}\,\varphi + \Delta X_1 \vartheta_{11}(1+\mu\varphi) = 0$$

$$\boxed{\Delta X_1 = \underbrace{\left(-\frac{\vartheta_{10}}{\vartheta_{11}}\right)}_{= X_{1,OC}} \cdot \frac{\varphi}{1+\mu\varphi} \; ; \quad M_B = M_1 \Delta X_1 = M_{B,OC,el} \cdot \frac{\varphi}{1+\mu\varphi}}$$



$\boxed{\text{staged construction}}$

$\boxed{\text{for systems of different age}}$



$$M_{tot} = M_0 + \cancel{X_1 M_1} + \underbrace{\Delta X_1}_{X_{1,oc}\cdot\frac{\varphi}{1+\mu\varphi}} \cdot M_1$$

$\cancel{X_1 M_1} = 0$ see previous slides

$$= M_0 + \underbrace{X_{1,oc}\cdot M_1}\,\frac{\varphi}{1+\mu\varphi}$$

$$M_{oc} = M_0 + X_{1,oc}\cdot M_1$$

$$\hookrightarrow \boxed{X_{1,oc}\cdot M_1} = M_{oc} - M_0$$

$$\Rightarrow M_{tot} = M_0 + (M_{oc} - M_0)\,\frac{\varphi}{1+\mu\varphi}$$

$$\boxed{M_{tot} = M_0\left(1 - \frac{\varphi}{1+\mu\varphi}\right) + M_{oc}\,\frac{\varphi}{1+\mu\varphi}}$$

Moment of different systems — Moment of system cast at once

(*) only forces on "new" part considered $t = t_s$ (time when new part is loaded)

Approximation:

$$M(t) = \sum M_i\left(1 - \frac{\varphi_i}{1+\mu\varphi_i}\right) + M_{oc}\,\frac{\varphi_0}{1+\mu\varphi_0}$$

$$= M_1\left(1 + \frac{\varphi_1}{1\ \mu\varphi_1}\right) + M_2\left(1 + \frac{\varphi_2}{1\ \mu\varphi_2}\right) + M_{oc}\,\frac{\varphi_0}{1+\mu\varphi_0}$$

for $t = t_s$: $\varphi = 0$, $M_B = 0.5\,M_{oc}$

$t \to \infty$: $\varphi = 2$, $M_B = 0.88\cdot M_{oc}$

Approximation "20 : 80"

$$M(t) = 0.2\sum M_i + 0.8\,M_{oc} = 0.2(M_1 + M_2) + 0.8\,M_{oc}$$

for $t = t_s$: $\varphi = 0$, $M_B = 0.5\,M_{oc}$

$t \to \infty$: $\varphi = 2$, $M_B = 0.9\cdot M_{oc}$

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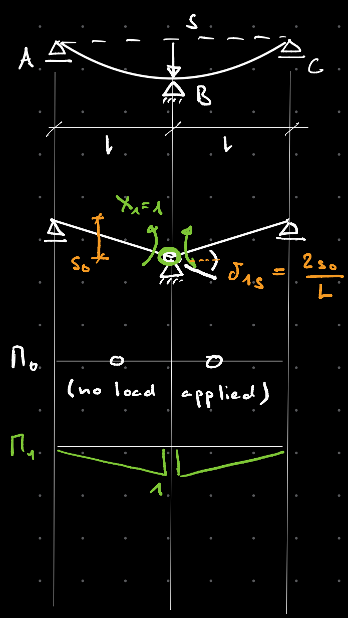
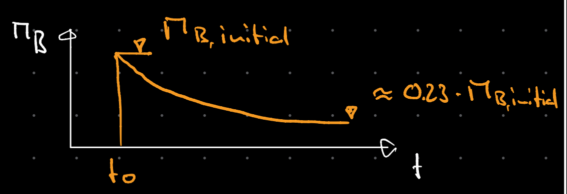
Support settlement
Effect of creep for fast / slow settlement



$$\delta_{10} = \int \frac{M_0 M_1}{EI} dx = \underline{0}$$

$$\delta_{11} = \int \frac{M_1 \cdot M_1}{EI} dx = \frac{2L}{3EI}$$

time-independent settlement ("fast" restraint)



"stc", $t = t_0 \Rightarrow \varphi = 0,\ \Delta X_1 = 0$

$$\delta_1 = \delta_{10}(1+\cancel{\varphi}^{0}) + X_1 \delta_{11}(1+\cancel{\varphi}^{0}) + \cancel{\Delta X_1 \delta_{11}(1+\mu\varphi)}^{=0} \overset{!}{=} \delta_{1s}$$

$$\Rightarrow X_1 = \boxed{\frac{\delta_{1s}}{\delta_{11}}}^{(*)} ; \quad M_{B,initial} = X_1 \cdot M_1$$

$$= \frac{3EI}{L^2} \cdot s_0$$

"tdc", $t > t_0$

$$\delta_1 = \cancel{\delta_{10}}^{=0}(1+\varphi) + X_1 \delta_{11}(1+\varphi) + \Delta X_1 \delta_{11}(1+\mu\varphi) \overset{!}{=} \delta_{1s}$$

$$\delta_{1s}(1+\varphi) + \Delta X_1 \delta_{11}(1+\mu\varphi) = \delta_{1s}$$

$$\Rightarrow \Delta X_1(t) = -\underbrace{\boxed{\frac{\delta_{1s}}{\delta_{11}}}^{(*)}}_{= X_{1,stc}} \cdot \frac{\varphi}{1+\mu\varphi} ; \quad \Delta M_B(t) = \Delta X_1 M_1$$

$$\boxed{M_B(t) = M_{B,initial} + \Delta M_B(t) = M_{B,initial}\left(1 - \frac{\varphi}{1+\mu\varphi}\right)}$$

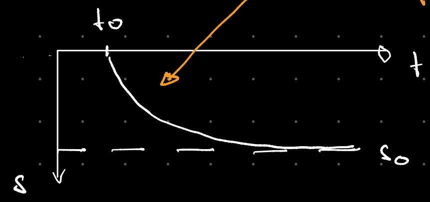
for $t \to \infty$: $\mu = 0.8,\ \varphi = 2$

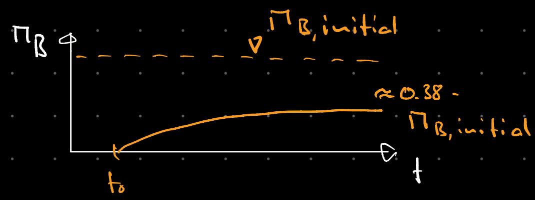
$$M_B(t) = M_{B,initial} \cdot 0.23$$

$\underbrace{\text{full initial restraint is reduced to 23\%}}_{\text{relaxation}}$

time-dependent settlement ("slow" restraint)

$$s(t) = s_0 \cdot \frac{\varphi(t)}{\varphi_\infty}$$





"stc", $t = t_0$

$\delta_1 = \ldots = 0$ since $s(t_0) = 0$

$\Rightarrow X_1 = 0$

$\Rightarrow M_B(t_0) = X_1 M_1 = 0$

"tdc", $t > t_0$

$$\delta_1 = \cancel{\delta_{10}}^{=0}(1+\varphi) + \cancel{X_1}^{=0} \delta_{11}(1+\varphi) + \Delta X_1(t) \delta_{11}(1+\mu\varphi) \overset{!}{=} \delta_{1s}(t)$$

$$\Delta X_1(t) = \underbrace{\boxed{\frac{\delta_{1s}}{\delta_{11}}}^{(*)}}_{= X_{1,stc}} \cdot \frac{\varphi}{\varphi_\infty (1+\mu\varphi)}$$

$$\Delta M_B(t) = \Delta X_1 M_1$$

$$\Rightarrow \boxed{M_B(t) = M_{B,initial} \cdot \frac{\varphi(t)}{\varphi_\infty (1+\mu\varphi(t))}}$$

for $t \to \infty$: $\varphi = 2,\ \mu = 0.8$

$\varphi(t) = \varphi_\infty$

$$M_B(t \to \infty) = M_{B,initial} \cdot 0.38$$

initially zero restraint builds up to 38% of full elastic restraint